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A Structural Model of Establishment and Industry Evolution

Evidence from Chile

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Abstract

Many recent models have been developed to fit the basic facts on establishment and industry evolution. While these models yield a simple interpretation of the basic features of the data, they are too stylized to confront the micro-level data in a more formal quantitative analysis. In this paper, the author develops a model in which establishments grow by innovating new products. By introducing heterogeneity to a stylized industry evolution model, the analysis succeeds in explaining several features of the data, such as the thick right tail of the size distribution and the relations between age, size, and the hazard rate of exit, which had eluded existing models. In the model, heterogeneity in producer behavior arises through a combination of exogenous efficiency differences and accumulated innovations resulting from past endogenous research and development investments. Integrating these forces allows the model to perform well quantitatively in fitting data on Chilean manufacturers. The counterfactual experiments show how producers respond to research and development subsidies and more competitive market environments.

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A Structural Model of Establishment and Industry Evolution: Evidence from Chile*

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1 Introduction

Empirical research using longitudinal firm or plant level data has shown strong regularities in establishment and industry evolution. Recently, researchers have started to build structural models to explain these regularities. However, being so stylized has made it difficult for these models to confront the micro-level data in a more formal quantitative analysis. In this paper, I present a model that explains several salient features of the data that had eluded the existing models. Furthermore, using a panel of Chilean Manufacturers, I provide an estimation of the model parameters and show its quantitative success in fitting the data.

This paper builds on the stylized model of establishment and industry evolution presented by Klette and Kortum (2004). In their model, an establishment is defined as a collection of products and each product evolves independently. Every product owned by an establishment can give rise to a new product as a result of a stochastic innovation process or can be lost to a competitor. This birth and death process for the number of products is the source of establishment and industry evolution. Through this parsimonious model of innovation, they explain various stylized facts that relate R&D, productivity, patenting, and establishment growth. Their model also generates heterogeneity in establishments' sizes and a skewed size distribution.

However, the Klette and Kortum (2004) model fails to capture several important features of the data. Through a parsimonious extension of their model, I succeed in explaining these features which are: the thick right tail of the size distribution, the independent relation between age, size and the hazard rate of exit, the relation between the variance of growth rate and size, and the pre-exit behavior of establishments in a birth cohort. At the root of the improvements over the Klette and Kortum (2004) model is the introduction of heterogeneity in producers' efficiency levels[1]. As a result of this heterogeneity, producers differ from each other in both their innovation rates and the revenues generated from each product.

The improvements and how they emerge through this extension can be explained as follows. The first improvement is in fitting the size distribution. In the model, more efficient producers are more innovative and they generate more revenue per product. This complementarity between the innovation rate and the revenue magnifies the variation among establishments' sizes

[1]Lentz and Mortensen (2008) also incorporates heterogeneity into Klette and Kortum (2004) model in a different way which I explain through the paper, is not sufficient for the improvements I specify here.

and leads to a closer fit to the observed size distribution, especially in the thickness of the right tail. On the other hand, the size distribution derived in the Klette and Kortum (2004) model is logarithmic. This distribution is skewed but the tail is much shorter than the one observed in the data. Through introducing random sized products, Lentz and Mortensen (2008) improve the size distribution that emerges in Klette and Kortum (2004), but still they cannot capture the thickness of the right tail. My model succeeds this without postulating any exogenous variation in the size of the products.

The second improvement caused by this extension is on the relation between the variance of growth rate and size. This relation has drawn much less attention than the relation between the growth rate and size, both in theoretical and empirical research. Some recent work by Stanley et al (1996), Bottazzi (2001), and Sutton (2002, 2007) illustrate that variance of growth rates declines at a rather slow rate as size of an establishment increases. The mixing of producers with different efficiency levels at any size allows my model to explain the flatness of this relation. On the other hand, the models of Klette and Kortum (2004) and Klepper and Thompson (2007) yield too steep slopes.

The third improvement of the model is on the independent relation between size, age, and the hazard rate of exit. Evidence shows that as size and age increase, the hazard rate of exit decreases[2]. In my model, exit is determined by the number of products owned by the producer. Mixing of establishments with different efficiency-types allows age and size to be correlated with the number of products through different channels. This yields a negative relation between the hazard rate of exit and size conditional on age and a negative relation between the hazard rate of exit and age conditional on size. Klette and Kortum (2004) and Luttmer (2007) can only explain the relation between the hazard rate of exit and size[3] and Klepper and Thompson (2007) explain both relations through introducing random sized products.

The final improvement is on the pre-exit behavior of establishments within a birth cohort. Evidence on Chilean manufacturers shows that there exists size dispersion among entrants and on average, establishments with larger startup sizes live longer than the smaller ones. I explain these observations through type-heterogeneity. All entrants start with a single product. But,

[2]This relation has been demonstrated by several studies including Dunne, Roberts, Samuelson (1988) and Evans (1987a, 1987b).

[3]Without conditioning on size, both models can generate a negative relation between the hazard rate of exit and age.

more efficient producers have larger startup sizes. Moreover, they are more innovative and face lower hazard rate of exit. As a result, conditional on when they will exit, larger producers survive longer.

Another novel feature of this paper is its quantitative strength. Most of the current models derive inferences about the heterogeneity in producer behavior by analyzing broadly defined sectors such as manufacturing, wholesale and retail, or service. Part of this observed heterogeneity could be purely due to different industry structures instead of the intrinsic efficiency differences across producers. To have a better identification of the source of heterogeneity across producers, I estimate the model's parameters separately on the five biggest 3-digit manufacturing industries in Chile. These industries differ in their size distributions, growth rate distributions, and entry rates. Estimation results show that model parameters can successfully explain different industry structures.

This paper also gives insight into the persistent differences in the performances of the establishments. It incorporates intrinsic exogenous efficiency differences that are determined before entering the economy and idiosyncratic innovations that endogenously accumulate during the life of an establishment. Both features have been used extensively to explain industry dynamics. My model incorporates both features in the producer's optimization problem. At the early stages of life, efficiency differences are the main contributors of the variation in size. At older ages, due to the selection of more efficient producers, contribution of the past innovations exceeds the contribution of the efficiency differences.

After capturing various features of the data, I perform counterfactual experiments. I analyze two policies that affect the innovation capacities of the producers. The model allows me to analyze how the policies affect producers at different sizes. In the first experiment, I look at the effects of an R&D subsidy. In the second experiment, I increase product market competition in the economy. In the model, innovations are made by incumbent producers which is in contrast with most previous models of creative destruction. Hence, the results of these experiments differ from their results. The experiments show that small producers are affected more from either policy change than the large producers.

The rest of the paper is organized as follows. Next, I summarize the related literature. Following that, I formulate the model and show its qualitative implications. Then, I estimate

the model using simulated method of moments and discuss the results. Following that, I perform a variance decomposition analysis of establishments' sizes and perform the counterfactual experiments. I finish the paper with some concluding remarks.

1.1 Related Literature

Industry evolution has drawn a lot of attention by researchers since late 1950s. At early stages, Simon and his coauthors (Simon and Bonnini (1958), Ijiri and Simon (1964, 1977)) succeeded in generating stochastic growth models providing a good approximation to the size distribution of large U.S. manufacturing firms. However, these models lacked structural foundations. The availability of detailed longitudinal panels since the 1980s accelerated the development of theoretical models based on optimizing agents and the analysis of the regularities in establishment and industry evolution. Sutton (1997) presents a detailed summary of the findings of this early literature.

Many recent studies seek to explain these regularities in a structural way. One of these models is introduced by Klette and Kortum (2004). Their model is based on the quality-ladder model of Grossman and Helpman (1991). Producers engage in innovation activity which results in Poisson arrivals of quality improvements over the existing products. The new quality leader of a product drives the incumbent producer out of the market and becomes the monopoly supplier. Lentz and Mortensen (2008) introduce heterogeneity into the Klette and Kortum (2004) model through different quality step choices of firms. This extension enables them to match several moments of the data and perform structural aggregate productivity decomposition. Another model that is related to Klette and Kortum (2004) is Luttmer (2008). In his model, innovations come as new varieties and the number of varieties in the economy grows at the rate of the population growth. He characterizes a balanced growth path for an economy where firms grow by developing new blueprints from their goods. High skilled entrepreneurs can also develop new blueprints from scratch and set up new firms.

This paper follows all three studies mentioned above. In the dynamics of establishment evolution, it follows Klette and Kortum (2004). In the way the innovations arrive, it follows Luttmer (2008). It is similar to Lentz and Mortensen (2008) in introducing heterogeneity into the Klette and Kortum (2004) setup.

However the way heterogeneity is introduced here follows from Melitz (2003). As in his model establishments are born with different efficiency levels. Here, this heterogeneity generates different innovation intensities across producers. Hence, I extend Melitz (2003) type static monopolistic competition models into a dynamic framework where establishments' sizes evolve over time. Moreover, compared to Melitz (2003) model, a smaller amount of dispersion in efficiency levels can generate a huge amount of size dispersion.

In the model, producers own multiple products. In a recent study, Bernard, Redding, and Schott (2006a, b) provide empirical evidence on how multi-product producers dominate total production in the U.S. economy. They also construct a static model of multi-product firms and analyze their behavior during trade liberalization. They introduce two margins (intensive and extensive) to expand size, and these margins are positively correlated with each other. However what causes size differences across producers in their model is different than the one presented here.

In another study, Klepper and Thompson (2007) construct a model that explains the subtle relations between size, age, growth, and survival. In their model, there is no heterogeneity across producers and their simple framework allows them to analytically characterize a wide range of regularities on industry dynamics. An establishment is a collection of random sized products and this randomness allows them to capture the independent relation between size, age, and the hazard rate of exit.

Fitting the observed size distributions has been an important feature of recent industry evolution models. Luttmer (2007) presents a model of firm and aggregate growth that is consistent with the observed size distribution of U.S. firms. Firms grow as a result of idiosyncratic productivity shocks, imitation by entrants, and selection. Using different mechanisms, both Luttmer (2007) and this model successfully capture the thick right tail of the size distribution. Luttmer (2007) also characterizes the balanced growth path of the economy while the focus here is on a single industry. Both models differ in their explanations of the relation between the variance of growth rates and size.

The model complements the work of Stanley et al (1996), Bottazzi (2001) and Sutton (2002, 2007) on explaining the relation between the variance of growth rates and size. All these studies propose simple statistical explanations to the observed relation in the data. Here, instead, I

Table 1: Industry Details

Industry Code	Before†	After*	Industry Name
311-312	3453	3244	Food Manufacturing and Food Products
321-322	1827	1735	Manufacturing of Textiles, Apparel except Footwear
331	1157	1092	Manufacturing of Wood and Cork Products except Furniture
341-342	710	646	Manufacturing of Paper, Paper Products, Printing, Publishing
381	1305	1212	Manufacturing of Fabricated Metal Products except Machinery

† Total number of observations in the original dataset. * Number of observations used in the analysis after the exclusion of observations due to missing variables and industry switches.

present a structural model incorporating optimizing firms.

1.2 Dataset

In this study, I use data from Chilean Manufacturing Census (Encuesta Nacional Industrial Anual, ENIA) which is provided by Chile's National Statistics Institute (INE). The dataset is an unbalanced panel of all establishments with 10 or more workers from 1979 to 1997. I use data on eight biggest 3-digit industries which are described in Table 1. Data is at the establishment level. In Chile, most of the firms had single establishments; hence the distinction between a firm and an establishment is not very crucial. Hsieh and Parker (2007) note that in 1984 only 350 establishments were associated with multi-establishment firms[4]. In the original dataset there were 8452 establishments and after excluding observations due to missing variables and industry switches, I used 7929 establishments in the analysis. Roughly the same 3-digit industries used here are analyzed in a study by Bergoeing, Hernando, Repetto (2005) who note that these industries represent around 60% of total value added in the Manufacturing Census. Table 2 shows the number of establishments of different entry cohorts observed during the span of the study.

[4]Moreover, Caves (1998) points out that most of the results on firm growth and turnover which form the main discussion in this paper, have been insensitive to the distinction between establishment and firm.

