Forecasting Consumption Growth with the Real Term Structure[*]

Kwok Ping Tsang[+]

Department of Economics, Virginia Tech

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Comments Welcome

Abstract: From the log-linearized consumption Euler equation, consumption growth of any horizon *m* is a function of the expected real return of maturity *m*, and they are linked through the elasticity of intertemporal substitution (EIS). Instead of using only the 1-period return and consumption growth, this result allows us to use the term structure of interest rates to identify the EIS. Using quarterly US data from 1954Q1 to 2007Q4, GMM results show that the real term structure is unrelated to future consumption growth: after controlling for small sample bias, we cannot reject the hypothesis that the EIS is zero. However, allowing a break in 1979 changes the results dramatically: the EIS is around 0.4 in the first period and it drops to around 0.2 in the second period. Not only is the EIS smaller, the out-sample forecasting power of ex post real return also drops in the second subsample compared to a simple AR(1) model for consumption growth. I find a lower EIS also for annual data.

JEL Classification: E21, E31, E43, E44, G12

Keywords: Consumption Euler Equation, Term Structure of Interest Rates, Inflation Forecast, Elasticity of Intertemporal Substitution, GMM

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[+] Department of Economics, Virginia Tech, Pamplin Hall (0316), Blacksburg, VA 24061

1. Introduction

The log-linearized Euler equation says that the *k*-period future consumption growth is driven by a *k*-period expected real return. Log-linearizing the Euler equation, the *k*-period future consumption growth can be written as a function of an intercept, a *k*-period expected return, the second moments and an expectation error. The coefficient on the expected return is the elasticity of intertemporal substitution (EIS). This paper looks at the size and statistical significance of the EIS over different horizons, or, equivalently, the expected real returns of different maturities as forecaster of consumption growth.

Using US aggregate quarterly data from 1954Q1 to 2002Q4, GMM results show that the EIS has decreased from around 0.4 for the subsample 1954Q1 – 1979Q3 to around 0.2 for the subsample 1979Q4 – 2002Q4. The response of consumption growth to real return is similar in magnitude over different horizons, and the results are not affected by whether the Euler equations for different maturities are estimated separately or estimated as a panel. Annual data on 1-year consumption growth and yield provide similar results. Not only is EIS smaller in the second subsample, out-sample forecasting for consumption growth shows that the performance of the Euler equation relatively to a simple AR(1) model worsens in the second sub-sample. The results suggest that the representative agent is less willing to substitute consumption over time since the 1980s, i.e. the key structural parameter EIS has changed.

Why Long Horizons?

What can be gained from looking at consumption growth of longer horizons? According to Parker and Julliard (2005)[1], there are two arguments that consumption growth of longer horizons will more accurately measure intertemporal substitution. First, as pointed out by Wilcox (1992), sampling error, imputation procedures and definitional difficulties lead to serially correlated measurement error in consumption, and the error leads to delayed measure of actual consumption. Second, if utility is not separable, the marginal utility of consumption will be a function of leisure, habits or other variables in the utility function. Provided that these variables are stationary and correlated with the real returns, they will have a similar impact as that of a serially correlated measurement error. For these three reasons, using consumption growth of longer horizons will provide a more accurate measure of how people substitute consumption over time. In addition to alleviating the effect of measurement error, looking at several horizons also allows us to estimate the Euler equations as a system and achieve better identification.

Related Works[2]

For empirical research on the EIS, there are two strands: papers that find a large and significant EIS and papers that find an EIS close to zero. A famous example of the finding of a small EIS is Hall (1988). Taking into account the time aggregation problem in consumption data, he shows that consumption growth is completely insensitive to interest rates. Campbell and Mankiw (1989) have similar results. Modifications of the simple representative consumer optimization problem usually give larger estimates of

[1] They explain stock returns using consumption growths of different horizons, which is different from my purpose here.
[2] Please see Section 5 for references on heterogeneous agents and the dependence of EIS on wealth.

EIS: Accounting for limited participation in asset market (Mankiw and Zeldes (1991)), non-separability between durables and nondurables (Ogaki and Reinhart (1998)) or non-separability between consumption and leisure (Basu and Kimball (2000)) are some examples.

The idea of linking the real term structure to consumption growths of different horizons originates from Harvey (1988). He tests whether the expected real term structure contains information on future consumption growth. He finds evidence for predictability during the 1970s and 1980s, and he shows that the real term structure has higher forecasting power than lagged consumption growth and lagged stock returns. Chapman (1997) also considers consumption growth and real yields of different maturities in a standard RBC model context.

2. Euler Equation

In the consumption capital asset pricing model (C-CAPM), the representative agent allocates resources to consumption and assets. The agent lives in an exchange economy and has the following objective function at time *t*:

$$\max_{\{C_{t+\tau}\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left[U\left(C_{t+\tau}\right) \right],$$

where β is the subjective discount rate and C_t is real consumption at time *t*. The agent can trade nominal bonds of different maturities with no risk of default (the presence of other assets do not concern us here). The agent uses the endowment and proceeds from the sale of bonds owned last period to finance consumption and bond acquisitions for the

current period. The first-order condition (Euler equation) from the optimization problem for 1 period is:

$$E_t\left[\beta\frac{U'(C_{t+1})}{U'(C_t)}\left(1+r_t^{(1)}\right)\right]=1 \text{ or } \beta E_t\left[U'(C_{t+1})\left(1+r_t^{(1)}\right)\right]=U'(C_t)$$

Since inflation from time t to time t+1 is unknown at time t, the one-period real return from time t to time t+1 $r_t^{(1)}$ is also unknown at time t. An optimizing consumer does not only equalize the marginal utilities of today and tomorrow but she also equalizes those of the more distant future. In general, the first-order condition for any horizon m holds:

$$E_t\left[\beta^m\frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right]=1 \text{ or } \beta^m E_t\left[U'(C_{t+m})\left(1+r_t^{(m)}\right)^m\right]=U'(C_t),$$

where $r_t^{(m)}$ is the unknown real return per period over the coming m periods. Following Hansen and Singleton (1983), the ratio of the marginal utility and the real return of any horizon m are assumed to be jointly log-normally distributed[3]:

$$\begin{aligned}&\ln E_t\left[\beta^m\frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right]=\\&E_t\left[\ln\left(\beta^m\frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right)\right]+\frac{1}{2}\text{var}_t\left[\ln\left(\beta^m\frac{U'(C_{t+m})}{U'(C_t)}\left(1+r_t^{(m)}\right)^m\right)\right]=0\end{aligned} \quad (1)$$

To simplify the exposition, let the utility function be the constant relative risk aversion (CRRA) type[4]:

$$U(C_t)=\frac{C_t^{1-\gamma}-1}{1-\gamma}.$$

[3] Harvey (1988) also makes use of the same assumption.
[4] It can be shown that we get the same reduced-form relationship with the ordinal certainty equivalence (OCE) or the Epstein-Zinn utility functions. See Appendix.

