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Endogenous Indexing and Monetary Policy Models

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Abstract:

Models in which firms use a rule of thumb or partial indexing in price setting are prominent in the recent monetary policy literature. The extent to which these firms adjust their prices to lagged inflation has been taken as fixed. We consider the implications of firms choosing the optimal degree of indexation so these simple pricing rules deliver prices as close as possible to those which would be chosen optimally. We find that the degree of indexation depends on the extent of persistence in the economy such that models with constant indexation are vulnerable to the Lucas critique. We also study the interactions between firms' price setting and the macroeconomic environment finding that, for the models which appear most plausible on microeconomic grounds, the Nash equilibrium between firms and the policy maker is characterised by zero indexation and zero macroeconomic persistence.

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Introduction

Models in which decision or optimisation costs introduce frictions to price changing and give rise to indexing or rule of thumb behavior by firms have been a central feature of the recent monetary policy literature. This contrasts with earlier models which emphasised price stickiness while assuming that price setting was optimal subject to constraints on when prices could be changed. The two most prominent models with indexing behaviour are Gali and Gertler ("GG", 1999) and Christiano, Eichenbaum and Evans ("CEE", 2005), the pricing mechanisms in which have also been used in Eichenbaum and Fisher (2003), Steinsson (2003) and Smets and Wouters (2003) amongst many others since these have become workhorse models (Woodford, 2007). They differ as to whether indexing behavior is combined with price stickiness or not but both give rise to the standard hybrid New Keynesian Phillips curve with forward looking expected inflation and lagged inflation.

A feature of the existing indexing and rule of thumb models is that the degree to which firms index to past inflation, which we call their indexing parameter, has been treated as an exogenous constant. The contribution of this paper is to explore the implications of varying that assumption[2]. It is natural to suppose that firms might consider the optimal or at least near optimal value of their indexing parameter so as to achieve higher profits by more closely matching their prices to the prices which would be set in the absence of decision or optimisation costs and thus come closer to constrained optimal behavior. It may not be appropriate to assume that firms would necessarily optimise their indexing parameter continuously, since infrequent re-optimisation is a maintained assumption of these models, but it seems plausible that they may review their indexing parameter periodically.

We study the implications of "endogenous indexing" behavior of this kind within the two

[2]In a discussion of this paper, Soderstrom (2007) quotes Lucas (1980), "I am attracted to the view that it is useful, in a general way, to be hostile toward theorists bearing free parameters..."

established models while adding a third, a variant of the CEE (2005) model which we show to be more efficient. All these models share a Calvo (1983) constant hazard structure, the Calvo signal being interpreted either as an opportunity to change price or as an opportunity to reoptimise prices depending on the model. Dennis (2006) reviews and empirically tests the CEE and GG models. The GG (1999) model assumes optimisation costs for a proportion of firms, who apply a rule of thumb in the setting of a new price when they may do so, while retaining the Calvo model's assumption that the firm's price remains fixed until the next Calvo signal. The rule of thumb determined price depends on a measure of lagged aggregate prices plus the product of an indexing parameter and lagged inflation. We find that this structure is constrained optimal and hence focus on the size of the indexing parameter within the rule of thumb. The scope of the CEE (2005) model is different in that firms may change price every period, the friction being that they do so fully optimally only when a Calvo signal arrives with indexing between signals. CEE assume an indexing structure in which the firm takes its own lagged price as a base and adds the product of an indexing parameter and lagged inflation. We refer to this as lagged own price indexing and find that this structure is not constrained optimal and a variant in which the firm uses the lagged aggregate price as a base - lagged aggregate price indexing (LAPI) - is generally superior. The decision problem for the firm is also considerably more complex in the CEE case.

A result common to all the models is that the optimal value of the indexing parameter depends on firms' beliefs about the degree of persistence in the economy. Intuitively if inflation is strongly persistent it is constrained optimal for an indexed or rule of thumb price to be strongly influenced by lagged inflation whereas weak persistence means that lagged inflation should not feature so prominently. This has two immediate implications for the standard versions of these models in which the indexing parameters are assumed fixed. First they are vulnerable to the Lucas critique since the value of the indexing parameter across firms influences the coefficients in the Phillips curve and hence a change in monetary policy regime which changes the degree of persistence will lead to

changes in those coefficients under endogenous indexing. Second, the derivation of microfounded loss functions for the indexing/rule of thumb models as a guide to optimal monetary policy on the assumption of fixed indexing parameters is questionable for the same reason that changes to the monetary policy regime may alter them. A further point is simply that the models with endogenous indexing allow one to check the plausibility of the particular values of the indexing parameters typically assumed.

The second set of results concerns the stability of the models and their ability to predict inflation persistence once we combine firms' choices with aggregate dynamics. We focus initially on the GG and LAPI models which appear to be the most plausible from the micro analysis. Under endogenous indexing one may think of a firm's choice of indexing parameter as a reaction function by which the indexing parameter responds to perceived persistence. Firms' choices collectively determine the Phillips curve which in turn acts as a constraint on the policy maker. Policy in part determines persistence given the Phillips curve (even if the degree of persistence is not a primary objective) so it may also be characterised as a reaction function specifying realised persistence as a function of the Phillips curve and thus indirectly firms' beliefs about persistence. Hence a natural question is the nature of the Nash equilibrium at which firms behavior is optimal given the degree of persistence at the macro level and policy is optimal given the Phillips curve which results from the behavior of firms. In particular it is interesting to ask at what level of persistence the models are stable in the sense that reaction functions intersect and a fixed point between firm and policy behavior is achieved. We find strong results with very mild restrictions on policy behavior that amount to ensuring stability. For both the Gali and Gertler (1999) and LAPI models the fixed point is zero persistence. This echoes a similar finding by Roberts (2006) who considers the possible interaction between indexing choices and outcomes in a reduced form model and finds a near-zero persistence equilibrium outcome.

For the CEE model we find that the optimal indexing parameter is state dependent and volatile over time, implying instability in the coefficients in the Phillips curve and the degree of persistence. In a long run expected sense, depending on the proportion of indexing firms assumed, inflation persistence may coexist with endogenous indexing but it is unclear how much weight to place on this result given the degree of instability in persistence.

Section 1 analyses the choice of indexing/rule of thumb structures and indexing parameters at the level of a single firm. Section 2 aggregates these to give the Phillips curves and Section 3 studies the interactions between firm and policy behavior. Section 4 concludes.

