CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES

CDMA07/03

Optimal Monetary and Fiscal Policy in an Economy with Non-Ricardian Agents*

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JANUARY 2007

ABSTRACT

The optimal policy mix maximizes a quadratic welfare objective which follows from the agents. utility function and depends only on inflation and output gap volatility. We analyze the optimal response of the economy to a rise in government spending. We find that the optimal economy moves along an analogue of a conventional inflation-output variance frontier, as the population share of non-Ricardian agents rises. Output should optimally vary more, as this is to the benefit of the liquidity-constrained agents via net real wages, while optimal inflation volatility falls as there is less of a need to use inflation to maintain fiscal solvency. There is little evidence that increased government spending would crowd in private consumption. The tax rate varies to contain pressures on prices by shifting the natural rate of output towards its preference-driven target level. We identify the size of the target deviation in output and the interest rate elasticity of demand as the key determinants of the optimal interest rate policy.

JEL Classification: E61, E63.
Keywords: Optimal Monetary and Fiscal Policy, Macroeconomic Stabilization, Non-Ricardian Agents.

* Acknowledgements: I should like to thank Jagjit S. Chadha for helpful comments on an earlier draft of this paper. Most of all, I am indebted to Charles Nolan for valuable suggestions and continuous encouragement. I should also like to acknowledge the financial support received from the Centre for Dynamic Macroeconomic Analysis at the University of St Andrews.

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1. Introduction

It has been known for some time that aggregate consumption data are difficult to reconcile with the life-cycle/permanent income hypothesis (LC-PIH). There is now substantial empirical evidence to support the notion that a large proportion of consumers in advanced economies behave in a non-Ricardian way (e.g., Campbell and Mankiw, 1989). For example, for the US economy, Mankiw (2000) suggests that as much as half of the aggregate consumption is inconsistent with LC-PIH.

The challenge for economists is to integrate the behavior of liquidity-constrained agents into modern frameworks developed to analyze optimal policy coordination. In this, we refer to setups following the pioneering work of Ramsey (1927).[1] His welfare-maximizing approach has been revived by Lucas and Stokey (1983), and then further extended to include market imperfections by, for instance, Aiyagari, Marcet, Sargent and Seppala (2001), Schmitt-Grohe and Uribe (2002), Benigno and Woodford (2003), and Siu (2004).[2] All of these models analyze optimal policy from the perspective of a representative agent.

In this paper, we allow for heterogeneity among agents and yet retain some of the valuable tractability of the representative agent set-up. We follow Amato and Laubach (2003), an interesting addition to the above listed literature, in that we derive a model with a continuum of identical agents who learn about their type (non-Ricardian versus Ricardian) at the beginning of each period. This approach generates an economy in which agents of a certain type represent a constant population share over time. As a result we may utilize some appealing aspects of recent advances in the analysis of optimal monetary and fiscal policy, while bringing the modeling environment in line with the empirical evidence on non-Ricardian behavior.

Our analysis is specific in several ways. First, our setup differs from Amato and Laubach (2003) in the definition of non-Ricardian behavior. We model non-Ricardian behavior via agents whose consumption is constrained by their contemporaneous after-tax wage income, and their choice as regards consumption and leisure is a result of an optimizing decision rather than simple adherence to a rule

[1] A popular alternative to the Ramsey-type analysis of optimal monetary (and fiscal) policy is the "optimal simple rules" approach. This approach involves supplementing a linear model of the economy with policy rules that are argued to be of descriptive value. Galí et al. (2004, 2007), for example, take this path to analyze various aspects of optimal policy in economies with liquidity-constrained agents.

[2] Barro (1979) also presents a conceptually similar framework, however, without direct analytical reference to individual utility maximization.

of thumb. Second, we endogenize the dynamics of the tax rate in order to have a richer environment in which jointly optimal monetary and fiscal policy strategies are determined to be pursued by the respective branches of government in a coordinated fashion. Third, we do not assume that a subsidy is available to eliminate the distortive effects of excess market power or government policies, commonly used in analyses of monetary policy following Rotemberg and Woodford (1999), and allow for large departures from the hypothetical steady state attained in first-best economies.[3] In our model, in which labor is the sole factor of production in the economy, the government uses distortive taxes on wage income to raise tax revenue. The market for intermediate goods is imperfectly competitive and the prices of (intermediate) goods are imperfectly flexible. Intermediate goods serve as an input in the production of a final good that is sold in a perfectly competitive environment.

We derive a quadratic policy objective and seek time-consistent optimal policies that maximize the welfare objective subject to the linear structural equations of the macroeconomy. This approach has been given sophisticated rationale in Benigno and Woodford (2003). As in their paper, our utility-based policy objective is a function of inflation and output gap variability only. To offer practical guidance for policy makers, we also derive a simple targeting rule in the spirit of Svensson (2002, 2003) and Taylor-like rules for monetary and fiscal policy that jointly implement the optimal policy.

The implications of our model are then illustrated numerically. A single, serially uncorrelated positive shock to government expenditures is the source of disturbance in the model. The main finding of the calibration exercise is that the optimal economy moves along an analogue of a conventional inflation-output variance frontier, as the population share of non-Ricardian agents rises. The optimal response in aggregate output increases because this is to the benefit of the liquidity-constrained households via net real wages. The net real wage rate is shown to be increasing and convex in aggregate output. It follows that those for whom net real wage is the sole determinant of welfare prefer more volatility in output. As the population share of non-Ricardian agents rises, this preference exerts greater influence on policy. The consequences of the fiscal shock for long-term fiscal sustainability, as expressed by a composite measure of "fiscal stress", then become less severe. There is then less of a need to use inflation to meet the fiscal solvency constraint. Optimal inflation volatility consequently falls.

[3] In an extension to their model, Amato and Laubach (2003) introduce inefficient supply shocks. However, they maintain the assumption of an efficient steady state.

In general, perfect stabilization of the tax rate is not optimal and the sign as well as the size of the response in the tax rate depends on the size of the preference-driven target response in output relative to the spending-induced shift in the natural rate of output. The resulting tax hike or tax cut then counteracts the arising cost-pull or cost-push pressure through its impact on the natural rate. The optimal interest rate response to a rise in government spending is negative. The relatively low interest rate elasticity of demand implied by our baseline calibration causes that the rate of interest associated with the target level of output is lower relative to the steady-state rate of interest determined by the rate of time preference. Hence, a cut in the interest rate is needed to stabilize the output gap. Furthermore, as the population share of the non-Ricardians rises, not only is it optimal to stabilize the economy around an increasing target level of output, but output also becomes less responsive to the interest rate. The interest rate associated with the target level of output correspondingly falls more below the steady-state rate. Somewhat surprisingly then, smaller responses in inflation are associated with larger cuts in the interest rate. Finally, our results do not imply that the inclusion of liquidity-constrained agents would bring about a rise in private consumption in response to an increase in government expenditures in the optimal economy. Our sensitivity analysis shows that these conclusions hold over a wide range of structural parameter values.

The rest of the paper is organized as follows. Section 2 sets out the micro-economic foundations of the model. Section 3 presents the model of the linear economy and the quadratic objective function of the policy maker that follow from the micro-foundations. Section 4 derives the optimal dynamics of endogenous variables following a shock to government expenditures, including the rules for optimal conduct of monetary and fiscal policy. The impact of the presence of non-Ricardian agents on optimal dynamics of the model and the implications for optimal policy are discussed in Section 5. In Section 5, we also check the sensitivity of the results with respect to some key structural parameter values. Section 6 concludes. Finally, the Appendix gives a detailed account of the derivations that underlie the macroeconomic model presented in Section 3.

2. The micro-foundations of the model

In this section, we present a general equilibrium framework that generates a constant population share of liquidity constrained agents in the economy. We allow for heterogeneity among agents in terms of access to the asset market but our

setup enables us to maintain much of the tractability of the representative agent framework. This feature of the analysis then facilitates the use of modern methods of optimal policy determination.

2.1. Consumers

Consider an economy inhabited by a continuum of identical, infinitely-lived agents indexed $k \in [0, 1]$. The agents' utility is increasing in consumption C and leisure $(1 - H)$. As in Galí et al. (2004), we assume the following functional form for the utility of agents

$$u = \frac{\left[C\left(1-H\right)^{\omega}\right]^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}} \tag{2.1}$$

with $\omega > 0$ and $\widetilde{\sigma}^{-1} > 0$. We further assume, similar to Amato and Laubach (2003), that the representative consumer receives information about whether she is able to smooth consumption over time or whether she has to rely only on current wage income to finance consumption at the beginning of each period. It is before this information regarding the current period is revealed that the agent makes her optimizing choice.

Let $\lambda \in [0, 1)$ denote the probability that the agent is unable to smooth consumption over time. λ is independent of the consumers' history. The law of large numbers then implies that λ will also stand for the proportion of "hand-to-mouth" agents in the economy.

We also assume that the newcomers to the asset market will be given an endowment, while the leavers will be charged a lump-sum tax by the government when leaving the asset market. Let the lump-sum transfer be on aggregate equal to the total asset holdings of the agents who are leaving the asset market, including the earned interest.[4] Let us also assume that access to the asset market is a requirement for receiving dividends.

At the beginning of each period, before the information about the type that each agent belongs to is revealed, there are two groups of consumers in the economy: those who were non-Ricardian and those who were Ricardian in the previous period. Their respective shares on the population are λ and $1 - \lambda$. Members of either group face two scenarios. Once the information about the type is revealed, they can become either non-Ricardian or Ricardian with probability λ and $1 - \lambda$ respectively. Therefore, after the information about access to the asset market

[4]This assumption is needed to ensure that the transversality condition for consumer asset holdings is satisfied.

is revealed, we shall have four types of agents in the economy: types A (D) had (no) access to the asset market in period $t-1$ and have (no) access to it in period t; types B and C represent the intermediate cases - the leavers and newcomers to the asset market. We assume that the agents know λ and observe total asset holdings in the economy (and all other aggregate variables).

Given these assumptions, we can write the expected budget constraint of those who were Ricardian in the previous period as follows

$$P_t C_t^{AB} + (1-\lambda) B_t^A = (1-\tau_t) W_t H_t^{AB} + (1-\lambda) P_t D_t^A + (1-\lambda)(1+i_{t-1}) B_{t-1}^A \tag{2.2}$$

in which $C_t^{AB} = (1-\lambda) C_t^A + \lambda C_t^B$ and an analogous relationship holds for the labor supply. $(1+i_{t-1}) B_{t-1}^A$ represents the asset holdings expected to be carried over from the previous period. i_t denotes the nominal interest rate and P_t is the general price level. D_t^A stands for dividends received by the agents in period t who continue to have access to the asset market. W is the nominal wage rate and τ is the flat tax rate on wage income.

Analogously, the expected budget constraint of those who were non-Ricardian in the previous period is given by

$$P_t C_t^{CD} + (1-\lambda) B_t^C = (1-\tau_t) W_t H_t^{CD} + (1-\lambda) P_t D_t^C + (1-\lambda)(1+i_{t-1}) B_{t-1}^C \tag{2.3}$$

In this case, $(1+i_{t-1}) B_{t-1}^C$ stands for the asset holdings expected to be acquired via the lump-sum transfer by the newcomers to the asset market from those previously Ricardian agents who will have left the asset market. Similarly, after gaining access to the asset market, the agents will also be entitled to receiving their share of dividends D_t^C.

From period $t+1$ onwards, regardless of whether they were Ricardian or non-Ricardian in $t-1$, the agents can belong to any of the above groups with probabilities (NR=non-Ricardian, R=Ricardian)

$$\begin{aligned}
&\text{type A: } P\left[R(T) \mid R(T-1)\right] = (1-\lambda)^2 = \pi_A \\
&\text{type B: } P\left[NR(T) \mid R(T-1)\right] = \lambda(1-\lambda) = \pi_B \\
&\text{type C: } P\left[R(T) \mid NR(T-1)\right] = \lambda(1-\lambda) = \pi_C \\
&\text{type D: } P\left[NR(T) \mid NR(T-1)\right] = \lambda^2 = \pi_D
\end{aligned}$$

for $T > t$. These probabilities also represent the respective population shares of agents of each type.

It follows that the aggregate budget constraint of consumers will be written as

$$P_t C_t + B_t = (1-\tau_t) W_t H_t + P_t D_t + (1+i_{t-1}) B_{t-1} \tag{2.4}$$

where the aggregate consumption is $C_t = \sum_s \pi_s C^s$ with $s = A, B, C, D$ and π_s are the respective probabilities given above. An analogous relationship holds for the labor supply H_t. Once redistribution via the lump sum tax has occured, only types A and C hold assets, therefore $B_t = \sum_s \pi_s B_t^s$ with $s = A, C$ and the same holds for dividends. This relationship holds for all t.

It is important to note that since we have assumed a population of measure one, the aggregate variables are also per capita levels as well as economy-wide averages. The constraint (2.4) will also be the expected budget constraint of the representative consumer.

