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Think Globally, Act Locally? Stock vs Flow Regulation of a Fossil Fuel

Jean-Pierre Amigues
Toulouse School of Economics

Ujjayant Chakravorty
University of Alberta

Michel Moreaux
Toulouse School of Economics

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Think Globally, Act Locally?
Stock vs Flow Regulation of a Fossil Fuel

by

Jean-Pierre Amigues, Ujjayant Chakravorty and Michel Moreaux[1]

Abstract

Regulation of environmental externalities like global warming from the burning of fossil fuels (e.g., coal and oil) is often done by capping both emission flows and stocks. For example, the European Union and states in the Northeastern United States have introduced caps on flows of carbon emissions while the stated goal of the Intergovernmental Panel on Climate Change (IPCC) which provides the science behind the current global climate negotiations is to stabilize the atmospheric stock of carbon. Flow regulation is often local or regional in nature, while stock regulation is global. How do these multiple pollution control efforts interact when a nonrenewable resource creates pollution? In this paper we show that local and global pollution control efforts, if uncoordinated, may exacerbate environmental externalities. For example, a stricter cap on emission flows may actually increase the global pollution stock and hasten the date when the global pollution cap is reached.

Keywords Dynamics, Environmental Regulation, Externalities, Nonrenewable Resources, Pollution

JEL Codes: Q12, Q32, Q41

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[1] Respectively, Toulouse School of Economics (INRA, IDEI and LERNA); University of Alberta, and Toulouse School of Economics (IUF, IDEI and LERNA). Correspondence: ujjayant@ualberta.ca

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1. **Introduction**

Environmental problems such as global warming are being addressed both globally through mechanisms such as the Kyoto Protocol and locally as in the United States and the European Union where many states and member countries have developed their own clean carbon policies and targets. For example in the US, at last count 29 states have implemented local measures that aim to reduce greenhouse gas emissions from electric utilities and trucks, buses and sport utility vehicles, as part of the Regional Greenhouse Gas Initiative (RGGI) and the Western Climate Initiative that includes several states such as California and Arizona as well as provinces from Canada. The European Union has implemented carbon emission caps under the European Trading System (ETS) which has been operational for some time.

On the other hand, a stated goal of the Intergovernmental Panel on Climate Change (IPPC) which provides the science behind the international negotiations on climate change is a stabilization of pollution concentration (for the IPCC's atmospheric stabilization goals, see IPCC (2001).[2] NASA Chief Climate Scientist James Hansen writes

"If humanity wishes to preserve a planet similar to that on which civilization developed and to which life on Earth is adapted, paleoclimatic evidence and ongoing climate change suggest that CO_2 will need to be reduced from its current 385 ppm to at most 350 ppm" (Hansen et al., 2008, pp. 1).

Since fossil fuels account for 75% of global emissions (rest is mainly deforestation), this target may be assumed to be a direct cap on the stock of carbon from the burning of fossil fuels. Thus we may have situations where the same externality is being subject to different regulatory policies at the same time.[3] In a recent policy paper on the architecture of global climate policy,

[2] although the actual agreement is subject to political considerations (some countries may be exempt from targets for a period) and may be cast in terms of emissions limits mainly to begin a period during which the necessary institutional mechanisms are put in place.

[3] Sometimes, the same source creates multiple externalities at the local and global level. For example, the burning of coal produces both sulphur and carbon. The flow of sulphur creates environmental externalities such as acid rain, and

Jeffrey Frankel (2009) suggests that one way to get developing countries to sign in is to have developed economies follow emission reduction targets for a period followed by global mandates for all countries. That is, a period of local regulation in regions such as the EU and North America, followed by global regulation.

The analogous issue of central and local regulation has been studied for transboundary pollution by List and Mason (2001) in a game theoretic context. However they do not consider a nonrenewable resource. In this paper, we model this local-global problem for a nonrenewable resource which creates an environmental externality – imagine the burning of coal to produce electricity. This externality is regulated by a social planner both globally by means of a stock constraint and locally through a flow constraint. We investigate the joint effects of these two regulatory instruments in a dynamic setting. The question we ask is: what is the impact of local and global regulation on the use of a polluting resource? Can action at one level mitigate the need for action at the other level? Under what conditions might we want an externality to be regulated both locally and globally or through only one of these two means?

These questions are important to ask because multiple regulatory instruments may be implemented in an uncoordinated fashion.[4] Furthermore these regulations have different costs associated with them. It may be less costly to implement local pollution standards in a state then develop global standards involving many countries, as we have seen from the protracted negotiations following the Kyoto Protocol.

An important methodological goal of the paper is to develop a model that can integrate strands of the literature on nonrenewable resources in which both stock (Chakravorty, Magne and Moreaux, 2006) and flow constraints (such as in Amigues et al (1998), Smulders and Van der Werf (2006)) have been used. We examine how these two types of caps jointly affect resource extraction and

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also adds to the stock of carbon in the atmosphere. Yang (2007) addresses the management of short-lived local externalities like sulphur and long-lived global externalities such as greenhouse gases.

[4] During the regime of President George W. Bush, while the US was cool towards negotiating an international climate treaty, several states went ahead unilaterally to start an emissions cap and trade program in the North-East. This program is ongoing and has resulted in significant emission reductions as well as generating modest revenues from the auctioning of carbon permits. It is not clear whether and how these regional efforts will ultimately be coordinated with any international action that may emerge in the near future.

under what conditions one of these instruments may work better than the other for managing externalities.

We show that while local regulation reduces the use of a polluting resource, it may delay the arrival of a clean substitute. On the other hand, flow regulation may make stock regulation redundant, i.e., the stock of pollution may remain under the globally mandated cap, especially if the endowment of the polluting resource is relatively small. When the reserves of the polluting resource (such as coal) are large, local regulation kicks in at the beginning of the time horizon and global regulation is imposed only later in time.[5] In general, local regulation when binding, helps slow the growth of global emissions and may delay the time when global regulation becomes binding. However once global regulation is in place, there is no role for local regulation anymore. The latter becomes redundant. This is because scarcity causes fossil fuel prices to rise, so the extraction rate and hence the flow of emissions declines over time. Once the flow is no longer binding, it does not bind in the future. From a policy point of view, these results suggest that it may be important to develop local instruments relatively earlier in the planning horizon, which in turn may provide policy makers the much needed breathing space to organize an agreement on global regulation of the pollution stock.

A counter-intuitive result from our analysis is that tightening regulation at one level may exacerbate pollution problems at another level. For example, if the regulator chooses a stricter global cap on the stock, that may reduce fossil fuel prices and increase pollution flows over some other time period. Moreover, if the atmospheric dilution of pollution is significant, then the rise in emissions may lead to a faster arrival at the global cap. A general result is that tightening the cap will lead to a longer duration at the cap and a delay in the complete transition to the clean substitute. These results suggest that pollution regulation at the local and global levels may need to be coordinated. In the absence of coordination, a policy change at one level may worsen pollution problems at another level, because of the dynamic effects of the policy change.

Section 2 develops the extended Hotelling model with local and global regulation. Section 3 discusses the effects of the two types of regulation on the use of the polluting resource and arrival

[5] In this case, Frankel's proposal may be efficient from an economic perspective.

of the clean substitute. Section 4 examines the effect of tightening the local and global regulation on the equilibrium stock of pollution and resource prices. Section 5 concludes the paper.

2. The Model

We adopt the standard Hotelling model in which a social planner derives utility from using a polluting nonrenewable resource. Let us call it coal. The utility function is given by $u(q)$ where q is the flow of energy at time t. This function is assumed strictly increasing and concave and satisfies the Inada condition $\lim_{q\downarrow 0} u'(q) = +\infty$. Let p denote the function $u'(q)$ and let the corresponding demand function, the inverse of $u'(q)$, be denoted by $D(p)$. Under the above assumptions, $D(p)$ is well defined over some (p_0, ∞) where $p_0 = \lim_{q\uparrow\infty} u'(q), \quad p_0 \geq 0$.

The initial stock of the nonrenewable resource is assumed known and is denoted by X_0. Let $X(t)$ be the residual stock of coal at time t and $x(t)$ be the instantaneous extraction rate. Then

$$\dot{X}(t) = -x(t). \qquad (1)$$

Let the extraction and processing cost of coal be a constant c_x. Let $z(t)$ be the emission from burning coal, and we assume that one unit of coal leads to b pollution units, that is $z(t) = bx(t)$. However, b does not really play any significant role in the subsequent analysis, so we equate it to unity by appropriate choice of units.[6] Then $z(t) = x(t)$. Each unit of coal creates one unit of pollution. Pollution emissions can thus be written in resource units.

Pollution flow is subject to a local cap $x(t) < \underline{x}$. In Appendix A, we show that multiple homogenous regions with emission caps can be aggregated into a single region. We interpret the emissions cap as 'local' regulation. However, this policy need not be strictly local. Any policy that limits emissions will suffice, such as the regulation of a pollutant (e.g., sulphur) whose emissions are perfectly correlated with the burning of the fossil fuel. Then the regulated level of coal use is given by $\underline{x}$ and we can define the corresponding price as $\underline{p} = u'(\underline{x})$. If $\underline{p}$ were lower

[6] The complete analysis for $b > 1$ can be obtained by writing to the authors.

than c_x then no coal will be burned since the price will be below the cost of extraction. The flow of emissions will not exceed the regulated level.

Apart from local externalities, the burning of coal leads to the build-up of the stock of carbon leading to global pollution, such as from warming of the planet. For simplicity, we assume that these global damages can only be regulated by some central authority. This may be a federal agency such as the Environmental Protection Agency (EPA) for domestic pollutants or a world body such as the IPCC that regulates global pollution stocks. Again, there is nothing sacred about this assumption but it helps to distinguish between the two policy instruments in our model. Define the stock of pollution at time t from coal burning by $Z(t)$. Then the build-up of pollution is given by

$$\dot{Z}(t) = x - \alpha Z, \tag{2}$$

where Z_0 is the initial stock of pollution, which is exogenously given. For simplicity, instead of assuming an explicit damage function for the global externality, we assume that damages are negligible when the stock is below some regulated level $\bar{Z}$ and infinite beyond. That is, $Z(t) \le \bar{Z}$ and $Z_0 < \bar{Z}$.[7] We can compute the maximum quantity of coal that can be used while satisfying the global constraint, derived from (1) as $\bar{x} \equiv \alpha \bar{Z}$. Define the corresponding price of energy as $\bar{p} = u'(\bar{x})$. We must have $c_x < \bar{p}$ in order to burn coal before the global cap is achieved. Note that $\bar{p} > \underline{p}$ iff $\bar{x} = \alpha \bar{Z} < \underline{x}$.

Let there be a backstop resource which is pollution free. Let $y(t)$ denote consumption of this resource and c_y its unit cost of production. For convenience we will call this clean resource solar energy. We make the usual assumption that the unit cost of extraction of coal is lower than the cost of using solar energy, i.e., $c_x < c_y$. These resources are perfect substitutes so that total

[7] An explicit damage function will introduce complications in the analysis, and its second and third derivative will play a critical role.

energy consumption at any time is given by $q(t) = x(t) + y(t)$. Let $\bar{y}$ be the given available flow of solar energy. Then we assume that $\bar{y} > D(c_y)$, i.e., there is sufficient solar energy to supply the entire demand at its unit cost. Under these assumptions, there is no scarcity rent in the solar industry.

Given a discount rate $r > 0$, the social planner chooses the energy mix by maximizing the discounted net surplus subject to the local and global caps, and solves

$$\underset{x,y}{Max} \int_0^{\infty} [u(x(t) + y(t)) - c_x x(t) - c_y y(t)] e^{-rt} dt \tag{3}$$

subject to (1) and (2). For convenience we use $-\mu$ as the multiplier function attached to (2) so that the value of μ is positive. The Lagrangian may then be written as

$$L = u(x+y) - (c_x x + c_y y) - \lambda x - \mu(x - \alpha Z) + \beta(\underline{x} - x) + \nu(\bar{Z} - Z) + \gamma_x x + \gamma_y y,$$

which yields the necessary conditions

$$u'(x+y) = c_x + \lambda + \mu + \beta - \gamma_x \tag{4}$$

$$u'(x+y) = c_y - \gamma_y \tag{5}$$

together with the complementary slackness conditions:

$$\begin{aligned} &\beta(\underline{x} - x) = 0,\ \underline{x} - x \geq 0,\ \gamma \geq 0 \\ &\gamma_x x = 0,\ x \geq 0,\ \gamma_x \geq 0 \\ &\gamma_y y = 0,\ y \geq 0,\ \gamma_y \geq 0. \end{aligned} \tag{6}$$

The dynamics of the costate variables is given by

$$\dot{\lambda} = r\lambda \tag{7}$$

$$\dot{\mu} = (\alpha + r)\mu - \nu \text{ and} \tag{8}$$

$$\nu(\bar{Z} - Z) = 0, \bar{Z} - Z \geq 0, \nu \geq 0. \tag{9}$$

Finally the transversality conditions are $\lim_{t\to\infty} \lambda(t)X(t) = 0$ and $\lim_{t\to\infty} \mu(t)Z(t) = 0$. The variable $\lambda(t)$ is the scarcity rent of the nonrenewable resource and $\mu(t)$ which is positive represents the tax on a unit of pollution as a result of the global cap. Note that we also have β which is the cost of the local cap – thus we have two types of taxes in this model. Both taxes are on per unit of pollution - μ is the tax per unit of emissions as a result of the global cap, and β is the tax from the local cap. If the global stock constraint is slack over any period of time $t \in [t_0, t_1]$ then $\nu(t) = 0$ so that $\mu(t) = \mu(t_0)e^{(\alpha+r)(t-t_0)}$ during that time period. If $Z_0 < \bar{Z}$, that is, if the initial stock of pollution is strictly lower than the regulated cap, then $\mu(t)$ must increase exponentially over an initial time period until the global ceiling is achieved. Finally, if the global ceiling is no longer binding beyond some time t, then the shadow price of the pollution stock must be zero beyond t.

3. Resource Use under Joint Regulation

Without any regulation on the flow or stock of emissions, the extraction path will be pure Hotelling. Coal will supply all the energy until it is completely exhausted and then solar energy will take over. Because of Hotelling, the price of coal will increase over time, hence extraction and emissions (which are proportional) will decrease. Thus emissions will be at a maximum at the initial time $t = 0$. This suggests that if local regulation were to be binding, it must be at the beginning of the program. But it is not clear whether the *stock* of pollution will also be maximum at the initial time. If at time $t = 0$, $x(0) \leq \alpha\bar{Z}$, that is, initial emissions are below the natural dilution level, then by (2) the stock of pollution will be permanently decreasing from its initial value Z_0. However if not, then the stock of pollution will increase initially.

To make the problem non-trivial, let us assume that the local and global regulated levels $\underline{x}$ and

$\bar{Z}$ are sufficiently strict, or alternatively, coal is abundant or dirty,[8] so that at least one of the caps will bind.

