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Knell, Markus

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The Intergenerational Distribution of Demographic Fluctuations in Unfunded and Funded Pension Systems

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The Intergenerational Distribution of Demographic Fluctuations in Unfunded and Funded Pension Systems

Markus Knell*
Oesterreichische Nationalbank
Economic Studies Division

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Abstract

In this paper I use a multi-period OLG model to study how a demographic shock is distributed among different generations. In particular, I investigate whether a funded pension system allows for a smoother adjustment than an unfunded system. The results suggest that the answer to this question depends on the specific organization of the funded system. If the contributions are only invested into a non-accumulated asset in fixed supply (e.g. into "land") and if the investment decisions are guided by fixed rules then the intergenerational distribution of the demographic shock is almost identical in the two systems. Assuming optimal investment decisions, on the other hand, will increase or decrease the fluctuations of the funded pillar (depending on the degree of risk aversion). It is only for the case where all savings are invested into accumulable, productive capital that the funded system will dampen the distributional consequences of a demographic shock.

Keywords: Pension System; Demographic Change; Financial Stability;
JEL-Classification: H55; J1; J18; J26

*OeNB, Economic Studies Division, Otto-Wagner-Platz 3, POB-61, A-1011 Vienna; Phone: (++43-1) 40420 7218, Fax: (++43-1) 40420 7299, Email: Markus.Knell@oenb.at. The views expressed in this paper do not necessarily reflect those of the Oesterreichische Nationalbank.

1 Introduction

Demographic developments have considerable effects on labor, goods and capital markets (cf. Bloom & Canning 2004, Poterba 2004). One area where this impact is immediately visible and manifest is the functioning of pension systems, both in its funded and in its unfunded form. In as far as the former is concerned there exists an important strand of literature (cf. Mankiw & Weil 1989, Abel 2003, Geanakoplos et al. 2004, Börsch-Supan 2004) that has looked at the possible consequences for asset markets. It has been shown that the ageing of the baby boom generation might first (during the "buying phase") be associated with a surge in house and stock market prices that is later followed by a corresponding drop (during the "sell-out phase"). A number of studies have found empirical support for the "asset-meltdown hypothesis" (cf. Favero et al. 2011), although the evidence still remains controversial (cf. Brooks 2002, Poterba 2004).

The consequences of demographic fluctuations for unfunded, pay-as-you-go (PAYG) pension system are often said to be even larger than the ones for asset-based funded systems. In a defined contribution (DC) system, e.g., a drop in the size of working cohorts has to be accompanied by a reduction in pension benefits in order to balance the budget of the social security system. For a defined benefit (DB) system, on the other hand, the adjustment has to involve an increase in the contribution rate. In both cases the effect of a baby-boom-and-bust-cycle might be considerable, although the adjustment burden is borne by different generations in the two variants of the PAYG system (younger generations in DB and older generations in DC systems). In fact, a number of authors and commentators have argued that unfunded pension systems are not really capable to deal with large demographic changes and that — even if they manage to keep the budget in balance — the adjustment processes will be associated with more pronounced swings in macroeconomic variables and in the intergenerational distribution than will be the case for funded systems. Modigliani et al. (2000), e.g., argue that the "funded system is more stable and financially resilient because under PAYG the required contribution rate depends on the rate of growth of payrolls"(p.2).[1] According to this line of reasoning a funded system is better able to smooth the burden of adjustment and to share it between a larger number generations by using the accumulation and decumulation of the capital stock. This argument, however, is less straightforward than it might look at first sight and it depends on a number of assumptions about the organization of the funded pillar and the saving and investment process (cf. Barr & Diamond 2009). In particular, it depends on

[1] Similar arguments can be found in Wilke (2008) and Börsch-Supan & Gasche (2010).

the assumption that the pension funds are invested into productive, accumulable capital. This, however, is not guaranteed. As noted, for example, by N. Barr and P. Diamond (2009): "The link between an increase in saving and increased investment is complex—some savings will simply increase prices of existing assets. Part of increased saving can drive up the prices of assets in limited supply, such as urban land" (p.71).

In this paper I analyze in detail how funded and unfunded pension systems react to demographic developments and whether or under which circumstances a funded system does in fact lead to a smoother reaction to demographic shocks. In particular, I present a multi-period overlapping generation model where cohorts might differ in their size. I study how a one-time shock in the size of one cohort affects the internal rate of return of the different generations. As the starting point of the analysis I will present a comparison between a benchmark unfunded and a benchmark funded system that challenges the conventional wisdom about the smoother adjustment properties of funded pillars. The benchmark PAYG system is a standard DC scheme. For the funded system, however, I use a framework that deviates from the standard model in a number of respects. First, instead of working with a representative agent in each generation I assume that there exist two classes of people with different degrees of risk aversion and time preference. Second, while the rich "upper class" is assumed to invest in accumulable real capital the (much larger) "working class" is assumed to invest its pension savings into a non-productive and non-produced asset ("gold"). Third, savings decisions are not guided by optimal behavior but rather follow fixed rules. I show that in this case the effect of demographic shocks on the intergenerational sharing of the burden is basically identical in unfunded and funded systems.

In later sections I look at each of the specific assumptions in this funded benchmark in order to see how sensitive the equivalence result reacts to each of these changes. The analysis shows that the implications of the model are almost unchanged if the non-accumulable asset also plays a role in production (i.e. it can be interpreted as "land" instead of "gold"). On the other hand, if people do not follow fixed investment rules but rather use intertemporal optimization, the funded system might be associated with smoother or wilder fluctuations, depending on the parameters of the utility function (in particular on the degree of risk aversion). It is only when one uses the standard assumptions of a representative individual that invests in real capital that one gets a smoother adjustment process for the funded system. This, however, is arguably a rather special case. Overall one can say that the effects of funding on the smoothness of adjustment might go in either direction and it would be a drastic simplification to base the argument

for a transition to a funded pension system on these ambiguous results.

There exist various strands of literature that are related to the present paper. One the one hand one can find a number of articles that look at the optimal design of PAYG systems in the presence of demographic fluctuations. Bohn (2001, 2009) and Thøgersen (1998), e.g., look at different types of PAYG systems and investigate how their organization (e.g. defined benefit vs. defined contribution or the indexation with prices or with wages) affects the risk-sharing properties of the system. A similar investigation, although in a multi-period setting, can be found in Auerbach & Lee (2011). They use stochastic simulations to analyze how different PAYG systems spread the risks that arise from demographic and economic shocks across generations and to compare the outcomes using expected utility calculations. Also related is the analysis by Ludwig & Reiter (2010) who show that the reaction of the existing German pension system to demographic shocks is relatively close to the second best solution of a Ramsey planner with full commitment. Different to this strand of literature I do not only focus on PAYG systems but also include a comparison with funded systems. Furthermore, I abstract from the difficult normative question of finding an optimal design of the pension system and rather concentrate on the positive effects of demographic shocks on the fluctuations of a popular and widely used intergenerational measure (the internal rate of return).

On the other hand, there also exists a line of research that looks at a comparison between funded and unfunded pension systems. Most papers in this literature focus on the different risk-return patterns of the two system, as is done, e.g., in Dutta et al. (2000), de Menil et al. (2006) and Knell (2010*b*). In these papers, however, it is always assumed that the funded pillar only invests in accumulable, productive capital. In this paper I show that in order to make an accurate comparison between unfunded and funded pillars it is important to also think about different *variants* of the funded pillar.

The paper is organized as follows. In section 2 I present the production and the household side of the model and I define the internal rate of return that is used as the measure of intergenerational distribution. In section 3 I study how the benchmark PAYG system and the benchmark funded system react to a one-time demographic shock. In section 4 I have a closer look at the funded system and I analyze how the pattern and the magnitude of the intergenerational fluctuations are affected if one alters the assumptions about the organization of the funded system. Section 5 concludes.

2 The model

I start by presenting the general structure of the production process of the economy and turn then to the behavior of households. Various specifications of the pension system will be introduced in sections 3 and 4.