1. Constrained Optimal Indexing Behavior

We consider the optimal choices of indexing/rule of thumb structures and indexing parameters for individual firms who take the macroeconomic environment as given. The framework is standard with monopolistically competitive firms and a Calvo constant hazard structure. The optimisation problem for the firm is given in general form by (1) where V is the firm's expected net present value of profits over the relevant decision horizon, β is the discount factor, α is one minus the (constant) probability of the Calvo signal (and hence the probability of the price not changing if prices are sticky or the probability of not reoptimising if they are flexible), X is the firm's price in levels, Q its output and C(Q) its cost function. The optimisation problem (1) when the Calvo signal arrives is subject to the usual constraint (2) which is the firm's demand curve given standard Dixit-Stiglitz preferences in which P is the aggregate price index and D an index of aggregate demand.

$$Max V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [X_{t+j} Q_{t+j} - C(Q_{t+j})] \tag{1}$$

$$Q_{t+j} = \left(\frac{X_{t+j}}{P_{t+j}}\right)^{-\eta} D_{t+j} \tag{2}$$

The models differ according to the restrictions placed on the evolution of the firm's price in (1). In a standard Calvo model it is simply fixed and determined fully optimally at time t. With GG (1999) firms it is also fixed but set according to a rule of thumb applied at time t. In this case firms do not optimise their price but under endogenous indexing we may consider them choosing an optimal formula by which that price is set. In the CEE (2005) model and the LAPI variant derived below the initial price, X_t is set optimally and then varies according to an indexing function. We refer to these mechanisms collectively as simple pricing rules and follow the literature in assuming that a simple rule uses lagged information only and hence cannot respond to the current value of any shock variables. In the framework that follows, lagged inflation is the only past dated state variable and hence conditioning a pricing rule on it remains "simple" in the sense of using only a small information set. In more general models with many past dated state variables an issue would arise of whether a simple rule can be conditioned on all of them or only a subset. If all past dated state variables may be included as here (and certainty equivalence applies) the optimal simple pricing rule will determine a price equal to the time t-1 rational expectation of the time t optimal price since the only constraint is the inability of the simple rule to react to contemporaneous (and assumed zero mean) shocks.

Before turning to the individual models we place further structure on the firms decision problem by assuming that inflation, π_t (=ln P_t-ln P_{t-1}), and the output gap in log deviation form, y_t, follow the processes (3) and (4) in which ϵ_t is a shock to marginal cost with variance σ^2. For the time being we assume that the firms know the ρ parameters in (3)-(4) with certainty.

$$\pi_t = \rho_\pi \pi_{t-1} + k\varepsilon_t \tag{3}$$

$$y_t = \rho_y \pi_{t-1} + c\varepsilon_t \tag{4}$$

At this stage it is simplest just to state (3) and (4) but later we show that they correspond to the reduced form of these variables in the models considered under plausible restrictions on policy. Similarly, for now we assert that both the ρ parameters would be zero if lagged inflation does not

appear in the Phillips curve and $\rho_\pi>0$, $\rho_y<0$ if it does (and with a positive coefficient). If lagged inflation is present in the Phillips curve and above target, for example, so all else equal current inflation will be above target $\rho_y<0$ implies that the current output gap will be lower than otherwise for given ϵ_t to reduce current inflation back to towards its target value, though not all in a single period so $\rho_\pi>0$.

A remaining common element to the models is the assumption that the log deviation of real marginal cost from its steady state, m, is linear in the output gap and the cost push shock to give (5).

$$m_t = \phi y_t + \varepsilon_t \tag{5}$$

1.1 Decision Costs with Sticky Prices

We consider the solution to (1) when price stickiness means that the firm's price remains fixed at X_t over the decision horizon but decision or optimisation costs give rise to that price being set by a rule of thumb. This corresponds to the scope of GG (1999) model. We report the solution to (1) for a fully optimising Calvo firm and then derive the optimal rule of thumb which delivers a price equal in expectation to the optimised price given the constraint of not using contemporaneous information.

For a standard Calvo firm, substituting (2) into (1) and imposing the restriction that the price remains fixed gives the problem (6) with first order condition (7).

$$Max_{X_t} V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [X_t^{1-\eta} D_{t+j} P_{t+j}^{\eta} - C([\tfrac{X_{t+j}}{P_{t+j}}]^{-\eta} D_{t+j})] \tag{6}$$

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j D_{t+j} P_{t+j}^{\eta} [(1-\eta) X_t + \eta \tfrac{\partial C(Q_{t+j})}{\partial Q_{t+j}}] \tag{7}$$

Following Goodfriend and King (1997), for example, (7) log linearises to (8) where lower case

implies the log of the variable.

$$x_t = (1 - \beta\alpha)E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [p_{t+j} + m_{t+j}] \tag{8}$$

Following the argument above, the optimal rule of thumb will efficiently use past dated information to achieve a price as close as possible to that in (8) and hence will amount to the time t-1 rational expectation of (8). An interim step is given by (9) which makes use of (3)-(5) and substituting (9) into the t-1 dated expectation of (8) and summing gives (10) where x^r indicates the optimal rule of thumb price and γ^r the indexing parameter with optimal value shown by (11).

$$E_{t-1}[p_{t+j} + m_{t+j}] = p_{t-1} + \pi_{t-1}[\rho_\pi (\frac{1 - \rho_\pi^{j+1}}{1 - \rho_\pi}) + \phi\rho_y \rho_\pi^j] \tag{9}$$

$$x_t^r = p_{t-1} + \gamma^r \pi_{t-1} \tag{10}$$

$$\gamma^r = \frac{\rho_\pi + (1 - \beta\alpha)\phi\rho_y}{1 - \beta\alpha\rho_\pi} \tag{11}$$

We next relate (10)-(11) to the GG (1999) framework. In their model the rule of thumb formula is expressed slightly differently but we show that it is equivalent to (10). A generalised version of the GG formula is (12), in which n_{t-1} are prices that were newly set the previous period, which reproduces their (23) except that we introduce the parameter γ^G which is implicitly set to unity in the original form.

$$x^G = n_{t-1} + \gamma^G \pi_{t-1} \tag{12}$$

New prices may be related to aggregate prices by (13) given the Calvo pricing rule.

$$p_t = (1 - \alpha)n_t + \alpha p_{t-1} \tag{13}$$

Lagging (13) one period and substituting into (12) gives (14).

$$x^G = p_{t-1} + (\gamma^G + \tfrac{\alpha}{1-\alpha})\pi_{t-1} \tag{14}$$

Comparison of (14) with (10) confirms that the GG rule of thumb structure (12) is constrained optimal in the sense of corresponding to the efficient forecasting structure of (10), while (10) and (11) permit an assessment of the appropriate size of γ^G. Setting this parameter to unity implies that the coefficient on lagged inflation in (14) is 1/(1-α). This will be optimal if the ρ parameters in (11) are such that the right hand side of that expression equals 1/(1-α) which implies the following condition.

$$\rho_\pi = \frac{1-(1-\alpha)(1-\beta\alpha)\phi\rho_y}{1-\alpha(1-\beta)}$$

With $\beta<1$ and $\rho_y<0$ this implies $\rho_\pi>1$ so inflation would have to be unstable for $\gamma^G=1$ to be optimal. A value of ρ_π in the more plausible range below unity would imply a lower value of γ^G and, as will be seen below, a smaller coefficient on lagged inflation in the Phillips curve.

