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Disentangling two causes of biased probability judgment: Cognitive skills and perception of randomness

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Kai Duttle

Disentangling Two Causes of Biased Probability Judgment – Cognitive Skills and Perception of Randomness

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Kai Duttle[1]

Disentangling Two Causes of Biased Probability Judgment – Cognitive Skills and Perception of Randomness

Abstract

This experimental study investigates the interaction of two influential factors of biased probability judgments. Results provide new insights on the preconditions for an application of either the gambler's fallacy or its exact opponent, the hot hand fallacy. The first factor is cognitive ability, measured in a cognitive reflection test. The second one is the level of perceived randomness in the observed outcomes. Probability judgments are found to vary significantly across salience of randomness treatments as well as across subgroups with high or low cognitive abilities. Like in previous research, subjects with higher cognitive skills are more likely to engage the gambler's fallacy, yet only if perception of sequential randomness is low. In a setting where randomness is very salient the exact opposite can be observed. Similarly surprising insights are revealed when controlling for cognitive abilities in the analysis of salience treatments. Past results are only confirmed for a subgroup with lower cognitive skills, while their peers' beliefs are completely opposite.

JEL Classification: C91, D84, J24

Keywords: Law of small numbers; gambler's fallacy; hot hand effect; cognitive reflection test

July 2015

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1. Introduction

The representativeness heuristic is one of the best documented behavioral biases in the psychology literature of judgment and decision making. A sizeable number of popular studies in economics and finance build their model assumptions on a representative agent that overinfers the representativeness of a small sample of observations. This is also called the *law of small numbers*. Particularly interesting thereby is biased probability judgment of binary random walk processes which is a unique topic in that there are two well-documented heuristics that hold completely opposite predictions for subjects' probability assessments. The gambler's fallacy predicts that agents believe in mean reversion of observed sequences over time. On the other hand, the hot hand fallacy assumes agents to believe in the continuation of streaks.

It is still not clear what triggers one or the other fallacy in subjects' probability judgments, yet there are several explanatory attempts in the literature. Burns and Corpus (2004) for instance find support for the gambler's fallacy when participants' perception of randomness in the sequence generating process is high, and for lower perception of randomness they find a dominant hot hand effect. Subjects in a study by Huber et al. (2010) display the gambler's fallacy in their own outcome predictions of coin flips, but rather rely on randomized "experts" who were successful in the past which is in line with a hot hand bias. Ayton and Fischer (2004) propose that the hot hand fallacy arises from the experience of positive serial correlations in human action sequences, while the gambler's fallacy arises from the experience of negative recency in sequences of natural events. An experimental study by Asparouhova et al. (2009) finds support for the Rabin (2002) model that predicts agents to expect short sequence-ending streaks to reverse (gambler's fallacy) but longer streaks to continue (hot hand fallacy). An increase in perceived randomness increases expectations of reversal to the mean of subjects in this study as well. Dohmen et al. (2009) explore the relationship of education and biased probability judgments, and also control for cognitive abilities. They find that subjects with higher education and better cognitive skills are more prone to the gambler's fallacy, while the hot hand fallacy is predominant in a group with lower cognitive abilities and less than 10 years of schooling. An experimental study by Xue et al. (2012) also suggests that subjects with higher cognitive abilities are more likely to engage the gambler's fallacy.

The current study takes up on those previous findings and aims to disentangle the effects of cognitive abilities and perception of sequential randomness on biased probability

judgments. For that purpose a cognitive reflection test controls for subjects' cognitive abilities, while the salience of randomness in the observed sequences is altered between treatments.

2. Experimental Design

A total of 66 subjects (26 males and 40 females) were recruited with ORSEE (online recruitment system for economic experiments; Greiner, 2004) among students of the German University Duisburg-Essen. The experiment was programmed and conducted with z-Tree (Fischbacher, 2007) at the Essen Laboratory for Experimental Economics (elfe). Payouts were labeled as "Taler", where 1 Euro equals 50,000 Taler.