$(1 + i_{t-1}) B_{t-1}$ represents the total asset holdings (of agents who were Ricardian in the previous period) in the economy at the beginning of period t. Once the information about access to the asset market in period t is revealed and redistribution occurs, these holdings will be split between those who stay Ricardian and the newcomers to the market who will receive the share of those who exit from the market. Since all agents are identical (in all aspects but access to the asset market) in our model, assets will be uniformly distributed across individuals who have access to the asset market. Hence, λ share of total assets carried over from the previous period will be held by newcomers to the asset market, while those who stay Ricardian will hold the rest. Now, since λ share of total assets will be held by a $\lambda(1-\lambda)$ fraction of the population, this implies that the newcomers must hold $\frac{\lambda}{\lambda(1-\lambda)} = \frac{1}{1-\lambda}$ of the average economy-wide holdings. Similarly, $(1-\lambda)$ share of total initial assets will be held by a $(1-\lambda)^2$ fraction of the total population. Therefore, those who stay Ricardian will also hold $\frac{(1-\lambda)}{(1-\lambda)^2} = \frac{1}{1-\lambda}$ of average holdings. It will therefore hold that $(1 + i_{t-1}) B_{t-1}^A = (1 + i_{t-1}) B_{t-1}^C = \frac{(1+i_{t-1})B_{t-1}}{1-\lambda}$. Agents will be aware of this, since aggregate holdings as well as λ are observed by everyone. Each agent's (expected rather than actual) initial asset holdings will thus be identical at the time of making the optimizing choice. Similar reasoning would apply to dividends.

Also, it can be shown that given a simple budget constraint that makes consumption equal to the after-tax wage, the period utility function of the form (2.1) is maximized if the liquidity-constrained agents supply labor

$$H^{NR} = \frac{1}{1+\omega} \tag{2.5}$$

and the optimal consumption of these agents is then going to be given by

$$C_t^{NR} = (1-\tau_t)\frac{W_t}{P_t}H^{NR} \tag{2.6}$$

for all t. We have used the superscript NR to denote the variables optimally chosen by agents who become non-Ricardian regardless of which group they belonged to in the previous period. The utility of the non-Ricardian agents, given this optimal choice, can be written using an indirect utility function $V^{NR}\left(\tau_t, \frac{W_t}{P_t}\right)$. Agents will take this optimal choice into account as a kind of an outside option, since the tax rate τ is set by the government, and we also assume perfectly competitive labor markets (and therefore that every single agent is a wage-taker).

Given the fact that the (expected) initial holdings of assets by both previous period's Ricardian and non-Ricardian agents are the same (see above), they both will face the same decision problem and choose a common optimal level of consumption, labor supply (labeled with the superscript R) and asset holdings for the case that they will have access to the asset market. The problem to be solved can thus be reduced to the problem of a representative agent choosing a sequence of $\left\{C_t^R, H_t^R, b_t\right\}$ to maximize

$$\max_{C_T^R, H_T^R, b_T} U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ (1-\lambda)\frac{\left[C_T^R\left(1-H_T^R\right)^{\omega}\right]^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}} + \lambda V^{NR}\left(\tau_t, \frac{W_t}{P_t}\right) \right\} \tag{2.7}$$

subject to

$$C_t + \frac{b_t}{1+i_t} = (1-\tau_t)\frac{W_t}{P_t}H_t + D_t + b_{t-1}\frac{P_{t-1}}{P_t} \tag{2.8}$$

for all $T \geqslant t$ in which now $C_t = \lambda C_t^{NR} + (1-\lambda)\, C_t^R$ and $H_t = \lambda H^{NR} + (1-\lambda)\, H_t^R$, with $b_t = (1+i_t)\,\frac{B_t}{P_t}$.

Combining the first-order conditions with respect to C_t^R and H_t^R yields

$$C_t^R = \frac{(1-\tau_t)}{\omega}\frac{W_t}{P_t}\left(1-H_t^R\right) \tag{2.9}$$

and we also obtain the Euler equation

$$\beta \frac{E_t\left(C_{t+1}^R\right)^{-\widetilde{\sigma}^{-1}}\left(1-H_{t+1}^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}}{\left(C_t^R\right)^{-\widetilde{\sigma}^{-1}}\left(1-H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}}\frac{(1+i_t)}{(1+E_t\pi_{t+1})} = 1 \tag{2.10}$$

in which $(1+E_t\pi_{t+1}) = \frac{P_{t+1}}{P_t}$.

In aggregate then, labor supply and consumption respectively are given by

$$H_t = \frac{\lambda}{1+\omega} + (1-\lambda) H_t^R \tag{2.11}$$

$$\begin{aligned} C_t &= \lambda C_t^{NR} + (1-\lambda) C_t^R \\ &= \frac{(1-\tau_t)}{\omega} \frac{W_t}{P_t} (1-H_t) \end{aligned} \tag{2.12}$$

Combining (2.5), (2.6), (2.9) and (2.12), yields

$$C_t^{NR} = \frac{\omega}{1+\omega} \frac{C_t}{(1-H_t)} \tag{2.13}$$

$$C_t^R = \frac{C_t}{(1-\lambda)} \left[1 - \frac{\omega\lambda}{(1-H_t)(1+\omega)} \right] \tag{2.14}$$

The expected $t-$period utility of the representative agent can then be written in terms of the aggregate variables as

$$\begin{aligned} u_t &= \frac{C_t^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}} \left\{ \lambda (1-H_t)^{\widetilde{\sigma}^{-1}-1} \left(\frac{\omega}{1+\omega} \right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)} \right. \\ &\left. + (1-\lambda)^{\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{(1-H_t)(1+\omega)} \right]^{1-\widetilde{\sigma}^{-1}} \left[1 - \frac{H_t}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)} \right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \right\} \end{aligned} \tag{2.15}$$

2.2. Firms

Let us assume a continuum of monopolistically competitive producers of differentiated intermediate goods (indexed j). These goods then serve as an input in the production of a single final good. The production technology of the final good - produced by a representative firm operating in a perfectly competitive environment - is described by a Dixit-Stiglitz (1977) aggregator

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{2.16}$$

where $y_t(j)$ is the quantity of an intermediate good used in the production of Y. A simple cost minimization exercise by final goods producers yields the expression for the demand for intermediate good j

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{2.17}$$

and the aggregate price index

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \tag{2.18}$$

Let us also assume that the production of the interemediate goods is described by the production function

$$y_t(j) = H_t(j)^{1/\alpha} \tag{2.19}$$

In equilibrium it holds that

$$\begin{aligned} H_t &= \int_0^1 H_t(j) \, dj \\ &= Y_t^{\alpha} \delta_t \end{aligned} \tag{2.20}$$

with $\delta_t = \int_0^1 \left(\frac{p_t(j)}{P_t} \right)^{-\theta\alpha} dj$ denoting price dispersion.

The producers of intermediate goods maximize profits given by

$$\Upsilon_t = p_t(j) \, y_t(j) - W_t H_t(j) \tag{2.21}$$

They do so in a forward-looking way, evaluating an expected stream of profits. We assume a price setting mechanism of the type put forward by Calvo (1983) with $\gamma \in (0,1)$ denoting the probability for a firm of charging unchanged prices in any period.

2.3. The government

Monetary and fiscal authorities, the two branches of the central government, coordinate their actions to ensure that social welfare given by (2.7) is maximized. The government raises revenues via distortive taxes on wage income to finance exogenous government spending G. It issues one-period nominal bonds to bridge

the gap between taxation and spending. The government therefore faces a flow budget constraint

$$B_t = (1 + i_{t-1}) B_{t-1} - P_t s_t \tag{2.22}$$

where B denotes the volume of one-period nominal bonds issued by the fiscal authority and s_t is the $t-$period primary budget surplus. This constraint can be rewritten as

$$\frac{b_t}{(1 + i_t)} = \frac{b_{t-1}}{(1 + \pi_t)} - s_t \tag{2.23}$$

3. The macroeconomic model

The micro-foundations discussed in detail in Section 2 imply a simple New Keynesian model of the macroeconomy. A straighforward log-linearization of the Euler equation (2.10) gives us the intertemporal IS relationship, which describes the demand side of the economy.

$$y_t = E_t y_{t+1} - \Phi \left(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^* \right) \tag{3.1}$$

y_t is the welfare-relevant output gap. It is defined as the difference between the actual and the target deviation of output from its steady-state value in response to a shock, with the target deviation being defined as some function of that shock and denoted $\widehat{Y}_t^*$.[5] $\widehat{Y}_t^*$ follows from the approximation to welfare (see below and in the Appendix). $\widehat{i}_t = \log \frac{1+i_t}{1+\bar{i}}$, where $\bar{i}$ is the steady state interest rate determined by the rate of time preference. $\widehat{r}_t^*$ represents the deviation in the interest rate that is consistent with $\widehat{Y}_t^*$. $\widehat{r}_t^*$ depends on exogenous real variables only and hence, cannot be affected by government policy. The details of the derivation are given in the Appendix.

The supply side of the economy is characterized by the following forward-looking New Keynesian Phillips curve

$$\pi_t = \kappa y_t + \chi_\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) + \beta E_t \pi_{t+1} \tag{3.2}$$

The supply equation links current inflation to the output gap, deviation in taxes and expected future inflation. $\widehat{\tau}_t^*$ is the deviation in the tax rate that would offset the cost-push (or cost-pull) pressure resulting from the increase in government

[5] In general, we shall denote the t-period percentage deviation of a variable x from its steady-state value $\overline{x}$ as $\widehat{x}_t = \frac{dx_t}{\overline{x}}$ with $dx_t = x_t - \overline{x}$.

spending. A cost-push (cost-pull) pressure follows a rise in government spending if the spending shock shifts the natural rate of output[6] less (more) relative to what is desirable from the viewpoint of welfare (the target deviation $\widehat{Y}_t^*$). A policy of $\widehat{\tau}_t^*$ then brings the natural rate in line with the target deviation and eliminates pressure on prices. The derivation of this structural equation from the firms' profit-maximizing decision as well as the definitions of parameter values are given in the Appendix.

The government's flow budget constraint can be shown to yield the following fiscal sustainability condition expressed in terms of the "gap-variables" in (3.2)

$$\begin{aligned} \widehat{b}_{t-1} - \pi_t + \chi_y y_t + \varphi_t &= (1-\beta)\left[f_y y_t + f_\tau\left(\widehat{\tau}_t - \widehat{\tau}_t^*\right)\right] \\ &\quad + \beta E_t\left[\widehat{b}_t - \pi_{t+1} + \chi_y y_{t+1} + \varphi_{t+1}\right] \end{aligned} \tag{3.3}$$

φ_t is the "fiscal stress" term introduced in Benigno and Woodford (2003) as a composite measure of the consequences for fiscal solvency of the spending shock in period t. The Appendix explains the steps involved in deriving (3.3) from (2.23). Coefficients f_y, f_τ and χ_y are also defined in the Appendix.

Finally, the central government conducts monetary and fiscal policy in a coordinated fashion to minimize the quadratic loss function

$$L_t = E_t \sum_{T=t}^{\infty} \beta^{T-t}\left\{\frac{1}{2} q_y y_T^2 + \frac{1}{2} q_\pi \pi_T^2\right\} \tag{3.4}$$

The Appendix provides a detailed account on how this loss function follows from the agents' expected utility function. Sutherland (2002) and Benigno and Woodford (2003) explain why this function represents a correct welfare ranking criterion to a solution resulting from the linear system of (3.1) and (3.3).

4. Optimal dynamics of the economy

It is now well understood that in the absence of full commitment the policy that maximizes (3.4) subject to (3.2) and (3.3) is time inconsistent. Woodford (2003) has therefore proposed formulating optimal policy from a "timeless perspective".

[6] The natural rate of output is defined as the level of output attained under flexible prices. The deviations in the natural rate depend on real exogenous disturbances and, in our case, on tax policy. See Woodford (2003, Chapter 3) for details.

This amounts to seeking such policies for the current period that one would have optimally committed oneself to implement in the distant past. This perspective on optimal policy rules out the possibility of policymakers 'exploiting' certain intial conditions (such as those that may indicate low inflation expectations, or a high value of nominal debt). In order to characterize this timelessly optimal policy, we need to restricts the policy choices for period t so that the policy maker uses the same procedure to formulate policy as in later periods. Specifically, this means that optimal (long-run) policy targets are derived from the first-order conditions for periods $T > t$ and the policy maker is then restricted to use the same mechanism for period t too. The problem facing the policymaker is, therefore, given by the following Lagrangian:

$$\begin{aligned} J_t &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \left(\frac{1}{2} q_y y_T^2 + \frac{1}{2} q_\pi \pi_T^2 \right) \right. \\ &\quad + \phi_{1,T} \left[\pi_T - \kappa y_T - \chi_\tau \left(\widehat{\tau}_T - \widehat{\tau}_T^* \right) - \beta E_t \pi_{T+1} \right] \\ &\quad + \phi_{2,T} \left[\widehat{b}_{T-1} - \pi_T + \chi_y y_T + \varphi_T - (1-\beta) f_y y_T - (1-\beta) f_\tau \left(\widehat{\tau}_T - \widehat{\tau}_T^* \right) \right. \\ &\quad \left. \left. - \beta \widehat{b}_T + \beta E_t \pi_{T+1} - \beta E_t \chi_y y_{T+1} \right] \right\} + \left(\phi_{2,t-1} - \phi_{1,t-1} \right) \pi_t + \chi_y \phi_{2,t-1} y_t \end{aligned} \tag{4.1}$$

The last terms represent the additional constraints concerning the initial period.

$$\pi_t = \overline{\pi}_t$$

$$y_t = \overline{y}_t$$

The correct specification of these conditions is discussed in Woodford (2003). In general, we want $\overline{\pi}_t$ and $\overline{y}_t$ to be predetermined, i.e. to be functions of lagged endogenous and/or exogenous variables. Benigno and Woodford (2005) explain why linear costraints of this form are sufficient to justify (3.4) being the correct second-order-accurate welfare objective and (4.1) the correct formulation of the policy problem. Svensson (2003) pointed out that adding the time-t terms to the policy Lagrangian internalizes the welfare effects of disappointing private sector expectations regarding policy in the initial period.

