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Limited Commitment and the Demand for Money in the U.K.

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Limited Commitment and the Demand for Money in the U.K.*

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Abstract

In the United Kingdom, money demand deviates from the convex relationship suggested by monetary theory. Limited commitment of borrowers via banks can explain this observation. Our finding is based on a microfounded monetary model, where a money market provides insurance against idiosyncratic liquidity shocks by offering short-term loans and by paying interest on money market deposits. We calibrate the model to U.K. data and show that limited commitment significantly improves the fit between the theoretical money demand function and the data. Limited commitment can also explain the "liquidity trap"; i.e., why the ratio of credit to M1 is currently so low, despite the fact that nominal interest rates are at their lowest recorded levels.

JEL classification: E4, E5, D9.
Keywords: money demand, money markets, financial intermediation, limited commitment.

1 Introduction

Monetary theory suggests a stable convex relationship between money demand and nominal interest rates. In the United Kingdom, however, this relationship looks different (see Figure 1): For low interest rates, money demand is high and its elasticity (denoted as ξ) is low. For intermediate rates, the elasticity of money demand is high and the level intermediate, while for higher rates, money demand and its elasticity are low. Standard monetary theory fails to replicate the U.K. money demand properties as shown in Figure 1 by the curve labeled "With Commitment".

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In this paper, we show that a model, where agents cannot commit to repay their debt (i.e., limited commitment), can significantly improve the fit between the theoretical money demand function and the data. In particular, it can explain the "odd" behavior of money demand in the United Kingdom. We derive the money demand in a microfounded monetary model, where agents face idiosyncratic payment shocks. A money market mitigates these shocks by offering short-term loans and by paying interest on money market deposits. We analyze and calibrate the model under two competing assumptions: Either agents can commit to repay their loans (with commitment) or they cannot (limited commitment). This distinction makes a difference to the shape of the money demand function, because limited commitment gives rise to an endogenous borrowing constraint which depends on monetary policy in an interesting way, as explained below.

The curve labeled "With Commitment" in Figure 1 displays the best-fit calibration when agents can fully commit to repay their debt. The curve labeled "Limited Commitment" in Figure 2 displays the best-fit calibration when agents cannot commit to repay their debt. In the calibration section, we show that the fit between the model-generated money demand function and the data is much better when we assume limited commitment.

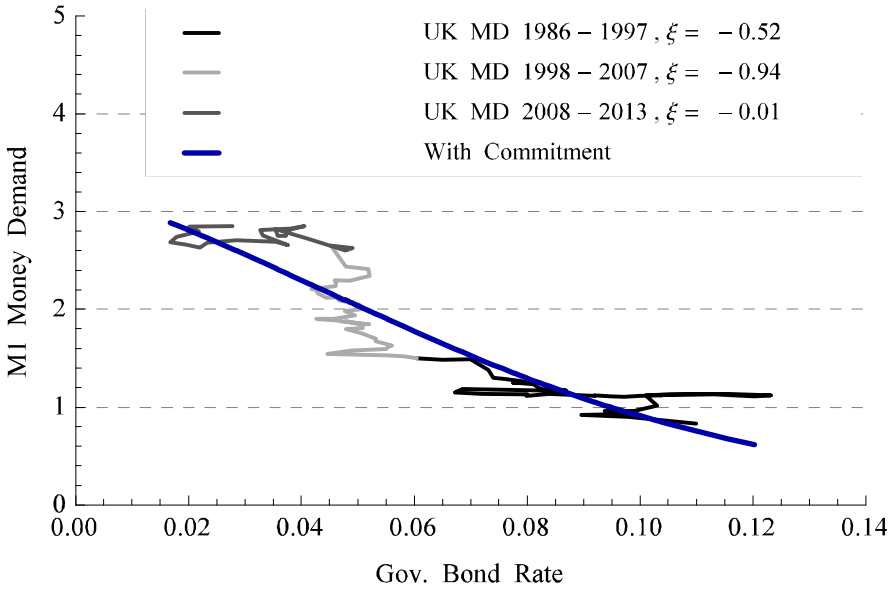

FIGURE 1: MONEY DEMAND IN THE U.K.

We model limited commitment under the assumption that the punishment for an agent who does not repay his loan is permanent exclusion from the money market. A borrower, thus, faces a classic trade-off: the short-term utility gain from not repaying his debt versus the discounted sum of utility losses from not being able to access the money market in the future. Financial intermediaries understand this trade off and are only willing to provide credit up to an endogenous upper-bound. This bound is the largest loan size that a borrower will pay back voluntarily.

Limited commitment affects the shape of the money demand function, because the endogenous upper-bound on borrowing depends on monetary policy in an interesting way. We find four regions: Region (1): For very low interest rates, the incentive to default is high, because the opportunity cost of holding money (the nominal interest rate) is low. Consequently, there is little borrowing and money demand is high, since agents hold large quantities of money in order to self-insure against the idiosyncratic liquidity shocks. Region (2): For low-to-intermediate interest rates, borrowing is rapidly increasing and the demand for money is decreasing in the interest rate. The reason is that an increase in the interest rate relaxes the endogenous borrowing constraint. Agents are able to borrow more and for that reason reduce their money holdings. Region (3): For intermediate interest rates, real borrowing and money demand are increasing simultaneously as the interest rate increases. In this region, the borrowing constraint is still binding and so an increase in the interest rate increases borrowing. The reason for the increase in real money holdings is that in this region the borrowing costs are increasing at a faster pace than the opportunity cost of holding money. Region (4): For high interest rates, the incentive to default is low, because the opportunity cost of holding money is high. Consequently, borrowing is unconstrained. Furthermore, the demand for money is decreasing in the interest rate.

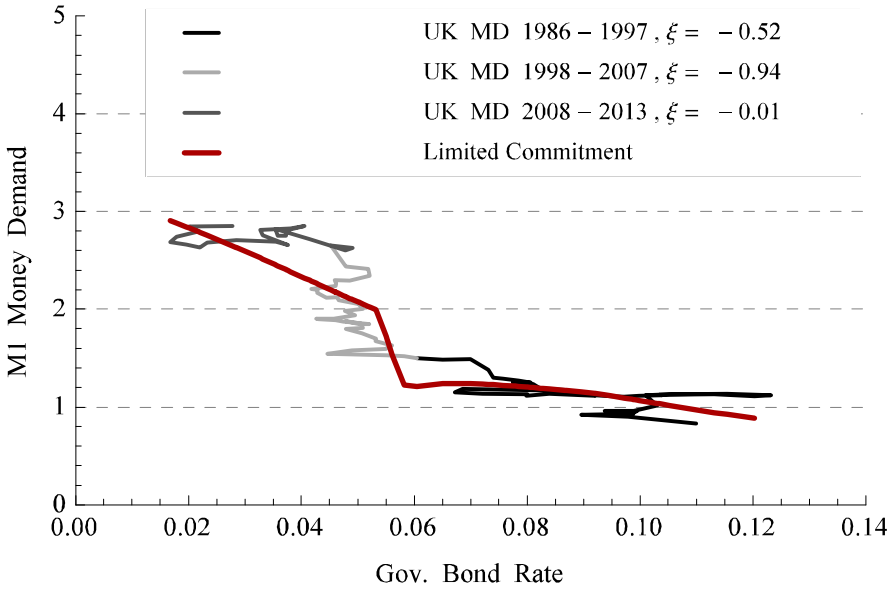

FIGURE 2: MONEY DEMAND IMPLIED BY OUR MODEL

To study to what extent limited commitment can account for the observed behavior of U.K. money demand, we calibrate the model by using U.K. data from 1986 to 2013. As shown in Figure 2, for interest rates below 4.9 percent (gray curve 2008-2013) we are in region 1, where interest rates are low, and so the incentive to default is high. Consequently, there is little borrowing and the demand for money is high. For interest rates between 4.9 and 6 percent (lighter gray curve 1998-2007), credit availability is rapidly increasing as the interest rate increases. As described above, under region 2

conditions, the result is a highly elastic money supply. Finally, for interest rates above 6 percent (black curve 1986-1997), we are in regions 3 and 4. There is a lot of borrowing because the incentives to default are low, and consequently the demand for money is low and its elasticity is moderate.

Our model implies that there is a positive relationship between nominal interest rates and credit activity. At first glance, these implications may sound somewhat counter intuitive, as one might think that low nominal interest rates should curb credit activity rather than the opposite. However, if limited commitment is indeed a serious issue, also the U.K. data should reflect this positive relationship. Figure 3 shows the ratio of credit to M1 as a function of nominal interest rates. In particular, it shows the total credit to the private non-financial sector and to households and non-profit institutions serving households, which is available from the Bank for International Settlements.

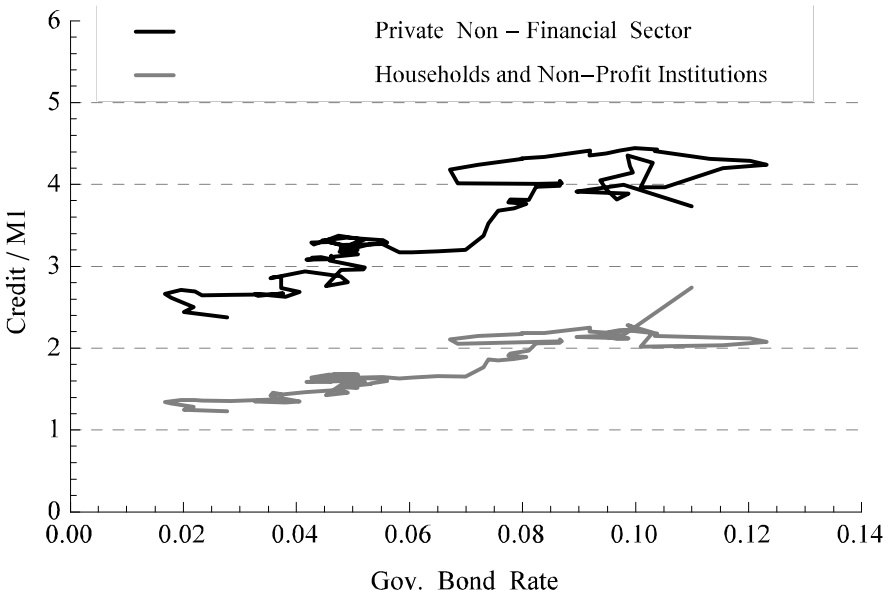

FIGURE 3: CREDIT-TO-M1 RATIO

Figure 3 shows that the supposed relationship is clearly positive in the U.K. economy. That is, we obtain a correlation between the ratio of credit to M1 and the nominal interest rates of 0.92 for the private non-financial sector and 0.93 for households and non-financial institutions, while our model estimates 0.87. Standard models which assume commitment of borrowers, have a hard time to replicate this fact. Limited commitment can thus also deliver an explanation for the "liquidity trap"; i.e., why the ratio of credit to M1 is currently so low, even though nominal interest rates are at their lowest recorded levels.