During the experiment subjects are subsequently presented with 100 independent sequences of eight changes drawn from a random walk with equal probabilities of positive and negative changes. Four groups of subjects observe those 100 sequences in a different manner. Group 2 observes the same sequences in the same order as group 1, however all changes are upside down. Groups 3 and 4 observe the changes of the first two groups in reverse order.[1] For every individual sequence subjects are asked to make a probability assessment of the next change being positive. These assessments are incentivized utilizing a variant of the quadratic scoring rule from Offermann and Sonnemans (2004).[2] In providing his probability assessment p of the next change being positive (in %) a representative subject is incentivized to disclose his true and honest estimate for the respective sequence by the following payout function. Payout (in Taler) equals $[10{,}000 - p^2]$ if the subsequent change is negative; payout equals $[10{,}000 - (100 - p)^2]$ if the subsequent change is positive. Feedback on earnings from the respective probability assessment is provided at the end of each period.

Two different treatments were conducted. 21 subjects were allocated to the first treatment while 45 subjects participated in the sessions under treatment 2. Instructions in treatment 1 were designed to make the random nature of the sequence generating process salient. They explained that each change is the outcome of a random draw from the computer, identical to the flip of a fair coin. It was clearly stated that statistical models are unable to predict future outcomes from past ones and, on average, there is no upward or downward trend even though random walk sequences almost always contain intervals of recognizable

[1] Sequences and the order in which they were presented to subjects are the same as in Asparouhova et al. (2009).
[2] Although the quadratic scoring rule assumes risk neutrality, deviations [of reported probabilities] from truth telling are not systematic. The scoring rule does not bias the results (Offerman & Sonnemans, 2001, p.10).

patterns. During treatment 2 on the other hand participants were not directly told that the observed changes were generated by a random walk process. Instead, instructions explained that they are presented with sequences of outcome surprises of a specific event, where a positive change indicates that the event turned out better than expected and a negative change indicates that it turned out worse than expected.[3]

Subjects' cognitive skills were measured in a 3-item questionnaire-type cognitive reflection test (CRT) which was completed at the end of the experiment. To each of the three CRT questions there exists an intuitive answer that is incorrect. Therefore the test measures a subject's ability to resist giving the intuitive wrong answer and cognitively reflect on the problem instead. Despite the simplicity of this procedure CRT scores were found to quite accurately predict subjects' performance in substantially more complex intelligence tests. Frederick (2005) reports significant, positive correlations with results from the Wonderlic Personnel Test (a popular 50-item test used to assess intellectual abilities), with the 18-item "need for cognition" scale (which quantitatively measures "the tendency for an individual to engage in and enjoy thinking"), and with the two most common college entrance examinations (the Scholastic Achievement Test and the American College Test).

3. Results

According to the law of small numbers people tend to believe that small samples replicate the probability distribution properties of the whole population and therefore overinfer the outcome of a random process after a small series of observations. This may happen in two different ways. People who exhibit the gambler's fallacy believe that, figuratively speaking, all outcomes are drawn from an urn without replacement. Thus with an increasing number of upward changes they would stepwise adjust their belief towards a higher probability of the next change going in the opposite direction. On the other hand, people who display a hot hand effect in their behavior adjust their prior beliefs about the relative proportions of outcomes in the urn. Thus with a growing number of positive changes their expectations of the next change being positive increases as well.