2.1 The production side

I assume that there are two types of assets in the economy: a produced and accumulable factor of production K_{t-1} (say capital) and a fixed stock of a non-producible factor $H_t = \bar{H}$ (say land/houses). Aggregate output is produced by combining these two factors with labor L_t:

$$Y_t = (A_t)^{1-a_1}(K_{t-1})^{a_1}(\bar{H})^{a_2}(L_t)^{1-a_1-a_2}, \tag{1}$$

where A_t is a labor-augmenting productivity term that grows with rate g, i.e.: $A_{t+1} = A_t(1+g)$. The timing of variables follows the "end-of-period" convention, i.e. stock variables are timed in the period when they are determined.

Capital depreciates at rate d and the aggregate capital stock thus accumulates as:

$$K_t = Y_t - C_t + (1-d)K_{t-1}, \tag{2}$$

where C_t is aggregate consumption. The output good can either be used for consumption or for the accumulation of the capital stock and it is used as the numéraire good. The relative price of land is denoted by P_t^H.

I assume that all markets are competitive and the factor prices are thus given by their marginal products:

$$r_t^K = a_1 \frac{Y_t}{K_{t-1}} \tag{3}$$

$$r_t^H = a_2 \frac{Y_t}{\bar{H}} \tag{4}$$

$$W_t = (1-a_1-a_2)\frac{Y_t}{L_t}. \tag{5}$$

Throughout the paper I will deal with situations where the population is ultimately stationary (although it might fluctuate in the short-run). Productivity, however, is allowed to grow over time. In order to be able to study the behavior on a balanced growth path

I will use the following normalizations:

$$\widetilde{Y}_t = \frac{Y_t}{A_t}, \widetilde{K}_{t-1} = \frac{K_{t-1}}{A_t}, \widetilde{W}_t = \frac{W_t}{A_t}, \widetilde{r}_t^H = \frac{r_t^H}{A_t}, \widetilde{P}_t^H = \frac{P_t^H}{A_t}. \tag{6}$$

The production function and the factor prices can then be written as:

$$\begin{aligned} \widetilde{Y}_t &= (\widetilde{K}_{t-1})^{a_1}(\bar{H})^{a_2}(L_t)^{1-a_1-a_2}, \\ r_t^K &= a_1 \frac{\widetilde{Y}_t}{\widetilde{K}_{t-1}}, \ \widetilde{r}_t^H = a_2 \frac{\widetilde{Y}_t}{\bar{H}}, \ \widetilde{W}_t = (1-a_1-a_2)\frac{\widetilde{Y}_t}{L_t}. \end{aligned}$$

On a balanced growth path the wage rate W_t grows with rate g and the same is also true for the relative price P_t^H and the rate of return r_t^H of the fixed factor of production.

An interesting special case occurs when $a_2 = 0$, i.e. when the fixed factor $\bar{H}$ does not play a role for production and can only be used as a store of value. In this case one can think of $\bar{H}$ as gold.

2.2 The social structure of households

For the modeling of the household side I use a slightly unusual (although not implausible) framework. In particular, I assume that there exist two classes of households that differ with respect to their risk preferences, their rate of time preference (or their planning horizons), their investment opportunities and their asset holdings.

2.2.1 Working Class

On the one hand, there is a large mass of households that are organized in a formation of multi-period, overlapping generations. In particular, I assume that each cohort has a (fixed and deterministic) life expectancy of Y and that it works for X years earning an annual wage of W_t. For the remaining $(Y - X)$ years individuals are retired and they have to live on pension income and on private savings. The size of each cohort is N_t and this size is allowed to fluctuate over time.

The intertemporal utility function of the representative member of generation t is given by:

$$U_t = \sum_{a=1}^{Y} \beta^{a-1} \frac{C_{a,t+a-1}^{1-\rho}}{1-\rho}, \tag{7}$$

where $\beta \leq 1$ is the time discount factor, $\rho \geq 1$ is the coefficient of relative risk aversion

(or the inverse of the intertemporal elasticity of substitution) and $C_{a,t+a-1}$ stands for the level of consumption at age a of the generation that is born in period t. In the benchmark model I assume that these "working class" households are only holding one type of asset (if at all): the non-produced fixed factor $\bar{H}$. I will discuss below the rationale behind this assumption.

These households' period budget constraints are then given by:

$$H_{a,t+a-1}P^H_{t+a-1} = I^W_a W_{t+a-1} + H_{a-1,t+a-2}\left(P^H_{t+a-1} + r^H_{t+a-1}\right) - C_{a,t+a-1}, \tag{8}$$

where $H_{a,t+a-1}$ stands for the total stock of land held by generation t at the end of period $t+a-1$ when they are a years old.[2] I^W_a is an indicator variable that is 1 for $1 \leq a \leq X$ and 0 for $X+1 \leq a \leq Y$. The agents enter the world without any asset holdings (i.e. $H_{0,t-1} = 0$) and from the assumption of a known year of death and the absence of a bequest motive it follows that they die without any asset holdings, i.e. $H_{Y,t+Y-1} = 0$.

2.2.2 Upper Class

Besides this quantitatively dominating group of "common people" — the "working class" — there also exists a tiny upper class that is in many dimensions the exact opposite of the majority. First, members of this upper class are assumed to have a dynastic perspective and they thus take the destiny of their offsprings into account. Second, they are less risk-averse than the ordinary people and I will work with the limiting case where they are completely risk-neutral (i.e. $\rho^{uc} = 0$). Third, they hold the entire stock of accumulated capital K_{t-1}. Fourth, they do not work and just live on capital income (i.e. $W^{uc}_t = 0$). Fifth, they are a very small group and they are not subject to the fertility fluctuations that characterize the rest of the society.[3] Due to their tiny size they do not influence the general trends in the development of the population and I will abstract from their small number when specifying the cohort size N_t and the size of the total population $\sum_{a=1}^{Y} N_{t-a+1}$.

Summarizing, I assume that one can write the intertemporal utility function of this

[2] At the beginning of time the entire stock of $\bar{H}$ is owned by the retired population.

[3] In fact, due to their special position it is not unreasonable to assume that they are exempt from the normal sources that lie behind the booms-and-bust in cohort sizes: war casualties, diseases, changing social norms etc. The support of nannies, domestic help etc. might also contribute to the immunity to societal fertility trends.

upper class as:

$$U_t^{uc} = \sum_{a=1}^{\infty} (\beta^{uc})^{a-1} \frac{\left(C_{t+a-1}^{uc}\right)^{1-\rho^{uc}}}{1-\rho^{uc}}, \tag{9}$$

where the assumption of risk-neutrality ($\rho^{uc} = 0$) implies that (9) becomes a linear function of consumption. The period budget constraint of the upper class is given by:

$$K_{t+a-1} = -C_{t+a-1}^{uc} + R_{t+a-1}^{K} K_{t+a-2}, \tag{10}$$

where $R_t^K = (1 + r_t^K - d)$ is the gross rate of return on capital between periods t and $t+1$ and K_0 is given. I also assume that the transversality condition for capital holds.

2.2.3 Rationale behind this set-up

Admittedly, this set-up is a caricature of the existing social structure. Nevertheless, it has a realistic flavor that captures with a rather broad brush a dividing line in economic and non-economic life styles that is present in many societies. The "superrich" often live a life of their own that is quite different from the lives of the "common people". They own a large part of total real wealth (if one abstracts from owner-occupied housing), their labor market income is negligible compared to their capital income, they have distinct reproductive patterns and often take a quite dynastic view and they are arguably also more risk-neutral than the rest of the population.[4]

One can speculate about possible reasons for the different investment behavior between socioeconomic groups, in particular for the fact that the majority of people does not hold any capital but only the non-accumulable asset $\bar{H}$. For example, this might have to do with limited knowledge, financial constraints or institutional obstacles. Furthermore, one must note that the model is set in a deterministic world and the assumed differential portfolio choices might also capture — as a short-cut — differences in risk-aversion. In the presence of uncertainty, any eventual differences in the returns r_t^K and $\widetilde{r}_t^H$ should be adjusted for their different risk and the dichotomous structure of asset holding might simply reflect the different degrees of risk aversion and the associated class-specific investment behavior.