While writing this paper, we read several studies that analyze U.K. money demand. Most of these studies are empirical and date back, at least, to Brown (1939). More recent studies comprise, but are not limited to, Friedman and Schwartz (1982), Hendry and Ericsson (1991), and Drake (1996). The main issue in this literature is parameter constancy, which is documented in Judd and Scadding

(1982) and Goldfeld and Sichel (1990). As pointed out by Ericsson (1998, p.299), "Non-constancy of estimated coefficients presents both economic and statistical difficulties in conducting any inferences from the empirical model. Constancy is a particularly important issue during periods of financial liberalization and innovation, and many countries have recently experienced such developments." Thus, when the economic environment changes due to developments in the financial industry, previously estimated models may become misspecified and their non-constancy is traced back to omitted variables. In contrast to this literature, we develop a microfounded model that embeds the key frictions that induce agents to trade in financial markets. This allows us to analyze the post-1985 data, where major financial innovations took place and where money demand deviates from the well-known convex relationship with nominal interest rates.[1]

Our paper belongs to the "new monetarist economics" literature. Profound discussions of this literature can be found in Williamson and Wright (2010), Nosal and Rocheteau (2011), and Lagos et al. (2015). Our model builds on Lagos and Wright (2005), where a centralized market and a decentralized market alternate each period. Berentsen et al. (2007) introduces a competitive money market into the Lagos and Wright (2005) framework. We also have this market, but unlike Berentsen et al. (2007), not all agents have access to the financial market. Our work is also related to the recent papers in this literature that study collateralization (e.g., Berentsen and Waller, 2011, Gu et al., 2013, and Bethune et al., 2014), and acceptability (e.g., Lagos and Rocheteau, 2008, Lester et al., 2012, and Hu and Rocheteau, 2013) of illiquid assets. However, unlike these papers, the main focus here is on the behavior of the money demand in the United Kingdom and how well our model can replicate it.

2 Environment

The basic setup follows Berentsen et al. (2015). The main difference is that we relax the full commitment assumption for financial transactions and study the implication of limited commitment for the shape of the money demand function.[2] There is a measure $[0, 1]$ of agents who live forever in discrete time. In each period, there are three markets that open and close sequentially. In the first market, agents can borrow and deposit money; in the second market, production and consumption of a specialized good takes place; in the third market, credit contracts are settled and a general good is produced and consumed. We call these markets *money market*, *goods market*, and *centralized market,* respectively. All goods are perfectly perishable in the sense that their value goes to zero if they are not consumed in the market where they are produced. This assumption rules out any form of commodity money. Finally, we assume that all goods are perfectly divisible.

At the beginning of each period, agents receive two idiosyncratic shocks. A preference shock determines whether an agent can consume or produce in the goods market: he can produce but not consume with probability n, or he can consume but not produce with probability $1 - n$. We refer to producers and consumers as *sellers* and *buyers*, respectively. An entry shock determines whether an

[1] See, for instance, Barnett et al. (1992) and Berentsen et al. (2015) for a more detailed description of the financial innovations that emerged in the 1980s and 1990s.

[2] The focus of Berentsen et al. (2015) is to investigate how financial innovations such as the introduction of money market deposit accounts affected the demand for money in the United States. Throughout the paper, they assume full commitment of borrowers via banks.

agent participates in the money market: he has access to the money market with probability π, or he does not have access with probability $1-\pi$. We refer to agents who have access as *active* and to agents who do not as *passive*.

In the goods market, buyers and sellers are matched according to the following reduced-form matching function, $\mathcal{M}(n, 1-n)$, where $\mathcal{M}$ denotes the number of matches in a period. We assume that $\mathcal{M}(n, 1-n)$ has constant returns to scale, and is continuous and increasing with respect to each of its arguments. The probability that a buyer is matched with a seller in the goods market is denoted by $\delta(n) = \mathcal{M}(n, 1-n)(1-n)^{-1}$, while the probability that a seller is matched with a buyer is denoted by $\delta^s(n) = \delta(n)(1-n)n^{-1}$. To simplify on notation, we shorten $\delta(n)$ and $\delta^s(n)$ as δ and δ^s, respectively.

A buyer enjoys utility $u(q)$ from consuming q units of the specialized good, where $u(q)$ satisfies the following properties: $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. A seller incurs a disutility $c(q) = q$ from producing q units of the specialized good. There is no record-keeping technology in this market and agents' actions are not publicly observed. This implies that a buyer's promise to pay his purchased goods in the future is not credible, hence trades must be settled immediately. Consequently, a medium of exchange is needed for transactions.

The centralized market is a frictionless market where agents can produce and consume a general good. No medium of exchange is needed for transactions in this market. Agents receive utility $U(x)$ from consuming x units of the general good, where $U(x)$ has the following properties: $U'(x)$, $-U''(x) > 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. They produce the general good according to a linear technology that transforms h hours of work into h units of the general good, suffering disutility h. Agents discount between, but not within, periods. Let $\beta \in (0,1)$ be the discount factor between two consecutive periods.

There exists a perfectly storable, divisible, intrinsically useless object in the economy, called money. Its supply evolves according to the low of motion $M_{t+1} = \gamma M_t$, where γ denotes the gross growth rate of money, and M_t the stock of money in period t. Also, there exists a central bank which injects (withdraws) money through a lump-sum transfer T_t to all agents in the centralized market, where $T_t = M_{t+1} - M_t = (\gamma - 1)M_t$. To economize on notation, we shorten $t+1$ and $t-1$ as $+1$ and -1, respectively.

Perfectly competitive financial intermediaries, or banks, take deposits and make loans in the money market, after agents learn their type (buyer or seller), but before they enter the goods market. Buyers and sellers have different liquidity needs in the money market: Buyers need more money than they have since they want to consume in the goods market, while sellers have excess money holdings since they can only produce. This generates a role for banks who can reallocate money from those who need less (i.e. sellers) to those who need more (i.e. buyers). Deposit and loan contracts are redeemed at the end of each period, in the centralized market. Banks also operate a costless, record-keeping technology of all financial transactions, but they cannot enforce loan repayment in the centralized market. Because of the record-keeping technology, banks perfectly know each agent's identity and credit history, but not his trade history. Finally, banks are perfectly competitive, which implies that the deposit rate, i_d, is equal to the loan rate, i_ℓ.

3 Agents' Decisions

We now describe the agent's decision problem in each market. To do so, we proceed backwards from the centralized market to the money market. All proofs are relegated to the Appendix.

The centralized market. In the centralized market, agents play different roles. First, they can consume and produce a general good, x. Second, they redeem their financial contracts: A seller receives money plus interest from his deposits, while a buyer pays back his loan plus interest. For now, we assume buyers always honor their obligations. Below, we relax this assumption and derive conditions such that loan repayment is voluntary. Finally, agents receive a lump-sum money transfer from the central bank, and choose the amount of money to take into the next period.

Let $V_3(m, \ell, d)$ be the value function of an agent entering the centralized market with m units of money, ℓ units of loans, and d units of deposits. Then, the agent's problem in the centralized market is

$$V_3(m, \ell, d) = \max_{x,h,m_{+1}} U(x) - h + \beta V_1(m_{+1}), \tag{1}$$

subject to the budget constraint

$$x + \phi m_{+1} = h + \phi m + \phi T + \phi (1 + i) d - \phi (1 + i) \ell, \tag{2}$$

where ϕ denotes the price of money in terms of the general good, and h denotes hours of work. A standard result in this literature is that the choice of m_{+1} is independent of m. This result comes from the quasi-linearity assumption in the consumption function and implies that the distribution of money holdings is degenerate at the end of each period.

The goods market. Let (q, z) be the terms of trade agreed within a meeting in the goods market, where q is the amount of goods produced by the seller and z is the amount of money exchanged. The terms of trade (q, z) are determined using the Kalai (1977), or *proportional*, solution to the bargaining problem[3], i.e.,

$$\begin{aligned} (q, z) &= \arg\max u(q) - \phi z \\ s.t. \quad u(q) - \phi z &= \theta [u(q) - q] \quad \text{and} \quad z \leq m. \end{aligned}$$

The equality constraint is the Kalai constraint which is the way agents split the trade surplus. The inequality constraint is the buyer's cash constraint according to which a buyer cannot give the seller more money than he has. If the buyer's cash constraint binds (i.e. $m = z$), then the solution to the above problem is

$$\phi m = g(q) \equiv \theta q + (1 - \theta) u(q). \tag{3}$$

If the buyer's constraint does not bind (i.e. $m > z$), then $q = q^*$, and $z = m^* = \frac{g(q^*)}{\phi}$.

[3] One of the desired properties of the Kalai solution is that it is *strongly monotonic* in the sense that no agent is made worse off from an expansion of the bargaining surplus.

The money market. In the money market, an agent can deposit or borrow money at the bank. The money market opens at the beginning of each period after agents learn their type (buyer or seller). Before the money market opens, agents also learn whether they will have access to this market or not. After agents have deposited and borrowed money, the money market closes.

Let $V_1^b(m)$ be the value function of an active buyer entering the money market with m units of money, and let $V_1^s(m)$ be that of an active seller. Also, let $V_2^b(m,\ell)$ be the value function of a (passive or active) buyer entering the goods market with m units of money and ℓ units of loans; note that active buyers never deposit money, so $d=0$. Also, $V_2^b(m,\ell)=V_2^b(m,0)$ for passive buyers since they do not participate in the money market. Similarly, let $V_2^s(m,d)$ be the value function of a seller entering the goods market with m units of money and d units of deposits; note that active sellers never borrow money; i.e., $\ell=0$. Moreover, $V_2^s(m,d)=V_2^s(m,0)$ for passive sellers since they do not participate in the money market.