The following analysis focusses on the number of upwards changes in a sequence (0-8) on subjects' relative expectations of the next change going upwards (0-50%). Two treatments

[3] Asparouhova et al. (2009) use the term "earnings surprise" in this context. As serial auto-correlation in earnings surprises (earnings momentum) is a real world phenomenon, this study uses a more neutral terminology.

alternated the salience of randomness in the sequence generating process. Treatment 1 presented changes as outcomes of coin flips. Treatment 2 labeled changes as outcome surprises of a specific event and thereby concealed the random nature of the underlying process. Figure 1 presents relative probability judgments in relation to the number of like changes for both treatments. All participants demonstrate a strong tendency to predict upwards changes. However, distributions of participants' probability assessments differed significantly among treatments (MWU test, p=0.07). The number of UP changes in a sequence was negatively correlated with subjects' expectations of a subsequent UP move in the coin flip treatment (−0.08, p<0.01), indicating behavior in line with the gambler's fallacy. In contrast, subjects' probability judgments were statistically unbiased in the outcome surprise treatment. These findings are in line with previous literature.

The average cognitive reflection test score in my data is 1.47 which ranks subjects of this study about equally with Frederick's (2005) sample of participants randomly picked among spectators of the July 4th Boston fireworks display. In order to investigate the effects of cognitive abilities on subjects' behavior, data is presented separately for a "low" cognitive skills group and a "high" abilities group in the following analyses. Similar to the practice in earlier studies participants with CRT scores of 0 or 1 are placed in the low CRT group (N=32), while those who correctly answered 2 or 3 questions belong in the high CRT group (N=34). Figure 1 illustrates graphical analyses of judgment biases of both groups in the two treatments. One striking treatment effect immediately catches the eye, which is the completely opposite pattern in relative probability assessments among the low CRT group members. This effect is highly significant (MWU test, p<0.01). In the coin flip treatment with high salience of sequential randomness decision makers exhibit a strong gambler's fallacy (correlation of −0.13, p<0.01), while in the outcome surprise treatment (which conceals the random nature of sequences) low CRT subjects are prone to the hot hand fallacy (correlation of 0.06, p<0.01). High CRT group members on the other hand are statistically unbiased in their probability judgments if salience of sequential randomness is high. In the outcome surprise treatment, in stark contrast to probability judgments by their fellow participants in the low CRT group, high CRT subjects exhibit the gambler's fallacy (correlation of −0.06, p<0.01).

Table 1

Linear regressions on the provided relative probability assessment for an UP move (in %).

Observations	All	CRT score ≤ 1	CRT score > 1
Outcome surprise treatment dummy	−7.50 (0.000)	−17.50 (0.000)	3.56 (0.115)
# UP changes (outcome surprise)	0.28 (0.238)	1.27 (0.002)	−0.55 (0.051)
# UP changes (coin flip)	−1.11 (0.002)	−2.25 (0.000)	0.30 (0.515)
# UP changes (male subjects)	−0.69 (0.000)	−0.73 (0.002)	−0.54 (0.001)
Adj. R^2	0.0071	0.0153	0.0046

Info: p-values in parentheses.

Table 1 presents the results of linear regressions on the relative probability assessment for a subsequent upwards move in the change sequence. The analysis considers all observations and each of the CRT groups individually. On top of the number of UP changes and the treatment effect it also controls for a gender effect which was found in earlier studies of the law of small numbers (Suetens & Tyran, 2012). Regression results strongly confirm what graphical analyses and correlations already suggested. Both the level of salience of sequential randomness and cognitive abilities of agents play crucial roles when it comes to the question under which circumstances either one or the other of the two opposing fallacies predominantly applies to probability judgment.

Table 2 sums up the results in a 2×2 matrix capturing treatments and CRT groups. In terms of a treatment effect, Burns and Corpus (2004) find support for the gambler's fallacy when the salience of randomness in sequences is high, and for a lower level of salience they find a dominant hot hand effect. This pattern is also confirmed in my data, yet it only applies to the subgroup of participants performing low in the cognitive reflection test. High CRT subjects display a reverse behavioral pattern and are more prone to the gambler's fallacy when salience of sequential randomness is low (first contradiction). As for the effect of cognitive abilities on probability forecasts, a number of studies suggest that decision makers with higher cognitive abilities are more likely to engage the gambler's fallacy (Dohmen et al., 2009; Xue et al., 2012). In the current study this correlation only holds when the random nature of the sequence generating process is concealed. In a treatment where sequential randomness is made salient to subjects the exact opposite relation holds, and a subgroup with lower cognitive skills engages the gambler's fallacy much stronger (second contradiction).