2.2.4 The determination of capital and the rate of return

Maximization of (9) leads to the first-order condition: $\left(\frac{C_{t+a}^{uc}}{C_{t+a-1}^{uc}}\right)^{\rho^{uc}} = \beta^{uc} R_{t+a}^K$. Using $\rho^{uc} = 0$ this leads to the equation $R_{t+a}^K = \frac{1}{\beta^{uc}}$. The assumption of risk-neutral superrich

[4]This, in fact, might exactly be the reason why they are rich in the first place.

thus leads to a situation where the rate of return on capital is constant over time and given by $1 + \bar{r}^K - d = \frac{1}{\beta^{uc}}$ or:

$$\bar{r}^K = \frac{1 - \beta^{uc}}{\beta^{uc}} + d. \tag{11}$$

The superrich households are willing to let their levels of consumption fluctuate widely between periods in order to smooth the real rate of return.[5] Note that for the limiting case where $\beta^{uc} \to 1$ it holds that $\bar{r}^K = d$. This is equal to the golden rule allocation where aggregate consumption is at its maximum.

Given $\bar{r}^K$ one can then use $r_t^K = a_1 \frac{\widetilde{Y}_t}{K_{t-1}}$ and the production function to calculate the level of capital K_{t-1} for every period (since $\bar{H}$ and L_t are also known in every period). It comes out as:

$$K_{t-1} = A_t \left(\frac{a_1}{\bar{r}^K}\right)^{\frac{1}{1-a_1}} \left(\bar{H}\right)^{\frac{a_2}{1-a_1}} (L_t)^{\frac{1-a_1-a_2}{1-a_1}}.$$

Using this in the production function (1) one can write:

$$Y_t = BA_t(\bar{H})^{\tilde{a}_2}(L_t)^{1-\tilde{a}_2}, \tag{12}$$

where $B \equiv \left(\frac{a_1}{\bar{r}^K}\right)^{\frac{a_1}{1-a_1}}$ and $\tilde{a}_2 \equiv \frac{a_2}{1-a_1}$. Using the normalization for productivity growth (6) this can also be expressed as $\widetilde{Y}_t = B(\bar{H})^{\tilde{a}_2}(L_t)^{1-\tilde{a}_2}$. From the perspective of the working class the economy thus appears as a world where output is produced with labor and with a non-producible factor that is in fixed supply. For the case where $a_2 = 0$ equation (12) reduces to the linear production function $Y_t = BA_tL_t$.[6]

[5] In this respect the present model is very different from a number of papers (cf. Krueger & Ludwig 2007, Fehr et al. 2008) that have tried to evaluate the impact of demographic changes on the global macroeconomy and in particular on factor prices. Most approaches have come to the conclusion that these demographic trends will lead to lower interest rates. The model presented here, however, shows that if there is a sufficient degree of heterogeneity between individuals and if the risky capital stock is primarily hold by the rather risk-neutral class, then the interest rate is fixed at $\bar{r}^K$. Demographic fluctuations might thus have a smaller effect on factor prices than is normally implied by these traditional analyses.

[6] Instead of the two-class structure one could also employ the popular small-open-economy assumption that has similar consequences as the model used in this paper. In particular, for a small, open economy the interest rate is given by the world interest rate r^W which is assumed to be constant. If one would assume in addition that the total capital stock is held by foreigners than the reduced form of this framework would closely correspond to my model. Underneath these similarities there are, however, crucial differences. First, the small-open-economy model has nothing to say about the determinants of the world interest rate while it is given by (11) in my model. Second, it always remains a partial equilibrium model which makes it difficult to study demographic changes as a *global* phenomenon. Third, it is harder to motivate why the capital is only owned by foreigners than to argue why it is the exclusive property of the superrich upper class.

2.3 The internal rate of return

Before turning to the discussion of different pension systems one has to specify how to measure the impact of demographic changes on the economic treatment of different cohorts. There exist various measures of intergenerational distribution that can be used to evaluate the distributional properties of existing pension schemes and the effects of proposed pension reforms. Four widely used measures are the internal rate of return, the present value ratio, the net present value and the implicit tax rate (cf. Geanakoplos et al. 1999, Fenge & Werding 2003). In this paper I focus on the internal rate of return (IRR) since it is the most commonly used indicator in this context.

As in the context of finance (where it is used in capital budgeting to compare the profitability of investments) the IRR is defined as the discount rate at which the net present value of costs (negative cash flows) and the one of benefits (positive cash flows) is equal. For pension systems the costs are the contributions paid into the system while the positive cash flows are the pension benefits. In other words, the internal rate of return δ_t for generation t is implicitly given by:

$$\sum_{a=1}^{Y} CF_{a,t+a-1} \left(\frac{1}{1+\delta_t} \right)^{a-1} = 0, \tag{13}$$

where $CF_{a,t+a-1}$ is the (positive or negative) cash flow that accrues for generation t when it is a years old.

3 Intergenerational burden-sharing in unfunded and funded pension systems

3.1 The PAYG system (benchmark case)

3.1.1 Assumptions

As a starting point I abstract first from the existence of land, i.e. I assume that $a_2 = 0$ and $\bar{H} = 0$. Using a normalization such that $B = 1$ equation (12) leads to $Y_t = A_t L_t$. The pension system is defined by two magnitudes: the fixed contribution rate $\hat{\tau}$ and the pension payment P_t, which is assumed to be identical for all retired cohorts. The budget

constraint of the households as stated in (8) then reduces to:

$$\begin{aligned} C_{a,t+a-1} &= (1-\hat{\tau})W_{t+a-1} \qquad \text{for } 1 \leq a \leq X, \\ C_{a,t+a-1} &= P_{t+a-1} \qquad \text{for } X+1 \leq a \leq Y. \end{aligned} \tag{14}$$

The households are in this case exempt from any meaningful economic decision and their utility (given by (7)) is just a function of the publicly determined PAYG system. This decision-free environment is a useful benchmark for later comparisons.

The pension in time t is determined in such a way that the budget of the social security system is permanently balanced. This means that $P_t = \hat{\tau} W_t \frac{R_t}{L_t}$, where $L_t = \sum_{a=1}^{X} N_{t-a+1}$ is the total labor supply, $R_t = \sum_{a=X+1}^{Y} N_{t-a+1}$ stands for the number of retired persons and $\frac{R_t}{L_t}$ is the dependency ratio. It can be easily checked that in this case the balanced budget condition $\hat{\tau} W_t L_t = P_t R_t$ is fulfilled.[7]

3.1.2 Results

The impact of fluctuations in the cohort size on the intergenerational pattern of the internal rates of return can best be illustrated by the use of a specific numerical example. This is done in Figure 1 for the case where the contribution rate is 25% ($\hat{\tau} = 1/4$), people start working at the age of 20, work for 45 years ($X = 45$), retire with 65 and die at the age of 80 ($Y = 60$).

In order to get a clear picture of how the size of a specific cohort influences the internal rate of return for another cohort I use the simple assumption of a one-time jump. This means that $N_t = \hat{N}$ for $t \neq 0$ and $N_t = \chi \hat{N}$ for $t = 0$. The graph in Figure 1 can thus be interpreted as an "impulse response function". It documents how the IRR of generations $\ldots, -2, -1, 0, 1, 2, \ldots$ react to an increase by the factor $\chi = 2$ in the size of generation N_0.

Insert Figure 1 about here

Figure 1 shows that the demographic shock hits different generations in a different manner. Some cohorts are granted a IRR that is higher than the steady state value while others are affected in a negative manner.

In order to understand the intuition behind the pattern of δ_t one can start with the generations that are born before $t = -60$. When the large cohort enters the labor market

[7] For details about this and for further variants of PAYG systems (besides the defined contribution scheme used in the present paper) see Knell (2010*a*).

