Learning-by-Sharing: Monetary Policy and the Information Content of Public Signals

(Job Market Paper)

Alexandre N. Kohlhas*

April 11, 2014

Abstract

This paper studies the effect of a central bank releasing public information about the state of the economy in a dispersed information business cycle model in which market participants learn from the distribution of prices, economy-wide output and the level of the interest rate. It demonstrates how central bank information disclosure can increase the information content of public signals by making expectations of future central bank actions closer to common knowledge. This effect is shown in a calibrated business cycle model to completely offset the standard learning externality of additional public information. Central bank information disclosure can thus decrease the level of uncertainty about the state of the economy for everyone – even the central bank itself. This upends the "anti-disclosure" result in Morris and Shin (2005) and Amador and Weill (2010).

(JEL codes: E52, D82, D83)

1 Introduction

The past two decades have seen a remarkable increase in the extent to which central banks publish their views about the state of the economy. What was previously rare has become commonplace.[1] Part of the reason for this increase in the dissemination of information has been a desire to more closely align private sector expectations with those of the central bank. But as Bernanke (2004) makes clear, another part of the explanation has been the hope that with access to more information people should make better choices.

The notion that with additional signals people should make more informed decisions is appealing. Indeed, if we treat all sources of information as exogenous, a rational Bayesian decision maker can never be made worse off after the release of additional signals, no matter how noisy they are. But in reality,

*I am indebted to Anezka Christovova, Giancarlo Corsetti, Stephen Morris, Kristoffer Nimark, Pontus Rendahl, Filip Rozsypal, Daniel Quigley, Frederik Toscani, Stephen Wright and seminar and workshop participants at the University of Cambridge and Universitat Pompeu Fabra for providing invaluable comments, improving the quality of this draft. I would especially like to thank my advisor, Donald Robertson, for his continued support and guidance. This research received generous support from the Cambridge-INET Institute.

Address: Institute for International Economic Studies, Stockholm University, SE-106 91 Stockholm, Sweden.
Email: kohlhas@gmail.com
Website: https://sites.google.com/site/alexandrekohlhas/

[1] See Dincer and Eichengreen (2009).

most types of information are not exogenous but endogenous; they are, for instance, GDP releases or financial market prices. And, as shown by Morris and Shin (2005) and Amador and Weill (2010), in an economy with endogenous public signals the simple intuition does not hold.[2] The creation of additional public signals by the central bank decreases the information content of the endogenous signals already present in the economy. In fact, this decrease in the informativeness of endogenous public signals can be so great as to increase uncertainty for everyone, despite the additional information.

To date, the debate about the role of central bank information has, however, predominately treated the central bank as being outside the economic system – not specifying what economic choices it makes and how these influence people's behavior. This paper makes up for this neglect. I show that once one accounts for the effects of monetary policy on people's choices, and hence how much information they convey through their actions to others, central bank information disclosure makes endogenous public signals *more* informative; not less. By sharing its information, even the central bank itself can end up learning more.

I study the effects of central bank information disclosure in a business cycle model with a Lucas (1972) island structure. In the model, households, firms and the central bank have imperfect information about a common fundamental, total factor productivity – neither of them knows its true value. Instead, market participants learn about the true state of the economy from two types of information: (1) public information (the distribution of prices, economy-wide output and the level of interest rates); and (2) private information (island inhabitants' and the central bank's own personal signals about the true state of the economy).

The presence of island-specific signals is what makes the endogenous public signals informative: they partially reflect the sum of island-specific private information. Yet the mix of dispersed information with learning from endogenous market outcomes also implies that the underlying fundamental is not the only unobservable that matters for individual choices. Knowledge of average and central bank beliefs, average expectations of central bank beliefs, and so on *ad infinitum*, also contributes to the dynamics of market outcomes – exactly as in Allen et al's (2006) "Keynesian beauty contest" (see also Townsend, 1983). I illustrate how the impact of central bank information disclosure in this paper depends critically on these higher-order expectations.

I embed the key features – learning from monetary policy stabilized variables and central bank private information – in a dispersed information business cycle model with nominal rigidities, similar to that of Lorenzoni (2009). To present my arguments, however, I initially consider a pared down version with only one action taken by the private sector. This version can be solved analytically and provides the essential intuition for the main results. I then proceed to calibrate the accuracy of private information in the full business cycle model using data on forecast precision derived from the "Greenbook" and the "The Survey of Professional Forecasters". This allows me to quantitatively assess the effect of central bank information disclosure.

I derive the model's implications for the informational consequences of central bank transparency. This reveals the critical role played by the setting of monetary policy. I show that there exist two mechanisms by which central bank policy affects the information content of endogenous public signals

[2] The learning externality discussed in these papers resembles that in Vives (1997) (see also Vives (2013)).

and demonstrate how central bank information disclosure weakens these.

The first mechanism – *the direct stabilization effect* – details the impact of *expected future interest rates* on the information content of prices and quantities. The effect of future interest rates, in essence, creates a separability problem: People's choices, pricing and output decisions, are a function of both their beliefs about the future fundamentals of the economy and their beliefs about future central bank actions. But when the central bank attempts to stabilize the economy, the latter offsets the former. A firm, for instance, receiving a signal of low future productivity may not choose to increase its price today in anticipation of future interest rate increases. Central bank stabilization policy makes people's pricing decisions, and hence economy-wide inflation, reflect less their expectations about total factor productivity. This decreases the informativeness of the price signals.[3]

The net-effect of the direct stabilization mechanism on market participants' and the central bank's uncertainty about the underlying fundamental, however, depends critically on the information structure. Because of nominal rigidities, the more the central bank stabilizes inflation, the more output (both economy-wide and island-specific) responds to productivity shocks. Thus, since everyone observes both economy-wide prices and output, the *direct stabilization cost* on prices is (partially) offset by a *direct stabilization benefit* on quantities. With a realistic information structure and productivity shocks the net-direct stabilization effect is *a priori* ambiguous. This distinction between how expected future policy affects the informativeness of prices and quantities has not previously been addressed by the literature.[4]

The second mechanism – *the two-sided learning externality* – describes how the setting of *current interest rates* uniformly decreases the informativeness of market outcomes. Contrary to the direct stabilization effect, this mechanism, however, leads to an equilibrium trade-off between the private sector's and the central bank's knowledge about the unobserved fundamental. The setting of current interest rates decreases private sector uncertainty; it is only the central bank itself which becomes more uncertain.

The driver of this trade-off is that market participants learn about the unobserved fundamental from the observation of private sector variables but also from the central bank's decision variable; the interest rate. Crucially, the informativeness of these two sources of information is related. By observing *current* changes in the interest rate, private sector agents learn about the central bank's private information.[5] This decreases their uncertainty about the state of the economy. However, it also implies that they attach less weight to their own private information compared to the situation where there was no central bank. Private sector agents therefore impart relatively less of their private information through their actions and more of the information from the interest rate that everyone already knows. As a result, the information content of endogenous price and output signals falls. This increases central bank uncertainty, all else equal. Put differently, if market participants learn about the unobserved fundamental from the setting of monetary policy, the economy is always somewhere in between the no-disclosure/full-disclosure case discussed by Morris and Shin (2005) and Amador and Weill (2010). With the setting of monetary

[3] Weiss (1982) and King (1982) provide key early contributions analyzing the informational effects of prospective feedback policy. Recently, Lorenzoni (2010) and Angeletos and La'O (2014) analyze models containing similar mechanisms.

[4] This also illustrates the importance of the exact source of persistent innovations to the economy. With persistent innovations in household preferences (or in the mark-ups charged by firms) only a direct stabilization cost exists: in this case, expectations of future central bank actions decrease simultaneously the responsiveness of both prices and output decisions.

[5] This signaling role of monetary policy is explored further in Baeriswyl and Cornand (2010).

policy, private sector agents always observe some noisy signal of central bank beliefs.

I demonstrate that central bank information disclosure weakens both of the mechanisms by which monetary policy changes the information content of endogenous public signals. Transparency partially separates the choice of monetary policy from its impact on the information content of variables. This, in turn, implies that central bank information disclosure can increase the information content of endogenous public signals.

To start with the two-sided learning externality, transparency completely separates the choice of monetary policy from its effect; there is no additional information to be learned from the setting of interest rates. The release of central bank private information provides the cleanest signal of what private sector agents could have learned from the interest rate. Central bank transparency therefore crowds out the most amount of private information from the endogenous variables, decreasing their informativeness by the largest possible amount. Central bank information disclosure, in other words, implies the largest possible two-sided learning externality. This demonstrates how the "anti-disclosure" result by Morris and Shin (2005) and Amador and Weill (2010) extends to situations where market participants observe signals of central bank beliefs.[6]

That said, central bank information disclosure also weakens the direct stabilization effect. And unlike the elimination of the two-sided learning externality, this can increase the (net-)informativeness of public signals by alleviating the separation problem caused by the direct stabilization effect. The existence of higher-order beliefs is crucial for this finding. Transparency implies that everyone knows what everyone else, and so on, expects of the central bank. This has offsetting effects on the informativeness of prices and output.

If everyone receives a private signal that productivity has decreased, firms would want to increase their prices. Their expectations about future central bank actions, however, counter-act this desire. Knowing by how much everyone else is not changing their price by because of what they expect the central bank to do in the future increases the informativeness of the price system: it makes it a cleaner signal of, in this case, market participants' private information about low productivity. Since the central bank accommodates innovations in productivity, transparency, however, has the opposite effect on the information content of output. The reason is as follows: with information disclosure everyone does not necessarily expect future output to decrease because of future central bank actions when they receive a private signal that productivity is low. This decreases the responsiveness of current output to market participants' private information, and hence its information content.

The relative magnitude of these effects is important. It determines whether central bank information disclosure by weakening the direct stabilization effect increases or decreases on a net-basis the informativeness of public signals, all else equal. I illustrate that, in the calibrated business cycle model, the net-direct stabilization effect is positive. Central bank information disclosure through the weakening of the direct stabilization effect improves the informativeness of endogenous public signals on a combined basis; the effect on prices is stronger than the effect on output. Indeed, the effect is so large that it

[6] Amador and Weill (2010) show in an effectively static setting that the decrease in the informativeness of public signals can, in fact, be so large as to outweigh the benefit of the additional signal for the private sector. For plausible parameter values, however, this does not occur in the dispersed information business cycle model. Private sector uncertainty always falls with central bank information disclosure.

almost completely offsets the negative impact from the two-sided learning externality. Unlike in Morris and Shin (2005) and Amador and Weill (2010), central bank information disclosure does not substantially alter the informativeness of public signals. A person with access to just the distribution of prices and economy-wide output has an estimate of total factor productivity that is only c. 0.4% worse (in mean-squared error sense) under central bank information disclosure than without. In fact, for plausible deviations from the baseline parameter values, central bank transparency improves (on a net-basis) the informational efficiency of public signals. By sharing its own view of the economy, even the central bank itself can learn more.

The baseline model moreover, if anything, represents a fairly conservative starting point for assessing the informational benefits of central bank information disclosure. The persistent innovation in the model is total factor productivity. This per definition creates both a direct stabilization cost and a direct stabilization benefit. With persistent innovations in demand (household preferences), a common source of innovations in business cycle models, only the direct stabilization cost remains: both output and inflation now become less informative due to market participants' expectations of future interest rates. This, in turn, implies a greater informational benefit of central bank information disclosure. I demonstrate that with persistent innovations in demand, transparency improves the informational efficiency of all public signals across a large range of coefficients of monetary policy activism.