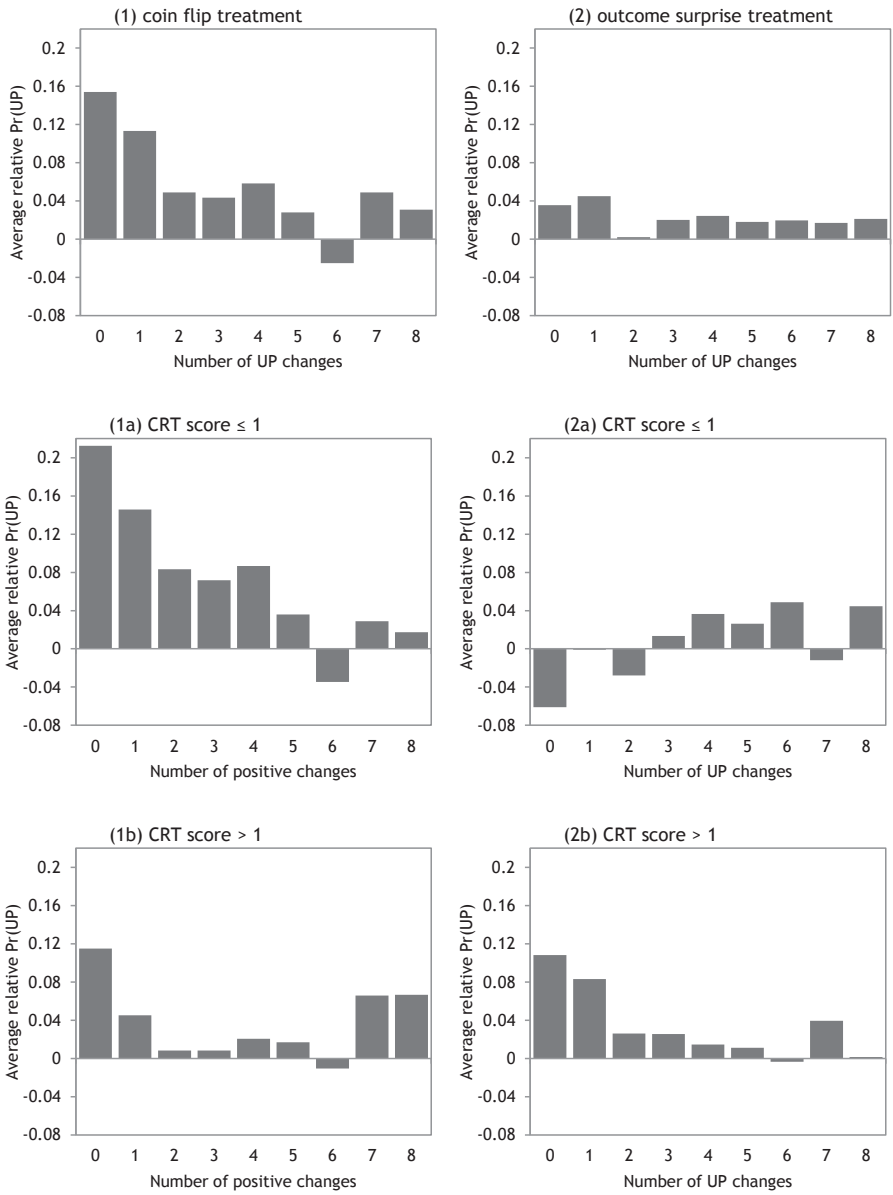
Table 2
Visualization of results.

