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by Ute Kapaun and Martin F. Quaas

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Does the optimal size of a fish stock increase with environmental uncertainties?

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September 25, 2012

Abstract. We analyze the effect of environmental uncertainties on optimal fishery management in a bio-economic fishery model. Unlike most of the literature on resource economics, but in line with ecological models, we allow the different biological processes of survival and recruitment to be affected differently by environmental uncertainties. We show that the overall effect of uncertainty on the optimal size of a fish stock is ambiguous, depending on the prudence of the value function. For the case of a risk-neutral fishery manager, the overall effect depends on the relative magnitude of two opposing effects, the 'convex-cost effect' and the 'gambling effect'. We apply the analysis to the Baltic cod and the North Sea herring fisheries, concluding that for risk neutral agents the net effect of environmental uncertainties on the optimal size of these fish stocks is negative, albeit small in absolute value. Under risk aversion, the effect on optimal stock size is positive for sufficiently high coefficients of constant relative risk aversion.

Keywords: fishery economics, environmental uncertainty, constant escapement, risk aversion, prudence

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Introduction

Environmental uncertainties have important effects on the development of fish stocks (Hilborn and Walters 1992). Accordingly, stochastic fluctuations in environmental variables such as temperature, salinity, or oxygen concentration, have to be taken into account when discussing optimal fishery management. For risk-neutral agents, Reed (1979) shows that optimal management of a fish stock is characterized by a constant escapement policy, i.e. it ensures that a constant proportion of the stock remains in the sea after fishing.[1] Reed (1979) also shows that the optimal constant escapement level in a stochastic environment is equal to, or larger than, the optimal escapement level in a deterministic setting where the unit harvesting cost function fulfills a number of regularity assumptions. Various articles have refined Reed's seminal work by adding multiple uncertainties (Clark and Kirkwood 1986, Sethi et al. 2005), costly capital adjustments (Singh et al. 2006), choice of regulatory instrument (Weitzman 2002), spatial structure of the resource, (Costello and Polasky 2008) and management with environmental prediction (Costello et al. 2001). All these studies assume risk-neutral decision-makers.[2]

One thing that most of these models have in common is that environmental stochasticity is modeled by an i.i.d. random variable z_t multiplied by the average stock-growth function $f(x_t)$ of the resource stock x_t at time t, i.e. $x_{t+1} = z_t \cdot f(x_t)$.[3]

The stock-growth function combines the different biological processes of re-

cruitment (young fish entering the harvested stock), survival, and growth in terms of weight. Multiplying the average stock-growth function by one random variable implies that all biological processes are equally affected by environmental fluctuations. From a biological point of view, however, it seems more plausible to assume that the respective processes would be influenced differently by fluctuations in the environmental conditions. In most ecological stock-assessment models of marine fish populations the reproduction process is considered to be more sensitive to environmental fluctuations than the survival of adult fish, which is usually assumed to be constant.[4] Accordingly, we split the stock-growth function into two functions describing the processes of recruitment and growth on the one hand and of adults surviving natural mortality on the other. The recruitment and growth are assumed to be stochastic, whereas natural mortality is assumed to be fixed.

We examine the effect of uncertainty on the optimal size of a fish stock under both risk neutrality and risk aversion. Whether or not optimal escapement increases with uncertainty is connected with the prudence (Kimball 1990) of the value function. If the value function of the fishery considered exhibits positive prudence, the optimal policy will involve precautionary savings in the natural capital stock and hence higher stock size under uncertainty than in the deterministic case. If the value function exhibits negative prudence, the optimal stock size under uncertainty will be lower than in the deterministic setting. As the value function depends on (i) the biomass growth function, (ii) the profit function, and (iii) the representative fisherman's utility function, all three have an influence on whether

optimal escapement increases or decreases with environmental uncertainty.

Considering risk-neutral stakeholders first, we show that the overall effect of uncertainty on the optimal size of a fish stock is ambiguous, depending on the relative magnitude of two opposing effects, the 'convex-cost effect' and the 'gambling effect'. The 'convex-cost effect' reduces optimal escapement under uncertainty over and against the deterministic case. Because harvesting costs are convex in the fish stock, expected harvesting costs are larger when fish stock growth is uncertain than they are at an expected stock level. The 'gambling effect', on the other hand, increases optimal escapement under uncertainty. It comes about because uncertainty is multiplicatively connected to stock growth, so the distribution of the next period's resource rents is positively skewed. This effect induces a risk-neutral fishery manager to 'bet' on favorable environmental conditions.

For the case of a risk-averse representative fisherman, we show for a special case that optimal escapement increases with uncertainty if the coefficient of relative risk aversion is large enough.

In quantitative terms, we apply the model to Eastern Baltic cod and North Sea herring fisheries and conclude that under risk neutrality the net effect of environmental uncertainties on the optimal size of these fish stocks is negative, albeit small in absolute value. Under risk aversion we observe a positive effect of uncertainty on optimal stock size for sufficiently high coefficients of constant relative risk aversion.

The paper is structured as follows: In the next section we set up the model

and solve the corresponding optimization problem. In Section 2 we consider optimal harvesting under risk neutrality, identifying the convex-cost effect and the gambling effect. The case of a risk-averse representative fisherman is discussed analytically in Section 3. We then apply our model to fisheries for Baltic cod (Section 4) and North Sea herring (Section 5), considering both risk neutrality and risk aversion. Section 6 summarizes and discusses the results.

1 A Fishery Model with Environmental Uncertainty

We consider a simple biomass model with stochastic recruitment in discrete time. The growth of biomass x_t from time step t to $t+1$ is described by the equation

$$x_{t+1} = g(s_t) + z_t\, r(s_t), \tag{1}$$

where s_t denotes the escapement, i.e. the biomass that remains in the ecosystem after harvest h_t so that $s_t = x_t - h_t$. The period between t and $t+1$ is divided into two parts. Harvesting takes place in the first part. In the second, the remaining fish biomass, i.e. the escapement, reproduces and grows in weight. A fraction of the fish dies by natural causes. The term $g(s)$ represents the survival of adult fish. The reproduction and growth process is represented as the stock-recruitment relationship $r(s)$.

Both functions are differentiable and non-decreasing. The expected recruitment at stock size s_t is described by the strictly concave function $r(s_t)$. Furthermore, the recruitment process is sensitive to environmental uncertainty, represented by the random variable z_t, which is independent and identically distributed over time

with an expected value equal to one, $E[z_t] = 1$. The survival of adult fish $g(s_t)$, by contrast, is deterministic. It can be concave or linear in s. In the latter case the natural mortality rate of adult fish is fixed. The assumtion of deterministic mortality at a constant rate is common in biological stock assessment models (Hilborn and Walters 1992, ICES 2011a,b). We briefly discuss the case of stochastic mortality in Appendix D.