Table 2: Number of Establishments Observed at Different Years

Cohorts	1980	1985	1990	1995	1997
- 1980	3225	2061	1452	1251	800
1981-1985		501	268	209	111
1986-1990			769	528	277
1991-1995				827	347

2 The Model

Klette and Kortum (2004) propose a stylized model with a simple interpretation of establishment and industry evolution. In Şeker (2006), I showed the quantitative strength of their qualitative results on establishment dynamics on a dataset of Chilean manufacturers. Here, I extend their work to be able to explain several other features of the data. To achieve this, I construct a model which combines the static setup of Melitz (2003) with the dynamic setup of Klette and Kortum (2004).

In the Klette and Kortum (2004) model, there is a fixed number of products and a producer expands into new markets through quality improvements on the existing products. Here, the producer grows by innovating new varieties.

This model focuses on solving the partial equilibrium for a single industry in steady state. The partial equilibrium analysis simplifies the analytical solution and the computation of the model and it is more appropriate to use since the focus is on a single industry rather than the whole economy. In this industry, total labor supply is fixed.

2.1 Producer's Problem

The industry consists of a large group of monopolistically competitive producers. Consumption of the composite good Y_t is determined by the CES production function given in equation 1

$$Y_t = \left(\int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The measure of the set J represents the mass of available products each of which is indexed by j. As a result of the steady state solution which will be derived below, J is fixed. Consumers have a taste for variety and consume $y_t(j)$ units of variety j. Goods are substitutes with elasticity of substitution $\sigma > 1$.

Producers are distinguished only by their efficiency levels, indexed by $\varphi > 0$ with their production functions given as $y_t = \varphi_t l_t$. Producers with same efficiency-types will charge the same price and make the same profit per product. However, the number of products they produce may vary as the result of the stochastic innovation process. The static profit maximization problem for any product market for a given wage rate $\bar{w}_t$ yields a price $p_t(\varphi) = \frac{\sigma \bar{w}_t}{(\sigma-1)\varphi_t}$ and revenue

$$\bar{r}_t(\varphi) = \left(\frac{p_t(\varphi)}{P_t}\right)^{1-\sigma} E_t \tag{2}$$

where $E_t = P_t Y_t$ is the aggregate expenditure of the composite good and P_t is the aggregate price index. For an establishment with n_t products, aggregate profit is

$$\bar{\pi}_t^{agg}(\varphi) = \bar{\pi}_t(\varphi)\, n_t. \tag{3}$$

Here $\bar{\pi}_t(\varphi)$ is the profit per product given as

$$\bar{\pi}_t(\varphi) = \left(\frac{\sigma \bar{w}_t}{(\sigma-1) P_t}\right)^{1-\sigma} \frac{E_t}{\sigma} \varphi_t^{\sigma-1}. \tag{4}$$

The efficiency levels of producers grow at an exogenous rate g_φ. This rate is fixed and common across all producers. As will be shown below, without this assumption, an average producer shrinks in size over time and the growth rate distribution cannot be fitted in the estimation. Efficiency growth is the only source of growth in the aggregate economy. Hence the aggregate expenditure E_t and the wage rate $\bar{w}_t$ grow at this rate.

The number of varieties n determines the portfolio of the producer. This portfolio increases by innovation of new products and it decreases by destruction of the current products. To succeed in innovation, the producer invests in R&D. This investment determines the Poisson arrival rate λn of new innovations and it is formulated as $nc(\lambda) = nc_0\lambda^{1+c_1}$ for c_0, $c_1 > 0$. This strictly increasing and convex cost function reflects the labor input required for R&D. Klette and Kortum (2004) provide motivation for incorporating the number of products in the R&D cost[5]. In the mean time, the producer faces a Poisson hazard rate μn of losing any product. The hazard rate μ is fixed and same for all establishments. Exit from the market occurs when

[5]Basically, n reflects the knowledge capital of the establishment which stands for the know-how and techniques the producer has learned with its previous innovations.

all products are destroyed. There is no reentering the market once exit takes place.

2.2 Value Function

The model can be solved using undetermined coefficients method. To have a simple analytical solution to the producer's value function, I will define $\pi(\varphi) = \frac{\bar{\pi}_t(\varphi)}{E_t}$ and $w = \frac{\bar{w}_t}{E_t}$. In the Appendix, I show how this assumption gives a stationary $\pi(\varphi)$, $p(\varphi)$, and P for $\forall\ \varphi$. I also show how $\bar{\pi}_t(\varphi)$, and E_t grow at the same rate. Moreover, in the Appendix, I present a simple formulation of how this industry of interest can be incorporated with the aggregate economy.

The state of the producer is determined by its number of products n. The dynamic maximization problem of a particular φ efficiency-type producer, for a constant interest rate r[6], is formulated in the following Bellman equation

$$rV_\varphi(n) = \max_{\lambda \geq 0} \left\{ \begin{array}{c} \pi(\varphi)\, n - wnc(\lambda) + \\ n\lambda\left[V_\varphi(n+1) - V_\varphi(n)\right] \\ +\mu n\left[V_\varphi(n-1) - V_\varphi(n)\right] \end{array} \right\}. \tag{5}$$

In this equation, the producer maximizes current profit net of R&D cost and its net future value.

To derive the solution of the Bellman equation, I conjecture that the value function is given as

$$V_\varphi(n) = \left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) n. \tag{6}$$

Here, $K(\varphi)$ is the continuation value of innovating. Substituting this value function into equation 5, I get the following equation, with the details of the derivation given in the Appendix,

$$(r+\mu)\, K(\varphi) = \max_{\lambda \geq 0} \left\{ \lambda\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) - wc(\lambda) \right\}. \tag{7}$$

Using this equation, I will find the value of $K(\varphi)$ and solve the stationary industry equilibrium. Before doing that, let's define the problem of the entrants.

[6] Constancy of r is shown in the Appendix.

2.3 New Entrants

Entry requires an innovation. Establishments discover their efficiency types when they enter the market. All entrants start with a single product but they don't necessarily have the same startup size because of their differing efficiency levels. More efficient establishments have larger product sizes which can be seen in equation 2. The potential entrants innovate at rate λ_e. They also face the same innovation cost function as the incumbents[7]. The free entry condition given below determines the entry rate in the industry

$$\underbrace{wc'(\lambda_e)}_{\text{marginal cost of innovation}} = \underbrace{\int V_\varphi \phi(\varphi)\, d\varphi}_{\text{net gain from innovation}} . \tag{8}$$

Here V_φ is the value of a single product for a φ efficiency-type producer, and $\phi(\varphi)$ is the efficiency-type distribution of entrants. Entry rate η is determined by the multiplication of λ_e and the constant measure of potential entrants M_e.

2.4 Stationary Industry Equilibrium

Recall that the interest rate r, wage w, and the hazard rate of exit μ are constant. A stationary equilibrium for this industry consists of innovation rate $\lambda(\varphi)$, for all efficiency-types and the entry rate η such that for any given w and r: (i) any φ efficiency-type incumbent producer solves equation 7 to maximize its value (ii) potential entrants solve equation 8 and break even in expectation.

Lentz and Mortensen (2005) provide a proof for the existence of equilibrium for a model closely related to mine. To guarantee the existence, I need that $\mu > \lambda(\varphi)$ for $\forall\ \varphi$. Otherwise size and age of some establishments diverge to infinity which precludes having a stationary size distribution[8]. The first order condition for equation 7 is

$$\lambda : \frac{\pi(\varphi)}{r+\mu} + K(\varphi) = wc'(\lambda) \tag{9}$$

The value of $K(\varphi)$ is derived from equation 7 as follows

[7] This is a simplifying assumption and the same structure for the innovation cost of entrants is used in Lentz and Mortensen (2008).

[8] The condition needed to guarantee $\mu > \lambda(\varphi)$ for $\forall\ \varphi$ is $wc'(\mu) > \frac{\pi - wc(\mu)}{r}$.

$$K(\varphi) = \max_{\lambda \geq 0} \left\{ \frac{\frac{\lambda \pi(\varphi)}{r+\mu} - wc(\lambda)}{r+\mu-\lambda} \right\}. \tag{10}$$

Implementing the value of $K(\varphi)$ into equation 9 and after some algebra, I get

$$wc'(\lambda) = \frac{\pi(\varphi) - wc(\lambda)}{r+\mu-\lambda}. \tag{11}$$

The right hand side of equation 11 is equal to the value of a single product V_φ. Klette and Kortum (2004) show that λ increases in π. In equation 4, π is increasing in φ. Hence, more efficient producers are more innovative. From equation 2, we know that they also generate more revenue from each product they produce. This complementarity between innovation rates and product sizes increase the size differences between low and high efficiency-type producers which consequently stretches out the right tail of the size distribution. It also implies that a smaller dispersion in efficiency levels (in models like Hopenhayn (1992) or Melitz (2003)) is required to explain the observed size dispersion. This property of the model is the main reason for obtaining the qualitative and quantitative success of the model.

2.5 Dynamics of Size Evolution

After solving the model for the optimal innovation rate, the evolution of an individual producer conditional on its efficiency-type can be characterized. For ease of notation, let λ denote the innovation rate $\lambda(\varphi)$ for a particular $\varphi-$type producer. At any moment, the number of products an establishment produces can stay the same, increase by one unit as a result of an innovation, or decrease by one unit due to the exogenous destruction rate. Denote $p_n(t; n_0|\varphi)$ as the probability that an establishment has n products at time t conditional on having n_0 products at time 0 and being type φ. This probability changes over time at rate $\dot{p}_n(t; n_0|\varphi)$. The following system of equations describe the evolution of an individual $\varphi-$type producer[9],

$$\begin{aligned} \dot{p}_n(t; n_0|\varphi) &= (n-1)\lambda p_{n-1}(t; n_0|\varphi) + (n+1)\mu p_{n+1}(t; n_0|\varphi) - n(\lambda+\mu) p_n(t; n_0|\varphi) \text{ for } \forall n \geq 1 \\ \dot{p}_0(t; n_0|\varphi) &= \mu p_1(t; n_0|\varphi). \end{aligned} \tag{12}$$

[9] A formal solution of this system of equations is given in Appendix C of Klette and Kortum (2004).

The solution to this set of coupled difference-differential equations yields a geometric distribution for establishment size at time t conditional on survival

$$\begin{aligned} \frac{p_n(t;1|\varphi)}{1-p_0(t;1|\varphi)} &= (1-\chi(t|\varphi))\,\chi(t|\varphi)^{n-1}, n \geq 1 \\ \text{where } \chi(t|\varphi) &= \frac{\lambda}{\mu} p_0(t;1|\varphi) = \frac{\lambda\left(1-e^{-(\mu-\lambda)t}\right)}{\mu-\lambda e^{-(\mu-\lambda)t}} \end{aligned} \tag{13}$$

where $\chi(a|\varphi)$ is the parameter of the size distribution. The solution of this system can be used to derive the moments of the growth rate of the number of products an establishment owns. The expected growth rate and the variance of growth rate of the number of products conditional on initial size n_0, are given as

$$E[G_t(\varphi)|N_0 = n_0] = e^{-(\mu-\lambda)t} - 1 \tag{14}$$

$$V[G_t(\varphi)|N_0 = n_0] = \frac{\lambda+\mu}{n_0(\mu-\lambda)} e^{-(\mu-\lambda)t}\left(1-e^{-(\mu-\lambda)t}\right). \tag{15}$$

The expected growth rate of total size relative to aggregate expenditure can also be determined. Size of a $\varphi-$type establishment is given as

$$S_t(\varphi) = \left(\frac{p(\varphi)}{P}\right)^{1-\sigma} n_t(\varphi). \tag{16}$$

$S_t(\varphi)$ grows due to the growth of the number of products.