	Treatment 1 – coin flip	Treatment 2 – outcome surprise	
low CRT	*gambler's fallacy*	*hot hand effect*	in line with Burns and Corpus (2004)
high CRT	*unbiased forecasts*	*gambler's fallacy*	**contradiction no. 2**
	contradiction no. 1	in line with Dohmen et al. (2009); Xue et al. (2012)	

Apart from the definitions used throughout this paper, there are also other approaches to model the hot hand and gambler's fallacies. For instance, one popular model considers the length of a sequence-ending streak (Barberis, Shleifer, & Vishny, 1998). In this case the gambler's fallacy predicts higher expectation of reversal with longer sequence-ending streaks, while under the hot hand fallacy a representative agent's expectation of streak continuation increases with the length of the streak. An analysis of sequence-ending streak length on subjects' expectation of reversal holds exactly the same implications as the results presented here. In treatment 1, subjects with lower cognitive abilities strongly believe in streak reversal and thus exhibit the gambler's fallacy, while probability judgments of subjects with higher abilities are unbiased. In treatment 2, low CRT group members, on average, strongly believe in continuation of streaks while high CRT subjects predominantly expect reversals.

Figure 1

Relative probability assessment–number of like changes graphs.



Info: Figures on the left display observations from the coin flip treatment 1, while those on the right present observations from the outcome surprise treatment 2. In the second row only probability judgments from subjects with a CRT score of 0 or 1 are taken into account, while the lower row figures display probability judgments from participants who answered 2 or 3 CRT questions correctly.

4. Conclusion

This experimental study investigates the interaction of two distinct factors that were found to influence decision makers' probability judgments of random walk processes. The first factor is cognitive ability, measured in a cognitive reflection test. The second one is the level of perceived randomness in the observed sequences. While one treatment deliberately concealed the random nature of the sequence generating process and entitles changes as outcomes of a specific event, a second treatment explained in detail the statistical characteristics of the underlying random walk.

Probability judgments vary significantly across treatments as well as across subgroups with high or low cognitive abilities. Previous research results stating that decision makers with higher cognitive skills are more likely to engage the gambler's fallacy are confirmed. Yet this only holds if perception of sequential randomness is low. In a setting where the random nature of the sequence generating process is very salient, the exact opposite can be observed. Low CRT subjects are much more prone to the gambler's fallacy. When explaining this contradictory finding, we have to be aware that all previous studies investigating the effect of cognitive skills on probability forecasts use a setting similar to treatment 2, where the random nature of changes is concealed. Thus we can conclude that previous findings cannot be applied to settings where the random walk of changes is salient. Instead, as in most studies investigating the effect of cognitive abilities on decision behavior (e.g., Hoppe & Kusterer, 2011; Oechssler, Roider, & Schmitz, 2009; Toplak, West, & Stanovich, 2011), subjects with higher cognitive skills are less prone to behavioral biases.

Controlling for cognitive abilities, the analysis of a treatment effect reveals similarly surprising insights. Past research results are verified for the low CRT group participants. These engage the gambler's fallacy when they are made aware of the random walk process, but display a hot hand effect when sequential randomness is concealed. A subgroup with higher cognitive abilities however displays a contradictory behavioral pattern; subjects engage the gambler's fallacy in the low salience treatment and their probability judgments are statistically unbiased when made aware of the random walk process. This second contradictory finding is particularly interesting with respect to a real life application. It is the result of two distinct effects: i. the behavior of high CRT subjects is much less influenced by a change in the salience of randomness, and ii. when randomness is concealed, high and low CRT groups show completely opposite behavioral patterns. While the explanation for the first effect is straight

forward, as high CRT subjects are found to be less affected by framing (Stanovich & West, 1998), the second effect cannot be easily explained without further research work. One approach can be found in Xue et al. (2012). According to the authors, the hot hand fallacy (i.e. positive feedback forecasts) is connected to affective / emotional decision making, while the gambler's fallacy strategy (i.e. make forecasts contrasting the history of changes) requires great cognitive control. Based on their findings, the authors state that it is either possible that the neural mechanisms of affective decision making are weaker in subjects with higher cognitive abilities to begin with, or they tend to exert stronger control over their emotional systems.