The price per unit of fish p is constant, i.e. the fishery is small compared to the overall market. We assume a generalized Schaefer production function for the instantaneous harvest rate $\tilde{h}$ in the fishing season, $\tilde{h} = q(\tilde{x})\, K_t$. We use $q(\tilde{x})$ to denote the catch per unit of effort, K_t to denote the effort of harvesting fish (which is assumed to be constant throughout the fishing season), and $\tilde{x}$ to denote the current stock size. Accordingly, $\tilde{x} = x_t$ at the beginning of year t's fishing season and $\tilde{x} = s_t$ at the end (Reed 1979, Clark 1990). Harvesting costs C_t are proportional to effort K_t, with ζ as the costs per unit effort $C_t = \zeta\, K_t$. Thus we obtain a unit cost function for harvesting fish, $c(\tilde{x}) = \zeta / q(\tilde{x})$. We assume that, in general, unit harvesting costs are weakly decreasing and weakly convex in the stock size, i.e. $c'(\tilde{x}) \leq 0$ and $c''(\tilde{x}) \geq 0$. This means that the catch per unit of effort $q(x)$ is non-decreasing with population abundance. Convexity also implies that the increase of the unit harvesting costs induced by a one-unit decrease in stock is greater for lower stock abundance than for higher stock abundance. A common specification of the harvesting cost function is $c(\tilde{x}) = c\, \tilde{x}^{-\chi}$ with $\chi > 0$. In that special case $\tilde{x}\, c(\tilde{x})$ is strictly concave (for $0 < \chi < 1$), constant (for $\chi = 1$),

or strictly convex (for $\chi > 1$). For $\chi = 0$, catch per unit effort is independent of stock abundance, the unit harvesting cost is constant with $C = \zeta$, and $\tilde{x}\, c(\tilde{x})$ is linear in stock. In empirical terms the most relevant case is $\chi \in (0, 1)$. In a study of 297 fisheries, Harley et al. (2001) find typical values of χ to be between 0.64 and 0.75.

During the harvesting season, each ton of fish caught reduces the stock by one ton. Therefore the aggregate annual profit Π_t is obtained by integrating the flow of profits over the whole fishing season $\Pi_t = \int_{s_t}^{x_t} (p - c(\tilde{x}))\, \mathrm{d}\tilde{x}$. The fishery manager aims to maximize the well-being of the representative fisherman earning his income from fishing profits:[5]

$$\max_{s_t} E\Big[\sum_{t=1}^{\infty} \rho^{t-1} u\left(\Pi_t\right) \mathrm{d}\tilde{x}\Big] \quad \text{subject to (1).} \tag{2}$$

Here the operator E denotes the expectation over the probability distribution of the random process $\{z_t\}$ and $\rho \in (0, 1)$ is the discount factor. Fishermen are typically averse to fluctuations in income. We find this reflected in the management plans for Baltic cod which contain rules to limit fluctuations in total allowable catches from year to year.[6] We take this effect into account by assuming that instantaneous utility $u(\Pi_t)$ derived from fishing income is increasing and weakly concave, $u'(\Pi_t) > 0$ and $u''(\Pi_t) \leq 0$.[7] For a risk-neutral fisherman, the instantaneous utility function is linear, $u(\Pi_t) \equiv \Pi_t$. For a risk-averse fisherman, the instantaneous utility function is strictly concave, $u''(\Pi_t) < 0$.

Using $J(x)$ to denote the value function associated with the stochastic opti-

mization problem (2), the Bellman equation reads[8]

$$J(x) = \max_s \Big\{ u\left(\int_s^x (p - c(\tilde{x}))\, \mathrm{d}\tilde{x} \right) + \rho\, E\big[J\big(g(s) + z\, r(s)\big) \big] \Big\} \tag{3}$$

In the following, we use $S^*(x)$ to denote the optimal feedback policy obtained as a solution of (3) for the stochastic case and $\bar{S}^*(x)$ to denote the optimal feedback policy for the corresponding deterministic model, where $z_t \equiv 1$ for all t. The question we are asking in this paper is whether, for a given stock size x, the solution $S^*(x)$ for the stochastic problem is larger than, equal to, or smaller than the solution $\bar{S}(x)$ for the deterministic model.

To adress this question, we consider the first-order condition for optimal escapement

$$u'\left(\int_s^x (p - c(\tilde{x}))\, \mathrm{d}\tilde{x} \right) \big(p - c(s)\big) = \rho\, E\left[\frac{d}{ds} J\big(g(s) + z\, r(s)\big) \right]. \tag{4}$$

This condition states that for the optimal escapement level s^* at a given stock size x the current marginal profits of the last unit of fish harvested (left-hand side, LHS) equal the discounted expected marginal profits of an additional unit of fish that escapes fishing (right-hand side, RHS). Uncertainty only makes a difference to the RHS of this equation. As the LHS is monotonically increasing in s, the optimal escapement level at a given stock size x will increase with the RHS of (4). Thus, the optimal escapement level will be higher (lower) under uncertainty than with the deterministic setting if the RHS of (4) is higher (lower) when z is stochastic than in the deterministic case $z \equiv 1$. This, in turn, depends on the curvature of the derivative of the value function with respect to the escapement level, $dJ\big(g(s) +$

$z\,r(s)\big)/ds$, in z. So the question whether or not optimal escapement increases with uncertainty is connected to the *prudence* (Kimball 1990) of the value function $J(g(s) + z\,r(s))$.

Prudence describes "the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to 'risk aversion', which is how much one dislikes uncertainty and would turn away from uncertainty if possible" (Kimball 1990:54). A positive prudence of the value function thus implies precautionary savings in the natural capital stock and hence higher stock size under uncertainty. Positive prudence is given if $dJ\big(g(s) + z\,r(s)\big)/ds$ is convex in the random variable z. The index of absolute prudence is defined by $\frac{d^2}{dz^2}\left(dJ\big(g(s) + z\,r(s)\big)/ds\right)$. If this index is positive (negative), we speak of positive (negative) prudence of the value function. Under positive (negative) prudence the optimal stock size under uncertainty will be higher (lower) than in the deterministic setting.

As the value function depends on (i) the biomass growth function, (ii) the profit function, and (iii) the representative fisherman's utility function, all three have an influence on whether optimal escapement increases or decreases with environmental uncertainty. For the detailed analysis of the combined effect we proceed in three stages. In Section 2 we study how optimal harvesting is affected by risk when fishermen are risk-neutral. Section 3 derives analytical results for risk-averse fishermen, but it requires relatively restrictive assumptions on biomass growth function, harvesting technology, and preferences to derive a closed-form expression for the value function. In section 4 and 5 we apply our analysis to the Baltic

cod and North Sea herring fisheries.

2 Optimal Harvesting under Risk Neutrality

For the risk-neutral case $u(\Pi_t) \equiv \Pi_t$, both the LHS and the RHS of (4) are independent of the current stock size x. Thus, the solution of this stochastic optimization problem is state-independent (see Appendix A). Optimal feedback policy $S^*(x)$ is the most rapid approach to the constant optimal escapement level s^*,

$$S^*(x) = s^* \quad \text{if} \quad x > s^* \quad \text{and} \quad S^*(x) = x \quad \text{otherwise.} \tag{5}$$

The optimal escapement level s^* is determined by the following condition (see Appendix B):

$$p - c(s^*) = \rho\, g'(s^*)\ \Big[p - E_z\Big[c\big(g(s^*) + z_t\, r(s^*)\big)\Big]\Big] + \rho\, r'(s^*)\ \Big[p - E_z\Big[z_t\, c\big(g(s^*) + z\, r(s^*)\big)\Big]\Big] \tag{6}$$

This condition states that for the optimal escapement level s^* current marginal profits from the last unit of fish harvested equal the discounted expected marginal profits from an additional unit that escapes fishing. The expected marginal profit on the RHS of (6) can be divided into two effects. The first term on the RHS represents the expected marginal profits from the additional surviving adults. The second term on the RHS stands for the expected marginal profits from additional recruits.

