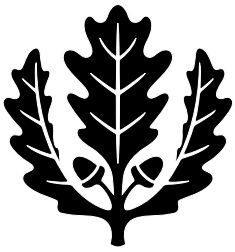

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The Making of Optimal and Consistent Policy: An Analytical Framework for Monetary Models

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Abstract

This paper shows that optimal policy and consistent policy outcomes require the use of control-theory and game-theory solution techniques. While optimal policy and consistent policy often produce different outcomes even in a one-period model, we analyze consistent policy and its outcome in a simple model, finding that the cause of the inconsistency with optimal policy traces to inconsistent targets in the social loss function. As a result, the central bank should adopt a loss function that differs from the social loss function. Carefully designing the central bank s loss function with consistent targets can harmonize optimal and consistent policy. This desirable result emerges from two observations. First, the social loss function reflects a normative process that does not necessarily prove consistent with the structure of the microeconomy. Thus, the social loss function cannot serve as a direct loss function for the central bank. Second, an optimal loss function for the central bank must depend on the structure of that microeconomy. In addition, this paper shows that control theory provides a benchmark for institution design in a game-theoretical framework.

Journal of Economic Literature Classification: E42, E52, E58

We presented an earlier version at Federal Reserve Bank of San Francisco. We thank Richard Dennis, Mark Spiegel, and Tao Wu for helpful comments and assistance. Yuan and Miller (2006) present a second method for the making of optimal and consistent policy. For convenience, we call the method in this paper, the analytical approach, and the second method, the implementation approach.

1. Introduction

The economic literature contains a strand that focuses on the optimality and consistency of decision making. Optimal plans lead inextricably to inconsistencies. An important part of this literature examines the optimality and consistency of microeconomic policy,[1] especially monetary policy.

Kydland and Prescott (1977) launch this whole literature by arguing that optimal policy proves inconsistent and showing that the inconsistency results from rational expectations. In a simple model of monetary policy making, the central bank needs some commitment technique to achieve optimal monetary policy over time. Absent the commitment technique, optimal monetary policy proves time inconsistent. The Kydland and Prescott (1977) thesis focuses on intertemporal issues and the need for commitment. While most of their analysis considers an intertemporal model, they do explore the issues within a simple sequential-decision, one-period model.

Barro and Gordon (1983a) build an analytical model for analyzing the inconsistency issue of monetary policy, by modifying a verbal and graphical model in Kydland and Prescott (1977).[2] Because of rational expectations, an inflation bias prevails under discretion (consistent policy), even though the optimal policy equals zero inflation. Barro and Gordon (1983b) prove that reputation can provide the commitment technique necessary to make consistent policy optimal, under certain conditions.

Based on Barro and Gordon's (1983a) standard monetary model, much of the literature

[1] Since the macroeconomic model involves the firm's and wage setter's decisions, we refer to the macroeconomic model as microeconomic model throughout the paper.

[2] Barro and Gordon (1983a) modify the social objective function, making both the deviations of inflation and unemployment from target quadratic terms, whereas the implied model in Kydland and Prescott (1977) enters the deviation of unemployment from target as a linear, and not quadratic, term. But, the model in Barro and Gordon (1983b) encompasses the verbal and graphical monetary model in Kydland and Prescott (1977).

provides solutions to the inconsistency problem in monetary policy. Before considering our solution, we first review and define the concepts of optimal and consistent policies. Then, we compute optimal policy and consistent policy in a simple model, using control-theory and game-theory solution techniques. By analyzing consistent policy and its outcome, we find that the source of inconsistency comes from *inconsistent targets*, an important observation in our method. That is, the two targets, the inflation rate and the employment level, in the social loss function prove inconsistent. As a result, the central bank should not adopt the social loss function as its own. Accordingly, we design the central bank loss function with consistent targets. Under the designed loss function, the optimal policy and consistent policy prove identical.

We compare and contrast our method to the solutions in the existing literature and get some interesting results. In addition, we apply our method to several different variants of the Barro and Gordon (1983a) type model with the same outcome. Yuan and Miller (2005) apply this method to the specific inflation-unemployment example in Kydland and Prescott (1977).

The paper unfolds as follows. Section 2 provides a brief review of the inconsistency of optimal plans. Section 3 develops the simple Barro and Gordon (1983a) type model and illustrates how consistent policy proves non-optimal. Section 4 discusses the design of the central bank loss function such that consistent policy proves optimal. Section 5 compares our solution to the inconsistency problem with other solutions in the existing literature. Section 6 repeats our method for a more complex, although still simple, model. Section 7 concludes.

2. Optimal Policy and Consistent Policy: A Review

Strotz (1955-56) first broached the subject of the inconsistency of optimal plans. Afterwards, much literature proves its existence, tries to determine its sources or causes, and provides its

solutions. To review this literature, we first clarify the concepts of optimal plans and consistent plans, making it easier to understand inconsistency of optimal plans.

Definitions of Optimal Policy and Consistent Policy

For optimal plans, the existing literature shares the same implicit definition, but uses different terms, for example, "commitment optimum path" (Pollak, 1968) and "Ramsey policy" (Chari, 1988). An optimal plan, defined by Strotz (1955-56), implies that an individual chooses over some future period of time to maximize the utility of the plan, evaluated in the present. The individual's choice, of course, conforms to certain constraints. Strotz's definition applies to one-person decision problems. In a game-theory model, especially a microeconomic model with a social planner (e.g., the government or the central bank), we define the *optimal policy* as the social planner's ex ante plan, if implemented, that produces a Pareto efficient outcome, according to some social welfare criterion.