Therefore, the value function of an agent at the beginning of each period is

$$V_1(m)=\pi\left[(1-n)V_1^b(m)+nV_1^s(m)\right]+(1-\pi)\left[(1-n)V_2^b(m,0)+nV_2^s(m,0)\right]. \tag{4}$$

An agent in the money market is an active buyer with probability $\pi(1-n)$, an active seller with probability πn, a passive buyer with probability $(1-\pi)(1-n)$, and a passive seller with probability $(1-\pi)n$. A passive agent can neither lend nor borrow money at the bank. Hence, he just waits for the goods market to open.

Loan repayment in the centralized market is voluntary. Hence, a buyer repays his loan if, and only if, it is profitable for him to do so; note that sellers have no obligation, so default is not an issue for them. In order to create an incentive to repay a loan, we assume a form of punishment for those who default. In particular, we assume that defaulters are banned from participating in the money market forever. This punishment is feasible, since banks perfectly know the identity of the defaulters and can (will) always refuse to trade with them.

A buyer who defaults on his debt enjoys a benefit and suffers a cost for doing so. On one hand, he has to work fewer hours in the centralized market, since the amount of money he needs as a defaulter is less than that of a non-defaulter. On the other hand, he will consume less in all future periods, since he can no longer insure himself against adverse liquidity shocks. If the cost associated with the punishment is higher than the benefit, a deviation is not profitable, and the buyer honors his obligation. We refer to a buyer who defaults on his debt as a *defaulter* (or *deviator*).

In what follows, we denote the variables associated with a defaulter by using a tilde "$\sim$". In the following Lemma, we establish a condition such that active buyers repay their loan voluntarily.

Lemma 1 *A buyer repays his loan if, and only if,*

$$\phi\ell\leq\phi\bar{\ell}, \tag{5}$$

where

$$\begin{aligned}\phi\bar{\ell} &= \frac{(\gamma-\beta)\left[g(\tilde{q})-g(q)\right]}{(1+i)(1-\beta)} \\ &\quad+\frac{\beta(1-n)\delta}{(1+i)(1-\beta)}\left\{\pi\left[u(\hat{q})-g(\hat{q})\right]+(1-\pi)\left[u(q)-g(q)\right]-\left[u(\tilde{q})-g(\tilde{q})\right]\right\},\end{aligned} \tag{6}$$

and where $\tilde{q}$ satisfies

$$\frac{\gamma-\beta}{\beta}=(1-n)\,\delta\left[\frac{u'(\tilde{q})}{g'(\tilde{q})}-1\right]. \tag{7}$$

We are now in the position to write the active buyer's maximization problem in the money market, i.e.,

$$V_1^b(m)=\max_{\ell} V_2^b(m+\ell,\ell) \tag{8}$$

subject to (5). The borrowing constraint (5) means that the real amount of loans a buyer can obtain is bounded above by $\phi\bar{\ell}$. A bank refuses to lend more than $\phi\bar{\ell}$, since that would imply non-repayment. Let λ_Φ denote the Lagrange multiplier on the borrowing constraint (5).

An active seller's problem in the money market is

$$V_1^s(m)=\max_{d} V_2^s(m-d,d) \quad s.t. \quad m-d\geq 0. \tag{9}$$

The constraint in (9) means that a seller cannot deposit more money than the amount he has. Let λ_s be the Lagrange multiplier on this constraint.

4 Equilibria

In what follows, we characterize three types of equilibria: an equilibrium where active sellers do not deposit all their money (i.e., $\lambda_s=0$) and the buyer's borrowing constraint is binding (i.e., $\lambda_\Phi>0$); an equilibrium where active sellers deposit all their money (i.e., $\lambda_s>0$) and the buyer's borrowing constraint is binding (i.e., $\lambda_\Phi>0$); an equilibrium where active sellers deposit all their money (i.e., $\lambda_s>0$) and the buyer's borrowing constraint is not binding (i.e., $\lambda_\Phi=0$).[4] We refer to these equilibria as type-0, type-I, and type-II, respectively.

Type-0 equilibrium. In a type-0 equilibrium, active sellers do not deposit all their money (i.e., $\lambda_s=0$) and the borrowing constraint is binding (i.e., $\lambda_\Phi>0$). For this to hold, sellers must be indifferent between depositing their money and not depositing it. This can be the case if, and only if, $i=0$.

Proposition 1 *A type-0 equilibrium is a list $\{i,\hat{q},\tilde{q},q,\phi\ell,\phi\bar{\ell}\}$ satisfying (6), (7), and*

$$g(\hat{q}) = g(q)+\phi\ell, \tag{10}$$

$$\phi\ell = \phi\bar{\ell}, \tag{11}$$

$$i = 0, \tag{12}$$

$$\frac{\gamma-\beta}{\beta} = (1-n)\,\delta\left\{\pi\left[\frac{u'(\hat{q})}{g'(\hat{q})}-1\right]+(1-\pi)\left[\frac{u'(q)}{g'(q)}-1\right]\right\}. \tag{13}$$

[4] In the Appendix, we also characterize an equilibrium where active sellers do not deposit all their money (i.e., $\lambda_s=0$) and the buyer's borrowing constraint is not binding (i.e., $\lambda_\Phi=0$). We refer to this equilibrium as the type-III equilibrium.

From (10), the real amount of money that an active buyer spends in the goods market, $g(\hat{q})$, is equal to the real amount of money spent as a passive buyer, $g(q)$, plus the real loan an active buyer obtains from the bank, $\phi\ell$. Equation (10) is derived from the active buyer's budget constraint and immediately shows that $\hat{q} > q$. An active buyer is borrowing-constrained in the type-0 equilibrium; he would like to borrow more than $\phi\bar{\ell}$, but the bank refuses to lend him more than $\phi\bar{\ell}$. This immediately implies that the marginal value of borrowing is higher than its marginal cost at $\phi\ell = \phi\bar{\ell}$. Equations (6) and (7) come from Lemma 1 and refer to the borrowing limit of an active buyer and the consumption quantity of a defaulter, respectively. Equation (12) comes from the active seller's decision in the money market. From (13), a passive buyer consumes an inefficiently low quantity of goods in the goods market unless $\gamma = \beta$. This last equation is derived from the choice of money holdings in the centralized market.

The system of equations in Proposition 1 admits at least one solution which is the straightforward solution $\hat{q} = q = \tilde{q}$. To see this, assume $\hat{q} = q$. Then, from (10), it holds that $\phi\ell = 0$. Furthermore, (13) collapses to (7), implying that $\tilde{q} = \hat{q}$. This means that the two terms on the right-hand side of (6) are both zero, and, thus, $\phi\bar{\ell} = 0$. Therefore, we conclude that the above-mentioned quantities are equilibrium quantities.

However, we cannot show analytically that no other equilibrium exists. In fact, to the contrary, we identify, numerically, equilibria where $q^* > \hat{q} > q$ and $\phi\ell = \phi\bar{\ell} > 0$.

Type-I equilibrium. In a type-I equilibrium, active sellers deposit all their money (i.e., $\lambda_s > 0$), and the active buyer's borrowing constraint is binding (i.e., $\lambda_\Phi > 0$). In a type-I equilibrium, the following proposition holds.

Proposition 2 *A type-I equilibrium is a list* $\left\{i, \hat{q}, \tilde{q}, q, \phi\ell, \phi\bar{\ell}\right\}$ *satisfying (6), (7), and*

$$g(\hat{q}) = g(q) + \phi\ell, \tag{14}$$

$$\phi\ell = \phi\bar{\ell}, \tag{15}$$

$$g(q) = (1-n)\, g(\hat{q}), \tag{16}$$

$$\frac{\gamma-\beta}{\beta} = \pi\left\{(1-n)\,\delta\left[\frac{u'(\hat{q})}{g'(\hat{q})} - 1\right] + ni\right\} + (1-\pi)(1-n)\,\delta\left[\frac{u'(q)}{g'(q)} - 1\right]. \tag{17}$$

All the equations in Proposition 2 have the same meaning as their counterparts in Proposition 1, except that (12) is now replaced by (16) which comes from the money market clearing condition. Equation (16) does not show up in Proposition 1, because sellers do not deposit all their money in a type-0 equilibrium, while they do so in a type-I equilibrium.

Type-II equilibrium. In a type-II equilibrium, active sellers deposit all their money (i.e., $\lambda_s > 0$), and the buyer's borrowing constraint is not binding (i.e., $\lambda_\Phi = 0$).

Proposition 3 *A type-II equilibrium is a list* $\left\{i, \hat{q}, \tilde{q}, q, \phi\ell, \phi\bar{\ell}\right\}$ *satisfying (6), (7), and*

$$g(\hat{q}) = g(q) + \phi\ell, \tag{18}$$

$$i = \delta\left[\frac{u'(\hat{q})}{g'(\hat{q})} - 1\right], \tag{19}$$

$$g(q) = (1-n)\, g(\hat{q}), \tag{20}$$

$$\frac{\gamma - \beta}{\beta} = \pi\delta\left[\frac{u'(\hat{q})}{g'(\hat{q})} - 1\right] + (1-\pi)(1-n)\,\delta\left[\frac{u'(q)}{g'(q)} - 1\right]. \tag{21}$$

All the equations in Proposition 3 have the same meaning as the respective equations in Proposition 2, except that (15) is now replaced by (19). The meaning of equation (19) is the following. In a type-II equilibrium, active buyers are not borrowing-constrained, which means that they borrow up to the point where the marginal cost of borrowing an additional unit of money (left-hand side) is equal to the marginal benefit (right-hand side). Note that $\delta\left[\frac{u'(\hat{q})}{g'(\hat{q})} - 1\right] > i$ in type-0 and type-I equilibria, since buyers are borrowing-constrained, and so they cannot borrow the desired amount of money.

4.1 Sequence of equilibria

We now discuss for which values of γ the different types of equilibria exist. For this purpose, we further divide the type-0 equilibrium into two subtypes: subtype-A and subtype-B. In the subtype-A equilibrium the money market interest rate is zero, $\phi\ell = \phi\bar{\ell} = 0$, and $\hat{q} = q$. In the subtype-B equilibrium, the money market interest rate is zero, $\phi\ell = \phi\bar{\ell} > 0$, and $q^* > \hat{q} > q$. The former equilibrium exists if $\beta < \gamma < \gamma_0$, while the latter exists if $\gamma_0 < \gamma < \gamma_1$. Note that for $\beta < \gamma < \gamma_0$, the equilibrium is a unique, while for $\gamma_0 < \gamma < \gamma_1$ both subtype-A and subtype-B equilibria coexist. Let γ_1 be the value of γ that separates the subtype-B and the type-I equilibria. Then, if $\gamma_1 < \gamma < \gamma_2$, the type-I equilibrium exists with $\phi\bar{\ell} > 0$ and $q^* > \hat{q} > q$. Let γ_2 be the value of γ that separates type-I and type-II equilibria. For $\gamma > \gamma_2$, the type-II equilibrium exists with $\phi\bar{\ell} > 0$ and $q^* > \hat{q} > q$. Both, the type-I and the type-II equilibria are characterized by a strictly positive money market interest rate. The sequence of equilibria is summarized in Table 1.