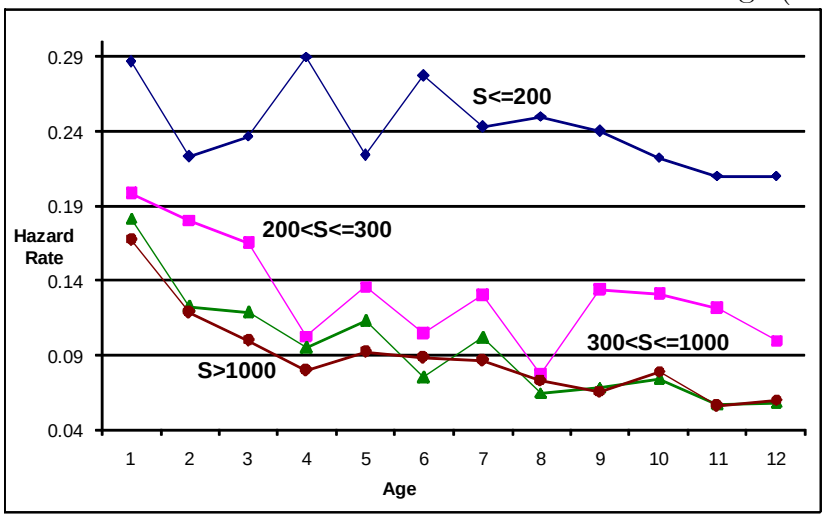
3 Model's Implications

The model has four novel implications, each of which explains a regularity observed in the Chilean dataset and several other empirical studies on industry evolution. Below, I describe three of these regularities, present evidence from te Chilean dataset on each one, and show how the model explains them. The fourth regularity, which is the long right tail of the size distribution, is explained in the simulation results section.

3.1 Effects of Size and Age on the Hazard Rate of Exit

Size and age are two important observable characteristics of establishments that have been extensively used to analyze their dynamics. Many studies have shown that the hazard rate of exit decreases as size and age increase[10]. Figure 1 shows the relation for the Chilean dataset including all industries. Each line represents a size cohort where size is measured as total sales. As both size and age increase, the hazard rate of exit decreases. The establishments in the smallest size cohort face higher exit rates than the other cohorts and the decline in hazard rate as age increases is slower in this size cohort. This is probably due to the small number of observations in the older ages for that size cohort. For the other cohorts, decrease in the hazard rate of exit is more pronounced.

Figure 1: Hazard Rate of Exit Conditional on Size and Age (Data)



The Klette and Kortum (2004) model can explain the relation between the hazard rate of exit and size. It also generates the negative relation between age and the hazard rate of exit but only because age is a proxy for size. Conditional on size, age has no effect on the hazard rate of exit. Introducing random sized products, Klepper and Thompson (2007) can explain both relations independently. In my model, the relation holds for a different reason. Establishments with different efficiency levels produce different numbers of products. Given size, there is a mixture of producers with different number of products and the older ones are more likely to

[10] Caves (1998) reviews the empirical literature on these relations.

have more products, hence are less likely to exit. Without the efficiency-type heterogeneity, all producers would have the same number of products at a given size and thus would face the same hazard rate of exit no matter how old they are.

I can derive an analytical formula that shows these relations. To construct this formula, at any age a, I need to know the hazard rate of exit, the number of products owned by the producer, and the efficiency-type distribution of producers. Using the solution of the system of equations for size evolution given in equation 12, having entered at size one, probability of exiting within one unit of time is given as

$$p_0\left(1;1|\varphi\right)=\frac{\mu\left(1-e^{-(\mu-\lambda)t}\right)}{\mu-\lambda e^{-(\mu-\lambda)t}} \tag{17}$$

From equation 13, the probability of having $n\geq 1$ products at age a is $\left(1-\chi\left(a|\varphi\right)\right)\chi\left(a|\varphi\right)^{n-1}$. A $\varphi-$type producer at size S has $n\left(\varphi\right)=\| \frac{S}{q\left(\frac{p}{P}\right)^{1-\sigma}} \|$ products. Age-conditional efficiency-type distribution with density $\phi^a\left(\varphi\right)$ can be derived using the type distribution at entry $\phi\left(\cdot\right)$ and the probability of surviving more than a years $1-p_0\left(a;1|\varphi\right)$, which is given as

$$\phi^a\left(\varphi\right)=\frac{\phi\left(\varphi\right)\left(1-p_0\left(a;1|\varphi\right)\right)}{\int\phi\left(\varphi\right)\left(1-p_0\left(a;1|\varphi\right)\right)d\varphi} \text{ for } \forall\ a\geq 1. \tag{18}$$

Using equations 17 and 18, the hazard rate of exit conditional on age a and size S, $H\left(a,S\right)$ can be derived as follows

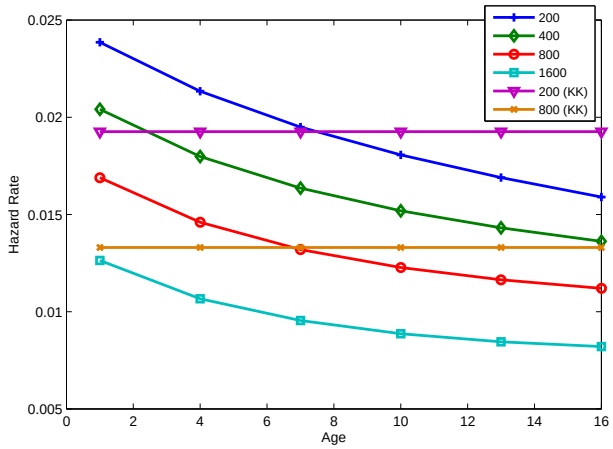
$$H\left(a,S\right)=\int p_0\left(1;1|\varphi\right)^{n(\varphi)}\left[\left(1-\chi\left(a|\varphi\right)\right)\chi\left(a|\varphi\right)^{n(\varphi)-1}\right]\phi^a\left(\varphi\right)d\varphi. \tag{19}$$

I plotted the graph of $H\left(a,S\right)$ for different size S and age a values in Figure 2[11]. For comparison, I also plotted the lines implied by the Klette and Kortum (2004) model which are labeled as "KK". Each plotted line shows the hazard rate for a different size level. For the Klette and Kortum (2004) model, I just showed two size levels 200 and 800. In their model, at any particular size, the hazard rate is independent of age but the hazard rate decreases as size increases. On the other hand, in my model both relations hold independently. Conditional on age, the hazard rate of exit decreases in size and respectively conditional on size, it decreases

[11]The parameter values used for the graph are from the simulation results of Food industry which will be explained in the empirical analysis part.

in age.

Figure 2: Hazard Rate of Exit Conditional on Size and Age (Model)



The analytical framework of the model allows me to prove the existence of these relations. In order to do that, first I present two lemmas which are used for the proof of the proposition.

Lemma 1 For all $t > 0$, $(\mu - \lambda) < \frac{e^{(\mu-\lambda)t}-1}{t}$ and $\frac{e^{(\mu-\lambda)t}-1}{t}$ is strictly increasing in t.

Proof. See Appendix. ■

Lemma 2 For all $t > 0$, the parameter of the size distribution $\chi(t)$ increases in innovation intensity λ (i.e. $\frac{\partial \chi(t)}{\partial \lambda} > 0$).

Proof. See Appendix. ■

Proposition 1 Hazard rate of exit $H(a, S)$ decreases in age conditional on size and decreases in size conditional on age for all ages $a > 0$ and sizes $S > 0$.

Proof. See Appendix. ■

The underlying reason for getting the independent relation between the hazard rate of exit and age is the heterogeneity in efficiency levels. Although more efficient establishments survive longer and grow faster, it's possible for a less efficient producer to get lucky and accumulate many products. This in return decreases the hazard rate of exit relative to a higher efficiency producer with only few products. Since it takes time to accumulate many products, that establishment will be older than the more efficient producer with a few products. Hence at any given size, it is possible to observe establishments with differing number of products and differing hazard rates of exit.

3.2 Effect of Size on the Variance of Growth Rates

In the literature on establishment and industry evolution, there are many studies that have analyzed the relation between size and the expected growth rate. However, only recently have there been studies that try to explain the dispersion in growth rates and how it changes with size. Stanley et al (1996), Bottazzi (2001), Sutton (2002, 2007) show that the variance of growth rates decreases as size increases[12]. The common feature of all these studies is the introduction of a statistical model to explain the relation observed in the data rather than using a structural model based on optimizing firm behavior. Three recent studies by Klette and Kortum (2004), Luttmer (2008) and Klepper and Thompson (2007) analyze this relation in a more structural setup. Although all three models qualitatively explain the negative relation, the slope of the relation implied by these models is too steep compared to the data.

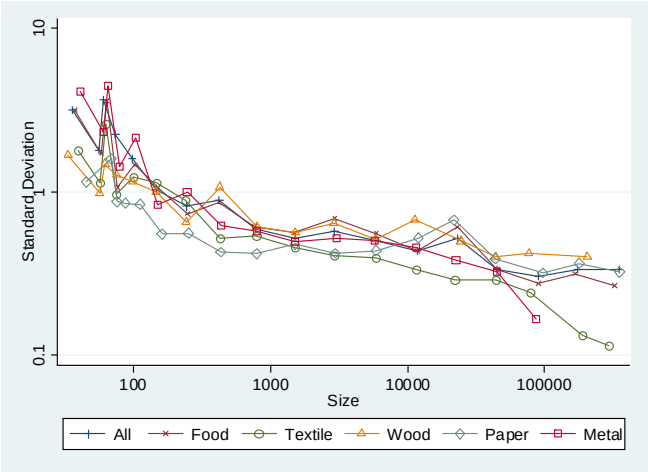
Figure 3 shows the relation between the log of the standard deviation of growth rates and the log of size observed in all data combined and in all five industries individually. Size is measured as total sales. Slopes of the fitted lines for each industry vary between -0.15 and -0.32. For the very small and very large size bins, deviations exist from a linear relation which is probably caused by the small number of observations in these bins. The relation observed for the specific industries is not different than the relation observed when all industries are merged. This fact suggests that the nature of this relation is due to some fundamental property of the economic dynamics and establishment behavior, which makes it appealing to identify.

Since the growth rate of efficiency is constant, the variance of growth rate of size is determined by the variation in the growth rate of the number of products. Emergence of the negative relation in the model is explained as follows. The evolution of an establishment is determined by combining the evolution of each of its products. Hence, the aggregate growth of an establishment is the average of the growth of these independent components. This leads to an inverse relation between the variance of the growth rates and initial size.

To make the analysis comparable to the empirical studies, I will look at the relation between the standard deviation of the growth rates and size. If all the producers had the same innovation

[12] Sutton (2002) shows that the slope of the fitted line between log of standard deviation of growth rates and log of size measured in sales varies between -0.15 and -0.21. Stanley et al (1996) performs the same analysis using employment as size and finds the slope as -0.16.

Figure 3: Standard Deviation of % Growth Rates (All Industries)



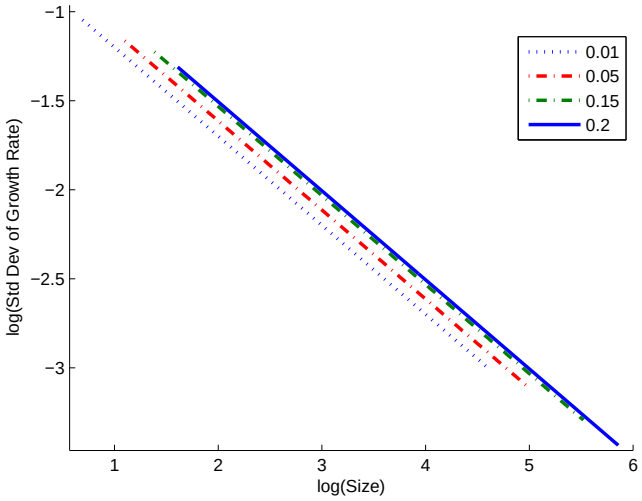
rate, as in Klette and Kortum (2004), this inverse relation would give a slope of -0.5. Figure 4 shows the relation for several innovation rate values. As the innovation rate increases larger establishments can exist in the market and the line shifts to the right. In my model, since producers differ in their efficiency levels, they have different innovation rates. Hence, the relation that emerges here is a mixture of the lines in Figure 4. Furthermore, since high-efficiency producers attain larger product sizes, they get even larger and this causes the line to extend even further to the right. This property plays an important role in generating a flatter relation[13] in the model.

In the model, as a result of the mixing, a high-efficiency producer with a single product can be in the same size bin with a low-efficient producer that luckily survived and gained many small products. The inefficient producer will exhibit lower variance in growth rate since it has many products. But the existence of high-efficient producer exhibiting high variance due to its single product will increase the total variance in that size bin. As a result, the mixing of producers with different efficiency levels generates a flatter relation between the variance of the growth rates and size.