A Strict Local Cap Makes Global Regulation Redundant

Consider the case $\underline{x} < \bar{x}$ or equivalently, $\underline{p} > \bar{p}$. This may occur when local regulation is strict relative to global regulation. For example, a region may care more for its own environmental quality than for the global stock of pollution, or pollution damages may be higher at the local level (e.g., densely populated areas) requiring a stricter cap. The above inequality implies that $\underline{x} < \bar{x} = \alpha \bar{Z}$ so that if local regulation is satisfied, then starting from the initial pollution stock $Z_0 < \bar{Z}$, the stock must always stay under the global cap $\bar{Z}$. Because of the local cap, emissions will never reach the level allowed at the global cap. The global regulation will never bind and it is therefore redundant. We can discuss the following three cases:

A Cheap Clean Substitute

It turns out that the order of extraction depends on the cost of solar energy. If solar were sufficiently cheap, satisfying the condition $c_y < \underline{p}$, then the program could use solar energy along with cheap coal (recall that $c_x < c_y$). Thus it is easy to conclude that coal will be used at the maximum regulated level $\underline{x}$, and residual demand will be supplied by the more expensive solar energy, given by $D(c_y) - \underline{x}$. This path is followed until all coal is exhausted at time T beyond which only solar energy is used. The price and extraction paths are shown in Fig. 1.

[Fig. 1 here]

The price of energy is constant and equal to c_y . It cannot rise above c_y because ultimately solar must become the sole supplier of energy. Solar energy must be employed at the onset because the price $\underline{p}$ when coal is constrained is higher than the price of solar energy ($c_y < \underline{p}$). Both the Hotelling solution (with no regulation) and the solution under a local cap are shown in Fig.1. The

[8] Since the pollution units are normalized so that $b = 1$, dirty coal would also mean abundance of the resource.

initial shadow price of coal λ_0^* must be lower than λ_0^H , its level in an unconstrained (without a local cap) model, where the superscript H represents the pure Hotelling solution with no regulation. The Hotelling shadow price must be bounded above by the extraction cost differential between the two resources, so we have $\lambda_0^* < \lambda_0^H < c_y - c_x$. As shown in the figure, solar energy kicks in from the beginning and the resource is exhausted later under regulation. Thus it takes longer for the clean energy to supply the complete market.

An Expensive Clean Substitute

We now consider the case $\underline{p} < c_y$, i.e., the price at which emissions are regulated is too low for solar to be competitive. This situation may arise when local regulation is relatively lax or the cost of solar energy is too high. Under regulation the only feasible option is to burn coal at the allowed level, until some time t_1. The price of energy during this first interval is the regulated price $\underline{p}$. Beyond this time, the price of energy rises and extraction of coal declines but solar is still not competitive since its price is higher. Finally coal is exhausted exactly when the price of energy reaches c_y and solar supplies all energy at that price. Fig. 2 shows the corresponding price and resource use paths.

[Fig. 2 here]

In the first phase when only coal is used, the price of energy is constant at $\underline{p}$. This price is higher than the unregulated price $c_x + \lambda_0^* e^{rt}$ and the difference is given by the local pollution tax $\beta(t) = \underline{p} - (c_x + \lambda_0^* e^{rt}) > 0$. At the end of the phase at time t_1 , the value of this multiplier is zero. The interval $[t_1, T)$ is a pure Hotelling phase in which the price of coal increases and extraction declines below the regulated level since the price of energy is now higher than the regulated price $\underline{p}$. The local tax is zero in this period and $\gamma_y(t) = c_y - (c_x + \lambda_0^* e^{rt}) > 0$. At T the price of energy finally reaches the cost of the backstop c_y and coal is completely exhausted at this instant, with all supply switching to the clean solar energy.

Compared to no cap, the local cap on emissions has the effect of initially decreasing coal use (check Fig. 2). Note that emission regulation actually delays the time when the clean resource becomes economically competitive because all of the fossil fuel must be used up. In both of these cases, local regulation is sufficient to prevent the stock of pollution from attaining the globally mandated cap. In the next section, we consider the case when local regulation is unable to prevent the global regulatory cap from becoming binding.

Both Local and Global Regulation Apply

We now assume that local regulation is not stringent enough to make global regulation redundant. More precisely, $\underline{p} < \bar{p}$ and equivalently $\bar{x} < \underline{x}$, that is, in spite of being under a local cap, the stock of pollution may still rise to hit the global cap. The analysis here depends again on the cost of the clean substitute. So we can discuss three cases: when the substitute is cheap, moderately expensive and expensive, respectively.

Cheap Substitute

By a cheap substitute, we mean the following inequality: $c_x < c_y < \underline{p} < \bar{p}$. Solar energy is cheap enough that it can be used under both local and global regulation. When $c_y < \underline{p}$, we saw above that coal is first used at the locally regulated rate with the residual demand supplied by solar energy. This path is maintained until coal is completely exhausted. The question that we need to ask now is: how is the stock of carbon affected and when is the globally mandated cap attained, if at all?

During the phase when coal is used under a local cap, we know from (2) that the stock of pollution accumulates according to the relationship $Z(t) = e^{-\alpha t}(Z_0 + \int_0^t \underline{x} e^{\alpha \tau} d\tau)$. The first exponential term on the right hand side represents the natural dilution effect over the pollution stock. The second term denotes aggregate emissions from burning coal. Solving this equation yields $Z(t) = (Z_0 - \frac{\underline{x}}{\alpha})e^{-\alpha t} + \frac{\underline{x}}{\alpha}$. Since $Z_0 < \bar{Z}$ and $\bar{x} < \underline{x}$, we have $Z_0 < \bar{Z} = \frac{\bar{x}}{\alpha} < \frac{\underline{x}}{\alpha}$. The first term on the right hand side in the equation for $Z(t)$ is negative and decreasing in absolute value. Thus

the aggregate stock of pollution $Z(t)$ is increasing and approaches $\frac{\bar{x}}{\alpha}$ which is greater than the global cap $\bar{Z}$.

Consider increasing the initial stock of coal, X_0. *Ceteris paribus*, if the stock is relatively small, the cumulative stock of pollution $Z(t)$ may not reach $\bar{Z}$. Coal may be exhausted before this level is reached. Then the global cap will never bind. The solution will be exactly as described in Fig.1. However with an increased initial endowment of coal, there exists a critical stock level (call it $\bar{X}_0$) such that for reserves higher than $\bar{X}_0$, the global regulation will indeed bind. Consider the time t_2 when the stock of pollution hits the regulated level $\bar{Z}$. Before t_2 the local emissions cap binds and after t_2 the global cap on stock binds. Beginning with time t_2 coal use must be constrained to $\bar{x}$, and the excess demand given by $D(c_y) - \bar{x}$ is supplied by solar energy. This period of joint use ends when coal is completely exhausted, as shown in Fig. 3.

[Fig. 3 here]

The new insight from this solution is that now that the global cap binds, the shadow price of pollution is not zero from the initial time until coal is exhausted. In the first period when $Z(t) < \bar{Z}$, by (8) and (9), $\mu(t) = \mu_0 e^{(\alpha+r)t}$. That is, the shadow cost of carbon in absolute value or equivalently, the pollution tax, rises exponentially. In the second time period starting from t_2, the tax starts declining since (9) holds and the multiplier on the global cap is strictly positive. The tax rate on pollution falls until it is zero at T when the cap ceases to bind and a complete transition to solar occurs.

Moderately Expensive Substitute

Moderately expensive solar energy implies in our framework the following inequality: $c_x < \underline{p} < c_y < \bar{p}$. Solar is too expensive to be used under local regulation but not under global regulation. To examine this case, consider the solution shown in Fig. 2 in which coal is used for a period of time at the local cap until a Hotelling segment kicks in. From (4), the price path once

the local cap is no longer binding and the global cap is not yet binding is given by $p(t) = c_x + \lambda_0 e^{rt} + \mu_0 e^{(\alpha+r)t}$. If initial endowments of coal increase, then the scarcity rent of coal must decline, leading to a decrease in the price of energy and an extended stay at the local cap. Recall that in this period coal use is constant at the local cap. As in the case of a cheap substitute, abundant coal will mean a prolonged first period, and will lead to a sufficiently large stock of pollution so that the global cap is binding. The question is when. Note that if no solar is used, then the price of energy must reach $\overline{p}$ but that is strictly greater than c_y. So the global ceiling must be achieved when the price of energy is c_y and some solar energy will be used at the global cap. Thus in order to attain the global ceiling, the energy price must rise from $\underline{p}$ to c_y as shown in Fig.4. Because both resources are being used at the global cap, the energy price is c_y which is lower than $\overline{p}$.

[Fig. 4 here]

In the figure, there is a first phase $[0, t_1]$ with constant extraction of coal at the local cap, then the price rises to equal the cost of the substitute during a second phase $[t_1, t_2]$, and finally the extraction of coal is limited to that allowed under the global cap, until complete exhaustion of coal and substitution by solar energy at time T. Local regulation helps delay the onset of global regulation by putting a cap on emissions early in the program. However, once the global regulation kicks in, there is no role for local regulation anymore.

Expensive Substitute

The case with an expensive substitute satisfies the following inequality: $c_x < \underline{p} < \overline{p} < c_y$. Thus solar energy is too expensive at the global cap, only coal must be used. When the price of energy increases from $\overline{p}$ to c_y, only coal must be used in this transition phase, since solar will still be economically infeasible. During this transition, price of coal will rise, hence emissions will be lower than the maximum allowed under global regulation, so the ceiling will no longer hold. The extraction profile and price paths are shown in Fig. 5.

[Fig. 5 here]

At the beginning, the local cap holds, followed by a period when prices rise until the global cap is attained during which coal is used at a constant maximal rate. Finally the price rises again and the stock of pollution declines from the regulated maximum until coal is completely exhausted exactly when the price equals the cost of the clean substitute.

In summary, what is the impact of joint local and global regulation on the extraction of a nonrenewable resource? Firstly, regulation of either type will postpone the date of exhaustion of the nonrenewable and delay the complete transition to the clean substitute. Secondly, local regulation always kicks in before global regulation takes effect. Both can not occur simultaneously. Third, local regulation may in some situations, render global regulation redundant. Fourth, regulation helps move up the deployment of the clean technology, especially when its costs are relatively low.

We can summarize as follows:

Proposition 1: *(a) Both local and global regulation postpone the date of exhaustion of the resource and (b) delay the full transition to the clean substitute. (c) Local regulation always binds before global regulation and (d) both can not be binding at the same time. (e) Local regulation may make global regulation redundant and finally (d) both regulations help move up the deployment of the clean substitute, especially when the substitute is low cost.*

3. Dynamic Interaction between the Local and Global Caps

In this section, we discuss how the local and global ceilings interact. Intuitively it seems that a local, or flow ceiling when binding will reduce carbon emissions and therefore will also lead to a reduction of the pollution stock. This idea is at the heart of current climate policies which try to reduce the carbon concentration in the atmosphere by reducing the emissions of greenhouse gases. However a flow cap even if binding only for a time will have dynamic effects upon the entire path of energy prices. We will show that a flow cap may lead to an increase in the global carbon tax, and conversely, a global cap may lead to a higher local tax.

Flow or Stock Regulation

In order to drive intuition, let us first consider each type of regulation in isolation and examine their effect on resource prices. Consider the case shown in Fig.2 where the clean substitute is costly.[9] Suppose the policy maker were to impose a stricter cap on emissions, $\underline{x}' < \underline{x}$.This would mean a higher regulated price $\underline{p}' > \underline{p}$. Intuitively, we expect that this stricter local cap will reduce the shadow price of the nonrenewable resource.[10] This implies a decline in energy prices once the cap is no longer binding, since $p(t) = c_x + \lambda_0 e^{rt}$. This has two consequences.

Firstly, since $\underline{p}$ is increased and $p(t)$ decreases, the period during which the local emissions cap binds will increase from t_1 to t_1' (see Fig. 6). As expected, a stricter local cap means that it binds for a longer time. However, with a lower price, the use of the resource increases and it is exhausted later in time.

[Fig. 6 here]

Moreover, a stricter cap causes resource use to be transferred to the future, when the cap is no longer binding. This suggests, as we will see later, that a stricter emissions cap may result in a higher stock of pollution. This is a first hint that tight local regulation may not contribute towards meeting a global pollution mandate. As shown in Fig.6, the full introduction of the clean substitute is delayed under a stricter emissions cap.

Now consider a stricter global cap on the pollution stock. Let $\bar{Z}' < \bar{Z}$ which in turn leads to a higher energy price at the ceiling denoted by $\bar{p}' > \bar{p}$. As before, the stricter cap means a lower scarcity rent for coal. On the other hand the more stringent cap will increase the carbon tax.

To fix ideas, consider the case of an expensive substitute shown in Fig.5. The global cap (in the period $[t_2, t_3]$) is sandwiched between two unconstrained periods, $[t_1, t_2]$ and $[t_2, T]$. Intuition suggests that lowering the cap will lead to a reallocation of coal use from the cap period to the adjoining two periods which are under no binding regulation. Since the post global cap phase is

[9] We do not consider all the possible solutions discussed earlier but focus on a few representative cases.
[10] We show this formally in Appendix B.

pure Hotelling, we can conclude that because a stricter cap means a lower resource rent, the overall use of coal will increase during this period.

Because the capped energy price $\bar{p}$ increases (see Fig.7) while the energy price is decreased during the phase $[t_3, T]$, time t_3 will increase to $t_3' > t_3$. The exhaustion of the nonrenewable resource is also delayed until $T' > T$. Tightening the global cap again delays the transition towards clean energy.

[Fig. 7 here]

The price and quantity effects during the pre cap period are more complex to describe. On the one hand the initial scarcity rent of coal λ_0 goes down because of the stricter cap. However, the stricter cap also implies a higher pollution tax μ_0. The overall effect on the price of energy before time t_2 given by $p(t) = c_x + \lambda_0 e^{rt} + \mu_0 e^{(\alpha+r)t}$ is ambiguous. Fig. 7 shows the case when the ceiling arrival occurs earlier in time as a result of the stricter cap.

Here the decline of the resource rent dominates the increase in the pollution tax. This will result in a decline in the initial price of energy. But the tax grows at rate ($\alpha + r$) which is higher than the growth rate of scarcity rent (rate r), hence the new price grows faster as shown. The new price path must cut the old path from below, since $\bar{p}' > \bar{p}$. Note from the figure that the net result is that a stricter cap implies higher coal use and emissions at the beginning, which is counter-intuitive.

The figure shows an earlier arrival at the global cap ($t_2' < t_2$). So the time spent at the global cap is higher. However, the reallocation of resources as a result of the tighter cap to the time periods before and after may also result in an alternative outcome – a late arrival at the cap with an ambiguous effect upon the time length of the ceiling period.[11]

In summary, the effect of a stricter global cap depends on several factors. There is a global

[11] This will *inter alia* depend on the curvature of the demand function in the neighborhood of the regulated price $\bar{p}$.

redistribution of resource use from before the cap to the post-cap period. The pollution tax increases and the scarcity rent decreases. The curvature of the demand function near the ceiling price $\bar{p}$ plays a role. The new energy price rises faster than the old one. However, the starting price may be lower, in which case initial emissions increase as a result of tightening regulation.