TABLE 1: SEQUENCE OF EQUILIBRIA[a]

Equilibria	γ	λ_Φ	λ_s	i	Real borrowing
subtype-A	$\beta < \gamma < \gamma_0$	$\lambda_\Phi > 0$	$\lambda_s = 0$	$i = 0$	$\phi\ell = \phi\bar{\ell} = 0$
subtype-B	$\gamma_0 < \gamma < \gamma_1$	$\lambda_\Phi > 0$	$\lambda_s = 0$	$i = 0$	$\phi\ell = \phi\bar{\ell} > 0$
type-I	$\gamma_1 < \gamma < \gamma_2$	$\lambda_\Phi > 0$	$\lambda_s > 0$	$i > 0$	$\phi\ell = \phi\bar{\ell} > 0$
type-II	$\gamma > \gamma_2$	$\lambda_\Phi = 0$	$\lambda_s > 0$	$i > 0$	$\phi\ell < \phi\bar{\ell} > 0$.

[a] Table 1 displays the sequence of equilibria. For low values of γ, the constraint on depositors is not binding and so the nominal interest rate is zero. Nevertheless, the borrowing constraint is binding. For intermediate values of γ, both constraints are binding, and for high values of γ only the constraint on deposits is binding.

The critical values of γ are derived as follows: γ_0 is the value of γ such that $\phi\bar{\ell} = 0$ in the type-0 equilibrium, γ_1 is the value of γ that solves $i = 0$ in the type-I equilibrium, while γ_2 is the value of γ that solves $i = \delta\left[u'(\hat{q})/g'(\hat{q}) - 1\right]$ in the type-I equilibrium.

4.2 Discussion

The quantity of specialized goods consumed by an active buyer, $\hat{q}$, a passive buyer, q, and a deviator, $\tilde{q}$, as a function of γ, are represented in Figure 4. To draw this figure, we assume $\theta = 1$ and a linear cost function $c(q) = q$. The left-hand side diagram refers to the case where all agents have access to the money market ($\pi = 1$). In this case, there are no passive agents, and a buyer can either be active, in which case he will consume $\hat{q}$, or banned from trading in the money market (because he defaulted), in which case he will consume $\tilde{q}$. The right-hand side diagram refers to the case where access to the money market is limited ($\pi < 1$). In this case, a buyer can be active, in which case he will consume $\hat{q}$; passive, in which case he will consume q; or banned from trading in the money market, in which case he will consume $\tilde{q}$. It is easy to see that, if $\pi = 0$, the money market is shut down and consumption is the same for all buyers.

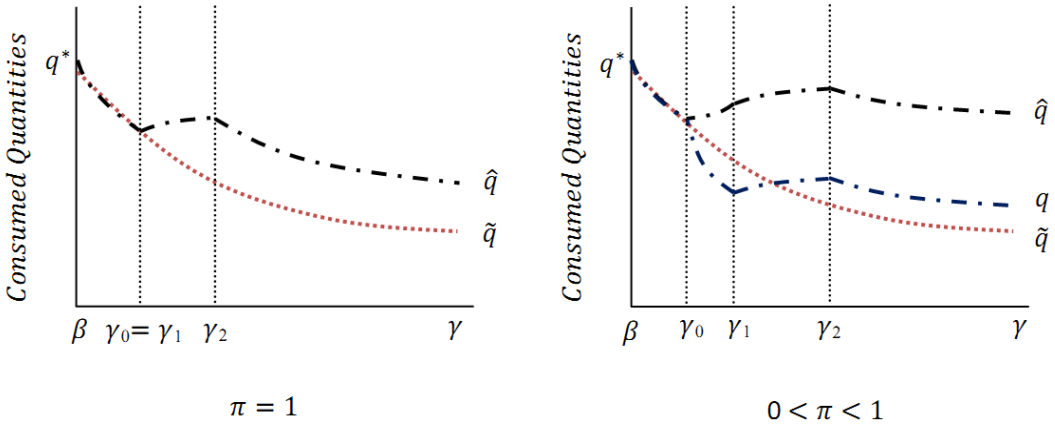

FIGURE 4: CONSUMED QUANTITIES

If all agents participate in the money market (i.e., $\pi = 1$), as in Berentsen et al. (2007), we have the following regions of equilibria. The subtype-A equilibrium, where $\phi\bar{\ell} = 0$ and the money market shuts down, occurs when $\beta < \gamma < \gamma_0 = \gamma_1$. When inflation is sufficiently low (i.e., $\beta < \gamma < \gamma_0 = \gamma_1$), the cost of carrying money is low, and so the benefit of participating in the money market is small. As a consequence, an active buyer finds it profitable to default. In this region, the allocation is the same as that in the absence of a money market, i.e., $\hat{q} = \tilde{q}$. The type-I equilibrium occurs when $\gamma_0 = \gamma_1 < \gamma < \gamma_2$. In this case, $\phi\bar{\ell}$ and $\hat{q}$ are increasing in γ. The reason for this is that an increase in the inflation rate relaxes the borrowing constraint, allowing the active buyer to borrow, and thus to consume more. Finally, the type-II equilibrium occurs for $\gamma > \gamma_2$. In this case, consumption by active and defaulting buyers is decreasing in γ because of the standard inflation-tax argument. Of course, it must be the case that $\hat{q} \geq \tilde{q}$ for any $\gamma \geq \beta$.

The diagram on the right-hand side of Figure 4 displays the consumed quantities for the limited participation case (i.e., $\pi < 1$). For $\gamma < \gamma_0$, we are in the subtype-A equilibrium, where financial intermediation shuts down, since $\phi\ell = \phi\bar{\ell} = 0$. Accordingly, the quantity consumed by active and

passive agents equals the quantity consumed by a deviator and is decreasing in γ. For $\gamma_0 < \gamma < \gamma_1$, the subtype-B equilibrium occurs where borrowing is constrained with $\phi\ell = \phi\bar{\ell} > 0$, and the consumption of active agents is increasing, while the consumption of passive agents is decreasing in γ. For $\gamma_1 < \gamma < \gamma_2$, the type-I equilibrium exists, where borrowing is constrained, and the consumption of active and passive agents is increasing in γ. For $\gamma > \gamma_2$, the type-II equilibrium exists, where borrowing is unconstrained, and the consumption of active and passive agents is decreasing in γ.

We now discuss the money demand. To do so, we first need to define the real output in the goods market and in the centralized market. The real output in the goods market is $Y_{GM} = (1-n)\,\delta\,[\pi\phi\hat{m} + (1-\pi)\,\phi m]$, where $\phi\hat{m} = g(\hat{q})$ and $\phi M_{-1} = \phi m = g(q)$, and the real output in the centralized market is $Y_{CM} = A$ for $U(x) = A\log(x)$. Accordingly, the total real output of the economy is $Y = Y_{GM} + Y_{CM}$, and the model-implied velocity of money is

$$v = \frac{Y}{\phi M_{-1}} = \frac{A + (1-n)\,\delta\,[\pi g(\hat{q}) + (1-\pi)\,g(q)]}{g(q)}. \tag{22}$$

The money demand can now be easily derived as the inverse of the velocity of money. In Figure 5, we show how money demand and money borrowing behave in the four regions described above for the limited participation case, $0 < \pi < 1$.

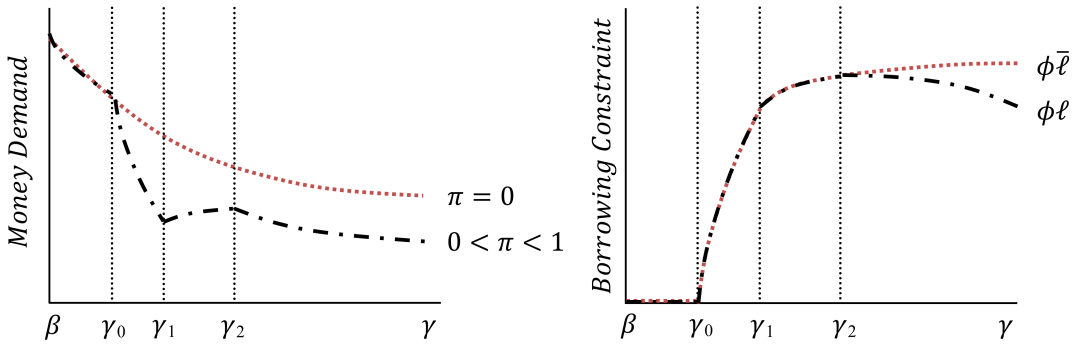

FIGURE 5: MONEY DEMAND AND BORROWING CONSTRAINT

For $\beta < \gamma < \gamma_0$, the demand for money is decreasing in γ and independent of π. This result is quite intuitive, since the money market shuts down for $\beta < \gamma < \gamma_0$ no matter what the value of π is.[5] For $\gamma_0 < \gamma < \gamma_1$, borrowing is constrained and ϕM_{-1} is decreasing in γ, hence the demand for money also declines. For $\gamma_1 < \gamma < \gamma_2$, borrowing is constrained, and the consumed quantity of active and passive agents is increasing in γ. Thus, money demand is increasing in this region. For $\gamma > \gamma_2$, borrowing is unconstrained, and the quantities consumed by active and passive agents are decreasing in γ. Therefore, money demand is declining in γ for $\gamma > \gamma_2$.

[5] In the theoretical model, there is no borrowing in this region, because we assume that all agents have limited commitment. In reality, there will be still some borrowing, because some credit activity is processed under full commitment and some borrowing is done against collateral.

5 Quantitative Analysis

We set a period length to one year. The following standard functional forms are assumed, $u(q) = q^{1-\alpha}/(1-\alpha)$, $U(x) = A\log(x)$, and $c(q) = q$. As in Kiyotaki and Wright (1993), we choose a matching function of the form $\mathcal{M}(B,S) = BS/(B+S)$, where $B = 1-n$ is the measure of buyers, and $S = n$ is the measure of sellers.