That the central bank is engaged in active stabilization policy and that all agents have access to independent private information therefore imply that central bank transparency can partially separate the choice of monetary policy from its informational repercussions *and*, at the same time, decrease uncertainty for everyone in the economy, even the central bank itself.

Last, this paper may also be of separate methodological interest to some: It describes how the solution technique in Kohlhas (2013), overcoming Townsend's (1983) "Problem of Infinite Regress" in a setting with asymmetric information and "informationally large agents" (meaning that their expectations are directly part of the state space), can be extended to the multivariate setting.

1.1 Related Literature

The related literature on learning from endogenous variables, as well as the effect of higher-order beliefs on equilibrium dynamics, is simply too vast to cite every worthy paper in the space of one article. That said, this work is most closely linked to four, occasionally overlapping, strands of literature.

First, this paper is related to the literature on the social value of information starting with the seminal work by Morris and Shin (2002). Vives (1997, 2013), Morris and Shin (2005) and Amador and Weill (2010, 2012) explore how the social value of information depends on the information content of endogenous public signals. But they critically do not control for how central bank information disclosure affects the informational repercussions of monetary policy. Important other contributions discussing how the social value of information depends on the "excess coordination" result in Morris and Shin (2002) are Hellwig (2004), Svensson (2006), Angeletos and Pavan (2007) and George-Marios Angeletos *et al.* (2013). Walsh (2007), Gosselin *et al.* (2007) and James and Lawler (2011, 2012) analyze the informational costs of central bank transparency in static settings, where the central bank, as in this paper, sets monetary

policy. Yet, these papers do not address how central bank stabilization policy affects the information content of endogenous public signals, or the impact of higher-order beliefs on uncertainty about the state of the economy.

Second, this paper complements the literature on the informational effects of government policies. King (1982) and Weiss (1982) provide early important contributions. More recently, Baeriswyl and Cornand (2010), Lorenzoni (2010) and Angeletos and La'O (2012, 2014) analyze how to optimally set monetary policy in effectively static settings with dispersed information. But as with the early contributions, they achieve tractability partially by limiting the role of higher-order beliefs in the equilibrium solution. In this paper, however, I allow higher-order beliefs to fully affect equilibrium outcomes. This, in turn, allows for a richer characterization of the impact of monetary policy on the information content of public signals. In particular, the positive effect of information disclosure from the weakening of the direct stabilization mechanism depends critically on higher-order thinking.

Third, this paper touches on the recent literature on dispersed information business cycle models and their solution. Lorenzoni (2009), Graham and Wright (2010) and Nimark (2012) all solve dynamic stochastic equilibrium models with dispersed information, tackling Townsend's (1983) "Problem of Infinite Regress". The novelty in this paper is that it shows how the truncated state-space solution technique in Kohlhas (2013), extending the work by Nimark (2011), can be used to solve dispersed information business cycle modes with asymmetric information, in which there is an agent, the central bank, which is "informationally large". Acharya (2013) presents a real business cycle model with dispersed information, but overcomes the infinite formation of higher-order beliefs by using the frequency domain techniques developed in Futia (1981) and Kasa (2000), and extended in Rondina and Walker (2012) and Kasa *et al.* (2012). The approach used in Acharya (2013) should be considered complementary to the one used in this paper.

Finally, this paper is related to recent work by Hellwig and Venkateswaran (2012). In their paper, they provide a "Hayekian Benchmark": conditions under which dispersed information still leads to the perfect information outcome, despite strategic interactions leading market-generated information to partially reflect higher-order beliefs. Like this paper, their work therefore analyzes the intersection between a "Hayekian" view of endogenous variables (they combine independent beliefs) and a "Keynesian" (they reflect higher-order beliefs).

1.2 Organization

The rest of this paper is organized as follows: Section 2 describes a basic model, allowing for an analytical characterization of the key insights of this paper. Section 3 presents the dispersed information business cycle model, while Section 4 outlines both the solution technique used to solve it as well as the calibration strategy employed. Section 5 and 6 demonstrate how the information content of endogenous public signals across information disclosure regimes depends on monetary policy. Section 7 concludes.

2 A Basic Model

I begin with a simple dispersed information model. The framework employed resembles a dynamic extension of Morris and Shin (2005) and James and Lawler (2011). The model has three main features: (1) a central bank which actively stabilizes an aggregate variable, used by all agents to learn about an unobserved fundamental driving the economy; (2) dispersed information – all agents, including the central bank, have private information about the unobserved fundamental; and last (3) intertemporal behavior – decisions today depend upon expected future actions. These three features allow me to analyze how monetary policy affects economic uncertainty, and hence illustrate the main points of this paper. Throughout this Section, I will be deliberately vague on the microfoundations behind the key equilibrium relationship; the next Section shows that this equation closely resembles the equilibrium conditions of a standard dispersed information business cycle model.

The model consists of a continuum of measure one of private sector individuals with imperfect information about some aggregate fundamental, θ_t. The action, a_{it}, of agent $i \in [0, 1]$ at time t is given by the linear decision rule:

$$a_{it} = \beta \mathbb{E}_{it} [a_{it+1}] + \theta_{it} + \gamma i_t, \tag{2.1}$$

where $\mathbb{E}_{it} [\cdot]$ denotes the expectation operator conditional on agent i's information set at time t; $0 < \beta < 1$ the discount factor; and $\theta_{it} \equiv \theta_t + \epsilon^{\theta}_{it}$, $\epsilon^{\theta}_{it} \sim WN(0, 1/p_{p\theta})$ an agent-specific noisy realization of θ_t. Last, i_t denotes the monetary authority's policy instrument and $\gamma \leq 0$. Depending on the exact interpretation of a_{it}, this setup can naturally be thought of as the reduced form of either the equilibrium condition determining how a firm sets prices under monopolistic competition (see Woodford (2002)) or a linearized consumer euler equation with demand shocks. In fact, in Section 3 below, a_{it} will be a vector containing both the price set by a firm under monopolistic competition and the consumption choice of a household. The exact details of the environment underlying (2.1) are, however, not crucial for the time being. What is important though is that the individual's action depends on the underlying fundamental, the central bank's policy instrument as well as the agent's expectation of next period's action. In particular the forward-looking component of (2.1) will be important for deriving our main results.

Equation (2.1) implies that the endogenous aggregate action, $a_t \equiv \int_0^1 a_{it} di$, equals:

$$a_t = \beta \bar{\mathbb{E}}_t [a_{t+1}] + \theta_t + \gamma i_t, \tag{2.2}$$

where $\bar{\mathbb{E}}_t [\cdot] \equiv \int_0^1 \mathbb{E}_{it} [\cdot] di$ is the average expectation operator across private sector individuals.[7] The central bank is described by a simple Taylor-type rule, in which i_t is set according to:

$$i_t = \phi \mathbb{E}_{cbt} [a_{t+1} - \theta_{t+1}] + \epsilon^{mp}_t, \ \epsilon^{mp}_t \sim WN(0, 1/p_{mp}), \tag{2.3}$$

where ϵ^{mp}_t denotes a monetary policy shock and $\phi > 0$ the degree of central bank activism.[8]

[7] If we interpret a_{it} as the price set by firm i at time t, then a_t corresponds to an aggregate price index. From (2.2) the aggregate price depends on the average expectation of future aggregate prices, an underlying shock (e.g. productivity) and last the central bank interest rate. The key to derive (2.2) from (2.1) is that the cross-sectional distribution of agents' actions is constant with a finite variance (see below). The strong law of large numbers therefore implies that: $\int_0^1 \mathbb{E}_{it} [a_{it+1}] di = \int_0^1 \mathbb{E}_{it} [a_{t+1}] di$.

[8] In (2.3) the central bank sets the policy instrument to stabilize expected deviations of the aggregate action from

Last, for simplicity, the unobserved underlying fundamental, θ_t, follows a random walk:

$$\theta_t = \theta_{t-1} + \epsilon_t^\theta,\ \epsilon_t^\theta \sim WN\left(0,\ 1/p_\theta\right). \tag{2.4}$$

2.1 Information Structure

Before we can investigate how central bank policy changes the information content of the endogenous aggregate, a_t, it is necessary to specify the information structure of the economy. I assume that all agents in the economy – both public and private – observe i_t in addition to a noisy realization of a_t. Each person also observes their own private information about θ_t as well as last period's fundamental, θ_{t-1}. The information set of private sector agent i at time t is therefore given by:

$$\mathcal{F}_{it} = \{\theta_{i\tau},\ \tilde{a}_\tau,\ i_\tau,\ \theta_{\tau-1}\}_{\tau=-\infty}^{\tau=t}$$

where $\theta_{i\tau}$ denote agents' private information about the state of the economy and $\tilde{a}_\tau \equiv a_\tau + \epsilon_\tau^a,\ \epsilon_\tau^a \sim WN\left(0, 1/p_a\right)$.

Turning to the central bank, I assume that its information set can be written as:

$$\mathcal{F}_{cbt} = \{\theta_{cb\tau},\ \epsilon_\tau^{mp},\ \tilde{a}_\tau,\ i_\tau,\ \theta_{\tau-1}\}_{\tau=-\infty}^{\tau=t},$$

where the central bank's private information about the underlying fundamental is defined by:

$$\theta_{cbt} \quad \equiv \quad \theta_t + \epsilon_{cbt}^\theta, \quad \epsilon_{cbt}^\theta \sim WN\left(0,\ 1/p_{cb\theta}\right).$$

Since $p_{cb\theta}$ is allowed to be greater than $p_{p\theta}$, this specification allows the central bank to potentially have more precise private information about the underlying fundamental than individual agents.[9]

The observation of the aggregate action in the economy is subject to "noise", ϵ_t^a. This is important; otherwise, both private sector agents and the central bank would be able to back out the underlying fundamental from their knowledge of (2.2) (see also Baxter *et al.* (2011)). While I further below give an economic interpretation of the analog to ϵ_t^a, the important role it plays in this model is to limit agents' ability to infer the true value of the underlying fundamental from the observation of endogenous variables. The use of "non-invertibility" shocks to maintain imperfect information in a rational expectations model is common in the literature. A similar modeling device is used in, for instance, Singleton (1987), Allen *et al.* (2006) and Lorenzoni (2009).

the unobserved fundamental. The crucial feature of (2.3) is, however, only that the central bank sets the interest rate depending on its expectation of θ_t. All of the below results therefore also carry through using the interest rate equations: $i_t = \bar{\phi}\mathbb{E}_{cbt}\left[a_{t+1}\right] + \epsilon_t^{mp}$ or $i_t = \bar{\phi}\mathbb{E}_{cbt}\left[\theta_t\right] + \epsilon_t^{mp}$, $\bar{\phi} > 0$.

[9] To ensure that central bank's estimates are more precise than the corresponding private sector ones it is necessary to assume that the central bank's private information is sufficiently more precise than the private sectors. For $p_{cb\theta} = p_{p\theta}$, the increased correlation between the monetary authority's signals, due to the impact of i_t on a_t, will cause the central bank to be worse at estimating θ_t than the private sector. Hellwig (1980) discusses a similar effect when there are informationally "large" players in asset markets.