Both Flow and Stock Regulation Combined

The goal of this section is to show that counterintuitive results may also arise when local and global regulation are considered jointly.

Tightening Local Regulation

Consider the solution shown in Fig.5. Over a first time interval $[0, t_1]$, only the flow cap is binding. During $[t_1, t_2]$, the flow cap is no longer binding but over $[t_2, t_3]$, the stock ceiling is binding. Finally over the interval $[t_3, T]$ resource extraction follows a Hotelling path with no binding regulation until transition to the backstop at time T. Suppose that we tighten the flow regulation from $\underline{x}$ to $\underline{x}' < \underline{x}$. Then we can summarize the results as follows:

Proposition 2: *Tightening local regulation implies (a) a decrease in resource rents and (b)an increase in emissions taxes; (c) both local and global caps end later in time; (d) the transition to the clean substitute is delayed; (e) the global pollution tax may increase and (f) the global cap may arrive later.*

Proof : See Appendix.

We use the insights obtained from the earlier discussion on the effect of the two individual caps to present some intuition.[12] As before, even with both caps in place, tightening the local cap will imply lower resource rents. This implies a lower production price of coal: $c_x + \lambda_0 e^{rt}$. So in the Hotelling phase, cumulative coal use will increase. Since $\bar{p}$ and c_y are unchanged, the global cap will end later in time and the transition to clean energy will be delayed.

[12] Technical material is relegated to the Appendix.

What is the effect of a stricter local cap on the global pollution tax μ? It all depends on how the tighter cap will shift resource use out of the local cap period and into other periods. For example, some resources are shifted to the last period $[t_3, T]$, and others to the intermediate period $[t_1, t_2]$ neither of which are subject to direct regulation. If a large amount of coal is transferred to the last period, less is moved to the initial period, then the pollution tax μ is increased (see Fig.8).

[Fig. 8 here]

However, if more coal is moved to the initial period, the pollution tax increase is small and is dominated by the fall in resource rents. Then the new energy price is lower but rises rapidly due to a higher μ. Then t_1 will increase and t_2 will decrease. That is, a tighter local cap leads to a faster accumulation of pollution and early arrival at the global cap. Several factors may lead to this outcome, including a higher natural dilution rate α. If this rate is high, it provides an incentive to increase the stock of pollution quickly, to make use of costless dilution. So more resources are transferred to the beginning than towards the end of the extraction period. This is shown in Fig. 8. However if the two price paths do not cut then the global cap comes later in time (not shown here).

A Stricter Global Cap

When both caps are imposed, tightening the global cap leads to the following:

Proposition 3: *Tightening global regulation implies (a) a decline in resource rents (b) increase in the global pollution tax (c) a delay in the arrival of the clean substitute (d) and the global cap ceases to bind later in time. It has an indeterminate effect on (e) the arrival time of the global cap and (f) the local pollution tax.*

Proof: See Appendix.

As before, resource rents decline with a stricter global cap and resources become cheaper. However, the global carbon tax must increase, since regulation is now tighter. Because of the cheaper fossil fuel, the pure Hotelling phase of resource extraction is pushed back in time as well

as the arrival of the clean substitute.[13]

Compared to the previous analysis, the difference here is that the price at the global cap is higher, as in Fig.7. So if more coal has to be burnt elsewhere, it has to be either after the global cap or only at the beginning of the unconstrained time period $[t_1, t_2]$ since the terminal price during this interval at time t_2 must be $\bar{p}'$, higher than before. If the first effect is relatively large, the carbon tax increase is small. So the fall in resource rent dominates and $p(t_1)$ declines. Since $\underline{p}$ stays the same, t_1 will increase. Because the energy price falls, the local tax β must increase.

On the other hand, a higher transfer to the last period will mean the opposite sequence of events – leading to an increase in t_1 and a lowering of the local tax β. As in the case with only the global cap, nothing can be said about the effect of a stricter global cap on its arrival date. This is shown in Fig. 9.

[Fig. 9 here]

It appears that the consequence of a stricter global cap over the beginning of the binding global ceiling phase depends crucially upon the properties of the demand function $D(p)$. If the demand is highly elastic, the direct effect of a stricter global cap over the price $\bar{p}$ will be low. The direct effect over $\bar{p}$ will thus be dominated by the indirect effect over the energy price before t_2 and t_2 will decrease. The converse will happen in the case of an inelastic demand function.[14]

Finally we show the case when a stricter global cap implies a longer stay under local regulation and a higher local tax (see Fig. 10).

[Fig. 10 here]

In summary, tightening either the local or the global cap delays the arrival of the clean substitute.

[13] In the Appendix, we show that the time duration of the Hotelling phase is reduced.
[14] The effect of a stricter global cap on the beginning of the binding global cap is independent of whether a local cap exists or not. It also applies when only a global cap is imposed, as we have seen earlier.

Both policies also reduce the resource rent and therefore may lead to an increase in coal consumption. However whether action at one level mitigates the need for action at the other level depends upon parameter values such as the local curvature properties of the demand curve and the dilution rate of the atmosphere.

4. Concluding Remarks

In this paper we have examined the effect of local regulation on emissions and a global regulation on the stock of pollution created by the extraction of a nonrenewable resource. We show that the effect of the two types of regulation on resource use may be quite different. In the case of a scarce resource, local regulation on emissions will always precede global regulation on the stock of pollution. If local regulation is sufficiently stringent, it will make global regulation unnecessary. We show that the effect of regulation is especially sensitive to the price of the clean backstop resource.

More importantly, we show that these regulations may have unintended consequences when the dynamic effects of resource allocation are taken into account. For example, tightening regulation on the pollution stock may lead to increased pollution at the local level. When both regulations are present, tightening emission caps at the local level may lead to a quicker build-up of pollution and an earlier arrival at the global cap. This is because pollution regulation leads to a decline in the value of the polluting resource and therefore under some conditions, consumption may actually increase. These insights are somewhat contrary to the impressions obtained from a static model, where different types of regulation may serve as substitutes. In a dynamic model, the effect of a policy intervention on the entire dynamic path is relevant and limiting pollution in one period may lead to an increase in another.

From a policy point of view, the joint effects of regulation at multiple levels may need to be coordinated so that the policy instruments act as substitutes and not as complements. As this paper shows, the precise qualitative effects of policy changes are difficult to compute and will depend strongly on parameter values.

But some key lessons can be learnt from this exercise. First the local cap reduces pollution

emissions and extends the life of the polluting resource and the arrival of the clean substitute. If the resource is abundant, the local regulation may not be able to prevent the stock of pollution from rising to the global cap. However, it may delay the time when this peak is achieved. What is interesting is that because the energy price in a model with resource scarcity always increases, once the global regulation is achieved, local regulation ceases to play a role.

These results are important because casual observation suggests that many local and regional jurisdictions are moving towards or have already implemented emission regulation, while waiting for a global consensus to emerge, such as from the process following the Kyoto Protocol. The current state of negotiations suggest that an effective global agreement may take a long time to emerge. In the absence of coordination of these multiple efforts within a country or globally, it is not immediately apparent whether all efforts are beneficial both at the local and global level. As we show in our analysis, in some situations, increased regulation may lead to an increase in pollution.

Our model is highly aggregated and we have not considered multiple heterogenous regions or strategic behavior both at the local and global level. In future work, the model could be extended to consider one global ceiling on the stock and possibly two regions such as the developed economies in the North and the developing economies in the South, each with its own emissions ceiling. How will region-specific emission caps affect resource use and the global stock of pollution when there is heterogeneity among regions? For example, the developed regions may have stringent local regulation (say, because of domestic pressures) while the developing countries may prefer lax local standards. The effect of this divergence in the regulatory environment on the build-up of the stock of pollution and the arrival of the clean substitute may inform the policy debate on the dynamic consequences of multiple levels of regulation.

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Appendix A: Equivalence between Multiple Symmetric Regions and a Single Region

We would like to show that multiple symmetric regions imposing identical emissions caps is equivalent to one region imposing a cap. Assume that there are n symmetric regions labelled $i = 1,..,n$. Let $v_i(q_i) = v(q_i)$ be the gross surplus of each region i given local energy consumption q_i. Let the function v be strictly increasing and concave with $\lim_{q_i \downarrow 0} v'(q_i) = \infty$. The average cost of coal is equal to c_x and same for all regions.

Because of the choice of units, $x_i(t)$ is also the pollution emission in region i. The flow cap $\underline{x}_i$ is the same in all the regions. The pollution stock dynamics is given by $\dot{Z}(t) = \sum_{i=1}^{n} x_i - \alpha Z$. The consumption of the clean substitute is given by $y_i(t)$ and its unit cost is c_y in all regions.

Under these assumptions, because of symmetry, aggregate coal consumption x must be allocated equally to all regions, i.e., $x_i = \frac{x}{n}$ and each region must consume the same quantity of solar energy, i.e., $y_i = \frac{y}{n}$ where y is aggregate solar consumption. Then aggregate surplus defined by $u(q)$ is given by $u(q) = nv(\frac{q}{n})$. This function has the same qualitative properties as the function v, i.e, it is strictly increasing ($u'(q) = v'(\frac{q}{n}) > 0$), strictly concave $u''(q) = \frac{1}{n} v''(\frac{q}{n}) < 0$ and satisfies the Inada property, $\lim_{q \downarrow 0} u'(q) = \lim_{\frac{q}{n} \downarrow 0} v'(\frac{q}{n}) = \infty$. The cap on emissions in each region may therefore be expressed as a cap on aggregate emissions, $x_i = \frac{x}{n} \leq \underline{x}_i$ is equivalent to $x = \sum_{i=1}^{n} x_i \leq n\underline{x}_i \equiv \underline{x}$. Finally note that the dynamics of the pollution stock $\dot{Z}(t)$ does not depend upon the way the aggregate consumption of coal is divided among the regions.

Appendix B: Proofs of Propositions

We perform a comparative dynamic exercise over the system of equations defining the vector $(\lambda_0, \mu_0, t_1, t_2, t_3, T)$ with respect to $\underline{x}$, the local cap, and $\bar{Z}$ the global cap. The variables $(\lambda_0, \mu_0, t_1, t_2, t_3, T)$ solve the following system:

$c_y = c_x + \lambda_0 e^{rT}$

$\bar{p} = c_x + \lambda_0 e^{rt_3}$ where $D(\bar{p}) = \alpha \bar{Z}$

$\bar{p} = c_x + \lambda_0 e^{rt_2} + \mu_0 e^{(\alpha+r)t_2}$

$\underline{p} = c_x + \lambda_0 e^{rt_1} + \mu_0 e^{(\alpha+r)t_1}$ where $D(\underline{p}) = \underline{x}$

$$X_0 = \underline{x} t_1 + \int_{t_1}^{t_2} D(c_x + \lambda_0 e^{rt} + \mu_0 e^{(\alpha+r)t}) dt + \alpha \bar{Z}(t_3 - t_2) + \int_{t_3}^{T} D(c_x + \lambda_0 e^{rt}) dt$$

$$\bar{Z} e^{\alpha t_2} = Z_0 + \underline{x} \frac{e^{\alpha t_1} - 1}{\alpha} + \int_{t_1}^{t_2} D(c_x + \lambda_0 e^{rt} + \mu_0 e^{(\alpha+r)t}) e^{\alpha t} dt.$$

Differentiating with respect to $(\lambda_0, \mu_0, t_1, t_2, t_3, T)$ and $(\underline{x}, \bar{Z})$, we get

$$0 = d\lambda_0 + r\lambda_0 dT$$

$$\frac{\alpha e^{-rt_3}}{D'(\bar{p})} d\bar{Z} = d\lambda_0 + r\lambda_0 dt_3$$

$$\frac{\alpha e^{-rt_2}}{D'(p_s)} d\bar{Z} = d\lambda_0 + e^{\alpha t_2} d\mu_0 + [r\lambda_0 + (\alpha + r)\mu_0 e^{\alpha t_2}] dt_2$$

$$\frac{e^{-rt_1}}{D'(\underline{p})} d\underline{x} = d\lambda_0 + e^{\alpha t_1} d\mu_0 + [r\lambda_0 + (\alpha + r)\mu_0 e^{\alpha t_1}] dt_1.$$

Next, from the resource stock condition, we have

$$0 = t_1 d\underline{x} + \underline{x} dt_1 + D(\bar{p}) dt_2 - D(\underline{p}) dt_1 + \int_{t_1}^{t_2} D'(p(t))[d\lambda_0 e^{rt} + d\mu_0 e^{(\alpha+r)t}] dt$$

$$+ \alpha(t_3 - t_2) d\bar{Z} + \alpha \bar{Z}(dt_3 - dt_2) + D(c_y) dT$$

$$- D(\bar{p}) dt_3 + \int_{t_3}^{T} D'(p(t)) d\lambda_0 e^{rt} dt.$$

Since $D(\bar{p}) = \alpha \bar{Z}$ and $D(\underline{p}) = \underline{x}$, this simplifies to

$$t_1 d\underline{x} + \alpha(t_3 - t_2) d\bar{Z} = I_\lambda^X d\lambda_0 + I_\mu^X d\mu_0 - D(c_y) dT, \quad \text{(A1)}$$

where we denote

$$I_\lambda^X \equiv -\left\{ \int_{t_1}^{t_2} D'(p) e^{rt} dt + \int_{t_3}^{T} D'(p) e^{rt} dt \right\} > 0 \text{ and} \quad \text{(A2)}$$

$$I_\mu^X \equiv -\int_{t_1}^{t_2} D'(p) e^{(\alpha+r)t} dt > 0, \quad \text{(A3)}$$

the signs of I_λ^X and I_μ^X obtained by using the fact that $D'(p) < 0$. Now differentiating the pollution stock condition yields

$$e^{\alpha t_2} d\bar{Z} + \alpha \bar{Z} e^{\alpha t_2} dt_2 = \frac{e^{\alpha t_1} - 1}{\alpha} d\underline{x} + \underline{x} e^{\alpha t_1} dt_1 + D(\bar{p}) e^{\alpha t_2} dt_2 - D(\underline{p}) e^{\alpha t_1} dt_1 + \int_{t_1}^{t_2} D'(p(t))[e^{rt} d\lambda_0 + e^{(\alpha+r)t} d\mu_0] e^{\alpha t} dt.$$

Since $D(\bar{p}) = \alpha \bar{Z}$ and $D(\underline{p}) = \underline{x}$, this simplifies to

$\frac{e^{\alpha t_1} - 1}{\alpha} d\underline{x} - e^{\alpha t_2} d\bar{Z} = I_\lambda^Z d\lambda_0 + I_\mu^Z d\mu_0$, where

$I_\lambda^Z \equiv -\int_{t_1}^{t_2} D'(p) e^{(\alpha+r)t} dt > 0$ and (A4)