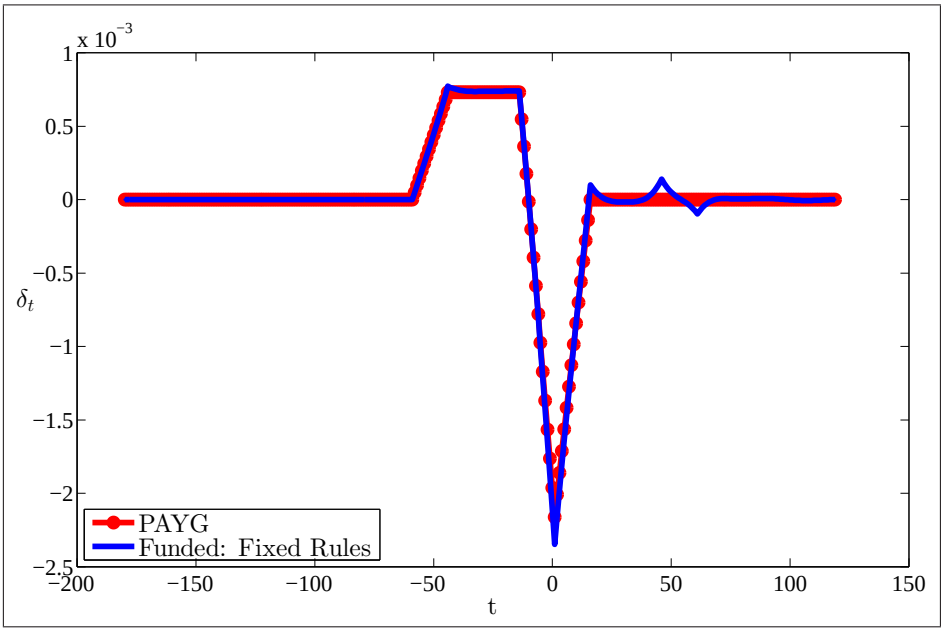


Figure 1: The IRR for a PAYG system and for a funded system where $a_2 = 0$ and where in both cases the contribution rate is fixed at $\hat{\tau} = 1/4$. Life expectancy is assumed to be $Y = 60$, workers retire at age $X = 45$ and the demographic development is given by $N_t = 1$ for $\forall t \neq 0$ and $N_t = 2$ for $t = 0$ (i.e. $\chi = 2$). For the funded system the fixed annuitization rule is given by $\mu = 1$.

these generations are already dead and they are thus unaffected and get the steady state IRR of $\delta_{ss} = 0$. For generation -59 this is, however, different since it has one year of retirement (at the age of 79) where its pension level is larger than normal ($\frac{P_0}{W_0} = 0.77$ instead of $\frac{P_{ss}}{W_{ss}} = 0.75$). This is reflected in the slightly higher IRR for this generation. The more years a generation spends in retirement while the large cohort is working the higher its IRR will rise. The generations between -45 and -15 are particularly favored since they receive the higher pension level for the whole period of their retirement. Starting with generation -14, however, this advantage is slowly turned upside down since these cohorts share some retirement years with the large cohort which depresses its pension level ($\frac{P_r}{W_r} = 0.7$ for $45 \leq r \leq 59$). The most "unfortunate" generation is the large cohort itself since it receives the low pension level for the entire duration of its retirement. This is reflected in the fact that δ_0 is the lowest value among all internal rates of return. Also the "neighboring" cohorts, however, are quite disadvantaged since they overlap for almost their total retirement span with the baby-boom generation. The IRR only returns to normal when a generation does not share any years in pension with this large cohort. The first generation for which this is the case is generation $+15$. This result is parallel for the one derived in a two-period framework in Bohn (2001).

3.2 The funded system (benchmark case): Fixed savings rules and investments in gold

In order to offer a systematic comparisons of the properties of the unfunded system with the ones of its funded pendants I use various environments that differ with respect to the type of assets that are available and with respect to the strategies that guide peoples' behavior. In this section I start with the benchmark case where the factor in limited supply plays no role in the production process ("gold") and where the contribution rate to the funded pillar and the rules of annuitization are fixed by the pension system. After having presented the results of this benchmark model I will then step by step lift the various assumptions in order to see how each of them affects the outcome.

Table 1 documents in which section (and in which figure) the various types of funded systems (that differ with respect to their investment assets and their investment behavior) are discussed.

Table 1: Different types of funded systems are discussed in the following sections

	Asset		
Investment Decision	Gold	Land	Capital
Fixed	3.2 (Fig. 1)	4.1 (Fig. 3)	4.3 (Fig. 5)
Optimal	4.2 (Fig. 4)	Not shown (see FN 12)	4.3 (Fig. 5)

3.2.1 Assumptions

It is assumed that the fixed factor is useless for production, i.e. $a_2 = 0$ and therefore also $r^H_{t+a-1} = 0$. Furthermore, the contribution rate of the funded system is assumed to be the same as in the PAYG system, i.e. $\tau_t = \hat{\tau}, \forall t$.[8] Consumption (cf. (8)) is thus given by:

$$C_{a,t+a-1} = (1-\hat{\tau})W_{t+a-1} \qquad \text{for } 1 \leq a \leq X. \tag{15}$$

The contributions to the pension fund buy $\frac{\hat{\tau}W_{t+a-1}}{P^H_{t+a-1}}$ units of gold. At the end of the working life the total stock of gold is given by $H_{X,t+X-1} = \hat{\tau}\sum_{a=1}^{X}\frac{W_{t+a-1}}{P^H_{t+a-1}}$. The level of consumption in old age is determined by the rules that govern the annuitization of this total pension capital. I assume that the disinvestment of the remaining capital stock $DI_{a,t+a-1}$ is given by:

$$\begin{aligned} DI_{a,t+a-1} &= \frac{\mu^{Y-a}}{Y-a+1}H_{a-1,t+a-1} \qquad \text{for } X+1 \leq a \leq Y, \\ C_{a,t+a-1} &= P^H_{t+a-1}DI_{a,t+a-1} \qquad \text{for } X+1 \leq a \leq Y. \end{aligned} \tag{16}$$

For $\mu = 1$ the formulation in (16) means that the pension system decumulates the capital stock in a linear fashion such that in each retirement period the capital stock is spread evenly over the remaining years. If $\mu > 1$ the disinvestment is "front-loaded" while for $\mu < 1$ it is "back-loaded". The annuitization in (16) guarantees that the capital stock is

[8]The fixed contribution rate $\hat{\tau}$ must not necessarily be related to the stipulations of a central pension fund but can also be interpreted as a short-cut for less-than-fully-rational behavior. In the presence of risk and uncertainty on the one hand and myopia and procrastination on the other hand, a fixed saving rule has often been suggested as an optimal strategy ("nudge"). The constant contribution rate $\hat{\tau}$ can thus also be understood as such a fixed commitment device.

completely depleted (i.e. $H_{Y,t+Y-1} = 0$).

3.2.2 Results

The asset price P_t^H follows from equalizing asset supply and asset demand, i.e.:

$$\bar{H} = \sum_{a=1}^{Y} H_{a,t+a-1} N_{t-a+1}. \tag{17}$$

In the demographic steady state where $N_t = \hat{N}, \forall t$ the normalized gold price $\widetilde{P}_{ss}^H = \frac{P_t^H}{A_t}$ is constant. For the case $\mu = 1$ one can derive (see appendix) that:

$$\widetilde{P}_{ss}^H = \frac{\hat{\tau} X Y \hat{N}}{2\bar{H}}. \tag{18}$$

As one would have expected, the equilibrium price of the asset in fixed supply increases with the size of the steady-state cohort $\hat{N}$, life expectancy Y, the retirement age X and the contribution rate $\hat{\tau}$ and it decreases with the amount of the available stock $\bar{H}$.

For the case of fluctuating cohort sizes one can use (17) to derive the time series of the asset price. Figure 2 shows the equilibrium path of $\widetilde{P}_t^H$ for the same numerical example that has been discussed above for the PAYG system (see Figure 1).[9]

`Insert Figure 2 about here`

These solutions of $\widetilde{P}_t^H$ can then be used in the equations for consumption during work and retirement to calculate the pattern of the internal rate of return δ_t for different generations. This is shown in Figure 1 for an annuitization rule of $\mu = 1$.[10]

Figure 1 contains a number of interesting results. The most striking observation is that the pattern of the generation-specific internal rates of return δ_t of the funded system is basically identical to the one of the PAYG system. The reason for this is as follows. At the time when the large cohort enters the labor market ($t = 0$) the price of the asset jumps upwards by about 2% since there is now a larger group of workers that want to save and thus a large volume of funds chases the given amount of available assets that are sold due to the fixed annuitization rules. This is clearly visible in Figure 2 (where the red line shows the normalized price $\widetilde{P}_t^H$ for the assumption $a_2 = 0$ and fixed savings rules).

[9] The figure also contains the asset price curves for other assumptions concerning asset availability and saving behavior that will be discussed in later sections.

[10] Different annuitization rules lead to almost identical results (not shwon).