2.2 Model Solution

To allow for a simple analytical characterization of the equilibrium of the model, I have above assumed that all agents in the economy, both public and private, perfectly observe last period's fundamental. Given the assumption that all agents observe θ_{t-1} at time t, the model boils down to a series of essentially static problems with a unique linear rational expectations solution.

That said, assuming that the central bank learns directly from a forward-looking endogenous variable complicates the solution of the model by implying a specific simultaneity problem: The endogenous variable, a_t, will depend on the current central bank estimate of the state via the interest rate decision, but the latter will also depend on the forward-looking variable. Appendix A shows how to solve this fixed-point problem using an approach similar to the one applied in Svensson and Woodford (2004). Proposition 1 details the solution of the model.

Proposition 1. *The unique linear rational expectation solution can be written as:*

$$\begin{aligned} \tilde{a}_t &= k_0\theta_{t-1} + k_1\epsilon_t^\theta + k_2 i_t + k_3\epsilon_t^a \\ i_t &= \bar{k}_0\theta_{t-1} + \bar{k}_1\epsilon_t^\theta + \bar{k}_2\epsilon_t^a + \bar{k}_3\epsilon_t^{mp} + \bar{k}_4\tilde{\theta}_{cbt}, \end{aligned}$$

where $k_j \in \mathbb{R}$, $j \in \{0, 1, 2, 3\}$, $\bar{k}_j \in \mathbb{R}$, $j \in \{0, 1, 2, 3, 4\}$ *and* $\tilde{\theta}_{cbt} \equiv \theta_{cbt} - \theta_{t-1}$.

Proof. See Appendix A.[10] □

Because individuals and the central bank have imperfect dispersed information, the solution coefficients in Proposition 1 partially depend on agents' signal extraction problem. The informativeness of the aggregate action, $\tilde{a}_t$, is thus determined by how much weight agents place on the various signals. These signal extraction weights will, in turn, depend on the solution coefficients due to the learning from the endogenous variable. Another fixed point problem arises.

To see how Proposition 1 can be used to find an analytical expression for the informativeness of the aggregate action, re-write (2.2), using the definition of $\tilde{a}_t$, as:

$$\begin{aligned} \tilde{a}_t - \gamma i_t &= \sum_{j=0}^{\infty} \beta^j \theta^{(j)}_{t+j|\ldots|t} + \gamma \sum_{j=1}^{\infty} \beta^j i^{(j)}_{t+j|\ldots|t} + \epsilon_t^a \\ &= \theta_t + \left(1 + \gamma\bar{k}_0\right) \sum_{j=1}^{\infty} \beta^j \theta^{(j)}_{t+j|\ldots|t} + \epsilon_t^a \end{aligned} \tag{2.5}$$

where $\theta^{(j)}_{t+j|\ldots|t} \equiv \bar{\mathbb{E}}_t\left[\theta^{(j-1)}_{t+j|\ldots|t+1}\right]$ with $\theta^{(0)}_{t|t} \equiv \theta_t$ and similarly $i^{(j)}_{t+j|\ldots|t} \equiv \bar{\mathbb{E}}_t\left[i^{(j-1)}_{t+j|\ldots|t+1}\right]$ with $i^{(0)}_{t|t} \equiv i_t$. $\theta^{(j)}_{t+j|\ldots|t}$ and $i^{(j)}_{t+j|\ldots|t}$ denote *dynamic higher-order average expectations* of θ_t and i_t: the average expectation at time t of the average expectation at time $t+1$, and so on, of the underlying fundamental or the interest rate at time $t+j$. Each dynamic higher-order expectation of θ_t will contain some information about the unobserved fundamental. This increases the informativeness of the aggregate action beyond the direct impact of the fundamental. Since all individuals in this model perfectly observe last period's

[10] For reference, the solution under full information is: $\tilde{a}_t = \frac{1-\gamma\phi-\gamma\phi\beta}{1-\gamma\phi-\beta}\theta_t + \gamma i_t + \epsilon_t^a$ and $i_t = \frac{\beta\phi}{1-\gamma\phi-\beta}\theta + \epsilon_t^{mp}$.

fundamental, θ_{t-1}, the infinite sequence of higher-order expectations in (2.5) collapses into an expression depending only on first-order beliefs.[11] Equation (2.5) becomes:

$$\tilde{a}_t - \gamma i_t = \theta_t + \beta \frac{1+\gamma \bar{k}_0}{1-\beta} \bar{\mathbb{E}}_t \left[\theta_t\right] + \epsilon_t^a. \tag{2.6}$$

The average expectation of the fundamental, $\bar{\mathbb{E}}_t \left[\theta_t\right]$, in (2.6) can, in turn, be split into the weighted sum of the average expectation conditional only on private information, $\tilde{\theta}_{it} \equiv \theta_{it} - \theta_{t-1}$, and the average expectation conditional on (some linear combination of) the public signals. But since all individuals in the economy observe the public signals – specifically, the interest rate and last period's fundamental – agents can deduct the importance of both θ_{t-1} and i_t on the informativeness of $\tilde{a}_t$. The additional information that agents learn from observing the aggregate variable is therefore only given by the expectation conditional on private information. The informativeness of the action, $\tilde{a}_t$, hence depends only on the informativeness of the variable s_t^a:

$$s_t^a \equiv \left(1 + \beta \frac{1+\gamma \bar{k}_0}{1-\beta} w_\theta\right) \epsilon_t^\theta + \epsilon_t^a,$$

where w_θ denotes the weight private sector individuals place on their private information, $\tilde{\theta}_{it}$, when forming their expectations, $\mathbb{E}_{it} \left[\theta_t\right]$. The signal-to-noise ratio of s_t^a – how informative our aggregate action is – depends on the expression: $\left(1 + \beta \frac{1+\gamma \bar{k}_0}{1-\beta} w_\theta\right)$. Referring back to Proposition 1, this term has to in equilibrium equal the ratio of coefficients k_1/k_3.

The informativeness of the aggregate action is thus determined by the unique solution to the equation:

$$\frac{k_1}{k_3} = 1 + \beta \frac{1+\gamma \bar{k}_0}{1-\beta} w_\theta = 1 + \beta \frac{1-\phi\gamma}{1-\beta-\phi\gamma} w_\theta \left(\frac{k_1}{k_3}, \phi\right), \tag{2.7}$$

where $\bar{k}_0 = \frac{\phi\beta}{1-\beta-\phi\gamma} > 0$ and I have denoted the dependence of the private sector's signal extraction weight on the ratio k_1/k_3 and monetary policy activism ϕ (see Appendix A).

Equation (2.7) illustrates that the informativeness of the action, a_t, depends upon monetary policy activism, ϕ; both through the *prospective feedback policy coefficient*, $\bar{k}_0$, and via the implicit dependence of w_θ on ϕ. The more weight private sector individuals place on their own private signal – that is, the larger w_θ is – the more the aggregate action reflects the sum of individual agents' private information; the truly new information that people can learn from $\tilde{a}_1$. But how much weight individuals place on their private information, amongst others, depends on the extent of monetary policy activism.

Furthermore, equation (2.7) also directly determines central bank uncertainty about the fundamental. Since the central bank can only learn additional information about the state of the economy from the endogenous aggregate, (2.7) determines the extent of central bank knowledge about θ_t. (The private sector also observes the interest rate, so the mapping between k_1/k_3 and private sector uncertainty is not one-for-one.)

[11] The term $\bar{\mathbb{E}}_t \bar{\mathbb{E}}_{t+1} \left[\theta_{t+2}\right]$, for instance, becomes $\bar{\mathbb{E}}_t \bar{\mathbb{E}}_{t+1} \left[\theta_{t+1}\right] = \bar{\mathbb{E}}_t \left[\theta_t + \bar{\mathbb{E}}_{t+1} \epsilon_{t+1}^\theta\right] = \bar{\mathbb{E}}_t \left[\theta_t\right]$.

2.3 The Direct Stabilization Effect à la King (1982)

The above outlined model contains two mechanisms through which changes in monetary policy affect the informativeness of the aggregate action. These two mechanisms correspond to the impact of *expected future* interest rates and *current* interest rates, respectively, on the information content of $\tilde{a}_t$. They also mirror the two channels identified above whereby changes to ϕ affect the ratio k_1/k_3. Depending on the exact parameter specification governing the economy, either of these mechanisms can dominate, with potentially quite different policy implications.

To start, I focus on the effect of expected future interest rates. I call this mechanism, for reasons which will be clear in a moment, the direct stabilization effect. Proposition 2 details its impact.

Proposition 2. *("The Direct Stabilization Effect") If $p_{mp} \to \infty$ and $\gamma < 0$ then $-1 < \gamma\bar{k}_0 < 0$. An increase in monetary policy activism (as measured by ϕ) will then decrease the informativeness of the action, $\tilde{a}_t$, increasing uncertainty about the underlying fundamental, θ_t, for everyone in the economy.*[12]

Proof. See Appendix A □

As (2.7) makes clear – for given private sector signal extraction weights – increases in monetary policy activism, ϕ, increase the prospective feedback policy coefficient, $\bar{k}_0$, leading to a fall in the right-hand side of the equation ($\partial\bar{k}_0 \setminus \partial\phi > 0$, $\partial\left(k_1 \setminus k_3\right) \setminus \bar{k}_0 < 0$). This decreases in equilibrium the informativeness of the endogenous public signal, and thus, all else equal, increases uncertainty for both the central bank and the private sector.

To better understand the intuition behind Proposition 2 consider equation (2.5). This equation illustrates two aspects of the the direct stabilization effect: First, in line with the conjecture in King (1982), if private sector agents also observe the policy instrument, *current feedback policy* does not impact the information contained in the endogenous aggregate. Since all agents observe the interest rate simultaneously with $\tilde{a}_t$, they can condition out the effect of changes in i_t on the endogenous aggregate, and thus on the information received when observing it.

Second, if $\gamma\bar{k}_0 < 0$ – which for instance can be the case if a_t is interpreted as aggregate inflation and θ_t as the negative of a productivity shock – *prospective feedback policy* will cause the endogenous aggregate to be less informative about the underlying fundamental. To see why this is the case, consider what happens when all private sector agents observe a large realization of $\theta_{it} > 0$. All else equal, agents will revise up their expectation of the entire future path of the fundamental. But if $\bar{k}_0 > 0$ they will also revise up their expectation of all future interest rates. If at the same time $\gamma < 0$ – such that increases in interest rates depress the aggregate action – this implies that in response to $\theta_{it} > 0$ agents will not expect *future* aggregate actions to increase greatly. Thus, the *current* aggregate action, a_t, will increase by less, revealing less information to all agents in the economy than it would have done otherwise – i.e. if $\gamma\bar{k}_0 = 0$. This clearly illustrates the direct stabilization effect: stabilization policy implies per definition more stable endogenous aggregates. But the more stable the endogenous aggregates are, the less informative they will be. Equation (2.5) makes it clear that it is *expected future policy actions* that cause the direct stabilization effect rather than the immediate impact of interest rate changes.[13]

[12] That $p_{mp} \to \infty$ ensures that changes in ϕ do not also change private sector signal extraction weights.

[13] That stabilization policy decreases the informativeness of prices (and other endogenous public signals) is an idea that

In principle, $\gamma\bar{k}_0$ can imply a sufficiently powerful offset on future aggregate actions from movements in the interest rate, such that $a_t - \gamma i_t$ will decrease in response to an increase in the fundamental. As Proposition 2 shows, however, the condition $-1 < \gamma\bar{k}_0 < 0$ will always hold in this simple model if γ is strictly negative. That is, if interest rate increases depress the aggregate action.