The first-order condition (with $T = t$) with respect to the deviation in outstanding liabilities implies that the Lagrange multiplier associated with the fiscal

constraint follows a Martingale process from period t onwards[7]

$$E_t\phi_{2,t+1} = \phi_{2t}. \tag{4.2}$$

The remaining first-order conditions imply

$$\pi_t = \omega_\phi \left(\phi_{2,t} - \phi_{2,t-1}\right) \tag{4.3}$$

$$y_t = m_\phi \phi_{2,t} + n_\phi \phi_{2,t-1}, \tag{4.4}$$

with $\omega_\phi = q_\pi^{-1}\left[1 + \frac{(1-\beta)f_\tau}{\chi_\tau}\right], m_\phi = q_y^{-1}\left[(1-\beta)\, f_y - \chi_y - \frac{(1-\beta)f_\tau\kappa}{\chi_\tau}\right], n_\phi = \frac{\chi_y}{q_y}$. It follows from (4.2), (4.3) and (4.4) that

$$E_t\pi_T = 0 \tag{4.5}$$

$$E_t y_T = (m_\phi + n_\phi)\, \phi_{2,t}, \tag{4.6}$$

for all $T > t$. With the shock being non-persistent, (3.2) written for period $t+1$ can be solved for the expected deviation in the tax rate

$$E_t\widehat{\tau}_{t+1} = -\frac{\kappa}{\chi_\tau} E_t y_{t+1}. \tag{4.7}$$

Substituting this into a forward-integrated version of (3.3), solving that equation forwards, and using (4.5) and (4.6), we obtain

$$\widehat{b}_t = n_b \phi_{2,t}, \tag{4.8}$$

where we have used n_b for $(m_\phi + n_\phi)\left(f_y - \frac{f_\tau\kappa}{\chi_\tau} - \chi_y\right)$. Outstanding liabilities therefore inherit the time series properties of the Lagrange multiplier ϕ_2. Also, (3.2) can be re-arranged to obtain an equation for the dynamics of the tax rate

$$\widehat{\tau}_t = \widehat{\tau}_t^* - \frac{\kappa}{\chi_\tau} y_t + \frac{1}{\chi_\tau}\pi_t. \tag{4.9}$$

Substituting this into a forward-integrated version of (3.3), and using (4.3) to (4.6), the dynamics of the Lagrange multiplier associated with the fiscal sustainability constraint will be

[7] Because of this time series property of the solution, restrictions must be placed on the number and/or the size of the shocks in order to keep the model in a close enough neighborhood of the initial steady state (see Benigno and Woodford, 2003, footnote 26). See also footnote 10 in this paper.

$$\phi_{2,t} = \phi_{2,t-1} + \frac{\varphi_t}{m_b + n_b}, \tag{4.10}$$

in which $m_b = \left[1 + (1-\beta)\frac{f_\tau}{\chi_\tau}\right]\omega_\phi - \left[(1-\beta) f_y - \frac{(1-\beta) f_\tau \kappa}{\chi_\tau} - \chi_y\right] n_\phi$. It is easy to understand that having $\varphi_t \gtrless 0$ will cause a level shift in ϕ_{2t} and then also a level shift in b_t. Plugging (4.10) back into (4.3) and (4.4) yields the dynamics of inflation and the output gap. (4.9) defines the dynamics of the tax rate. For $m_\phi \neq -n_\phi$, this implies a unit root in output gap and hence also in the tax rate.

4.1. Optimal rules for monetary and fiscal policy

To offer some guidance for optimal policy conduct in practice, we translate the above optimal paths into policy response functions. We first derive a targeting rule in the sense of Svensson (2002, 2003) that the authorities should jointly aim to satisfy in every period. Then we follow Taylor (1997), who uses a pair of interest-rate and budget surplus rules to assess policy conduct in the US, and derive rules for individual branches of public authority that will implement the optimal policy as expressed by the targeting rule.[8]

The first-order conditions (4.3) and (4.4) imply the targeting rule

$$\pi_t + \frac{n_\phi}{m_\phi}\pi_{t-1} - \frac{\omega_\phi}{m_\phi}(y_t - y_{t-1}) = 0. \tag{4.11}$$

Monetary and fiscal authorities should co-ordinate their action to ensure this rule is satisfied in every period.

4.1.1. Monetary policy

We shall use (4.11) together with the IS curve (3.1) to derive a monetary policy rule. First, it is easy to see that (3.1), combined with (4.5) directly yields

$$i_t = \widehat{r}_t^* + \Phi^{-1}(E_t y_{t+1} - y_t). \tag{4.12}$$

Together with (4.11) this leads us to

$$i_t = \widehat{r}_t^* + \Phi^{-1}\frac{n_\phi}{\omega_\phi}\pi_t. \tag{4.13}$$

[8] See Woodford (2003, Chapter 7) for discussion on the merits of alternative approaches to formulating optimal policy rules.

4.1.2. Fiscal policy

The fiscal control variable in our model is the tax rate. Its optimal dynamics is given by (4.9). It is, however, more in line with real-world policy conduct to think of the primary budget surplus as the primary indicator of fiscal policy stance in the short term. We therefore derive the optimal path for the primary fiscal surplus which, together with the interest rate rule, ensures (4.11) is satisfied in every period. We show in the Appendix that the fiscal solvency constraint can be written as

$$W_t = s_t \left(C_t^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} + \beta E_t W_{t+1},$$

where $W_t = \frac{b_{t-1}}{1+\pi_t} \left(C_t^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}$. Solving this for s_t and using (2.10) leads us to

$$s_t = \frac{b_{t-1}}{\Pi_t} - \frac{b_t}{R_t}, \tag{4.14}$$

where $R_t = 1 + i_t$. The first-order approximation to (4.14) can be written as

$$\widehat{s}_t = \frac{1}{1-\beta}\left(\widehat{b}_{t-1} - \pi_t - \beta\widehat{b}_t + \beta\widehat{i}_t\right) + O\left(2\right). \tag{4.15}$$

Using (4.8) and (4.13), we can now easily derive the optimal path for the surplus

$$\widehat{s}_t = \frac{1}{1-\beta}\left\{\widehat{b}_{t-1} - \left[1 + \frac{\beta n_b n_\phi}{\omega_\phi\left(m_\phi + n_\phi\right)} - \beta\Phi^{-1}\frac{n_\phi}{\omega_\phi}\right]\pi_t - \frac{\beta n_b}{m_\phi + n_\phi} y_t + \beta\widehat{r}_t^*\right\}. \tag{4.16}$$

where we have used

$$\phi_{2,t} = \frac{1}{m_\phi + n_\phi} y_t + \frac{n_\phi}{\omega_\phi\left(m_\phi + n_\phi\right)}\pi_t. \tag{4.17}$$

which follows from (4.3) and (4.4). (4.16) can also be for the surplus-to-GDP ratio d_t as follows

$$\begin{aligned}\widehat{d}_t = \frac{1}{1-\beta}\Bigg\{&\widehat{b}_{t-1} - \left[1 + \frac{\beta n_b n_\phi}{\omega_\phi\left(m_\phi + n_\phi\right)} - \beta\Phi^{-1}\frac{n_\phi}{\omega_\phi}\right]\pi_t \\ &- \left(\frac{\beta n_b}{m_\phi + n_\phi} + 1 - \beta\right) y_t + \epsilon_t\Bigg\}.\end{aligned} \tag{4.18}$$

where $\epsilon_t = \beta\widehat{r}_t^* - (1-\beta)\,\widehat{Y}_t^*$ is a shock term. Chadha and Nolan (2007) examine a fiscal rule of a similar functional form in the context of the UK economy.

5. Numerical results

In this section, we present the numerical results for the dynamics of the economy in response to a single, serially uncorrelated positive shock to government spending of 1 percent of steady-state output. We first evaluate the optimal impulse responses in the absence of non-Ricardian consumers. Then, we investigate how the optimal dynamics of the economy varies with the share of the non-Ricardians in the economy.

5.1. Calibration

We calibrate the model using conventional structural parameter values. The parameters such as the tax rate or the steady-state price markup were set so that the steady-state fiscal outcomes take on reasonable values. β, the discount rate, is set to 0.99. The consumption share of national income is 0.8. The value of $\widetilde{\sigma}^{-1}$ is set so that $-\frac{u_{cc}\overline{Y}}{u_c}$ equals 0.157, which is the value estimated in Rotemberg and Woodford (1999). Similarly, the value of ω is calibrated so that $\frac{u_{ll}\bar{l}}{u_l}$ - with $l = 1 - H$ representing leisure - takes on the value of -0.473, as used in Benigno and Woodford (2003). We assume an approximate 9 percent price markup in the product market, arising due to imperfect competition among intermediate goods producers. $\alpha = 1.25$ so that the production function governing the production of intermediate goods is of decreasing-returns-to-scale type. The price stickiness parameter in the Calvo-pricing model γ has been set to 0.75. The steady state labor income tax rate is 30 percent. These parameter values imply a steady-state surplus-to-GDP ratio of 0.02 and hence $\frac{\bar{b}}{\overline{Y}} = 2$. The steady-state aggregate output, as shown in the Appendix, is independent of λ.

The sensitivity of the results to changes in some of these parameter values are reported in a separate section.

5.2. The economy with Ricardian agents only

Let us first consider the limit of our model when all the agents are Ricardian. Output is demand-determined in New Keynesian models. Therefore, the story in a general case would be told as follows. Following a spending shock, firms need to hire more labor to be able to meet the increased demand. However, since labor has diminishing marginal productivity, the economy will become more labor-intensive, average productivity will fall and marginal cost will rise. Since prices are just a simple markup over marginal costs, inflation will be generated. When prices are

fully flexible ($\gamma = 0$), all prices adjust immediately to their optimal level following the shock, price dispersion never occurs and inflation is therefore not costly to the agent ($q_\pi = 0$). The supply curve becomes vertical. Solving our supply equation with $\zeta = \infty$ yields that the policy of $\widehat{\tau}_t^*$ represents the optimal response to a spending shock in an economy with perfectly flexible prices, stabilizing output at its target level $\widehat{Y}_t^*$ and maximizing welfare. (4.12) tells us that the corresponding action on the monetary side is given by $\widehat{r}_t^*$.

In models with endogenous tax dynamics, the fiscal constraint needs to be satisfied too. Under perfect price flexibility, it is high inflation, offsetting "fiscal stress" plus any inherited deviation in liabilities, that makes the $\widehat{\tau}_t^*$ tax plan feasible. Under conditions of imperfect price flexibility, the policy that is optimal under perfect flexibility fully stabilizes output at its target level only provided the price level is fully stabilized. It is, however, easy to see from the fiscal solvency constraint (3.3) that with no inflation, output at its target level and no "tax gap", the requirement of fiscal sustainability will not be satisfied if φ_t ($+\widehat{b}_{t-1}$, if any) is non-zero. Hence, in an environment with sticky prices, the endogeneity of the tax dynamics and the associated requirement of fiscal solvency precludes full stabilization of the economy via price-level targeting. The degree to which full stabilization is prevented is determined by the "fiscal stress" φ_t ($+\widehat{b}_{t-1}$, if any). An internally consistent mix of tax and interest rate policy different from $\widehat{\tau}_t^*$ and $\widehat{r}_t^*$ will then be needed to satisfy both structural constraints (3.2) and (3.3) simultaneously, yielding non-zero responses in inflation and the output gap.

Figure 5.1[9] plots the optimal paths under price stickiness for $\lambda = 0$. We see that the optimal volatility of both endogenous variables that appear in the objective function - inflation and output gap - is very small. The results are similar to those reported in Benigno and Woodford (2003) who analyze a shock to government transfers rather than government spending and show how optimal volatility of inflation changes depending on the degree of price stickiness. As pointed out in the previous section, outstanding liabilities, output gap and the tax rate are all non-stationary. Schmitt-Grohe and Uribe (2002) observe that when price changes are costly, governments finance an innovation in government spending partly by raising debt and partly via higher taxes. To avoid a large one-off hike in taxes in the period when the shock hits the economy, the government spreads the tax inrease over many periods, which optimally implies a permanently higher tax rate and public debt level. The higher tax rate in turn keeps output

[9] 1 on the vertical axes stands for 1 percent deviation. The inflation and interest rates are quarterly. We have assumed the economy had been in its steady state before the shock occured.

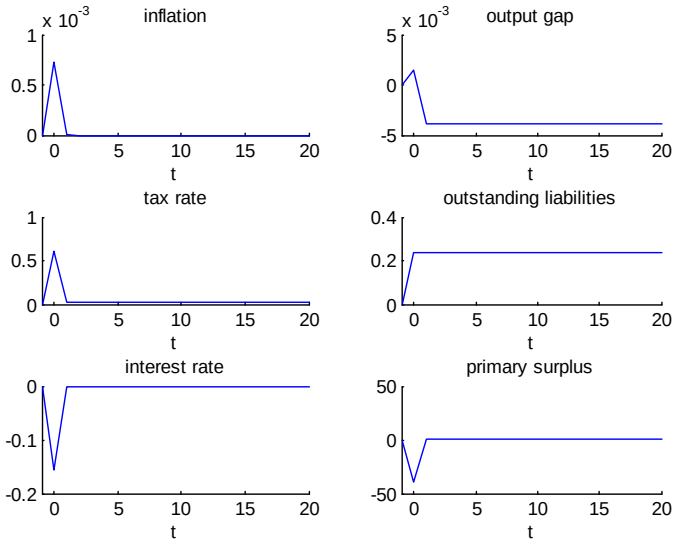

Figure 5.1: Optimal paths - Ricardian economy

below its pre-shock steady-state level in the long run.[10]

As for the policy variables, the primary surplus initially falls, since the spending shock is not fully covered by extra tax revenues. The surplus is also non-stationary. It jumps to a new level that is consistent with the higher long-run level of public debt.