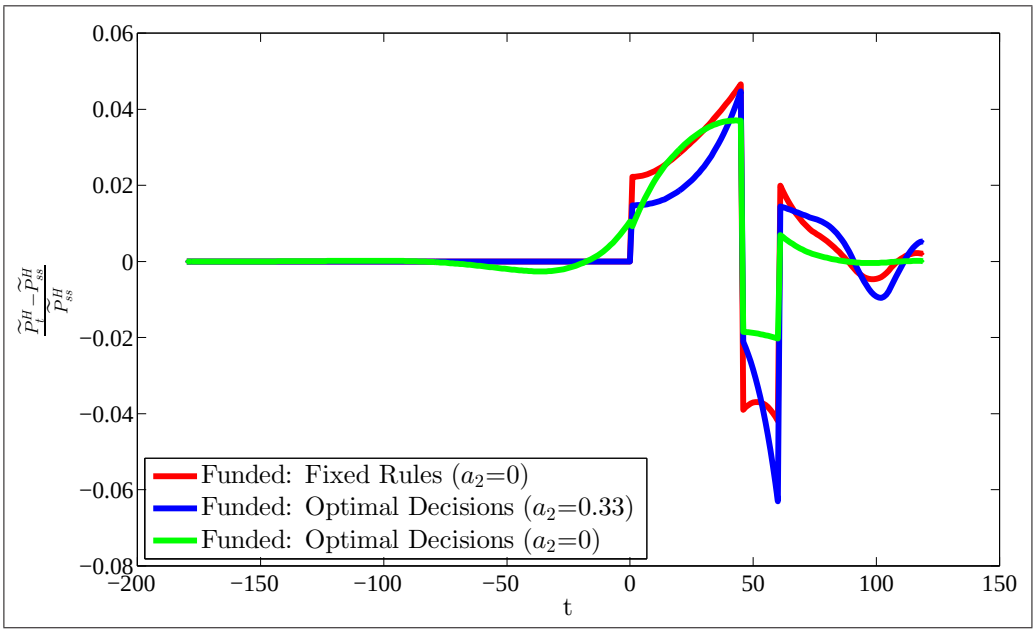


Figure 2: The figure shows the percentage deviation of $\widetilde{P}_t^H$ from its steady state value under various assumptions concerning asset availability and investment behavior. The first case ($a_2 = 0$ and fixed decisions) is discussed in this section, the second ($a_2 = 1/3$ and fixed decisions) in section 4.1 and the case with optimal decision rules in section 4.2. The rest of the parameter values are: $Y = 60$, $X = 45$, $g = 0$, $\chi = 2$.

This price increase evidently has a positive effect on the rate of return of the persons who are already retired. Even the pensioner that is in the last year of his life (i.e. generation -59) gets a small increase in its IRR. In the next period ($t = 1$) the stock of assets in the pension fund is smaller than in the period before. The reason for this is that the cohort that has just retired (i.e. generation -44) has already faced one period of its working life (the previous period $t = 0$) where the asset price $\widetilde{P}_t^H$ has been higher and where the fixed volume of savings $\hat{\tau}W_0$ has bought less pieces of gold than in the steady state constellation. Given the annuitization rule (16) of the pension fund this means that the first pension installment of generation -44 is lower and thus the total supply of assets is also lower. The total demand for assets, however, is as large as in period $t = 0$ since the labor force has the same size as before ($L_0 = L_1 = 44 \times \hat{N} + 1 \times 2\hat{N}$) and there is a fixed contribution rate. This disparity of asset supply and asset demand further drives up the asset price (see Figure 2) which itself increases the mismatch in the following period $t = 2$ and so on up to period $t = 45$. All generations that are lucky enough to be in retirement or enter retirement in these periods of price hikes benefit from these developments and face a higher IRR. Most privileged are the cohorts -45 to -15, each of which has faced full 15 periods of retirement with asset prices that have been higher than normal. These generations differ in the number of working periods with inflated asset prices and also in the exact magnitude of these elevated prices. It is all the more astonishing that the resulting internal rates of return are very similar and there is only a slight decrease from generation -45 (for which $\delta_{-45} = 0.000772$) to generation -15 (where $\delta_{-15} = 0.000741$).

In period $t = 45$ the large cohort N_0 retires and this leads to a collapse in the asset price (by 8%). This "asset meltdown" is due to the fact that the demand for capital (by the remaining workers) is back to normal while the asset supply is hugely increased by the large cohort of new pensioners. The IRR of generation -14 shrinks by quite a bit since its last period of retirement coincides with the period of this price decrease. In the years that follow the asset price shows a little hump shape. This follows from the interplay of two effects. First, the cohorts that die have less and less capital (since they have faced steadily increasing asset prices when accumulating). This alone would increase asset prices. On the other hand, when cohort $+1$ retires it had at least one working period with low asset prices (namely period $t = 45$) in which it could accumulate a higher pension capital. These effects interact and result in the hump-shaped pattern.

When the large cohort N_0 dies in period $t = 60$ the asset price jumps up again. This is due to the fact that the cohorts that remain in retirement had to pay for the larger part of their working lives higher prices for their assets and have thus accumulated a

smaller stock of capital. These generations, however, also die away and with this process also the asset price decreases until period $t = 98$. The influence of asset prices during accumulation (lower prices = a larger stock of assets) and during decumulation (lower prices = a lower return) creates a number of "echo effects" of this one-period shock that are reflected both in the path of the asset price P_t^H and in the pattern of δ_t. In fact, these variables will oscillate around the steady state without reaching it again in finite time. Note that in the case of the PAYG system these oscillation do not occur and the steady state is again attained in period $t = 15$.

Despite the slight differences in the pattern of the internal rates of return it is still surprising how close the two lines in Figure 1 are even though they are based on completely different calculations. One pattern follows from the stipulations of the PAYG scheme and the other from the ups and downs of asset prices. The logic behind both patterns is, however, basically the same. The generations -60 to -15 are the clear "winners" of the baby-boom cohort N_0. They receive higher pensions in the unfunded and in the funded system where in the first case this is due to the advantageous fluctuations in the dependency ratio $\frac{R_t}{L_t}$ and in the second case due to equally advantageous fluctuations in the asset price P_t^H. Both systems are also characterized by the fact that the boom generation itself is the "loser" that shows the lowest IRR among all generations ($\delta_0 = -0.0024$). The doubling of the own cohort size thus reduces the IRR by about a quarter of a percentage point.[11]

Summing up, the results of Figure 1 demonstrate that the often-heard claim that a PAYG system is more receptive and vulnerable to demographic fluctuations and that a funded system will necessarily dampen or even counteract these shocks (eg. Modigliani et al. 2000, Börsch-Supan & Gasche 2010) is not correct. Under the assumptions used in this section the difference between funding and non-funding basically vanishes. In the next section I will analyze how sensitive the basic result about the equivalence of unfunded and funded systems reacts to changes in the underlying assumptions.

[11] Another way to make sense of the results in Figure 1 is to recognize that for $a_2 = 0$ the non-produced factor ("gold") could also be viewed as a perpetual bond (cf. Geanakoplos et al. 1999, Lindbeck & Persson 2003).

4 Extensions

4.1 A funded system with fixed rules and investments in land

As a first case I look at the situation where the asset in fixed supply is not only used as a store of value but also plays some role in the production process (i.e. $a_2 > 0$). As a consequence, its rate of return will not only depend on the development of the asset price P_t^H but also on the marginal productivity r_t^H (see (8)). It is best to think of $\bar{H}$ in this case as a fixed stock of land that is needed to build factories, to accommodate offices and workers etc.

The basic difference to the case with $a_2 = 0$ lies in the fact that now a demographic shock will also change the wage rate (see (5)) while before the wage has been fixed at $W_t = A_t$. With substitutable factors of production an increase in the size of one cohort will also increase the size of the labor force thereby lowering its marginal product and thus the wage rate. On the other hand, such a development will make the second factor of production relatively more scarce which will increase its value. In the appenidx I show how the main equations have to be changed in order to accomodate this case.

The change in factor prices will change the pattern of δ_t for both the funded and the unfunded system. The impact on the latter might at first sound surprising. It follows, however, from the simple fact that now the development of W_t is affected by the change in cohort size while for $a_2 = 0$ the wage rate had been independent of demographics. This is illustrated in Figure 3. The main difference is that for $a_2 = \frac{1}{3}$ the IRR increases for the generations between -60 and -15 (while for $a_2 = 0$ it is flat for generations -45 to -15) and it "overshoots" for generations $+15$ to $+45$. The reason for this is that the boost in the pension level of the earlier generations (the ones born between -60 and -15) is weaker than for $a_2 = 0$ due to the fact that the wages of the working population decrease during the years when generation 0 is part of the labor force. For later generations (the ones born after $t = 15$) the development of the wage rate has a positive impact since their pension level is based on the wage that has returned to the (higher) normal level while their contributions had been a constant fraction of the lower wage level.