We identify a set of parameters which are defined as follows: (i) preference parameters β, A, and α; (ii) technology parameters n and π; (iii) bargaining weight θ; and (iv) policy parameter i_b. The parameters are identified using quarterly U.K. data, from 1986 to 2013.[6] In Table 2, we show the identification restrictions for the respective parameters.

Table 2: Calibration targets

Parameter	Target description	Target value
β	average real interest rate	0.034
i_b	average government bond yield	0.062
θ	average markup	0.15

The policy parameter $i_b = \gamma/\beta - 1 = 0.062$ matches the average yield on government bonds. The preference parameter $\beta = (1+r)^{-1} = 0.968$ is chosen such that the real interest rate in the model replicates that in the data, which is measured as the difference between the average yield on government bonds and the annual change in the consumer price index. The technology parameter n is set to maximize the number of matches; i.e., $n = 0.5$.

The remaining unknown parameters A, α, and θ are identified by minimizing the sum of squared differences between the model-implied and the observed money demand, and by matching the goods market mark-up simultaneously.[7] The elasticity of money demand and the level of money demand are not used as a target. As a result, there may be a discrepancy between the average values of the velocity of money and the elasticity of money demand in the data and in the model. We will show later that this discrepancy occurs, but that its magnitude is reasonably small.[8] The remaining target for the calibration is the goods market markup, μ. Martins et al. (1996) estimate a value of μ equal to 0.15, and we borrow this value from them.

The velocity of money in the model is given by (22). After simulating the model, the elasticity of money demand is estimated in the same way as the empirical elasticity; i.e., by ordinary least squares and a log-log specification. The goods market markup is a weighted average of the real money holdings —divided by the cost of production— exchanged by active and passive buyers in a match, $\mu = \pi g(\hat{q})/c(\hat{q}) + (1-\pi)\, g(q)/c(q) - 1$.

[6] The data source is provided in the Appendix. For consistency, all data used in this paper was obtained from the Federal Reserve Bank of St. Louis FRED database. As the quarterly time series for M1 is only available for the post-1985 period, we focus on this sample period in the entire paper.

[7] We also tried to calibrate the model in the standard way; i.e., identifying A and α such that model replicates the average velocity of money and the elasticity of money demand. The results were qualitatively the same, so we decided to omit them to economize on space.

[8] For example, when we set $\pi = 0.65$, the average values of the velocity of money and elasticity of money demand in the model are $v = 0.57$ and $\xi = -0.72$, while those in the data are $v = 0.65$ and $\xi = -0.74$.

The set of targets described above identifies all parameters, with the exception of the money market access probability, π. In Table 3, we present the calibration results for different values of π compared to the best-fit calibration with full commitment, denoted as FC.[9]

TABLE 3: CALIBRATION RESULTS[a]

Value of π	FC	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
Parameters:											
A	0.062	0.058	0.057	0.066	0.091	0.142	0.023	0.014	0.015	0.007	0.010
α	0.167	0.164	0.163	0.184	0.256	0.540	0.132	0.111	0.117	0.096	0.107
θ	0.761	0.760	0.752	0.748	0.782	0.912	0.673	0.650	0.643	0.624	0.619
Critical Values:											
i_0	-	0.057	0.056	0.051	0.048	0.045	0.042	0.039	0.037	0.035	0.034
i_1	-	0.067	0.061	0.057	0.054	0.060	0.044	0.040	0.038	0.036	0.034
i_2	-	0.114	0.102	0.085	0.069	0.064	0.093	-	0.111	-	-
Miscellaneous:											
s_{GM}	0.428	0.436	0.470	0.526	0.580	0.613	0.685	0.710	0.738	0.783	0.788
$1-\Delta$	0.170	0.127	0.125	0.116	0.089	0.047	0.157	0.174	0.169	0.194	0.180
Σ sq. diff.	7.39	6.46	6.19	6.07	8.08	8.49	15.71	19.25	20.96	25.27	27.85

[a] Table 3 displays the calibrated values for the key parameters A, α and θ for different values of π. Table 3 also displays the values of the critical interest rates, i_0, i_1 and i_2; the goods market share on total output, s_{GM}, and the welfare cost of inflation, $1-\Delta$. The welfare cost of inflation is defined as the percentage of total consumption that agents would be willing to give up in order to be in a steady state with a nominal interest rate of 3 percent instead of 13 percent. The table also shows the sum of squared differences (Σ sq. diff) between the model-implied money demand and the data.

We find that a value of $\pi = 0.65$ minimizes the sum of the squared differences between the model-implied and the observed money demand. The sum is roughly 18 percent lower compared to the sum obtained for FC. Table 3 shows that the calibrated parameter values of A, θ, and α increase for values up to $\pi \approx 0.75$ and decreases thereafter.[10] Furthermore, higher values of π result in a reduction of i_0 and i_1, while i_2 declines for values up to $\pi \approx 0.75$ and increases thereafter. For high values of π, we find that the U.K. economy remains in the type-I equilibrium for $i > i_1$ (except for $\pi = 0.90$).

[9] We decided against stating the calibration and simulation results for $\pi < 0.55$, as the effect on money demand is relatively modest for low values of π.

[10] The reason for this behaviour of the calibrated parameter values is that, for values of $0.6 \leq \pi \leq 0.75$, the subtype-B equilibrium exists in a region where we observe a high elasticity of money demand in the data; i.e., for interest rates between 4.9 and 6.0 percent. In this region, a higher value of α expands the region of existence of the subtype-B equilibrium, and thus improves the fit of the model with the data; a higher value of α implies that also A and θ need to be higher in order to match the targets. In contrast, for values of $\pi > 0.75$, the subtype-B equilibrium exists for interest rates below 4.9 percent. However, in the data, the high-elasticity region is observed for interest rates between 4.9 and 6 percent. Thus, in order to improve the fit with the data, it is optimal to have a very small region of existence of the subtype-B equilibrium.

For intermediate values of π, we find that the welfare cost of inflation, $1-\Delta$, is lower than that for FC.[11] For example, for $\pi = 0.75$, we find that $1-\Delta$ is roughly 72 percent lower as compared to that for FC.[12] What is the reason for a lower welfare cost of inflation for intermediate values of π? One reason is that, for $i_1 < i < i_2$, borrowing is constrained, and thus the consumed quantity of active and passive agents is increasing in i. Furthermore, consumed quantities (by active and passive agents) are generally higher for intermediate values of π than they are for FC. As another argument, note that, for intermediate values of π, the calibrated value of α is higher than that for FC. As a result, the utility function exhibits a higher concavity, and thus higher nominal interest rates have a less adverse effect on consumption.

Table 4: Money demand properties[a]

π	Data	FC	0.55	0.575	0.60	0.625	0.65	0.675	0.70	0.725	0.75
1986-1997:											
v	0.89	0.88	0.92	0.92	0.91	0.89	0.86	0.83	0.80	0.77	0.76
ξ	-0.52	-1.52	-0.66	-0.65	-0.54	-0.48	-0.48	-0.48	-0.35	-0.17	-0.13
1998-2007:											
v	0.52	0.49	0.48	0.48	0.48	0.49	0.50	0.50	0.51	0.50	0.50
ξ	-0.94	-0.65	-0.65	-0.80	-1.05	-1.38	-1.75	-2.13	-2.21	-1.90	-1.74
2008-2013:											
v	0.36	0.40	0.39	0.39	0.39	0.40	0.40	0.39	0.40	0.41	0.42
ξ	-0.01	-0.30	-0.30	-0.31	-0.30	-0.29	-0.27	-0.25	-0.21	-0.14	-0.13
Entire Period											
v	0.65	0.57	0.57	0.57	0.57	0.57	0.57	0.56	0.56	0.56	0.56
ξ	-0.74	-0.82	-0.80	-0.81	-0.77	-0.74	-0.72	-0.69	-0.62	-0.55	-0.53
Miscellaneous:											
Σ sq. diff.		7.39	6.46	6.31	6.19	5.94	6.07	6.97	8.08	8.38	8.49
Σ sq. Mom.		1.19	0.20	0.13	0.10	0.28	0.73	1.47	1.69	1.08	0.82

[a] Table 4 displays the money demand properties, v and ξ, of the data and the model for each sub-period. The table reports the sum of squared differences (Σ sq. diff.) between the model-implied and the observed money demand, and the sum of squared differences between the generated moments (v and ξ) of the model and the data in the three sub-periods, denoted as Σ sq. Mom.

It seems evident from Figure 1 that the U.K. money demand exhibits different properties in the three sub-periods (i.e. 1986-1997, 1998-2007, 2008-2013) that characterize different interest rate

[11] The welfare cost of inflation is calculated as the percentage of total consumption that agents would be willing to give up in order to be in a steady state with a nominal interest rate of 3 percent instead of 13 percent.

[12] The absolute value of the welfare cost of inflation is relatively high, because we assume bargaining in the goods market, which generates the well-known hold-up problem for $i > 0$. See, for instance, Rocheteau and Wright (2005), Aruoba et al. (2007), and Craig and Rocheteau (2008).

regimes. Here, we are interested to see whether our model is able to replicate the money demand properties in those sub-periods. For this purpose, we recalibrate the model for values of π between 0.55 and 0.75; namely, we are taking a closer look at what happens in the region where the model fits the data well. Then, we simulate the model-implied money demand for different values of π and present the results in Table 4. Let "Σ sq. Mom." denote the sum of squared differences of v and ξ between the model and the data, in the three sub-periods, and "Σ sq. diff." be the sum of squared differences between the model-implied and the observed money demand. We use these two variables as alternative indicators of how well the observed money demand is replicated by the model.

A value of $\pi = 0.625$ succeeds in replicating the money demand properties in each subperiod to a large extent. It minimizes the term "Σ sq. diff." and also generates a low value of "Σ sq. Mom.". In the first subperiod (i.e., 1986-1997), the model proves competent in replicating the low money demand and the low elasticity. In the second subperiod (i.e., 1998-2007), the model generates a higher elasticity than that observed in the data, but replicates the level of money demand quite well. In the third sub-period (i.e., 2008-2013), the model generates a higher money demand, and the model-implied elasticity remains too high. Table 4 shows that over the entire period under consideration, the model-implied level of the velocity of money is generally lower than that observed in the data. However, the average elasticity of money demand generated by the model comes close to that in the data. Thus, the discrepancy in the average value of v and ξ between the model and the data is relatively small for $\pi = 0.625$.