$I_\mu^Z \equiv -\int_{t_1}^{t_2} D'(p) e^{(2\alpha+r)t} dt > 0$, (A5)

the positive signs obtained from $D'(p) < 0$. Lastly, let $k(t) \equiv r\lambda_0 + (\alpha + r)\mu_0 e^{\alpha t} > 0$. Expressing the derivative in matrix form gives

$$\begin{bmatrix} 1 & 0 & r\lambda_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & r\lambda_0 & 0 & 0 \\ 1 & e^{\alpha t_2} & 0 & 0 & k(t_2) & 0 \\ 1 & e^{\alpha t_1} & 0 & 0 & 0 & k(t_1) \\ I_\lambda^X & I_\mu^X & -D(c_y) & 0 & 0 & 0 \\ I_\lambda^Z & I_\mu^Z & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\lambda_0 \\ d\mu_0 \\ dT \\ dt_3 \\ dt_2 \\ dt_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{e^{-rt_1}}{D'(\underline{p})} \\ t_1 \\ \frac{e^{\alpha t_1} - 1}{\alpha} \end{bmatrix} dx_f + \begin{bmatrix} 0 \\ \frac{\alpha e^{-rt_3}}{D'(\bar{p})} \\ \frac{\alpha e^{-rt_2}}{D'(\bar{p})} \\ 0 \\ \alpha(t_3 - t_2) \\ -e^{\alpha t_2} \end{bmatrix} d\bar{Z}.$$

Denote by Δ the determinant of the first matrix on the left hand side. Then $\Delta \equiv k(t_1)k(t_2)r\lambda_0 \left\{ r\lambda_0 [I_\lambda^X I_\mu^Z - I_\lambda^Z I_\mu^X] + D(c_y) I_\mu^Z \right\}$. Appendix C shows that $\Delta > 0$.
We can state the following:

Lemma 1. *Let* $d\underline{x} > 0$. *Then* $\frac{d\lambda_0}{d\underline{x}} > 0, \frac{dt_1}{d\underline{x}} < 0, \frac{dt_3}{d\underline{x}} < 0, \frac{dT}{d\underline{x}} < 0$ *and the signs of* $\frac{d\mu_0}{d\underline{x}}, \frac{dt_2}{d\underline{x}}$ *are indeterminate.*

Proof: Consider

$$\frac{d\lambda_0}{d\underline{x}} = \frac{\Delta_\lambda^0}{\Delta} \equiv \frac{1}{\Delta}\begin{vmatrix} 0 & 0 & r\lambda_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r\lambda_0 & 0 & 0 \\ 0 & e^{\alpha t_2} & 0 & 0 & k(t_2) & 0 \\ \frac{e^{-rt_1}}{D'(\underline{p})} & e^{\alpha t_1} & 0 & 0 & 0 & k(t_1) \\ t_1 & I_\mu^X & -D(c_y) & 0 & 0 & 0 \\ \frac{e^{\alpha t_1}-1}{\alpha} & I_\mu^Z & 0 & 0 & 0 & 0 \end{vmatrix}$$

so that $\Delta_\lambda^0 = k(t_1)k(t_2)(r\lambda_0)^2\left[t_1 I_\mu^Z - \frac{e^{\alpha t_1}-1}{\alpha} I_\mu^X\right]$ using (A3) and (A5), where the sign of Δ_λ^0 is the sign of $\Delta_\lambda^1 \equiv -\int_{t_1}^{t_2} D'(p) e^{(\alpha+r)t}\left[t_1 e^{\alpha t} - \frac{e^{\alpha t_1}-1}{\alpha}\right]dt.$ (A6)

Let $f(t) \equiv t_1 e^{\alpha t} - (e^{\alpha t_1}-1)/\alpha$, $t \in [t_1, t_2]$. Note that $f'(t) > 0$. Next, define $g(\tau) = \tau e^{\alpha\tau} - (e^{\alpha\tau}-1)/\alpha$ so that $g(0) = 0$ and $g'(\tau) = \alpha\tau e^{\alpha\tau} > 0$. Thus $t_1 > 0$ implies $g(t_1) = f(t_1) > 0$ so that $f(t) > 0$, $t \in [t_1, t_2]$. Hence $\Delta_\lambda^1 > 0$ implies that $\Delta_\lambda^0 > 0$ which yields $d\lambda_0 / d\underline{x} > 0$. Since T and t_3 only depend upon λ_0 and $dT / d\lambda_0 = dt_3 / d\lambda_0 = -1/(r\lambda_0) < 0$, $\frac{d\lambda_0}{d\underline{x}} > 0$ implies that $\frac{dT}{d\underline{x}} = \frac{dt_3}{d\underline{x}} = -\frac{1}{r\lambda_0}\frac{d\lambda_0}{d\underline{x}} < 0.$

Note that

$$\frac{d\mu_0}{d\underline{x}} = \frac{\Delta_\mu^0}{\Delta} = \frac{1}{\Delta}\begin{vmatrix} 1 & 0 & r\lambda_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & r\lambda_0 & 0 & 0 \\ 1 & 0 & 0 & 0 & k(t_2) & 0 \\ 1 & \frac{e^{-rt_1}}{D'(\underline{p})} & 0 & 0 & 0 & k(t_1) \\ I_\lambda^X & t_1 & -D(c_y) & 0 & 0 & 0 \\ I_\lambda^Z & \frac{e^{\alpha t_1}-1}{\alpha} & 0 & 0 & 0 & 0 \end{vmatrix} \quad (A7)$$

so that $\Delta_\mu^0 = k(t_1)k(t_2)r\lambda_0\left\{r\lambda_0\left[I_\lambda^X \frac{e^{\alpha t_1}-1}{\alpha} - I_\lambda^Z t_1\right] + D(c_y)\frac{e^{\alpha t_1}-1}{\alpha}\right\}$. Using (A2), (A4) and the

definition of $f(t)$, the sign of Δ_μ^0 is the sign of

$$\Delta_\mu^1 = \frac{e^{\alpha t_1}-1}{\alpha}\left[D(c_y) - r\lambda_0 \int_{t_3}^{T} D'(p)e^{rt}dt \right] + r\lambda_0 \int_{t_1}^{t_2} D'(p) f(t) e^{rt} dt. \qquad \text{(A8)}$$

Since $D'(p) < 0$, the first term of the above sum is positive while the second term is negative. We conclude that $d\mu_0 / d\underline{x}$ has an indeterminate sign. Finally,

$$\frac{dt_1}{d\underline{x}} = \frac{\underline{\Delta}^0}{\Delta} \equiv \frac{1}{\Delta} \begin{vmatrix} 1 & 0 & r\lambda_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & r\lambda_0 & 0 & 0 \\ 1 & e^{\alpha t_2} & 0 & 0 & k(t_2) & 0 \\ 1 & e^{\alpha t_1} & 0 & 0 & 0 & \frac{e^{-rt_1}}{D'(\underline{p})} \\ I_\lambda^X & I_\mu^X & -D(c_y) & 0 & 0 & t_1 \\ I_\lambda^Z & I_\mu^Z & 0 & 0 & 0 & \frac{e^{\alpha t_1}-1}{\alpha} \end{vmatrix}. \qquad (A9)$$

Appendix D shows that $\underline{\Delta}^0 < 0$ so that $dt_1 / d\underline{x} < 0$. It is straightforward to see that the sign of $dt_2 / d\underline{x}$ is indeterminate. ■

To examine the effect of a stricter local cap $d\underline{x} < 0$ on μ_0 and on the energy price on the interval $[t_1, t_2)$, we state the following Lemma.

Lemma 2. *Let* $d\underline{x} < 0$. *Define* $H^X = -\int_{t_3}^{T} D'(p)e^{rt}dt + D(c_y)/(r\lambda_0)$,

$\hat{H}_p \equiv -\dfrac{\alpha e^{-\alpha t_2}}{e^{\alpha t_1}-1}\displaystyle\int_{t_1}^{t_2} D'(p) f(t) e^{rt}(e^{\alpha t_2} - e^{\alpha t})dt$ *and* $\hat{H}_\mu \equiv -\dfrac{\alpha}{(e^{\alpha t_1}-1)}\displaystyle\int_{t_1}^{t_2} D'(p) f(t) e^{rt} dt$ *where* $f(t) \equiv t_1 e^{\alpha t} - (e^{\alpha t_1}-1)/\alpha$. *Then* $\hat{H}_p < \hat{H}_\mu$ *and (i) if* $H^X < \hat{H}_p$ *then* $d\mu_0 / |d\underline{x}| > 0$, $dt_2 / |d\underline{x}| < 0$ *and the energy price over* $[t_1, t_2]$ *first falls then rises; (ii) if* $\hat{H}_p < H^X < \hat{H}_\mu$ *then* $d\mu_0 / |d\underline{x}| > 0$, $dt_2 / |d\underline{x}| > 0$ *and energy prices fall everywhere in the interval* $[t_1, t_2]$ *; and (iii) if* $\hat{H}_\mu < H^X$ *then* $d\mu_0 / |d\underline{x}| < 0$, $dt_2 / |d\underline{x}| > 0$ *and energy prices fall again during* $[t_1, t_2]$.

Proof: Define $\delta \equiv (I_\lambda^X + D(c_y)/r\lambda_0) I_\mu^Z - I_\mu^X I_\mu^Z$. Appendix C shows that $\delta > 0$. Simplifying when $d\underline{x} < 0$ from (A7), we have

$$\frac{d\mu_0}{|d\underline{x}|} = -\frac{1}{\delta}\left\{ \frac{e^{\alpha t_1}-1}{\alpha}\left[I_\lambda^X + \frac{D(c_y)}{r\lambda_0} \right] - t_1 I_\lambda^Z \right\} = \frac{1}{\delta}\left\{ t_1 I_\lambda^Z - \frac{e^{\alpha t_1}-1}{\alpha}\left[I_\lambda^X + \frac{D(c_y)}{r\lambda_0} \right] \right\}. \qquad \text{(A10)}$$

Let $K_\lambda^X \equiv -\int_{t_1}^{t_2} D'(p)e^{rt}dt = \int_{t_1}^{t_2} \frac{\partial x(t)}{\partial \lambda_0}dt$ and $H^X \equiv -\int_{t_3}^{T} D'(p)e^{rt}dt + \frac{D(c_y)}{r\lambda_0} = \int_{t_3}^{T} \frac{\partial x(t)}{\partial \lambda_0}dt - \frac{dT}{d\lambda_0}D(c_y)$.

Here K_λ^X measures the first order change in aggregate resource use from varying λ_0 during the period $[t_1, t_2]$, and H^X for the period $[t_3, T]$. From the definition of I_λ^X, we get $I_\lambda^X + D(c_y)/(r\lambda_0) = K_\lambda^X + H^X$. Thus (A10) is equivalent to

$$\frac{d\mu_0}{d|\underline{x}|} = \frac{1}{\delta}\left\{t_1 I_\lambda^Z - \frac{e^{\alpha t_1}-1}{\alpha}K_\lambda^X - \frac{e^{\alpha t_1}-1}{\alpha}H^X\right\} = \frac{1}{\delta}\left\{-\int_{t_1}^{t_2} D'(p)f(t)e^{rt}dt - \frac{e^{\alpha t_1}-1}{\alpha}H^X\right\}, \quad \text{(A11)}$$

where we use the definition of $f(t)$. Since $f(t) > 0$ as shown before and $D'(p) < 0$ we conclude that absent any quantity effect after t_3, μ_0 should increase following a decrease in $\underline{x}$. Let

$$\hat{H}_\mu \equiv -\frac{\alpha}{(e^{\alpha t_1}-1)}\int_{t_1}^{t_2} D'(p)f(t)e^{rt}dt. \quad \text{(A12)}$$

We have from (A11) that

$$\frac{d\mu_0}{d|\underline{x}|}\begin{cases} > 0 & \text{if} \quad H^X < \hat{H}_\mu \\ \\ \leq 0 & \text{if} \quad H^X \geq \hat{H}_\mu \end{cases}.$$ Next, on the interval $[t_1, t_2]$, we can write:

$$\frac{dp(t)}{|d\underline{x}|} = e^{rt}\left[\frac{d\lambda_0}{|d\underline{x}|} + e^{\alpha t}\frac{d\mu_0}{|d\underline{x}|}\right] = \frac{e^{rt}}{\delta}\left\{\frac{e^{\alpha t_1}-1}{\alpha}I_\mu^X - t_1 I_\mu^Z + e^{\alpha t}\left[I_\lambda^Z t_1 - \frac{e^{\alpha t_1}-1}{\alpha}\left(K_\lambda^X + H^X\right)\right]\right\}$$

$$= \frac{e^{rt}}{\delta}\left\{\frac{e^{\alpha t_1}-1}{\alpha}\left[I_\mu^X - e^{\alpha t}K_\lambda^X\right] - t_1\left[I_\mu^Z - e^{\alpha t}I_\lambda^Z\right] - e^{\alpha t}\frac{e^{\alpha t_1}-1}{\alpha}H^X\right\},$$

using (A6) and (A8). Define $A_p^X(t) \equiv I_\mu^X - e^{\alpha t}K_\lambda^X = \int_{t_1}^{t_2} D'(p)(e^{\alpha t} - e^{\alpha\tau})e^{r\tau}d\tau$,

$A_p^Z(t) \equiv I_\mu^Z - e^{\alpha t}I_\lambda^Z = \int_{t_1}^{t_2} D'(p)(e^{\alpha t} - e^{\alpha\tau})e^{(\alpha+r)\tau}d\tau$, and

$$A_p(t) \equiv \frac{e^{\alpha t_1}-1}{\alpha}A_p^x(t) - t_1 A_p^Z(t) = \int_{t_1}^{t_2} D'(p)f(\tau)(e^{\alpha\tau} - e^{\alpha t})e^{r\tau}d\tau$$

from the definition of $f(\tau)$. Then $\frac{dp(t)}{|d\underline{x}|} = \frac{e^{rt}}{\delta}\left\{A_p(t) - e^{\alpha t}\frac{e^{\alpha t_1}-1}{\alpha}H^X\right\}$. (A13)

Note that $D'(p) < 0$ implies that $A_p(t)$ is an increasing function of time, $e^{\alpha\tau} - e^{\alpha t_1} > 0$ and $e^{\alpha\tau} - e^{\alpha t_2} < 0$, $\tau \in (t_1, t_2)$ which together imply that $A_p(t_1) < 0$ and $A_p(t_2) > 0$. So $A_p(t)$ changes sign during the interval $(t_1, t_2]$ and there exists a unique $\underline{\hat{t}} \in (t_1, t_2)$ such that

$$A_p(t)\begin{cases} < 0 & \text{if} \quad t < \underline{\hat{t}} \\ \\ \geq 0 & \text{if} \quad t \geq \underline{\hat{t}} \end{cases}.$$

We conclude that, absent any quantity effect after t_3, the energy price during $[t_1, t_2]$ should first decrease and then increase. But the quantity effect after t_3 given by H^X has a negative impact

upon the energy price during $[t_1, t_2]$. Thus $A_p(t_1) < 0$ in all cases. So $dp(t_1)/|d\underline{x}| < 0$. Since a stricter local cap means a higher $\underline{p}$ during $[0, t_1)$, t_1 should increase. The decrease in the price before t_1 combined with the increase of $\underline{p}$ resulting from a stricter local cap together imply that the local tax β rises. Finally let $\hat{H}_p \equiv -\frac{\alpha e^{-\alpha t_2}}{e^{\alpha t_1} - 1}\int_{t_1}^{t_2} D'(p) f(t) e^{rt}(e^{\alpha t_2} - e^{\alpha t})dt$. From (A13), we have

$$\frac{dp(t_2)}{d|\underline{x}|}\begin{cases} > 0 & \text{if} \quad H^X < \hat{H}_p \\ \\ \leq 0 & \text{if} \quad H^X \geq \hat{H}_p \end{cases}$$

. It is immediately checked that $\hat{H}_\mu > \hat{H}_p$. This completes the proof of Lemma 2. ■

We now check that the proof of Proposition 2 is complete. For $d\underline{x} < 0$, Lemma 1 gives $d\lambda_0/|d\underline{x}| < 0$ which establishes part (a). Using $p(t) = c_x + \lambda_0 e^{rt} + \mu_0 e^{(\alpha+r)t}$, Lemma 2 yields $dp(t)/|d\underline{x}| < 0$, $\forall t \in [0, t_1]$. Since $\beta(t) = \underline{p} - p(t)$, $t \in [0, t_1)$, we have $d\beta(t)/|d\underline{x}| > 0$ establishing part (b). Lemma 1 also shows that $dt_1/|d\underline{x}| > 0$ and $dt_3/|d\underline{x}| = dT/|d\underline{x}| > 0$, which prove parts (c), (d) and (e). Lastly, applying Lemma 2, we see that for a sufficiently low level of H^X, $d\mu_0/|d\underline{x}| > 0$ and $dt_2/|d\underline{x}| < 0$, proving parts (f) and (g).