`Insert Figure 3 about here`

The results for the funded system are also shown in Figure 3. One observes that the path of IRR for the funded system again follows the pattern of the PAYG system although the differences are now slightly larger than for $a_2 = 0$. Nevertheless, for all practical

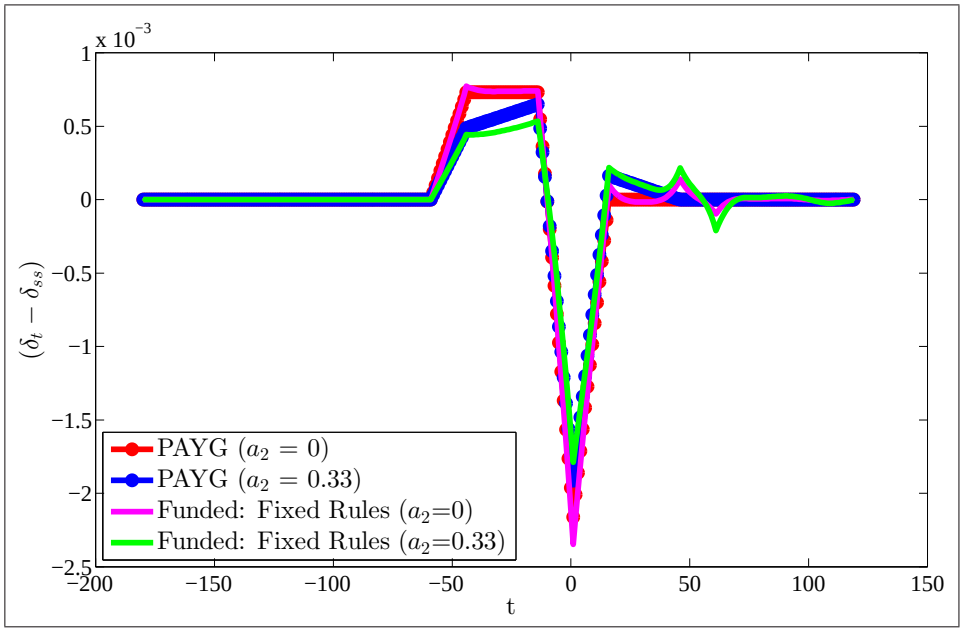


Figure 3: The figure compares the development of the IRR for funded systems where either $a_2 = 1/3$ or $a_2 = 0$ and where the pension fund has a fixed contribution rate $\hat{\tau} = 1/4$ and a fixed annuitization rule with $\mu = 1$. Since the steady state values for the IRR depend on a_2 the lines show $\delta_t - \delta_{ss}$, where $\delta_{ss} = 0$ (for $a_2 = 0$) and $\delta_{ss} = 0.034$ (for $a_2 = 1/3$). The other parameter values are the same as in Figure 1.

purposes the pattern of the two curves is so close that the differences between the two systems can be regarded as negligible. The mechanism behind the pattern of δ_t for the funded system is that the change in W_t has a negative impact on earlier generations (they have less to invest) and a positive impact on later generations (they face a more "solvent" group of buyers). The oscillations that are visible in Figure 3 are again the consequence of counteracting price effects and their echoes on the accumulation and decumulation of funds. The rest of the intuition is the same as for the case of gold ($a_2 = 0$).

Summing up, the differences between the case where workers invest in an unproductive (gold, $a_2 = 0$) or productive (land, $a_2 = \frac{1}{3}$) asset are qualitatively similar. Although the precise intergenerational pattern of the IRR depends on the role the fixed factor plays in production, the differences to the unfunded pension system are in both cases small. Again, one cannot conclude that funding leads to "smoother" reactions to demographic changes than PAYG pension systems. If anything, the patterns in Figure 3 suggest that the fluctuations of the funded system are somewhat *larger* than for the unfunded one.

4.2 A funded system with optimal investment behavior

So far I have assumed that the contributions to and dissipation of the pension capital follow fixed rules associated with the contribution rate $\hat{\tau}$ (for accumulation) and the annuitization rate μ (for decumulation). As I have argued above this can be due to the institutionally determined stipulations of a funded pension system or it might be a short-cut that captures the rules-of-thumb used by feeble and less-than-fully-rational individuals that try to commit themselves. In this section I look at the case where individuals choose their consumption plans in an optimal way by maximizing their intertemporal utility function (7).

The only difference to the cases discussed in sections 3.2 and 4.1 is that the consumption levels are not longer given by (15) and (16) but rather by the Euler equation:

$$C_{a+1,t+a} = \left(\beta \frac{P^H_{t+a} + r^H_{t+a}}{P^H_{t+a-1}}\right)^{\frac{1}{\rho}} C_{a,t+a-1}, \tag{19}$$

where $\frac{P^H_{t+a}+r^H_{t+a}}{P^H_{t+a-1}}$ can be regarded as the total rate of return when one unit of the asset is held from period $t+a-1$ to period $t+a$. In addition, it is still true that the individual does not own anything when he or she is born ($H_{0,t-1} = 0$) and that at the end of life — due to the absence of any bequest motive — the stock of assets will be completely

depleted. The rest of the equations (especially (4), (5) and (17)) are the same as before.

In Figure 4 I show the pattern of δ_t for the assumption of optimizing behavior for $a_2 = 0$. For the preference parameters I use $\rho = 2.5$ and $\beta = 0.97$. These magnitudes are in line with assumptions in the related literature (de Menil et al. 2006, Kulish et al. 2010).

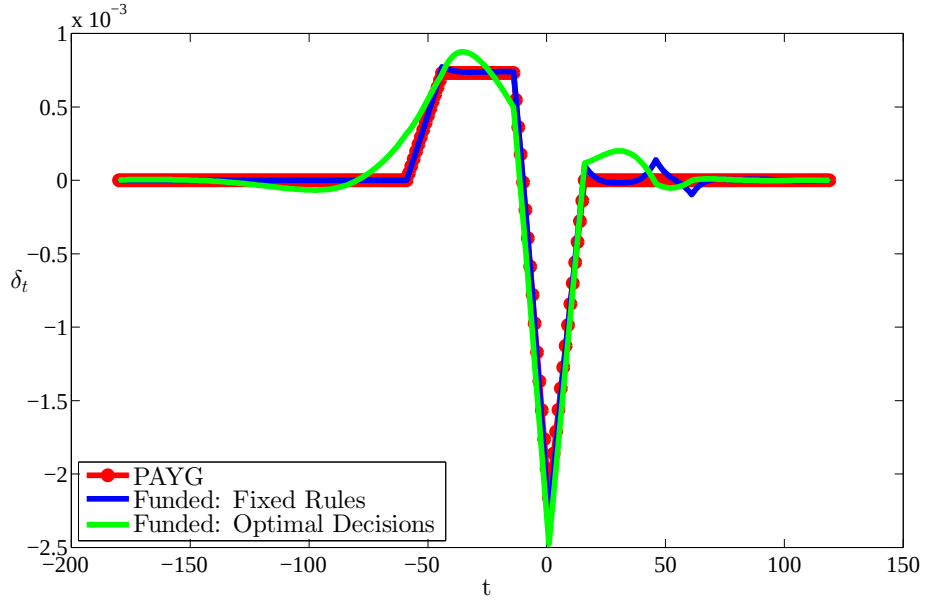
Insert Figure 4 about here

The first thing to note by looking at Figure 4a is that optimizing behavior by the agents has no effect on the qualitative nature of how the demographic shock is distributed among the different generations. The IRR is highest for the generations ranging from about -45 to -15 and lowest for the "shock generation" 0 and the generations in its temporal vicinity. There are, however, also a number of differences.