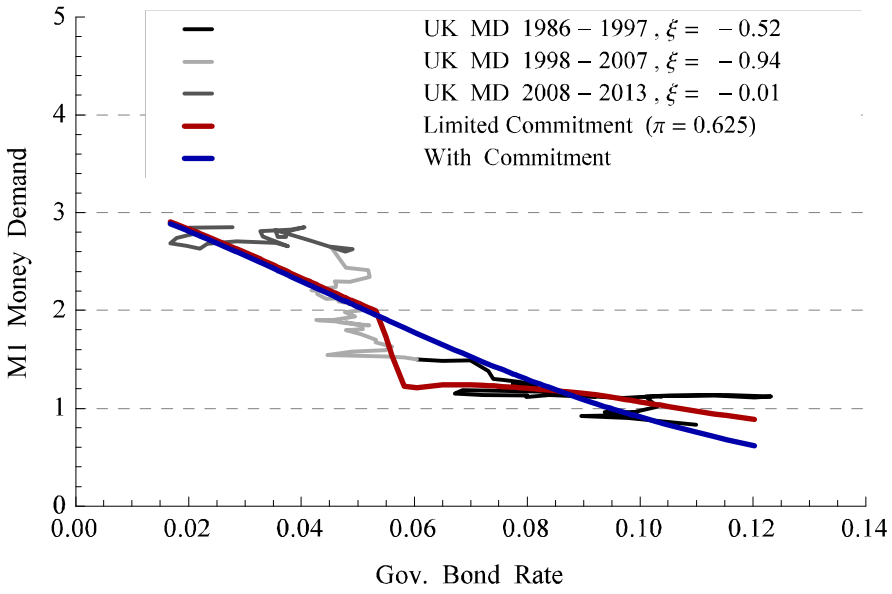

FIGURE 6: MONEY DEMAND FOR $\pi = 0$ AND $\pi = 0.625$

As a benchmark, it is useful to make a comparison with the model-implied money demand under the assumption of full commitment of borrowers via banks. The money demand properties of the model with full commitment and with limited commitment and $\pi = 0.625$ are shown in Figure 6.

We check the model performance also for higher values of π, and the results are promising. According to Table 4, the model succeeds in replicating the data at $\pi = 0.75$ better than at FC. Graphically, the diagram in Figure 7 shows how well the money demand is predicted at $\pi = 0.75$ as opposed to FC. Under $\pi = 0.75$, the model does a reasonable job in replicating the very low elasticity of money demand in the first and third subperiods. It also does reasonably well in matching the high elasticity of money demand in the second subperiod.

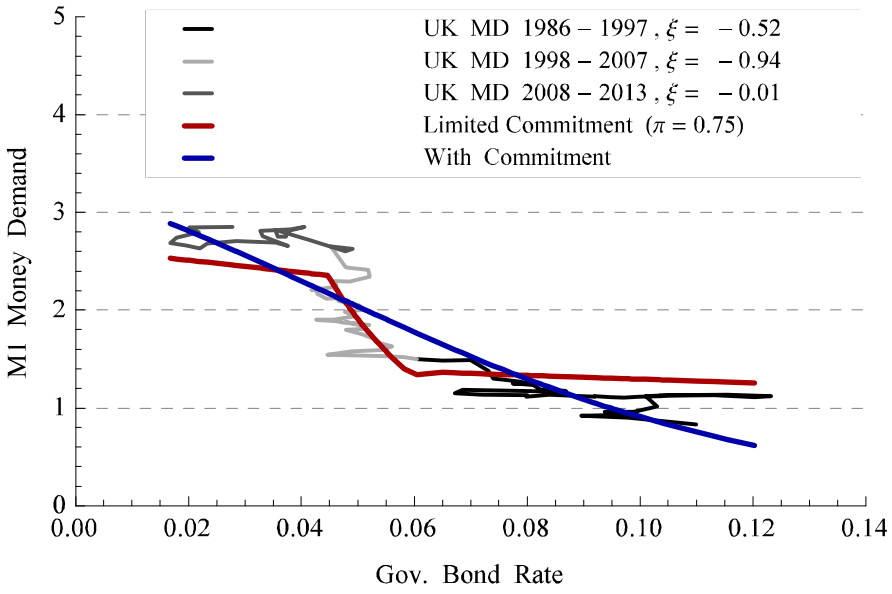

FIGURE 7: MONEY DEMAND FOR $\pi = 0$ AND $\pi = 0.75$

For the best fitting simulation, with $\pi = 0.625$, we find $i_0 = 0.053$ and $i_1 = 0.059$. Hence, our findings indicate that the financial sector in the U.K. has major difficulties in fulfilling its fundamental function in reallocating liquidity for nominal interest rates below 5.3 percent. We believe this finding helps to explain the irregular relationship between money demand and nominal interest rates in the United Kingdom, which contrasts sharply with what we observe in the United States, where money demand and nominal interest rates have featured a stable, convex relationship since the early 1980s (see for instance Berentsen et al., 2015).

5.1 Full Commitment

Hereafter, we briefly discuss how we derived the best-fit calibration with full commitment (FC). In Berentsen et al. (2015), we derive the model under full commitment and refer the interested reader to this paper for more details. We follow the same calibration procedure as before; i.e., we identify A, α, and θ by minimizing the sum of squared differences between the model-implied and the observed money demand, and by matching the goods market mark-up simultaneously. Table 5 presents the calibration and simulation results for different values of $\pi \in (0, 1)$.

Table 5: Calibration results: full commitment[a]

Value of π	0.0	0.2	0.4	0.6	0.8	1.0
Parameters:						
A	0.062	0.031	0.015	0.007	0.003	0.003
α	0.167	0.131	0.106	0.087	0.074	0.076
θ	0.761	0.719	0.675	0.632	0.591	0.567
Entire Period						
v	0.57	0.57	0.58	0.59	0.60	0.64
ξ	-0.82	-0.83	-0.80	-0.77	-0.68	-0.52
Σ sq. diff.	7.39	8.18	9.64	12.17	16.01	22.68

[a] Table 5 displays the calibrated values for the key parameters A, α and θ for different values of π. Table 5 also displays the money demand properties, v and ξ, of the model over the entire period and the sum of squared differences (Σ sq. diff.) between the model-implied and the observed money demand.

The above table shows that the sum of the squared differences between the model-implied and the observed money demand is minimized for an access probability of $\pi = 0$. Thus, the calibration results labeled as FC in the previous section are the ones obtained for full commitment with zero access to the money market.

Our best-fit calibration indicates that assuming full commitment is a bad choice. Under this assumption, the best-fit calibration yields $\pi = 0$; i.e., no financial intermediation. This clearly contradicts the data, as the U.K. economy features a highly developed financial industry. Thus, this finding strengthens our conclusion that limited commitment plays a crucial role in explaining the nonconvex shape of the U.K. money demand.

6 Conclusion

Using a "new monetarist economics" approach, we build a model that incorporates a money market where agents can borrow and deposit money. We first show in theory how limited commitment results in a nonconvex relationship between money demand and nominal interest rates. We then calibrate the model and demonstrate that limited commitment significantly improves the fit between the theoretical money demand function and the historical money demand data of the U.K. for the post-1985 period.

Our model also implies that there is a positive correlation between nominal interest rates and credit activity, a finding that is consistent with the U.K. data. Standard models, which assume commitment of borrowers, have a hard time in replicating this fact. Limited commitment thus can also deliver an explanation for the "liquidity trap"; i.e., why the ratio of credit to M1 is currently so low, even though nominal interest rates are at their lowest recorded levels.

7 Appendix I: Proofs

Proof of Lemma 1. Since a buyer has to work to repay his debt, he may decide to default in the centralized market. Here, we derive conditions such that debt repayment is voluntary. A defaulting buyer's value function at the beginning of the centralized market is

$$\tilde{V}_3(m) = U(x^*) - \tilde{h} + \beta \tilde{V}_{+1}(\tilde{m}_{+1}),$$

and his budget constraint is $x^* + \phi \tilde{m}_{+1} = \tilde{h} + \phi m + \phi T$. Note that non-repayment only affects hours of work and the amount of money a buyer takes into the next period. By eliminating $\tilde{h}$ using the budget constraint, the value function $\tilde{V}_3(m)$ can be rewritten as

$$\tilde{V}_3(m) = U(x^*) - x^* - \phi \tilde{m}_{+1} + \phi m + \phi T + \beta \tilde{V}_{+1}(\tilde{m}_{+1}).$$

The value function of a buyer who repays his loan in the centralized market is

$$V_3(m) = U(x^*) - h + \beta V_{+1}(m_{+1}),$$

and his budget constraint is $x^* + \phi m_{+1} = h + \phi m + \phi T - \phi(1+i)\ell$. By eliminating h using the budget constraint, we can rewrite $V_3(m)$ as

$$V_3(m) = U(x^*) - x^* - \phi m_{+1} + \phi m + \phi T - \phi(1+i)\ell + \beta V_{+1}(m_{+1}).$$

A buyer repays his loan if, and only if, $V_3(m) \geq \tilde{V}_3(m)$, which implies

$$\phi(1+i)\ell \leq \phi \tilde{m}_{+1} - \phi m_{+1} + \beta \left[V_{+1}(m_{+1}) - \tilde{V}_{+1}(\tilde{m}_{+1}) \right]. \tag{23}$$

Let us now derive $\tilde{V}_{+1}(\tilde{m}_{+1})$ and $V_{+1}(m_{+1})$.