Now, turn to a consistent plan. Strotz (1955-1956) defines a consistent plan as the best plan among those that an individual will actually follow. Pollak (1968) argues, however, that Strotz's consistent plan, which corresponds to Pollak's "naïve optimum path", could not actually occur. Pollak defines another term "sophisticated optimum path," the correct definition of a consistent plan. The sophisticated optimum path captures the same idea as subgame perfect equilibrium and/or sequential equilibrium in game-theory models, though Pollak's model encompasses only a one-person decision problem. Kydland and Prescott (1977) define consistent policy much like Pollak's "naïve optimum path." Kydland (1977) suggests "operational characteristics of economics models…point strongly toward an equilibrium concept for dynamic dominant-player models… This solution is called the feedback solution… it has the property that the original plan is consistent under replanning." Chari and Kehoe (1989) define time consistent

policy as a sustainable plan. Sustainability closely relates to subgame perfection and sequential equilibrium. In sum, in microeconomic models with more than one decision maker, we define a *consistent policy* as the government's (or the central bank's) plan, which, together with the strategies of other decision makers, constitutes an equilibrium. The equilibrium can include a Nash equilibrium, a subgame perfect equilibrium (Selten, 1965), or a sequential equilibrium (Kreps and Wilson, 1982). At this point, we do not discuss, in detail, which equilibrium concept proves more adequate, because different equilibrium concepts correspond to different types of game-theory models. Also, the concept of equilibrium continuously evolves. Loosely speaking, an equilibrium contains a strategy profile that results in "an outcome that satisfies mutually consistent expectations." (Shubik, 1998, p. 6)

Existence and Sources of Inconsistency

With clear definitions of optimal plans and consistent plans, we can now more easily grasp the nature of the inconsistency of optimal plans from perspective of game theory, where equilibria often prove Pareto inefficient. Now, given that optimal plans generally prove inconsistent, the literature studies the sources of inconsistency. For one-person decision problems, inconsistency may arise from an "intertemporal tussle" (Strotz, 1955-1956) and the specific functional form of utility.[3] Thus, for example, Calvo (1978a), Rodriguez (1981), and Leininger (1985) show that consistent and optimal plans exist in an important class of economies with special functional forms for utility (e.g., stationary period or instantaneous utility). Other researchers, such as Dasgupta (1974), demonstrate that an inadequate social welfare criterion can lead to inconsistent optimal plans. In sum, inconsistency can occur for different specific reasons in different specific

[3] Actually, the "intertemporal tussle" in Strotz (1955-1956) results from the non-exponential discount function, which also captures a specific functional form of utility.

models. This view, we argue also applies to more-than-one-person decision problems (i.e., game-theory models).

Kydland and Prescott (1977) first perceived the inconsistency that resulted from rational expectations.[4] Rational expectations imply the important notion of equilibrium in game theory. As defined above, an equilibrium outcome reflects rational players' mutually consistent expectations. We argue that they correctly recognize the source of inconsistency of the optimal plan. Then Kydland and Prescott (1977) conclude, "there is no way control theory can be made applicable to economic planning when expectations are rational." (p. 473) But in Kydland and Prescott (1980), they also indicate, "Even though there is little hope of the optimal plan being implemented—because of its time inconsistency—we think the exercise is of more than pedagogical interest. The optimal plan's return is a benchmark with which to compare the time consistent solution..." (p. 79) In other words, control theory can identify the optimal plan and, thus, the optimal economic outcomes. Then, we can seek a consistent plan that coincides with the optimal plan through institutional design. That is, the optimal plan can indicate how to design the optimal institution, through which we implement the optimal plan with a consistent plan. This task encompasses the rest of this paper and the other paper by Yuan and Miller (2006).

Solutions to Inconsistency of Optimal Plan

We classify the solutions to the inconsistency of optimal plans into three types: rules, reputation, and delegation.

Kydland and Prescott (1977) argue for "rules rather than discretion." That is, rules can provide the commitment technique to achieve optimal policy. And the literature provides many illustrations that economies perform better under rules than under consistent policy (i.e.,

[4] Calvo (1978b) independently perceived this point.

discretion). As a result, a literature exists on the design of policy rules. In monetary models, they include McCallum (1988), Taylor (1993), Svensson (1999), and so on. Even if a central bank voluntarily adopts a rule, it still faces the commitment issue – time consistency. Delegation of a rule to the central bank can address this commitment issue. In our context, we delegate a loss function, not a rule, to the central bank.

As we know, equilibria often prove Pareto inefficient (i.e., consistent policy generally proves not optimal). Game theory suggests, however, that an equilibrium outcome may prove optimal under certain conditions, if the game repeats and reputation plays a role. That is, reputation can provide a commitment technique to attain optimal policy in repeated games. Barro and Gordon (1983b) construct such a model to show that optimal policy proves implementable and consistent under certain conditions. Backus and Driffill (1985) demonstrate that reputation, based on the concept of Kreps and Wilson's (1982) sequential equilibrium, makes optimal policy credible. We, however, do not advocate the reputation approach, which we explain below.

In our context and in monetary models, delegation means that the government delegates a monetary policy objective to the central bank. In a broad sense, delegation implies mechanism or institutional design. When establishing a specific institution (e.g., the central bank), the government must delegate an appropriate objective. Rogoff (1985), Walsh (1995), Svensson (1997), Chortareas and Miller (2003), and so on fall broadly into the delegation approach.

Rogoff (1985) suggests the appointment of a "conservative" central banker. That is, appoint someone who places a higher weight on reducing inflation than society. We note for later reference that this suggestion implies that the central banker use a loss function that differs from society. Rogoff's conservative central banker cannot completely eliminate the inflation bias,

which prevails under consistent policy. That is, consistent policy still does not prove optimal under Rogoff's conservative central banker.

Svensson (1997) delegates an inflation target that differs from society's target. Once again, the central banker possesses a loss function that differs from the social loss function. Svensson's inflation target can completely eliminate the inflation bias, if we simplify Svensson's model to the basic model without employment persistence. That is, consistent policy proves optimal under inflation targeting for the simplified model.

Walsh (1995) introduces an incentive contract that penalizes the central banker for deviations from the target inflation rate. The proper choice of the penalization rate completely eliminates the inflation bias. Walsh implicitly assumes in his derivation that the government places no weight on the cost of the incentive contract. Chortareas and Miller (2002) show that if the government places some weight on the cost of the incentive contract that the contract cannot completely eliminate the inflationary bias. As an alternative to the inflation contract, Chortareas and Miller (2002) propose an output contract for the central banker that penalizes deviations of output from the natural rate. The proper choice of the penalty rate completely eliminates the inflation bias, even if the government cares about the cost of the contract.

In sum, delegation solutions to the inconsistency problem adopt a central bank loss function that differs from society's loss function. We develop a general method of institutional design or target delegation that causes consistent policy to prove optimal (i.e., Section 4). Then we compare our general solution to the other delegation solutions offered in the existing literature (i.e., Section 6). But, first, we construct a standard model where consistent policy proves non-optimal in the next section.

3. Optimal Policy and Consistent Policy in a Simple Model

Barro and Gordon (1983a) introduce a basic model for analyzing the inconsistency issue in monetary policy. We adopt one-period model with complete information. Reasons follow.

First, understanding our analytical method becomes less difficult in the simplest models. Thus, we attack the problem one piece at a time.