In the risk-neutral case, the curvature properties of the marginal cost function are essential to determine the prudence of the value function. To compare the op-

timal escapement level in the stochastic case with the optimal escapement level $\bar{s}^*$ in the deterministic case (i.e., for $z_t \equiv 1$ in Equation (1)), we consider the following equivalent to condition (6) in the deterministic setting:

$$p - c(\bar{s}^*) = \rho\, g'(\bar{s}^*)\, \Big[p - c\Big[g(\bar{s}^*) + r(\bar{s}^*)\Big]\Big] + \rho\, r'(\bar{s}^*)\, \Big[p - c\Big[g(\bar{s}^*) + r(\bar{s}^*)\Big]\Big]. \quad (7)$$

We obtain a higher (lower) optimal escapement level when the next period's expected marginal costs are lower (higher) than the marginal costs at the expected next period's stock level (which coincides with the deterministic case as we have $E_z[z] = 1$). We consider the effects for additional surviving adults and for additional recruitment separately. The first term on the RHS of condition (6) in the stochastic case is smaller than the first term on the RHS of condition (7) in the deterministic case. This is due to the convexity of the marginal harvesting cost function. We refer to this effect as the 'convex-cost effect'. The second term on the RHS of condition (6) will be larger than the second term on the RHS of condition (7) if function $\tilde{x}\, c(\tilde{x})$ is concave. We refer to this effect as the 'gambling effect'.

2.1 Convex-cost effect

If marginal harvesting costs are convex in the fish stock, they will also be convex in the random variable. Expectation $E\Big[c\big(g(s^*) + z_t\, r(s^*)\big)\Big]$ is over a convex function of the random variable, so expected marginal harvesting costs are greater than the marginal costs at expected stock growth $c\big(g(s^*) + r(s^*)\big)$ as $E[z] = 1$. The convex marginal cost function implies that the increase in marginal harvesting

costs for a stock growth below the mean is greater than the decrease in marginal harvesting costs for a stock growth above the mean. Accordingly, the expected marginal harvesting costs are greater under uncertainty than in the deterministic case. We refer to this effect, which tends to reduce the optimal escapement level, as the convex-cost effect.

Intuitively, the effect of convex marginal harvesting costs is similar to the effect of risk aversion, so it is optimal to invest less if the asset is risky. Accordingly, it is also intuitive that under uncertainty the convex-cost effect will reduce optimal escapement over and against the deterministic model. More precisely, the convexity of marginal harvesting costs reduces the prudence of the value function. If the convex-cost effect were the only effect present, the prudence of the value function would be unambiguously negative, and the optimal escapement level would decrease with environmental uncertainty.

2.2 Gambling effect

If the function $\tilde{x}\,c(\tilde{x})$ is convex, the second term on the RHS of condition (6) will be lower than the second term on the RHS of (7). The prudence of the value function would be unambiguously negative, so the optimal escapement level under uncertainty would be unambiguously lower than in the deterministic setting.

As set out earlier, the more relevant case in empirical terms is where $\tilde{x}\,c(\tilde{x})$ is concave in $\tilde{x}$. Here the expression $z_t\,c\big(g(s^*)+z_t\,r(s^*)\big)$ is a concave function in the random variable z_t. The expected marginal costs $E\Big[z_t\,c\big(g(s^*)+z_t\,r(s^*)\big)\Big]$ will then

be smaller than the marginal costs at expected stock growth $c\big(g(s^*) + E[z]\, r(s^*)\big)$ (which coincides with the deterministic case).

The economic intuition for this effect is as follows: As uncertainty is multiplicatively connected with stock growth, the distribution of next period's resource rents is positively skewed and has a fat tail at high rents. Under favorable environmental conditions, a marginal increase in current escapement will result in a strong marginal increase in the fish stock, producing both a large harvest and low marginal harvesting costs in the next period. Under adverse environmental conditions, marginal harvesting costs in the next period will be high because of the low fish stock, but this effect is dampened by the fact that the harvest is small. So the expected marginal increase in harvesting costs with a marginal increase in escapement is lower than the marginal increase in harvesting costs under expected stock growth. In other words, a risk-neutral fishery manager will tend to bet on favorable environmental conditions. This is why we call this effect the gambling effect.

The gambling effect tends to increase the prudence of the value function, i.e. to increase the optimal escapement level under environmental uncertainty.[9]

Since the convex-cost effect and the gambling effect work in opposite directions, the overall result is ambiguous. The optimal escapement level could be either higher or lower than in the deterministic case.[10]

2.3 A special case: when survival is proportional to recruitment

In our model, a special case arises if survival is proportional to recruitment, i.e. if there exists some $\kappa \in [0,1)$ such that $(1-\kappa)\, g(s) = \kappa\, r(s)$. In this case, biomass growth (equation 1) can be written as

$$x_{t+1} = \tilde{z}_t\, f(s_t), \tag{8}$$

where $\tilde{z} = \kappa + (1-\kappa)\, z$ is a random variable with mean 1 and $f(s) \equiv r(s)/(1-\kappa)$ is the expected biomass growth function. This special case deserves attention, as the model for it is equivalent to the model studied by Reed (1979). In the latter case the optimality condition (6) simplifies to

$$p - c(s^*) = \rho f'(s^*) \Big(p - E_{\tilde{z}}\Big[z\, c\big(\tilde{z}\, f(s^*)\big)\Big]\Big). \tag{9}$$

If the function $\tilde{x}\, c(\tilde{x})$ is concave in $\tilde{x}$, the optimal escapement level in the stochastic model will be unambiguously higher than in the deterministic case.[11] In other words the gambling effect will outweighs the convex-cost effect.

If we further specify the unit cost function $c(x) = c\, x^{-\chi}$ with $\chi \in (0,1)$ and assume a log-normal distribution of environmental stochasticity $\tilde{z}$ with mean $\mu_{\tilde{z}} = 1$ and standard deviation $\sigma_{\tilde{z}}$, condition (9) can be written as follows (see Appendix C):

$$p - c(s^*) = \rho f'(s^*) \left(p - \frac{c\, f(s^*)^{-\chi}}{(1+\sigma_{\tilde{z}}^2)^{\frac{\chi(1-\chi)}{2}}}\right), \tag{10}$$

The left hand-side of this equation increases with the degree of uncertainty, as measured by the variance $\sigma_{\tilde{z}}^2$. Accordingly, the optimal escapement level increases

monotonically with uncertainty. In quantitative terms, the influence of uncertainty on the optimal escapement level will typically be small. It is maximal for $\chi = 1/2$, as then the exponent of the factor $(1+\sigma_{\tilde{z}}^2)^{\chi\,(1-\chi)/2}$ on the RHS of (10) reaches its maximum for all values of $\chi \in (0,1)$. But even in this case and for an unrealistically high degree of uncertainty $\sigma_{\tilde{z}}^2 = 1$, this factor changes the RHS of (10) by less than ten percent, as $(1+\sigma_{\tilde{z}}^2)^{\chi\,(1-\chi)/2} = 2^{1/8} < 1.10$.

3 Optimal Harvesting under Risk Aversion

In the case of risk aversion, it is in general not possible to solve the Bellman equation (3) analytically. For special cases an analytical solution is however feasible.[12] To study the case of a risk-averse representative fisherman analytically, we thus have to further specify the model.

First, we neglect harvesting costs in this section, i.e. we assume $c(\tilde{x}) \equiv 0$. This not only simplifies the analysis, it also enables us to better isolate the effect of risk aversion. The point is that in the absence of harvesting costs, both the convex-cost effect and the gambling effect vanish, and optimal escapement for the risk-neutral case would be independent of uncertainty.

Second, we focus on the case where survival is proportional to recruitment and the biomass growth function is given by (8). We furthermore assume that the biomass growth function $f(s)$ has the functional form

$$f(s) = \left(\alpha\, s^{1-\phi} + \alpha\, \beta^{1-\phi}\right)^{\frac{1}{1-\phi}} \tag{11}$$

with positive constants α, β, and ϕ. A special case of this biomass growth function

is the Beverton-Holt function $f(s) = (s/\alpha)/(1 + (s/\beta))$ obtained by setting $\phi = 2$.