I derive the formula for the variance of growth rates as follows. Suppose that there are K different efficiency types in the industry denoted as $\{\varphi_k\}_{k=1}^{K}$for $K \in \mathbb{Z}^+$. For each type, define the variance of growth rates conditional on initial size S as $V\left(G\left(\varphi_k\right)|\tilde{S}=S\right)$. Total variation

[13]Lentz and Mortensen (2008) introduces heterogeneity in producer innovation intensities however the slope is still too steep.

Figure 4: Standard Deviation of % Growth Rates vs Size (Different Innovation Rates)



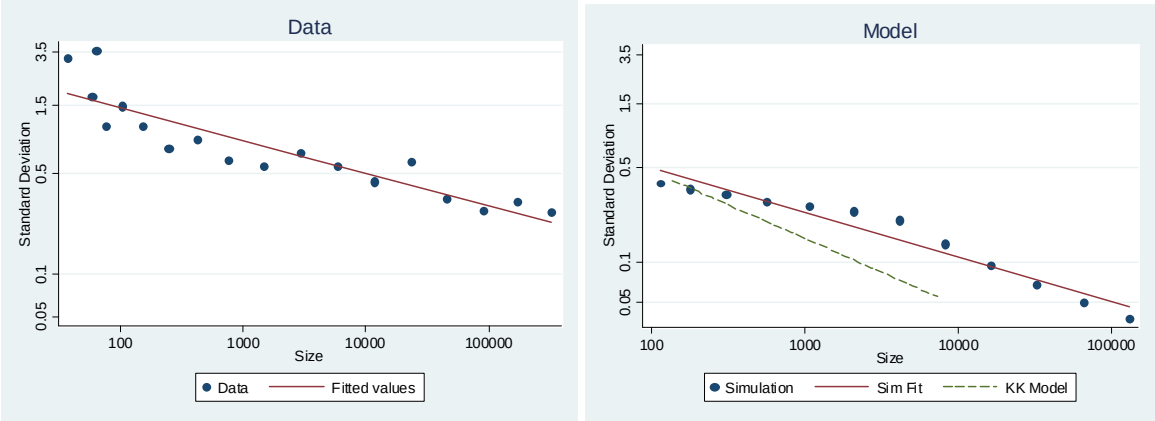
can be written as

$$V\left(G|\tilde{S}=S\right)=\sum_{k=1}^{K}p\left(\varphi_k|\tilde{S}=S\right)\left[V\left(G\left(\varphi_k\right)|\tilde{S}=S\right)+\left(E[G(\varphi_k)]-\bar{E}\right)^2\right] \tag{20}$$

where $p\left(\varphi_k|\tilde{S}=S\right)$ is the probability of being φ_k-type conditional on having initial size S, $E[G(\varphi_k)]$ is the expected growth rate for the φ_k-type producer, and $\bar{E}$ is the expectation taken over all K types. Recall that formula for $V\left(G\left(\varphi_k\right)|\tilde{N}=n\right)$ where n is the number of products was given in equation 15. Since each product size is fixed, it is straight forward to show that $V\left(G\left(\varphi_k\right)|\tilde{S}=S\right)$ is equal to $V\left(G\left(\varphi_k\right)|\tilde{N}=n\right)$ for all efficiency-types. Hence I can simulate the model's results easily.

In the estimation part that will be discussed below, this slope was not targeted to match in the data. However the relation implied by the model improves the Klette and Kortum (2004) result which is shown in Figure 5. The graph on the left shows the data for the food industry and the fitted line which has a slope of -0.23. The graph on the right shows the simulated data and the fitted line to it with slope -0.33. The line labeled as "KK Model" shows the Klette and Kortum (2004) result with slope -0.5. The model clearly improves Klette and Kortum (2004) results. However the standard deviation of growth rates for small establishments is much larger in the data than in the simulation.

Figure 5: Standard Deviation of % Growth Rates for Food Industry: Data vs Model



3.3 Life Cycle of a Birth Cohort

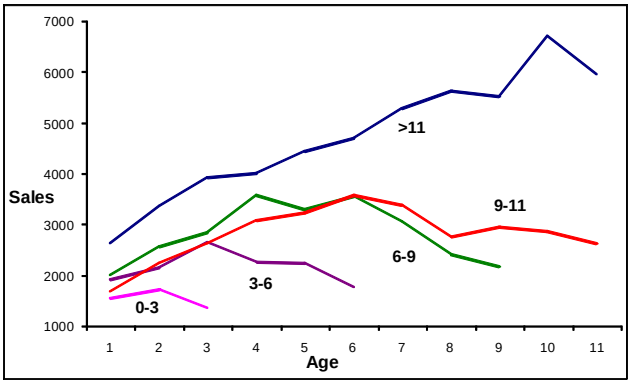
There are various factors that affect the post-entry performance of establishments such as the amount of sunk costs, as argued by Dixit (1989) and Hopenhayn (1992), and the innovative environment of the industry, as observed by Geroski (1995). Empirical evidence on these hypotheses has been provided by Audretsch (1991, 1995) and Baldwin (1995). On the other hand, Audretsch (1995) summarize other studies which argue that characteristics specific to establishments also influence their post-entry performances. Since size is the most important observable characteristic specific to an establishment, its value at the startup could be signaling important information about the evolution process. To show how important the startup size is, in Figure 6, I plot the size evolution of establishments grouped with respect to the age at which they exit. The graph combines all industries and all birth cohorts from 1980 to 1997. A similar but noisier picture emerges when Figure 6 is drawn for single industries or for specific birth cohorts due to small number of observations. To get a nicer picture of the relation, I combined all birth cohorts in all industries.

The graph shows that the establishments that will survive longer are larger in terms of sales than the exiting establishments within the same birth cohort at all ages including the startup. It also shows the shadow of death effect; establishments which will exit in the future start to shrink in size several years before their exit.

The model captures these two features of the data, as shown in Figure 7[14]. In the model,

[14]I estimate my model for the chosen 3-digit industries individually. This graph shows the model's result

Figure 6: Pre-Exit Behavior of Establishments (Data)



heterogeneity in size at the startup occurs by the variation in efficiency levels. The model captures the shadow of death effect especially for the 3-6 and 6-9 year cohorts, but in the data this effect is more pronounced. In Klette and Kortum (2004), all producers start with one product; hence, there is no dispersion in size at startup. Lentz and Mortensen (2008) introduce randomness in size of each product and heterogeneity. However, there is no relation between the startup size and the post-entry performance. In the proposition below, I show how the model explains this relation.

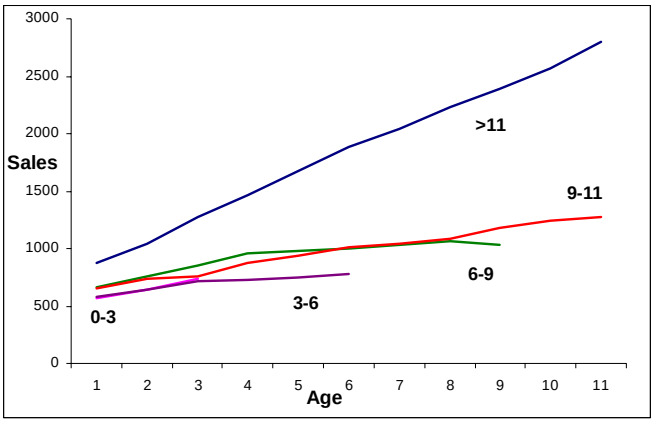
Proposition 2 Consider a cohort of establishments all entering at the same time. At any age $a \geq 1$, within this cohort, establishments that survive longer are larger in size than the exiting establishments (i.e. for S representing the establishment size, t_x representing the time of the exit, $\Delta > 0$, $0 \leq a < t$, $E\left[S\left(a\right)|t_x = t\right] \leq E\left[S\left(a\right)|t_x = t + \Delta\right]$).

Proof. See Appendix. ■

The positive relation between the startup size and the likelihood of survival is not unique to Chilean Data. Audretsch (1995) performs a logit estimation using US establishments and concludes that startup size has an impact on the likelihood of survival. Similar results are found in Dunne, Roberts, and Samuelson (1989). This evidence shows that it's an important feature that needs to be understood in order to explain causes of size dispersion, heterogeneous responses of producers to exogenous shocks, and responses to policy changes. The model has the potential to explain these issues.

for Food industry not the all industries combined because I intend to show how model explains this relation qualitatively.

Figure 7: Pre-Exit Behavior of Establishments (Simulation)



4 Empirical Analysis of Industry Evolution

In this section, I estimate the model separately for five 3-digit industries in the Chilean Manufacturing sector. I analyze whether the dispersion observed in establishment behavior in the aggregate manufacturing sector also holds at the 3-digit industries. All the values of sales and wages are given in thousands of 1986 real Chilean peso. The nominal values are deflated by the aggregate GDP deflator from World Bank Development Indicators database.

4.1 Industry Comparison

Five industries that will be analyzed, their 3-digit SIC codes, and the total numbers of establishments observed were given in Table 1. Doing the same estimation exercise five times aims to capture the flexibility of the model in explaining different industry structures.

In the Appendix, I show the data analysis on the shape of the size distributions, on turnover, and on growth rates across industries. The size distribution of each industry is estimated nonparametrically (using Kernel regressions) (see Figure 10). The shapes of these distributions change very little over time. Food and Paper industries differ from the other three industries in several ways with the most outstanding difference being on the shape of the size distribution. The average sales and the variation of sales are larger and the coefficient of skewness of the log size distributions is higher in these two industries (see Figure 11). The ranking of the industries is preserved for the variation in size when I analyze the industry data with respect

to their means (see Figure 11). This implies that the differences in size distributions are not purely caused by economies of scale but are due to some intrinsic differences across industries.

Further industry analysis shows that Food and Paper industries have lower turnover rates (see Figure 11). Capital intensities vary across industries but do not explain the differences in the shape of the size distribution. Using establishment data for the U.S. economy, Rossi-Hansberg and Wright (2007) show that the larger the capital intensity in a sector, the thinner is the right tail of the size distribution. They define sectors at 2-digit SIC level. The industries analyzed here are more narrowly defined and at this level the relation suggested by their model does not hold. I looked at the average capital intensities of establishments that were in the market in 1979 and that entered in 1980[15]. I found the capital-output ratio of every establishment and then plotted the average ratio for every industry (see Figure 12). The Paper industry, which is the most capital intensive, has the thickest right tail. The other industry with a thick right tail is the Food industry and its capital intensity is among the lowest. Hence capital intensity does not play a distinguishing role in explaining the differences in the shape of size distributions for narrowly defined Chilean industries.

4.2 Estimation

The model introduced above is estimated using simulated method of moments. With this method, I try to find the values of the model parameters that bring the vector of moments from the simulated data closest to those from the data.

4.2.1 Data Moments

Eight moments are chosen for parameter estimation. They are prominent in explaining the industry structure and they also reflect the cross-industry differences. As one of the targets of this paper is fitting the industry size distribution, 10, 25, 50, 75 and 99^{th} percentiles are chosen. The shape of the size distribution is affected by the exogenous destruction rate of products, aggregate expenditure, efficiency type distribution, and innovation cost parameters. Another moment is the entry rate which plays an important role in identification of the destruction rate. Two other moments, the mean and the variance of establishment growth rates, are

[15] Capital data was only available for these establishments in the dataset.

closely affected by the innovation cost parameters, destruction rate, and the efficiency type distribution.

The values of these moments for Food and Food Products industry (SIC 311-312) are given in Table 3. Following Horowitz (2001), the standard errors of the moments are estimated by 1000 bootstrap repetitions which are given in parentheses. Since the model incorporates growth in average establishment size, only year 1979 is used to estimate the moments for the size distribution. The rest of the moments are constructed by averaging over the 1979 -1997 period[16]. The annual values of these moments didn't show a strong trend over time which is in accordance with the steady state assumption of the model. The growth rate distribution is found as the annual increase in sales and it includes the exiting establishments (i.e. -1 is placed in the year that the establishment exits). Moment vectors for the other industries are given in the Appendix.

The parameter vector to be identified is $\theta = \left\{\sigma, \mu, c_0, c_1, \mu_\varphi, \sigma_\varphi, \bar{\varphi}, E\right\}$ which are described in Table 4. Since there are just eight moments identifying eight parameters, the system is just identified. Efficiency types are lognormally distributed ($\varphi \sim LN\left(\mu_\varphi, \sigma_\varphi\right)$) with $\bar{\varphi}$ representing the minimum efficiency level.