Derivation of $\tilde{V}_{+1}(\tilde{m}_{+1})$. A deviator is banned forever from the money market. The next-period value function of a deviator is

$$\tilde{V}_{+1}(\tilde{m}_{+1}) = \frac{1}{1-\beta} \left[(1-n)\delta u(\tilde{q}) - n\delta^s \bar{q} + U(x^*) - \tilde{h} \right],$$

where $\bar{q} \equiv \pi \hat{q} + (1-\pi) q$ is the expected (or average) quantity he produces if he is a seller; with probability π the buyer he meets is active, in which case he produces $\hat{q}$, while with probability $1-\pi$ the buyer is passive, in which case he produces q. The first two terms within brackets are the expected net payoff in the goods market, while the third and fourth terms equal the net payoff in the centralized market. Expected hours of work for a defector in the centralized market are $\tilde{h} = (1-n)\tilde{h}_b + n\tilde{h}_s$, where $\tilde{h}_b$ and $\tilde{h}_s$ are expected hours of work of a buyer and a seller, respectively, and are defined as

$$\begin{aligned} \tilde{h}_b &= \delta \left[x^* + \phi_{+1} \tilde{m}_{+2} - \phi_{+1} \tilde{m}_{+1} - \phi_{+1} T_{+1} + g(\tilde{q}) \right] \\ &\quad + (1-\delta) \left[x^* + \phi_{+1} \tilde{m}_{+2} - \phi_{+1} \tilde{m}_{+1} - \phi_{+1} T_{+1} \right] \\ &= x^* + \phi_{+1} \tilde{m}_{+2} - \phi_{+1} \tilde{m}_{+1} - \phi_{+1} T_{+1} + \delta g(\tilde{q}) \end{aligned}$$

and

$$\begin{aligned}\tilde{h}_s &= \delta^s\left[x^* + \phi_{+1}\tilde{m}_{+2} - \phi_{+1}\tilde{m}_{+1} - \phi_{+1}T_{+1} - \bar{g}\right] \\ &\quad + (1-\delta^s)\left[x^* + \phi_{+1}\tilde{m}_{+2} - \phi_{+1}\tilde{m}_{+1} - \phi_{+1}T_{+1}\right] \\ &= x^* + \phi_{+1}\tilde{m}_{+2} - \phi_{+1}\tilde{m}_{+1} - \phi_{+1}T_{+1} - \delta^s\bar{g},\end{aligned}$$

respectively. If the deviator is a seller in the next period, then he receives, in the goods market, an average amount of money, in real terms, equal to $\bar{g} \equiv \pi g(\hat{q}) + (1-\pi) g(q)$. Hence, using $(1-n)\delta^s = n\delta$, expected hours of work for a deviator can be rewritten as

$$\begin{aligned}\tilde{h} &= (1-n)\tilde{h}_b + n\tilde{h}_s \\ &= x^* + \phi_{+1}\tilde{m}_{+2} - \phi_{+1}\tilde{m}_{+1} - \phi_{+1}T_{+1} + (1-n)\delta\left[g(\tilde{q}) - \bar{g}\right].\end{aligned}$$

Moreover, using $\tilde{m}_{+2} = \gamma\tilde{m}_{+1}$ and $T_{+1} = (\gamma-1)m_{+1}$, we can rewrite $\tilde{h}$ as follows

$$\begin{aligned}\tilde{h} &= x^* + (\gamma-1)\phi_{+1}\tilde{m}_{+1} - (\gamma-1)\phi_{+1}m_{+1} + (1-n)\delta\left[g(\tilde{q}) - \bar{g}\right] \\ &= x^* + (\gamma-1)\left[g(\tilde{q}) - g(q)\right] + (1-n)\delta\left[g(\tilde{q}) - \bar{g}\right].\end{aligned}$$

Substituting $\tilde{h}$ into $\tilde{V}_{+1}(\tilde{m}_{+1})$ yields

$$\tilde{V}_{+1}(\tilde{m}_{+1}) = \frac{1}{1-\beta}\left\{\begin{array}{c}(1-n)\delta u(\tilde{q}) - n\delta^s\bar{q} + U(x^*) - x^* \\ -(\gamma-1)\left[g(\tilde{q}) - g(q)\right] - (1-n)\delta\left[g(\tilde{q}) - \bar{g}\right]\end{array}\right\}.$$

Derivation of $V_{+1}(m_{+1})$. Let $\bar{u} \equiv \pi u(\hat{q}) + (1-\pi) u(q)$ be the expected utility of a non-defaulting buyer in the goods market. If the buyer is active, he enjoys utility $u(\hat{q})$; if he is passive, he enjoys utility $u(q)$. The next-period value function of a non-deviator is

$$V_{+1}(m_{+1}) = \frac{1}{1-\beta}\left\{(1-n)\delta\bar{u} - n\delta^s\bar{q} + U(x^*) - h\right\}.$$

Note that the average disutility, $\bar{q}$, suffered by a seller in the goods market depends on his trading partner's participation status, active vs passive, and not on his participation status. The expected hours of work of a non-deviator in the centralized market are $h = (1-n)h_b + nh_s$, where

$$\begin{aligned}h_b &= \delta\left[x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1} + \bar{g}\right] \\ &\quad + (1-\delta)\left[x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1}\right] \\ &= x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1} + \delta\bar{g}\end{aligned}$$

and

$$\begin{aligned}h_s &= \delta^s\left[x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1} - \bar{g}\right] \\ &\quad + (1-\delta^s)\left[x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1}\right] \\ &= x^* + \phi_{+1}m_{+2} - \phi_{+1}m_{+1} - \phi_{+1}T_{+1} - \delta^s\bar{g}.\end{aligned}$$

Hence, average hours of work for a non-deviator are

$$\begin{aligned} h &= (1-n)\, h_b + n h_s \\ &= x^* + \phi_{+1} m_{+2} - \phi_{+1} m_{+1} - \phi_{+1} T_{+1} \\ &= x^*, \end{aligned}$$

where we have used $T_{+1} = m_{+2} - m_{+1}$. By replacing h in $V_{+1}(m_{+1})$, we obtain

$$V_{+1}(m_{+1}) = \frac{1}{1-\beta}\left\{(1-n)\,\delta\bar{u} - n\delta^s\bar{q} + U(x^*) - x^*\right\}.$$

Using the above expressions to eliminate $V_{+1}(m_{+1})$ and $\tilde{V}_{+1}(\tilde{m}_{+1})$ in (23), we obtain

$$\begin{aligned} \phi(1+i)\,\ell &\leq \phi\tilde{m}_{+1} - \phi m_{+1} + \beta\left[V_{+1}(m_{+1}) - \tilde{V}_{+1}(\tilde{m}_{+1})\right] \\ &= \gamma\left[\phi\tilde{m} - \phi m\right] + \frac{\beta}{1-\beta}\left\{\begin{array}{c}(1-n)\,\delta\left[\pi u(\hat{q}) + (1-\pi)\,u(q)\right] \\ -n\delta^s\bar{q} + U(x^*) - x^*\end{array}\right\} \\ &\quad + \frac{\beta}{1-\beta}\left[\begin{array}{c}-(1-n)\,\delta u(\tilde{q}) + n\delta^s\bar{q} - U(x^*) + x^* \\ +(\gamma-1)\left[g(\tilde{q}) - g(q)\right] + (1-n)\,\delta\left[g(\tilde{q}) - \bar{g}\right]\end{array}\right] \end{aligned}$$

or, after further simplification,

$$\begin{aligned} \phi\ell &\leq \frac{(\gamma-\beta)\left[g(\tilde{q}) - g(q)\right]}{(1+i)(1-\beta)} \\ &\quad + \frac{\beta(1-n)\,\delta\left\{\pi\left[u(\hat{q}) - g(\hat{q})\right] + (1-\pi)\left[u(q) - g(q)\right] - \left[u(\tilde{q}) - g(\tilde{q})\right]\right\}}{(1+i)(1-\beta)} \end{aligned}$$

where $\tilde{q}$ satisfies (7).

Derivation of (7). The envelope condition for a defector in the money market is

$$\frac{\partial\tilde{V}_1}{\partial m} = (1-n)\,\frac{\partial\tilde{V}_2^b}{\partial m} + n\frac{\partial\tilde{V}_2^s}{\partial m},$$

which, substituting $\partial\tilde{V}_2^b/\partial m$ and $\partial\tilde{V}_2^s/\partial m$, can be written as

$$\frac{\partial\tilde{V}_1}{\partial m} = (1-n)\,\phi\left[\delta\frac{u'(\tilde{q})}{g'(\tilde{q})} + 1 - \delta\right] + n\phi.$$

Updating the previous equation one period ahead, and using the first-order condition in the centralized market, we obtain (7). ■

Proof of Proposition 1. Equations (6), (7), and (10)-(13) hold in a type-0 equilibrium. The derivation of (6) and (7) is in the proof of Lemma 1. Equation (10) means that the real amount of money an active buyer spends in the goods market, $g(\hat{q})$, is equal to the real amount of money spent as a passive buyer, $g(q)$, plus the real loan an active buyer receives from the bank, $\phi\ell$. Equation (11) is straightforward and means that the real loan which a buyer receives from the bank is equal to the maximum amount he can obtain. This is a direct consequence of the fact that his borrowing

constraint is binding in the type-0 equilibrium. Before we derive equations (12) and (13), we first characterize the solutions to the agent's decision problems. The first-order conditions of the agent's problem (1) are

$$U'(x) = 1, \qquad \text{and} \qquad \frac{\beta \partial V_1}{\partial m_{+1}} = \phi. \tag{24}$$

The term $\beta \partial V_1 / \partial m_{+1}$ reflects the marginal value of taking one additional unit of money into the next period, and ϕ is the marginal cost of doing so. As in Lagos and Wright (2005), the choice of m_{+1} is independent of m. As a result, each agent exits the centralized market with the same amount of money. The envelope conditions are

$$\frac{\partial V_3}{\partial m} = \phi, \quad \frac{\partial V_3}{\partial d} = \phi (1+i), \quad \text{and} \quad \frac{\partial V_3}{\partial \ell} = -\phi (1+i). \tag{25}$$

The marginal value of money at the beginning of the centralized market is equal to the price of money in terms of general goods. This implies that the value function V_3 is linear in m. The value function for a buyer in the goods market is

$$V_2^b(m, \ell, 0) = \delta \left[u(q) + V_3(m - z, \ell, 0) \right] + (1 - \delta) V_3(m, \ell, 0).$$

The buyer's envelope conditions are

$$\frac{\partial V_2^b}{\partial m} = \delta \left[u'(q) \frac{\partial q}{\partial m} + \phi \left(1 - \frac{\partial z}{\partial m} \right) \right] + (1 - \delta) \phi, \quad \text{and} \quad \frac{\partial V_2^b}{\partial \ell} = -\phi (1+i).$$

If the buyer's cash constraint is not binding, then $\frac{\partial q}{\partial m} = 0$ and $\frac{\partial z}{\partial m} = 0$. In this case, the buyer's first envelope condition reduces to $\frac{\partial V_2^b}{\partial m} = \frac{\partial V_3}{\partial m} = \phi$. If the constraint is binding, then $\frac{\partial q}{\partial m} = \frac{\phi}{g'(q)}$ and $\frac{\partial z}{\partial m} = 1$. In this case, the buyer's envelope conditions in the goods market become

$$\frac{\partial V_2^b}{\partial m} = \delta \phi \frac{u'(q)}{g'(q)} + \phi (1 - \delta), \quad \text{and} \quad \frac{\partial V_2^b}{\partial \ell} = -\phi (1+i). \tag{26}$$