1.2 Decision Costs with Flexible Prices

We turn to the situation where decision/optimisation costs continue to motivate simple pricing rule behavior but prices are flexible. The Calvo signal is now interpreted as an opportunity to optimize the firm's price but prices may still change (according to the simple pricing rule) each period in the absence of that signal. This corresponds to the scope of the CEE (2005) model but we derive the constrained optimal solution first before contrasting it with that framework.

If prices are flexible the optimisation (1) can reduce to a static problem since with flexible prices there is no inherent reason for the choice of price in one period to affect future periods. Hence a baseline result is the fully optimal flexible price using full information which, in log linear form, is given by x^f in (15). This is a standard result and may be derived following the steps above or simply taken from (8) with α=0.

$$x_t^f = p_t + m_t \tag{15}$$

Given (15) the constrained optimal indexed price, x^a, will be the t-1 expectation of (15) given by (16) and (17) which make use of (3)-(5).

$$x_t^a = p_{t-1} + \gamma^a \pi_{t-1} \tag{16}$$

$$\gamma^a = \rho_\pi + \phi\rho_y \tag{17}$$

Hence firms will set the fully optimal flexible price in (15) when the Calvo signal permits them to optimise with contemporaneous information and the indexed price in (16) between signals, noting that the constrained optimal indexing formula (16) involves the use of the lagged aggregate price as a base to which the indexing term is added and hence we refer to this model as lagged aggregate price indexing (LAPI).

This contrasts with the CEE (2005) indexing formula in which the firm sets a price, X^c , when the Calvo signal arrives and subsequently indexes to its own lagged price rather than the aggregate lagged price until the next signal, hence we refer to lagged own price indexing. The indexing formula takes the form (18) in levels or (19) in logs as in Woodford's (2003, chapter 3, eqn. 3.4) version of the model, also used in Smets and Wouters (2003), which generalises the original CEE formulation (CEE 2005, eqn 8) in which γ^c was implicitly set to unity.

$$X_{t+j}^c = X_{t+j-1}^c \left(\frac{P_{t+j-1}}{P_{t+j-2}}\right)^{\gamma^c} \tag{18}$$

$$x_{t+j}^c = x_{t+j-1}^c + \gamma^c \pi_{t-1} \tag{19}$$

The CEE firm's problem is hence (1) subject to (2), plus the need to use a simple pricing rule based

on t-1 information which is assumed to take the form (18). Here the structure of (18) or (19), in which the lagged own price appears, is imposed as a constraint whereas in the LAPI model the indexing formula (16) emerged as part of the solution and did not involve the lagged own price. Hence the constraint (18) binds and we find detailed results for the CEE model differ from the LAPI framework. Given that the latter maximised (1) subject to (2) and the need to use a simple pricing rule, the CEE outcome for firms will be inferior due to the presence of the extra constraint. This questions the use of the CEE model if we seek constrained optimal behavior unless it is argued that the firm's own lagged price is more readily observable than the lagged aggregate price and hence simpler to use. That simplicity, however, comes at the cost both of the inferior performance already noted and a considerably more complex optimisation problem under endogenous indexing. Using (18) with (2), the CEE firm's optimisation problem (1) is given by (20).

$$Max_{X_t^c,\gamma} V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [(X_t^c (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma})^{1-\eta} D_{t+j} P_{t+j}^{\eta} - C(Q_{t+j})] \tag{20}$$

The partial derivative of (20) with respect to X^c at time t (and hence treating γ^c as fixed) gives the first order condition (21) for the initial price set when the Calvo signal is received which linearises to (22).

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j P_{t+j}^{\eta} D_{t+j} (\frac{P_{t+j-1}}{P_{t-1}})^{-\gamma\eta} [(1-\eta) X_t^c (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma} + \eta P_{t+j} M_{t+j}] \tag{21}$$

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [-x_t^c + p_{t+j} + m_{t+j} - \gamma(p_{t+j-1} - p_{t-1})] \tag{22}$$

We simplify (22) using (3)-(5) to express the forward looking components of (22) in terms of time t information and the ρ parameters. As an initial step (3)-(5) imply (23)-(25).

$$p_{t+j} = p_t + \pi_t \rho_{\pi} (\frac{1-\rho_{\pi}^j}{1-\rho_{\pi}}) + k\Sigma_{i=1}^{j} \varepsilon_{t+i} (\frac{1-\rho_{\pi}^{j+1-i}}{1-\rho_{\pi}}) \tag{23}$$

$$\pi_{t+j} = \rho_\pi^j \pi_t + k\Sigma_{i=1}^{j} \varepsilon_{t+i} \rho_\pi^{j-i} \tag{24}$$

$$m_{t+j} = \phi\rho_y \pi_{t+j-1} + (1+\phi c)\varepsilon_{t+j} \tag{25}$$

From these (22) implies (23) for the optimal initial price when the Calvo signal is received.

$$x_t^c = p_t + (1-\beta\alpha)m_t - \frac{\beta\alpha\pi_t}{1-\beta\alpha\rho_\pi}[\gamma - \rho_\pi - \phi\rho_y(1-\beta\alpha)] \tag{26}$$

For the choice of the indexing parameter the natural assumption is that firms would choose a value for γ each time they re-optimise their price. Denoting this by γ^c, the partial derivative of (20) with respect to γ^c is given by (27) which linearises to (28).

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j P_{t+j}^{\eta} D_{t+j} \ln(\frac{P_{t+j-1}}{P_{t-1}})(\frac{P_{t+j-1}}{P_{t-1}})^{-\gamma^c\eta}[(1-\eta)X_t^c(\frac{P_{t+j-1}}{P_{t-1}})^{\gamma^c} + \eta P_{t+j} M_{t+j}] \tag{27}$$

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j (p_{t+j-1} - p_{t-1})[-x_t^c + p_{t+j} + m_{t+j} - \gamma^c(p_{t+j-1} - p_{t-1})] \tag{28}$$

Using (23)-(25) and substituting for x_t from (26) gives (29).

$$\gamma_t^c = \rho_\pi + \frac{\begin{array}{l} k^2\sigma^2\beta\alpha(1-\beta\alpha\rho_\pi)[1-\beta\alpha\rho_\pi^2 + \phi\rho_y(1-\beta\alpha)] \\ + (1-\beta\alpha)^2[\pi_t^2\phi\rho_y(1-\beta\alpha\rho_\pi - \beta\alpha(1-\beta\alpha\rho_\pi^2)) - \pi_t m_t(1-\beta\alpha\rho_\pi)(1-\beta\alpha\rho_\pi^2)] \end{array}}{k^2\sigma^2\beta\alpha(1-\beta\alpha^2\rho_\pi^2) + \pi_t^2(1-\beta\alpha)^2} \tag{29}$$

Hence γ^c depends on the values of marginal cost and inflation at the time the Calvo signal arrives. Given that in the CEE model prices may be indexed for multiple periods there will be multiple values of γ^c present in the price structure and hence the Phillips curve which will then have time varying coefficients. We elaborate on this in the following section.