Other than these industry specific moments, the real interest rate r is fixed at 5%. In the dataset, I had information on the average wage rates and the numbers of blue and white collar workers. Using that information, I found the average annual wage rate for each industry in thousands of real 1986 Chilean peso. Real wages for the industries are: 306.8 for Food, 303 for Textile, 271 for Wood, 489 for Paper, and 371 for Metal industries. Since aggregate expenditure growth in an industry is equal to exogenous efficiency growth rate g_φ, this parameter is directly estimated from the data with values of 7.8% for Food, 5.5% for Textile, 12% for Wood, 11% for Paper, and 9.5% for Metal industries.

4.2.2 Simulation Method and Algorithm

The data is comprised of sales for all producers. Denote this panel by $\Delta = \left\{\left\{Y_{i,t}\right\}_{i=1}^{N_t^D}\right\}_{t=1979}^{1997}$ where i refers to the producer, t refers to the year, and N_t^D is the number of observations

[16] Chile went through a financial crisis in 1982-1983. I excluded these years from finding the average values of the moments.

Table 3: Data Moments for Food Industry

Moment	Definition	Value*
pctile(10)	10^{th} Percentile	128.80 (4.72)
pctile(25)	25^{th} Percentile	207.43 (4.85)
pctile(50)	50^{th} Percentile	346.15 (10.58)
pctile(75)	75^{th} Percentile	894.56 (72.30)
pctile(99)	99^{th} Percentile	31291.11 (2846.05)
E[g]	Average Growth Rate	0.027 (0.0057)
Std[g]	Std Dev of Growth Rate	0.75 (0.047)
η	Entry Rate	0.054 (0.0018)

*Values in the parentheses show the standard errors.

Table 4: Model Parameters

c_0, c_1	Innovation cost
μ	Destruction rate
$\mu_\varphi, \sigma_\varphi$	Efficiency distribution
$\bar{\varphi}$	Minimum efficiency level
σ	Elasticity of substitution
E	Aggregate Expenditure

in the data at time t. Using the panel, I calculated the vector of data moments denoted as $\hat{\beta}(\Delta)$. Then, given a vector of parameters θ, I simulated a panel of sales $\Delta^s = \left\{Y_{i,t}^s\right\}_{i=1}^{N^S}$ for $N^S = 10,000$ establishments and repeated the simulation for $S = 10$ times. Using the simulated panel, I found the moment values and averaged them over the simulation size giving $\hat{\beta}^S(\theta)$ as follows:

$$\hat{\beta}^S(\theta) = \frac{1}{S}\sum_{s=1}^{S} \hat{\beta}\left(\Delta^s(\theta)\right).$$

Finally, the estimator $\hat{\theta}$ is found as the solution to the following criterion function,

$$\hat{\theta} = \arg\min_{\theta} \left(\hat{\beta}^S(\theta) - \hat{\beta}(\Delta)\right)' I \left(\hat{\beta}^S(\theta) - \hat{\beta}(\Delta)\right)$$

where I is the identity matrix. Since the system is exactly identified, identity matrix is used as the weighting matrix which gives the equally weighted minimum distance (EWMD) estimator. Standard errors of the estimator are estimated by bootstrap method. Given the data Δ, a bootstrap sample Δ^b is drawn with replacement. The draws are made block over time. This means that if a particular establishment is selected, the entire time series for this establishment

is included in the constructed sample. Then for each drawn sample[17], the estimator vector $\hat{\theta}^b$ is found by solving

$$\hat{\theta}^b = \arg\min_{\theta} \left(\hat{\beta}^s\left(\theta^b\right) - \hat{\beta}\left(\Delta^b\right)\right)' I \left(\hat{\beta}^s\left(\theta^b\right) - \hat{\beta}\left(\Delta^b\right)\right).$$

The model is highly nonlinear. Hence, down-hill simplex method (amoeba) is used for optimization. The steps to compute the industry equilibrium are given as follows:

1. The parameter vector is initialized and the vertices of the simplex are determined.
2. For each parameter vector, an upper bound of the efficiency type distribution satisfying $\lambda(\varphi) < \mu$ for $\forall\varphi$ is determined.
3. Given the bounds of the efficiency type distribution and the parameter vector, an aggregate price index P is found.
4. For 10,000 establishments, the value function is solved and innovation intensities are obtained.
5. Using these values, establishment and industry related moment values are determined.
6. The value of the criterion function is checked and using the amoeba routine, the simplex of parameter vectors is updated.
7. The system is iterated until either the value of the criterion function or the parameter vector converged.

4.2.3 Simulation Results

The estimated parameter vectors for each industry analyzed are given in Table 5. The standard errors[18] are given in parentheses. Simulation results show that the parameter values for Food and Paper industries differ from the other industries. These two industries have lower innovation costs which allow them to have more innovative producers. Combined with the higher efficiency levels, this contributes to the existence of larger producers. Lower destruction rate explains the

[17] For this exercise 50 bootstrap repetition is used.

[18] I found the standard errors for only two of the industries (Food and Wood).

smaller turnover rates. The stochastic innovation process together with the higher variation in efficiency levels explains the greater dispersion in size for these industries. It is also seen that part of the differences in the size distribution is attributed to the differences in aggregate expenditures. On the other hand, the variation in the elasticity of substitution and the minimum efficiency level parameters are relatively small across industries.

Table 5: Parameter Estimates for All Industries

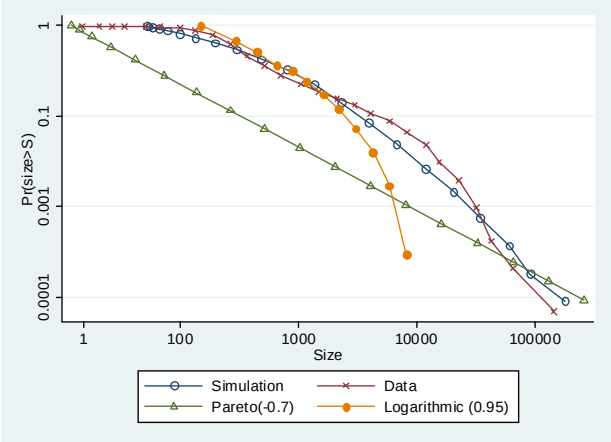
	Food	Paper	Textile	Wood	Metal
σ	2.19	2.37	2.6	2.4	2.3
	(0.44)	(1.09)	(0.46)	(0.69)	(0.48)
c_0	47085.71	44713.4	71896.1	71999.6	74367.9
	(1569)	(1846.9)	(1514.8)	(6081)	(3214.2)
c_1	3.95	3.82	5.45	5.73	5.32
	(0.02)	(0.4)	(0.06)	(0.11)	(0.25)
μ	0.125	0.13	0.184	0.18	0.176
	(0.002)	(0.01)	(0.002)	(0.01)	(0.01)
μ_φ	2.03	1.82	0.87	1.55	0.88
	(0.17)	(0.22)	(0.22)	(0.33)	(0.42)
σ_φ	4.59	4.25	1.77	1.11	1.43
	(0.36)	(1.05)	(0.17)	(0.39)	(0.27)
$\bar{\varphi}$	1.19	1.21	2	1.43	1.62
	(0.04)	(0.17)	(0.07)	(0.17)	(0.14)
E	1293.2	2239.3	931.7	517.6	827
	(46)	(100.4)	(73.1)	(47.6)	(74)

The values in the parentheses show the standard errors.

Estimation results for each industry are given in the Appendix. Figure 8 shows the observed and simulated size distributions for Food industry. I also plot the logarithmic size distribution implied by the Klette and Kortum (2004) model and the Pareto distribution. As the graph shows, the model captures the fat right tail quite well. Logarithmic distribution cannot generate enough variation in size and cannot generate very large sized establishments. The Pareto distribution which was used by Luttmer (2007) to fit the size distribution of the U.S firms, can generate large sizes[19]. But it cannot fit the data at medium-sized classes. Graphs for other industries with the simulation results are given in the Appendix. Although the model performs well in capturing the shape of the size distributions, it cannot generate very small sized producers (i.e. with total sales less than 100).

[19]Pareto distribution used here has coefficient 0.7 which corresponds to a straight line with slope -0.7.

Figure 8: Size Distribution for Food Industry



In the model, innovation is done by the incumbent producers. When innovation cost is low, these producers get more innovative and there is less room for entry. Hence larger industries have lower turnover rates as observed in the data. The model performs relatively well in capturing the entry rates across industries. However, it underestimates the growth rate distribution.

5 What Causes Size Dispersion?

In this section, I elaborate a novel feature of the model. The model incorporates two forces that generate persistent differences in establishments' performances. In his review of models on establishment evolution, Sutton (1997) lists these forces as: (i) intrinsic efficiency differences that are determined before entering the economy (ii) differences that are generated through idiosyncratic innovations that accumulate through the life of the establishment. Both views have drawn great attention in the literature[20]. Among the first group of models, Lucas (1978) and Jovanovic (1982) link the differences in efficiencies to the differences in the skills of entrepreneurs. In the second group of models, performance is driven by producer specific learning, R&D, and innovation. Some recent models that follow this view are Ericson and Pakes (1995), Klepper (1996), Klette and Kortum (2004), and Klepper and Thompson (2007).

The model introduced here distinguishes the contributions of exogenous idiosyncratic efficiency differences and accumulated innovations to explain size dispersion. It generates disper-

[20] For a review and comparison of both types of models see Klette and Raknerud (2002).

sion in the startup sizes due to efficiency differences. As establishments grow old, the innovation process induces dispersion among producers of the same efficiency-type. As a result, both factors contribute to the total variation of size.

To see how the contribution of each part evolves over time, I analyze the life-cycle of producers within a birth cohort. First, I look at the variation at birth (age=0). All establishments start with a single product. Hence, within type variation, Var^0_{Within} is zero. Total variation in size only reflects dispersion between efficiency types, which is given as

$$Var^0_{Total} = \int \left(s\left(\varphi\right) - Es\right)^2 \phi\left(\varphi\right) d\varphi \tag{21}$$

where $s\left(\varphi\right)$ is sales per product for a $\varphi-$type producer, $Es = \int s\left(\varphi\right)\phi\left(\varphi\right)d\varphi$ is the expected size at the entry, and $\phi\left(\varphi\right)$ is the probability density at the entry.

As the establishments in the same birth cohort grow older, as a result of the stochastic innovation process, dispersion will emerge among the establishments of the same efficiency type. Let n^a be the number of products owned by a $\varphi-$type producer at age a, which is a random draw from the geometric distribution with parameter $\chi\left(a|\varphi\right)$. This distribution has mean $\frac{1}{1-\chi(a|\varphi)}$ and variance $\frac{\chi(a|\varphi)}{(1-\chi(a|\varphi))^2}$. Conditional on type, the expected size and the variance of size at age a are given as

$$E\left[s\left(\varphi\right)n^a|\varphi\right] = s\left(\varphi\right)\frac{1}{1-\chi\left(a|\varphi\right)} \tag{22}$$

$$Var[s\left(\varphi\right)n^a|\varphi] = s\left(\varphi\right)^2\frac{\chi\left(a|\varphi\right)}{\left(1-\chi\left(a|\varphi\right)\right)^2}. \tag{23}$$

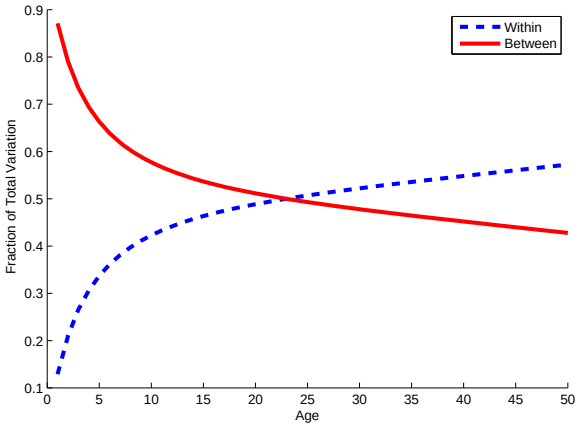
Total size is $\bar{E} = \int E\left[s\left(\varphi\right)n^a|\varphi\right]\phi^a\left(\varphi\right)d\varphi$. Using equations 22 and 23, total variation in size at age a is determined as

$$Var^a_{Total} = \underbrace{\int Var\left[s\left(\varphi\right)n^a|\varphi\right]\phi^a\left(\varphi\right)d\varphi}_{Var^a_{Within}} + \underbrace{\int\left(E\left[s\left(\varphi\right)n^a|\varphi\right] - \bar{E}\right)^2\phi^a\left(\varphi\right)d\varphi}_{Var^a_{Between}}. \tag{24}$$

The total variation in size is plotted in Figure 9. The graph shows that, as the establishments get older, the share of within type dispersion increases. It increases monotonically and exceeds the between type dispersion after 23 years. At the end of 50 years of survival, within type

variation accounts for 57% of the total variation in size. Since producers with low efficiency don't live as long, between type heterogeneity decreases at older ages. Hence more of the total variation is explained by within type dispersion. This analysis shows that both the intrinsic efficiency differences and the accumulated innovations contribute to explain the variation in size, but their contributions change with the establishments' ages.