The response of the interest rate might appear somewhat counter-intuitive. There is, however, an intuitive explanation. Given the minute deviation of inflation and the output gap in the optimal economy, the optimal deviation in the interest rate is primarily determined by the size and the sign of $\widehat{r}_t^*$ - the deviation in the interest rate consistent with the target deviation in output. $\widehat{r}_t^*$ is a composite measure and, as shown in the Appendix, is a net effect of two components. One is the effect on the interest rate of a pure demand shock induced by the rise in government expenditures. This acts to push the interest rate upwards. The

[10] In Horvath (2006), we argue that the reported non-stationarity in the behavior of endogenous variables is embodied in the type of the welfare objective one uses as a welfare-ranking criterion. If one maximizes the policy Lagrangian over unconditional rather than conditional expectations, which amounts to looking for an optimum of the welfare objective for all initial states of nature, the shadow value of government revenue ϕ_2 will follow a stationary AR process and all policy-relevant endogenous variables will then become stationary.

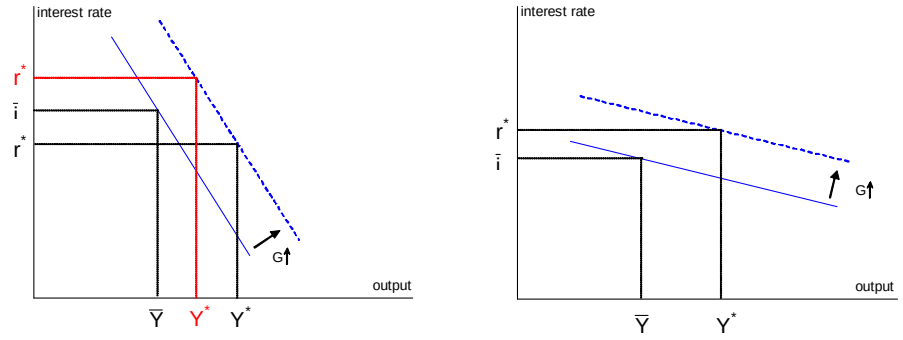

Figure 5.2: Determinants of $\widehat{r}_t^*$

second component represents the drag on the interest rate arising from the need to bring the output of the economy to its target level. The size and the sign of $\widehat{r}_t^*$ then crucially depend on the size of $\widehat{Y}_t^*$ and the elasticity in the IS relationship Φ. In our calibration, the second component dominates mainly because of the relatively small value of Φ compared with the slope coefficients derived in frameworks such as Benigno and Woodford (2003).[11] Hence, the interest rate deviation consistent with the target deviation of output is below the zero-inflation steady state level of interest. This causes the unexpected sign of the deviation in the policy rate needed to be implemented to stabilize the output gap. Figure 5.2 presents a simplified diagrammatical analysis of the determinants of $\widehat{r}_t^*$. The left panel displays the case described above alongside a hypothetical situation in which the target deviation in output would be smaller. The right panel shows another hypothetical case with demand being more responsive to changes in the interest rate (higher Φ). In both alternative cases, $\widehat{r}_t^*$ would be positive.

5.3. The role of non-Ricardian agents

The general part of the discussion in the previous section on flexible-price versus sticky-price economies also holds in the presence of non-Ricardian agents. Here we investigate the impact of the presence of non-Ricardian agents in the economy on the optimal deviations in our endogenous variables in the period when the shock

[11] The difference has roots in the functional form of the utility function (2.1) and the definition of marginal utility of consumption that follows from it.

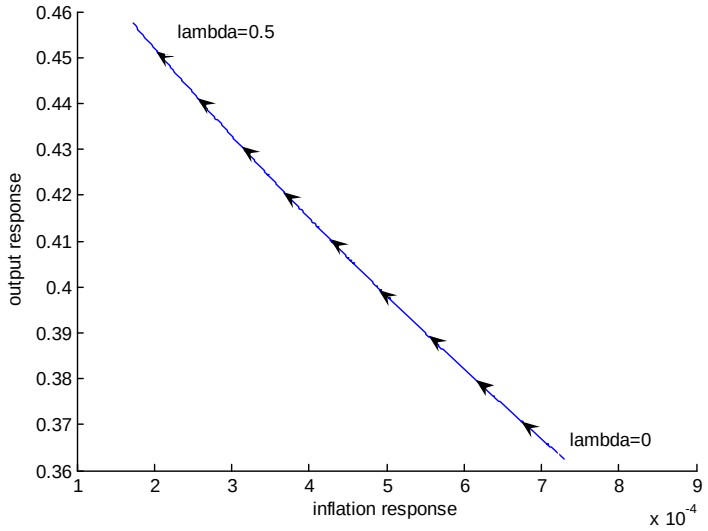

Figure 5.3: Optimal inflation and output volatility

occurs.

Our main result can be visualized using an analogue of a conventional inflation-output variance frontier. Figure 5.3 shows how the optimal deviation in inflation and output in the period when the shock occurs varies with the share of non-Ricardian agents in the economy.[12] We see that the optimal economy moves along the frontier from a state of (relatively) higher inflation coupled with low output volatility towards an optimum with lower inflation and higher output volatility, as the share of the non-Ricardian agents in the population rises.

The above developments can be traced back to changes in individual preferences. Figure 5.4 gives further insights. One needs to note that a spending-induced increase in output has a positive social welfare effect via the consumption of the non-Ricardians, the sole determinant of which is the economy-wide net real wage rate. An approximation to the net real wage rate given by (2.12) would reveal that it - and hence, non-Ricardians' consumption and welfare, is convex in aggregate output. Hence, the non-Ricardians will prefer more output volatility to less. Also, a reduction in the volatility of output has favourable level effects (as seen from (6.56) in the Appendix). The corresponding welfare effects are higher when the

[12] We have used a range for λ that we perceive as realistic and useful for our analysis.

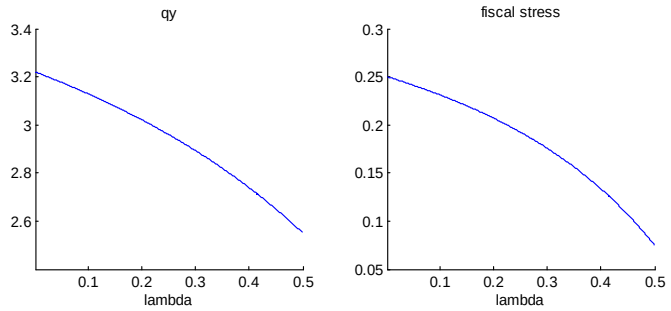

Figure 5.4: The main underlying causes of optimal outcomes

share of the non-Ricardians is low and thus, more stability in the level of output is desirable when λ is small.[13] Overall then, as the presence of the non-Ricardians in the population increases, q_y - the measure of public dislike for output volatility - falls. A given output deviation is then less costly in welfare terms, and the optimal deviation rises.

Figure 5.4 also reveals that the "fiscal stress" associated with a given spending shock becomes smaller when λ rises. A given rise in government spending has less severe consequences for fiscal sustainability when the associated response in output is larger. The need to use inflation to ensure the requirement of fiscal solvency is met then diminishes.

The optimal initial responses in the policy instruments and policy-related variables reflect the optimal outcomes described above. $\widehat{r}_t^*$ is affected both by the optimality of stabilizing output around an increasing target level and the declining elasticity in the IS relationship when the share of liquidity-constrained agents rises (Φ falls with rising λ). Both influences act to make $\widehat{r}_t^*$ more negative. A stronger anti-inflationary policy stance is therefore paradoxically associated with a larger cut in the policy rate.

The size and the sign of the response in the tax rate varies depending on the size of the target deviation in output relative to the impact of the rise in government spending on the natural rate of output. For low population shares of the non-Ricardians, the target deviation in output is smaller compared with the spending-induced shift in the natural rate and a tax hike is required to offset the resulting cost-pull pressure. For larger values of λ, with a rising target response,

[13] Such first-order effects are generally ignored if one assumes the steady state of the economy is efficient or only mildly inefficient (see Woodford, 2003).

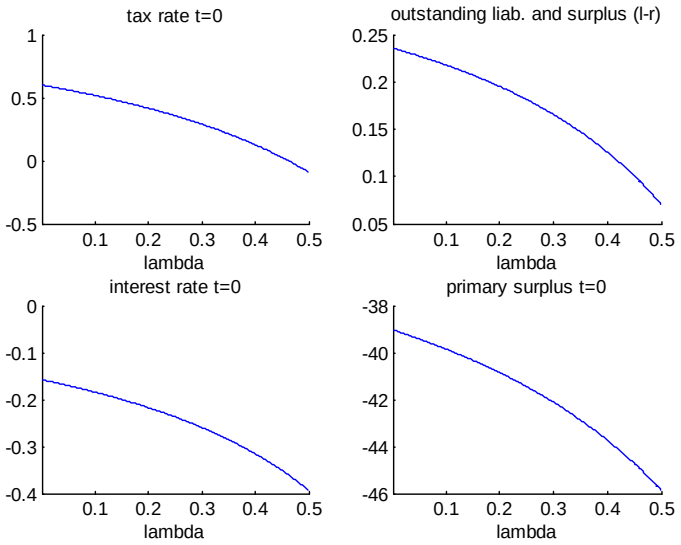

Figure 5.5: Optimal responses in policy-related variables

ultimately the opposite will hold. The tax rate will need to be cut to contain the cost-push pressure. We include Figure 5.6 to show how varying the tax rate can be used to contain pressures on prices.[14] The responses in oustanding liabilities and the primary fiscal surplus follow from the above.

Another variable that deserves close attention is aggregate private consumption. Several papers (see for instance Muscatelli et al., 2004, Coenen and Straub, 2005 or Galí et al., 2007) have sought to see whether the inclusion of non-Ricardian agents in models sheds new light on the debate regarding the potential positive effects of a rise in government spending on aggregate consumption. The conclusion that seems to emerge from the mentioned literature is that the presence of liquidity-constrained agents in the economy, perhaps alongside other factors, might help reconcile the results obtained from micro-founded New Keynesian models of the macroeconomy with empirical evidence of a crowding-in effect. In our analysis, we do not find a positive consumption response for a spending shock of a commonly-used size (1 percent of steady-state output), not even for higher

[14] In the diagram, we use a simplified, static plot of a rather special case when prices can be fully stabilized as $\widehat{b}_{t-1}$ exactly offsets the fiscal stress φ_t. This case represents a simple way of explaining how $\widehat{\tau}_t^*$ is determined in our model.

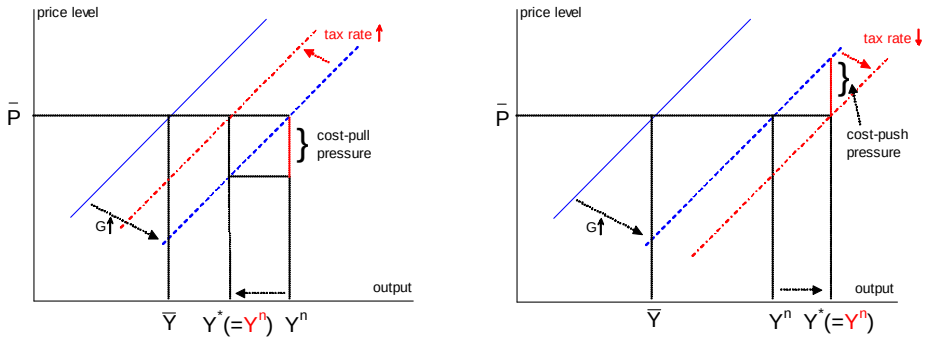

Figure 5.6: Using tax policy to stabilize the price level

values of λ. This result is robust to the choice of parameter values, as explained in the next section. We suspect the difference in the results is due to a fundamental conceptual difference between our model and the mentioned analyses, namely our use of welfare-relevant output gaps. The preference-driven target deviation in output plays a crucial role in defining the optimal dynamics of output and hence also consumption. The target deviation is not large enough to be consistent with a positive private consumption response under our baseline calibration. In what follows, we check the robustness of this claim.

5.4. Sensitivity analysis

We check the sensitivity of our results to changes in the values of structural parameters. In particular, we focus on the parameters determining the shape of the utility function. First, Figures 5.7 and 5.8 show the change in results resulting from different values of $-\frac{u_{cc}\overline{Y}}{u_c}$ and $-\frac{u_{ll}\bar{l}}{u_l}$. It is easy to see that the patterns identified in the previous section hold for a wide range of parameter values. When preferences approach the state of being log-linear in consumption, however, some of the conclusions from the previous sections get reversed.