Third, we assume an instantaneous utility function with constant relative risk aversion $\vartheta > 0$,

$$u(\Pi(x, s)) = \frac{\nu}{1 - \vartheta} \, (x - s)^{1-\vartheta}, \tag{12}$$

with $\nu > 0$. This model is analytically solvable for the special case $\phi = \vartheta$. In Appendix E we show that the value function is

$$J(x) = \frac{\psi_1}{1 - \vartheta} \, x^{1-\vartheta} + \psi_2 \tag{13}$$

with some constants $\psi_1 > 0$ and ψ_2, and that the optimal escapement rule is

$$s^* = \left(\rho \, \alpha \, E[z^{1-\vartheta}]\right)^{\frac{1}{\vartheta}} \, x. \tag{14}$$

It is obvious that if $\vartheta < 1$ ($\vartheta > 1$), optimal escapement will be lower (higher), the higher the uncertainty is. $\vartheta < 1$ means not only that risk aversion is relatively low but also that the curvature of the biomass growth function is relatively high.[13]

We can again connect this result to the prudence of the value function $J(\tilde{z} \, f(s))$. In this case, the function inside the expectation operator on the RHS of (4) is $dJ(\tilde{z} \, f(s))/ds = \psi_1 \, z^{1-\vartheta} \, \alpha \, s^{1-\vartheta}$. It is concave (convex) in ϑ if $\vartheta < 1$ ($\vartheta > 1$). Thus, a value function that exhibits negative (positive) prudence will give rise to a lower (higher) optimal escapement for $\vartheta < 1$ ($\vartheta > 1$).

4 Quantitative Example I: Baltic Cod Fishery

Our first quantitative example is the Baltic cod fishery. The water in the Baltic Sea is brackish, making it a marginal area for cod. The fish population depends

on fluctuating fresh water inflows from the North Sea to increase the salinity level to a degree where their eggs can hatch (Rockmann et al. 2007). Accordingly, the recruitment process for Baltic cod is highly uncertain and represents a useful example for the stochastic recruitment model.

To estimate the biological growth function we use stock assessment data (years 1966-2009) from the International Council of the Exploration of the Sea (ICES 2011). We combine the data for total stock biomass in year t, X_t, total harvest H_t (as the sum of official landings and discards) and natural mortality (assumed to be fixed at $M = 0.2$, as in the ICES stock assessments) to obtain the escapement S_t and the recruitment biomass R_t as follows

$$S_t = X_t - H_t,$$

$$R_t = X_{t+1} - e^{-0.2} S_t.$$

The recruitment variable R_t thus encompasses both the reproduction process and growth in weight. It is calculated as the total stock biomass at the beginning of period $t+1$ minus the fraction of period t's escapement that survives natural mortality. We assume that recruitment follows a stochastic Beverton-Holt (1957) stock-recruitment function $r(s_t) = z_t \, \alpha_1 \, s_t/(1 + \alpha_2 s_t)$.[14] Using the Levenberg-Marquardt algorithm for nonlinear least squares, we estimated the equation

$$\ln(R_t) = \ln\left(\frac{\alpha_1 \, S_t}{1 + \alpha_2 \, S_t}\right) + \varepsilon_t, \tag{15}$$

assuming that ε_t is an independent and identically normally distributed random variable with zero mean.[15] We obtain estimates $\hat{\alpha}_1 = 1.189$ with a standard error

of 0.111, $\hat{\alpha}_2 = 1.525$ million tons with a standard error of 0.496 million tons, and an estimate $\hat{\sigma}^2 = 0.083$ million tons for the standard deviation of $z_t = \exp(\varepsilon_t)$. Because of the logarithmic specification of (15), estimate $\hat{\alpha}_1$ is biased. For our numerical computations, we use the adjusted value $\alpha_1 = \hat{\alpha}_1 \exp(-0.5\,\hat{\sigma}^2) = 1.140$.

To estimate the parameters of the harvesting function, we specify the cost function $C = \zeta K_t$ and $q(\tilde{x}) = q_0\, x^{\chi}$. With this specification, total fishing effort in year t is $K_t = \int_{s_t}^{x_t} q(\tilde{x})^{-1} d(\tilde{x}) = \frac{1}{q_0\,(1-\chi)}[x^{1-\chi} - s^{1-\chi}]$ (Clark 1990). We use data on effort as days at sea for the Danish fleet from (Fiskeriregnskabsstatistik 2007, Fiskeridirektoratet 2007) and (ICES 2009) data for total biomass and escapement. Using the Levenberg-Marquardt algorithm for nonlinear least squares we obtain $q_0 = 5.162$ tons per days at sea (standard error 2.173), and $\chi = 0.953$ (standard error 0.213). For prices and unit effort cost ζ, we use the estimate from Kronbak (2002; 2005) and Quaas et al. (2010). Normalizing the price of a million tons of harvest to unity, we obtain a unit effort cost parameter of $\zeta = 0.554$, measured in billions of Euros at million days at sea. For the unit harvesting cost parameter we thus have $\zeta/q_0 = 0.107$ Euros/kg.

In sum, the functional specifications we use are

$$g(s_t) = e^{-0.2}\, s_t$$

$$r(s_t) = \frac{1.140\, s_t}{1 + 1.525\, s_t}$$

$$z_t \propto \text{LOGN}(1, 0.083)$$

$$p - c(\tilde{x}) = 1 - 0.107\, \tilde{x}^{-0.953}$$

$$\rho = \frac{1}{1 + 0.05}$$

Our first step in the quantitative analysis for Baltic cod is to consider the optimal escapement levels for a risk-neural representative fisherman. To determine the optimal escapement levels in the deterministic and stochastic recruitment models, we solve conditions (6) and (7) using these specifications numerically. For our sensitivity analysis, we use random samples of 1000 sets of parameter values for α_1, α_2, q_0 and ζ, assuming that the parameter values are independently normally distributed with means and standard deviations as obtained from the estimations (or variance-covariance matrix from the estimation for α_1 and α_2). For each parameter set, we compute the optimal escapement and determine the standard deviation of the sample of optimal escapement levels thus obtained.

The optimal escapement level for the deterministic model (where $z \equiv 1$) is $\bar{s}^* = 0.904$ million tons, with a standard deviation of 0.257 million tons. For the stochastic model we obtain $s^* = 0.902$ million tons as the optimal escapement, with a standard deviation of 0.255 million tons. Thus we have a slightly lower optimal escapement level in the stochastic model than its deterministic counterpart, but the difference of 1 619 tons (with a standard deviation of 657 tons) is small. This

indicates that while model uncertainty is substantial, it is not so large as to make the quantitative results completely unreliable.

Next, we analyze how risk aversion influences the results. For this purpose, we assume an instantaneous utility function with constant relative risk aversion $u(\Pi_t) = \Pi_t^{1-\vartheta}/(1-\vartheta)$ and use the same biomass growth function, marginal cost function, and discount rate as before. We solve the stochastic optimization problem (2) numerically for different risk-aversion coefficients ϑ by numerically computing the value function $J(x)$. To do so, we use the collocation method (Miranda and Fackler 2002), where the value function $J(x)$ is approximated by a finite linear combination of Chebychev polynomials.[16]

The results are shown in Figure 1.