The value function for a seller in the goods market is

$$V_2^s(m, 0, d) = \delta \left[-q + V_3(m + z, 0, d) \right] + (1 - \delta) V_3(m, 0, d),$$

and envelope conditions are

$$\frac{\partial V_2^s}{\partial m} = \phi, \quad \text{and} \quad \frac{\partial V_2^s}{\partial d} = \phi (1+i). \tag{27}$$

The first-order condition of the buyer's problem (8) is

$$\frac{\partial V_2^b}{\partial m} + \frac{\partial V_2^b}{\partial \ell} = \lambda_\Phi. \tag{28}$$

where λ_Φ denotes the Lagrange multiplier on the buyer's borrowing constraint (5). The first-order condition of the seller's problem (9) in the money market is

$$-\frac{\partial V_2^s}{\partial m}+\frac{\partial V_2^s}{\partial d}=\lambda_s. \tag{29}$$

The envelope condition of (4) is

$$\frac{\partial V_1}{\partial m}=\pi\left[(1-n)\frac{\partial V_1^b}{\partial m}+n\frac{\partial V_1^s}{\partial m}\right]+(1-\pi)\left[(1-n)\frac{\partial V_2^b}{\partial m}+n\frac{\partial V_2^s}{\partial m}\right].$$

Applying the envelope theorem to (8) and (9), the above envelope condition can be rewritten as

$$\frac{\partial V_1}{\partial m}=\pi\left[(1-n)\left(\frac{\partial V_2^b}{\partial m}+\lambda_\Phi\right)+n\left(\frac{\partial V_2^s}{\partial m}+\lambda_s\right)\right]+(1-\pi)\left[(1-n)\frac{V_2^b}{\partial m}+n\frac{V_2^s}{\partial m}\right]. \tag{30}$$

Derivation of (12). If $\lambda_\Phi>0$ and $\lambda_s=0$, then (28) and (29) can be written as

$$\delta\left[\frac{u'(\hat{q})}{g'(\hat{q})}-1\right]=\frac{\lambda_\Phi}{\phi}+i,\quad \text{and}\quad (12), \tag{31}$$

respectively, where we have used (26) and (27) to eliminate $\frac{\partial V_2^b}{\partial m}$, $\frac{\partial V_2^b}{\partial \ell}$, $\frac{\partial V_2^s}{\partial m}$, and $\frac{\partial V_2^s}{\partial d}$. The first expression in (31) gives us the value of the multiplier λ_Φ.

Derivation of (13). Use $\lambda_s=0$ and the envelope conditions in the goods market, (26) and (27), to rewrite the money market envelope condition (30) as follows

$$\frac{\partial V_1}{\partial m}=\phi(1-n)\,\delta\left\{\pi\left[\frac{u'(\hat{q})}{g'(\hat{q})}-1\right]+(1-\pi)\left[\frac{u'(q)}{g'(q)}-1\right]\right\}+\phi.$$

Finally, update this expression by one period and replace $\frac{\partial V_1}{\partial m_+}$ using (24), to obtain (13). ■

Proof of Proposition 2. Equations (6), (7), and (14)-(17) hold in a type-I equilibrium. The derivation of (6) and (7) is in the proof of Lemma 1. Equations (14) and (15) are identical to (10) and (11), respectively.

Derivation of (16). In a type-I equilibrium, active sellers deposit all their money at the bank; i.e., $d=m$. Moreover, active buyers carry $\hat{m}$ units of money out of the money market, where $\hat{m}=m+\ell$, and the market clearing condition in the money market requires that total deposits must be equal to total loans; i.e., $\pi n d=\pi(1-n)\ell$. Using $d=m$ and $\hat{m}=m+\ell$, the market clearing condition in the money market can be rewritten as $m=(1-n)\hat{m}$. Multiplying each side of the last equation by ϕ, and using (3), we obtain (16).

Derivation of (17). Use (26) and (27) to substitute $\frac{\partial V_2^b}{\partial m}$ and $\frac{\partial V_2^b}{\partial \ell}$ in (28), and $\frac{\partial V_2^s}{\partial m}$ and $\frac{\partial V_2^s}{\partial d}$ in (29) to obtain

$$\delta\left[\frac{u'(\hat{q})}{g'(\hat{q})}-1\right]=\frac{\lambda_\Phi}{\phi}+i\quad\text{and}\quad \phi i=\lambda_s, \tag{32}$$

respectively. Again, eliminate $\frac{\partial V_2^b}{\partial m}$, $\frac{\partial V_2^s}{\partial m}$, and λ_s into (30) using (26), (27), and (32), respectively, to obtain

$$\frac{\partial V_1}{\partial m}=\pi\phi\left\{(1-n)\left[\delta\frac{u'(\hat{q})}{g'(\hat{q})}+1-\delta\right]+n(1+i)\right\}+(1-\pi)\,\phi\left\{(1-n)\left[\delta\frac{u'(q)}{g'(q)}+1-\delta\right]+n\right\}.$$

Updating this expression by one period, and using (24) to replace $\frac{\partial V_1}{\partial m_{+1}}$, we obtain (17). ■

Proof of Proposition 3. Equations (6), (7), and (18)-(21) hold in a type-II equilibrium. The derivation of (6) and (7) is in the proof of Lemma 1. Equations (18) and (20) are identical to (14) and (16), respectively.

Derivation of (19). In a type-II equilibrium, active sellers deposit all their money (i.e., $\lambda_s > 0$) and the buyer's borrowing constraint is not binding (i.e., $\lambda_\Phi = 0$). Substituting $\lambda_\Phi = 0$ in (28), we obtain

$$\frac{\partial V_2^b}{\partial m} + \frac{\partial V_2^b}{\partial \ell} = 0. \tag{33}$$

Then, using (26) and (27) to substitute $\frac{\partial V_2^b}{\partial m}$, $\frac{\partial V_2^b}{\partial \ell}$, $\frac{\partial V_2^s}{\partial m}$, and $\frac{\partial V_2^s}{\partial d}$ in (33) and (29), we obtain

$$(19), \qquad \text{and} \qquad i = \frac{\lambda_s}{\phi}, \tag{34}$$

respectively. The second equation just gives us the value of the multiplier λ_s.

Derivation of (21). Eliminate $\frac{\partial V_2^b}{\partial m}$, $\frac{\partial V_2^s}{\partial m}$, and λ_s in (30) using (26), (27), and (34), respectively, to get

$$\frac{\partial V_1}{\partial m} = \phi \left\{ i\pi + (1-\pi)(1-n)\,\delta \left[\frac{u'(q)}{g'(q)} - 1 \right] + 1 \right\}.$$

Updating this expression by one period, and using (24) to replace $\frac{\partial V_1}{\partial m_{+1}}$, we obtain (21). ■

8 Appendix II: Type III equilibrium

For completeness, we analyze here a fourth equilibrium, called type-III equilibrium, which we have not discussed in the main text of the paper. This equilibrium has the following properties: active sellers do not deposit all their money (i.e., $\lambda_s = 0$), and the buyer's borrowing constraint is not binding (i.e., $\lambda_\Phi = 0$).

Proposition 4 *A type-III equilibrium is a tuple* $\left\{ i, \hat{q}, \tilde{q}, q, \phi\ell, \phi\overline{\ell} \right\}$ *satisfying (6), (7), and*

$$g(\hat{q}) = g(q) + \phi\ell, \tag{35}$$

$$i = \delta \left[\frac{u'(\hat{q})}{g'(\hat{q})} - 1 \right], \tag{36}$$

$$i = 0, \tag{37}$$

$$\frac{\gamma - \beta}{\beta} = (1-\pi)(1-n)\,\delta \left[\frac{u'(q)}{g'(q)} - 1 \right]. \tag{38}$$

The meaning of all equations in Proposition (4) is the same as that of their counterparts in Proposition (3), except that (20) is now replaced by (37). Active buyers consume the first-best quantity and the nominal interest rate is zero in a type-III equilibrium. Since buyers are not borrowing-constrained, relaxing the borrowing constraint does not affect their decision.

Proof of Proposition 4. Equations (6), (7), and (35)-(38) hold in a type-III equilibrium. The derivation of (6) and (7) is in the proof of Lemma 1. Equations (35)-(37) are identical to (10), (12) and (19), respectively.

Derivation of (38). From (36) and (37), $u'(\hat{q}) = g'(\hat{q})$. Use $\lambda_s = 0$, (26), (27), and $u'(\hat{q}) = g'(\hat{q})$, to rewrite (30) as follows:

$$\frac{\partial V_1}{\partial m} = \phi + (1-\pi)(1-n)\phi\delta\left[\frac{u'(q)}{g'(q)} - 1\right].$$

Update this expression by one period and replace $\frac{\partial V_1}{\partial m_+}$ using (24), to obtain (38).

Let γ_3 be the value of γ that separates the type-III and type-II equilibria. Table A.1 below summarizes the values of the multipliers and the amount of real borrowing in these equilibria. ■

TABLE A.1: TYPE-II AND TYPE-III EQUILIBRIA

Equilibrium	γ	λ_Φ	λ_s	Real borrowing
type-III	$\beta < \gamma < \gamma_3$	$\lambda_\Phi = 0$	$\lambda_s = 0$	$\phi\ell < \phi\bar{\ell}$
type-II	$\gamma > \gamma_3$	$\lambda_\Phi = 0$	$\lambda_s > 0$	$\phi\ell < \phi\bar{\ell}$.

9 Appendix III: Data sources

The data we use for the calibration is downloadable from the Federal Reserve Bank of St. Louis FRED database. Table A.2. gives a brief overview of the data sources.

TABLE A.2: DATA SOURCE

Description	Identifier
Consumer price index for the U.K.	GBRCPIALLQINMEI
M1 for the U.K.	MANMM101GBM189S
Gross domestic product for the U.K.	UKNGDP
Long-term government bond yield	IRLTLT01GBQ156N
Total Credit to Private Non-financial Sector	CRDQGBAPUBIS
Total Credit to Households & Non-profit Institutions Serving Households	CRDQGBAHUBIS

For all time series, we use quarterly data for the period from 1986:Q4 to 2013:Q4.

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