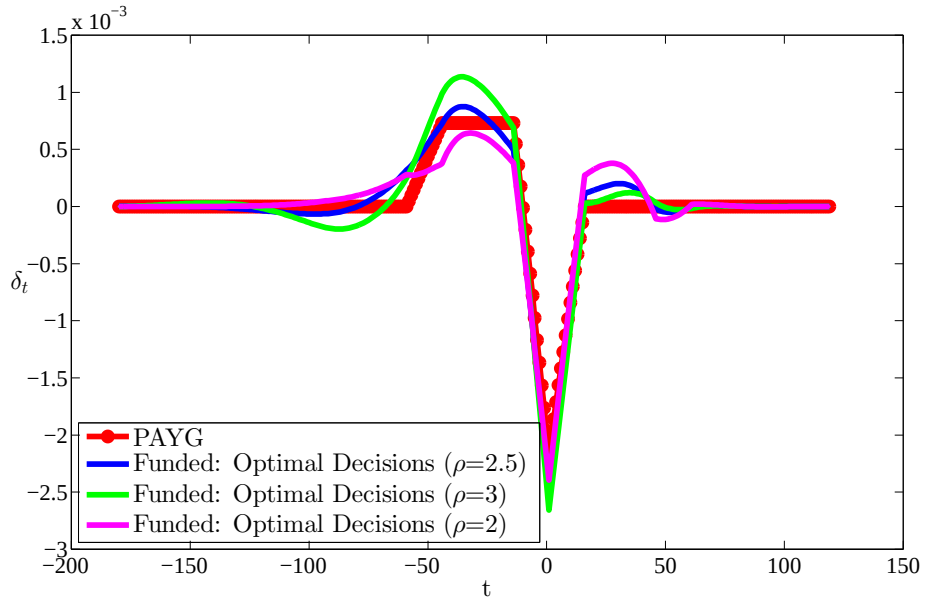
First, the forward-looking optimal behavior has the effect that even generations that do not share a single common period with the large cohort are affected by the demographic shock since they adjust their behavior in anticipation of the expected changes in the asset price. For the example in Figure 4a the early-born generations will decrease their asset demand in reaction to the price hike that will happen when the large cohort enters the labor market. This early reaction has an effect on the asset price even before period $t = 0$ as shown in Figure 2. These price movements are reflected in the shape of the IRRs. Second, the amplitude of the pattern of δ_t depends on the assumptions about the intertemporal utility function as shown in Figure 4b. For higher values of ρ the IRR of earlier generations are larger. This stems from the fact that with higher ρ people are less willing to substitute consumption between periods and they are thus more exposed to the price changes. On the other hand, a higher willingness to shift consumption between periods means that prices will fluctuate less since the adjustment is more concentrated on changes in asset holdings. A similar, although less pronounced results holds for the rate of time preference β where a higher degree of patience leads to smaller fluctuations in P_t^H and δ_t (not shown). Third, the IRRs for the generations -45 to -15 are no longer flat but rather show a hump-shape, where the highest IRR is for generation -36 ($\rho = 3$) or -32 ($\rho = 2$).[12]

Summing up, the results of this section imply that the comparison between funded and unfunded systems in the presence of optimizing individual behavior is sensitive to the choice of the preference parameters and to the importance of the factor in the production

[12]For the case where the non-accumulated factor is used for production ($a_2 = \frac{1}{3}$) the results are qualitatively the same as in Figure 4 (not shown). The swings in δ_t are, however, larger than for the case with $a_2 = 0$.



(a) Optimal decisions vs. fixed rules



(b) Different values for ρ

Figure 4: The figures show the IRR for a funded system where $a_2 = 0$ and where investment behavior is determined by utility maximization with $\rho = 2.5$ and $\beta = 0.97$. In panel (a) this is compared to the case of fixed investment rules (for $\hat{\tau} = 1/4$ and $\mu = 1$) and the case of a PAYG system. Panel (b) shows the results for two alternative values of ρ. The other parameter values are the same as in Figure 1.

function. The patterns of the fluctuations in the IRR after the demographic shock are, however, similar (both in shape and in magnitude) to the ones with fixed investment rules. Only for quite risk-averse individuals the fluctuations might become smaller than for the case of a PAYG system.

4.3 A funded system and investments in capital

So far I have assumed that in society there are two types of classes: an "upper class" of superrich people that holds the entire stock of real, accumulable capital K_{t-1} and a "working class" that only transfers resources between different periods of life by buying and selling units of the non-accumulable stock but that — as a whole — does not accumulate any wealth besides $\bar{H}$. In this section I study how the results change when using the standard framework of homogeneous households.

In particular, I now assume that there is just one class of people, that real capital is the only available asset to absorb their savings and that their investments are guided by optimizing behavior. This closely corresponds to the typical one-good benchmark economy that is used in the majority of the related literature.[13]

Under these assumptions the relative price of capital is fixed at 1 and the period budget constraint (8) can thus be easily transformed to the present case by substituting "K" for "H" and by noting that $P_t^K = 1$. Details can be found in the appendix. The rest of the model is the same as in the previous section. I will again deal with the cases where there are fixed savings rules (described by the parameters $\hat{\tau}$ and μ) and where saving is chosen in an optimal manner.

I have used the same parameterization as before to solve the model numerically and to calculate from this the path of the IRR for different generations. The results are shown in Figure 5 together with the curves for the case of two PAYG systems.[14] The first PAYG system is the benchmark case of a defined contribution scheme where the contribution rate is fixed and the pension payments are adjusted. For the alternative PAYG scheme I assume, on the other hand, that both changes in the contribution rate and in the pension level are used in order to balance the budget. In Knell (2010*a*) I discuss this issue in more

[13] Alternatively, one could also look at a set-up where in addition to investment opportunities in real capital there also exists a PAYG pillar and/or a second non-produced asset (land). The latter case has, e.g., been studied by Imrohoroglu et al. (1999).

[14] In order to make these cases comparable I assume that in both cases with funding the coefficient of the asset in the production function is one third and that in the case of the PAYG system the stock of assets ($\bar{H}$ or K) is constant.

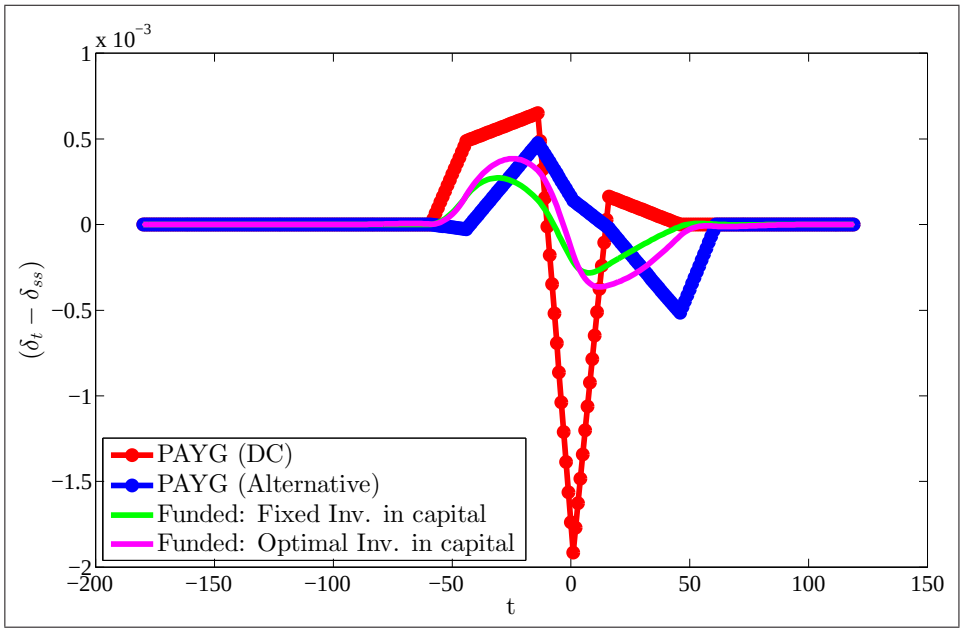


Figure 5: The figures report the development of the IRR for a funded system where the funds are invested into real capital with $a_1 = 1/3$ (and $a_2 = 0$). For reasons of comparisons the figures also contain the lines for the benchmark PAYG system (defined contribution) and for an alternative PAYG system (where both the contribution rate and the pension level are changed in proportion 30:70, respectively). One curve shows the case with fixed investment rules (with $\hat{\tau} = 1/4$ and $\mu = 1$) while the other the situation where investment behavior is determined by utility maximization with $\rho = 2.5$ and $\beta = 0.97$. The depreciation rate is set equal to $d = 0.05$ and the other parameter values are the same as in Figure 1.

details and I show that the specific variant used in Figure 5 is the one that is associated with the minimal intergenerational fluctuations after a demographic shock.