Remark. (Concern or Benefit?) The condition $\gamma\bar{k}_0 < 0$ is likely to hold if we think of a_t as being aggregate inflation and θ_t as the negative of total factor productivity. If a_t, however, instead is interpreted as aggregate output, the sign of the direct stabilization effect flips around. In a standard business cycle model with nominal rigidities, output responds too sluggishly to productivity shocks. It is therefore optimal for the central bank to partially accommodate these innovations (see Gali (2008)). If monetary policy behaves optimally, it is then more likely that $\bar{k}_0 < 0$, and hence from (2.5) that prospective feedback policy helps the dissemination of information via aggregate variables. Whether economic agents observe prices or quantities (or both) therefore determine whether increases in monetary policy activism imply more or less uncertainty about the driving forces of the economy.[14]

Proposition 2 shows that the direct stabilization effect operates similarly across both the private sector and the central bank. Increases in monetary policy activism, ϕ, lead to similar changes in uncertainty for both individual agents and the monetary authority. However, when private sector agents also try to learn the central bank's private information via the setting of *current* interest rates, there also exists another channel by which changes in the aggressiveness of monetary policy affect economic uncertainty. And, unlike the direct stabilization effect, this channel has disparate effects across the private sector and the central bank.

2.4 A Two-sided Learning Externality

To show that there exists an informational trade-off between the private sector, on the one hand, and the central bank, on the other hand, I set $\gamma = 0$. This allows us to focus on the two-sided learning externality – the effect of *current* interest rates – without having prospective central bank policy directly muting the information content of the aggregate variable. In terms of (2.7), setting $\gamma = 0$ allows us to analyze how changes in ϕ affect k_1/k_3 through the signal extraction weight, w_θ, keeping $\gamma\bar{k}_0$ constant. Lemma 1 and Proposition 3 detail the impact of increases in monetary policy activism when the direct stabilization effect is absent.

Lemma 1. *If $\gamma = 0$, the informativeness of the interest rate, i_t, and hence the weight attached to it in the private sector's signal extraction problem, is strictly increasing in monetary policy activism, ϕ. The weights attached to the aggregate action, $\tilde{a}_t$, and the private signal, θ_{it}, on the contrary, are strictly decreasing. In response to increases in ϕ, the central bank also places less weight on $\tilde{a}_t$ in its own signal extraction problem. Unlike individual agents, though, it places more weight on its private signal θ_{cbt}.*

can be traced back to Hayek (1945):"*We must look at the price system as such a mechanism for communicating information if we want to understand its real function – a function which, of course, it fulfills less perfectly as prices grow more rigid* (and stable)" p. 526.

[14]The difference between the informational impact of policy depending on whether agents observe prices or quantities is quite general. It will, for instance, also exist in asset markets, where agents have access to the total volume traded. However, the sign of the direct stabilization effect is quite sensitive to the assumption of the underlying shock. Replacing the productivity shock in our thought experiment with a demand shock would imply that both output and prices become less informative as we increase monetary policy activism. To the best of my knowledge, this difference between quantities and prices as well as demand and supply shocks has not previously been studied in the literature.

Proof. See Appendix A. □

Proposition 3. *("The Two-sided Learning Effect") If $\gamma = 0$, an increase in monetary policy activism (as measured by ϕ) improves the private sector's knowledge about the underlying fundamental, θ_t, but generates less precise information for the central bank. The informativeness of the aggregate action, $\tilde{a}_t$, decreases.*

Proof. See Appendix A. □

To understand these changes to the signal extraction weights, and their corresponding impact on the informativeness of $\tilde{a}_t$, re-write the interest rate equation, using Proposition 1, as:

$$\begin{aligned} i_t &= \bar{k}_0 \mathbb{E}_{cbt}\left[\theta_t\right] + \epsilon_t^{mp} \\ &= \bar{k}_0 \left[\theta_{t-1} + w_{cba} s_t^a + w_{cb\theta} \tilde{\theta}_{cbt}\right] + \epsilon_t^{mp}, \end{aligned} \tag{2.8}$$

where w_{cba} and $w_{cb\theta}$ denote central bank signal extraction weights and $\tilde{\theta}_{cbt} = \theta_{cbt} - \theta_{t-1}$. The signal private sector individuals receive from observing the interest rate, conditional on the rest of their information sets, therefore equals:

$$s_t^i \equiv \bar{k}_4 \tilde{\theta}_{cbt} + \epsilon_t^{mp}, \tag{2.9}$$

where $\bar{k}_4 = \bar{k}_0 w_{cb\theta}$ and $\partial \bar{k}_4 / \partial \phi > 0$ (see Appendix A).

Equation (2.9) allows us to illustrate the intuition behind Lemma 1 and Proposition 3: When the central bank becomes more active, it becomes easier for the private sector to disentangle the central bank's expectation about the fundamental, and thus the central bank's private signal, $\tilde{\theta}_{cbt}$, from the policy instrument. This increase in the signal-to-noise ratio of the interest rate implies a greater weight attached to it in the private sector's signal extraction problem. But the corollary of the greater weight placed on the interest rate by private sector agents is a lower weight attached to their own private information, $\tilde{\theta}_{it} (\partial w_\theta / \partial \phi < 0)$. Since private sector individuals use less their private signal when determining their own action, the endogenous aggregate reflects less the combined independent information of the private sector – the truly new information that people could learn – and more the information from the public interest rate that everyone already knows. This decreases the information content of $\tilde{a}_1$. The monetary authority hence increasingly learns only its own private information from the observation of the endogenous aggregate and correspondingly attaches less weight to it in its own signal extraction problem.

In equilibrium, increases in monetary policy activism therefore lead to less private sector uncertainty about the underlying fundamental, as the increase in the signal-to-noise ratio of the interest rate improves the ability of private sector agents to infer the true value of the fundamental. The central bank, knowing its own private information, however, only suffers: the decreased imparting of private information decreases the informational efficiency of the endogenous aggregate, and thus, all else equal, increases central bank uncertainty (see 2.7).[15]

[15] The central bank's and individual $i's$ mean-squared error of the current value of θ_t is given by, respectively:

$$\begin{aligned} MSE_{cbt}(\theta_t) &= \left[p_\theta + p_{cb\theta} + (k_1/k_3)^2 p_a\right]^{-1} \\ MSE_{it}(\theta_t) &= \left[p_\theta + p_{cb\theta} + (k_1/k_3)^2 p_a + \frac{p_{cb\theta} p_{mp} \bar{k}_4^2}{p_{cb\theta} + p_{mp} \bar{k}_4^2}\right]^{-1}, \end{aligned}$$

In sum, if private sector individuals learn from the setting of current interest rates and the central bank has access to private information, deciding on the optimal degree of monetary policy activism is complicated. Not only does the central bank face the usual optimal stabilization problem but it also has to internalize the informational consequences of its actions. Like with expected future interest rates, the setting of current interest rates decreases the information content of $\tilde{a}_t$. But unlike the direct stabilization effect, the two-sided learning externality implies an informational trade-off: more active central bank policy leads, on the one hand, to more informed private sector decisions, but, on the other hand, also to less informed public policy.[16]

2.5 Central Bank Information Disclosure

The above showed how monetary policy changes the information content of endogenous public signals through current and expected future interest rates. In this subsection, I illustrate how central bank information disclosure – the public dissemination of the central bank's private signal – alters these mechanisms. Specifically, I demonstrate that central bank information disclosure substantially mutes the strength of both mechanisms by which monetary policy decreases the information content of $\tilde{a}_t$. This, in turn, implies that – depending on whether the two-sided learning externality or the direct stabilization effect dominates – information disclosure itself increases or decreases the informativeness of public signals.[17]

To illustrate how central bank information disclosure limits the above effects, I consider a simplified two-period version of the baseline model without perfect state revelation – that is when θ_{t-1} is not part of agents' information set. Without the perfect state revelation assumption the infinite-horizon model suffers from Townsend's (1983) "Problem of forecasting the forecast of others": the infinite sequence of higher-order beliefs in the forward solution in (2.5) does not collapse into an expression involving only first-order expectations. Similar to Keynes' (1936) beauty contest, the underlying state driving a_t and i_t is comprised of all higher-order average and central bank beliefs of θ_t *ad infinitum*. Limiting the model to only two periods circumvents the problem of infinite regress of expectations; from (2.5) only the first two segments of higher-order beliefs matter for the equilibrium solution. The presence of higher-order beliefs is important – it is what will drive the weakening of the direct stabilization effect.

The equations of motion for the aggregate action, a_t, and the interest rate, i_t, are:

$$a_t = \beta\bar{\mathbb{E}}_t\left[a_{t+1}\right] + \gamma i_t, \tag{2.10}$$

$$i_t = \phi\mathbb{E}_{cbt}\left[\theta\right] + \epsilon_t^{mp}, \quad \epsilon_t^{mp} \sim N\left(0,\, 1/p_{mp}\right), \tag{2.11}$$

where $t = \{1,\, 2\}$, $\theta \sim N\left(0,\, 1/p_\theta\right)$ and by assumption $a_3 = \theta$. Equation (2.10) has no direct impact of

where $\bar{k}_4 = \bar{k}_0 w_{cb\theta}$. Since the signal-to-noise ratio of $\tilde{a}_t$, determined by k_1/k_3, decreases as w_θ drops (see 2.7), central bank uncertainty *increases*. But as shown in Appendix A, the increase in $\bar{k}_4$ – the increase in the signal-to-noise ratio of the interest rate – is sufficiently strong to offset the decrease in k_1/k_3, resulting in a *decrease* in private sector uncertainty.

[16] In a setting where $\gamma \neq 0$ and p_{mp} is finite, the mechanism behind the two-sided learning externality also has implications for the direct stabilization effect. The more active the central bank is, the cleaner a signal the interest rate is of its beliefs. This decreases private sector uncertainty about central bank expectations, which, in turn, decreases the strength of the direct stabilization effect; it causes agents to use their private information less when inferring central bank beliefs. The effect is, however, of second-order importance (see below).

[17] The central bank does not actually have to disclose its private information. It is sufficient that it releases its forecast $\mathbb{E}_{cbt}\left[a_{t+1}\right]$.

the fundamental, θ, on the action, a_t. This allows us to focus on the role of higher-order thinking for the informativeness of the aggregate action.

The private sector's and the central bank's information sets are given by:

$$\mathcal{F}_{it} = \{\theta_i,\, \tilde{a}_1,\, i_1\}\,, \quad \mathcal{F}_{cbt} = \{\theta_{cb},\, \tilde{a}_1\}\,,$$

where $\tilde{a}_1 \equiv a_1 + \epsilon_1^a$, $\epsilon_1^a \sim WN\,(0,\, 1/p_a)$, $\theta_i \equiv \theta + \epsilon_i^\theta$, $\epsilon_i^\theta \sim WN\,(0,\, 1/p_c)$ and $\theta_{cb} \equiv \theta + \epsilon_{cb}^\theta$, $\epsilon_{cb}^\theta \sim WN\,(0,\, 1/p_c)$. p_c here denotes the common precision of the private sector's and the central bank's respective private signals. I thus assume that both the private sector and the central bank only observe information in the first period; in the second period they do not observe any additional signals. This assumption is not crucial for establishing the key result. As Section 5 below shows, the same mechanism will exist in a model where everyone receives new information each period. The only aspects of the model that are essential is that higher-order beliefs matter for the equilibrium solution, and that the central bank influences the economy.