Plugging the utility function into (1) and taking logarithm, we have the log-linearized form of the Euler equation (logged consumptions are in lowercase):

$$-\gamma\left(E_t c_{t+m} - c_t\right) + m\ln\beta + E_t m\ln\left(1+r_t^{(m)}\right) + \frac{1}{2}\mathrm{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+r_t^{(m)}\right)^m\right)\right] = 0 \qquad (2)$$

For a small real return, I can use the approximation $\ln\left(1+r_t^{(m)}\right) \approx r_t^{(m)}$. Divide both sides of (2) by $m\gamma$. To obtain further insights from the Euler equation, the real return is decomposed into the nominal return and inflation: the term

$$\mathrm{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+r_t^{(m)}\right)^m\right)\right]$$

can be rewritten as

$$\mathrm{var}_t\left[\ln\left(\beta^m\left(\frac{C_{t+m}}{C_t}\right)^{-\gamma}\left(1+i_t^{(m)}\right)^m\left(1+\pi_t^{(m)}\right)^{-m}\right)\right]$$

where $\pi_t^{(m)}$ is the inflation over the coming *m* periods in annual term, and $i_t^{(m)}$ is the nominal yield per period of maturity *m* that is known today. For small nominal return and inflation, the variance term is approximately equal to

$$\mathrm{var}_t\left[m\ln\beta - \gamma\left(c_{t+m} - c_t\right) + mi_t^{(m)} - m\pi_t^{(m)}\right].$$

We can now write expected consumption growth as a function of the expected real return, subjective discount rate and some second moments:

$$\begin{aligned}
&\frac{1}{m}\left(E_t c_{t+m} - c_t\right) \\
&= \frac{\ln\beta}{\gamma} + \frac{1}{\gamma}E_t r_t^{(m)} + \left(\frac{\gamma}{2m}\mathrm{var}_t\left(c_{t+m}\right) + \frac{m}{2\gamma}\mathrm{var}_t\left(\pi_t^{(m)}\right) + \mathrm{cov}_t\left(c_{t+m}, \pi_t^{(m)}\right)\right) \qquad (3) \\
&= \frac{\ln\beta}{\gamma} + \frac{1}{\gamma}E_t r_t^{(m)} + v_t
\end{aligned}$$

The effect of the expected real return on the expected consumption growth is determined by the elasticity of intertemporal substitution (EIS)$1/\gamma$ (the inverse of which is the coefficient of relative risk aversion for the CRRA case).

The linearized consumption Euler equation (3) can be estimated in two ways. According to the optimization problem, for any horizon *m* the coefficient on the expected real return is the same EIS parameter $1/\gamma$, and we can stack (3) for all horizons and estimate them as a system. Alternatively, the restriction that the slope coefficient is the same $1/\gamma$ can be relaxed and we can estimate (3) separately for each horizon.

3. Data

We use US quarterly data from 1954Q1 to 2007Q4. Since we consider consumption growth up to 5 years, the effective end period of the sample is 2002Q4. Nominal zero-coupon bond yields of maturities 1 quarter, 1 year, 2 years, 3 years, 4 years and 5 years are from the Center for Research in Security Prices (CRSP) database. Observations are taken at the last trading day of each quarter. Real per capita consumption expenditure of nondurable goods and service is from the Bureau of Economic Analysis (BEA). To be consistent with the type of consumption we use, price level is defined by the weighted average of the price index of nondurable goods and the price index of service. Both indexes are from the BEA and the weight is calculated by the amount of the two types of expenditure.

Figure 1a plots the 1-quarter consumption growth and 1-quarter *ex post* real return, and the co-movement between the two variables illustrates the main point of this paper.

4. Estimation Method and Results

One way to estimate (3) is to generate inflation forecasts for different horizons and subtract them from the nominal yields to get the real term structure. This method has two concerns: 1) a model for inflation is needed and any test on the EIS becomes a joint hypothesis that the inflation model is correctly specified and 2) the generated real term structure is a generated variable and the standard errors for equation (3) have to be corrected. To be avoid ad hoc modeling of inflation, equation (3) is estimated by GMM. The moment conditions for (3) can be written as:

$$E_t\left[\left(\frac{1}{m}\left(c_{t+m}-c_t\right)-\alpha^{(m)}-\frac{1}{\gamma}r_t^{(m)}\right)\mathbf{z}_t\right]=\mathbf{0}\text{, for } m=1,\ldots,M \qquad (4)$$

where $\mathbf{z}_t$ is the instrument set. The instruments are 2 lags of the 1-quarter ex post real return and the nominal term spread (i.e. the difference between the 5-year and 1-quarter yields)[5]. For both single equation and system estimations, equation (4) is estimated by 2-step efficient GMM.

A Monte Carlo Experiment

When m>1, observations on the LHS variable are overlapping over time (the Hansen-Hodrick problem). Conventionally, to deal with the resulting serial correlation in the error term, the Newey-West procedure is used to calculate the covariance matrices. The sample size employed in this paper is only slightly larger than 200. The Newey-West procedure, which is a large sample result, might lead to incorrect inference for my small

[5] Results are very similar using instrument sets that include lagged consumption or more lags.

sample size. To deal with the potential small sample bias I conduct a Monte Carlo experiment with the following DGP:

$$c_{t+1} - c_t = \mu_c + \varepsilon_{t+1},\ \varepsilon_{t+1} \sim i.i.d.N\left(0, \sigma_c^2\right)$$

$$\pi_{t+1} = \mu_\pi + \phi_1 \pi_t + \phi_2 \pi_{t-1} + u_{t+1},\ u_{t+1} \sim i.i.d.N\left(0, \sigma_\pi^2\right)$$

$\mathrm{cov}\left(\varepsilon_t, u_{t+j}\right) \neq 0$ only when j=0

Parameters for the above DGP are estimated from the data, and consumption growth and inflation of any horizon can be generated. To generate nominal yields, I adopt the factor model in Diebold, Rudebusch and Aruoba (2006) and generate the level, slope and curvature factors from a VAR(1) model using their parameter estimates[6]. The three shocks to the three term structure factors are uncorrelated to the shocks to consumption growth and inflation.