Figure 9: Total Variation in Size over Time



6 Counterfactual Experiments

A distinguishing feature of Klette and Kortum (2004) model from the earlier work in endogenous growth models is the research done by the incumbent establishments. In the models of Grossman and Helpman (1991) and Aghion and Howitt (1992), innovations are done only by new establishments and it is hard to reconcile this property with the persistence of large establishments in industries[21]. With this feature of the model, I will demonstrate the effects of two policy changes on the establishments' innovation capacities. In the first experiment, I introduce a 25% subsidy on research investment. In the second experiment I decrease the price-cost margin by 10%. In the model, for each product, establishments charge the same markup. Hence price-cost margin is equal to $\frac{\sigma}{\sigma-1}$ and is constant. 10% decrease in the price-cost margin can be generated with a 15% increase in the elasticity of substitution. The results of these counterfactual experiments are given in Table 6. Table shows the percentage changes

[21] See Klette and Griliches (2000) for further discussion of this difference between the previous literature and the new studies.

Table 6: Counterfactual Experiments

	25% Research Subsidy (% change)	10% decrease in PCM (% change)
Average Growth*	4	-3
Std Dev of Growth	0.07	-0.07
Entry Rate	-30	17
Exit Rate	-10	7
(By Size Quartiles)		
1^{st} Quartile		
Size	111	-63
Research Int	45	-40
Innov Rate	33	-26
2^{nd} Quartile		
Size	81	-53
Research Int	40	-36
Innov Rate	27	-21
3^{rd} Quartile		
Size	47	-37
Research Int	34	-32
Innov Rate	20	-15
4^{th} Quartile		
Size	17	-13
Research Int	11	-21
Innov Rate	6	-0.05

*Average growth rate is calculated conditional on survival

in the selected variables after the policy change. I divide the establishments into size quartiles both before and after the change and show how the evolution of the different parts of the size distribution change. For each quartile, I look at the change in average size, research intensity, and innovation rate.

In most of the existing models of creative destruction, since the innovation is done by the outside firms, subsidies encourage them to do more research. The model's implications are in contrast with these views. Here, subsidies on R&D investment increase the innovative capacity of the incumbent establishments. With higher innovation intensities, incumbent establishments grow faster by 4% and survive longer (exit rate decreases by 10%). This leaves less room for

potential entrants to enter the market. The effect of the policy change varies for different parts of the size distribution. Although it improves size, R&D intensity and innovation rate for all parts of the distribution, the gain is larger for the small producers. Due to the larger increase in their innovation rates, grow rates of small producers also increase more.

The impact of competition on R&D expenditures and the rate of innovation has been debated for a long time. Increased elasticity of substitution will cause tougher competition in the product market which will lower the flow of profits of the incumbent producers. Lower profits will lead to lower R&D expenditures and less innovation. As a result, average growth rate of establishments is going to decrease by 3%. Having lower innovation rates, they will exit more often (7% more) and there will be more room for new entrants to the industry. Looking at the quartiles of the size distribution, it's seen that tougher competition hurts the small establishments the most. Average establishment size and the innovation rates in the first quartile shrink by around five times more than the respective variable in the fourth quartile.

7 Conclusion

This study improves recent models of industry evolution in explaining several regularities observed in the data which have been hard to capture by the existing models. Through a parsimonious extension of a highly stylized model introduced by Klette and Kortum (2004), I construct a model that succeeds in explaining: (i) the fat tail in the size distribution, (ii) the independent relation between size, age, and the hazard rate of exit, (iii) post-entry performance of a birth cohort, and (iv) the negative relation between the variance of the growth rates and size. The model is consistent with many empirical regularities. It demonstrates a good framework for understanding the micro foundations of industry evolution and it is analytically tractable.

In this paper, I also intended to show the quantitative strength of the model in explaining the data. The model performs well in capturing various moments of the size distribution, entry rates, and growth rates for five 3-digit industries in Chilean Manufacturing sector. Comparison of the five industries shows that innovation structure, the destruction rate, and efficiency type distribution play a role in explaining the differences across industries.

In the model, establishments are defined as legal entities formed of multiple products. Their evolution is the sum of the evolution of each of their products. In this respect, the model

complements several existing models explaining product scope. The way the multi-product producers are modeled here is closest to the one introduced in Bernard, Redding, and Schott (2006a). Although the factors driving variation in size are different, both models introduce two margins that contribute to expand establishment size. However, their model lacks a dynamic framework and focuses on an analysis of trade liberalization.

The model has several interesting extensions that are worth pursuing. Introducing aggregate uncertainty to the economy can extend our understanding of the response of the economy to negative shocks and the recovery from economic slowdowns. Another fruitful area is adding the financial side to the model. This would bring a more comprehensive understanding of the establishment dynamics. One promising work in this field is done by Cooley and Quadrini (2001). They show how the combination of persistent shocks and financial frictions can account for the simultaneous dependence of the establishment dynamics on size conditional on age and on age conditional on size. However, their model has some limitations such as not being able to predict the effect of age on the hazard rate of exit. Finally the model could be carried into an open economy to understand how the technology imported through intermediate products affects the establishment and industry evolution.

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A Growth rate of $\pi(\varphi)$

Aggregate industry production function was given as

$$Y_t = \left(\int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}.$$

Taking the derivative with respect to time, I get

$$\dot{Y} = \frac{\sigma}{\sigma-1} \left(\int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}-1} \left(\int_{j \in J} \frac{\sigma-1}{\sigma} y_t(j)^{\frac{\sigma-1}{\sigma}-1} y_t(j) \frac{\dot{y}_t(j)}{y_t(j)} dj \right).$$

Since for every product j, $y_t(j) = \varphi_t l$ and l is constant, I get $\frac{\dot{y}_t(j)}{y_t(j)} = g_\varphi$ and $\frac{\dot{Y}}{Y} = g_\varphi$. Since E_t is also growing at rate g_φ and $E_t = P_t Y_t$, $P_t = \left(\int_{j \in J} p_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$ is constant. This implies that $p_t(j) = \frac{\sigma \bar{w}_t}{(\sigma-1)\varphi_t}$ has to be constant. Hence, the wage rate $\bar{w}_t$ must grow at the same rate as the efficiency level φ. It's also seen that $\bar{\pi}_t(\varphi) = \left(\frac{p_t(\varphi_t)}{P_t} \right)^{1-\sigma} \frac{E_t}{\sigma}$ grows at rate g_φ and hence $\pi(\varphi) = \frac{\bar{\pi}(\varphi)}{E_t}$ is constant.

B Aggregate Economy

A possible setup for the aggregate economy is given below.

Consumer's Problem

The economy consists of a unit continuum of consumers. Intertemporal utility of a representative consumer is

$$U_t = \int_t^\infty e^{-\rho(\tau - t)} \ln C_\tau d\tau$$

where ρ is the discount rate and C_τ is the aggregate consumption at time τ. $\ln C_\tau$ measures the instantaneous utility at time τ. Every consumer maximizes utility subject to an intertemporal budget constraint

$$\int_t^\infty e^{-[R_\tau - R_t]} P_\tau^D C_\tau d\tau \leq \int_t^\infty e^{-[R_\tau - R_t]} \bar{w}_\tau d\tau + W_t$$

where $R_\tau = \int_0^\tau r_z dz$ is the aggregate interest rate up to time τ, P^D is the price of the final consumption good D, $\bar{w}_\tau$ is the wage rate, and W_t is the value of the household's asset holdings. Total value of spending at time τ is $I_\tau = P_\tau^D C_\tau$. The optimization problem of the consumer yields

$$\frac{\dot{I}}{I} = r_\tau - \rho.$$

Final Good Producer

The final good sector is perfectly competitive. Cobb-Douglas production function for this sector is given as

$$D_t = Z_t^\beta Y_t^{1-\beta}$$

where Z_t is the consumption of the homogeneous good and Y_t is the consumption of the composite good. Let P_t^Z and P_t represent the prices of these goods respectively. Profit maximizing allocations of these goods are given as

$$Z_t = \beta \frac{P_t^D D_t}{P_t^Z} \text{ and } Y_t = (1-\beta) \frac{P_t^D D_t}{P_t}.$$

The only factor in production is labor which is perfectly mobile across sectors and across establishments in the composite good sector. Homogeneous good sector is also perfectly competitive and one unit of output requires a single unit of labor implying $P_t^Z = \bar{w}_t$.

In this setup, the relation between the growth rates of composite good industry and aggregate economy can be easily acquired. Implementing the demand values of Z_t and Y_t into aggregate production function, I get

$$D_t = \left(\beta \frac{P_t^D D_t}{P_t^Z}\right)^\beta \left((1-\beta) \frac{P_t^D D_t}{P_t}\right)^{1-\beta}$$

which can be simplified to

$$\beta^\beta (1-\beta)^{1-\beta} P_t^D = \left(P_t^Z\right)^\beta P_t^{1-\beta}.$$

Taking the time derivatives of both sides of this equation, I get

$$\frac{\dot{P}_t^D}{P_t^D} = \beta \frac{\dot{P}_t^Z}{P_t^Z} + (1-\beta) \frac{\dot{P}_t}{P_t}.$$

Since wage grows at rate g_φ and $\frac{\dot{P}_t}{P_t} = 0$, I get $\frac{\dot{P}_t^D}{P_t^D} = \beta g_\varphi$. Then using $P_t Y_t = E_t = (1-\beta) P_t^D D_t$,

I find the growth rate of the final good

$$g_D = \frac{\dot{D}_t}{D_t} = \frac{\dot{E}_t}{E_t} - \frac{\dot{P}_t^D}{P_t^D} = g_\varphi - \beta g_\varphi = (1-\beta) g_\varphi.$$

Following this result, growth rate of the homogenous good industry is

$$\begin{aligned} g_D &= \beta g_Z + (1-\beta)\, g_Y \\ g_Z &= \frac{1}{\beta}\left((1-\beta) g_\varphi - (1-\beta)\, g_\varphi\right) = 0. \end{aligned}$$

Finally, the growth rate of $I_t = P_t^D D_t = E_t/(1-\beta)$, as defined in the consumer's problem is equal to g_φ. This implies that $r_\tau = \rho$ and it is constant.

C Value Function Solution

Implementing the conjectured value function into equation 5, I get

$$\begin{aligned} r\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) n &= \max_{\lambda \geq 0} \{\pi(\varphi)\, n - nwc(\lambda) \\ &\quad + n\lambda\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) - \mu n\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right)\}. \end{aligned}$$

After cancelling n in both side of the equation, I get

$$\begin{aligned} (r+\mu)\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) &= \max_{\lambda \geq 0}\left\{\pi(\varphi) - wc(\lambda) + \lambda\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right)\right\} \\ (r+\mu) K(\varphi) &= \max_{\lambda \geq 0}\left\{\lambda\left(\frac{\pi(\varphi)}{r+\mu} + K(\varphi)\right) - wc(\lambda)\right\} \end{aligned}$$

D Relations between Hazard Rate of Exit, Size and Age

Lemma 1 For all $t > 0$, $(\mu - \lambda) < \frac{e^{(\mu-\lambda)t}-1}{t}$ and $\frac{e^{(\mu-\lambda)t}-1}{t}$ is strictly increasing in t.