Solving equation (2.10) forward for $t = 1$, and incorporating the "noise" term ϵ_1^a , gives:

$$\tilde{a}_1 - \gamma i_1 = \beta^2 \bar{\mathbb{E}}\bar{\mathbb{E}}\,[\theta] + \beta\gamma\phi\bar{\mathbb{E}}\mathbb{E}_{cb}\,[\theta] + \epsilon_1^a. \tag{2.12}$$

The endogenous public signal is therefore a function of only two higher-order beliefs: the average expectation of the average expectation of the fundamental and the average expectation of the central bank's expectation of the fundamental.

In the following, I first show how central bank information disclosure weakens the two mechanisms whereby monetary policy changes the information content of the public signal. I then proceed to compare the informativeness of $\tilde{a}_1$ across information disclosure regimes.

2.5.1 Information Disclosure and the Effects of Monetary Policy

I start with the two-sided learning externality. This effect is driven by the non-nestedness of information sets between the (average) private sector and the central bank. Introducing central bank transparency to the model therefore fundamentally alters the impact of the two-sided learning externality.

With central bank information disclosure, the information set of individual i at time t becomes:

$$\mathcal{F}_{it}^D = \{\theta_i,\, \tilde{a}_1,\, i_1, \theta_{cb}\}\,,$$

where D denotes information disclosure, while the central bank's information set stays the same: $\mathcal{F}_{cbt}^D = \mathcal{F}_{cbt}$. Thus, for all t:

$$\mathcal{F}_{cbt}^D \subset \mathcal{F}_{it}^D. \tag{2.13}$$

When the central bank discloses its information, the private sector has no need to use the policy instrument in its signal extraction problem – it already knows the central bank's private information. Thus, a change in the aggressiveness of the central bank has no informational consequences when $\gamma = 0$

(i.e. when the direct stabilization effect is also absent). And hence, in general, the central bank does not need to trade-off its stabilization motive with the two-sided informational consequences described above. The key to this result is (2.13) – that the release of the central bank's private information causes its information set to be nested within the private sector's, and hence dispenses with the need for two-sided learning.

Second, transparency also eliminates the direct stabilization effect in the two-period model. Central bank information disclosure implies that the average private sector expectation of next-period's interest rate $\bar{\mathbb{E}}[i_2] = \phi\bar{\mathbb{E}}\mathbb{E}_{cb}[\theta]$ – the key driver of the direct stabilization effect – becomes common knowledge ($\bar{\mathbb{E}}\mathbb{E}_{cb}[\theta] = \mathbb{E}_{cb}[\theta] \in \mathcal{F}_{it}^{D}$). Everyone knows what everyone else expects of the central bank in the future. This nullifies the direct stabilization effect in (2.12).

To understand the intuition behind this result, consider the situation where everyone in the economy observes a large positive realization of θ_i. If the central bank does not disclose its private information, private sector agents will revise up their beliefs about the central bank's expectation of the fundamental, and hence increase their expectations of next-period's interest rate ($\phi > 0$). This decreases the information content of the endogenous public signal, all else equal. However, under central bank transparency, the private sector does not change its beliefs about next-period's interest rate. Everyone already knows what the central bank is planning to do. This removes the negative effect of prospective policy on the informativeness of the aggregate action.

The above two-period model is stylized: the central bank only observes new information in the first period and not the second. In a more general setting, average private sector expectations of future central bank beliefs, and hence future interest rates, can be separated into two components: (1) average private sector expectations (of average expectations etc.) of *current* central bank beliefs of the fundamental; and (2) average private sector expectations (of average expectations etc.) of *future* central bank signals (see 2.5). If the central bank reveals its private information, everyone in the economy knows what everyone else (and so on) expects of the first component. Current central bank beliefs become common knowledge. This decreases the strength of the direct stabilization effect, but does not completely remove it. If everyone observes a large positive private signal, agents would still expect future interest rates to increase because of the second component – albeit substantially less than under opacity.

Remark. (Concern or Benefit?) I have assumed that $\phi > 0$. That is that there is a direct stabilization concern rather than a benefit. The weakening of the direct stabilization effect, however, still holds for $\phi < 0$, but the sign of the expected interest rate change in the above explanation switches.

In sum, central bank information disclosure (partially) divorces monetary policy from its informational repercussions. The two-sided learning externality is eliminated and the direct stabilization effect is (in general) weakened. This allows the central bank to, all else equal, focus more on its stabilization objectives. Proposition 4 summarizes the effects in the two-period model.

Proposition 4. *("A Separation Principle in the two-period model") If the central bank releases its private information, θ_{cb}, an increase in monetary policy activism (as measured by ϕ) has no impact on the information content of the endogenous public signal, $\tilde{a}_1$.*

Proof. See Appendix A. □

2.5.2 Information Disclosure and the Informativeness of Public Signals

The elimination of the two-sided learning externality and the weakening of the direct stabilization effect change the informativeness of the endogenous aggregate $\tilde{a}_1$ compared to the no-disclosure case. In particular, I show that the elimination of the two-sided learning externality generalizes the negative effect of information disclosure described by Morris and Shin (2005) and Amador and Weill (2010). Unlike the results in those papers, however, central bank information disclosure can in this model improve the informational efficiency of endogenous variables. This is due to the weakening of the direct stabilization effect.

To demonstrate the changes in the information content of $\tilde{a}_1$, I follow the same steps as in Proposition 1. This allows me to derive analytical expressions for how informative $\tilde{a}_1$ is across information disclosure regimes. The equations are given by:

$$(k_1/k_3)^{ND} = \beta^2 w_\theta^2 + \beta\gamma\phi w_\theta^{cb} w_{\theta cb}, \quad w_\theta = \frac{p_c}{p_\theta + p_c + (\varsigma^{ND})^2 p_a + \mu} \tag{2.14}$$

$$(k_1/k_3)^{D} = \beta^2 \left(w_\theta^D\right)^2, \quad w_\theta^D = \frac{p_c}{p_\theta + 2p_c + (\varsigma^{D})^2 p_a}, \tag{2.15}$$

where k_1 and k_3 denote the coefficients on the fundamental and the noise term in the linear solution, respectively; $\mu = \frac{p_c\left(\phi w_\theta^{cb}\right)^2 p_{mp}}{p_c + \left(\phi w_\theta^{cb}\right)^2 p_{mp}} < p_c$ the precision of the interest rate signal; and last w_θ^{cb} the signal extraction weight private sector agents place on their own private information when estimating the central bank's private signal.

The Anti-Disclosure Result ($\gamma = 0$): The two equations (2.14) and (2.15) allow for a simple illustration of how the elimination of the two-sided learning externality ties in with the Morris and Shin (2005) "anti-disclosure" result (see also Amador and Weill (2010) and Vives (2013)). This, in turn, demonstrates that the partial separation of monetary policy from its effects on economic uncertainty does not necessarily come without an informational cost.

To illustrate the anti-disclosure result, I set $\gamma = 0$. This allows me to focus on the effects of information disclosure without having the direct stabilization effect muting the information content of $\tilde{a}_1$. The only difference between the two expressions for k_1/k_3 in this case is the difference in the signal extraction weights placed on private information. And since, $w_\theta > w_\theta^D > 0$ for any given k_1/k_3, and since both $\partial w_\theta/\partial(k_1/k_3)$ and $\partial w_\theta^D/\partial(k_1/k_3)$ are negative, we can immediately conclude that:

$$(k_1/k_3)^{ND} > (k_1/k_3)^{D} > 0.$$

Central bank information disclosure decreases the information content of the aggregate variable. This increases central bank uncertainty and partially offsets the beneficial effects of information disclosure for the private sector. The intuition behind the anti-disclosure result is clear and similar to that of the two-sided learning externality: disseminating central bank private information creates a new public signal. This additional public signal reduces the private sector's weight on its own private information (amplified by the new signal being common knowledge) leading to less independent information being imparted into the endogenous variable. This decreases its informativeness – it reflects less the sum of

independent private information, the truly new bits of information that people could learn from it, and more the central bank's information that everyone already knows.

However, when private sector agents observe changes in the policy instrument, the central bank will always reveal some of its private information through its actions. Moreover, the more active monetary policy is, the clearer a signal the private sector receives of the central bank's private information from the observation of the interest rate – even if the monetary authority does not actually release this information. To see that, in this model, the limit of the two-sided learning externality corresponds to full information disclosure consider equation (2.11):

$$\begin{aligned} i_1 &= \phi \mathbb{E}_{cbt}\left[\theta\right] + \epsilon_1^{mp} \\ &= \phi\left[w_{\theta cb}\theta_{cb} + w_{acb}a_1\right] + \epsilon_1^{mp}, \end{aligned}$$

where $\partial\left(\phi w_{\theta cb}\right)/\phi > 0$. The limit of monetary policy activism, ϕ, is central bank transparency – the signal-to-noise ratio of θ_{cb} in the interest rate equation goes to infinity. And when $\gamma = 0$, the level of ϕ, and hence the two-sided learning externality, can be interpreted as directly varying the strength of the Morris and Shin anti-disclosure result.

The separation principle, in this model, therefore comes at the informational cost of the largest possible two-sided learning externality. Central bank information disclosure can lead to a situation where the central bank knows less about the underlying fundamental.

The Weakening of the Direct Stabilization Effect ($\gamma < 0$): But central bank information disclosure does not only eliminate the two-sided learning externality but also weakens the direct stabilization effect. This, in turn, makes it possible for $\tilde{a}_1$ to be more informative with central bank information disclosure than without. To demonstrate this, I allow $\gamma < 0$ and let $p_{mp} \to 0$. The latter simplifies the exposition by turning-off the two-sided learning externality. Proposition 5 details the effect.

Proposition 5. *("An Offset to Morris and Shin (2005)") If* $\gamma \in \left[-\frac{\beta}{\phi},\ \frac{\beta}{\phi}\left\{\left(\frac{p_\theta + p_c}{p_\theta + 2p_c}\right)^2 - 1\right\}\right]$ *and* $p_{mp} \to 0$*, then* $0 < \left(k_1/k_3\right)^{ND} < \left(k_1/k_3\right)^{D}$*. Central bank information disclosure increases the information content of the endogenous public signal,* $\tilde{a}_1$*.*[18]

When the central bank attempts to stabilize the economy, information disclosure can make the endogenous public signal more informative – not less. This is despite the Morris and Shin (2005) anti-disclosure result still being active. Central bank information disclosure can decrease uncertainty for everyone in the economy – even the central bank itself.

The reason behind this offset is the weakening of the direct stabilization effect: Central bank information disclosure does not only change private sector agents' signal extraction problem but also the state vector. Intuitively, letting the central bank disclose its private information implies that the private sector can condition out the effect of future interest rates, and thus the impact of the "cross-expectation"

[18] The proof is simple: For values of γ in this range the right-hand side of (2.14) is always positive but lower than the right-hand side of (2.15). This necessitates that in equilibrium: $0 < \left(k_1/k_3\right)^{ND} < \left(k_1/k_3\right)^{D}$. More negative values of γ generate negative values of $\left(k_1/k_3\right)^{ND}$ – the action a_1 becomes a negative signal of the fundamental θ. Central bank uncertainty is given by: $MSE_{cb}^{j}(\theta) = \left(p_\theta + \left(\left(k_1/k_3\right)^j\right)^2 p_a + p_c\right)^{-1}, \quad j = \{ND,\ D\}$.

term, $\bar{\mathbb{E}}\mathbb{E}_{cb}[\theta]$, on the dynamics of $\tilde{a}_1$ (see 2.12).