Second, we see that the response in aggregate consumption takes on consistently negative values. This result is particularly robust with respect to the degree of price stickiness. The upper-right panel of Figure 5.9 plots the initial response in aggregate consumption for different λ (as before, on the horizontal axis) and varying values of γ, ranging from 0.1 to 0.9. Parameters in the utility function vary as before. The difference in the response is barely visible. Hence, unlike Galí

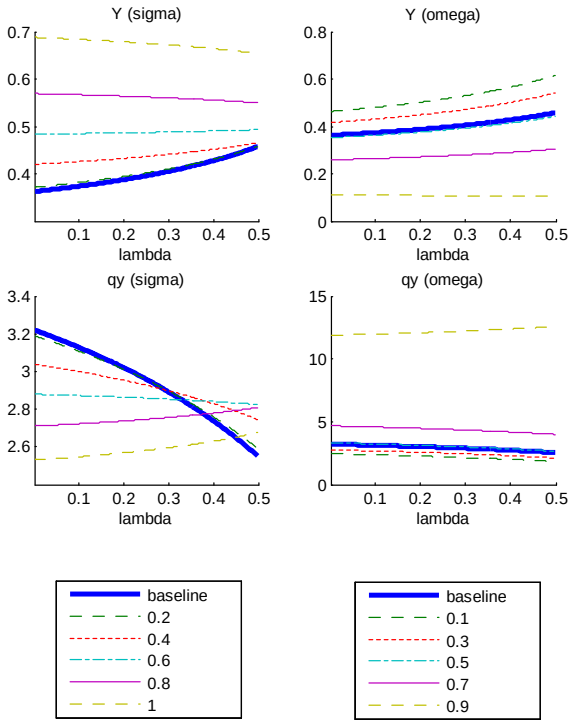

Figure 5.7: Sensitivity analysis: initial aggregate output response and the welfare costs of output deviation

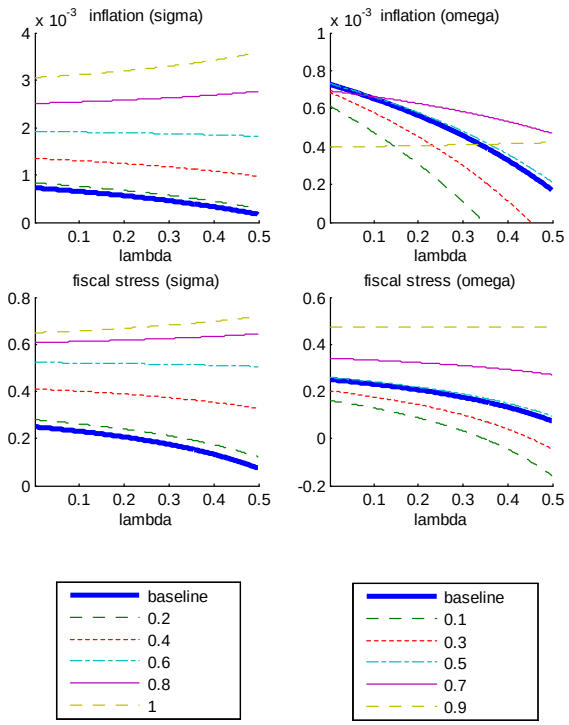

Figure 5.8: Sensitivity analysis: initial inflation response and the degree of "fiscal stress"

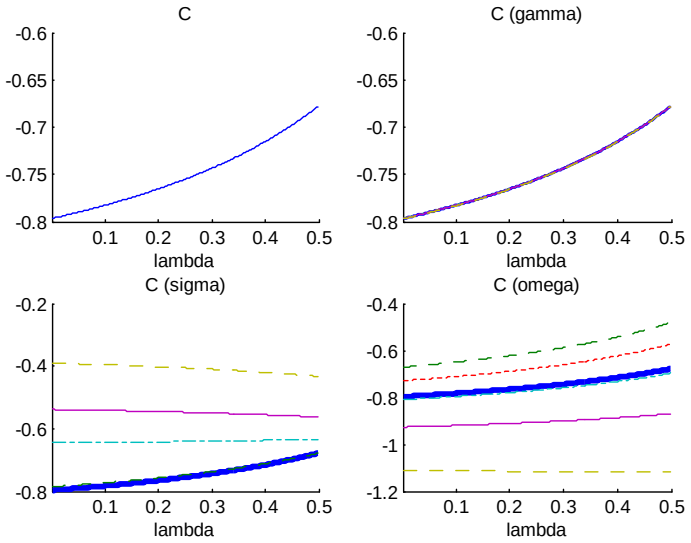

Figure 5.9: The response in aggregate consumption to increased government spending and its robustness to parameter values

et al. (2007), we do not find that the degree of price stickiness could significantly affect the sign of the response in aggregate consumption to an increase in government spending in the optimal economy. The reason is that γ only determines the size of the optimal output gap which has a very small influence on the optimal output response relative to the influence of the preference-drive target deviation, which is independent of price stickiness. Overall then, the optimal response in private consumption is consistently negative across a wide range of $\lambda, \widetilde{\sigma}^{-1}$ and ω.

6. Concluding remarks

We have used a New-Keynesian linear-quadratic setting to highlight some optimal policy implications of the presence of non-Ricardian agents in the economy. Building upon the earlier literature, we include liquidity-constrained consumers into a model with endogenous tax dynamics and derive a second-order-accurate quadratic welfare-ranking criterion for the case of large distortions to the economy's steady state.

We find that the optimal deviation in output in response to a rise in govern-

ment spending increases as the population share of non-Ricardian agents rises, while optimal inflation volatility decreases. We have demonstrated that these results can be traced back to individual preferences. Our model does not signal any potential positive private consumption response to an increase in government spending in the optimal economy and this result is robust to a wide range of parameter values governing individual preferences or nominal rigidity in the model. We obtain that potentially controversial policy responses, such as a cut in the tax rate or accommodative monetary policy stance, might in some cases be the optimal policy response to a rise in government spending. The associated analysis offers interesting observations regarding the determinants of optimal policy in a broader sense.

In this paper, as in Galí et al. (2004), we compromise on generality by assuming a utility function of a specific form. This facilitates aggregation in the model and enables us to obtain a tractable solution. It remains to be seen if the results hold in a more general setup.

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Appendix

A second-order approximation to the aggregate supply relationship

The aggregate supply relationship can be derived from the firm's optimization problem as follows. The profit function of a firm producing intermediate goods is given by (2.21). We assume Calvo pricing with γ being the probability of leaving prices unchanged in a given period. The firm is choosing the optimal price and the intertemporal first-order condition is written as

$$E_t \sum_{T=t}^{\infty} \gamma^{T-t} Q_{t,T} Y_T \left(\frac{p_t(i)}{P_T} \right)^{-\theta} \left[(1-\tau_T) - \mu \frac{W_T}{P_T} \alpha Y_T^{\alpha-1} \left(\frac{p_t(i)}{P_T} \right)^{-\theta(\alpha-1)-1} \right] = 0 \tag{6.1}$$

$Q_{t,T}$ is the stochastic discount factor which can be derived from (2.10). We can define $\omega_p = \alpha - 1$ and μ will replace $\frac{\theta}{\theta-1}$ as the price markup due to imperfect competition in the intermediate goods market. After substituting from (2.9) for the real wage rate and using $p_t(i) = p_t^*$, we obtain a closed-form solution

$$\frac{p_t^*}{P_T} = \left(\frac{K_t}{F_t} \right)^{\frac{1}{1+\theta\omega_p}} \tag{6.2}$$

with

$$K_t = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \left(C_T^R \right)^{\left(1-\widetilde{\sigma}^{-1}\right)} \left(1 - H_T^R \right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)-1} \frac{\mu\alpha\omega Y_T^{\alpha}}{(1-\tau_T)} \left(\frac{P_T}{P_t} \right)^{\theta(1+\omega_p)} \tag{6.3}$$

$$F_t = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} \left(C_T^R \right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_T^R \right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} Y_T \left(\frac{P_T}{P_t} \right)^{\theta-1} \tag{6.4}$$

The price index evolves according to

$$P_t = \left[(1-\gamma) p_t^{*1-\theta} + \gamma P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{6.5}$$

which together with (6.2) implies the following implicit definition of inflation $\Pi_t = \frac{P_t}{P_{t-1}}$

$$\left[\frac{1-\gamma\Pi_t^{\theta-1}}{1-\gamma} \right]^{\frac{1+\theta\omega_p}{\theta-1}} = \frac{F_t}{K_t} \tag{6.6}$$

The law of motion for price dispersion as defined in (2.20) is

$$\delta_t = \gamma \Pi_t^{\theta(1+\omega_p)} \delta_{t-1} + (1-\gamma) \left[\frac{1-\gamma \Pi_t^{\theta-1}}{1-\gamma} \right]^{-\frac{\theta(1+\omega_p)}{1-\theta}} \tag{6.7}$$

Let us further define

$$\begin{aligned} D_t &= \left[\frac{1-\gamma \Pi_t^{\theta-1}}{1-\gamma} \right]^{-\frac{1+\theta\omega_p}{\theta-1}} && (6.8) \\ &= \frac{K_t}{F_t} && (6.9) \end{aligned}$$

As derived in Benigno and Woodford (2005), a second-order approximation to D_t as given by (6.9) can be written as

$$\begin{aligned} \frac{\widehat{D}_t}{1-\beta\gamma} + \frac{1}{2} Z_t \widehat{D}_t &= z_t + \frac{\gamma}{1-\gamma\beta} E_t \left[\widehat{D}_{t+1} + (1+\theta\omega_p) \pi_{t+1} \right] + \frac{1}{2} z_t X_t \\ &\quad + \frac{1}{2} \gamma\beta E_t Z_{t+1} \left[\widehat{D}_{t+1} + (1+\theta\omega_p) \pi_{t+1} \right] \\ &\quad - \frac{1}{2} \frac{\gamma}{1-\gamma\beta} (1-2\theta-\theta\omega_p) E_t \pi_{t+1} \left[\widehat{D}_{t+1} + (1+\theta\omega_p) \pi_{t+1} \right] \\ &\quad + O(3) \end{aligned} \tag{6.10}$$

Z_t, z_t and X_t are expressions and will be defined later. $E_t \pi_{t+1}$ refers to expected inflation. A second-order expansion of (6.8) can in turn be written as

$$\frac{\widehat{D}_t}{1+\theta\omega_p} = \frac{\gamma}{1-\gamma} \pi_t - \frac{1}{2} \frac{\gamma(1-\theta)}{(1-\gamma)^2} \pi_t^2 + O(3) \tag{6.11}$$

Substituting for $\widehat{D}_t$ in (6.10) leads us to

$$V_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \zeta z_T + \frac{1}{2} \zeta z_T X_T + \frac{1}{2} \theta (1+\omega_p) \pi_T^2 \right\} + O(3) \tag{6.12}$$

where $\zeta = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma(1+\theta\omega_p)}$ and V_t stands for

$$V_t = \pi_t - \frac{1}{2} \frac{(1-\theta)}{(1-\gamma)} \pi_t^2 + \frac{1}{2} (1-\beta\gamma) \pi_t Z_t + \frac{1}{2} \theta (1+\omega_p) \pi_t^2 \tag{6.13}$$

In line with Beningno and Woodford (2004), we write (6.3) as $K_t = E_t \sum_{T=t}^{\infty} (\gamma\beta)^{T-t} k_{t,T}$ in which $k_T = k_T P_{t,T}^{-\theta(1+\omega_p)}$. The definition of $k_{t,T}$ follows from (6.3). $P_{t,T}$ stands for the dispersion terms in $k_{t,T}$. k_T then collects the remaining terms from $k_{t,T}$. We can analogically decompose F_t. z_T and X_T in (6.12) are then defined as

$$z_T = \widehat{k}_T - \widehat{f}_T \tag{6.14}$$

$$X_T = \widehat{k}_T + \widehat{f}_T \tag{6.15}$$

Using (2.11) and (2.14), k_T can be written as

$$\begin{aligned} k_T &= \left\{ \frac{C_T}{(1-\lambda)} \left[1 - \frac{\omega\lambda}{(1-H_T)(1+\omega)} \right] \right\}^{\left(1-\widetilde{\sigma}^{-1}\right)} \\ &\quad * \left[1 - \frac{H_T}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)} \right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)-1} \frac{\mu\alpha\omega Y_T^{\alpha}}{(1-\tau_T)} \end{aligned} \tag{6.16}$$

and f_T becomes

$$\begin{aligned} f_T &= \left\{ \frac{C_T}{(1-\lambda)} \left[1 - \frac{\omega\lambda}{(1-H_T)(1+\omega)} \right] \right\}^{-\widetilde{\sigma}^{-1}} \\ &\quad * \left[1 - \frac{H_T}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)} \right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} Y_T \end{aligned} \tag{6.17}$$