Proof. Using l'Hopital's rule, $\lim_{t\to 0}\frac{e^{(\mu-\lambda)t}-1}{t} = \lim_{t\to 0}\frac{(\mu-\lambda)e^{(\mu-\lambda)t}}{1} = (\mu-\lambda)$. Taking the derivative of the term with respect to t, I get

$$\begin{aligned} \frac{\partial}{\partial t}\left(\frac{e^{(\mu-\lambda)t}-1}{t}\right) &= \frac{e^{(\mu-\lambda)t}(\mu-\lambda)\, t - \left(e^{(\mu-\lambda)t}-1\right)}{t^2} \\ &= \frac{e^{(\mu-\lambda)t}((\mu-\lambda)\, t - 1) + 1}{t^2}. \end{aligned}$$

It will be sufficient to show that for $\forall t > 0$, $e^{(\mu-\lambda)t}((\mu-\lambda)\, t - 1) + 1 > 0$. Defining $x = (\mu-\lambda)\, t > 0$, I need to show $e^x x - e^x + 1 > 0$

Taylor series expansion of $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}...$ for $-\infty < x < \infty$. Implementing this into the inequality above, I get

$$\begin{aligned} & x\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right) - \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}...\right) + 1 \\ = \; & x^2\left(1 - \frac{1}{2!}\right) + x^3\left(\frac{1}{2!} - \frac{1}{3!}\right) + ... + x^n\left(\frac{1}{(n-1)!} - \frac{1}{n!}\right) + ... \end{aligned}$$

Since $\left(\frac{1}{(n-1)!} - \frac{1}{n!}\right) > 0$ for $n > 1$, the inequality holds. Note that $x \to \infty$ only when $t \to \infty$ and as $t \to \infty$, $\lim_{t\to\infty} \frac{e^{(\mu-\lambda)t}-1}{t} = \lim_{t\to\infty} \frac{(\mu-\lambda)e^{(\mu-\lambda)t}}{1} = \infty$ under which the inequality also holds. ■

Lemma 2 For all $t > 0$, the parameter of the size distribution $\chi(t)$ increases in innovation intensity λ (i.e. $\frac{\partial\chi(t)}{\partial\lambda} > 0$).

Proof. In equation 13, it was shown that the parameter of the size distribution $\chi(t) = \frac{\lambda\left(1-e^{-(\mu-\lambda)t}\right)}{\mu-\lambda e^{-(\mu-\lambda)t}}$. Then

$$\begin{aligned} \frac{\partial\chi(t)}{\partial\lambda} &= \frac{\left(1 - e^{-(\mu-\lambda)t} - \lambda e^{-(\mu-\lambda)t}t\right)\left(\mu - \lambda e^{-(\mu-\lambda)t}\right) - \left(\lambda - \lambda e^{-(\mu-\lambda)t}\right)\left(-e^{-(\mu-\lambda)t} - \lambda e^{-(\mu-\lambda)t}t\right)}{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right)^2} \\ &= \frac{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right) + \left(-e^{-(\mu-\lambda)t} - \lambda e^{-(\mu-\lambda)t}t\right)\left(\mu - \lambda e^{-(\mu-\lambda)t} - \lambda + \lambda e^{-(\mu-\lambda)t}\right)}{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right)^2} \\ &= \frac{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right) + \left(-e^{-(\mu-\lambda)t} - \lambda e^{-(\mu-\lambda)t}t\right)(\mu - \lambda)}{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right)^2}. \end{aligned}$$

After the cancellations the derivative simplifies to

$$\frac{\partial\chi(t)}{\partial\lambda} = \frac{\mu\left(1 - e^{-(\mu-\lambda)t}\right) - \lambda e^{-(\mu-\lambda)t}t(\mu - \lambda)}{\left(\mu - \lambda e^{-(\mu-\lambda)t}\right)^2}.$$

I need to show that this term is greater than zero. Since $\left(\mu - \lambda e^{-(\mu-\lambda)t}\right)^2 > 0$, I only need to show that

$$\begin{aligned} \mu\left(1 - e^{-(\mu-\lambda)t}\right) &> \lambda e^{-(\mu-\lambda)t}t(\mu - \lambda) \\ \frac{\mu\left(e^{(\mu-\lambda)t} - 1\right)}{t} &> \lambda(\mu - \lambda). \end{aligned}$$

Using the result of Lemma 1, as $t \to 0$, $\frac{e^{(\mu-\lambda)t}-1}{t} \to (\mu - \lambda)$. Hence, the inequality holds. Since $\frac{\partial}{\partial t}\left(\frac{e^{(\mu-\lambda)t}-1}{t}\right) > 0$, the inequality holds for $\forall t > 0$. ■

Proposition 1 Hazard rate of exit $H(a, S)$ decreases in age conditional on size and decreases in size conditional on age for all ages and sizes ($a > 0$ and $S > 0$).

Proof. The proposition has two parts. First I will prove that conditional on size, hazard rate of exit decreases in age.

Klette and Kortum (2004) show that $\chi'(t) > 0$. In Lemma 2, I showed that $\frac{\partial\chi(t)}{\partial\lambda} > 0$. Also,

Klette and Kortum (2004) prove that λ is uniquely determined for the profit per product π values. In this model, since π is uniquely determined by the efficiency level φ, χ is increasing in φ.

At any size S,

$$n = \frac{S}{q\left(\frac{(\sigma-1)\varphi P}{\sigma w}\right)^{\sigma-1}}.$$

Among the establishments with size S, more efficient producers (with higher φ) will own fewer products.

To simplify the proof, I assume that there are two types of establishments with low and high efficiency levels $\{\varphi_l, \varphi_h\}$. Denote the age of the low-type producer as a_l and high-type as a_h. For any $a_h \geq a_l$, $\chi_h(a_h) \geq \chi_l(a_h) \geq \chi_l(a_l)$. Since χ is the parameter of the geometric distribution[22], this relation implies that size distribution for high type producers stochastically dominates the size distribution of low types.

$$\chi_h(a_h) > \chi_l(a_l) \Rightarrow 1 - \chi_h(a_h)^n < 1 - \chi_l(a_l)^n.$$

Note that χ is a monotonically increasing function in time. Hence low efficiency producers being more likely to have more products than the high efficiency producers is possible when $a_l > a_h$. In this case, when the difference between a_l and a_h is large enough

$$\begin{aligned} \chi_h(a_h) &< \chi_l(a_l) \\ 1 - \chi_l(a_l)^n &< 1 - \chi_h(a_h)^n. \end{aligned}$$

Klette and Kortum (2004) show that the hazard rate of exit at age a is $\mu(1 - \chi(a))$. Using the conclusion that at among the producers with size S, $a_l > a_h$ and $\chi_l(a_l) > \chi_h(a_h)$, the hazard rate of exit is lower for the older producers,

$$\mu(1 - \chi_l(a_l)) < \mu(1 - \chi_h(a_h)).$$

The second part of the proposition is more straightforward. At any age a, $\chi_h(a) > \chi_l(a)$ because χ is increasing in φ. Hence, more efficient producers are more likely to have more products and are less likely to exit

$$\mu(1 - \chi_h(a)) < \mu(1 - \chi_l(a)).$$

■

[22] Probability mass function of geometric distribution is $\Pr(X = n) = (1-p)\,p^{n-1}$. Cummulative distribution is $\Pr(X \leq n) = 1 - p^n$.

E Pre-exit Behavior of Establishments

Proposition 2 Consider a cohort of establishments all entering at the same time. At any age $a \geq 1$, within this cohort, establishments that survive longer are larger in size than the exiting establishments (i.e. for S representing the establishment size, t_x representing the time of the exit, $\Delta > 0$, $0 \leq a < t$, $E[S(a)|t_x = t] \leq E[S(a)|t_x = t + \Delta]$).

Proof. At the startup ($a = 0$), all establishments start with a single product. From equation 16, size of a $\varphi-$type establishment relative to aggregate expenditure at the startup will be $S_1(\varphi) = \left(\frac{p(\varphi)}{P}\right)^{1-\sigma}$. For any $\varphi_h > \varphi_l$, $E[S_1(\varphi_h)] > E[S_1(\varphi_l)]$. Also, since φ_h-type producers are more innovative[23], they're less likely to exit than the φ_l-type producers. Hence, more efficient producers start larger and they are more likely to survive longer. This generates the dispersion at the startup.

This difference between high and low efficient producers persists through their life spans. Klette and Kortum (2004) show that expected size of an establishment at any age a, conditional on survival is given as

$$\sum_{n=1}^{\infty} n \frac{p_n(a;1|\varphi)}{1 - p_0(a;1|\varphi)} = \frac{1}{1 - \chi(a|\varphi)}.$$

At any age $a > 0$, comparing establishments with efficiency levels $\varphi_h > \varphi_l$, $E[n(a)|\varphi_h] > E[n(a)|\varphi_l]$ and $\mu(1 - \chi(a, \varphi_h)) < \mu(1 - \chi(a, \varphi_l))$. Since the revenue per product is also higher for the more efficient producers, we get $E[S_a(\varphi_h)] > E[S_a(\varphi_l)]$.

Now consider any two establishment at age a of the same efficiency level. For an establishment that exits at time $t > a$ and $\Delta > 0$, instantaneous hazard rate of exit is given as[24] $\Pr(t_x = t) = \frac{\dot{p}_0(t;1)}{1 - p_0(t;1)} = \mu(1 - \chi(t))$. This probability increases in time. Then the expected size of a $\varphi-$type establishment at age a conditional on exiting at $t > a$ is

$$E[n(a)|t_x = t] = \sum_{n=1}^{\infty} n \frac{p_n(a;1)}{\mu(1 - \chi(t))}.$$

For $\forall\ t, \Delta > 0$

$$\frac{1}{\mu(1 - \chi(t))} < \frac{1}{\mu(1 - \chi(t + \Delta))},$$

which leads to $E[n(a)|t_x = t] < E[n(a)|t_x = t + \Delta]$ for $\forall a > 0$. ■

[23] Klette and Kortum (2004) show that innovation rate λ increases in profit π, which increases in efficiency rate φ.

[24] For simplicity I drop the efficiency level from the equations that will follow.