`Insert Figure 5 about here`

The pattern for the IRR is considerably smoother in the case where the pension fund invests in accumulable capital rather than in non-accumulable land. The reason for this outcome is that by accumulating and decumulating capital the pension fund (and this means the economy at large) can dampen the fluctuation in factor prices that would occur otherwise and can thus also mitigate the differential impact of the demographic shock on different generations.

The pattern of IRR for the case with optimizing savings behavior is qualitatively similar to the pattern with fixed decision rules. Optimizing behavior does seem to be associated with larger swings in δ_t, a phenomenon that could already be observed in

section 4.2. This result, however, depends again on the degree of relative risk aversion. Smaller values of ρ will lead to smaller fluctuations (not shown).

The availability of real capital thus confirms the claim that funding leads to a smoother adjustment process. Investments in capital and the possibility to accumulate help to dampen the ups and downs of asset prices, rates of returns and the associated IRRs. This is clearly visible in Figure 5 where, e.g., the drop in the IRR of generation 0 is much less pronounced than before. At the same time, however, the comparison with the previous sections show that this result depends crucially on the assumption that all savings are channeled into a productive and accumulable capital stock and not into a non-accumulated asset.

As an aside, I want to note, however, that there exist other variants of PAYG systems that also lead to smaller fluctuations than a DC scheme. The alternative PAYG scheme shown in Figure 5, e.g., that uses changes in the contribution rate and in the pension level (in a mixture of 70% and 30%, respectively) to balance the budget also reduces the intergenerational differences in δ_t for a PAYG system. For this case the differences in the smoothness of adjustment between funded and unfunded systems look much smaller.

5 Conclusion

Advocates of the funded system often claim that even a well-designed PAYG system will necessarily be less able to deal with demographic shocks than a funded system since it will cause large intergenerational differences in the mix of contributions and pension payments and in the internal rate of return and it will thus lead to a less equitable outcome. In this paper I have shown that this assertion is not generally true in a model that allows for heterogeneous individuals, for different types of assets and different determinants of savings plans. To the best of my knowledge, the interplay between these factors has so far not been studied systematically in the related literature.

The findings of this paper suggest that the intergenerational pattern of δ_t in funded systems and its difference to unfunded systems crucially depends on a number of factors. In particular: the social structure of society ("representative generations" vs. a "dichotomous structure"); the types of asset in which people invest (accumulable vs. non-accumulable assets); the choice of the intertemporal consumption plan (optimal behavior vs. fixed rules); and the assumptions about individuals' preferences (risk aversion, time preference, planning horizon).

In this paper I have mostly used different combinations of these extreme cases in order to study and describe the main mechanisms at work. In reality one typically observes less clear-cut situations. For example, different socioeconomics groups typically hold both types of assets, although in very different compositions. For future work it would be promising to develop a calibrated version of the model that allows for these mixed cases and that could be used to derive an quantitative assessment of the likely impact of demographic changes (e.g. on the real interest rate).

Moving one step further, there exists of course an even larger list of factors and assumptions that have to be taken into account when talking about the likely effects of a funded system on the smoothness of adjustment to demographic shocks. For example, empirical studies show that savings decisions do not follow the pattern implied by the life-cycle hypothesis. They are, e.g., also influenced by precautionary behavior and by bequest motives. Furthermore, the retirement age is not necessarily a given parameter but is often also chosen in an optimal manner. Finally, even in the case where individuals' savings are invested into real capital one has to think about the kind of capital in which this is done. In the simple deterministic one-good model the investment decision is a rather mechanistic process. Accumulation is here instantaneous and the success immediate. In reality, however, there is a large amount of uncertainty involved. Apparently reasonable capital investments could turn out to be useless just a couple of years later. The generations that have invested into these dubious capital goods will face a detrimental shock to their IRR. The study of these issues is also left for future research.

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Appendices

A Details on section 3

A.1 Section 3.2

For $\mu = 1$ (17) reduces to:

$$\bar{H} = \hat{N}\left(\sum_{a=1}^{X}\hat{\tau}\sum_{s=1}^{a}\frac{1}{\widetilde{P}_{ss}^{H}} + \sum_{a=X+1}^{Y}\hat{\tau}\frac{Y-a}{Y-X}\sum_{s=1}^{X}\frac{1}{\widetilde{P}_{ss}^{H}}\right).$$

This can be solved to give equation (18).

Fluctuating cohort sizes lead to deviations of $\widetilde{P}_t^H$ from $\widetilde{P}_{ss}^H$. As an example, say that the system is in the steady state for $t \neq 0$ with $N_t = \hat{N}$ but that in $t = 0$ the cohort size changes to $N_0 = \chi\hat{N}$. Before this shock period the asset price has been given by $\widetilde{P}_{ss}^H$ (see (18)). The new equilibrium price in period $t = 0$ can now be calculated from:

$$\bar{H} = \frac{\hat{\tau}}{\widetilde{P}_0^H}(\chi + X - 1) + \hat{N}\sum_{a=2}^{X}\sum_{s=1}^{a-1}\frac{\hat{\tau}}{\widetilde{P}_{ss}^H} + \hat{N}\sum_{a=X+1}^{Y}\frac{Y-a}{Y-X}\sum_{s=1}^{X}\frac{\hat{\tau}}{\widetilde{P}_{ss}^H}.$$

The asset price comes out as $\widetilde{P}_0^H = \widetilde{P}_{ss}^H\frac{X+\chi-1}{X}$. A positive demographic shock ($\chi > 1$) will thus push up the asset price since there are now more people around who want to invest. The following development of the asset price can be calculated in a recursive manner although the analytical expressions get more complicated. This is shown in the main text in figure 2.

A.2 Section 4.1

The central equations of the funded system are basically the same as before. The only difference is that in the budget constraint (8) now $r_t^H \neq 0$ and the annuitization equation (16) has to be adapted to:

$$DI_{a,t+a-1} = \frac{\mu^{Y-a}}{Y-a+1}H_{a-1,t+a-1}\left(1+\frac{r_{t+a-1}^H}{P_{t+a-1}^H}\right) \qquad \text{for } X+1 \leq a \leq Y,$$
$$C_{a,t+a-1} = P_{t+a-1}^H DI_{a,t+a-1} \qquad \text{for } X+1 \leq a \leq Y. \qquad (20)$$

The contribution rate is again assumed to be $\hat{\tau}$ and thus the equation (15) that determines the consumption of young workers is also the same as before. The path of the asset price is still described by equation (17), i.e. by the value P_t^H that equates asset demand and asset supply. The factor price r_t^H, on the other hand, is given by (4).

A.3 Section 4.3

The period budget constrant is now given by:

$$K_{a,t+a-1} = I_a^W W_{t+a-1} + K_{a-1,t+a-2} R_{t+a-1}^K - C_{a,t+a-1}, \tag{21}$$

where $R_t^K = 1 + r_t^K - d$ is the rate of return, d is the depreciation rate and r_t^K is the marginal product of capital given by (3). $K_{a,t+a-1}$ are the units of capital owned by generation t at age a. As before I use the end-of-period notation for capital and it holds that workers enter and leave their lives without any capital holdings (i.e. $K_{0,t-1} = K_{Y,t+Y-1} = 0$). The aggregate capital stock is given by:

$$K_{t-1} = \sum_{a=1}^{Y} K_{a,t-1} N_{t-a+1}. \tag{22}$$

As far as the case with fixed decisions is concerned, consumption while young and old is given by expressions similar to (15) and (20). In particular: $C_{a,t+a-1} = (1 - \hat{\tau}) W_{t+a-1}$ for $1 \leq a \leq X$ and $C_{a,t+a-1} = \frac{\mu^{Y-a}}{Y-a+1} K_{a-1,t+a-1} R_{t+a-1}^K$ for $X + 1 \leq a \leq Y$.

As far as optimal behavior is concerned the Euler equation (19) has to be slightly adapted to: $C_{a+1,t+a} = \left(\beta R_{t+a-1}^K\right)^{\frac{1}{\rho}} C_{a,t+a-1}$. The rest of the model is again the same as in section 4.2.