We prove the following Lemma for the effect of a change in the global cap:

Lemma 3. *When* $d\bar{Z} > 0$, $\frac{d\lambda_0}{d\bar{Z}} > 0, \frac{d\mu_0}{d\bar{Z}} < 0,$ *the signs of* $\frac{dt_1}{d\bar{Z}}, \frac{dt_2}{d\bar{Z}}$ *are indeterminate, and* $\frac{dt_3}{d\bar{Z}} < 0, \frac{dT}{d\bar{Z}} < 0.$

Proof: Consider

$$\frac{d\lambda_0}{d\bar{Z}} = \frac{\Delta_\lambda^2}{\Delta} \equiv \frac{1}{\Delta}\begin{vmatrix} 0 & 0 & r\lambda_0 & 0 & 0 & 0 \\ \frac{\alpha e^{-rt_3}}{D'(\bar{p})} & 0 & 0 & r\lambda_0 & 0 & 0 \\ \frac{\alpha e^{-rt_2}}{D'(\bar{p})} & e^{\alpha t_2} & 0 & 0 & k(t_2) & 0 \\ 0 & e^{\alpha t_1} & 0 & 0 & 0 & k(t_1) \\ \alpha(t_3 - t_2) & I_\mu^X & -D(c_y) & 0 & 0 & 0 \\ -e^{\alpha t_2} & I_\mu^Z & 0 & 0 & 0 & 0 \end{vmatrix}.$$

The sign of $d\lambda_0 / d\bar{Z}$ is the sign of $\Delta_\lambda^2 = k(t_1)k(t_2)(r\lambda_0)^2\left[\alpha(t_3 - t_2)I_\mu^Z + e^{\alpha t_2} I_\mu^X\right] > 0$ which yields $d\lambda_0 / d\bar{Z} > 0$. Since $dT / d\bar{Z} = -(d\lambda_0 / d\bar{Z})(r\lambda_0)^{-1}$, we obtain $dT / d\bar{Z} < 0$. Moreover

$\frac{dt_3}{d\bar{Z}} = \frac{1}{r\lambda_0}\left\{\frac{\alpha e^{-rt_3}}{D'(\bar{p})} - \frac{d\lambda_0}{d\bar{Z}}\right\}$ and $D'(p) < 0$ give $dt_3 / d\bar{Z} < 0$. Since $\bar{p}$ decreases when the global cap is relaxed, the impact on t_3 and T are not equal, i.e., $\frac{d(T-t_3)}{d\bar{Z}} = \frac{dT}{d\bar{Z}} - \frac{dt_3}{d\bar{Z}} = -\frac{\alpha e^{-rt_3}}{r\lambda_0 D'(\bar{p})} > 0$. Relaxing the global cap leads to a longer Hotelling phase $[t_3, T)$. Next

$$\frac{d\mu_0}{d\bar{Z}} = \frac{\Delta_\mu^2}{\Delta} = \frac{1}{\Delta}\begin{vmatrix} 1 & 0 & r\lambda_0 & 0 & 0 & 0 \\ 1 & \frac{\alpha e^{-rt_3}}{D'(\bar{p})} & 0 & r\lambda_0 & 0 & 0 \\ 1 & \frac{\alpha e^{-rt_2}}{D'(\bar{p})} & 0 & 0 & k(t_2) & 0 \\ 1 & 0 & 0 & 0 & 0 & k(t_1) \\ I_\lambda^X & \alpha(t_3 - t_2) & -D(c_y) & 0 & 0 & 0 \\ I_\lambda^Z & -e^{\alpha t_2} & 0 & 0 & 0 & 0 \end{vmatrix}$$ so that

$\Delta_\mu^2 = k(t_1)k(t_2)r\lambda_0\left\{-r\lambda_0\left[I_\lambda^X e^{\alpha t_2} + I_\lambda^Z \alpha(t_3 - t_2)\right] - e^{\alpha t_2} D(c_y)\right\} < 0$ which yields $d\mu_0 / d\bar{Z} < 0$. It is straightforward to check that the signs of $dt_1 / d\bar{Z}$ and $dt_2 / d\bar{Z}$ are indeterminate, which completes the proof of Lemma 3. ■

For a stricter global cap, we examine its effect on the energy price during the period $[t_1, t_2]$, summarized as follows:

Lemma 4. *When* $d\bar{Z} < 0$, *there exists*
$\hat{H}_{\bar{Z}} \equiv e^{-\alpha(t_1+t_2)}\int_{t_1}^{t_2} D'(p)e^{r\tau}\left[e^{\alpha t_1} - e^{\alpha\tau}\right]\left\{\alpha(t_3 - t_2)e^{\alpha\tau} + e^{\alpha t_2}\right\}d\tau, H^X = -\int_{t_3}^{T} D'(p)e^{r\tau}d\tau + \frac{D(c_y)}{r\lambda_0}$ *such that (a) if* $H^X < \hat{H}_{\bar{Z}}$, *the price of energy first falls then rises in the interval* $[t_1, t_2)$, *i.e.,* $dp(t_1)/|d\bar{Z}| < 0$, $dt_1/|d\bar{Z}| > 0$ *and* $d\beta/|d\bar{Z}| > 0$. *(b) If* $\hat{H}_{\bar{Z}} < H^X$ *the price of energy rises,* $dp(t_1)/|d\bar{Z}| > 0$, $dt_1/|d\bar{Z}| < 0$ *and* $d\beta/|d\bar{Z}| < 0$

Proof: Differentiating the energy price we have

$$\frac{dp(t)}{|d\bar{Z}|} = e^{rt}\left[\frac{d\lambda_0}{d|\bar{Z}|} + \theta e^{\alpha t}\frac{d\mu_0}{|d\bar{Z}|}\right] = \frac{e^{rt}}{\delta}\left\{-\alpha(t_3 - t_2)I_\mu^Z - e^{\alpha t_2}I_\mu^X + e^{\alpha t}\left[\alpha(t_3 - t_2)I_\lambda^Z + \left(I_\lambda^X + \frac{D(c_y)}{r\lambda_0}\right)e^{\alpha t_2}\right]\right\}$$

$$= \frac{e^{rt}}{\delta}\left\{-\alpha(t_3 - t_2)\left[I_\mu^Z - e^{\alpha t}I_\lambda^Z\right] - e^{\alpha t_2}\left[I_\mu^X - e^{\alpha t}K_\lambda^X\right] + e^{\alpha(t+t_2)}H^X\right\}$$

$$= \frac{e^{rt}}{\delta}\left\{-\alpha(t_3 - t_2)A_p^Z(t) - e^{\alpha t_2}A_p^X(t) + e^{\alpha(t+t_2)}H^X\right\}, \qquad \text{(A14)}$$

where $A_p^X(t)$ and $A_p^Z(t)$ have been defined earlier. Let $B_p(t) \equiv -\left[\alpha(t_3 - t_2)A_p^Z(t) + e^{\alpha t_2}A_p^X(t)\right]$. Since $e^{\alpha\tau} - e^{\alpha t_1} > 0$, $\tau \in [t_1, t_2]$ we have $A_p^X(t_1) > 0$, $A_p^Z(t_1) > 0$ and $B_p(t_1) < 0$. Conversely $e^{\alpha\tau} - e^{\alpha t_2} < 0$, $\tau \in [t_1, t_2]$ implies that $A_p^X(t_2) < 0$, $A_p^Z(t_2) < 0$ and $B_p(t_2) > 0$. Note that $A_p^X(t)$ and $A_p^Z(t)$ both decrease while $B_p(t)$ is strictly increasing during $[t_1, t_2]$. In summary, absent any quantity effect beyond t_3, the energy price should pivot in the interval $[t_1, t_2]$, first falling after t_1 then rising near t_2. Since $H^X > 0$, a low level of H^X will not qualitatively alter this result while a high level of H^X will induce a global increase of $p(t)$. Using the expressions for $A_p^X(t)$, $A_p^Z(t)$ and (A14), we can compute the critical level $\hat{H}_{\bar{Z}}$ of H^X such that $dp(t_1)/d\bar{Z} = 0$. Then if $H^X < \hat{H}_{\bar{Z}}$, $p(t_1)$ decreases which implies that $dt_1/|d\bar{Z}| > 0$ and $d\beta/|d\bar{Z}| > 0$. The converse occurs when $H^X > \hat{H}_{\bar{Z}}$. This completes the proof of Lemma 4.

■

Finally for $dt_2/|d\bar{Z}|$, we can state:

Lemma 5. *Let $d\bar{Z} < 0$. Then (a) if $d\bar{p}/|d\bar{Z}| > \lim_{t \uparrow t_2} dp(t)/|d\bar{Z}|$, then $dt_2/|d\bar{Z}| > 0$. The direct effect of $d\bar{Z} < 0$ is higher than the indirect effect induced by the energy price variation on $[t_1, t_2]$. So the global cap will start later in time. (b) If $d\bar{p}/|d\bar{Z}| < \lim_{t \uparrow t_2} dp(t)/|d\bar{Z}|$, then $dt_2/|d\bar{Z}| < 0$. The direct effect of $d\bar{Z} < 0$ is dominated by the indirect effect over the energy price path before the global ceiling period, so t_2 must decrease.*

Proof: Differentiating at time t_2 gives

$$-\frac{\alpha}{D'(p_s)}|d\bar{Z}| = d\lambda_0 e^{rt_2} + e^{(\alpha+r)t_2}d\mu_0 + [r\lambda_0 e^{rt_2} + (\alpha+r)\mu_0 e^{(\alpha+r)t_2}]dt_2$$ which implies

which in turn yields
$$-\frac{\alpha}{D'(\bar{p})} = \left\{\frac{d\lambda_0}{|d\bar{Z}|}e^{rt_2} + e^{(\alpha+r)t_2}\frac{d\mu_0}{|d\bar{Z}|}\right\} + \left[r\lambda_0 e^{rt_2} + (\alpha+r)e^{(\alpha+r)t_2}\right]\frac{dt_2}{|d\bar{Z}|}$$
$$= \lim_{t \uparrow t_2}\frac{dp(t)}{|d\bar{Z}|} + \lim_{t \uparrow t_2}\frac{dp(t)}{dt}\frac{dt_2}{|d\bar{Z}|}$$

so that $\frac{dt_2}{|d\bar{Z}|} = \frac{1}{\dot{p}_-(t_2)}\left[\frac{d\bar{p}}{|d\bar{Z}|} - \lim_{t \uparrow t_2}\frac{dp(t)}{|d\bar{Z}|}\right]$ where we denote the left hand side limit of $\dot{p}(t)$ by $\dot{p}_-(t_2)$ which is positive. Recall that as a function of time, $p(t)$ is not differentiable at $t = t_2$. The first term inside the bracket on the RHS is positive since $D'(p) < 0$ and so is the second term since we have shown that $dp(t_2)/|d\bar{Z}| > 0$ for all H^X. Hence the difference of the two terms is negative and t_2 declines. ■

In order to complete the proof of Proposition 3, check that from lemma 3, $d\lambda_0/|d\bar{Z}| < 0$ and $d\mu_0/|d\bar{Z}| > 0$ which establish claims 3(a) and 3(b). We also have $dT/|d\bar{Z}| > 0$ and $dt_3/|d\bar{Z}| > 0$ which establish 3(c) and 3(d). From Lemma 4, the sign of $dt_1/|d\bar{Z}|$ is indeterminate which proves

3(e). Lemma 5 shows that the time t_2 may either increase or decrease, which is 3(f). Finally, $\beta(t),\ t \in [0, t_1]$ may either increase or decrease from Lemma 4, which gives (g).

Appendix C: Computation of the sign of Δ

Recall that $\Delta \equiv k(t_1)k(t_2)r\lambda_0\left\{r\lambda_0[I_\lambda^X I_\mu^Z - I_\lambda^Z I_\mu^X] + D(c_y)I_\mu^Z\right\}$. In order to show that $\Delta > 0$, it is sufficient to show $\Delta^0 \equiv [I_\lambda^X I_\mu^Z - I_\lambda^Z I_\mu^X] > 0$. Comparing (A3) and (A4) for I_μ^X and I_λ^Z yields $I_\mu^X = I_\lambda^Z$. Writing $|D'| \equiv -D'(p(t))$ yields

$$\Delta^0 = \left[\int_{t_1}^{t_2} |D'| e^{rt} dt + \int_{t_3}^{T} |D'| e^{rt} dt\right]\left[\int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt\right] - \left[\int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} dt\right]^2$$

$$= \left\{\left[\int_{t_1}^{t_2} |D'| e^{rt} dt\right]\left[\int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt\right] - \left[\int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} dt\right]^2\right\}$$

$$+\left[\int_{t_3}^{T} |D'| e^{rt} dt\right]\left[\int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt\right].$$

Let $\Delta^1 \equiv \left[\int_{t_1}^{t_2} |D'| e^{rt} dt\right]\left[\int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt\right] - \left[\int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} dt\right]^2$, $a(t) \equiv |D'(p(t))|^{1/2} e^{rt/2}$

$b(t) \equiv D'(p(t)|^{1/2} e^{\frac{2\alpha+r}{2}t}$ and

$a^2(t) = |D'| e^{rt} b^2(t) = |D'| e^{(2\alpha+r)t}; a(t)b(t) = |D'|^{1/2} e^{rt/2} |D'|^{1/2} e^{\frac{2\alpha+r}{2}t} = |D'| e^{(\alpha+r)t}$.