Second, for a one-period game, the inconsistency of optimal plans generally exists, no matter whether players' decisions occur simultaneously or sequentially.[5] The prisoner's dilemma provides an example of the simultaneous-decision, one-period game model. Before the game begins, both suspects know that their optimal strategy equals "confess;" their rational and consistent strategy equals "defect," once the game starts. Other examples of sequential-decision, one-period games exist, where such inconsistency prevails.

Third, some multi-period models in the literature basically reduce to one-period models for a stationary period function and a discount function of the form δ^t, where $0 \leq \delta \leq 1$ equals discount factor. Such models include the inflation-unemployment example in Kydland and Prescott (1977) and the Barro and Gordon (1983a,b) model.

Fourth, a repeated game creates other difficulties, making them greatly different from the one-period game. The Folk Theorem indicates that equilibrium outcomes of the game only require that each player's payoff exceeds the player's max-min payoff. No definite method predicts which equilibrium gets chosen, however. Moreover, the equilibrium outcomes in a repeated game may depend on psychology and culture. As a result, the equilibrium becomes

[5] Kydland (1977, p313) already notes that even in the first period, the dominant player may deviate from the original policy, implying inconsistency even in the one-period game. He refers to the deviant policy as the closed-loop policy. A similar deviation occurs for open-loop policy—the optimal policy.

unreliable.[6] But if we make optimal policy consistent in a one-period game, then in the repeated game, the equilibrium (consistent) policy always proves optimal.

Fifth, we can consider a repeated game with incomplete information as a one-period game with complete information. As the game repeats, players adjust their beliefs in a Bayesian fashion and approach complete information as the game progresses.

The Basic Model

We begin with a standard version model of the microeconomy and a quadratic social loss function in terms of the inflation rate and employment.[7] That is,

(1) $$L = \chi\pi^2 + (\ell - \bar{\ell})^2,$$

where π equals the inflation rate, ℓ equals the logarithm of employment level, and $\bar{\ell}$ equals the logarithm of "full employment," which we assume higher than the logarithm of the natural level, $\tilde{\ell}$. The social loss function implies that the society considers two targets – a zero inflation rate and "full employment." The weight that society places on the inflation target relative to the employment target equals χ, the trade-off parameter. To simplify, we consider only a one-period social loss function, allowing us to omit the period subscript *t*. We also assume that the central bank directly control inflation rate, π.

Now, we model the microeconomic structure with an expectations-augmented Phillips curve and rational expectations. That is,

(2) $$\ell = \tilde{\ell} + \beta(\pi - \pi^e) - u \text{ and}$$

(3) $$\pi^e = E(\pi),$$

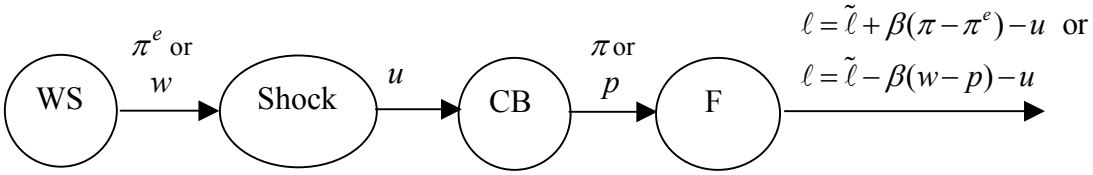
---

[6] But Barro and Gordon (1983b) show that reputation plays a role and indicate the conditions under which the consistent policy proves optimal.

[7] See, for example, Barro and Gordon (1983a), Rogoff (1985), Flood and Isard (1989), Lohnman (1992), Walsh (1995), Persson and Tabellini (1993), Svensson (1997), and so on.

where β equals the responsiveness of employment to unexpected inflation, u equals an independently and identically distributed negative supply shock with mean 0 and variance σ^2, and π^e equals the wage setter's expectation of the inflation rate.

The private sector's behavioral equations (2) and (3), the firm (F) and the wage setter (WS), respectively, conform to the following logic. The wage setter and the firm sign a wage contract, where the wage setter sets the nominal wage, w, and the firm sets the labor amount, ℓ, that it hires. After signing the wage contract, a negative shock, u, may occur. Then the central bank (CB) implements its policy decision, π, minimizing the social loss function. Since the contract fixes the nominal wage, the wage setter must form a rational expectation of the inflation rate before setting the wage rate, contingent on that inflation forecast (i.e., the wage setter uses behavioral equation 3). Finally, given the firm's decision, a certain employment level emerges from the firm's behavioral equation (2). The timing of the sequential decisions in this one-period model unfolds according to the following chart:[8]



Further, we assume that the participants in the economy (i.e., central bank, wage setter, and firm) view the model as common knowledge (i.e., the social loss function and the two behavioral equations of the private sector).

For the convenience of extending the standard model below, we transform the model as follows, without changing any essence of the model. We replace π with $p–p_0$, because our model

[8] We will shortly modify the model and replace the inflation rate with the change in the price level from its starting value and the expected inflation rate with the nominal wage rate. That is, the wage setter will attempt to keep the real wage rate constant on an expected basis.

is a one-period game. Thus,

$$\pi = \left(P - P_0\right)/P_0 = d\ln P = dp = p - p_0, \tag{4}$$

where P and p equal the price level and its natural logarithm, respectively, and the subscript zero indicates their starting values. In addition, $L_{WS} = \left(w - p\right)^2$ equals the loss function of the wage setter, who intends to hold the real wage level constant. Because the supply shock happens after the signing of the wage contract, the wage setter must form a rational expectation of the inflation rate and accordingly must set an optimal nominal wage to minimize the expected loss $E\left(L_{WS}\right)$.

The modified model becomes the following:

$$\begin{aligned} &\min_p L = \chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2 \\ &s.t. \begin{cases} \ell = \tilde{\ell} - \beta(w - p) - u \\ \min_w E\left(L_{WS}\right) = E\left[\left(w - p\right)^2\right] \end{cases} \end{aligned} \tag{5}$$

Now we compute this model's optimal policy and consistent policy with control-theory and game-theory solution techniques.

Optimal Policy

As stated above, control theory can provide a useful benchmark. With control theory, we determine the optimal plan and, thus, the optimal economic outcome. This provides a benchmark for policy making. The benchmark case assumes complete information and decisions made by one person before the game starts. That is, we assume that the optimal policy is an ex ante plan made by a social planner with complete information. The optimal policy and outcome for model (5) reduce to the following results:[9]

$$p = p_0 + \frac{\beta}{\chi + \beta^2}u \text{ and} \tag{6}$$

[9] See Appendix A for the derivation.