FIGURE 1 about here

The upper panel shows the optimal feedback policies $S^*(x)$ under uncertainty ($\sigma_z^2 = 0.0834$) and $\bar{S}^*(x)$ in the deterministic case for three different coefficients of risk aversion: the risk-neural case $\vartheta = 0$, slight risk aversion $\vartheta = 0.1$, and stronger risk aversion $\vartheta = 0.5$. For the risk-neutral case, the optimal policy is the most rapid approach to constant escapement (cf. section 2). The higher the risk aversion, the smoother the optimal policy becomes: escapement is relatively lower (harvest is higher) at relatively low stock sizes, and relatively higher (harvest is lower) at higher stock sizes.

For all three coefficients of risk aversion, there is hardly any difference between the optimal policies under uncertainty and in the deterministic setting. The lower

panel in Figure 1 shows the difference $S^*(x) - \bar{S}^*(x)$ for three different stock levels, $x \in \{0.15, 0.50, 1.8\}$ million tons. Note that the scale of the y axis in the lower panel is in thousands of tons, while in the upper panel it is in millions of tons. This shows that the difference is well below 1 percent of the optimal escapement. Although the overall effect is small, the effect of increasing risk aversion is unambiguous. The higher the coefficient of risk aversion is, the higher is the difference in optimal escapement under uncertainty and in the deterministic case. The two lower stock sizes considered ($x = 0.15$ and $x = 0.50$ million tons) are smaller than the optimal escapement s^* in the risk-neutral case. For these stock sizes, the optimal escapement for $\vartheta \to 0$ is the same with and without uncertainty, as it simply equals the current stock size. The higher stock size $x = 1.8$ million tons is above s^*. In this case, the optimal escapement for a risk-neutral representative fisherman is lower under uncertainty than in the deterministic case (see above). With increasing risk aversion, the difference also becomes positive for this stock size .

5 North Sea Herring Fishery

Our second case study is the North Sea herring fishery. Here we use the same functional specifications as for the Baltic cod fishery. For North Sea herring, we use the price and cost function from Nostbakken (2008), where $p = 2.465$ NOK/ kg, $\zeta = 1,189,565$ NOK/per vessel-year and a catchability per vessel-year of $q_0 = 0.0011$. We again normalize the price to unity and obtain $\zeta/(p*q_0) = 0.439$

Euros/kg and $\chi = 1$ as parameters of the cost function.

To estimate the parameters of the biological growth function, we use ICES (1998; 2007) data for the period 1947-2005 to calculate the escapement S_t as the product of the total biomass X_t and e^{-F}, where F is the mean fishing mortality rate for age classes from 2 to 6. We again assume a deterministic natural mortality with a rate of $M = 0.16$ (as in the ICES stock assessments for herring) and a Beverton-Holt function for the stock-recruitment relationship. Using the same model (15) and regression method as for cod, we obtain estimates $\hat{\alpha}_1 = 2.048$ (standard error 0.266), $\hat{\alpha}_2 = 0.956$ million tons (standard error 0.204 million tons), and $\hat{\sigma}^2 = 0.104$ million tons for the standard deviation of $z_t = \exp(\varepsilon_t)$. Again, we use the adjusted value $\alpha_1 = \hat{\alpha}_1 \exp(-0.5\,\hat{\sigma}^2) = 1.9445$ for our numerical analysis.[17]

For the risk-neutral case we compute an optimal escapement level $s^* = 2.769$ million tons (with a standard deviation of 0.302 million tons) in the stochastic model. In the deterministic model the optimal escapement level is $\bar{s}^* = 2.780$ million tons.Thus we again observe a slightly lower optimal escapement level in the stochastic case. The difference is larger than with the Baltic cod fishery but at 10 356 tons (with a standard deviation of 4 784 tons) still quite small.

In Figure 2 we show the optimal policies for risk-averse fishermen.

FIGURE 2 about here

The results are similar to those obtained for Baltic cod. The differences in optimal escapements between the stochastic and deterministic cases are small in absolute value. The unambiguous effect of risk aversion is that difference increases and,

for sufficiently high degrees of risk aversion, optimal escapement is higher under uncertainty than in the deterministic case.

6 Discussion and Conclusion

In this paper we studied the effects of environmental uncertainties on optimal fishery management for both risk-neutral and risk-averse fishermen. To account for natural mortality and recruitment, we split the stock growth function of the fish stock into two processes. Following the biological approach taken in stock assessment models, we assume that natural mortality is fixed at a given value. The recruitment process, by contrast, depends on stochastically fluctuating environmental conditions.

We have demonstrated that the optimal escapement level can be higher or lower than in the deterministic setting, depending on the prudence of the value function. This in turn depends on (i) the biomass growth function, (ii) the profit function and (iii) the representative fisherman's utility function. Positive prudence gives rise to higher optimal escapement, whereas negative prudence results in a lower optimal escapement level.

For risk-neutral fisherman we showed that the question whether or not the optimal escapement increases with uncertainty is influenced by two counteracting cost effects: the convex-cost effect and the gambling effect. The convex-cost effect results from higher expected marginal costs due to the convexity of the cost function and tends to reduce optimal escapement. Intuitively, the convex-cost effect

means that optimal investment is smaller when the asset is risky. The gambling effect results from lower expected marginal costs under uncertainty because the harvestable biomass increases more strongly under favorable environmental conditions than under adverse environmental conditions. The gambling effect thus tends to increase optimal escapement. The finding that the effect of uncertainty on the optimal size of the fish stock is ambiguous under risk neutrality is in contrast to Reed's (1979) unambiguous result that that the optimal constant escapement level in a stochastic environment is equal to, or larger than, the optimal escapement level in a deterministic setting. This is because we have allowed that the processes of recruitment and adult survival are affected differently by environmental uncertainties. In the paper we have considered the case where recruitment is uncertain, but survival is deterministic. In Appendix D we show that the effect of uncertainty is ambiguous also in the case of stochastic survival. Under risk aversion, the effect of uncertainty on optimal escapement is still ambiguous, depending on the stock growth function parameter and the coefficient of constant relative risk aversion.

To quantify the effect of uncertainty both under risk neutrality and risk aversion, we applied the model to the Baltic cod and the North Sea herring fisheries. Under risk neutrality we observed in both fisheries lower optimal escapement in the stochastic environment than in the deterministic setting, but the difference is small in absolute value. In the setting with risk-averse fishermen we found higher optimal escapement levels for sufficiently high coefficients of constant relative risk aversion. Again, the difference is very small, and well below one percent of optimal

escapement.

This finding is in line with previous results of Sethi et al. (2005) who analyze the effect of multiple uncertainties on optimal fishery management. Their simulations show that stock growth uncertainty has only minor effects on the optimal fishery management. Our paper completes their findings: By introducing the two cost-effects and refering to the notion of prudence, we provide a deeper understanding and an intuitive explanation of the influence of environmental uncertainties on the optimal fishery management. We extend the analysis to the case of risk aversion, both analytically and numerically. Furthermore by applying our model to two real fisheries, incorporating the estimated variance in stock growth and the estimated cost function, we provide valid results for fishery management.

Our quantiative results show that the optimal feedback policy is not very strongly affected by environmental uncertainties. Our results suggest that even if fluctuations in environmental conditions would increase, for example due to climate change, there is little need to adapt fishery management recommendations. One should keep in mind, however, that other forms of uncertainty may have a much stronger effect, including measurement uncertainty with regard to the size of the fish stock (Sethi et al. 2005), or the possibility of regime shifts (Polasky et al. 2011, de Zeeuw and Zemel 2012).