Then $\Delta^1 = \left[\int_{t_1}^{t_2} a^2(t) dt\right]\left[\int_{t_1}^{t_2} b^2(t) dt\right] - \left[\int_{t_1}^{t_2} a(t)b(t) dt\right]^2$. Using the Cauchy-Schwartz inequality for integrals, we obtain $\left(\int a(t)b(t)dt\right)^2 \le \left(\int a^2(t)dt\right)\left(\int b^2(t)dt\right)$ which gives $\Delta^1 \ge 0$. This implies that $\Delta^0 = \left[\int_{t_3}^{T} |D'| e^{rt} dt\right]\left[\int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt\right] + \Delta^1 > 0$. Hence $\Delta > 0$. ■

Appendix D: Computation of the sign of $\underline{\Delta}^0$

From (A9) we have

$$\underline{\Delta}^0 = k(t_2)r\lambda_0 \begin{vmatrix} 1 & 0 & r\lambda_0 & 0 \\ 1 & e^{\alpha t_1} & 0 & \frac{e^{-rt_1}}{D'(\underline{p})} \\ I_\lambda^X & I_\mu^X & -D(c_y) & t_1 \\ I_\lambda^Z & I_\mu^Z & 0 & \frac{e^{\alpha t_1}-1}{\alpha} \end{vmatrix} \equiv k(t_2)r\lambda_0\underline{\Delta}^1$$

and

$$\underline{\Delta}^1 = \begin{vmatrix} e^{\alpha t_1} & 0 & \frac{e^{-rt_1}}{D'(\underline{p})} \\ I_\mu^X & -D(c_y) & t_1 \\ I_\mu^Z & 0 & \frac{e^{\alpha t_1}-1}{\alpha} \end{vmatrix} + r\lambda_0 \begin{vmatrix} 1 & e^{\alpha t_1} & \frac{e^{-rt_1}}{D'(\underline{p})} \\ I_\lambda^X & I_\mu^X & t_1 \\ I_\lambda^Z & I_\mu^Z & \frac{e^{\alpha t_1}-1}{\alpha} \end{vmatrix} \equiv -D(c_y)\underline{\Delta}^2 + r\lambda_0\underline{\Delta}^3,$$

where

$$\underline{\Delta}^2 \equiv \left[e^{\alpha t_1}\left(\frac{e^{\alpha t_1}-1}{\alpha}\right) - I_\mu^Z \frac{e^{-rt_1}}{D'(p_f)} \right] > 0,$$

since $D'(p) < 0$, and $\underline{\Delta}^3 \equiv \frac{e^{-rt_1}}{D'(\underline{p})}\left[I_\lambda^X I_\mu^Z - I_\lambda^Z I_\mu^X\right] - t_1\left[I_\mu^Z - I_\lambda^Z e^{\alpha t_1}\right] + \frac{e^{\alpha t_1}-1}{\alpha}\left[I_\mu^X - I_\lambda^X e^{\alpha t_1}\right].$

In Appendix B we have shown that $\Delta^0 \equiv I_\lambda^X I_\mu^Z - I_\lambda^Z I_\mu^X > 0$. So

$$I_\mu^Z - I_\lambda^Z e^{\alpha t_1} = \int_{t_1}^{t_2} |D'| e^{(2\alpha+r)t} dt - e^{\alpha t_1}\int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} dt = \int_{t_1}^{t_2} |D'| e^{(\alpha+r)t}[e^{\alpha t} - e^{\alpha t_1}]dt, \text{ and}$$

$$I_\mu^X - I_\lambda^X e^{\alpha t_1} = \int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} dt - e^{\alpha t_1}\int_{t_1}^{t_2} |D'| e^{rt} dt - e^{\alpha t_1}\int_{t_3}^{T} |D'| e^{rt} dt$$

$$= \int_{t_1}^{t_2} |D'| e^{rt}(e^{\alpha t} - e^{\alpha t_1}) dt - e^{\alpha t_1}\int_{t_3}^{T} |D'| e^{rt} dt.$$

Let $h(t) \equiv e^{\alpha t} - e^{\alpha t_1} > 0$ for $t \in [t_1, t_2)$. Then

$$\underline{\Delta}^3 = \frac{e^{-rt_1}}{D'(\underline{p})}\Delta^0 - t_1\int_{t_1}^{t_2} |D'| e^{(\alpha+r)t} h(t) dt + \frac{e^{\alpha t_1}-1}{\alpha}\left[\int_{t_1}^{t_2} |D'| e^{rt} h(t) dt - e^{\alpha t_1}\int_{t_3}^{T} |D'| e^{rt} dt\right]$$

$$= \frac{e^{-rt_1}}{D'(\underline{p})}\Delta^0 - \int_{t_1}^{t_2} |D'| e^{rt} h(t)\left[t_1 e^{\alpha t} - \frac{e^{\alpha t_1}-1}{\alpha} dt\right] - \frac{e^{2\alpha t_1} - e^{\alpha t_1}}{\alpha}\int_{t_3}^{T} |D'| e^{rt} dt.$$

From Appendix B, $f(t) \equiv t_1 e^{\alpha t} - (e^{\alpha t_1} - 1)/\alpha > 0$, $t \in [t_1, t_2)$. Thus $\underline{\Delta}^3 < 0$ since $D'(p) < 0$ and $\Delta^0 > 0$. Thus $\underline{\Delta}^1 = -D(c_y)\underline{\Delta}^2 + r\lambda_0\underline{\Delta}^3 < 0$, since $\underline{\Delta}^2 > 0$ and $\underline{\Delta}^3 < 0$. Hence we conclude that $\underline{\Delta}^0 = k(t_2) r\lambda_0 \underline{\Delta}^1 < 0$. ■

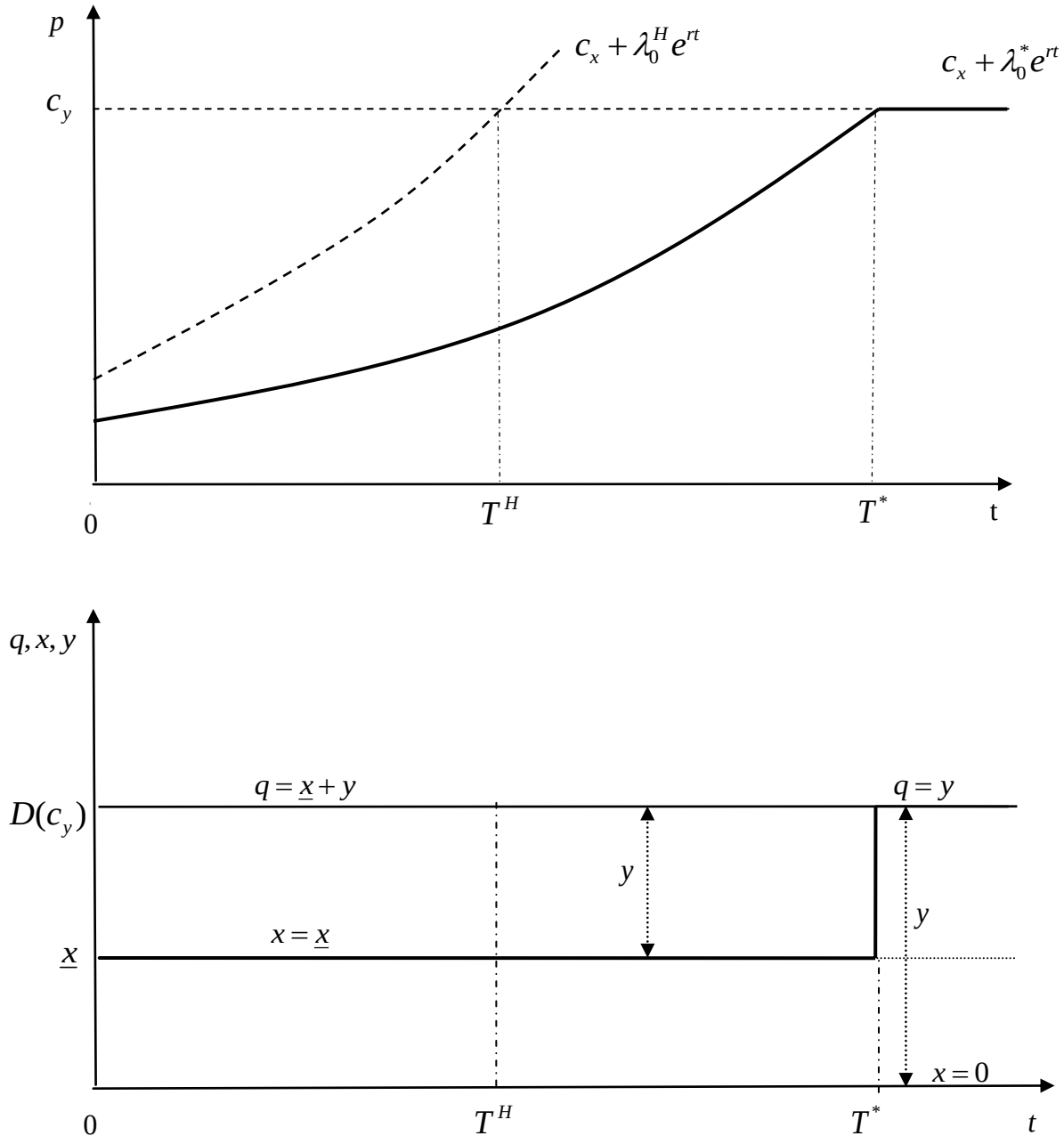

Fig.1. Local Regulation with a Cheap Substitute: Both Coal and Solar are used at the beginning

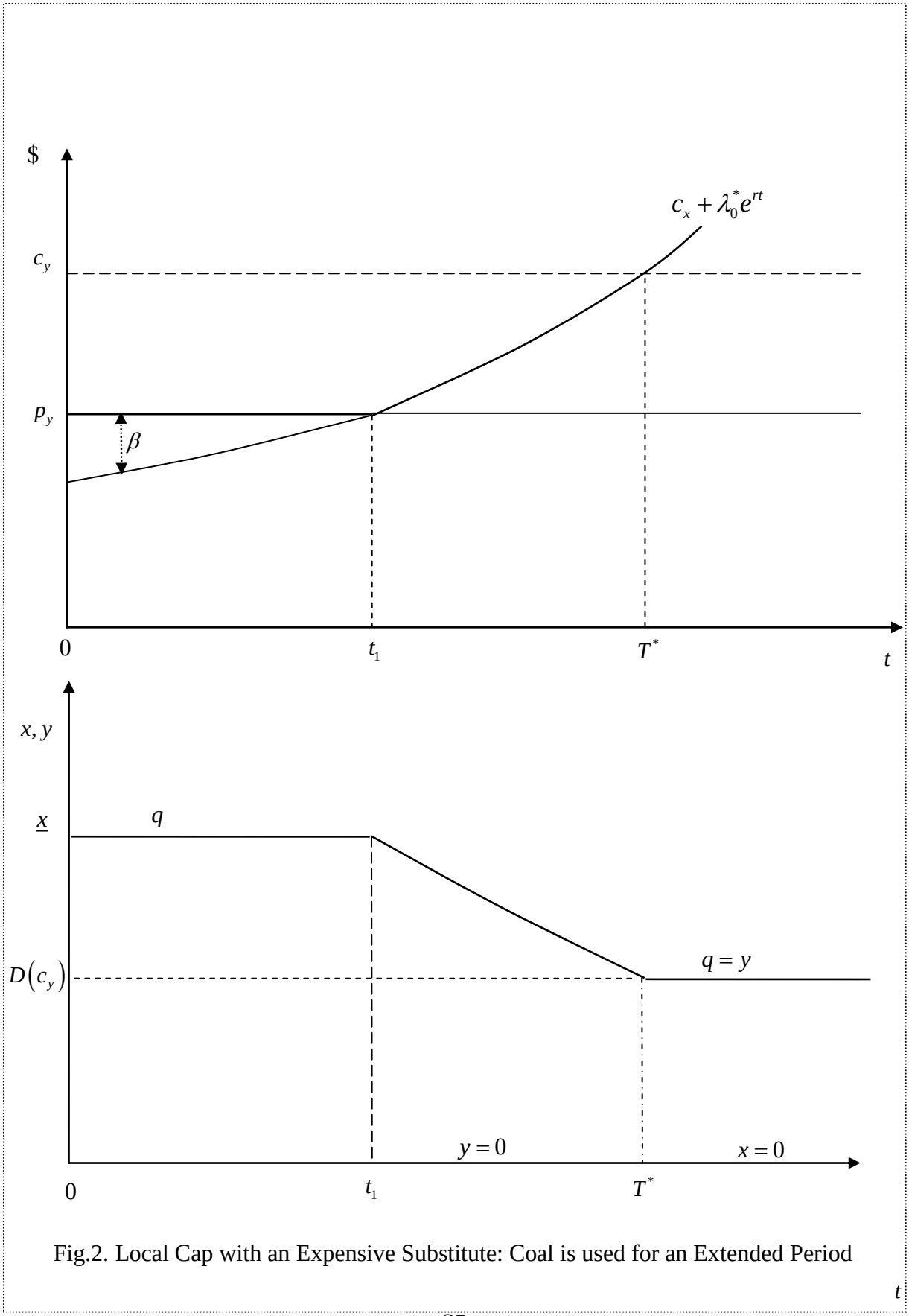

Fig.2. Local Cap with an Expensive Substitute: Coal is used for an Extended Period