(7) $$E(L) = \frac{\chi}{\chi + \beta^2}\sigma^2 + k^2 .$$

Consistent Policy

Using backward induction to solve the problem expressed in model (5), we first must solve for the central bank's optimal decision for the price level. That is, given the nominal wage and the supply shock, the central bank chooses the p to minimize the social loss function, yielding the following relationship:

(8) $$p = \frac{\chi}{\chi + \beta^2} p_0 + \frac{\beta^2}{\chi + \beta^2} w + \frac{\beta}{\chi + \beta^2} k + \frac{\beta}{\chi + \beta^2} u ,$$

where $k \equiv \overline{\ell} - \tilde{\ell}$ equals the employment bias.

With the central bank's reaction function in equation (8), the wage setter's expected loss equals the following relationship:

(9) $$E(L_{WS}) = \left[\frac{\chi}{\chi + \beta^2}(w - p_0) - \frac{\beta}{\chi + \beta^2} k \right]^2 + \left(\frac{\beta}{\chi + \beta^2} \right)^2 \sigma^2 .$$

Therefore, the equilibrium nominal wage equals[10]

(10) $$w = p_0 + \frac{\beta}{\chi} k .$$

Substituting equation (10) into equation (8) yields the equilibrium price level as follows:

(11) $$p = p_0 + \frac{\beta}{\chi} k + \frac{\beta}{\chi + \beta^2} u .$$

As observed in the standard literature, there exists an inflationary bias, $E(p - p_0) = \frac{\beta}{\chi} k$.

With the equilibrium nominal wage and price level, we get the equilibrium employment

[10] We obtain equation (8) by setting $dE(L_{WS})/dw = 0$.

(12) $$\ell = \widetilde{\ell} - \frac{\chi}{\chi + \beta^2} u \,,$$

which leads to the expected social loss as follows:

(13) $$E(L) = \frac{\chi}{\chi + \beta^2}\sigma^2 + \left(1 + \frac{\beta^2}{\chi}\right)k^2 .$$

Compared with the optimal policy and outcomes in equations (6) and (7), the consistent policy and outcomes generate the inflationary bias (i.e., a higher price level than the initial one) in equation (11) and a larger social loss in equation (13).

Two important points deserve comment. First, the two targets in the social loss function, p_0 and $\overline{\ell}$, actually conflict with each other, given the microeconomic structure in equations (2) and (3). If the central bank wants to achieve full employment, it must inflate the economy, meaning that the central bank cannot achieve the zero inflation-rate target. If the central bank, on the other hand, wants to hit the zero inflation-rate target, then it cannot raise the employment level above the natural level. So the two targets, p_0 and $\overline{\ell}$, prove incompatible. Does it make sense to delegate incompatible targets to the central bank? No. Then how can we define compatible targets for the central bank? We assume that compatible targets, p^* and ℓ^*, exist. Such compatible targets must conform to the structure of the microeconomic model that underlies the central bank optimization problem. The next section formalizes how we specify an optimization scheme that delegates compatible or consistent targets to the central bank, with which the central bank optimizes the social loss function. We cannot ignore the microeconomic model of the economy when we stipulate or delegate the two economic targets for the central bank.

Second, we observe that the employment target, $\overline{\ell}$, proves overambitious and

unattainable under the assumptions of the microeconomic model because

$$E(\ell) = E\left[\tilde{\ell} - \beta(w-p) - u\right] = \tilde{\ell} - \beta\left[w - E(p)\right] - E\left[u\right] = \tilde{\ell}\,, \tag{14}$$

which means that the level of employment can only equal the natural level, on average. According to equations (11) and (12), we also know that

$$\begin{aligned} E(p) &= p_0 + \frac{\beta}{\chi}k \neq p_0 \text{ and} \\ E(\ell) &= \tilde{\ell} \neq \overline{\ell} \end{aligned} \tag{15}$$

.

The above inequalities mean that, on average, the central bank cannot achieve each of its targets, which seems illogical. Society should not delegate such targets to the central bank. A more sensible approach makes the following assumptions about delegating targets to the central bank

$$p^* = E(p) \text{ and } \ell^* = E(\ell)\,. \tag{16}$$

That is, proper targets should allow the central bank to achieve them. We call such targets *consistent targets*. The assumptions in equation (16) prove essential for determining optimal and consistent monetary policy in Section 4. Actually, the assumptions in (16) hold when choosing parameters, p^* and ℓ^*, to minimize the central bank loss function.[11]

In sum, the central bank should not adopt or get delegated the social loss function as its own.

4. Central Bank Loss Function Design

The analysis of the prior section leads to two fundamental conclusions about why the monetary policy action leads to an inflationary bias. First, the targets of monetary policy identified in the social loss function prove inconsistent with each other in the context of the microeconomic model. Second, the central bank uses the social loss function as its own loss function with the

[11] See equations (36) and (37).

inconsistent targets.

Our method requires that the central bank appointed possesses a loss function, or gets delegated a loss function, that differs from the social loss function in that the targets of policy used by the central bank prove consistent with the microeconomic model. Ample precedent exists for the use of a central bank loss function that differs from the social loss function – Rogoff (1985), Svensson (1997), Walsh (1995), Chortareas and Miller (2003), and so on. These authors, however, do not consider the most important lesson from our prior analysis, delegating consistent targets for the central bank.

Consider the social loss function in equation (1). Society possesses two targets – a zero inflation rate, $p_{0,}$ and full employment, $\overline{\ell}$, as well as the relative weight that the society places on the two targets, χ. That is, three important parameters exist -- χ, p_0, and $\overline{\ell}$ -- in the social loss function. Similarly, we assume that these same three parameters, possibly with different values, enter the central bank's loss function, denoted respectively as χ^*, p^*, and ℓ^*. That is, we assume the following central bank loss function:

$$L_{CB} = \chi^*\left(p - p^*\right)^2 + \left(\ell - \ell^*\right)^2 . \tag{17}$$