From (29) it is also clear that the optimal indexing parameter differs from that in the LAPI model (17) which confirms that constraint (18) binds in the CEE model. Since (17) resulted from optimal indexing given (3)-(5), outcomes in the CEE case will generally be inferior. We highlight two reasons for this, both of which reflect the CEE indexing structure (18) or (19) being based on the firm's own price the previous period rather than the lagged aggregate price as in the LAPI model (16). First consider the optimal price set when the Calvo signal arrives (26). If inflation is non-zero at that time, the value of γ^c influences the price that is set since that price will be indexed by that period's inflation the following period, which will in turn be indexed by future inflation in future periods. Hence there is a linkage between indexing in the future and the current optimal price given (19) which is not present in the LAPI model since the firm may set the price optimal for that period only. Second, consider shocks during the indexing period before the price is reoptimised. In the first instance both models will fail to adjust the firm's price at the time when a shock occurs since by assumption the indexing formulae use lagged information only. Thereafter we may distinguish between two objectives for the indexing formula which are that the firm's price should "catch up" with the effects of the shock and then "forecast" the ongoing effects of the shock if there is persistence. In the LAPI structure (16) these roles are separated since indexing to the lagged aggregate price implies automatic catch up so the indexing parameter is geared solely towards the forecasting role (and hence is zero if there is no persistence). In (18) both roles must be addressed by the indexing parameter since the indexed price is based on the lagged own price.

Proposition 1. Under endogenous indexing, the constrained optimal price setting behavior of firms with significant decision or optimisation costs depends on their perception of the degree of persistence in inflation and the output gap/marginal cost.

In the models above, significant decision or optimisation costs leads firms to employ a rule of thumb or indexing formula when not resetting their prices optimally. From (11), (17) and (29) the indexing parameters depend on the coefficients in (3)-(4).

We also note that in both the GG and LAPI models, the indexing parameters in (11) and (18) are zero if there is no persistence.

2. Phillips Curves

This section presents the Phillips curves for the three simple pricing models considered above. The main motivation is to facilitate the analysis of the following section which studies the interactions between firms' choices of indexing parameters at the micro level considered above and the aggregate dynamics resulting from those choices via the Phillips curves. On a notational point we denote the indexing parameters in each of the Phillips curves by γ^A so as to distinguish between the parameter in the Phillips curve and the value of γ that would be chosen by a single firm as above. In equilibrium these will be equal under endogenous indexing but it is helpful to keep them separate for the time being.

This section also generalises the CEE model to allow for a proportion of non-indexing firms, modeled as standard Calvo firms. This addresses a criticism of the model, discussed in Dennis (2006), that it assumes too high a degree of price flexibility (since all indexing firms change price each period) whereas micro data suggests at least some price stickiness. We also show the Phillips

curve for the LAPI model which is new.

First, the Gali and Gertler (1999) Phillips curve is given by (30) which is simply their equation (24) with notation adjusted to allow for the more general γ considered in (10), rather than their value of unity for γ^G in (12), and N for the share of rule of thumb firms rather than GG's ω (these firms being mixed with standard Calvo firms in this model). These are notational changes and a generalisation of the indexing parameter, the only difference in the framework above being the potential endogeneity of the latter. In the Gali-Gertler model, rule of thumb firms are mixed with standard Calvo firms in the proportions N and (1-N) and hence if N=0, (30) reduces to the standard New Keynesian Phillips curve.

$$\pi_t = \frac{\beta\alpha E_t[\pi_{t+1}] + N\gamma^A(1-\alpha)\pi_{t-1} + (1-N)(1-\alpha)(1-\beta\alpha)(\phi y_t + \varepsilon_t)}{\alpha + N(1-\alpha)(1+\beta\alpha\gamma^A)} \qquad (30)$$

We briefly note two properties of (30). First it fails the weak form of the natural rate hypothesis, in the sense of the sum of coefficients on inflation on each side of the equation not being equal, even if β=1 unless γ^A equals 1/(1-α) which is the value obtained from Gali and Gertler's assumption of γ^G=1 in (12). It was argued that this value was implausible under endogenous indexing unless inflation was believed to be extremely persistent. Second, if empirical estimates of the coefficient on lagged inflation are used to infer possible values of the structural parameters as in Gali and Gertler (1999), a lower value of γ^A (reflecting more plausible beliefs about inflation persistence) would require a larger implied proportion of rule of thumb firms, N.

The Phillips curve for the LAPI model is new but may readily be derived. In principle it would be appropriate to mix the LAPI firms with some standard Calvo firms but the Phillips curve in that case is cumbersome (it involves tracking the inflation rates in the two sectors separately) and the interest in this model is chiefly that it represents the efficient indexing version of the CEE model rather than

promoting it as a convincing model in a general sense. For LAPI firms prices are set optimally according to (15) if the Calvo signal occurs (with probability 1-α) and set according to the indexing function (16) if not (with probability α). If we assume that there are a large number of firms these probabilities translate into proportions and hence the price level is given by (31) from which the Phillips curve is (32) using (15) and (16).

$$p_t = (1-\alpha)x_t^f + \alpha x_t^a \tag{31}$$

$$\pi_t = \gamma^A \pi_{t-1} + \tfrac{1-\alpha}{\alpha}(\phi y_t + \varepsilon_t) \tag{32}$$

We note that the very simple form of (32) without any forward looking expectations reflects the absence of any price staggering when all prices are flexible and backward looking firms use the lagged aggregate price as an indexing base. It also violates the weak form of the natural rate hypothesis unless $\gamma_A=1$ which is implausible from (17) given $\rho_y<1$ unless inflation is again unstable with $\rho_\pi>1$.

