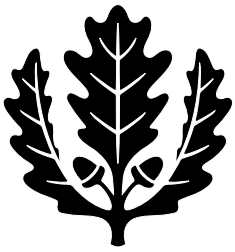

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Consistent Targets and Optimal Monetary Policy: A Note

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Abstract

Kydland and Prescott (1977) develop a simple model of monetary policy making, where the central bank needs some commitment technique to achieve optimal monetary policy over time. Although not their main focus, they illustrate the difference between consistent and optimal policy in a sequential-decision one-period world. We employ the analytical method developed in Yuan and Miller (2005), whereby the government appoints a central bank with consistent targets or delegates consistent targets to the central bank. Thus, the central bank s welfare function differs from the social welfare function, which cause consistent policy to prove optimal.

Journal of Economic Literature Classification: E42, E52, E58

1. Introduction

Kydland and Prescott (1977) illustrate the time inconsistency of optimal policy. That is, the central bank needs some commitment technique to achieve optimal monetary policy over time. Absent a commitment technique, optimal monetary policy proves time inconsistent. Their thesis focuses on intertemporal issues and the need for commitment. They illustrate the difference between consistent and optimal policy in a sequential-decision, one-period framework, where that difference hinges on whether the central bank incorporates how the private sector responds to changes in central bank actions.

Our paper shows how the analytical method developed in Yuan and Miller (2005) applies to the Kydland and Prescott (1977) sequential-decision one-period model. To wit, the government appoints a central bank with the correct (optimal) objective function that includes consistent targets or delegates to the central bank that correct (optimal) objective function, which causes a convergence of the consistent to the optimal monetary policy.

2. Basic Model

Kydland and Prescott (1977, pp. 477-480) develop a one-period exposition of their point about the difference between consistent and optimal policy with their inflation-unemployment model.

They begin with an expectations augmented Phillips curve as follows:

$$u_t = \lambda(x_t^e - x_t) + \tilde{u}, \tag{1}$$

where u equals the unemployment rate, x equals the inflation rate, x^e equals the expected inflation rate, and $\tilde{u}$ equals the natural rate of unemployment.

Next, they impose rational expectations or, given that no stochastic elements exist in the model, perfect foresight on the formation of inflation expectations. Thus,

$$x_t^e = Ex_t = x_t . \tag{2}$$

They verbally describe the social objective function, given as follows:

(3) $$S(x_t, u_t),$$

which they illustrate in Figure 1 (p. 479). The following social loss function captures the characteristics of their description and figure:

(4) $$L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u}),$$

where α equals the weight that society gives to combating inflation relative to unemployment deviations from the natural rate.

Consistent policy chooses the inflation rate to minimize the social loss function subject to the Phillips curve and the perfect foresight inflation rate of the private sector, but where the central bank does not consider how the perfect foresight inflation rate responds to the central bank's policy choice. That is, we substitute the Phillips curve, but not the perfect foresight inflation rate (i.e., x_t), into the social loss function and take the derivative with respect to the inflation rate. The solution of the optimization yields the following:

(5) $$x_t = \lambda/\alpha;\ x_t^e = \lambda/\alpha;\ u_t = \tilde{u};\ \textit{and}\ \ L_t^S = \lambda^2/\alpha.$$

Consistent policy, as defined by Kydland and Prescott (1977), produces an inflationary bias at the natural rate of unemployment. As such, the result matches the Barro and Gordon (1983a, 1983b) bias.

Optimal policy chooses the inflation rate to minimize the social loss function subject to the Phillips curve and the perfect foresight inflation rate of the private sector, but where the central bank does consider how the perfect foresight inflation rate responds to its policy choice. That is, we substitute the Phillips curve and the perfect foresight inflation rate into the social loss function and take the derivative with respect to the inflation rate. The solution of the

optimization yields the following:

(6) $$x_t = 0;\ x_t^e = 0;\ u_t = \tilde{u};\ and\ \ L_t^S = 0.$$

The optimal policy outcomes do not include an inflationary bias and the social loss equals zero, smaller than the social loss under consistent policy.

Therefore, the inconsistency of optimal policy exists in this simple model. The inconsistency emerges because the central bank shares the same loss function with society, a loss function with inconsistent targets. Given the economic structure, equations (1) and (2), the unemployment level always equals the natural rate. The central bank, however, with a loss function linear in the unemployment rate decreases the unemployment rate to as low a level as possible[1] and, thus, always possesses the incentive to inflate. At the same time, the central bank uses a zero inflation rate target. As a result, the central bank faces the dilemma of inconsistent targets.

In addition, the inflation bias, λ/α, persists, despite the zero inflation target, and natural unemployment level persists, even with the "minus infinite" unemployment target. Logic dictates that the government delegate to the central bank achievable, consistent targets.