F Industry Comparison

Figure 10: Size Distribution of Industries

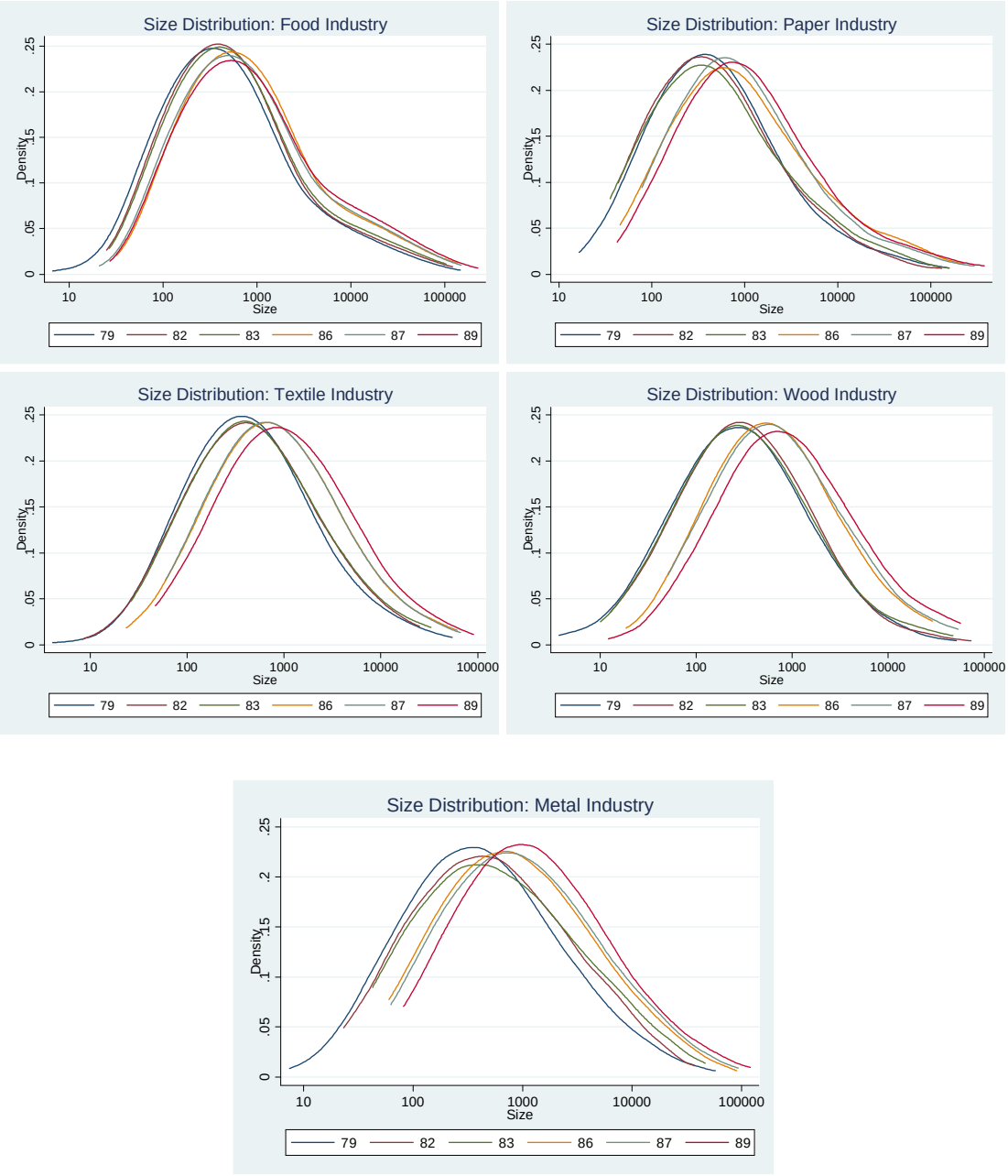
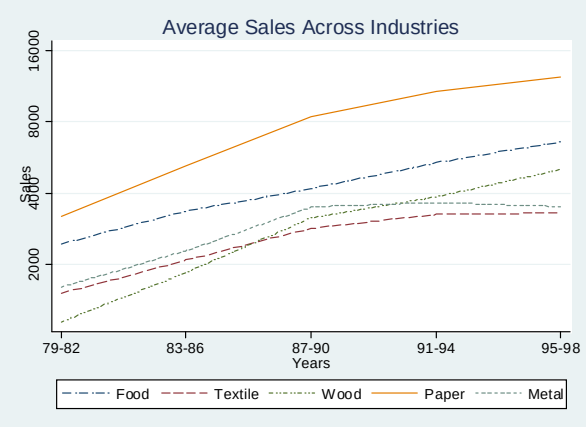
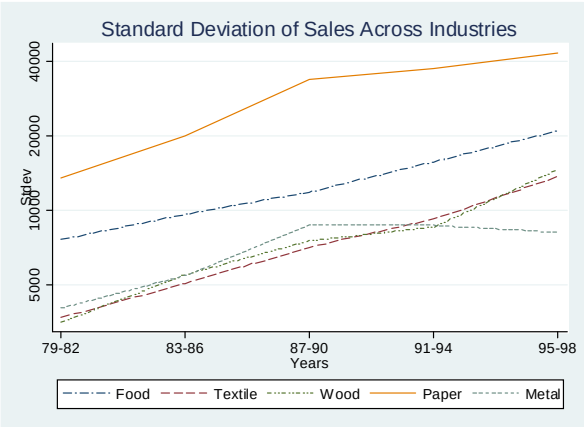
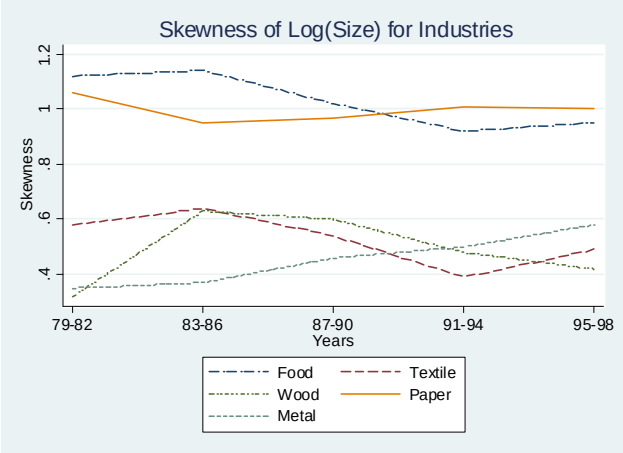
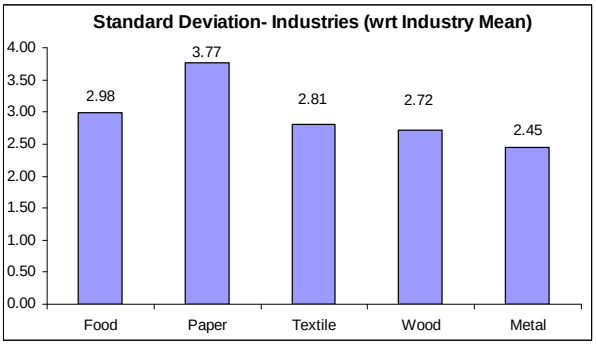


Figure 11: Graphs on Industry Comparison









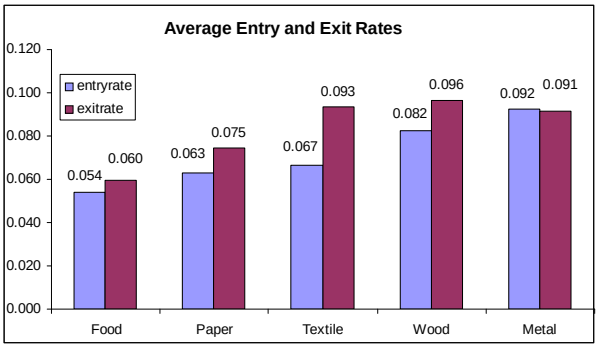
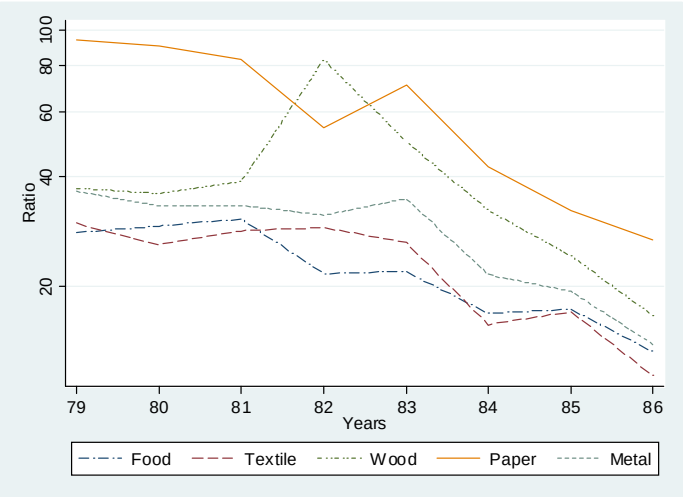


Figure 12: Capital-Output Ratio across Industries



G Estimation Results for Industries

Table 7: Simulation Results

	Food (311)			Paper (341)	
Moments	Data	Simulation		Data	Simulation
pctile(10)	128.80	73.2		107.36	45.5
pctile(25)	207.43	133		186.74	88.1
pctile(50)	346.15	368.2		380.76	295.4
pctile(75)	894.56	1217.2		1026.46	1305.7
pctile(99)	31291.11	31276		44379.6	44370
η	0.054	0.043		0.063	0.064
E[g]	0.027	0.011		0.02	0.02
Std[g]	0.75	0.39		0.5	0.40
E[Y]	2427.0	2117.1		3192.10	2964.3
Std[Y]	7569.0	9912.3		13549.7	18172

	Textile(321)			Wood(331)			Metal (381)	
Moments	Data	Simulation		Data	Simulation		Data	Simulation
pctile(10)	106.94	118		62.50	72.3		86.2	143
pctile(25)	193.09	180.1		119.52	125.5		154.3	213.4
pctile(50)	370.63	357.2		267.54	267.8		343.8	400
pctile(75)	890.32	900.1		673.43	660.2		1021.5	908.7
pctile(99)	20498.3	20498		13421.26	13422		16141.43	16144
η	0.067	0.058		0.082	0.044		0.092	0.06
E[g]	-0.01	-0.02		0.05	0.045		0.017	0.015
Std[g]	0.70	0.47		0.90	0.48		0.83	0.47
E[Y]	1502.6	1530.5		1136.8	1007		1598.1	1361.4
Std[Y]	3716.4	7407.7		3552.5	4069		4052.9	6205

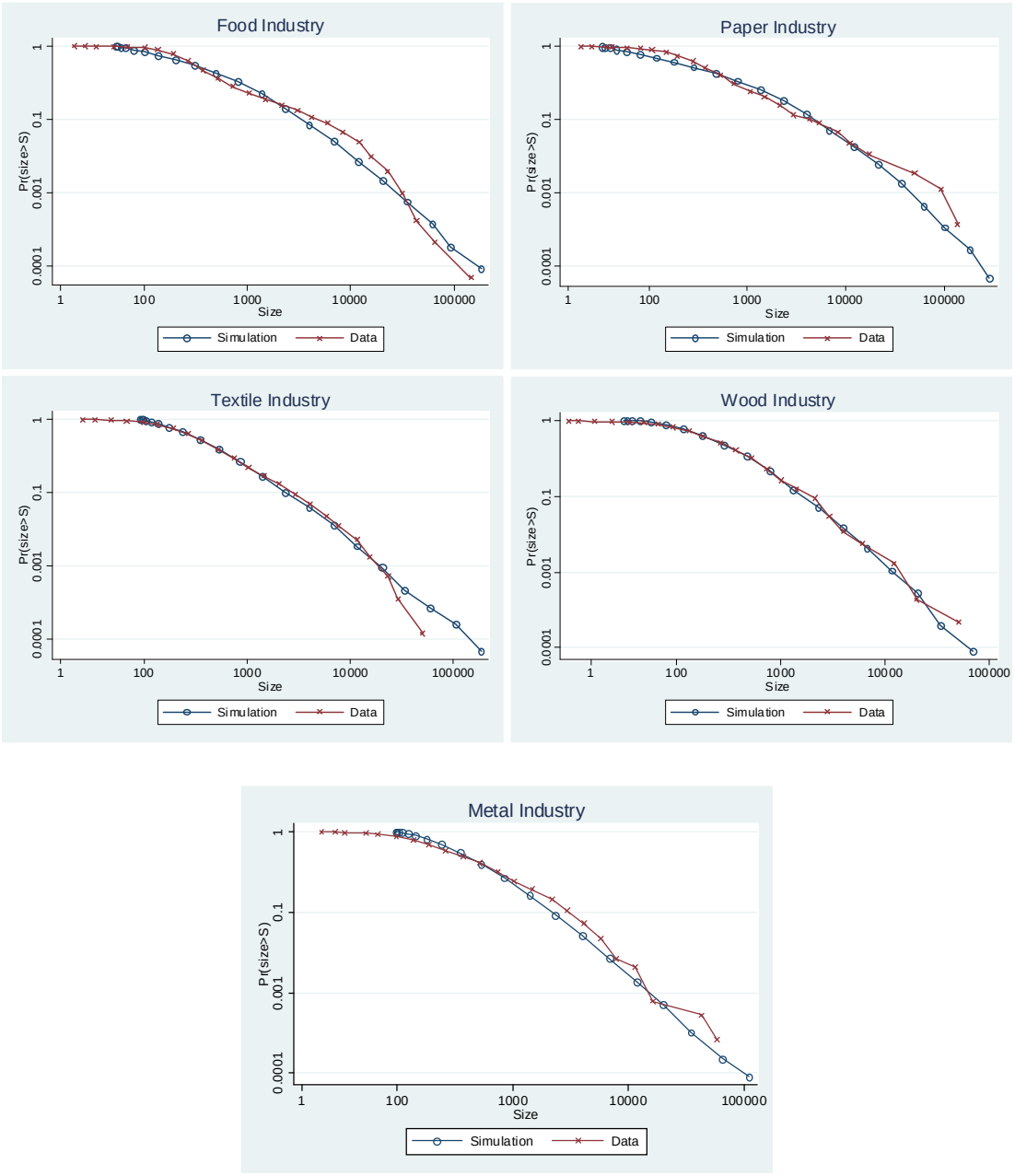



Figure 13: Industry Size Distributions: Model Fit