In what follows, we also make use of the following second-order expansions. $C_t = Y_t - G_t$ can be approximated by

$$\widehat{C}_t = c^{-1}\left(\widehat{Y}_t - \widehat{G}_t\right) + \frac{1}{2}c^{-1}\left(1-c^{-1}\right)\widehat{Y}_t^2 + c^{-2}\widehat{Y}_t\widehat{G}_t + t.i.p. + O(3) \tag{6.18}$$

c is the consumption share of income in the steady state and $t.i.p.$ refers to terms independent of policy. $(1-\tau_t)$ can be expanded as follows

$$\widehat{(1-\tau_t)} = -\omega_\tau \widehat{\tau}_t - \frac{1}{2}\frac{\omega_\tau}{(1-\overline{\tau})}\widehat{\tau}_t^2 + O(3) \tag{6.19}$$

where $\omega_\tau = \frac{\overline{\tau}}{1-\overline{\tau}}$. Similarly, the expansion for leisure $(1-H_t)$ is written as

$$\widehat{(1-H_t)} = -\omega_H \widehat{H}_t - \frac{1}{2}\frac{\omega_H}{\left(1-\overline{H}\right)}\widehat{H}_t^2 + O(3) \tag{6.20}$$

with $\omega_H = \frac{\overline{H}}{\left(1-\overline{H}\right)}$. From (2.20), it follows that

$$\widehat{H}_t = \alpha \widehat{Y}_t + \widehat{\delta}_t \tag{6.21}$$

The second-order expansions of (6.7) is written as

$$\widehat{\delta}_t = \gamma \widehat{\delta}_{t-1} + \frac{1}{2}\frac{\gamma}{(1-\gamma)}\theta\left(1+\omega_p\right)\left(1+\omega_p\theta\right)\pi_t^2 + O\left(3\right) \tag{6.22}$$

We see that the expansion does not contain linear terms in inflation and hence, when multiplied with other first-order terms, the product will be accurate to the third order. Therefore, also

$$\widehat{H}_t^2 = \alpha^2 \widehat{Y}_t^2 \tag{6.23}$$

An infinite discounted sum of (6.22) will be equal to

$$\begin{aligned} E_t \sum_{T=t}^{\infty} \beta^{T-t} \widehat{\delta}_t &= \frac{1}{2}\frac{\gamma}{(1-\gamma)(1-\beta\gamma)}\theta\left(1+\omega_p\right)\left(1+\omega_p\theta\right) E_t \sum_{T=t}^{\infty} \beta^{T-t}\pi_T^2 \\ &\quad + t.i.p. + O\left(3\right) \end{aligned} \tag{6.24}$$

We can also establish that

$$\left[1 - \widehat{\frac{\omega\lambda}{(1-H_t)(1+\omega)}}\right] = -\Omega_H \widehat{H}_t - \frac{1}{2}\Omega_H \left\{ \frac{(1+\omega)\left(1-\overline{H}^2\right) - \omega\lambda}{\left(1-\overline{H}\right)\left[(1+\omega)\left(1-\overline{H}\right) - \omega\lambda\right]} \right\} \widehat{H}_t^2 + O\left(3\right)$$

$$\left[1 - \frac{H_t}{1-\lambda} + \widehat{\frac{\lambda}{(1-\lambda)(1+\omega)}}\right] = -\chi_H \widehat{H}_t - \frac{1}{2}\chi_H\left(1+\chi_H\right)\widehat{H}_t^2 + O\left(3\right)$$

We have defined $\Omega_H = \frac{\omega_H \omega\lambda}{\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]}$ and $\chi_H = \frac{(1+\omega)\overline{H}}{(1+\omega)\left(1-\lambda-\overline{H}\right)+\lambda}$.

Using the above derivations, it is now straightforward to derive the aggregate supply relationship. For z_T and X_T, we have

$$\begin{aligned} z_T &= \widehat{C}_T + \left[1 - \widehat{\frac{\omega\lambda}{(1-H_T)(1+\omega)}}\right] + (\alpha - 1)\widehat{Y}_t \\ &\quad - \left[1 - \frac{H_T}{1-\lambda} + \widehat{\frac{\lambda}{(1-\lambda)(1+\omega)}}\right] - \widehat{(1-\tau_t)} \end{aligned} \tag{6.25}$$

$$
\begin{aligned}
X_T &= \left(1-2\widetilde{\sigma}^{-1}\right)\widehat{C}_T+\left(1-2\widetilde{\sigma}^{-1}\right)\left[1-\frac{\widehat{\omega\lambda}}{\left(1-H_T\right)\left(1+\omega\right)}\right] \\
&\quad+\left[2\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right]\left[1-\frac{H_T}{1-\lambda}+\frac{\widehat{\lambda}}{\left(1-\lambda\right)\left(1+\omega\right)}\right] \\
&\quad+\left(\alpha+1\right)\widehat{Y}_t-\left(\widehat{1-\tau_t}\right) \qquad (6.26)
\end{aligned}
$$

It follows that

$$
\begin{aligned}
V_t &= E_t\sum_{T=t}^{\infty}\beta^{T-t}\zeta\left\{z_y\widehat{Y}_T-c^{-1}\widehat{G}_T+\omega_\tau\widehat{\tau}_T\right. \\
&\quad+\frac{1}{2}\left(z_{yy}+z_yx_y\right)\widehat{Y}_T^2+\frac{1}{2}\omega_\tau\left(\frac{1+\overline{\tau}}{1-\overline{\tau}}\right)\widehat{\tau}_T^2 \\
&\quad+\omega_\tau\left\{\left(1-\widetilde{\sigma}^{-1}\right)\left(c^{-1}-\alpha\Omega_H\right)+\alpha-\alpha\chi_H\left[\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right]\right\}\widehat{\tau}_T\widehat{Y}_T \\
&\quad+c^{-1}\left[c^{-1}-\left(1-2\widetilde{\sigma}^{-1}\right)\left(c^{-1}-\alpha\Omega_H\right)-\alpha\left(1-\widetilde{\sigma}^{-1}\right)\left[\chi_H\left(1-\omega\right)+1\right]-\widetilde{\sigma}^{-1}\right]\widehat{Y}_T\widehat{G}_T \\
&\quad\left.-c^{-1}\omega_\tau\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T+\frac{1}{2}\frac{\theta\left(1+\omega_p\right)}{\zeta}\left(1+\chi_H-\Omega_H\right)\pi_T^2\right\} \\
&\quad+t.i.p.+O\left(3\right) \qquad (6.27)
\end{aligned}
$$

where we have used

$$
z_y=c^{-1}-\alpha\Omega_H+\alpha\chi_H+\alpha-1 \qquad (6.28)
$$

$$
x_y=\left(1-2\widetilde{\sigma}^{-1}\right)\left(c^{-1}-\alpha\Omega_H\right)-\left[2\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right]\alpha\chi_H+\alpha+1 \qquad (6.29)
$$

$$
\begin{aligned}
z_{yy} &= c^{-1}\left(1-c^{-1}\right)-\Omega_H\alpha^2\frac{\left[\left(1+\omega\right)\left(1-\overline{H}^2\right)-\omega\lambda\right]}{\left(1-\overline{H}\right)\left[\left(1+\omega\right)\left(1-\overline{H}\right)-\omega\lambda\right]} \\
&\quad+\chi_H\left(1+\chi_H\right)\alpha^2 \qquad (6.30)
\end{aligned}
$$

To the first order, this can be written as a difference equation of the following form

$$
\pi_t=\zeta\left[z_y\widehat{Y}_t-c^{-1}\widehat{G}_t+\omega_\tau\widehat{\tau}_t\right]+\beta E_t\pi_{t+1}. \qquad (6.31)
$$

From this we can infer that the immediate effect of the government spending shock is to raise the natural rate of output by $\frac{c^{-1}}{z_y}\widehat{G}_t$. See Woodford (2003, Chapter 3) on details regarding the natural rate of output.

The fiscal solvency condition

We start with a simple flow government budget constraint

$$B_t = (1 + i_{t-1}) B_{t-1} - P_t s_t \tag{6.32}$$

B_t is the nominal value of public debt in period t and s_t is the $t-$period primary surplus of the government budget ($s_t = T_t - G_t$). Following Benigno and Woodford (2003), this flow budget constraint implies the following sustainability condition

$$\frac{b_{t-1}}{\Pi_t} \left(C_t^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} = E_t \sum_{T=t}^{\infty} \beta^{T-t} s_T \left(C_T^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_T^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \tag{6.33}$$

which requires current outstanding real liabilities to be offset by the discounted sum of future primary surpluses. We have used (2.10) to substitute for the discount factors. We can denote the left-hand side as W_t

$$\begin{aligned} W_t &= \frac{b_{t-1}}{\Pi_t} \left\{ \frac{C_t}{(1-\lambda)} \left[1 - \frac{\omega\lambda}{(1-H_t)(1+\omega)} \right] \right\}^{-\widetilde{\sigma}^{-1}} \\ &\quad * \left[1 - \frac{H_t}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)} \right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \end{aligned} \tag{6.34}$$

(6.33) can be written as a difference equation

$$W_t = s_t \left(C_t^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} + \beta E_t W_{t+1} \tag{6.35}$$

In the steady state, it holds that

$$(1-\beta)\overline{W} = \overline{s} \left(\overline{C}^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - \overline{H}^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \tag{6.36}$$

To the first order, W_t can be approximated as

$$\widehat{W}_t = \widehat{b}_{t-1} - \pi_t + \chi_y \widehat{Y}_t + \widetilde{\sigma}^{-1} c^{-1} \widehat{G}_t + O(2) \tag{6.37}$$

with $\chi_y = \widetilde{\sigma}^{-1}\alpha\Omega_H - \omega\left(1-\widetilde{\sigma}^{-1}\right)\alpha\chi_H - \widetilde{\sigma}^{-1}c^{-1}$. A second-order expansion of the right-hand sides of (6.33) can in turn be expressed as

$$\begin{aligned} s_t\left(C_t^R\right)^{-\widetilde{\sigma}^{-1}}\left(1-H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} &= (1-\beta)\,\overline{W}\left\{1-\left(\widetilde{\sigma}^{-1}c^{-1}-f_H\overline{H}\alpha\right)\widehat{Y}_t+\widetilde{\sigma}^{-1}c^{-1}\widehat{G}_t+\widehat{s}_t\right. \\ &\quad +f_H\overline{H}\widehat{\delta}_t-\left(\widetilde{\sigma}^{-1}c^{-1}-f_H\overline{H}\alpha\right)\widehat{s}_t\widehat{Y}_t+\widetilde{\sigma}^{-1}c^{-1}\widehat{s}_t\widehat{G}_t \\ &\quad -\frac{1}{2}\left[\widetilde{\sigma}^{-1}c^{-1}-\widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2}-f_H\overline{H}\alpha^2\right. \\ &\quad \left.\left.-f_{HH}\overline{H}^2\alpha^2+2\widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha\right]\widehat{Y}_t^2\right. \\ &\quad \left.-\widehat{Y}_t\widehat{G}_t\left[\widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2}-\widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha\right]\right\}+t.i.p.+O\left(3\right) \end{aligned} \tag{6.38}$$

We have used $(1-\beta)\,\overline{W}f_H$ to denote $\frac{\partial\left[(1-\beta)\overline{W}\right]}{\partial\overline{H}}$ and $(1-\beta)\,\overline{W}f_{HH} = \frac{\partial^2\left[(1-\beta)\overline{W}\right]}{\partial\overline{H}^2}$.

In an economy with tax on wage income, the total tax revenues are going to be given by

$$\begin{aligned} T_t &= \tau_t\frac{W_t}{P_t}H_t \\ &= \omega\frac{\tau_t}{1-\tau_t}\frac{H_t}{1-H_t}C_t \end{aligned} \tag{6.39}$$

In the steady state, total tax revenues equal

$$\overline{T} = \omega\omega_\tau\omega_H\overline{C} \tag{6.40}$$

$s_t = T_t - G_t$ can then be approximated by

$$\begin{aligned} \widehat{s}_t &= \frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_t+d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}}+c^{-1}\right)\widehat{Y}_t-\left(d_\tau^{-1}c^{-1}+d^{-1}\right)\widehat{G}_t \\ &\quad +\frac{d_\tau^{-1}}{1-\overline{H}}\widehat{\delta}_t+\frac{1}{2}d_\tau^{-1}\left[c^{-1}+\frac{\left(1+2\omega_H\right)\alpha^2}{1-\overline{H}}+\frac{\alpha c^{-1}}{1-\overline{H}}\right]\widehat{Y}_t^2 \\ &\quad +\frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_t^2+\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}}+c^{-1}\right)\widehat{\tau}_t\widehat{Y}_t \\ &\quad -\frac{d_\tau^{-1}}{1-\overline{\tau}}c^{-1}\widehat{\tau}_t\widehat{G}_t-\frac{d_\tau^{-1}}{1-\overline{H}}\alpha c^{-1}\widehat{Y}_t\widehat{G}_t \\ &\quad +t.i.p.+O(3) \end{aligned} \tag{6.41}$$