In this DGP *ex post* real return $i_t^{(1)} - \pi_{t+1}^{(1)}$ are correlated to consumption growth $c_{t+1} - c_t$ due to $\mathrm{cov}\left(\varepsilon_t, u_{t+j}\right) \neq 0$, and *ex ante* real return, which uses information up to time *t* only, should not explain consumption growth. The same argument applies to longer horizons. For 10,000 Monte Carlo runs, I generate sample sizes of 250 and 125 from the DGP and estimate (3) as both single equations and system, using the 2 lags of the 1-quarter *ex post* real return and the nominal term spread as instruments. I use the empirical distributions of the *t* statistic for the null that $1/\gamma = 0$ for inference.

[6] The reader may notice that Diebold, Rudebusch and Aruoba (2006) are using monthly data and hence their parameter estimates are inappropriate for my purpose. I use their parameter estimates just for convenience, and the Monte Carlo results reported below are not sensitive to the parameters in the VAR(1) factor model.

Main Results

Table 1 shows the main results of this paper[7]. Columns 2 to 4 in the first panel show the 2-step GMM results for the full sample. Using the empirical distribution obtained from the Monte Carlo experiment, except for the 5-year horizon we cannot reject the null hypothesis that $1/\gamma = 0$. Not only is the EIS not statistically significant, economically the magnitude of the EIS is also small: a 1% increase in the real return is related to around 0.1% increase in consumption for all horizons.

Next the sample is separated into two: 1954Q1 to 1979Q3 and 1979Q4 to 2002Q4. The two subsamples are of roughly the same length, and the quarter 1979Q3 is also when Volcker became the Fed chair and monetary policy started to undergo a substantial change. Splitting the sample around that period does not affect the results. Two other robustness checks are performed to ensure that the break point is not arbitrary. First, I perform the Andrews (1993) GMM structural break test to search for the break point statistically. Though the estimated break points differ slightly among maturities, they are all close to the date 1979Q4 selected here. Second, results from rolling regressions with a window size of 80 quarters for all maturities again show that the break point is around 1979Q4. See Figure 3 for the rolling regression results. Panels 2 and 3 in Table 1 show the GMM results for the two subsamples, and the empirical distributions are obtained from a Monte Carlo experiment with a sample size of 125. In the first subsample, the EIS is much more significantly both statistically and economically. The EIS estimates for the short horizons and the system are significant at 5% or 10% levels, and they are *big*:

[7] Similar regressions have been run using *ex post* S&P stock return, and the estimated EIS is always smaller than that using real yield. A similar same break in the EIS is also found in the late 1970s. The results for stock return may suffer from the problem of weak instrument though: it is very hard to find variables that can explain *ex post* stock return. For this reason the results are not reliable and are hence not reported.

a 1% percent increase in the real return is related to a 0.4%-0.5% increase in consumption growth. In the second sample, the EIS estimates are similarly significant statistically, but they are much smaller in magnitude: a 1% percent increase in the real return is related to a 0.2%-0.4% increase in consumption growth. Consumption growth is much less responsive to changes in real return in the second subsample[8].

Why do the EIS estimates for the full sample not "lie between" those from the two subsamples? As explained in Section 2, the intercept term in (3) contains the second moments of consumption and inflation. When the second moments are time-varying, equation (3) has an omitted variable. When periods of high volatility are also periods of low real return (e.g. when periods of high inflation are also periods of high inflation volatility), the EIS estimate in (3) will be biased downward. Breaking the sample into two alleviates this omitted variable problem, and lead to EIS estimates that are larger. In fact, the estimate of the constant term in the first subsample is much larger statistically and economically than that in the second subsample.

To give more intuition to the results in Table 1, Figure 2 plots the estimates from Table 1 for the two subsamples and I build the 95% confidence intervals using the Monte Carlo distributions. We can interpret the plots as showing the response of consumption growth of different horizons to a 1% upward shift of the real term structure. The response of consumption growth is larger in the first subsample, though the uncertainty of the response in the longer horizons is smaller in the second subsample.

[8] A usual criticism of the empirical results is that the DGP in the Monte Carlo experiment is wrong. As a robustness check, I use the first quarter of each year to create a non-overlapping sample for 1-year horizon. Consumption growth is regressed on a constant, a post-80 dummy, the ex post real return and the interaction of the real return and the post80 dummy. Estimation is done in GMM, and the result does not depend on whether the post-80 dummy is included in the instrument set. I find that the EIS drops from 1 in the first subsample to around 0 in the second subsample. The lower EIS is not s spurious result due to overlapping data. The same robustness check cannot be done for longer horizons due to insufficient observations.

As a digression, notice that in Table 1 the performance of the Newey-West procedure performs poorly even for a sample of 250. A forecast horizon of 4 quarters the rejection rate if the true null that $1/\gamma = 0$ is well over 10%. The performance worsens as the forecast horizon increases and especially when the Euler equations are estimated as a system. Based on the same Monte Carlo experiment, for the Newey-West to achieve rejection rate close to 5% requires about 2,000 observations. The poor performance of the Newey-West standard error is also documented in Nelson and Kim (1993) in a different context.

Evidence from Out-Sample Forecasting

So far only the in-sample fit of the model is considered. The smaller response of consumption growth to real return may also lead to poorer out-sample forecasting power in the second subsample. In this section I compare the forecasting power of *ex post* real return to that of lag consumption growth.

For each of the two subsamples, I estimate (3) for 1-quarter horizon using the subsample except the last 20 quarters. Based on the parameter estimates, out-sample forecast is made for the last 20 quarters in each subsamples. The expected 1-quarter inflation is proxied by the fitted value for inflation by an AR(1) model using the whole sample (i.e. I assume the consumer knows the parameters in the AR(1) model)[9]. Similarly, an AR(1) model for consumption growth is estimated and serves as the "benchmark" forecasting model. The models are estimated only once and not recursively for each subsample.

[9] Estimating the AR(1) model for inflation separately for the two subsamples changes the results little.

Table 3 shows the ratio of the root mean squared error (RMSE) and mean absolute error (MAE) of (3) to the AR(1) model, and a number larger than 1 means the AR(1) model performs better. In both subsamples the Euler equation performs worse than the AR(1) model. In the second subsample both models perform better, but, comparing to the first subsample, the Euler equation performs relatively worse than the AR(1) model. The contrasting forecast performance of the two models is consistent with the main result of this paper: consumption growth responds less to real return since the 1980s.