In the first stage, the central bank sets the price level, p, in the loss function (17), subject to the constraint of the microeconomic model as follows:

$$\ell = \widetilde{\ell} - \beta(w - p) - u , \qquad \text{and} \tag{18}$$

$$\min_{w} E\left(L_{WS}\right) = E\left[(w - p)^2\right]. \tag{19}$$

Given the solution to the first-stage problem, the second stage requires that the chosen χ^*, p^* and ℓ^* minimize the expected social loss. That is, the appointed central banker possesses that precise loss function that includes these parameter values or the central bank gets delegated

these parameter values for its loss function. Therefore, the central bank optimizes with the following model:

$$\min_{p^*,\ell^*,\chi^*} E(L) = E\left[\chi\left(p-p_0\right)^2+\left(\ell-\overline{\ell}\right)^2\right]$$

$$s.t.\begin{cases}\min\limits_{p} L_{CB} = \chi^*\left(p-p^*\right)^2+(\ell-\ell^*)^2 \\ s.t.\begin{cases}\ell=\tilde{\ell}-\beta(w-p)-u \\ \min\limits_{w} E\left(L_{WS}\right)=E\left[\left(w-p\right)^2\right]\end{cases}\end{cases} \quad (20)$$

The model's solution involves two steps. First, solve the following partial model:

$$\min_{p} L_{CB} = \chi^*\left(p-p^*\right)^2+(\ell-\ell^*)^2$$

$$s.t.\begin{cases}\ell=\tilde{\ell}-\beta(w-p)-u \\ \min\limits_{w} E\left(L_{WS}\right)=E\left[\left(w-p\right)^2\right]\end{cases} \quad (21)$$

The solution process of this partial model exactly mirrors the solution in Section 3, producing the following results:

$$w = p^* + \frac{\beta}{\chi^*}\left(\ell^*-\tilde{\ell}\right), \quad (22)$$

$$p = p^* + \frac{\beta}{\chi^*}\left(\ell^*-\tilde{\ell}\right)+\frac{\beta}{\chi^*+\beta^2}u\,, \quad (23)$$

$$\ell = \tilde{\ell}-\frac{\chi^*}{\chi^*+\beta^2}u \text{ and} \quad (24)$$

$$E\left(L\right) = EL(\mathrm{I})+EL(\mathrm{II})\,, \quad (25)$$

where $EL(\mathrm{I})=\left[\chi\left(\frac{\beta}{\chi^*+\beta^2}\right)^2+\left(\frac{\chi^*}{\chi^*+\beta^2}\right)^2\right]\sigma^2$ and $EL(\mathrm{II})=\chi\left[p^*+\frac{\beta}{\chi^*}\left(\ell^*-\tilde{\ell}\right)-p_0\right]^2+k^2$ equals the expected social losses, related and unrelated to the shock. Notice that we calculate the expected social loss $E(L)$, not the central bank loss L_{CB}. The central bank, now, must minimize the expected social loss. Minimizing the central bank's loss function in the first step only aids in

minimizing the social loss in the second step. When the central bank minimizes its loss function (equation 17), the expected social loss equals the expression in equation (25), which we desire to minimize in the second step.

Second, we select χ^*, p^*, and ℓ^* to minimize the expected social loss in equation (25). Consider *EL*(II). Absent a shock, the relative importance between inflation target and employment target, χ^*, does not matter. Rather, we minimize the expected social loss, given no shock, when $p^* + \frac{\beta}{\chi^*}\left(\ell^* - \tilde{\ell}\right) - p_0 = 0$. If a shock occurs, what happens to χ^*? Solving the simultaneous equations generated by the first order conditions as follows:

$$\text{(26)} \qquad \begin{cases} \dfrac{\partial E(L)}{\partial \chi^*} = 0 \\ \dfrac{\partial E(L)}{\partial p^*} = 0, \\ \dfrac{\partial E(L)}{\partial \ell^*} = 0 \end{cases}$$

we obtain

$$\text{(27)} \qquad \chi^* = \chi \qquad \text{and}$$

$$\text{(28)} \qquad p^* + \frac{\beta}{\chi}\left(\ell^* - \tilde{\ell}\right) = p_0 .$$

The condition that $\chi^* = \chi$ means that to minimize the effect of supply shock, the central bank's optimal preference χ^* between inflation target and employment target should equal that of society. This result contradicts Rogoff's (1985) proposed solution to the inflationary bias of appointing a conservative central banker, whereby ($\chi^* > \chi$).

We can also prove that χ^* must equal χ by apagoge. If $\chi^* > \chi$, that is, the central banker places more importance on inflation target than does society, then the central banker will under-

inflate when observing a large negative shock. Thus, the employment level, which falls significantly due to the large negative shock, cannot reach the level that society desires. Therefore, social welfare falls because of the central bank's conservatism when a large negative shock occurs. In contrast, if $\chi^* < \chi$, that is, the central banker places more importance on the employment target than does society, then the central banker will over-inflate when facing a small negative shock to raise the employment level. This also reduces social welfare because the central bank over reacts to a small negative shock. Because society cannot forecast whether the shock will occur and how large the shock is, society should stipulate an identical preference parameter χ^* for the central bank as that of society, χ.

In sum, the conservative central banker approach does not directly attack the cause of the inconsistency in monetary policy. If Rogoff's approach could solve the inconsistency in monetary policy, then that inconsistency resulted from a larger weight on the employment target relative to the inflation target (i.e., a smaller weight on the inflation target). Much literature (e.g., Lohnmann 1992, Svensson 1997, Blinder 1998, Chortareas and Miller 2003, and so on), however, argues that the inconsistency in monetary policy reflects an over ambitious employment target. Therefore, the inappropriate employment target itself causes the inconsistency, not the larger weight on the employment target.

In addition, our result coincides with and receives support from McCallum's (1997) following argument about society's and central banker's preferences and the relative weight attached to the inflation versus employment deviations from target:

> "A related disagreement with the standard literature involves the notion that it is useful to conduct analysis, involving institutional design, under the presumption that central banks can have preferences that are systematically

different from the society's. This might occasionally be the case in some nations, but on average I would expect that the relative importance given to inflation and unemployment avoidance will be approximately the same by a central bank and the society of which it is a part. In democracies, central banks will tend to be aware of and reflect the preferences of the population. That tendency might be discouraged in various ways, but I would expect that (for example) attempts to appoint governors with tastes more anti-inflationary than society's would often result in ex post surprises about these tastes. And I would expect legislation to be overturned fairly promptly if it were truly inconsistent with the preferences of the nation's population. In any event, it would seem to be asking for trouble if institutions were designed under the presumption that CB preferences differ from those of the public at large." (p. 107)

Equation (28) shows that the two targets of the central bank specifically relate to each other. When we stipulate the targets for the two economic variables (p and ℓ), we should not ignore the microeconomic model that implicitly links the two. Equation (28) identifies how the two targets must relate to each other.