For the CEE model with an indexing parameter not restricted to unity, the Phillips curve is presented by Woodford (2003, chapter 3, eqn 3.6) and may be expressed as (33) using the notation above.

$$\pi_t = \frac{\beta E_t[\pi_{t+1}] + \gamma^A \pi_{t-1} + (\tfrac{1-\alpha}{\alpha})(1-\beta\alpha)(\phi y_t + \varepsilon_t)}{1+\beta\gamma^A} \tag{33}$$

If the CEE firms are mixed with standard Calvo firms (in proportions N and 1-N), adressing the empirical criticism that the CEE model assumes too much price flexibility, the Phillips curve becomes (34) in which γ^A is replaced by $N\,\gamma^A$.[3] These two parameters appearing solely as their

[3]Aggregation of the two sets of firms to derive (34) assumes that the probability of a price change for the Calvo firms is equal to the probability of a price re-optimisation for the CEE firms. In general the two probabilities may differ, in which case the inflation rates for each of the two sectors appears in the Phillips curve, but the assumption of equality allows for a simple analysis of the effects of N<1.

product implies that they would not be separately identifiable from empirical information but they may be separately calibrated which is done below. In turn (34) implies that values of γ^A calculated from empirical estimates of the lagged inflation coefficient in (33) would be too high if N<1.

$$\pi_t = \frac{\beta E_t[\pi_{t+1}] + N\gamma^A \pi_{t-1} + (\frac{1-\alpha}{\alpha})(1-\beta\alpha)(\phi y_t + \varepsilon_t)}{1+\beta N\gamma^A} \tag{34}$$

Following the original CEE model, the Phillips curves (33) and (34) have been presented as if there was a single indexing parameter shared by all firms at all times and hence without a time subscript. Under endogenous indexing, however, (29) showed that the indexing parameter will be state dependent and thus vary over time (though not across firms choosing its value at the same time). Since CEE prices may be indexed for multiple periods, prices at time t will be indexed using values set at different times in the past. Allowing for the probability of indexing rather than re-optimising a price, α, the "aggregate" γ^A at time t is given by (35) where the right hand γ terms are given by (29) at time t-j. A further note is that the value in (35) would appear in the numerator of (33)-(34) and the same expression one period later in the denominators since that refers to indexing from t to t+1.

$$\gamma_t^A = \frac{(1-\alpha)}{\alpha}\sum_{j=1}^{\infty}\alpha^j \gamma_{t-j}^A \tag{35}$$

Proposition 2. Under endogenous indexing the structure and coefficients of the Phillips curve depend on firms' perceptions of the degree of persistence in inflation and the output gap.

From Proposition 1, perceptions of the degree of persistence determine the indexing parameters in (11), (17) and (24). These appear in the Phillips curves respectively (25), (27) and (28) and hence perceptions of the degree of persistence partly determine the Phillips curve coefficients. In the special case where the γ parameters are zero, lagged inflation no longer appears in the Phillips curve.

Corollary 1. Under endogenous indexing, the indexing or rule of thumb models are vulnerable to the Lucas critique.

A change in monetary policy regime which affects the persistence parameters in (3)-(4) will change the γ parameters by Proposition 1 and hence the Phillips curve by Proposition 2 in which case the Phillips curves for these models are not invariant to monetary policy.

Remark 1. Under endogenous indexing, the derivation of appropriate microfounded loss functions for these models would be more complex than if the indexing parameters are fixed as has typically been assumed (Steinsson 2003, Woodford 2003). We leave this issue to future research.

3. Policy and Stability Analysis

We consider the interaction between firms price setting behavior from Section 1, which depends on macroeconomic persistence, and the degree of persistence which depends on the Phillips curve and hence the underlying price setting behavior as shown in Section 2. In particular we derive the fixed point between the ρ parameters in (3)-(4) which guide price setting and the values which must obtain given the Phillips curve. We focus primarily on the GG and LAPI models since the CEE model appears to be less compelling at the microeconomic level.

A prior step is to note that from the Phillips curves the relevant state variables are lagged inflation and the current value of the cost push shock. We assume that policy itself does not introduce additional state variables. This will be valid if a simple Taylor rule combined with an IS relationship without additional lagged variables is used, or a simple quadratic loss function in inflation and the output gap is minimised under discretion. This assumption supports the original assumption of the processes for inflation and the output gap in (3)-(4). It may be noted that it would be violated if policy is implemented under commitment since the presence of the forward looking inflation term

in the Phillips curves will give rise to the lag of the output gap in the reduced form of the system. Efficient simple pricing rules would then incorporate the lagged output gap in firms' indexing formulae so that variable would appear in the Phillips curve which would in turn affect the nature of optimal policy and so on. This example suggests that there may be rich interactions between the set of variables included in simple pricing rules and the set of state variables in the reduced form of the system as policy interacts with those pricing rules via the Phillips curve. We focus on the simplest fixed point of that interaction in which firms perceive the reduced form processes (3)-(4) and policy does not add additional state variables. This case is the simplest with which to explore the dynamics of the models under endogenous indexing and corresponds to the standard indexing/rule of thumb models.

Beyond the assumption concerning the relevant state variables we find that strong results are obtained from the Phillips curves combined with the indexing choices at the micro level of Section 1 with very little additional structure on the policy process. In particular two restrictions are required. First that if the coefficient on lagged inflation in the Phillips curve is zero the ρ parameters in (3)-(4) will also be zero. This may be justified on standard MSV grounds (McCallum 1983, 1999). Second we assume that if the coefficient on lagged inflation is positive, ρ_π will be positive and ρ_y negative. The former simply follows from inflation generally being brought back to target gradually when lagged inflation appears in the Phillips curve and the latter is necessary to ensure that. Given that these mild restrictions are sufficient for our results we present them first before illustrating them with a particular policy model.

Hence we proceed by seeking the simultaneous solution to the relevant Phillips curve from Section 2 together with the relevant γ parameter from Section (1) and the processes (3)-(4). As a first step we substitute (3)-(4) into the relevant Phillips curve until only terms in the two state variables remain. Since that equation must be satisfied for any values of the state variables the coefficient on

each of them must be zero which gives two equations of which that from the lagged inflation term is informative for the ρ parameters. This may then be compared with the determinants of the γ parameters from the micro analysis.

This procedure is most easily introduced using the LAPI model. Substituting (3)-(4) into (32) until only terms in the two state variables remain gives (36) in which the two square bracketed expressions must be zero for a solution to obtain.

$$0 = \pi_{t-1}[\rho_\pi - \gamma^A - (\tfrac{1-\alpha}{\alpha})\phi\rho_y] + \varepsilon_t[k - (\tfrac{1-\alpha}{\alpha})(1 + c\phi)] \tag{36}$$

The first square bracket in (36) is informative about the relationship between the ρ parameters in (3)-(4) and the ρ parameters in (17) which must be equal at a fixed point between the price setting behavior of firms and the degree of persistence at the macro level. Substituting (17) into the first term in (36) gives (37) in which γ^A is the indexing parameter in the Phillips curve and γ^a the value which would be chosen by a firm given the ρ parameters from (17).

$$\gamma^a - \gamma^A = \tfrac{1}{\alpha}\phi\rho_y \tag{37}$$

A Nash equilibrium between firm's choices and aggregate dynamics occurs when the two γ parameters are equal since otherwise γ^A would change as firms collectively change γ^a. From (37) this can only occur when $\rho_y=0$ which in turn will only occur when $\rho_\pi=0$ and lagged inflation does not appear in the Phillips curve so there is no persistence. Furthermore, since ρ_y is negative away from that equilibrium it implies that $\gamma^a<\gamma^A$ and hence the latter will fall towards zero over time.