Further Evidence from Annual Data

I use the CPI inflation, real per capita consumption and 1-year interest rate from the updated Shiller (1989) annual dataset for the period 1954-2004[10]. Figure 1b plots the 1-year consumption growth and the 1-year *ex post* real return. GMM is used and the instruments are 2 lags of the *ex post* 1-year real return and the difference between the 10-year and 1-year yields. Since the sample size is small, the GMM results are only suggestive and might be unreliable.

The results are reported in Table 2 for the full sample, the subsample 1954-1979 and the subsample 1980-2004. For each 1% increase in the 1-year real return, there will be a 0.4% increase in consumption growth for the coming year in the full sample. For the subsample 1954-1979 the increase in consumption growth is 0.9%, and it drops to 0.01% (which is not statistically significant) in the subsample 1980-2004. The same drop in the EIS appears in the annual data.

[10] The data are available from Shiller's website http://www.econ.yale.edu/~shiller/data.htm.

6. Concluding Remarks

This paper measures the EIS by using the real term structure to explain consumption growth of various horizons. For all horizons, I find that the full quarterly sample of 1954Q1 to 2002Q4 gives us a small and statistically insignificant estimate of the EIS. Splitting the sample in the late 1970s changes the results dramatically: the EIS is roughly 0.4 in the first subsample 1954Q1-1979Q3 and drops to roughly 0.2 in the second subsample 1979Q4-2002Q4.

In a recent work Guvenen (2006) points out that heterogeneity can explain the low estimate of the EIS using aggregate data: nondurable and service consumption is more equality distributed than income or wealth. If the EIS decreases with wealth[11], then estimate of the EIS using aggregate data captures the intertemporal substitution behavior of the poorer majority. A more unequal distribution of wealth since the 1980s may explain the reduced estimate of the EIS.

[11] See Blundell , Browning and Meghir (1994) and Attanasio and Browning (1995) for evidence using household data.

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Figure 1a: Plot of 1-Quarter Consumption Growth and Ex Post Real Return (Quarterly Data)

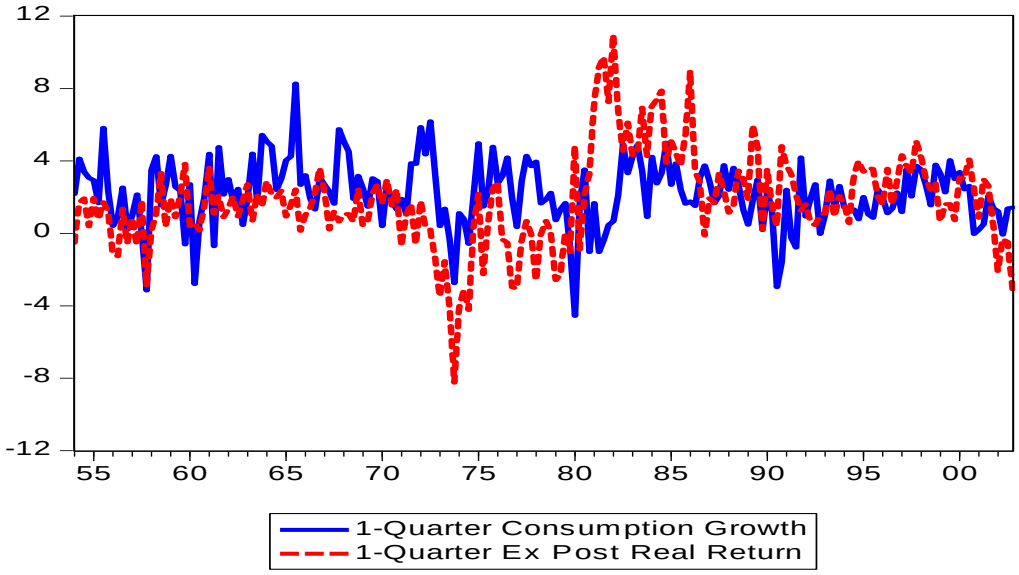
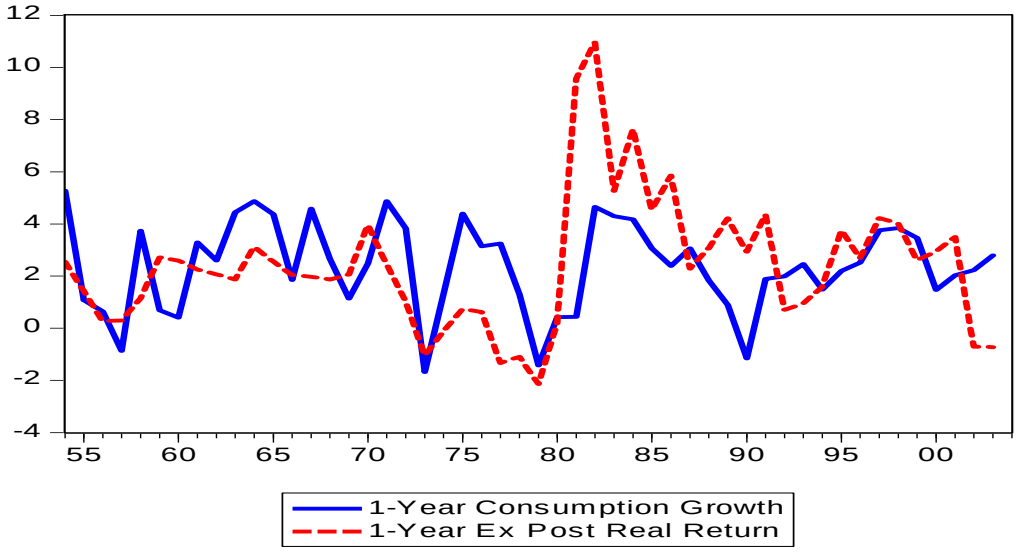
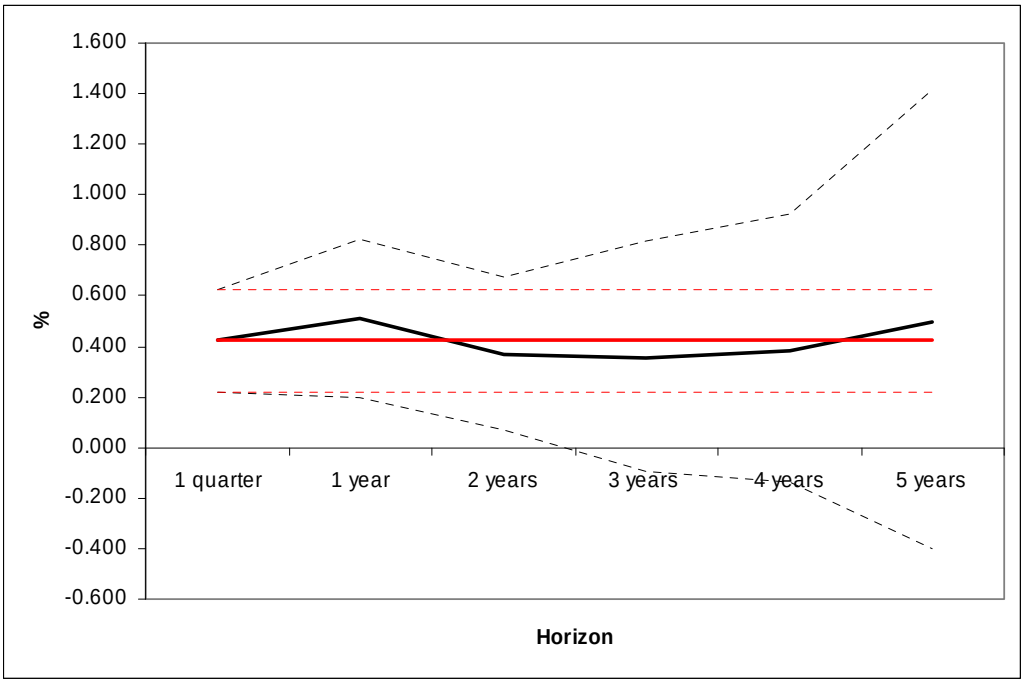