Without central bank disclosure, private sector agents need to use their private information to infer the path of future interest rates, and hence second guess central bank beliefs. This imparts additional private information into the endogenous aggregate that cannot be separated from the private information that is infused by average higher-order expectations. But since the "cross-expectation" term enters negatively in the endogenous aggregate – there is a direct stabilization concern rather than benefit – this use of private information to second guess public opinion deducts from its informativeness. It is this lack of use of private information to second guess future policy decisions that counteracts the Morris and Shin (2005) effect, and eventually causes the endogenous public signal to better reflect the fundamental under disclosure than without.

In essence, central bank information disclosure alleviates the identification problem created by the direct stabilization effect: observing, for instance, no change in $\tilde{a}_1$ can both be because everyone received private information about the fundamental in line with their prior, or because everyone received new information but expects the central bank in response to change interest rates next period, offsetting the effect on the action today. By making the central bank's current beliefs about the fundamental common knowledge, disclosure offers this distinction. Central bank information disclosure implies that individuals know more about what everyone else expects of future interest rates. This increases the information content of aggregate variables by removing the direct stabilization effect (weakening it in the more general case where new information is available in each period).

The implicit assumption is that $\phi > 0$ – there is a direct stabilization concern and not a benefit. If the central instead attempts to accommodate the underlying fundamental ($\phi < 0$), the mechanism reverses. The expectation of future interest rates, in this case, improves the dissemination of information through $\tilde{a}_1$. Central bank information disclosure then (partially) destroys the amplification coming from expectations of future interest rates. This increases uncertainty for everyone in the economy by lowering the information content of $\tilde{a}_1$. Whether there is a direct stabilization concern or benefit therefore also determines whether central bank information disclosure – through the elimination of the direct stabilization mechanism – offsets or amplifies the Morris and Shin (2005) anti-disclosure result.

In sum, to understand the effects of central bank information disclosure it is critical to account for the channels through-which monetary policy changes the information content of endogenous variables. Specifically, if the central bank attempts to stabilize the economy, central bank information disclosure can partially separate the choice of monetary policy from its informational repercussion *and* decrease uncertainty for everyone in the economy – even the central bank itself. The separation result from the previous subsection does not necessarily come at an informational cost. But the behavior of the monetary authority is crucial for this situation to occur.

The baseline model used so far is highly stylized. To analyze whether the direct stabilization effect and the two-sided learning externality are economically meaningful for realistic parameter values, I need to move beyond the baseline model and investigate how central bank stabilization policy affects uncertainty in a standard microfounded business cycle model. This will also allow us to assess whether the offset to the Morris and Shin (2005) result is indeed important. The next Section starts with this task. I show how an equation resembling (2.2) for the endogenous aggregate is indeed the equilibrium outcome of a standard dispersed information business cycle model.

3 A Dispersed Information Business Cycle Model

This Section presents a dispersed information business cycle model, similar to those used in Lorenzoni (2009), Graham and Wright (2010) and Angeletos and La'O (2013), but with learning from endogenous public signals. The key equilibrium relationships of the model closely resemble the mathematical structure outlined in the previous Section. Importantly though, introducing the business cycle model allows us to quantify the magnitude of the two separate effects, identified above, on the informational efficiency of public signals. This makes it possible to assess the size and direction of the impact of central bank information disclosure on economic uncertainty.

Similar to Lucas (1972), the economy is comprised of a continuum of informationally separated islands subject to both common and island specific shocks that cannot be distinguished by direct observation. Instead, inhabitants on each island – firms and households – must estimate the persistent common components to make optimal decisions. Furthermore, across the economy, a central bank sets the interest rate to reduce deviations of aggregate variables, output and inflation, from target levels. Last, I deviate from the three equation model outlined above by no-longer assuming that agents know the previous period's value of the underlying fundamental.

3.1 Households

There is a continuum of measure one of islands indexed by $i \in [0,\, 1]$. On each island there is a representative household that consumes goods and supplies labor in a competitive market to the firms on that island. The preferences of household i are given by:

$$\mathbb{E}_{i0} \sum_{t=0}^{\infty} \beta^t U\left(C_{it},\, N_{it}\right) \tag{3.1}$$

$$U\left(C_{it},\, N_{it}\right) = exp\left(d_{it}\right)\left[\frac{\left(C_{it}-hC_{t-1}\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{1}{1+\sigma_l} N_{it}^{1+\sigma_l}\right], \tag{3.2}$$

where $\mathbb{E}_{i0}\left[\cdot\right]$ denotes the expectation operator conditional on household i's information set; β the discount factor; C_{it} the consumption of a composite good (defined below) for household i at time t; C_{t-1} the amount of aggregate consumption across islands at time $t-1$; σ_c the coefficient of relative risk-aversion; and d_{it} an idiosyncratic intertemporal substitution shock. When $h > 0$, (3.2) allows for (external) habit formation in household preferences. Finally, N_{it} denotes the total amount of labor supplied by household i and σ_l the inverse Frisch elasticity.

Since each island produces its own variety of final consumption goods that is sold to all households across islands, the composite good, C_{it}, is given by:

$$C_{it} \equiv exp\left\{\int_0^1 logC_{jit}dj\right\},$$

where C_{jit} is the consumption of final good production from island j by the consumer from island i at time t. The associated welfare based consumption price-index is defined by:

$$\bar{P}_{it} \equiv exp\left\{\int_0^1 logP_{jt}dj\right\} = P_t.$$

The household on island i owns all firms on that island, and thus receives the profits $\int_0^1 \Pi_{lit}dl$, where Π_{lit} is the profit of firm l on island i at time t. Following Lorenzoni (2009), nominal one-period bonds are the only financial assets traded across islands. Due to the presence of island-specific shocks, household i is subject to uninsurable idiosyncratic income risk. The budget constraint for consumer i is:

$$Q_t B_{it+1} + \bar{P}_{it} C_{it} = B_{it} + W_{it} N_{it} + \int_0^1 \Pi_{lit} dl, \tag{3.3}$$

where B_{it} denotes the holdings of nominal bonds; $Q_t \equiv exp(i_t)$ the associated price, where i_t is the central bank interest rate; and W_{it} the island specific wage.

In equilibrium, household i maximizes (3.1) with regards to consumption, hours worked and bond holdings subject to (3.2), (3.3) and a No-Ponzi-Game condition.

3.2 Firms

On island i, a final consumption good, Y_{it}, is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of imperfectly substitutable island-specific intermediate goods, indexed by $l \in [0, 1]$, using the technology:

$$Y_{it} = \left[\int_0^1 (Y_{lit})^{\frac{1}{1+\lambda_{ipt}}} dl\right]^{1+\lambda_{ipt}},$$

where Y_{lit} denotes the input of intermediate goods by firm l and λ_{ipt} the random degree of substitutability. Because the representative firm takes output and input prices as given, profit maximization implies that the demand for Y_{lit} by the final goods firm on island i is:

$$Y_{lit} = \left(\frac{P_{lit}}{P_{it}}\right)^{-\frac{1+\lambda_{ipt}}{\lambda_{ipt}}} Y_{it}, \tag{3.4}$$

such that the final goods price can be written as:

$$P_{it} = \left[\int_0^1 (P_{lit})^{-\frac{1}{\lambda_{ipt}}} dl\right]^{-\lambda_{ipt}}.$$

The intermediate goods firm $l \in [0, 1]$ operates in an island-specific monopolistically competitive market, where firms are price-setters à la Calvo (1983). Each period, a fraction $1 - \mu$ of all intermediate good firms on island i are allowed to freely adjust their price. Firms that cannot re-optimize, adjust their prices according to the indexation rule:

$$P_{lit} = \left(\frac{P_{it-1}}{P_{it-2}}\right)^{\gamma_p} P_{lit-1},$$

where $\gamma_p \in [0, 1]$ denotes the extent of partial indexation. Intermediate goods production is done using

the technology:

$$Y_{lit} = \Theta_{it} N_{lit}, \tag{3.5}$$

where N_{lit} is the labor input hired in the perfectly competitive island-specific labor market and Θ_{it} the level of island-specific productivity. The problem of intermediate good firm l on island i is to maximize:

$$\mathbb{E}_{it} \sum_{k=0}^{\infty} \mu^k Q_{it+k|it} \left[P^{\star}_{lit} \left(\frac{P_{it-1+k}}{P_{it-1}} \right)^{\gamma_p} Y_{lit+k|lit} - W_{it+k} N_{lit+k} \right]$$

with regards to $P^{\star}_{lit}$ subject to the technological constraint (3.5) and the demand relation (3.4). The firm takes as given the stochastic processes for P_t, W_{it} and Y_{it}, as well as the stochastic discount factor of household i :

$$Q_{it|it+k} \equiv \beta^k \left(\frac{C_{it+k} - hC_{t+k-1}}{C_{it} - hC_{t-1}} \right)^{-\sigma_c} \left(\frac{\bar{P}_{it}}{\bar{P}_{it+k}} \right) \left(\frac{exp\left(d_{it+k}\right)}{exp\left(d_{it}\right)} \right).$$

3.3 Aggregates, Monetary Policy and Shocks

The economy-wide output, Y_t, is defined using the log-aggregator over individual island's final goods production; i.e:

$$Y_t \equiv exp \left\{ \int_0^1 log Y_{it} di \right\}.$$

The corresponding economy-wide price index, P_t, is given by:

$$P_t \equiv exp \left\{ \int_0^1 log P_{it} di \right\} = \bar{P}_{it}. \tag{3.6}$$

Because households consume goods from all islands, there will be no difference between island i's consumer price index and the aggregate economy-wide producer price index. Aggregate (gross) inflation is equal to: $\Pi_t \equiv P_t / P_{t-1}$.

The central bank, in the model, is described by a simple forward-looking Taylor Rule, targeting deviations of aggregate variables from target levels. Specifically, I assume that i_t is set according to:

$$i_t = \rho_{mp} i_{t-1} + \phi_{\pi} \mathbb{E}_{cbt} \left[\pi_{t+1} \right] + \phi_y \mathbb{E}_{cbt} \left[y_{t+1} - y^{flex}_{t+1} \right] + \epsilon^{mp}_t, \tag{3.7}$$

where lower-case letters denote the log of a variable.[19] $\rho_{mp} \in [0, 1]$ is here defined as the degree of interest rate smoothing; ϕ_{π} and ϕ_y the coefficients chosen by the monetary authority; y^{flex}_{t+1} the economy's flexible price full-information equilibrium level of output at time $t+1$; and last $\epsilon^{mp}_t \sim WN\left(0, \sigma^2_{mp}\right)$ a monetary policy shock. $\mathbb{E}_{cbt}\left[\cdot\right]$ denotes the expectation operator conditional on the central bank's information set at time t.[20]

[19] For ease of notation, I omit constant terms in linear equations throughout this paper.

[20] (3.7) shows that the central bank targets expected deviations of inflation from target as well as the difference between equilibrium output and the corresponding flex-price level. George-Marios Angeletos (2013) show in a dispersed information business cycle model, similar to the one considered in this paper, that optimal monetary policy under dispersed information still seeks to replicate the flexible-price allocation, albeit it is no-longer first best efficient. In addition, they illustrate that the optimal policy must "lean against the wind". See also Lorenzoni (2010) (and George-Marios Angeletos (2013) and Angeletos and La'O (2014)) for optimal monetary policy in a model that also includes dispersed information on the household side, as in this case.