Notes

[1]Reed assumes an objective function that is linear in harvest, which implies that neither the consumer nor the fishermen show risk aversion

[2]Pindyck (1984) studies the management of renewable resources under uncertainty in continuous time by adding a stochastic differential equation of the Ito type, assuming a downward sloping demand function. This can be interpreted as reflecting risk-aversion of consumers.

[3]Costello et al. (2001) do not restrict the disturbance to be multiplicative to the stock growth, but still assume that z_t is i.i.d with mean one, which implies that z_t is somehow multiplicatively connected.

[4]The influence of environmental conditions differ for different species. For Baltic cod and North Sea herring see ICES Advice 2010a, 2010b, Books 6 (herring) and 8 (cod).

[5]We follow the convention of the previous literature and chose the escapement as the control variable.

[6]Council Regulation (EC) No 1098/2007 of 18. September 2007.

[7]The assumption of a strictly concave utility function is sensible if fishermen have imperfect access to capital markets. Experimental evidence suggests that a typical value for the coefficient of relative risk aversion is about 0.74 (Andersen et al. 2008).

[8]Because the optimization problem (2) is autonomous, the value function $J(x)$ does not depend on time.

[9]It may appear contradictory that the 'gambling effect' leads to a higher degree of prudence. However, the term *prudence* has been coined because of its effect – a higher degree of prudence induces higher precautionary savings – and not because of a particular motivation why an agent saves more under uncertainty.

[10]Under constant unit harvesting costs both the gambling and the convex-cost effect vanish and the optimal escapement level in the stochastic and deterministic model are the same.

[11]Reed's (1979) intuitive explanation for a higher optimal escapement level is that "the marginal average annual harvesting cost in the stochastic model resulting from an increase in the escape-

ment level [...] is, because of the averaging process, less than the corresponding marginal cost in the deterministic model."

[12]To our knowledge, no analytically solvable model has been available for the discrete-time model so far. For a similar problem in continuous time, Pindyck (1984) provides three examples of analytically solvable models.

[13]If we again assume that z is log-normally distributed with variance σ_z^2, we obtain $(E[z^{1-\vartheta}])^{1/\vartheta} = (1+\sigma_z^2)^{(1-\vartheta)/2}$ (see appendix C).

[14]We also estimated the more general growth function of $r(s_t) = \left(\alpha\, s_t^{1-\phi} + \alpha\,\beta^{1-\phi}\right)^{\frac{1}{1-\phi}}$, but found the parameters to be not significant.

[15]A Durbin-Watson test shows no autocorrelation in the error terms ($DW = 1.718$, $p = 0.312$).

[16]For cod (herring), we use 101 (212) collocation nodes on the interval $x \in [0.1, 5]$ million tons ($x \in [0.1, 8]$ million tons). The optimization routines were implemented in Matlab. All program codes will be made available as online supporting material.

[17] The Durbin-Watson test revealed autocorrelation in the error term ($DW = 1.014$, $p = 3.647 * 10^{-5}$). As Nostbakken (2008) states, "[s]ome problems of autocorrelation are [...] inevitable when using a simple surplus growth model to explain the complex dynamics of the fish stock". Considering an autoregressive model for the error term means that another stock variable has to be included in the model. This would greatly increase the complexity of the stochastic optimization problem, which is becond the scope of the present paper.

Appendix

A Dynamic Programming and Constant Escapement Policy

We consider the problem for a finite time horizon T. The result for an infinite time horizon is then obtained by letting $T \to \infty$. The risk neutral fishery manager faces the following maximization problem:

$$\max_{s_t} E\Big[\sum_{t=1}^{T} \rho^{t-1} \int_{x_t - h_t}^{x_t} p - c(\tilde{x})\,\mathrm{d}\tilde{x}\Big] \tag{16}$$

$$\text{s.t. } x_{t+1} = g(s_t) + z_t\, r(s_t). \tag{17}$$

By letting $\Pi(x_t, s_t) = \pi(x_t) - \pi(s_t) = \int_{s_t}^{x_t} \pi'(v)\,\mathrm{d}v$ and $\pi'(v) = p - c(v)$ equation (16) can be also expressed as

$$\max_{s_t} E\Big[\sum_{t=1}^{T} \rho^{t-1}\big(\pi(x_t) - \pi(s_t)\big)\Big].$$

To demonstrate that the optimal management approach is of the constant escapement type analogous to Reed (1979) we solve the Bellman equation by backward induction:

$$J_n(x_{T-(n-1)}) = \max_{s_{T-(n-1)}} \Big\{\big(\pi(x_{T-(n-1)}) - \pi(s_{T-(n-1)})\big) + \rho E\big[J_{n-1}\big(x_{T-n}\big)\big]\Big\}, \tag{18}$$

where the operator E denotes the expectation over the probability distribution of the random variable z_t. First we solve the problem for the last period T:

$$J_1(x_T) = \max_{s_T}[\pi(x_T) - \pi(s_\infty)],$$

where s_T is assumed to be the escapement level corresponding to the open-access

fishery s_∞. Now we consider the problem for the penultimate period:

$$J_2(x_{T-1}) = \max_{S_{T-1}} \Big\{ \big(\pi(x_{T-1}) - \pi(s_{T-1})\big) + \rho E\big[\pi(x_T) - \pi(s_\infty)\big] \Big\},$$

where s_∞ is a constant and $x_T = g(s_{T-1}) + z_t r(s_{T-1})$, such that the previous equation can be written as

$$J_2(x_{T-1}) = \max_{S_{T-1}} \Big\{ \big(\pi(x_{T-1}) - \pi(s_{T-1})\big) + \rho E\big[\pi\big(g(s_{T-1}) + z_t r(s_{T-1})\big)\big] \Big\} + \text{const.} \quad (19)$$

Under the assumptions on the curvature properties of $g(\cdot)$, $r(\cdot)$ and $c(\cdot)$, this problem has a unique maximum at an escapement level which we denote by s^*. This escapement level is optimal if the stock at the beginning of the period (x_t) is greater than s^*. The optimal policy is a most rapid approach strategy to s^*. This can be validated by inserting s^* in $J_2(x_{T-1})$ and substituting the result in $J_3(x_{T-2})$:

$$J_2(x_{T-1}) = \big(\pi(x_{T-1}) - \pi(s^*)\big) + \rho E\big[\pi\big(g(s^*) + z_t r(s^*)\big)\big] + \text{const.}$$

Since s^* is constant, in particular note, that s^* is independent of stock, x_t, the function can be rewritten as

$$J_2(x_{T-1}) = \pi(x_{T-1}) + \text{const.}$$

Now we consider $J_3(x_{T-2})$:

$$J_3(x_{T-2}) = \max_{S_{T-2}} \Big\{ \big(\pi(x_{T-2}) - \pi(s_{T-2})\big) + \rho E\big[J_2(x_{T-1})\big] \Big\}. \quad (20)$$

Inserting $J_2(x_{T-1}) = \pi(x_{T-1}) + \text{const}$ yields

$$J_3(x_{T-2}) = \max_{S_{T-2}} \Big\{ \big(\pi(x_{T-2}) - \pi(s_{T-2})\big) + \rho E\big[\pi\big(g(s_{T-2}) + z_t r(s_{T-2})\big)\big] \Big\} + \text{const.} \quad (21)$$

Since $J_3(x_{T-2})$ is equivalent to $J_2(x_{T-1})$ in (19) except for the constant the constant escapement strategy s^* is also valid for the period $T-2$. By complete induction the same holds for all periods.