The results described in this paper contribute to the ongoing research on biases in probability judgments. In particular, they provide new insights on the preconditions for an application of either the gambler's fallacy or its exact opponent, the hot hand fallacy. A best possible knowledge about these preconditions is key for comprehending many phenomena in economics and finance that are assumed to root in people's biased probability judgments.

References

Asparouhova, E., Hertzel, M., & Lemmon, M. (2009). Inference from streaks in random outcomes: Experimental evidence on beliefs in regime shifting and the law of small numbers. *Management Science*, *55*(11), 1766–1782.

Ayton, P., & Fischer, I. (2004). The hot hand fallacy and the gambler's fallacy: Two faces of subjective randomness? *Memory & Cognition*, *32*(8), 1369–1378. http://doi.org/10.3758/BF03206327

Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics*, *49*(3), 307–343.

Burns, B. D., & Corpus, B. (2004). Randomness and inductions from streaks: "Gambler's fallacy" versus "hot hand." *Psychonomic Bulletin & Review*, *11*(1), 179–184.

Dohmen, T., Falk, A., Huffman, D., Marklein, F., & Sunde, U. (2009). Biased probability judgment: Evidence of incidence and relationship to economic outcomes from a representative sample. *Journal of Economic Behavior & Organization*, *72*(3), 903–915. http://doi.org/10.1016/j.jebo.2009.07.014

Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, *10*, 171–178.

Frederick, S. (2005). Cognitive Reflection and Decision Making. *The Journal of Economic Perspectives*, *19*(4), 25–42.

Greiner, B. (2004). An Online Recruitment System for Economic Experiments. *Forschung Und Wissenschaftliches Rechnen 2003. GWDG Bericht*, *63*, 79–93.

Hoppe, E. I., & Kusterer, D. J. (2011). Behavioral biases and cognitive reflection. *Economics Letters*, *110*(2), 97–100.

Huber, J., Kirchler, M., & Stöckl, T. (2010). The hot hand belief and the gambler's fallacy in investment decisions under risk. *Theory and Decision*, *68*(4), 445–462. http://doi.org/10.1007/s11238-008-9106-2

Oechssler, J., Roider, A., & Schmitz, P. W. (2009). Cognitive abilities and behavioral biases. *Journal of Economic Behavior & Organization*, *72*(1), 147–152.

Offerman, T., & Sonnemans, J. (2001). *Is the quadratic scoring rule behaviorally incentive compatible?* (Working Paper).

Offerman, T., & Sonnemans, J. (2004). What's Causing Overreaction? An Experimental Investigation of Recency and the Hot-hand Effect. *Scandinavian Journal of Economics*, *106*(3), 533–554. http://doi.org/10.1111/j.0347-0520.2004.t01-1-00376.x

Rabin, M. (2002). Inference by Believers in the Law of Small Numbers. *The Quarterly Journal of Economics*, *117*(3), 775–816. http://doi.org/10.1162/003355302760193896

Stanovich, K., & West, R. F. (1998). Individual differences in framing and conjunction effects. *Thinking & Reasoning*, *4*(4), 289–317.

Suetens, S., & Tyran, J.-R. (2012). The gambler's fallacy and gender. *Journal of Economic Behavior & Organization*, *83*(1), 118–124. http://doi.org/10.1016/j.jebo.2011.06.017

Toplak, M. E., West, R. F., & Stanovich, K. E. (2011). The Cognitive Reflection Test as a predictor of performance on heuristics-and-biases tasks. *Memory & Cognition*, *39*(7), 1275–1289. http://doi.org/10.3758/s13421-011-0104-1

Xue, G., He, Q., Lei, X., Chen, C., Liu, Y., Chen, C., … Bechara, A. (2012). The Gambler's Fallacy Is Associated with Weak Affective Decision Making but Strong Cognitive Ability. *PLoS ONE*, *7*(10), e47019. http://doi.org/10.1371/journal.pone.0047019