In addition to the monetary policy innovation in (3.7), the economy is subject to three other types of disturbances. First, island-specific productivity, θ_{it}, is set to be the sum of a common component, θ_t, and an island specific component, ϵ^{θ}_{it}:

$$\theta_{it} = \theta_t + \epsilon^{\theta}_{it}, \quad \epsilon^{\theta}_{it} \sim WN\left(0, \sigma^2_{p\theta}\right),$$

where θ_t – *the underlying aggregate fundamental* – follows an AR(1):

$$\theta_t = \rho_\theta \theta_{t-1} + \epsilon^{\theta}_t, \quad \epsilon^{\theta}_t \sim WN\left(0, \sigma^2_{\theta}\right). \tag{3.8}$$

Second, the idiosyncratic island-specific intertemporal substitution shock equals:

$$d_{it} = d_t + \epsilon^{d}_{it}, \quad \epsilon^{d}_{it} \sim WN\left(0, \sigma^2_{pd}\right),$$

where the common component across islands is defined as:

$$d_t = \epsilon^{d}_t, \quad \epsilon^{d}_t \sim WN\left(0, \sigma^2_{d}\right). \tag{3.9}$$

And last, the random degree of substitutability between inputs in final goods production on island i, λ_{ipt}, follows:

$$\lambda_{ipt} = \lambda_p + \xi^{p}_{it},$$

where:

$$\xi^{p}_{it} = \xi^{p}_t + \epsilon^{p}_{it}, \quad \epsilon^{p}_{it} \sim WN\left(0, \sigma^2_{pp}\right),$$

and the common component equals:

$$\xi^{p}_t = \epsilon^{p}_t, \quad \epsilon^{p}_t \sim WN\left(0, \sigma^2_{p}\right). \tag{3.10}$$

I thus assume that aggregate demand and markup shocks, d_t and ξ^p_t, fluctuate in a purely unpredictable manner, while aggregate productivity follows an AR(1). These assumptions are made for two reasons. First, while it is common to model markup shocks as idiosyncratic disturbances (see e.g. Smets and Wouters (2007)), modeling demand shocks along the same lines is less so. But, as will become clear further below, the role of d_t and ξ^p_t in this model is, in general, to limit island inhabitants' ability to infer the value of the underlying fundamental, θ_t, from the observation of aggregate output, y_t, and inflation, π_t – in a similar manner to the "noise" shock ϵ^a_t in Section 2. In a simple model, with few aggregate fundamentals, it is fairly straightforward for island inhabitants to back out the driving forces from the observation of endogenous aggregates (see Baxter *et al.* (2011)). In reality, however, the ability of households to infer the economy's fundamentals is likely impaired by the presence of structural breaks, potential model misspecification, noisy statistics, and by the presence of a larger number of disturbances with richer serial correlation patterns. Following Lorenzoni (2009) and Graham and Wright (2010), the approach taken in this paper is to keep the model relatively simple, assuming rational expectations and only one underlying fundamental, but to complicate the household inference problem by, in essence,

adding idiosyncratic noise to local and aggregate information.

Second, from a computational perspective, allowing for persistence in d_t and ξ_t^p would also complicate the solution of the model, without adding fundamentally new insight about the transmission of information via endogenous public signals. Changing the set of underlying fundamentals in the economy from including just total factor productivity to also including demand and mark-up shocks would imply having to also track household and central bank higher-order expectations of d_t and ξ_t^p, in addition to θ_t. All else equal, the curse-of-dimensionality (see Kohlhas (2013)) would start to bite at noticeably lower orders of expectation, complicating numerically the task of finding a satisfactory solution to the model.

3.4 Equilibrium

Unlike the model presented in Section 2, the equilibrium dynamics of the endogenous aggregates can no longer be characterized analytically; numerical solution techniques are needed. To study the rational expectations sequence of market equilibrium, I use a log-linear approximation of the equilibrium relationships. In a setup with imperfect information, a log-linear approximation greatly simplifies the inference problem as it allows for the applicability of standard linear projection techniques, as well as facilitating aggregation.[21]

3.4.1 Log-Linear Island-Specific Equilibrium Conditions

Following standard steps, the log-linear approximation to household $i's$ first-order condition for the optimal intertemporal allocation of consumption is:

$$\mathbb{E}_{it}\left[\Delta c_{it+1} - h\Delta c_t\right] = \frac{1-h}{\sigma_c}\left(i_t - \mathbb{E}_{it}\left[\pi_{t+1}\right]\right) + \frac{1-h}{\sigma_c} d_{it}, \tag{3.11}$$

where Δ denotes the first difference operator. Similarly, log-linearizing the household's labor supply condition equating the marginal disutility of labor supply with the marginal benefit of increased consumption gives:

$$\sigma_l n_{it} = w_{it} - p_t - \frac{\sigma_c}{1-h}\left(c_{it} - hc_{t-1}\right). \tag{3.12}$$

Last, to complete the characterization of the household side, the budget constraint is in log-linear form:

$$\beta h_{it+1} = h_{it} - c_{it} + y_{it},$$

where $h_{it} = B_{it}/P_t Y_t$ is kept in levels as it can take both positive and negative values.

On the firm side, the equilibrium price for the final good produced by island i can be log-linearized to give an island i Phillips Curve, relating the marginal cost, mc_{it}, and the mark-up disturbance, ξ_{it}^p, to expected future, current and past island i inflation:

$$\pi_{it} = \frac{\gamma_p}{1+\gamma_p\beta}\pi_{it-1} + \frac{\beta}{1+\gamma_p\beta}\mathbb{E}_{it}\left[\pi_{it+1}\right] + \lambda mc_{it} + \xi_{it}^p, \tag{3.13}$$

[21] Following Lorenzoni (2009) and Nimark (2012), I log-linearize around the stochastic equilibrium of an economy with no heterogeneity and full information (where all the shocks except σ_θ^2 are set to zero).

where $\lambda \equiv (1-\mu)(1-\mu\beta)/\left[\mu(1+\gamma_p\beta)\right]$ and the marginal cost on island i is equal to:

$$mc_{it} \quad = \quad w_{it} - \theta_{it} - p_{it} \tag{3.14}$$

3.4.2 Log-Linear Aggregate Equilibrium Conditions:

Integrating (3.11) across all islands $i \in [0,\, 1]$, imposing market clearing, $y_t = c_t$, and using the strong law of large numbers, gives us the economy-wide dynamic IS-equation:

$$y_t = \frac{h}{1+h}y_{t-1} + \frac{1}{1+h}\bar{\mathbb{E}}_t\left[y_{t+1}\right] - \frac{1-h}{(1+h)\,\sigma_c}\left(i_t - \bar{\mathbb{E}}_t\left[\pi_{t+1}\right]\right) + \frac{1-h}{(1+h)\,\sigma_c}\epsilon_t^d, \tag{3.15}$$

relating current period aggregate output to lagged and average expected future output, average expected future inflation, the demand shock and the interest rate. Likewise integrating (3.13) across islands, using (3.14) and (3.12), gives the economy-wide Phillips Curve:

$$\begin{aligned} \pi_t \quad &= \quad \frac{\gamma_p}{1+\gamma_p\beta}\pi_{t-1} + \frac{\beta}{1+\gamma_p\beta}\bar{\mathbb{E}}_t\left[\pi_{t+1}\right] \\ &+ \quad \lambda\left[\left(\sigma_l + \frac{\sigma_c}{1-h}\right)\left(y_t - y_t^{flex}\right) - \frac{\sigma_c h}{1-h}\left(y_{t-1} - y_{t-1}^{flex}\right)\right] + \epsilon_t^p, \end{aligned} \tag{3.16}$$

where y_t^{flex} denotes the economy's natural rate of output, and is given by:

$$y_t^{flex} = \frac{\sigma_c h}{\sigma_l(1-h)+\sigma_c}y_{t-1}^{flex} + \frac{1+\sigma_c}{\sigma_l + \sigma_c/(1-h)}\theta_t. \tag{3.17}$$

Equation (3.15) and (3.16) comprise the system of linear expectational difference equations, which, along with the interest rate equation (3.7) and the natural rate of output (3.17), describe the evolution of the aggregate variables in the economy.[22]

3.5 Local and Aggregate Information

The inhabitants on each island have full information about all local variables. This allows each island inhabitant access to the island-specific signals, θ_{it}, d_{it} and λ_{ipt}. Since these signals contain a common component across islands, local variables will be informative about the dynamics of the aggregate state. Furthermore, each island inhabitant has access to information about aggregate output, y_t, inflation, π_t and the economy-wide interest rate, i_t.[23] Thus, the information set of the inhabitants on island i is given by:[24]

$$\mathcal{F}_{it} = \{\theta_{i\tau},\, d_{i\tau},\, \lambda_{ip\tau},\, y_\tau,\, \pi_\tau,\, i_\tau\}_{\tau=-\infty}^{\tau=t}.$$

[22] The equations of motion for aggregate output and inflation, given by (3.15) and (3.16), contain both habit persistence and partial indexation. These features are common in medium-scale DSGE models (see Smets and Wouters (2007)) and are important for assessing the effect of central bank stabilization policy on economic uncertainty (see Appendix D).

[23] I assume that agents learn from aggregate inflation rather than the aggregate price level since this simplifies the signal extraction problem by keeping all indicators stationary. Reformulating the information set in this way does not change anything substantial in the model.

[24] In addition to these signals, inhabitants on island i also observe total production of the island-specific good, y_{it}, and the consumer price index, $\bar{p}_{it}$. But since these indicators reduce to linear combinations of y_t and p_t in equilibrium, they are not included in the above information set.

Turning to the central bank, I assume that it has access to private signals, θ_{cbt}, d_{cbt} and λ_{cbpt}, with potentially better signal precision about the underlying disturbances than the private sector, in addition to also observing the economy-wide aggregates. Hubert (2011), summarizing the literature on the relative accuracy of central bank versus private sector forecasts started by Romer and Romer (2000), argues that central bank forecast dominate private sector ones for several of the most important macroeconomic variables. And that the main reason for this forecast dominance is access to superior private information about the shocks impacting the economy. The central bank's information set can therefore be written as:

$$\mathcal{F}_{cbt} = \{\theta_{cb\tau},\, d_{cb\tau},\, \lambda_{cbp\tau},\, \epsilon^{mp}_{\tau},\, y_{\tau},\, \pi_{\tau},\, i_{\tau}\}^{\tau=t}_{\tau=-\infty},$$

where:

$$\begin{aligned}
\theta_{cbt} &= \theta_t + \epsilon^{\theta}_{cbt}, \quad \epsilon^{\theta}_{cbt} \sim WN\left(0,\, \sigma^2_{cb\theta}\right) \\
d_{cbt} &= d_t + \epsilon^{d}_{cbt}, \quad \epsilon^{d}_{cbt} \sim WN\left(0,\, \sigma^2_{cbd}\right) \\
\lambda_{cbpt} &= \xi^{p}_{t} + \epsilon^{p}_{cbt}, \quad \epsilon^{p}_{cbt} \sim WN\left(0,\, \sigma^2_{cbp}\right),
\end{aligned}$$

completing the description of the model.