Using equations (27) and (28) causes equations (22), (23), (24), and (25) to reduce to the following equations:

(29) $$w = p_0 ,$$

(30) $$p = p_0 + \frac{\beta}{\chi + \beta^2} u ,$$

(31) $$\ell = \tilde{\ell} - \frac{\chi}{\chi + \beta^2} u , \text{ and}$$

(32) $$E(L) = \frac{\chi}{\chi + \beta^2}\sigma^2 + k^2 .$$

Notice that consistent policy and the resulting outcome contained, respectively, in equations (30) and (32), under the loss function (17) whose parameters satisfy equations (27) and (28), prove optimal, as shown in equations (6) and (7).

Equation (28), however, allows an infinite set of price and employment targets that optimize the social welfare function. Can we reduce the degree of uncertainty and pin down precise values? We need another condition. We noted above that it seems illogical to delegate to the central bank targets that it cannot expect to achieve. Rather, the assumptions contained in equation (16) specify the rules that choose targets that the central bank can achieve. In the present context, that means the following:

(33) $$p^* = E(p) = p_0 \text{ and}$$

(34) $$\ell^* = E(\ell) = \tilde{\ell} .$$

Moreover, $p^* = p_0$ and $\ell^* = \tilde{\ell}$ together satisfy the target-relationship contained in equation (28).

For the present problem, equations (33) and (34) define *consistent targets*. C*onsistent targets* exist if, and only if, they satisfy target-relationship equation (28) and the assumptions contained in equation (16). That is, *consistent targets* should prove compatible with each other, and should not prove too ambitious or too modest. Moreover, they should prove attainable by the central bank.

Viewing the problem somewhat differently, but leading to the same conclusion, *consistent targets* optimize the central bank welfare function. To see this, consider the value of the central bank loss function, given the solutions to the minimization of the social loss function. Note that the only ambiguity relates to the values of the target price and employment levels. That

is, equation (27) indicates that the weight associated with the inflation term in the social loss function equals the weight in the central bank loss function. Thus, we need to choose the target price and employment levels to minimize the following central bank loss function:

$$\min_{p^*,\ell^*} EL_{CB} = E\left[\chi\left(p_0 + \left\{\frac{\beta}{\chi+\beta^2}\right\}u - p^*\right)^2 + \left(\tilde{\ell} - \left\{\frac{\chi}{\chi+\beta^2}\right\}u - \ell^*\right)^2\right], \tag{35}$$

$$= \chi(p_0 - p^*)^2 + (\tilde{\ell} - \ell^*)^2 + \left\{\frac{\chi}{\chi+\beta^2}\right\}\sigma^2.$$

This minimization problem produces the following solutions:

$$p^* = p_0 \text{ and} \tag{36}$$

$$\ell^* = \tilde{\ell}\,. \tag{37}$$

Thus, *consistent targets* also minimize the central banks loss function as well as the social loss function. The central bank's minimum loss equals the following:

$$EL_{CB} = \left\{\frac{\chi}{\chi+\beta^2}\right\}\sigma^2. \tag{38}$$

In sum, given the social loss function and the microeconomic equations contained in model (5), the central bank's loss function, which helps to optimize the social welfare and which uses *consistent targets*, equals the following:

$$L_{CB} = \chi\left(p - p_0\right)^2 + \left(\ell - \tilde{\ell}\right)^2. \tag{39}$$

Under the central bank's loss function (equation 39), the consistent policy and outcome, contained in equations (30) and (32) prove optimal, as shown in equations (6) and (7).

5. Compare and Contrast to the Existing Literature

As noted above, Rogoff (1985), Svensson (1997), Walsh (1995), and Chortareas and Miller (2002) each address the inherent inflation bias in the basic Barro and Gordon (1983a,b) model,

offering different solutions. This section compares our findings with an optimal objective function and consistent targets.

Rogoff's (1985) solution proves inconsistent with our findings. That is, he alters the central bank objective function by appointing a conservative central banker. Within the context of the model used above, he did not adopt consistent targets. Moreover, he also did not adopt an optimal central bank objective function. Our findings for optimal monetary policy require that equations (27) and (28) hold. Rogoff (1985) appoints a central banker for whom the trade-off coefficient between price and employment stability exceeds that for society. We find that the trade-off coefficient should not change. Moreover, Rogoff (1985) maintains Barro and Gordon's target values for the price and employment levels, which prove inconsistent in our framework.

Svensson's (1997) solution does adopt an optimal central bank objective function, but he chooses price and employment level targets that prove inconsistent. His loss function takes the following form in our context:

$$L_{CB}\left(\text{Svensson}\right) = \chi\left[\left(p - p_0\right) - \pi^*\right]^2 + \left(\ell - \overline{\ell}\right)^2, \tag{40}$$

where π^* equals the inflation target. Svensson (1997) determined that the optimal inflation target π^* equals $-\frac{\beta}{\chi}k$. Interestingly, his targets $p^* = p_0 - \frac{\beta}{\chi}k$ and $\ell^* = \overline{\ell}$ satisfy the optimal target relationship identified in equation (28), but $p^* \neq E\left(p\right)$ and $\ell^* \neq E\left(\ell\right)$, which means that he uses inconsistent targets.

By using $\pi^* = -\frac{\beta}{\chi}k$, we can transform equation (40) as follows:

$$L_{CB}\left(\text{Svensson}\right) = L_{CB} + \left[\left(1 + \frac{\beta^2}{\chi}\right)k^2 + 2\beta k\left(w - p_0\right) + 2ku\right], \tag{41}$$

where L_{CB} equals the optimal central bank loss function in equation (39).

Walsh (1995) introduces a central banker inflation contract, which penalizes the central banker for producing an inflation rate different from its target value. His central bank loss function takes on the following form in our context:

$$L_{CB}\left(\text{Walsh}\right)=\chi\left(p-p_0\right)^2+\left(\ell-\overline{\ell}\right)^2+2f\left(p-p_0\right), \tag{42}$$

where f measures the penalty (fine) imposed on the magnitude once the central bank deviation from the zero inflation rate target. Walsh determined that the optimal penalty, f, equals βk. Substituting $f=\beta k$ into equation (42) and transforming the result produces the following:

$$L_{CB}\left(\text{Walsh}\right)=L_{CB}+\left[k^2+2\beta k\left(w-p_0\right)+2ku\right], \tag{43}$$

where L_{CB} equals the optimal central bank loss function in equation (39).