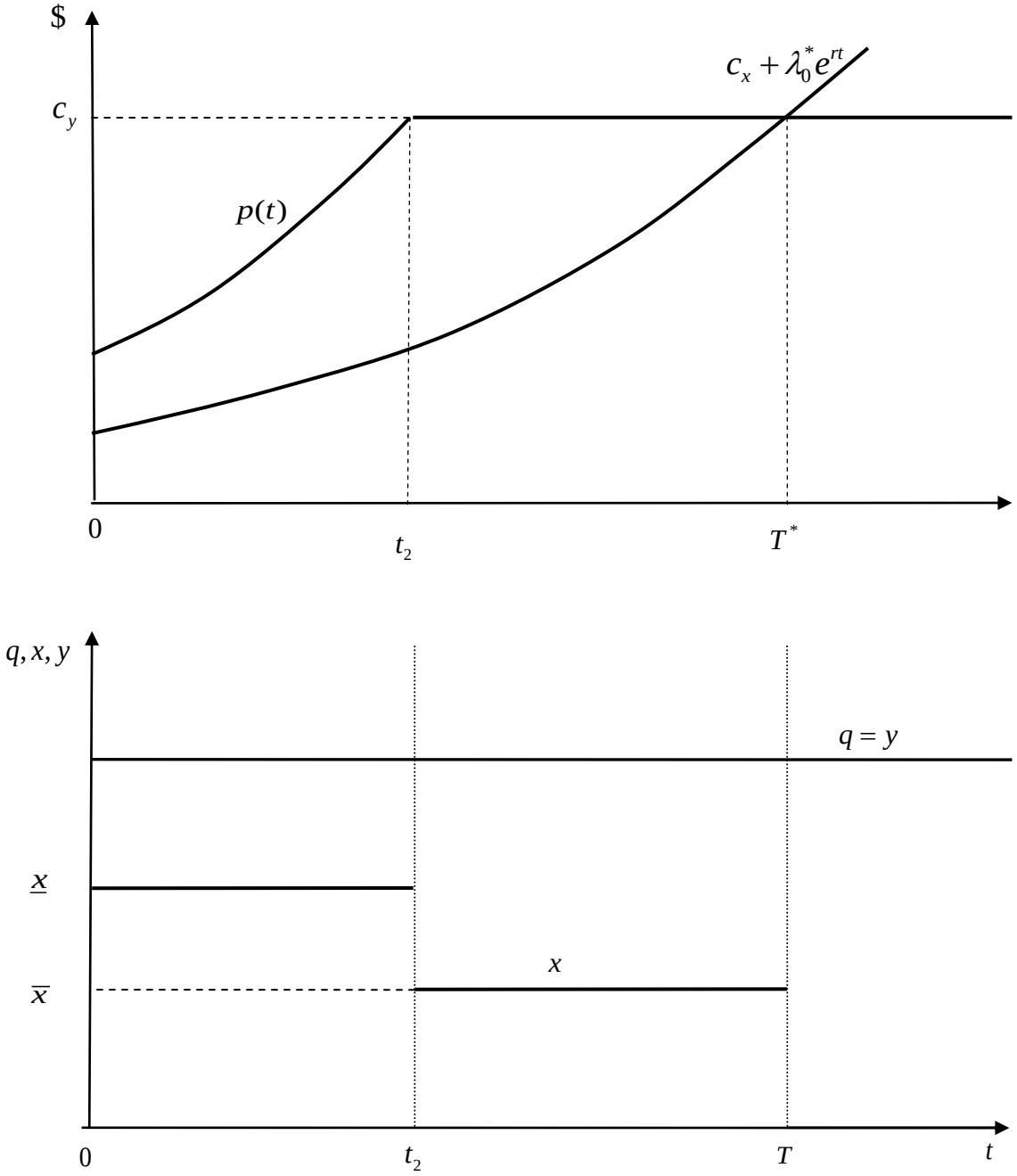

Fig.3. Resource Use under both Local and Global Caps: Abundant Coal and Cheap Substitute

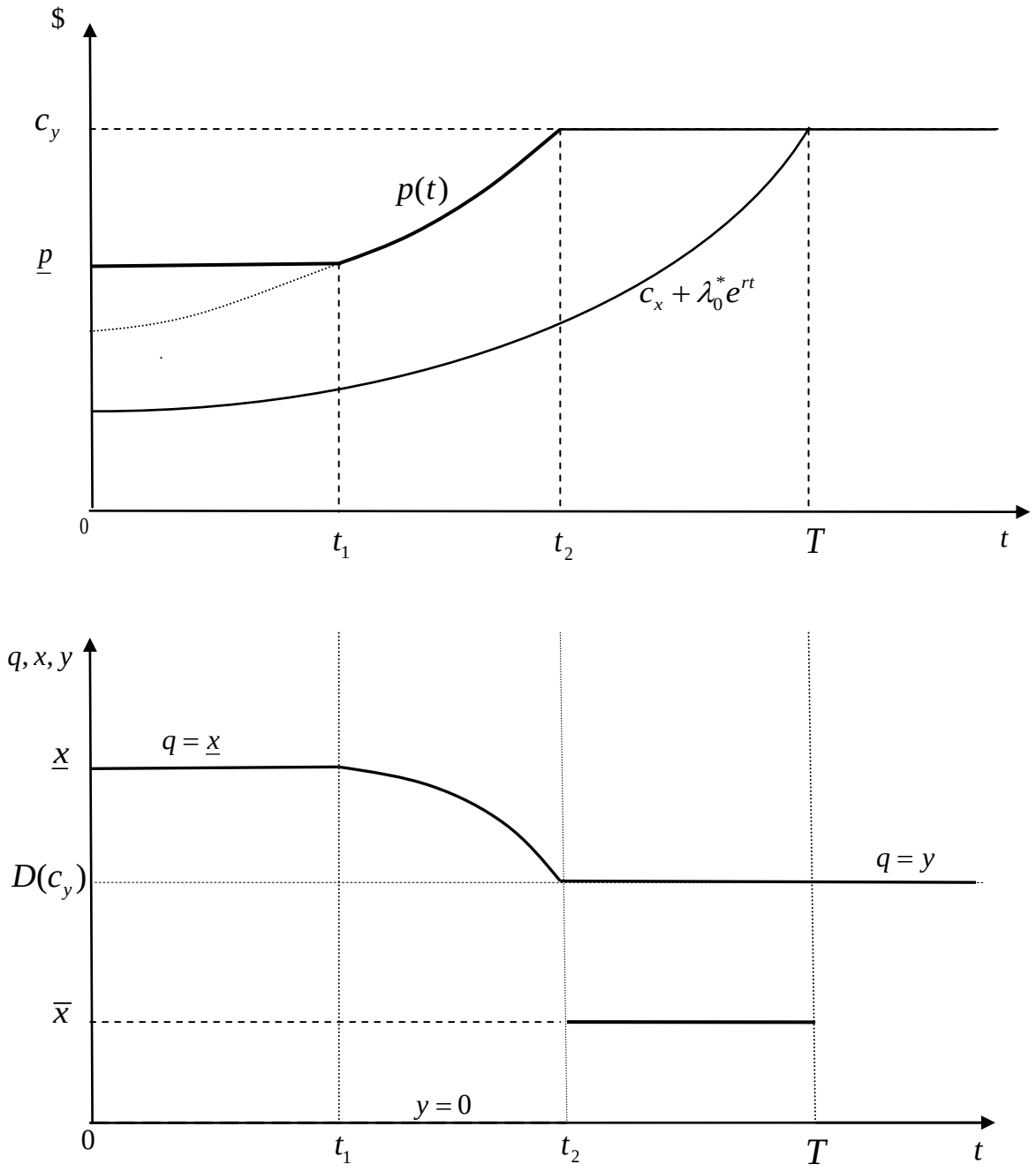

Fig.4. Both Local and Global Regulation and a Moderately Expensive Substitute with Abundant Coal: Both Resources are used at the Global Cap.

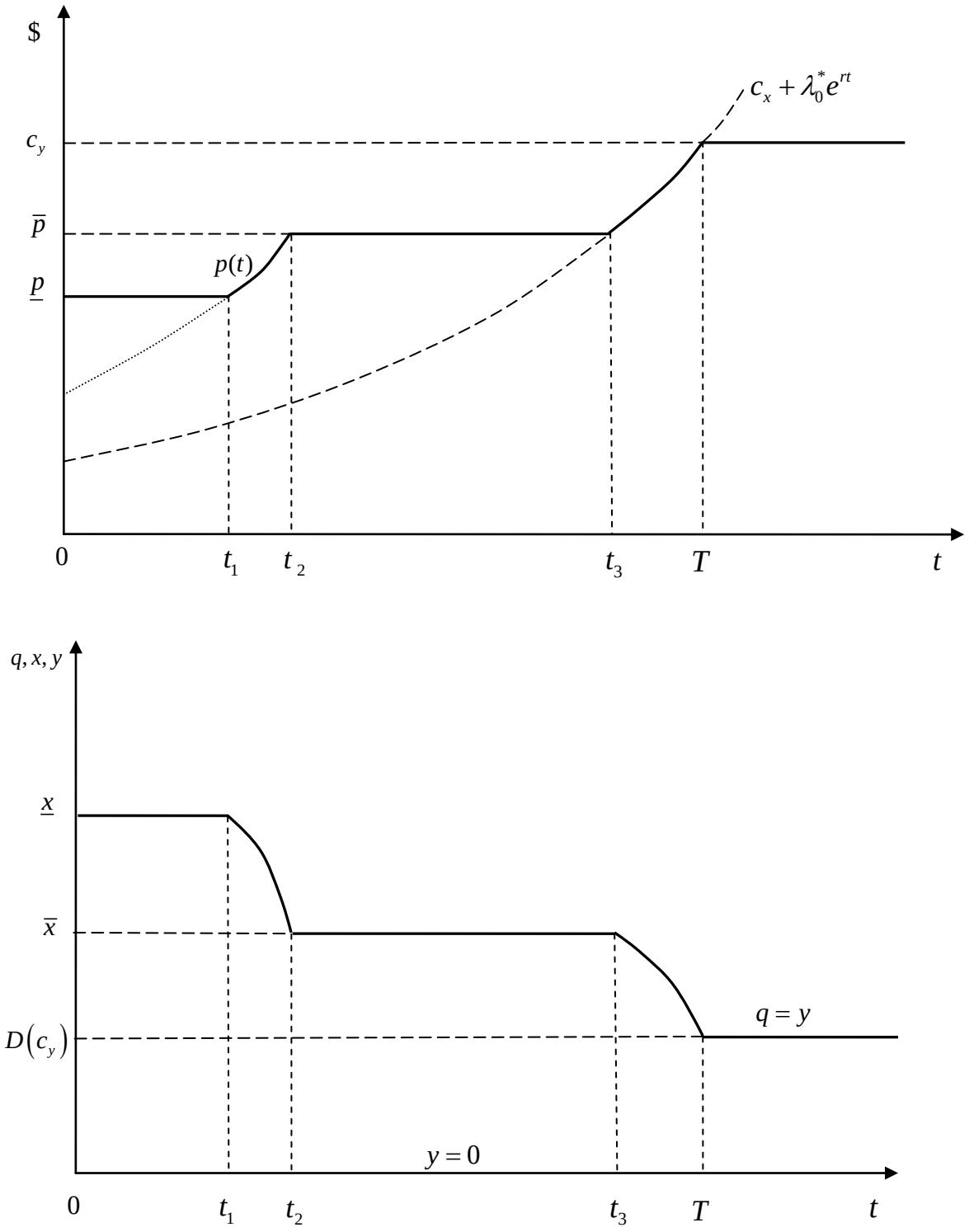

Fig.5. Both Local and Global Caps with Abundant Coal and an Expensive Substitute

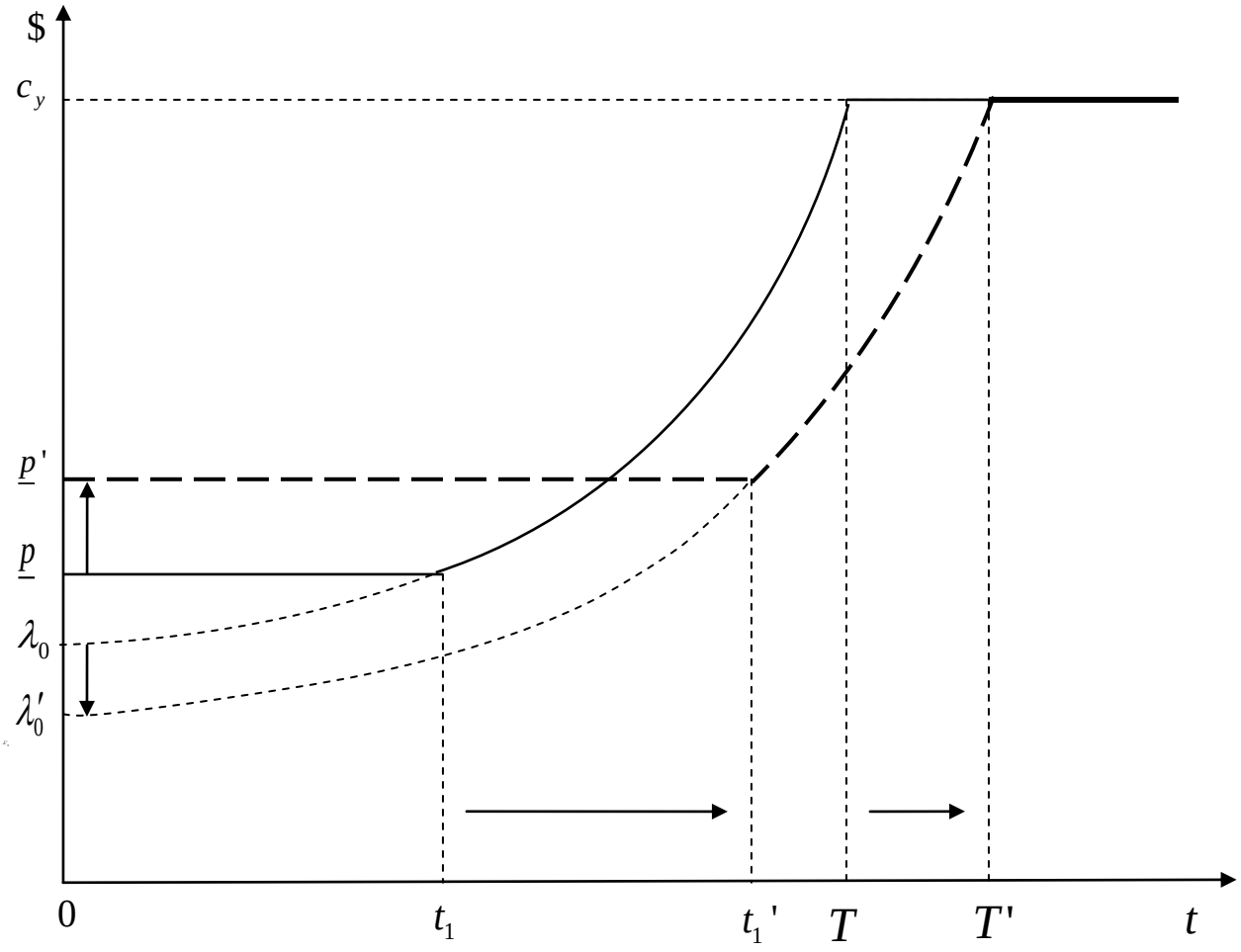

Fig. 6. Stricter Local Cap leads to a Longer Period when the Cap binds and a Delay in the Arrival of the Clean Substitute