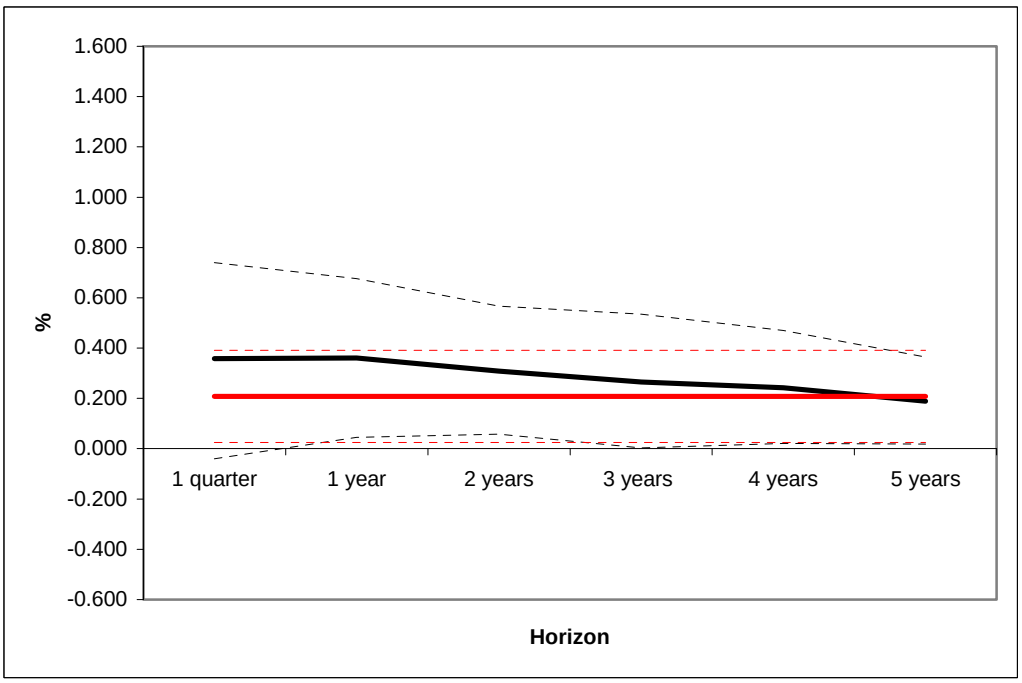
Figure 1b: Plot of 1-Year Consumption Growth and Ex Post Real Return (Annual Data)



Note: See Section 3 for details on the data.

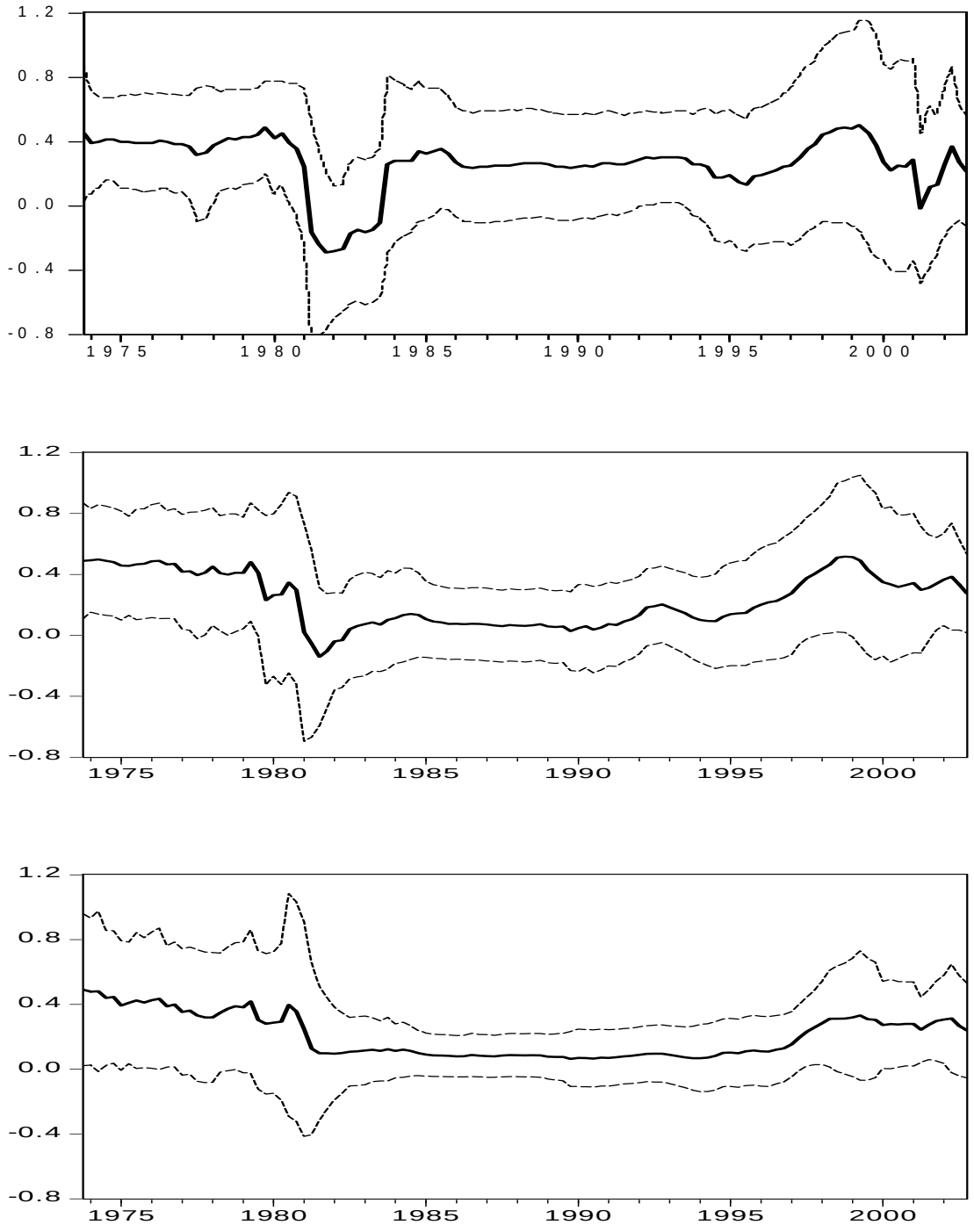
Figure 2 – Reaction of Consumption Growth to a 1% Upward Shift of the Real Term Structure in the Two Subsamples 1954Q1-1979Q3 and 1979Q4-2002Q4