Chortareas and Miller (2002, C&M) consider a central banker employment contract, which penalizes the central banker for producing and employment level different from its target value. Their central bank loss function takes on the following form in our context:

$$L_{CB}\left(\text{C\&M}\right)=\left[\chi\left(p-p_0\right)^2+\left(\ell-\overline{\ell}\right)^2\right]-\xi\left(tr\right), \tag{44}$$[12]

where $tr=t_0-t\left(\ell-\tilde{\ell}\right)$ equals the incentive scheme, t_0 equals a fixed payment, t equals the marginal penalization rate, and ξ equals the weight that the central banker attaches to the incentive scheme. Chortareas and Miller (2002) determine that the optimal incentive scheme takes the form:

$$tr=t_0-\left(2/\xi\right)k\left(\ell-\tilde{\ell}\right). \tag{45}$$

[12] In fact, Chortareas and Miller (2002) use a utility function where the incentive scheme enters with a positive sign and the loss function enters with a negative sign. We multiply by minus one to convert into the loss function used in our work.

Substituting equation (45) into equation (44) and transforming the result produces the following:

$$L_{CB}\left(\text{C\&M}\right) = L_{CB} - \left(\xi t_0 - k^2\right), \tag{46}$$

where, once again, L_{CB} equals the optimal central bank loss function in equation (39).

Excluding Rogoff (1985), we see that the modified central bank loss functions all contain our optimal central bank loss function plus some additional terms. But these additional terms do not depend on the price level. Thus,

$$\frac{\partial L_{CB}\left(\text{Walsh}\right)}{\partial p} = \frac{\partial L_{CB}\left(\text{Svensson}\right)}{\partial p} = \frac{\partial L_{CB}\left(\text{C\&M}\right)}{\partial p} = \frac{\partial L_{CB}}{\partial p}. \tag{47}$$

Therefore, when implementing *consistent policy*, the equilibrium outcomes prove identical under the four loss functions -- $L_{CB}\left(\text{Walsh}\right)$, $L_{CB}\left(\text{Svensson}\right)$,$L_{CB}\left(\text{C\&M}\right)$ and L_{CB}. In that sense, we can consider the four loss functions as equivalents. That is,

$$L_{CB}\left(\text{Walsh}\right) \cong L_{CB}\left(\text{Svensson}\right) \cong L_{CB}\left(\text{C\&M}\right) \cong L_{CB}. \tag{48}$$

6. An Alternative Microeconomic Model

Our analysis so far relies on the microeconomic equations contained in model (5). We modify the wage setter's loss function to include a preference for a higher real wage rate as well as higher employment than that natural rate. We leave the term contained in the original wage setter's loss function in model (5) so that we can derive the solution from that original model as a special case of this extended model. That is, setting the two parameters of the wage setter's loss function equal to zero produces the results of the system contained in model (5). Moreover, setting one of the two parameters equal to zero, but not the other parameter generates the results from two additional special cases – wage setter wants a higher real wage or wage setter wants higher employment than the natural rate. The model structure generates the following problem:

$$\min_{p} E(L) = E\left[\chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right]$$

$$s.t.\begin{cases} \ell = \tilde{\ell} - \beta(w - p) - u \\ \min_{w} E\left(L_{WS}\right) = E\left[-2\gamma(w - p) + \left(w - p\right)^2 + \lambda(\ell - \bar{\ell})^2\right] \end{cases} . \tag{49}$$

Its optimal policy and outcome equal the following:[13]

$$p = p_0 + \frac{\beta}{\chi + \beta^2} u \text{ , and} \tag{50}$$

$$E(L) = \frac{\chi}{\chi + \beta^2}\sigma^2 + \frac{(k + \beta\gamma)^2}{(1 + \lambda\beta^2)^2} . \tag{51}$$

Applying our method (i.e., Section 4) to the alternative model leads to the following problem:

$$\min_{p^*, \ell^*, \chi^*} E(L) = E\left[\chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right]$$

$$s.t.\begin{cases} \min_{p} L_{CB} = \chi^*\left(p - p^*\right)^2 + (\ell - \ell^*)^2 \\ s.t.\begin{cases} \ell = \tilde{\ell} - \beta(w - p) - u \\ \min_{w} E\left(L_{WS}\right) = E\left[-2\gamma(w - p) + \left(w - p\right)^2 + \lambda(\ell - \bar{\ell})^2\right] \end{cases} \end{cases} . \tag{52}$$

Carrying out the first optimization problem of the central bank and the wage setter generates the following results that correspond to equations (22), (23), and (24):

$$w = p^* + \frac{\beta}{\chi^*}\left[\left(\ell^* - \tilde{\ell}\right) + \frac{\beta(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)}\right] + \frac{(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)} , \tag{53}$$

$$p = p^* + \frac{\beta}{\chi^*}\left[\left(\ell^* - \tilde{\ell}\right) + \frac{\beta(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)}\right] + \frac{\beta}{\chi^* + \beta^2} u \text{ , and} \tag{54}$$

$$\ell = \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)} - \frac{\chi^*}{\chi^* + \beta^2} u . \tag{55}$$

[13] See Appendix B for the derivation.

Note that if γ and λ equal zero, equations (53), (54), and (55) reduce to equations (22), (23), and (24). Further, if only γ equals zero, then employment exceeds the natural rate and if only λ equals zero, then employment falls below the natural rate.

Now, to carry out the second optimization, we choose the starred values of the price and employment levels and the trade-off parameter to maximize the social welfare function, yielding the following results that correspond to equations (27) and (28):

(56) $$\chi^* = \chi \qquad \text{and}$$

(57) $$p^* + \frac{\beta}{\chi}\left[\left(\ell^* - \tilde{\ell}\right) + \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)}\right] = p_0 .$$

Once again, if γ and λ equal zero, equations (56) and (57) reduce to equations (27) and (28).

Substituting equations (56) and (57) into equations (53), (54), and (55) generates the following solutions:

(58) $$w = p_0 + \frac{(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)},$$

(59) $$p = p_0 + \frac{\beta}{\chi + \beta^2} u \text{ , and}$$

(60) $$\ell = \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)} - \frac{\chi}{\chi + \beta^2} u .$$

Thus, the real wage will rise (fall) and the employment level will fall below (rise above) the natural rate when $\gamma > \lambda\beta k$ ($\gamma < \lambda\beta k$), ignoring the shock.