We have defined $d_\tau = \frac{\overline{s}}{\overline{\tau}}$ and $d = \frac{\overline{s}}{\overline{Y}}$. Using this in (6.38) yields

$$\begin{aligned} s_t \left(C_t^R\right)^{-\widetilde{\sigma}^{-1}} \left(1 - H_t^R\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} &= (1-\beta)\,\overline{W}\left\{1 + \frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_t \right. \\ &+ \left[f_H \overline{H}\alpha - \widetilde{\sigma}^{-1}c^{-1} + d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\right]\widehat{Y}_t \\ &- \left(d_\tau^{-1}c^{-1} + d^{-1} - \widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_t + \left(\frac{d_\tau^{-1}}{1-\overline{H}} + f_H\overline{H}\right)\widehat{\delta}_t \\ &+ \frac{1}{2}f_{YY}\widehat{Y}_t^2 + \frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_t^2 \\ &+ \frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}} + c^{-1} - \widetilde{\sigma}^{-1}c^{-1} + f_H\overline{H}\alpha\right)\widehat{\tau}_t\widehat{Y}_t \\ &\left. - \frac{d_\tau^{-1}c^{-1}}{1-\overline{\tau}}\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_t\widehat{G}_t - f_{YG}\widehat{Y}_t\widehat{G}_t\right\} \\ &t.i.p. + O\left(3\right) \end{aligned} \tag{6.42}$$

with

$$\begin{aligned} f_{YY} &= d_\tau^{-1}\left[c^{-1} + \frac{\left(1+2\omega_H\right)\alpha^2}{1-\overline{H}} + \frac{\alpha c^{-1}}{1-\overline{H}}\right] - \widetilde{\sigma}^{-1}c^{-1} + \widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2} + f_H\overline{H}\alpha^2 \\ &+ f_{HH}\overline{H}^2\alpha^2 - 2\widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha - 2d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right)\left(\widetilde{\sigma}^{-1}c^{-1} - f_H\overline{H}\alpha\right) \end{aligned} \tag{6.43}$$

$$\begin{aligned} f_{YG} &= \frac{d_\tau^{-1}}{1-\overline{H}}\alpha c^{-1} - \left(\widetilde{\sigma}^{-1}c^{-1} - f_H\overline{H}\alpha\right)\left(d_\tau^{-1}c^{-1} + d^{-1}\right) \\ &- \widetilde{\sigma}^{-1}c^{-1}d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}} + c^{-1}\right) + \widetilde{\sigma}^{-1}\left(\widetilde{\sigma}^{-1}+1\right)c^{-2} - \widetilde{\sigma}^{-1}c^{-1}f_H\overline{H}\alpha \end{aligned} \tag{6.44}$$

Plugging (6.42) back into (6.33), and using (6.24)

$$\begin{aligned}\widehat{W}_t &= (1-\beta)\,E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\left[f_H\overline{H}\alpha-\widetilde{\sigma}^{-1}c^{-1}+d_\tau^{-1}\left(\frac{\alpha}{1-\overline{H}}+c^{-1}\right)\right]\widehat{Y}_T\right.\\ &+\frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_T-\left(d_\tau^{-1}c^{-1}+d^{-1}-\widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_T+\frac{1}{2}f_{YY}\widehat{Y}_T^2\\ &+\frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_T^2+\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}}+c^{-1}-\widetilde{\sigma}^{-1}c^{-1}+f_H\overline{H}\alpha\right)\widehat{\tau}_T\widehat{Y}_T\\ &-\frac{d_\tau^{-1}c^{-1}}{1-\overline{\tau}}\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T-f_{YG}\widehat{Y}_T\widehat{G}_T\\ &\left.\frac{1}{2}\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)\frac{\theta\left(1+\omega_p\right)}{\zeta}\pi_T^2\right\}+t.i.p.+O\left(3\right)\end{aligned}\tag{6.45}$$

With (6.37) on the left-hand side, this can be written with first-order accuracy as

$$\begin{aligned}\widehat{b}_{t-1}-\pi_t+\chi_y\widehat{Y}_t+\widetilde{\sigma}^{-1}c^{-1}\widehat{G}_t &= (1-\beta)\left[f_y\widehat{Y}_t+f\tau\widehat{\tau}_t\right.\\ &\left.-\left(d_\tau^{-1}c^{-1}+d^{-1}-\widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_t\right]\\ &+\beta E_t\left[\widehat{b}_t-\pi_{t+1}+\chi_y\widehat{Y}_{t+1}+\widetilde{\sigma}^{-1}c^{-1}\widehat{G}_{t+1}\right]\end{aligned}\tag{6.46}$$

The definitions of f_y and $f\tau$ follow from (6.45).

Approximation to utility

We have derived in the text that the agent's utility function can be written in terms of aggregate variables as follows

$$\begin{aligned}u_t &= \frac{C_t^{1-\widetilde{\sigma}^{-1}}}{1-\widetilde{\sigma}^{-1}}\left\{\lambda\left(1-H_t\right)^{\widetilde{\sigma}^{-1}-1}\left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)}\right.\\ &\left.+\left(1-\lambda\right)^{\widetilde{\sigma}^{-1}}\left[1-\frac{\omega\lambda}{\left(1-H_t\right)\left(1+\omega\right)}\right]^{1-\widetilde{\sigma}^{-1}}\left[1-\frac{H_t}{1-\lambda}+\frac{\lambda}{\left(1-\lambda\right)\left(1+\omega\right)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}\right\}\end{aligned}\tag{6.47}$$

The second order approximation to the discounted sum of expected utilities given by this expression can be written as

$$\begin{aligned} U_t &= \overline{C}^{1-\widetilde{\sigma}^{-1}} E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \left(Kc^{-1} + L\overline{H}\alpha \right) \widehat{Y}_T + \right. \\ &+ \frac{1}{2} \left[Kc^{-1} \left(1 - \widetilde{\sigma}^{-1} c^{-1} \right) + L\overline{H}\alpha \left(\alpha + 2 \left(1 - \widetilde{\sigma}^{-1} \right) c^{-1} \right) + M\overline{H}^2 \alpha^2 \right] \widehat{Y}_T^2 \\ &\left. + \left[K\widetilde{\sigma}^{-1} c^{-2} - L\overline{H}\alpha c^{-1} \left(1 - \widetilde{\sigma}^{-1} \right) \right] \widehat{Y}_T \widehat{G}_T + \frac{1}{2} L\overline{H} \frac{\theta \left(1 + \omega_p \right)}{\zeta} \pi_T^2 \right\} \\ &+ t.i.p. + O\left(3 \right) \end{aligned} \tag{6.48}$$

where

$$\begin{aligned} K &= \lambda \left(1 - \overline{H} \right)^{\widetilde{\sigma}^{-1} - 1} \left(\frac{\omega}{1 + \omega} \right)^{(1+\omega)\left(1 - \widetilde{\sigma}^{-1}\right)} + \\ &(1 - \lambda)^{\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{\left(1 - \overline{H} \right) (1 + \omega)} \right]^{1 - \widetilde{\sigma}^{-1}} \left[1 - \frac{\overline{H}}{1 - \lambda} + \frac{\lambda}{(1 - \lambda)(1 + \omega)} \right]^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)} \end{aligned} \tag{6.49}$$

$$\begin{aligned} L &= \lambda \left(1 - \overline{H} \right)^{\widetilde{\sigma}^{-1} - 2} \left(\frac{\omega}{1 + \omega} \right)^{(1+\omega)\left(1 - \widetilde{\sigma}^{-1}\right)} \\ &- \left[1 - \frac{\overline{H}}{1 - \lambda} + \frac{\lambda}{(1 - \lambda)(1 + \omega)} \right]^{\omega\left(1 - \widetilde{\sigma}^{-1}\right)} \left[1 - \frac{\omega\lambda}{\left(1 - \overline{H} \right)(1 + \omega)} \right]^{-\widetilde{\sigma}^{-1}} * \\ &* \left\{ \frac{\lambda\omega \left(1 - \lambda \right)^{\widetilde{\sigma}^{-1}}}{(1 + \omega)\left(1 - \overline{H} \right)^2} + \frac{\omega \left(1 - \lambda \right)^{\widetilde{\sigma}^{-1}} \left[(1 + \omega)\left(1 - \overline{H} \right) - \omega\lambda \right]}{\left(1 - \overline{H} \right) \left[\left(1 - \lambda - \overline{H} \right)(1 + \omega) + \lambda \right]} \right\} \end{aligned} \tag{6.50}$$

$$
\begin{aligned}
M &= \lambda\left(\frac{\omega}{1+\omega}\right)^{(1+\omega)\left(1-\widetilde{\sigma}^{-1}\right)}\left(2-\widetilde{\sigma}^{-1}\right)\left(1-\overline{H}\right)^{\widetilde{\sigma}^{-1}-3} \\
&+\left[1-\frac{\overline{H}}{1-\lambda}+\frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}\left[1-\frac{\omega\lambda}{\left(1-\overline{H}\right)(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}}\frac{\omega(1-\lambda)^{\widetilde{\sigma}^{-1}}}{\left(1-\overline{H}\right)}* \\
&*\left\{\frac{2\omega\lambda\left(1-\widetilde{\sigma}^{-1}\right)}{\left(1-\overline{H}\right)\left[\left(1-\lambda-\overline{H}\right)(1+\omega)+\lambda\right]}-\frac{2\lambda}{(1+\omega)\left(1-\overline{H}\right)^{2}}\right. \\
&-\frac{\omega\widetilde{\sigma}^{-1}\lambda^{2}}{(1+\omega)\left(1-\overline{H}\right)^{2}\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]} \\
&\left.+\frac{\left[\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right](1+\omega)\left[(1+\omega)\left(1-\overline{H}\right)-\omega\lambda\right]}{\left[\left(1-\lambda-\overline{H}\right)(1+\omega)+\lambda\right]^{2}}\right\}
\end{aligned}
\tag{6.51}
$$

The policy objective function

We have derived above that a second-order approximation to the utility function can be written as

$$
\begin{aligned}
U_t &= \overline{C}^{1-\widetilde{\sigma}^{-1}}E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\Theta_Y\widehat{Y}_T+\right. \\
&+\frac{1}{2}\left[Kc^{-1}\left(1-\widetilde{\sigma}^{-1}c^{-1}\right)+L\overline{H}\alpha\left(\alpha+2\left(1-\widetilde{\sigma}^{-1}\right)c^{-1}\right)+M\overline{H}^{2}\alpha^{2}\right]\widehat{Y}_T^{2} \\
&\left.+\left[K\widetilde{\sigma}^{-1}c^{-2}-L\overline{H}\alpha c^{-1}\left(1-\widetilde{\sigma}^{-1}\right)\right]\widehat{Y}_T\widehat{G}_T+\frac{1}{2}L\overline{H}\frac{\theta(1+\omega_p)}{\zeta}\pi_T^{2}\right\} \\
&+t.i.p.+O(3)
\end{aligned}
\tag{6.52}
$$

where $\Theta_Y=\left(Kc^{-1}+L\overline{H}\alpha\right)$. The approximation contains a linear term in one of the endogenous variables, which means that policies aiming at stabilization of deviations of fundamentals from their steady-state values not only affect the variation of the variables but also have significant level effects. However, as explained in Sutherland (2002), (6.52) will not serve as a correct, second-order-accurate welfare-ranking criterion for optimal policies obtained as a solution to a system of linear structural equations (derived above). To obtain a welfare function with second-order terms, one has to use second-order approximations to structural equations to substitute out the linear term from (6.52). This way, a correct welfare-ranking criterion is obtained, while the level-effects are preserved in an implicit form.

The second-order approximations to the fiscal solvency condition and the aggregate supply relationship can be written as

$$\begin{aligned}\frac{\widehat{W}_t}{(1-\beta)} &= E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{f_y\widehat{Y}_T+\frac{d_\tau^{-1}}{1-\overline{\tau}}\widehat{\tau}_T-\left(d_\tau^{-1}c^{-1}+d^{-1}-\widetilde{\sigma}^{-1}c^{-1}\right)\widehat{G}_T\right.\\ &+\frac{1}{2}f_{YY}\widehat{Y}_T^2+\frac{1}{2}\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(1+2\omega_\tau\right)\widehat{\tau}_T^2\\ &+\frac{d_\tau^{-1}}{1-\overline{\tau}}\left(\frac{\alpha}{1-\overline{H}}+c^{-1}-\widetilde{\sigma}^{-1}c^{-1}+f_H\overline{H}\alpha\right)\widehat{\tau}_T\widehat{Y}_T\\ &-\frac{d_\tau^{-1}c^{-1}}{1-\overline{\tau}}\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T-f_{YG}\widehat{Y}_T\widehat{G}_T\\ &\left.\frac{1}{2}\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)\frac{\theta\left(1+\omega_p\right)}{\zeta}\pi_T^2\right\}+t.i.p.+O\left(3\right)\end{aligned}\tag{6.53}$$

$$\begin{aligned}\frac{V_t}{\zeta} &= E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{z_y\widehat{Y}_T-c^{-1}\widehat{G}_T+\omega_\tau\widehat{\tau}_T\right.\\ &+\frac{1}{2}\left(z_{yy}+z_yx_y\right)\widehat{Y}_T^2+\frac{1}{2}\omega_\tau\left(\frac{1+\overline{\tau}}{1-\overline{\tau}}\right)\widehat{\tau}_T^2\\ &+\omega_\tau\left\{\left(1-\widetilde{\sigma}^{-1}\right)\left(c^{-1}-\alpha\Omega_H\right)+\alpha-\alpha\chi_H\left[\omega\left(1-\widetilde{\sigma}^{-1}\right)-1\right]\right\}\widehat{\tau}_T\widehat{Y}_T\\ &+c^{-1}\left[c^{-1}-\left(1-2\widetilde{\sigma}^{-1}\right)\left(c^{-1}-\alpha\Omega_H\right)-\alpha\left(1-\widetilde{\sigma}^{-1}\right)\left[\chi_H\left(1-\omega\right)+1\right]-\widetilde{\sigma}^{-1}\right]\widehat{Y}_T\widehat{G}_T\\ &\left.-c^{-1}\omega_\tau\left(1-\widetilde{\sigma}^{-1}\right)\widehat{\tau}_T\widehat{G}_T+\frac{1}{2}\frac{\theta\left(1+\omega_p\right)}{\zeta}\left(1+\chi_H-\Omega_H\right)\pi_T^2\right\}\\ &+t.i.p.+O\left(3\right)\end{aligned}\tag{6.54}$$