B Optimal Escapement Level

We now consider the optimal escapement level in the stochastic growth model. By differentiating equation (18) with respect to $s_{T-(n-1)}$ we get the following condition:

$$\pi'(s^*) = \rho\, E\Big[\pi'\big(g(s^*) + z_t\, r(s^*)\big)\big(g'(s^*)\ + z_t\, r'(s^*)\big)\Big].$$

Substituting $p - c(v)$ for $\pi'(v)$ and rearranging we get

$$p - c(s^*) = \rho\, g'(s^*) \left(p - E\Big[c\big(g(s^*) + z_t\, r(s^*)\big)\Big] \right) + \rho\, r'(s^*) \left(p - E\Big[z_t\, c\big(g(s^*) + z\, r(s^*)\big)\Big] \right). \quad (22)$$

C Increasing uncertainty when survival is proportional to recruitment

Inserting the cost function $c(\tilde{x}) = \zeta\, \tilde{x}^{-\chi}$ in equation (9) and rearranging we get

$$p - c(s^*) = \rho f'(s^*) \Big(p - \frac{\zeta}{f(s^*)^{\chi}} E_{\tilde{z}}[\tilde{z}^{1-\chi}] \Big).$$

With $\tilde{z}$ as a lognormally-distributed random variable the expectation can be calculated as

$$E_{\tilde{z}}[\tilde{z}^{1-\chi}] = \int_0^\infty \tilde{z}^{1-\chi} \frac{1}{\tilde{z}\sqrt{2\pi s_t^2}} \exp\Big(-\frac{(\ln z - m_{\tilde{z}})^2}{2s_t^2}\Big) \mathrm{d}z.$$

With $\mu_{\tilde{z}} = 1$, the parameters are $m_{\tilde{z}} = -\frac{1}{2}\ln\big(1+\sigma_{\tilde{z}}^2\big)$ and $s_{\tilde{z}}^2 = \ln\big(1+\sigma_{\tilde{z}}^2\big)$. Substituting $\ln(z) = q$ we get

$$E_{\tilde{z}}[\tilde{z}^{1-\chi}] = \int_0^\infty \exp\big((1-\chi)q\big) \frac{1}{\sqrt{2\pi s_t^2}} \exp\Big(-\frac{(q - m_{\tilde{z}})^2}{2s_t^2}\Big) \mathrm{d}q,$$

which yields

$$E_{\tilde{z}}[\tilde{z}^{1-\chi}] = \Big(1+\sigma_{\tilde{z}}^2\Big)^{\frac{\chi(1-\chi)}{2}}.$$

For $\chi \in (0,1)$ the expectation decreases with increasing variance σ^2. This results in lower expected costs and higher expected profits and gives thus an incentive to choose a higher escapement level.

D Optimal escapement with both mortality and recruitment stochastic

In this appendix we briefly consider the case where both processes, survival and recruitment, are stochastic, i.e. we replace (1) by

$$x_{t+1} = w_t\, g(s_t) + z_t\, r(s_t), \tag{23}$$

with two i.i.d. random variables w_t and z_t. The condition for the optimal escapement level (6) under risk-neutrality then generalizes to

$$p - c(s^*) = \rho\, g'(s^*)\ \Big[p - E_{w,z}\Big[w_t\, c\big(w_t\, g(s^*) + z_t\, r(s^*)\big)\Big]\Big] + \rho\, r'(s^*)\ \Big[p - E_{w,z}\Big[z_t\, c\big(w_t g(s^*) + z\, r(s^*)\big)\Big]\Big]. \quad (24)$$

If w_t and z_t are perfectly correlated, this model is equivalent to Reed's (1979) model, and the optimal escapement level under uncertainty is unambiguously larger than in the deterministic case. If w_t and z_t are not perfectly correlated, the effect of uncertainty on the optimal escapement level is ambiguous. This holds in particular for the case where w_t and z_t are independent. To show this, we consider the example of the Baltic Cod fishery considered in Section 4. The only modification is that we assume that the two random variables $z_t \propto \text{LOGN}(1, \sigma_z^2)$ and $w_t \propto \text{LOGN}(1, \sigma_w^2)$ are independently distributed with $\sigma_w^2 \geq 0$. For $\sigma_z^2 = \sigma_w^2 = 0$, we obtain $\bar{s}^* = 0.904$ million tons (see Section 4). For the example considered here, we set $\sigma_z^2 = 0.5$, slightly higher than the empirical value. For $\sigma_w = 0.1$, we get $s^* = 0.901 < \bar{s}^*$. For $\sigma_w = 0.5$, we get $s^* = 0.911 > \bar{s}^*$. Thus, the effect of uncertainty is ambiguous.

E Optimal escapement with risk aversion

Using $c(\tilde{x}) \equiv 0$, the biomass growth equation (8) with the specification (11), $u(\Pi) = \nu\, \Pi^{1-\vartheta}/(1-\vartheta)$, and $\phi = \vartheta$ in (4), the first-order condition for optimal

escapement becomes

$$\frac{\nu\, p^{1-\vartheta}}{(x-s)^{\vartheta}} = \rho\, E\left[\frac{d}{ds} J\left(z\ \left(\alpha\, s^{1-\vartheta} + \alpha\, \beta^{1-\vartheta}\right)^{\frac{1}{1-\vartheta}}\right)\right] \tag{25}$$

Guessing $s = \delta\, x$ with some $\delta > 0$ and (13) for the value function, condition (25) becomes

$$\frac{\nu\, p^{1-\vartheta}}{((1-\delta)\, x)^{\vartheta}} = \rho\, \psi_1\, E\left[z^{1-\vartheta}\right]\ \alpha\, (\delta\, x)^{-\vartheta} \tag{26}$$

$$\Leftrightarrow \qquad \nu\, p^{1-\vartheta} \left(\frac{\delta}{1-\delta}\right)^{\vartheta} = \rho\, \alpha\, E\left[z^{1-\vartheta}\right]\ \psi_1 \tag{27}$$

The Bellman-equation reads

$$J(x) = \frac{\nu\, p^{1-\vartheta}}{1-\vartheta}\ ((1-\delta)\, x)^{1-\vartheta} + \rho\, E\left[J(z\, f(\delta\, x))\right]$$

Using the guess (13) again, we obtain

$$\frac{\psi_1}{1-\vartheta}\, x^{1-\vartheta} + \psi_2 = \frac{\nu\, p^{1-\vartheta}}{1-\vartheta}\ ((1-\delta)\, x)^{1-\vartheta} + \rho\, E\left[\frac{\psi_1}{1-\vartheta}\, z^{1-\vartheta}\ \left(\alpha\, (\delta\, x)^{1-\vartheta} + \alpha\, \beta^{1-\vartheta}\right) + \psi_2\right]$$

$$\psi_1 = \nu\, p^{1-\vartheta}\, (1-\delta)^{1-\vartheta} + \rho\, E[z^{1-\vartheta}]\, \alpha\, \delta^{1-\vartheta}\, \psi_1 \tag{28}$$

$$\psi_2 = \frac{\rho}{1-\rho}\, E[z^{1-\vartheta}]\, \alpha\, \beta^{1-\vartheta}\, \psi_1 \tag{29}$$

Using (28) in (27), we obtain (14).