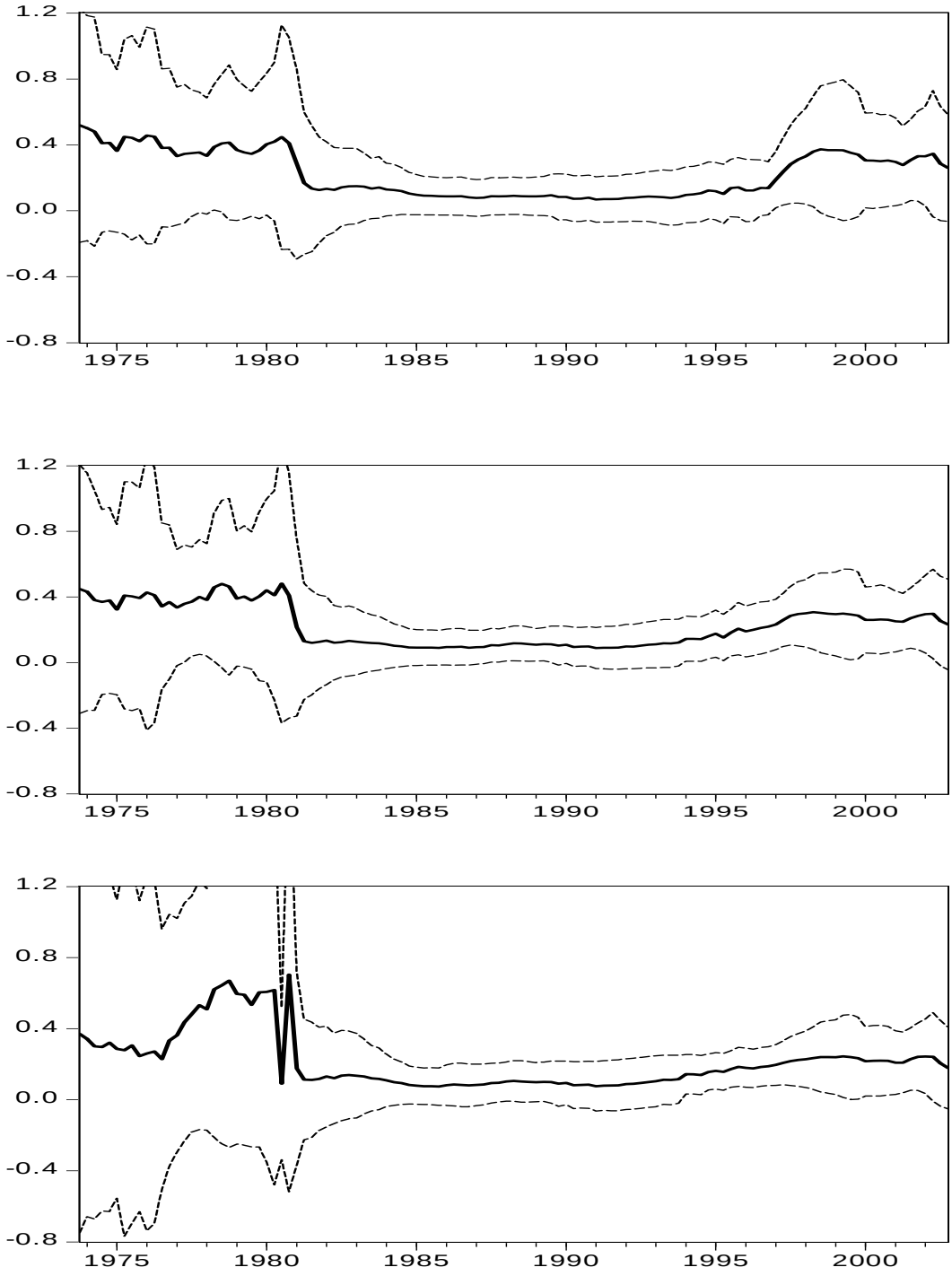
Note: The estimates for the EIS are the same as those from Table 1. The 95% confidence intervals are calculated using the Monte Carlo distributions. The black lines show the estimates from the single equation estimates, and the red lines show the system estimates.

Figure 3a – Rolling Regression Results for 1-Quarter, 1-Year and 2-Year Horizons



Note: The window size is 80 quarters, and the estimate shown in the graph is the end-of-period estimate, i.e. estimate for 1990Q1 uses 80 quarters of data ending at 1990Q1. The 95% confidence intervals are constructed using the critical values obtained from the same Monte Carlo experiment in Table 1 for a sample of 80 quarters.

Figure 3b – Rolling Regression Results for 3-Year, 4-Year and 5-Year Horizons



Note: The window size is 80 quarters, and the estimate shown in the graph is the end-of-period estimate, i.e. estimate for 1990Q1 uses 80 quarters of data ending at 1990Q1. The 95% confidence intervals are constructed using the critical values obtained from the same Monte Carlo experiment in Table 1 for a sample of 80 quarters.