The task is to find a linear combination of these two equations such that contains coefficients at linear terms of endogenous variables equal to those in (6.52), which will enable us to substitute out the linear term in (6.52). We are therefore looking for a vector of coefficients $\mathbf{v}'=(v_1,v_2)$ such that satisfy the following system of equations

$$\begin{pmatrix} f_y & z_y \\ \frac{d_\tau^{-1}}{1-\overline{\tau}} & \omega_\tau \end{pmatrix}\mathbf{v}=\begin{pmatrix}\Theta_Y\\0\end{pmatrix}$$

We find that

$$\mathbf{v} = \begin{pmatrix} -\frac{\Theta_Y}{\Omega}\omega_\tau\left(1-\overline{\tau}\right) \\ \frac{\Theta_Y}{\Omega}d_\tau^{-1} \end{pmatrix} \tag{6.55}$$

$\Omega = z_y d_\tau^{-1} - \omega_\tau\left(1-\overline{\tau}\right) f_y$. It therefore holds that

$$\begin{aligned} E_t\sum_{T=t}^{\infty}\beta^{T-t}\Theta_Y\widehat{Y}_T &= E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\mathbf{v}'\boldsymbol{\xi}_Y\widehat{Y}_T + \mathbf{v}'\boldsymbol{\xi}_\tau\widehat{\tau}_{L,T}\right\} \\ &= -E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\frac{1}{2}\left[v_1 f_{YY} + v_2\left(z_{yy}+z_y x_y\right)\right]\widehat{Y}_T^2\right. \\ &\quad -\left(v_1 f_{YG} - v_2 x_{YG}\right)\widehat{Y}_T\widehat{G}_T \\ &\quad \left.+\frac{1}{2}\frac{\theta\left(1+\omega_p\right)}{\zeta}\left[v_1\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)+v_2\left(1+\chi_H-\Omega_H\right)\right]\right\}\pi_T^2 \\ &\quad +v_1\frac{\widehat{W}_t}{(1-\beta)}+v_2\frac{V_t}{\zeta}+t.i.p.+O\left(3\right) \end{aligned} \tag{6.56}$$

where $\boldsymbol{\xi}_Y' = (f_y, z_y)$ and $\boldsymbol{\xi}_\tau' = \left(\frac{d_\tau^{-1}}{1-\overline{\tau}}, \omega_\tau\right)$. The definition of x_{YG} follows from the (6.54). We can now substitute this into (6.52) to obtain

$$\begin{aligned} U_t &= -\overline{C}^{1-\widetilde{\sigma}^{-1}}E_t\sum_{T=t}^{\infty}\beta^{T-t}\left\{\frac{1}{2}q_y\widehat{Y}_T^2 - q_{YG}\widehat{Y}_T\widehat{G}_T + \frac{1}{2}q_\pi\pi_T^2\right\} \\ &\quad +v_1\overline{C}^{1-\widetilde{\sigma}^{-1}}\frac{\widehat{W}_t}{(1-\beta)}+v_2\overline{C}^{1-\widetilde{\sigma}^{-1}}\frac{V_t}{\zeta}+t.i.p.+O\left(3\right) \end{aligned} \tag{6.57}$$

The coefficients are defined as follows

$$\begin{aligned} q_y &= \frac{\Theta_Y}{\Omega}d_\tau^{-1}\left(z_{yy}+z_y x_y\right)-\frac{\Theta_Y}{\Omega}\overline{\tau}f_{YY}-Kc^{-1}\left(1-\widetilde{\sigma}^{-1}c^{-1}\right) \\ &\quad -L\overline{H}\alpha\left(\alpha+2\left(1-\widetilde{\sigma}^{-1}\right)c^{-1}\right)-M\overline{H}^2\alpha^2 \end{aligned} \tag{6.58}$$

$$q_{YG} = K\widetilde{\sigma}^{-1}c^{-2}-L\overline{H}\alpha c^{-1}\left(1-\widetilde{\sigma}^{-1}\right)-\frac{\Theta_Y}{\Omega}\overline{\tau}f_{YG}-\frac{\Theta_Y}{\Omega}d_\tau^{-1}x_{YG} \tag{6.59}$$

$$q_\pi = \frac{\theta\left(1+\omega_p\right)}{\zeta}\left[\frac{\Theta_Y}{\Omega}\left(1-\overline{\tau}\right)\left(\frac{d_\tau^{-1}}{1-\overline{H}}+f_H\overline{H}\right)-L\overline{H}\right] \tag{6.60}$$

(6.57) can also be expressed as

$$\begin{aligned} U_t &= -\overline{C}^{1-\widetilde{\sigma}^{-1}} E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \frac{1}{2} q_y \left(\widehat{Y}_T - \widehat{Y}_T^* \right)^2 + \frac{1}{2} q_\pi \pi_T^2 \right\} \\ &\quad + v_1 \overline{C}^{1-\widetilde{\sigma}^{-1}} \frac{\widehat{W}_t}{(1-\beta)} + v_2 \overline{C}^{1-\widetilde{\sigma}^{-1}} \frac{V_t}{\zeta} + t.i.p. + O(3) \end{aligned} \tag{6.61}$$

where we have defined

$$\widehat{Y}_T^* = \frac{q_{YG}}{q_y} \widehat{G}_T \tag{6.62}$$

$\widehat{Y}_T^*$ represents the efficient deviation of output from its steady state. $\widehat{Y}_T - \widehat{Y}_T^*$ is then the difference between the actual and what we refer to as target deviation, which in the context of our model corresponds to the welfare-relevant output gap.[15] The linear terms in the second line represent the welfare effect of the policy in the initial period t. This is a transitory component. Benigno and Woodford (2003) explain why this component can be treated as predetermined in what they call the second stage of the Ramsey problem.[16].

We can now re-write all our structural equations in terms of the welfare-relevant output gap. The price aggregate supply relationship (6.31) becomes

$$\pi_t = \kappa y_t + \chi_\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) + \beta E_t \pi_{t+1} \tag{6.63}$$

$y_t = \widehat{Y}_T - \widehat{Y}_T^*$, $\kappa = \zeta z_y$ and $\chi_\tau = \zeta \omega_\tau$. $\widehat{\tau}_t^* = \frac{z_y}{\omega_\tau} \left(\frac{c^{-1}}{z_y} - \frac{q_{YG}}{q_y} \right) \widehat{G}_t$ is the deviation in the tax rate that exactly offsets any cost-push (or cost-pull shock) induced by the shock to government expenditures. We see that it is proportional to the difference between the target deviation in output and the spending-induced shift in the natural rate of output.

The fiscal solvency condition becomes

$$\begin{aligned} \widehat{b}_{t-1} - \pi_t + \chi_y y_t + \varphi_t &= (1-\beta) \left[f_y y_t + f\tau \left(\widehat{\tau}_t - \widehat{\tau}_t^* \right) \right] \\ &\quad + \beta E_t \left[\widehat{b}_t - \pi_{t+1} + \chi_y y_{t+1} + \varphi_{t+1} \right] \end{aligned} \tag{6.64}$$

[15] For detailed discussion of what one should consider as the welfare-relevant output gap under various assumptions about the size of the distortions to steady state see Woodford (2003, Chapter 6).

[16] See also discussion in Woodford (2003, Chapter 6).

$$\begin{aligned}\varphi_t &= \left(\chi_y \frac{q_{YG}}{q_y} + \widetilde{\sigma}^{-1} c^{-1}\right) \widehat{G}_t \\ &\quad + (1-\beta)\, E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ d_\tau^{-1} c^{-1} + d^{-1} - \widetilde{\sigma}^{-1} c^{-1} \right. \\ &\quad \left. - f_y \frac{q_{YG}}{q_y} - \frac{f\tau}{\omega_\tau} \left(c^{-1} - z_y \frac{q_{YG}}{q_y} \right) \right\} \widehat{G}_T \end{aligned} \tag{6.65}$$

is the composite measure of "fiscal stress" introduced in Benigno and Woodford (2003).

The IS relationship

The relationship defining the stochastic discount factor (2.10) can be written in terms of the aggregate variables as follows

$$\beta \frac{E_t \left(C_{t+1}\right)^{-\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{(1-H_{t+1})(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}} \left[1 - \frac{H_{t+1}}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}}{C_t^{-\widetilde{\sigma}^{-1}} \left[1 - \frac{\omega\lambda}{(1-H_t)(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}} \left[1 - \frac{H_t}{1-\lambda} + \frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}} \frac{(1+i_t)}{(1+E_t\pi_{t+1})} = 1 \tag{6.66}$$

A straightforward log-linearization of this relationship yields

$$\Phi^{-1} \widehat{Y}_t = \Phi^{-1} E_t \widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t \pi_{p,t+1}\right) - \widetilde{\sigma}^{-1} c^{-1} \left(E_t \widehat{G}_{t+1} - \widehat{G}_t\right) \tag{6.67}$$

in this we have assumed that $\widehat{\delta}_{-1} = 0$ or equivalently, that t is large enough so that $\gamma^t \widehat{\delta}_{-1}$ is insignificantly different from 0. Φ^{-1} will now be defined as

$$\Phi^{-1} = \widetilde{\sigma}^{-1} c^{-1} - \widetilde{\sigma}^{-1} \alpha \Omega_H + \omega\alpha \left(1 - \widetilde{\sigma}^{-1}\right) \chi_H \tag{6.68}$$

Rewriting (6.67) in terms of the welfare-relevant output gap, we obtain

$$y_t = E_t y_{t+1} - \Phi \left(\widehat{i}_t - E_t \pi_{p,t+1}\right) + \left(\frac{q_{YG}}{q_y} - \Phi \widetilde{\sigma}^{-1} c^{-1}\right) \left(E_t \widehat{G}_{t+1} - \widehat{G}_t\right) \tag{6.69}$$

Assuming no persistence in the shock, we obtain the equation in the text with $\widehat{r}_t^* = \left(\widetilde{\sigma}^{-1} c^{-1} - \Phi^{-1} \frac{q_{YG}}{q_y}\right) \widehat{G}_t$.

The steady state

In the steady state, prices are stable and hence there is no dispersion. (6.2) becomes

$$1 = \frac{\mu\alpha\omega\overline{C}^{R}\overline{Y}^{\alpha-1}}{(1-\overline{\tau})\left(1-\overline{H}^{R}\right)}. \tag{6.70}$$

which can also be written in terms of aggregate variables as follows

$$1 = \frac{\mu\alpha\omega c\left[1-\frac{\omega\lambda}{\left(1-\overline{Y}^{\alpha}\right)(1+\omega)}\right]\overline{Y}^{\alpha}}{(1-\lambda)(1-\overline{\tau})\left[1-\frac{\overline{Y}^{\alpha}}{1-\lambda}+\frac{\lambda}{(1-\lambda)(1+\omega)}\right]} \tag{6.71}$$

where we have used $c = \frac{\overline{C}}{\overline{Y}}$ and $\overline{H} = \overline{Y}^{\alpha}$. This relationship defines the steady-state level of output. It is easy to show that this relationship yields two solutions for the steady-state output, one of which represents a special case with the Ricardian agents consuming no leisure ($\overline{H}^{R} = 1$) so that $\overline{H} = \frac{1+\omega-\omega\lambda}{1+\omega}$. This implies a corner solution case, a case of zero consumption for Ricardians in the steady state. A positive deviation from this steady state then implies an infinite increase in utility for Ricardian agents and for the whole economy too. We therefore concentrate on the other solution which can be written as

$$\overline{Y} = \left[\frac{-b_Y - \sqrt{b_Y^2 - 4a_Y c_Y}}{2a_Y}\right]^{\frac{1}{\alpha}} \tag{6.72}$$

where

$$a_Y = (1+\omega)(1-\overline{\tau}+\mu\alpha\omega c)$$
$$b_Y = \mu\alpha\omega c(\omega\lambda-1-\omega)-(1-\overline{\tau})(1+\omega)(2-\lambda)-(1-\overline{\tau})\lambda$$
$$c_Y = (1-\overline{\tau})[(1-\lambda)(1+\omega)+\lambda]$$

(6.72) can be shown to be equal to

$$\overline{Y} = \left[\frac{1-\overline{\tau}}{\mu\alpha\omega c+1-\overline{\tau}}\right]^{\frac{1}{\alpha}}$$

which is independent of λ.

Next, (2.10) implies that in the steady state

$$\overline{R} = \overline{I} = \frac{1}{\beta} \tag{6.73}$$

$\overline{R} = (1 + \overline{r})$ and $\overline{I} = \left(1 + \bar{i}\right)$, $\overline{r}$ and $\bar{i}$ are real and nominal interest rates, respectively. Finally, (6.35) implies

$$\begin{aligned}
(1-\beta)\,\overline{W} &= \overline{s}\left(\overline{C}^{R}\right)^{-\widetilde{\sigma}^{-1}}\left(1-\overline{H}^{R}\right)^{\omega\left(1-\widetilde{\sigma}^{-1}\right)} \\
&= \overline{s}\left(\frac{\overline{C}}{1-\lambda}\right)^{-\widetilde{\sigma}^{-1}}\left[1-\frac{\omega\lambda}{\left(1-\overline{H}\right)(1+\omega)}\right]^{-\widetilde{\sigma}^{-1}}\left[1-\frac{\overline{H}}{1-\lambda}+\frac{\lambda}{(1-\lambda)(1+\omega)}\right]^{\omega\left(1-\widetilde{\sigma}^{-1}\right)}
\end{aligned}$$

which, given the definition of $\overline{W}$, gives us

$$\overline{s} = (1-\beta)\,\overline{b} \tag{6.74}$$

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