In sum, the government should choose a central bank, or delegate to the central bank, a loss function that differs from the social loss function.

3. Optimal Objective and Consistent Targets

The central bank confronts the problem of inconsistent targets in the above optimization when delegated the social loss function contained in equation (4). We hypothesize a central bank loss function of the same form, but with different choices for the targets and the trade-off parameter.

[1] The loss function means that the unemployment rate target equals minus infinity. As a practical matter, negative unemployment cannot occur and the actual unemployment rate proves positive because of the penalty imposed by higher inflation as the unemployment rate falls.

We then choose the optimal targets and trade-off parameter for the central bank loss function that minimizes the social loss function. The following system captures the complete problem:

$$\min_{\alpha^*, x^*, u^*} L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u})$$

$$(7) \qquad s.t.: \begin{cases} \min_{x_t} L_t^{CB} = \alpha^*(x_t - x^*)^2 + 2(u_t - u^*) \\ s.t.: \begin{cases} u_t = \lambda(x_t^e - x_t) + \tilde{u} \quad and \\ x_t^e = x_t. \end{cases} \end{cases}.$$

The optimization proceeds in two steps. First, we minimize the central bank loss function subject to the Phillips curve and the perfect foresight inflation rate of the private sector, but where the central bank does not consider how the perfect foresight inflation rate responds to its policy choice (i.e., the central bank employs consistent policy). Second, we substitute the solutions from the first optimization into the social loss function and optimize the social loss function by choosing optimal values for the target inflation and unemployment rates and the trade-off parameter.

Consider the first optimization. Substitute the Phillips curve into the central bank loss function and minimize with respect to the inflation rate and apply rational expectations (perfect foresight). The following solution emerges:

$$(8) \qquad x_t = x_t^* + \lambda/\alpha^*;\ x_t^e = x_t^* + \lambda/\alpha^*;\ u_t = \tilde{u};\ and\ \ L_t^S = \alpha\left(x^* + \lambda/\alpha^*\right)^2.$$

Now, consider the second optimization.

$$(9) \qquad \min_{\alpha^*, x^*, u^*} L_t^S = \alpha\left(x^* + \lambda/\alpha^*\right)^2.$$

Optimization with respect to the target inflation rate (i.e., x^*) and the trade-off parameter (i.e., α^*) gives an infinite number of combinations for the target inflation rate and the trade-off parameter as follows:

(10) $$x^* = -\lambda / \alpha^*, \quad where \quad \alpha^* \neq 0.$$

This outcome proves similar to Svensson's (1997) inflation targeting solution and to the output bias in the Barro and Gordon (1983a, 1983b) model. Substituting the solution for the target inflation rate into the solutions of the first-stage optimization produces the overall solution as follows:

(11) $$x_t = 0; \; x_t^e = 0; \; u_t = \tilde{u}; \; and \; L_t^S = 0.$$

In other words, consistent policy in the Kydland and Prescott (1977) definition yields the optimal policy, when the central banker possesses or gets delegated the optimal targets and trade-off parameter.

Note again that an infinite number of combinations of the inflation rate and the trade-off parameter exist, and nothing pins down the target unemployment rate. Can we reduce the degree of uncertainty and pin down precise values? We need another condition. As noted above, logic dictates that the government should appoint a central bank with consistent targets, or delegate consistent targets to a central bank so that the central bank can expect to achieve those targets. We define consistent targets, as opposed to consistent policy, to mean that the expected values of the target variables equal the targeted values. That is,

(12) $$Ex_t = x^* \quad and \; Eu_t = u^*.$$

Thus,

(13) $$\alpha^* = \infty, x^* = 0 \; and \; u^* = \tilde{u}.$$

The first condition pins down the value of α^*, which must equal infinity. That is, the optimal central bank loss function only includes the square of the deviation of the inflation rate from its target of zero. As a side result, the consistent targets minimize the central bank loss function, in

this case $L_t^{CB} = 0$ (see the equation in model 8).

If society desires an unemployment rate as low as possible, as in equation (4), then the optimal central bank loss function cannot exhibit any weight on the unemployment component, otherwise the central bank always possesses an incentive to inflate. As a result, the optimal loss function for the central bank puts infinite weight on the inflation component relative to the unemployment component (or zero weight on the unemployment component relative to the inflation component) to counteract the social loss function.

4. Conclusion

Although Kydland and Prescott (1977) focus on time inconsistency of monetary policy, they do provide an illustration of their dynamic result in a sequential-decision, one-period framework. Within that framework, consistent policy does not yield optimal policy, because the targets in the social welfare (loss) function prove inconsistent. We show that if the central banker possesses consistent targets, then consistent policy proves optimal.

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