Table 1: Efficient GMM Estimates (Full Sample, Subsample 1954Q1-1979Q3 and Subsample 1979Q4-2002Q4)

	Coefficient	S.E.	*t*-stat	1.25%	2.50%	5.00%	95.00%	97.50%	98.75%
1 quarter	0.184	0.090	2.053	-2.864	-2.170	-1.823	1.746	2.121	2.884
1 year	0.132	0.068	1.946	-3.280	-2.471	-2.020	1.957	2.398	3.419
2 years	0.114	0.041	2.757	-3.916	-2.789	-2.281	2.280	2.806	3.813
3 years	0.119	0.036	3.275	-4.211	-3.127	-2.523	2.560	3.181	4.319
4 years	0.108	0.030	3.563	-4.605	-3.310	-2.743	2.786	3.403	4.775
5 years	0.078	0.029	2.661	-4.784	-3.578	-2.897	2.916	3.625	4.993
System	0.092	0.021	4.307	-7.279	-5.399	-4.467	4.492	5.347	7.080

	Coefficient	S.E.	*t*-stat	1.25%	2.50%	5.00%	95.00%	97.50%	98.75%
1 quarter	0.423	0.093	4.566	-2.988	-2.241	-1.883	1.771	2.144	3.086
1 year	0.510	0.118	4.327	-3.743	-2.682	-2.233	2.129	2.675	3.773
2 years	0.369	0.093	3.982	-4.579	-3.232	-2.680	2.722	3.320	4.644
3 years	0.356	0.121	2.947	-5.198	-3.709	-3.061	3.049	3.832	5.762
4 years	0.385	0.128	3.007	-5.803	-4.103	-3.289	3.388	4.194	5.996
5 years	0.499	0.205	2.437	-6.120	-4.385	-3.467	3.592	4.488	6.522
System	0.423	0.037	11.378	-7.139	-5.455	-4.595	4.514	5.438	7.567

	Coefficient	S.E.	*t*-stat	1.25%	2.50%	5.00%	95.00%	97.50%	98.75%
1 quarter	0.358	0.178	2.013	-2.988	-2.241	-1.883	1.771	2.144	3.086
1 year	0.361	0.118	3.070	-3.743	-2.682	-2.233	2.129	2.675	3.773
2 years	0.309	0.078	3.983	-4.579	-3.232	-2.680	2.722	3.320	4.644
3 years	0.265	0.071	3.752	-5.198	-3.709	-3.061	3.049	3.832	5.762
4 years	0.243	0.054	4.478	-5.803	-4.103	-3.289	3.388	4.194	5.996
5 years	0.189	0.039	4.864	-6.120	-4.385	-3.467	3.592	4.488	6.522
System	0.208	0.034	6.200	-7.139	-5.455	-4.595	4.514	5.438	7.567

Note: Columns 2 to 4 are results from 2-step efficient GMM using 2 lags of *ex post* 1-quarter real return and nominal term spread as instruments. Standard errors are estimates by the Newey-West procedure. The empirical distributions of the *t* statistic are obtained from the Monte Carlo experiment as described in Section 4. The *J* test does not reject any of the estimations and so the *J* statistics are not reported. Estimates for the intercepts are not shown as they are not informative.

Table 2: GMM Result for the Relationship between 1-Year Consumption Growth and 1-Year Ex Post Real Return with Annual Data (1951-2004)

Period	Coefficient	S.E.	*t*-stat
1954-2003	0.429	0.087	4.912
1954-1979	0.926	0.228	4.056
1980-2003	0.011	0.200	0.057

Note: Data are from the Shiller (1989) annual dataset. Instruments are two lags of the *ex post* real return and the difference between the 10-year and 1-year yields are used. The *J* test does not reject any of the estimations and so the *J* statistics are not reported. Estimates for the intercepts are not shown as they are not informative.

Table 3: Out-Sample Forecasting Results for the Two Subsamples

Forecast Period	RMSE Euler	MAE Euler	RMSE AR(1)	MAE AR(1)	RMSE Ratio	MAE Ratio
1969Q4-1979Q3	1.794	1.441	1.695	1.348	1.058	1.069
1993Q1-2002Q4	1.077	0.902	0.986	0.794	1.092	1.136

Note: For each of the two subsamples, an AR(1) model for 1-quarter consumption growth and equation (3) for 1-quarter horizon are estimated using eachsubsample except the last 20 quarters. For the forecast in (3) the ex post return at each period is assumed to be known. Based on the parameter estimates, out-sample forecast is made for the last 20 quarters in each subsamples. The ratios in the table are those of the root mean squared error (RMSE) and mean absolute error (MAE) of equation (3) to the AR(1) model, and a number larger than 1 means the AR(1) model performs better

Appendix

A.1 The Ordinal Certainty Equivalence (OCE) Case

The OCE model proposed by Seldon (1978) avoids the inverse relationship between the EIS and risk aversion parameter. Now the problem becomes:

$$\max_{\left\{C_{t+\tau}^{*}\right\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t\left[U\left(C_{t+\tau}^{*}\right)\right]$$

where

(6) $$V\left(C_{t+\tau}^{*}\right) = E_t\left[V\left(C_{t+\tau}\right)\right]$$

Now the argument that enters the utility function directly is the "certain equivalent" consumption $C_{t+\tau}^{*}$. The "certain equivalent" is defined by (6). The curvature of the function U controls the level of intertemporal substitution and the curvature of the function V controls the level of risk aversion (See Attanasio and Weber (1989) for details). Define the inverse function $G = V^{-1}$, I write down the Euler equation for this problem as:

$$\beta^{m} U'\left(G'\left(E_t\left[V'\left(C_{t+m}\right)\left(1+r_t^{(m)}\right)^{m}\right]\right)\right) = U'\left(G'\left(V'\left(C_t\right)\right)\right)$$

Following the convention, I use the CRRA functional forms for both functions: $V\left(C\right) = \frac{C^{1-\theta}-1}{1-\theta}$ and $U\left(C^{*}\right) = \frac{C^{*1-\gamma}-1}{1-\gamma}$. The above Euler equation can be rewritten as (using the facts that $G' = V'^{-1}$ and $C_t = C_t^{*}$):

$$\beta^{m} C_{t+m}^{*-\gamma} C_{t+m}^{*\theta} E_t\left[C_{t+m}^{-\theta}\left(1+r_t^{(m)}\right)^{m}\right] = C_t^{-\gamma}$$