For the GG framework, substituting (3)-(4) into (30) until only terms in the state variables remain yields (38).

$$0 = \begin{matrix} \epsilon_t[\ k[\beta\alpha(\rho_\pi - N\gamma^A(1-\alpha) - [1-(1-\alpha)(1-N)]] + (1+\phi c)(1-\alpha)\alpha[1-\beta\alpha] \\ \pi_{t-1}[\rho_\pi[\beta\alpha(\rho_\pi - N\gamma^A(1-\alpha) - [1-(1-\alpha)(1-N)]] + \phi\rho_y(1-\alpha)(1-N)[1-\beta\alpha] + \gamma^A N(1-\alpha)] \end{matrix} \quad (38)$$

Furthermore we may factorise the second line of (38) using (11) to give (39) in which γ^A is the indexing parameter in the Phillips curve as before and γ^r is the optimal indexing parameter at the micro level from (11). In equilibrium these must be equal in which case (39) shows that ρ_y and ρ_π must be zero also. Hence this replicates the zero persistence result found for the LAPI model above and lagged inflation will once again disappear from the Phillips curve. Furthermore, away from this equilibrium the top line in (39) will be negative so the second line must be positive with $\gamma^r < \gamma^A$ and the latter will again be pulled down towards zero as firms set γ^r.

$$0 = \begin{matrix} \phi\rho_y(1-\alpha)[1-\beta\alpha] - \rho_\pi\alpha(1-\beta\rho_\pi) \\ + (\gamma^A - \gamma^r)N(1-\alpha)[1-\beta\alpha\rho_\pi] \end{matrix} \quad (39)$$

Proposition 3. In the Gali and Gertler (1999) and LAPI models the unique Nash equilibrium between the pricing behavior of firms based on the degree of macroeconomic persistence and the latter resulting from that pricing behavior is one with zero persistence and lagged inflation is no longer present in the Phillips curve.

This follows from the discussion above with the stability restrictions imposed on the ρ parameters.

Having demonstrated these zero persistence results using only information from the micro analysis and the Phillips curve, we illustrate them by conducting a policy simulation while also being able to say more about the indexing parameter for the CEE case (29) by simulating the terms in inflation and marginal cost.

We assume the standard quadratic loss function (40) to represent policy preferences and minimise this subject to the Phillips curves (30), (32) and (34) in turn for each of the models. Since this is for

illustration we do not explore the question of whether sophisticated policy should seek to influence firms indexing behaviour through policy choices.

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \lambda y_{t+j}^2) \tag{40}$$

We note that the Phillips curves are encompassed by the form (41) in which the F, B and M coefficients may be retrieved by inspection from the individual Phillips curves.

$$\pi_t = FE_t[\pi_{t+1}] + B\pi_{t-1} + M(\phi y_t + \varepsilon_t) \tag{41}$$

Following McCallum and Nelson (2004) we use (3) to substitute out the forward looking expectation in (41) and use the resulting expression as the constraint in a Lagrangean for the minimisation of (40). The resulting first order conditions for inflation and the output gap may be combined to give (42) from which we may use (3)-(4) to substitute out variables other than the state variables to give (43) in which each of the two square bracketed terms must equla zero for the expression as a whole to be satisfied since the state variables may potentially take any values. The two equations implicit in (43) combined with the two implicit in (38) for the GG model, (36) for LAPI and their equivalents from substituting (3)-(4) into (34) for CEE (not shown) constitute four equations in the four unknown coefficients in (3)-(4) which we solve numerically.

$$\pi_t + \frac{\lambda}{M\phi}[y_t(1 - F\rho_\pi) - \beta B E_t[y_{t+1}]] = 0 \tag{42}$$

$$\begin{aligned} &\pi_{t-1}[\rho_\pi(M\phi - \beta\lambda\rho_y B) + \rho_y\lambda(1 - F\rho_\pi)] \\ &+ \varepsilon_t[k(M\phi - \beta\lambda\rho_y B) + c\lambda(1 - F\rho_\pi)] = 0 \end{aligned} \tag{43}$$

We present the results of this exercise in a sequence of figures, in each case plotting the indexing

parameter firms would choose given the coefficients in (3)-(4) against the indexing parameter γ^A in the Phillips curves. Values for the latter are assumed and used to generate the coefficients in (3)-(4) which then generate the γ parameters from the micro results. Hence for different values of γ^A the figures are informative about whether that value is likely to rise or fall as a consequence of firms' choices with a fixed point where the two are equal.

Figures 1-2 for the GG and LAPI models respectively illustrate the results above. In each case the fixed point where $\gamma^A=\gamma^r$ and $\gamma^A=\gamma^a$ is where both are zero and the lines are below the 45 degree line so in each case the "micro" γ will be lower than the aggregate or pre-existing γ so the latter will be brought down by firms' choices. As would be expected the disparity between the two becomes more marked in the GG model as the share of indexing/rule of thumb firms, N, declines.

For the CEE model we follow a similar procedure in assuming a sequence of values for γ^A in (34), minimise the policy loss function with respect to that constraint, and see the implications of the coefficient values in (3)-(4) that result from that for the optimal indexing parameter from the micro analysis, in this case given by (29). Since (29) involves variables as well as parameters, however, we simulate inflation and marginal cost using the derived values for the reduced form coefficients and examine the properties of the indexing parameter in (29) over that generated sample. We simulate 5100 periods for inflation and marginal cost using shocks drawn from a normal distribution, ignoring the first hundred periods so the influence of assumed initial conditions (both variables set to zero) will have become very small. This exercise sheds some light on the properties of the indexing parameter in (29) in relation to the assumed aggregate indexing parameter in (34), and to an extent is informative about the values at which the two may be equal, but it is only a partial simulation of the system since it holds γ^A fixed while γ^c varies when in practice the latter will influence the former through (35) and the system as a whole will have rich dynamics.