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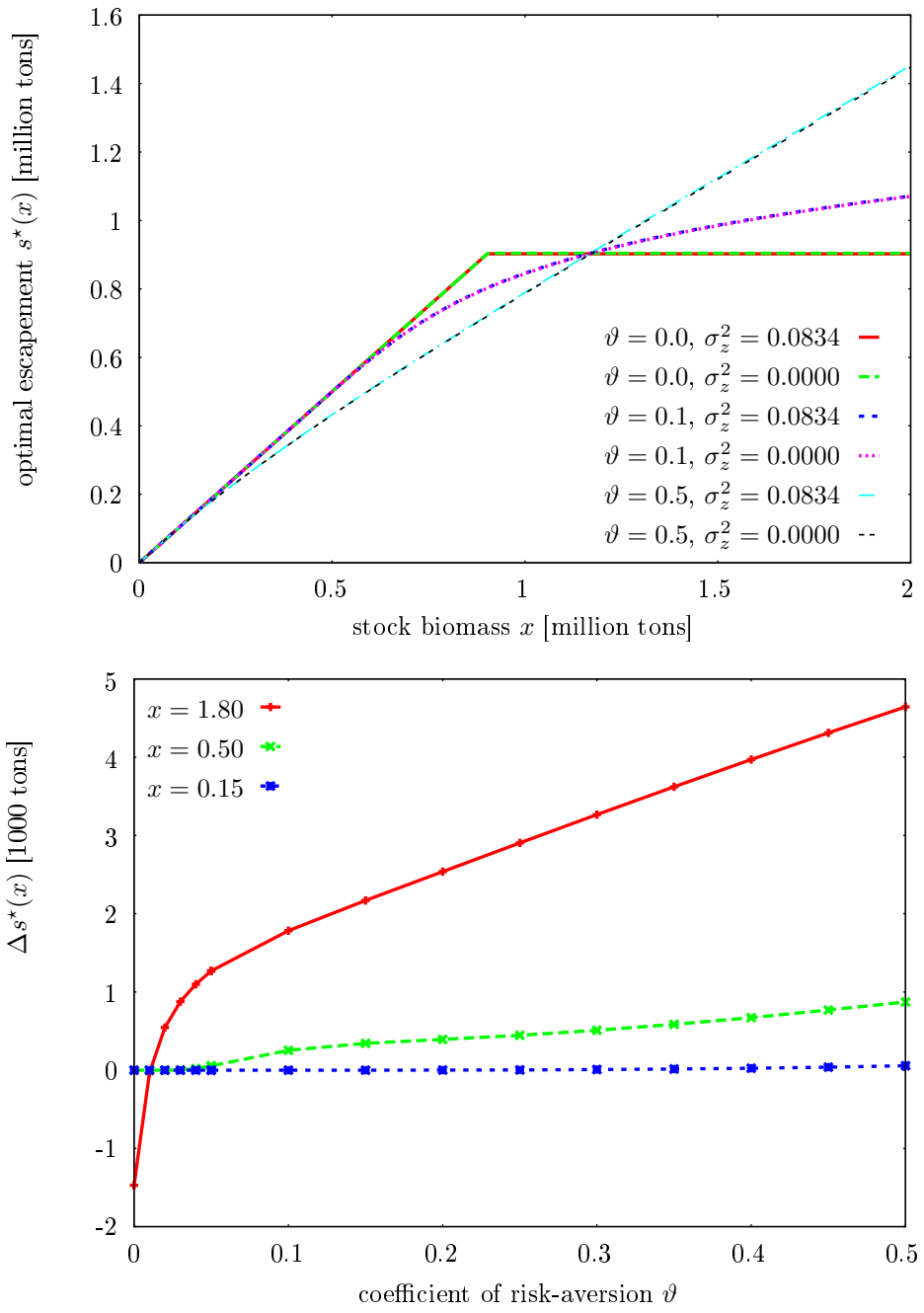

Figure 1: Optimal escapement for Eastern Baltic cod as a function of the representative fisherman's coefficient of risk aversion ϑ. Note that the scale on the y-axis in the upper panel is in millions of tons, while it is in thousands of tons in the lower panel.

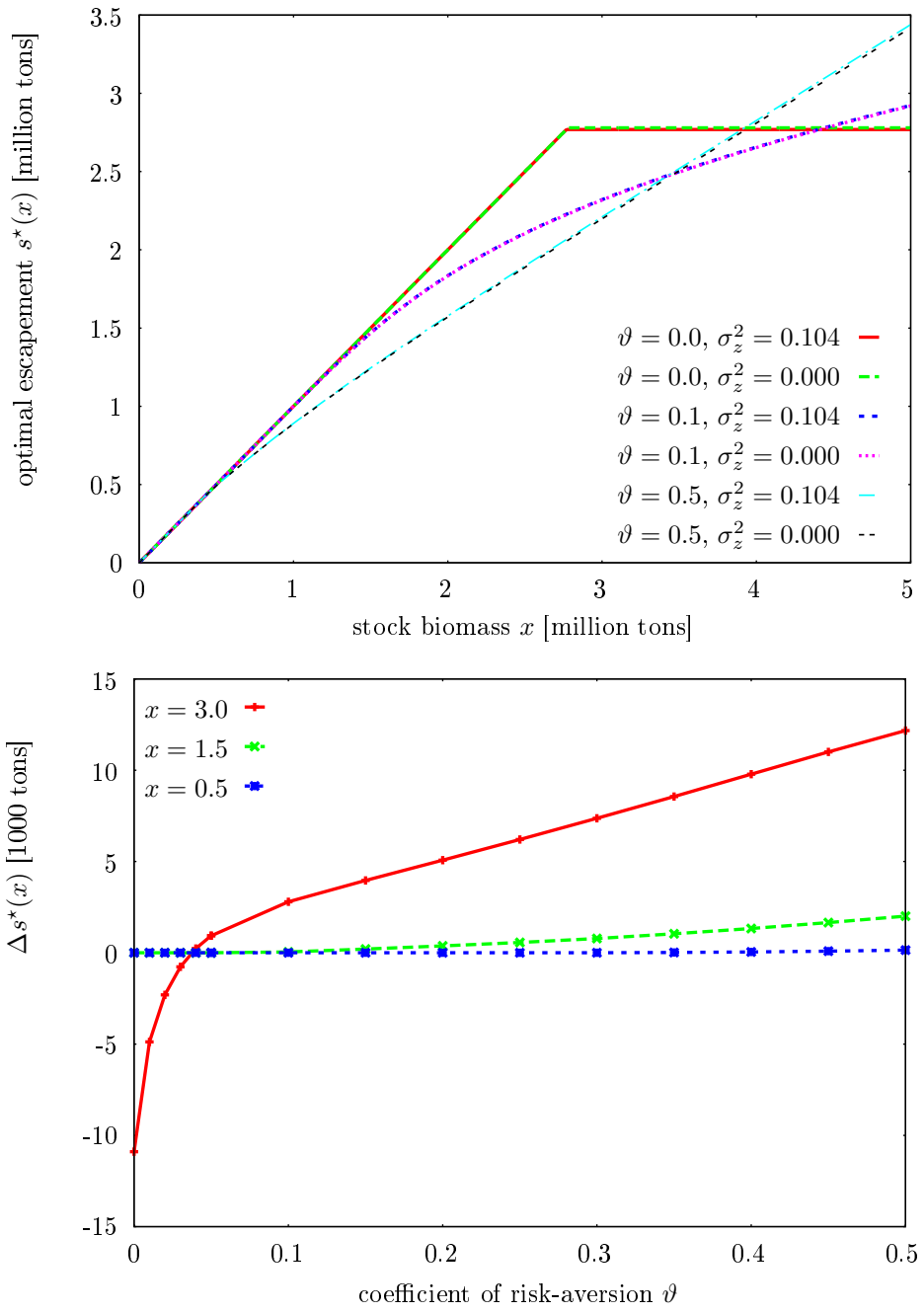

Figure 2: Optimal escapement for North Sea herring as a function of the representative fisherman's coefficient of risk aversion ϑ. Note that the scale on the y-axis in the upper panel is in millions of tons, while it is in thousands of tons in the lower panel.