Writing in only observable consumption:

$$\beta^m E_t\left[\left(\frac{C_{t+m}}{C_t}\right)^{-\theta}\left(1+r_t^{(m)}\right)^m\right]E_t\left[\left(\frac{C_{t+m}}{C_t}\right)^{1-\theta}\right]^{(\theta-\gamma)/(1-\theta)}=1$$

Of course, if $\theta=\gamma$ I get back the same Euler equation as in the CRRA case in Section 2. Again, when both the terms in the expectation are log-normally distributed, I have the similar linear approximation:

$$m\ln\beta-\theta\left(E_t c_{t+m}-c_t\right)+mE_t r_t^{(m)}+\frac{1}{2}\mathrm{var}_t\left(\ln\left(\left(\frac{C_{t+m}}{C_t}\right)^{-\theta}\left(1+r_t^{(m)}\right)^m\right)\right)$$
$$+\frac{\theta-\gamma}{1-\theta}\left[\left(1-\theta\right)\left(E_t c_{t+m}-c_t\right)+\frac{1}{2}\mathrm{var}_t\left(\ln\left(\left(\frac{C_{t+m}}{C_t}\right)^{1-\theta}\right)\right)\right]=0$$

$$m\ln\beta-\gamma\left(E_t c_{t+m}-c_t\right)+mE_t r_t^{(m)}+\frac{1}{2}\mathrm{var}_t\left(-\theta\left(c_{t+m}-c_t\right)+mr_t^{(m)}\right)$$
->
$$+\frac{\theta-\gamma}{1-\theta}\left[\frac{1}{2}\mathrm{var}_t\left(\left(1-\theta\right)\left(c_{t+m}-c_t\right)\right)\right]=0$$

$$\gamma\left(E_t c_{t+m}-c_t\right)$$
->
$$=m\ln\beta+mE_t r_t^{(m)}+\frac{m^2}{2}\mathrm{var}_t\left(r_t^{(m)}\right)-\theta m\,\mathrm{cov}\left(c_{t+m},r_t^{(m)}\right)+\frac{1}{2}\left(\theta-\gamma+\theta\gamma\right)\mathrm{var}_t\left(c_{t+m}\right)$$

$$\frac{1}{m}\left(E_t c_{t+m}-c_t\right)$$
->
$$=\frac{1}{\gamma}\ln\beta+\frac{1}{\gamma}E_t r_t^{(m)}+\frac{m}{\gamma 2}\mathrm{var}_t\left(r_t^{(m)}\right)-\frac{\theta}{\gamma}\mathrm{cov}\left(c_{t+m},r_t^{(m)}\right)+\frac{1}{2\gamma m}\left(\theta-\gamma+\theta\gamma\right)\mathrm{var}_t\left(c_{t+m}\right)$$

-> $$\frac{1}{m}\left(c_{t+m}-c_t\right)=\alpha^{(m)}+\frac{1}{\gamma}E_t r_t^{(m)}+\varepsilon_{t+m}$$

Now $\alpha^{(m)}$ is a function of γ, θ and the second moments, and the coefficient on the ex ante real return is again the elasticity of intertemporal substitution. The only difference between the OCE model and the CRRA model is the different interpretation of the intercept term.

A.2 The Epstein-Zin (1989) Recursive Utility Case

Notice that the above results are not sensitive to the choice of a utility function. For example, if we have the Epstein-Zin (1989) utility function (an extension of the Kreps and Porteus (1978) model):

$$U_t = \left[C_t^{1-\gamma} + \beta E_t \left(U_{t+1}^{1-\theta} \right)^{(1-\gamma)/(1-\theta)} \right]^{1/(1-\gamma)}$$

Now the parameter γ is the reciprocal of the EIS, and the parameter θ is the coefficient of relative risk aversion. Epstein and Zin (1989) show that two Euler equations can be derived:

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}} \left(1+\overline{r}\right)^{\frac{\gamma-\theta}{1-\gamma}} \left(1+r_t^{(1)}\right) \right] = \beta^{\frac{(1-\theta)}{(\gamma-1)}}$$

and

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}} \left(1+\overline{r}\right)^{\frac{1-\theta}{1-\gamma}} \right] = \beta^{\frac{(1-\theta)}{(1-\gamma)}}$$

The term $\overline{r}$ stands for the unobservable return to aggregate wealth (which includes human capital). The first Euler equation can be rewritten in the multi-period version as:

$$E_t \left[\left(\frac{C_{t+m}}{C_t} \right)^{\frac{(1-\theta)\gamma}{(\gamma-1)}} \left(1+\overline{r}\right)^{\frac{m(\gamma-\theta)}{1-\gamma}} \left(1+r_t^{(m)}\right)^m \right] = \beta^{\frac{m(1-\theta)}{(\gamma-1)}}$$

Assuming consumption growth, return to aggregate wealth, and real return are jointly lognormally distributed, Attanasio and Weber (1989) show that the Euler equation can be written as:

$$\frac{1}{m}\left(c_{t+m}-c_t\right)=\alpha^{(m)}+\frac{1}{\gamma}E_t r_t^{(m)}+\varepsilon_{t+m}$$

where

$$\alpha^{(m)}=\frac{\ln\beta}{\gamma}+\frac{1}{m\gamma}\frac{1}{2}\left[\left(\frac{\gamma-\theta}{1-\gamma}\right)\mathrm{var}_t\left(1+\overline{r}\right)-\mathrm{var}_t\left(r_t^{(m)}\right)-\gamma^2\left(\frac{1-\theta}{1-\gamma}\right)^2\mathrm{var}_t\left(c_{t+m}\right)\right]$$
$$-\gamma\left(\frac{1-\theta}{1-\gamma}\right)\mathrm{cov}\left(r^{(m)},c_{t+m}\right)+\frac{\gamma-\theta}{1-\gamma}\mathrm{cov}_t\left(r_t^{(m)},1+\overline{r}\right)$$

and $\varepsilon_{t+m}=\frac{1}{m}\left(c_{t+m}-E_t c_{t+m}\right)$

Using the Epstein-Zin utility function only leads to a different interpretation of the coefficients: 1) the intercept term is a more complicated function of the discount rate and conditional variance and covariance terms, and 2) the effect of the expected real return on consumption growth is controlled by the EIS $1/\gamma$, and its reciprocal does not reflect the degree of risk aversion. The linear relationship between consumption growth and real yield still holds.