FIGURE 1

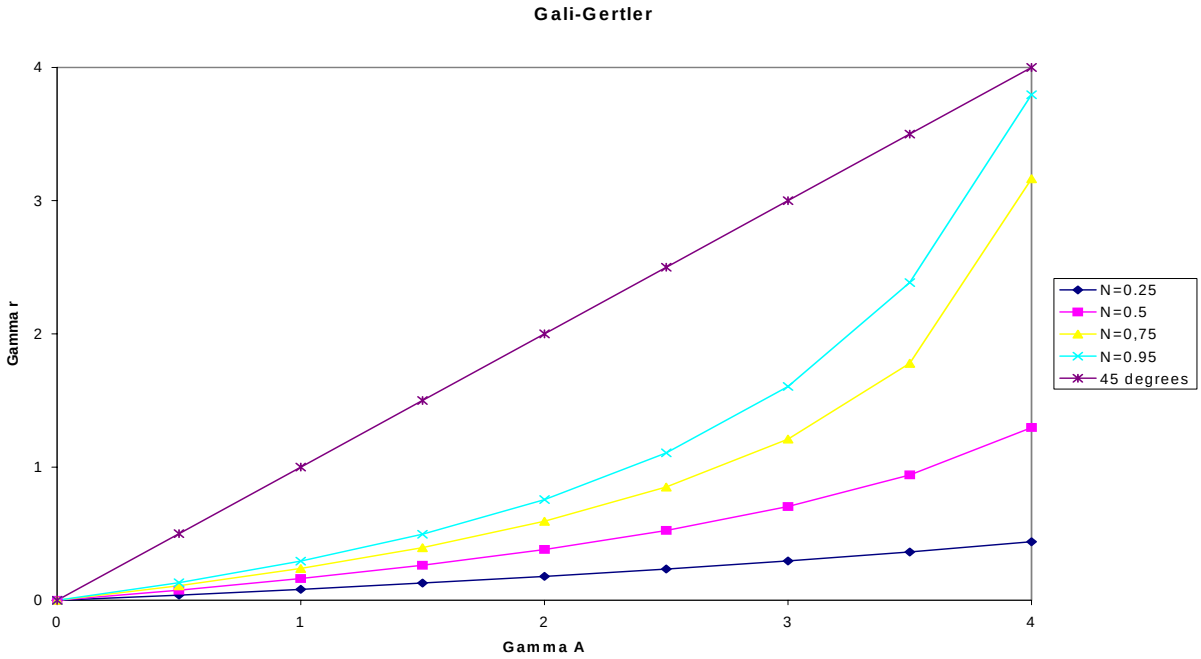


FIGURE 2

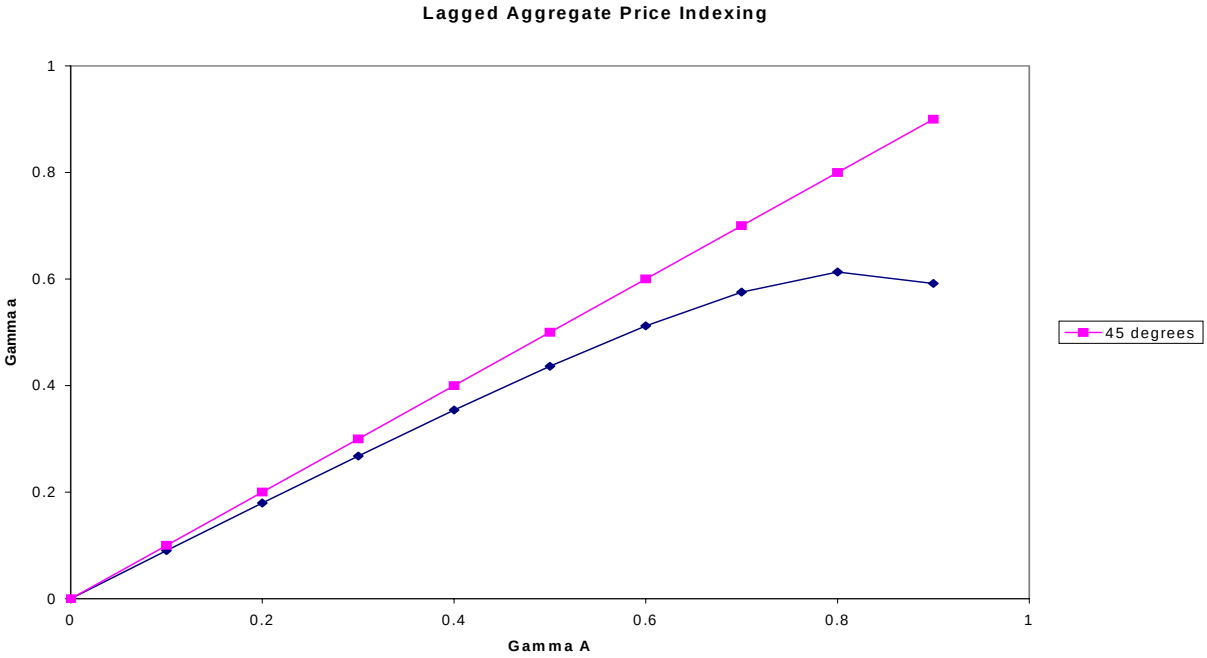


Figure 3 presents the result of this exercise for the CEE model when the share of indexing firms, N, is unity followed by Figure 4 where N=0.5. In each case the solid line is the average realisation for γ^c from (29) on the vertical axis for given values of γ^A on the horizontal axis. The dashed 45 degree line is where the two equate. The dotted lines above and below the solid line show plus and minus one standard deviation in the aggregate γ measure (35). Hence this is not the standard deviation of γ^c itself, which is two to three times as large, but the standard deviation of the aggregate over different values of γ^c which appears in the Phillips curve. Hence for a given initial value of γ^A, movements in the average value of γ^c above and below the solid line towards the dotted lines are typical and the size of that vertical movement gives an indication of the horizontal move in γ^A that can be expected once the feedback from firms choices to the Phillips curve are allowed for. Given the spread of the standard deviation lines, considerable volatility in the Phillips curve coefficients is likely. We do not model those dynamics explicitly, partly because this model was less convincing on microeconomic grounds given that the indexing formula assumed is inefficient, but it seems clear that the CEE model with endogenous indexing, at least in the way modeled above, will demonstrate considerable instability.

In Figure 3 the solid line is a little above the 45 degree line until very high levels of persistence are reached (a value of γ^A of unity corresponds to a value of ρ_π close to 0.9) so long run expected outcomes will tend to that level but at the same time the line is only just above the 45 degree line and hence leftward movements will be almost as common as rightward movements.

In Figure 4 with a share of indexing firms of 0.5 rather than unity, the long run expected position is reversed with a tendency towards low levels of persistence, though once again the standard deviation lines are far apart and hence there will be considerable volatility.