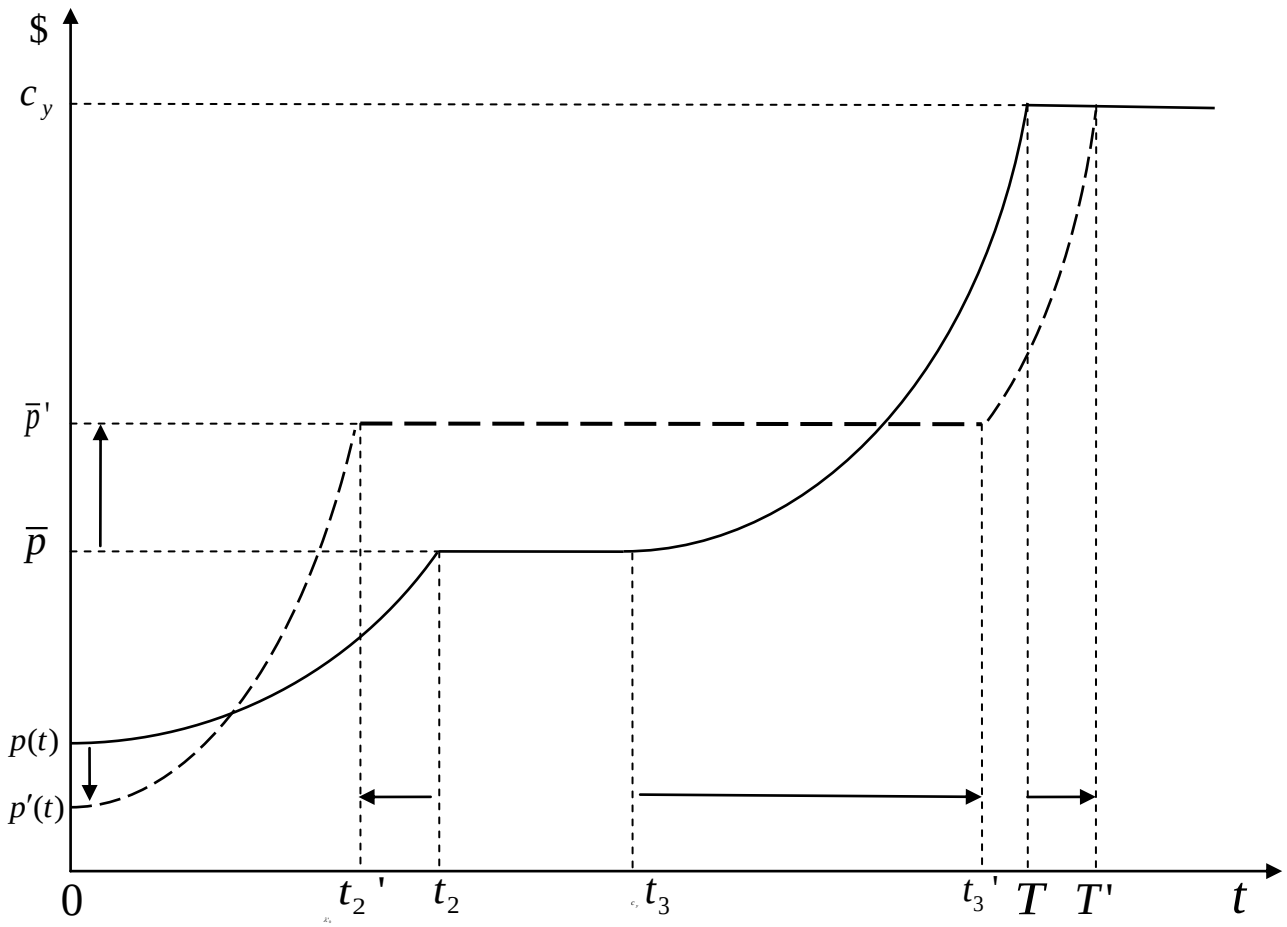

Fig.7. Stricter Global Cap leads to higher Emissions at the Beginning and an Early Arrival at the Cap

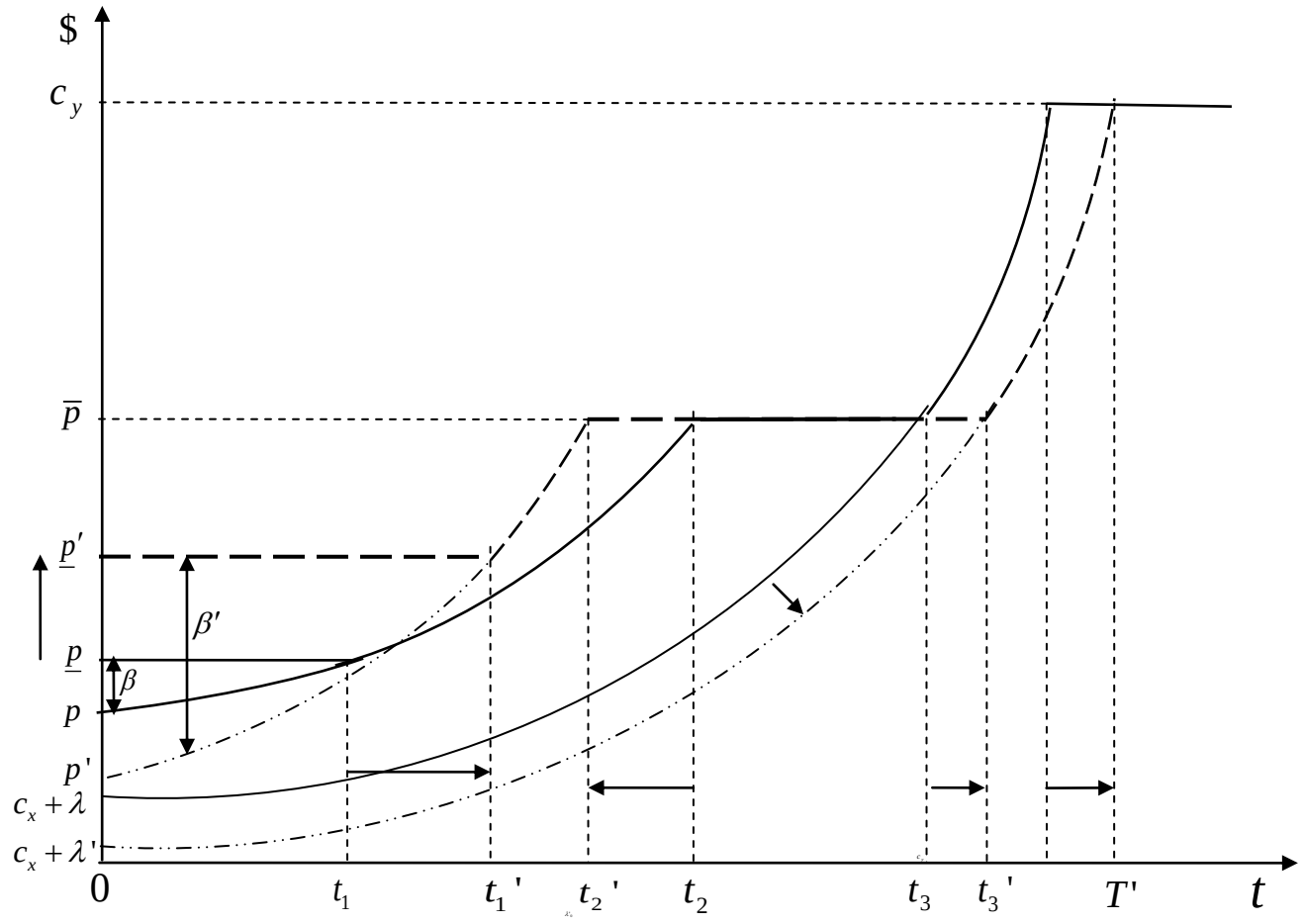

Fig 8. The Global Cap is reached earlier with a stricter Local Cap. Global and Local Taxes both go up.

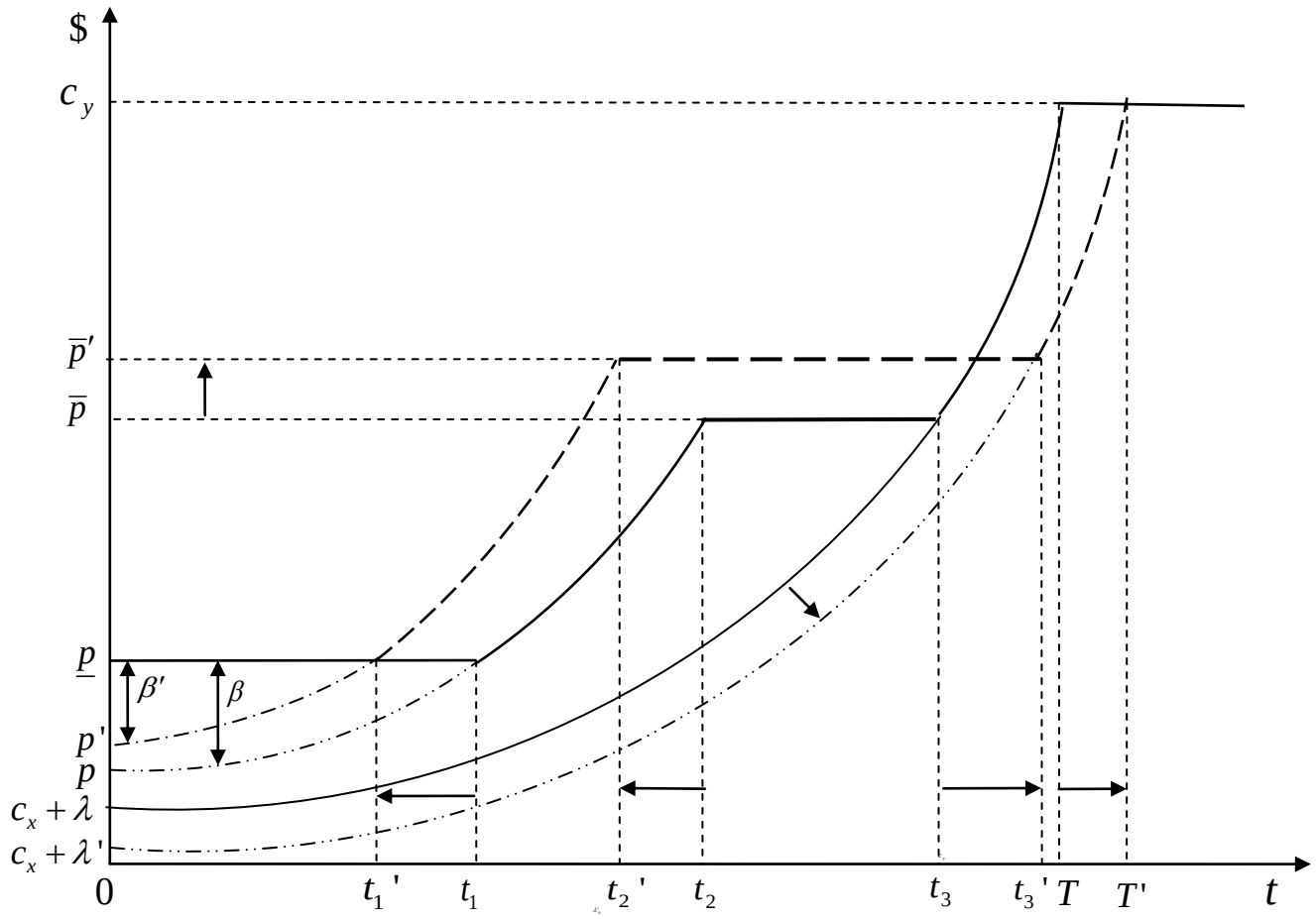

Fig 9. A stricter Global Cap may also imply a shorter stay under the Local Cap and a smaller Local Tax.

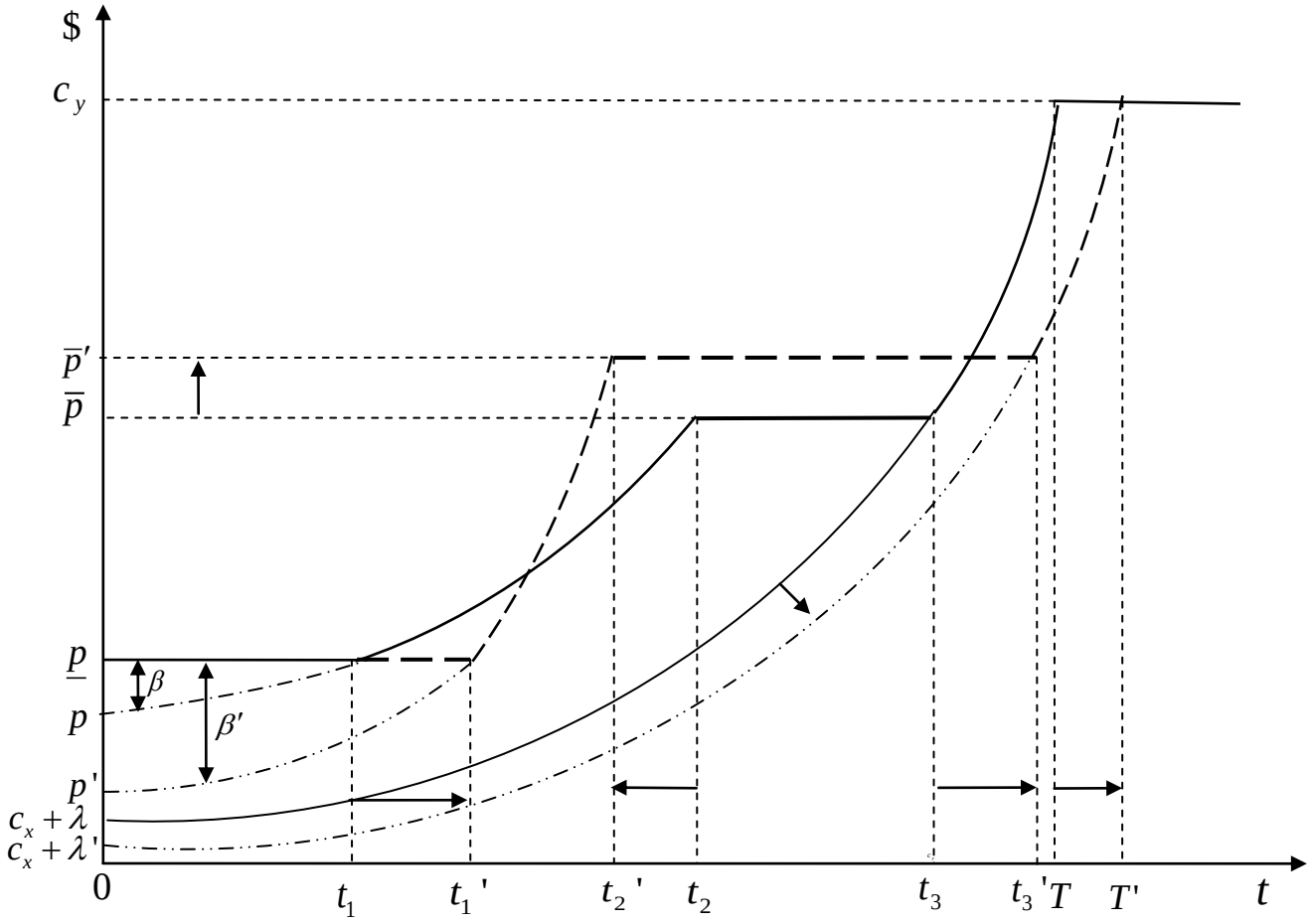

Fig 10. A stricter Global Cap may imply a longer stay under Local Regulation and a higher Local Tax.

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