Substituting the solutions for the price and employment levels into the social welfare function generates the following social loss:

(61) $$E(L) = \frac{\chi}{\chi + \beta^2}\sigma^2 + \frac{(k + \beta\gamma)^2}{(1+\lambda\beta^2)^2} .$$

Now, equation (57) permits an infinite number of combinations of the target price and employment levels. Applying the concept of *consistent targets* or choosing the target price and employment levels to minimize the central bank loss function yields the following results:

(62) $$p^* = E(p) = p_0$$ and

(63) $$\ell^* = E(\ell) = \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)}.$$[14]

Therefore, the optimal central bank loss function equals the following:

(64) $$L_{CB} = \chi\left(p - p_0\right)^2 + \left\{\ell - \left[\tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)}\right]\right\}^2 .$$

Under this central bank loss function, the consistent policy and outcome, contained in equations (59) and (61), prove identical to the optimal policy and outcome, shown in equations (50) and (51).

7. Conclusion

Since the work of Strotz (1955-1956), economists continue to struggle with the consistency of optimal plans. Kydland and Prescott (1977) link the problem to the time consistency of optimal economic policy, showing that consistent policy proves non-optimal, in a game framework. Much work focuses on the simple one-period, game-theory problem developed by Barro and Gordon (1983a,b). Within that model, an inflationary bias exists, since consistent policy proves non-optimal.

Our paper develops a general method for making consistent policy, optimal, in simple models. We adopt institutional design or central bank delegation to solve the inconsistency problem. That is, we propose appointing a central banker who possesses consistent targets or delegate consistent targets to the central banker to make consistent policy optimal.

[14] These two target values also minimize the central bank's loss function.

The control-theory solution provides the benchmark optimal policy with which we evaluate the consistent policy emerging from our delegated (designed) central bank loss function. Carefully designing the central bank's loss function can make optimal policy and consistent policy identical. This desirable result requires an understanding the two following points. First, the determination of the social loss function reflects a normative process. The social loss function only provides a criterion for designing a public institution, not the direct loss function for this specific institution.[15] Second, the microeconomic structure proves key to determining the central bank loss function. That is, an optimal loss function for the central bank must depend on the structure of the microeconomy.

What moral can we draw from our story? If the social welfare criterion incorporates inconsistent targets, then the central bank improves social welfare by adopting a different welfare criterion with consistent targets, a second-best solution. That is, even though the central bank's performance depends on the target-inconsistent social welfare criterion, it can improve its performance by employing a target-consistent welfare function. Finally, if the central bank's performance depends on its own target-consistent welfare function, it can further improve its performance. That is, the best outcome occurs if society replaces its target-inconsistent welfare function with a target-consistent welfare function, the first-best solution. The third-best solution occurs when the central bank uses toe target-inconsistent welfare criterion as its own.

In sum, the correct central bank loss function depends on two factors -- the social welfare criterion and the structure of the microeconomy. Future research should focus on the following. First, consider our method with alternative normative social welfare criteria and with alternative

[15] A popular view takes optimizing the representative household's utility as the social welfare criterion. This view, however, does not permit differences between private and social interests. Social welfare criteria can also capture the ideas of Rawls' maximin criterion (Rawls, 1971), the Bergson-Samuelson social welfare function (Pollak, 1979),

microeconomic structures. Second, and more important, evaluate our method within a dynamic model, since economic variables generally prove persistent.

Appendix A:

For the two-stage stochastic optimization problem associated with model (5), the first stage obtains the optimal value of deterministic variable, *w*, under certainty.[16] That is, solving the following model

$$\text{(A-1)} \qquad \min_{w,p} L = \chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2 \quad s.t. \begin{cases} \ell = \tilde{\ell} - \beta(w - p) \\ w = p \end{cases},$$

with the constraint, $w = p$, coming from the first-order condition of the wage-setter's loss function, we find that

$$\text{(A-2)} \qquad w = p = p_0 \quad and \quad \ell = \tilde{\ell}.$$

The second-stage adjusts the price level, *p*, after the first-stage optimal value under certainty, when a shock, *u*, occurs. Because the loss function equals a quadratic form with linear constraints, the augmentation of *p* must include a term linear in *u*. Thus,

$$\text{(A-3)} \qquad p = p_0 + au \text{ ; and}$$

the second-stage problem equals the following:

$$\text{(A-4)} \qquad \min_{a} EL = E\left[\chi\left(p_0 + au - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right] \quad s.t. \begin{cases} \ell = \tilde{\ell} - \beta\left[w - \left(p_0 + au\right)\right] - u \\ w = p_0 \end{cases},$$

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or Arrow's social welfare function (Arrow 1951), and so on. Thus, we argue that constructing the social welfare criterion reflects a normative problem in philosophy.

[16] See Kolbin (1977) and Marti (2005) for the two-stage solution technique with a stochastic optimization. In sum, the approach solves the optimization under certainty and then incorporates the random shock in the second stage as done in Appendix A and B.

which generates $a = \dfrac{\beta}{\chi + \beta^2}$. That is, the optimal policy equals equation (6).

Appendix B:

For the two-stage stochastic optimization problem associated with model (49), the first-stage obtains the optimal value of deterministic variable, *w*, under certainty. That is, solving the follow model

$$\text{(B-1)} \qquad \min_{w,p} E(L) = E\left[\chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right] \quad s.t. \begin{cases} \ell = \tilde{\ell} - \beta(w - p) \\ -\gamma + \left(w - p\right) - \lambda\beta\left(\ell - \overline{\ell}\right) = 0 \end{cases},$$

with the constraint, $-\gamma + \left(w - p\right) - \lambda\beta\left(\ell - \overline{\ell}\right) = 0$, coming from the first-order condition of the wage-setter's loss function, we find that

$$\text{(B-2)} \qquad w = p_0 + \frac{(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)}, p = p_0, \text{ and } \ell = \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)}.$$

The second-stage adjusts the price level, *p*, after the first-stage optimal value under certainty, when a shock, *u*, occurs. Similarly, the augmentation of *p* must include a linear term in *u* as follows:

$$\text{(B-3)} \qquad p = p_0 + bu \text{ ; and}$$

now, the second-stage problem equals the following:

$$\text{(B-3)} \qquad \min_{b} EL = E\left[\chi\left(p_0 + bu - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right] \quad s.t. \begin{cases} \ell = \tilde{\ell} - \beta\left[w - \left(p_0 + bu\right)\right] - u \\ w = p_0 + \dfrac{(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)} \end{cases},$$

which generates $b = \dfrac{\beta}{\chi + \beta^2}$. That is, the optimal policy equals equation (50).

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