FIGURE 3

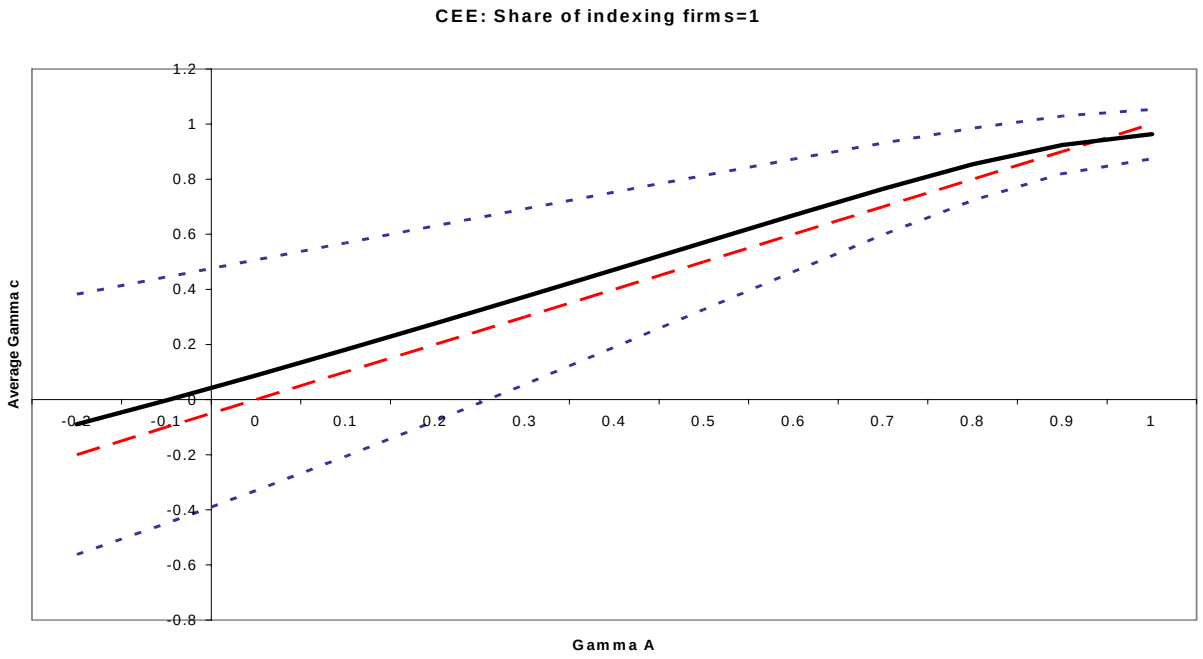
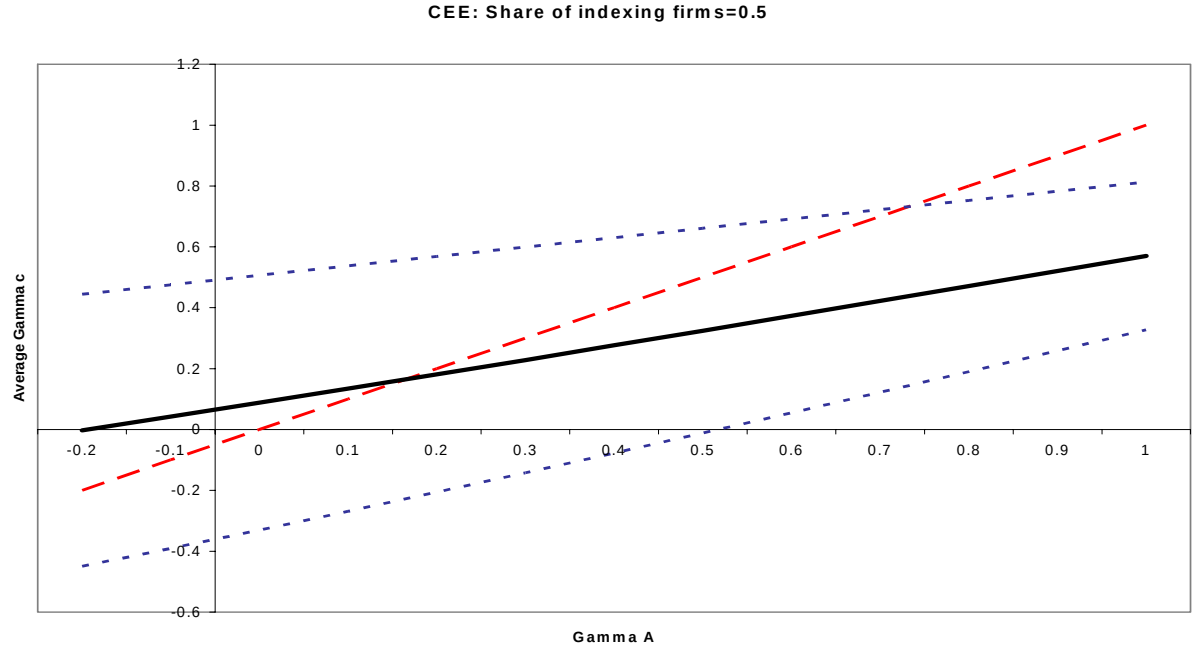


FIGURE 4



4. Conclusion

The paper has analysed the implications of endogenising the extent to which firms adjust their prices to lagged inflation when applying a rule of thumb or indexing formula. The motivation for this behavior by firms simply being to achieve a simple pricing rule which would deliver outcomes as close as possible to those which would obtain under full optimisation. The wider motivation of the paper was to explore the suitability of these models for monetary policy analysis when firms behavior is constrained optimal given the need to employ a simple pricing rule rather than them simply choosing arbitrary values for their indexing parameters.

The key results were that first, constrained optimal rules of thumb/indexing formulae depend on the degree of persistence in the macroeconomic environment, which implies that these models are vulnerable to the Lucas critique (and microfounded loss functions which assume constant indexing parameters appear questionable). Second, the indexing structure in Christiano, Eichenbaum and Evans (2005) appears less plausible on microeconomic grounds than the simpler lagged aggregate price indexing framework, and the introduction of some standard Calvo firms to match micro data better reduces the coefficient on lagged inflation in the Phillips curve and thus weakens the model's ability to reproduce persistence.. Third, the Gali and Gertler and LAPI models are unable to reproduce inflation persistence under endogenous indexing when a fixed point is reached between pricing behavior and macroeconomic persistence. The Christiano, Eichenbaum and Evans model appears to be highly volatile under endogenous indexing with state dependent and thus time varying coefficients in the Phillips curve. If the share of indexing firms in the price structure is large the model may be able to reproduce inflation persistence in a long run average sense but with continuing volatility.

These results appear to strongly question the suitability of these models for monetary policy analysis,

partly due to the Lucas critique vulnerability, and either due to their inability to match persistence at least in the long run when learning dynamics may have worked themselves out, or because they may imply implausibly high volatility in the coefficients of the Phillips curve and the outcomes that would obtain. Lastly we note that the paper has been concerned with analytical rather than empirical issues but there may be interesting empirical questions to be addressed once the links between outcomes and pricing behaviour provided by endogenous indexing are considered.

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