4 Model Solution and Calibration

The model for the endogenous aggregates consisting of (3.15), (3.16) along with (3.7) and (3.17) contains two non-standard features rendering common solution techniques inapplicable: (1) dispersed information combined with no perfect state revelation, and (2) a non-atomistic agent with potentially superior private information. The combination of dispersed information about the underlying common fundamental and no perfect state revelation has since Townsend (1983) been known to imply an infinite state-vector consisting of higher-order beliefs of the common fundamental. The problem of infinite regress of expectations arises in this setting because agents need to "forecast-the-forecast-of-others" to make optimal consumption and pricing decisions.

Like in Section 2, assuming that the central bank learns from endogenous aggregates also complicates the solution of the model as it implies a particular simultaneity problem. With forward-looking variables among the observables, the endogenous variables will depend on the current central bank estimate of the state, but the latter will also depend on the forward-looking variables. This circularity problem, however, now telescopes, when compared to the one encountered previously, due to the infinite state vector.

To overcome these complications, this Section shows how to extend the solution technique for private information models with atomistic agents in Nimark (2011) to accommodate the non-atomistic asymmetric information case discussed above. The approach used follows closely Kohlhas (2013). Details are relegated to Appendix B. The Section ends with a discussion of how to calibrate the extent of asymmetric information between the monetary authority and the private sector.

4.1 Solving the Dispersed Asymmetric Information Model

To find the linear rational expectations equilibrium for the endogenous aggregates, y_t, π_t and i_t, I conjecture (and verify in Appendix B) that the solution takes the form:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = J \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + AX_t^{(0:\infty)} + Bi_t + Cu_t, \tag{4.1}$$

$$i_t = T \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + DX_t^{(0:\infty)} + Eu_t, \tag{4.2}$$

$$X_t^{(0:\infty)} = MX_t^{(0:\infty)} + Nu_t, \tag{4.3}$$

where the matrices J, A, B, C, T, D and E are functions of the structural parameters of the model. $X_t^{(0:\infty)}$ denotes the infinite dimensional expectational state vector (defined below), while u_t denotes the vector of aggregate shocks given by: $u_t = \begin{bmatrix} \epsilon_t^{\theta}, \epsilon_t^{mp}, \epsilon_t^{d}, \epsilon_t^{p}, \epsilon_{cbt}^{\theta}, \epsilon_{cbt}^{d}, \epsilon_{cbt}^{p} \end{bmatrix}'$. In words, I conjecture that the endogenous aggregates can be described by a linear combination of previous periods' values and the components of $X_t^{(0:\infty)}$. The expectational state vector is assumed to follow a VAR(1).

To verify the above conjuncture, and find the equilibrium of the model, I use a two-part solution strategy, defining an infinite-dimensional mapping from $\{M, N\} \to \{M, N\}$; the fix-point of which will provide the linear rational expectations equilibrium of our model.[25] The first part of the solution procedure consists of finding the coefficient matrices determining the evolution of the aggregate variables given a law of motion for the expectational state. To find these, I use the Method of Undetermined Coefficients (see Appendix B for a full description of the first part). The second step of the procedure consists of finding expressions for M and N, conditional on the above matrices for the solution of the endogenous variables. To solve for the state transition equation, I first describe the elements included in the infinite dimensional state vector. Proposition 6 provides the composition of the hierarchy of expectations, $X_t^{(0:\infty)}$:

Proposition 6. *The expectational state vector, $X_t^{(0:\infty)}$, is given by:*

$$X_t^{(0:\infty)} \equiv \begin{bmatrix} X_t^{(0)} \\ X_t^{(1)} \\ \vdots \end{bmatrix}, \quad X_t^{(k)} \equiv \begin{bmatrix} \bar{\mathbb{E}}_t X_t^{(k-1)} \\ \mathbb{E}_{cbt} A_{nk} X_t^{(k-1)} \end{bmatrix}, \; k \geq 1, \tag{4.4}$$

where A_{nk} denotes an annihilation matrix that removes the last $2f_k$ rows from $X_t^{(k-1)}$ and $X_t^{(0)} \equiv x_t \equiv \begin{bmatrix} y_t^{flex} & \theta_t \end{bmatrix}'$.[26]

[25] Clearly, the solution to the model given by equation (4.1) to (4.3) is not practically feasible due to the infinite state vector. The next subsection shows how an implementable approximate solution can be found.

[26] To get a better feel for the structure of $X_t^{(0:\infty)}$, I simplify the notation by following Woodford (2002) and denote current period average higher-order expectations as: $x_t^{(k)} \equiv \bar{\mathbb{E}}_t \left[x_t^{(k-1)} \right]$. Similarly, I denote the central bank's expectation of, for instance, x_t as: $x_t^{(cb)} \equiv \mathbb{E}_{cbt} [x_t]$. "Cross-expectations", such as the central bank's expectation of the average expectation,

Proof. See Appendix B. □

In Proposition 6, A_{nk} removes the last $2f_k$ rows from $\mathbb{E}_{cbt}\left[X_t^{(k-1)}\right]$ as these are superfluous: they consist of elements given by the central bank's expectation of its own expectation, which, by The Law of Iterated Expectations, can be reduced to its own expectation, already defined higher up in the hierarchy. Interestingly, f_k follows the Fibonacci Sequence and therefore evolves according to Binet's formula: $f_k = \frac{z_1^k - z_2^k}{z_1 - z_2}$, where $z_1 = \frac{1+\sqrt{5}}{2}$ and $z_2 = \frac{1-\sqrt{5}}{2}$. Given (4.4), an alternative, and computationally more useful, expression for the hierarchy of expectations is:

$$X_t^{(0:\infty)} = H \begin{bmatrix} x_t \\ \bar{\mathbb{E}}_t X_t^{(0:\infty)} \\ \mathbb{E}_{cbt} A_n X_t^{(0:\infty)} \end{bmatrix}, \tag{4.5}$$

where $H : \mathbb{R}^\infty \to \mathbb{R}^\infty$ is a permutation matrix that re-orders the elements of the right-hand side vector, while A_n is a block-diagonal matrix that has the annihilation matrices A_{nk}, $k = 1, 2, \ldots$ along its diagonal.

From (4.5) it is clear that to verify the state transition equation (4.3), we must find expressions for $\bar{\mathbb{E}}_t\left[X_t^{(0:\infty)}\right]$ and $\mathbb{E}_{cbt}\left[A_n X_t^{(0:\infty)}\right]$ as linear combinations of $X_{t-1}^{(0:\infty)}$ and u_t. This requires adding some more structure to the formation of higher-order beliefs. Following Nimark (2011), I assume that it is common knowledge among agents that expectations are formed rationally. This assumption implies that not only are first-order expectations model consistent estimates of x_t but second-order expectations are likewise model consistent optimal forecasts of $X_t^{(1)}$. Continuing with this argument implies that all segments of higher-order beliefs are model consistent optimal forecasts of the previous segment. Imposing this assumption allows to find the law of motion in (4.3) by using the Kalman Filter, taking as given the state transition matrices M and N. The Kalman Filter thus plays a dual role in this model: it both determines how individual agents estimate the state as well as (via (4.5)) the law of motion of the very same state that the agents are estimating.

As outlined in detail in Appendix B, combining the Kalman Filter with generalized inverses allows us to derive expressions for $\bar{\mathbb{E}}_t\left[X_t^{(0:\infty)}\right]$ and $\mathbb{E}_{cbt}\left[A_n X_t^{(0:\infty)}\right]$ as linear functions of $X_{t-1}^{(0:\infty)}$ and u_t. Subsequently, these expressions can be stacked via (4.5) to give a VAR(1) form for the evolution of the state – exactly the same form as our conjecture in (4.3).

Two complications, however, arise when solving the private sector's and the central bank's signal

are accordingly defined as: $x_t^{(cb)(1)} \equiv \mathbb{E}_{cbt}\bar{\mathbb{E}}_t[x_t]$. Using this notation allows us to express $X_t^{(0:\infty)}$ as:

$$X_t^{(0:\infty)} = \begin{bmatrix} x_t \\ \hline x_t^{(1)} \\ x_t^{(cb)} \\ \hline x_t^{(2)} \\ x_t^{(1)(cb)} \\ x_t^{(cb)(1)} \\ \hline \vdots \end{bmatrix},$$

where the horizontal bars separate the orders of expectation, k. This representation of $X_t^{(0:\infty)}$ perhaps more easily shows how the number of elements in each order of expectation increase according to the Fibonacci Sequence (see also Kohlhas (2013)).

extraction problem. First, similar to the problem encountered in Section 2, since the central bank learns directly from the forward-looking endogenous variables there is a simultaneity problem: the endogenous variables will depend on the current central bank estimate of the state via the interest rate decision, but the latter will also depend on the forward-looking variables. To solve this fixed-point problem, I iterate repeatedly on the central bank's expectation of $X_t^{(0:\infty)}$ until convergence; i.e. until the conjectured central bank expectation of the state equals the actual expectation from the Kalman Filter.[27]

Second, unlike in standard signal extraction problems, agents in this economy receive both signals of the underlying state, $X_t^{(0:\infty)}$, and of the innovations to the state, u_t. The inhabitants on island i, for instance, both observe θ_{it}, a signal of an element in $X_t^{(0:\infty)}$, and d_{it}, a signal of an element in u_t. Likewise, the central bank perfectly observes the monetary policy shock, ϵ_t^{mp}, in addition to observing its own private information about θ_t. This problem can, however, quite straight-forwardly be solved by re-writing the signal extraction problem such that it is directly suitable for standard Kalman Filtering (see Appendix B).

4.2 A Feasible Solution to the Model

Because of the infinite length of the state vector, the equilibrium of the model given by (4.1) to (4.3) is unfortunately not practically feasible. That said, as shown in Nimark (2011), and for the case of asymmetric dispersed information in Kohlhas (2013), it is possible to find an arbitrarily precise approximate solution to the linear rational expectations equilibrium by truncating the state-space after $\bar{k}$ segments of higher-order expectations. The approximate truncated state-space solution is in our case given by:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \cong J_{\bar{k}} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + A_{\bar{k}} X_t^{(0:\bar{k})} + B_{\bar{k}} i_t + C_{\bar{k}} u_t, \tag{4.6}$$

$$i_t \cong T_{\bar{k}} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + D_{\bar{k}} X_t^{(0:\bar{k})} + E_{\bar{k}} u_t, \tag{4.7}$$

$$X_t^{(0:\bar{k})} \cong M_{\bar{k}} X_t^{(0:\bar{k})} + N_{\bar{k}} u_t, \tag{4.8}$$

where $X_t^{(0:\bar{k})}$ is a vector that includes $\bar{k}$ segments of current period higher-order expectations of x_t, while subscript $\bar{k}$ denotes the corresponding elements of the solution matrices.

The reason that it is possible to directly truncate the state-space of the model to find an arbitrarily precise approximate solution is twofold. First, conditional on the underlying fundamental being stationary, the variance of each component of the hierarchy of expectations, $X_t^{(0:\infty)}$, is a "part-wise" non-increasing sequence. Unlike the result in Nimark (2011), I cannot claim that the variance of *each* element in $X_t^{(0:\infty)}$ is non-increasing; the central bank's expectation of x_t might, for instance, be more precise than the average private sector estimate. But I can conclude that the variance of "*part-wise*" elements, where part-wise refers to the average private sector and central bank expectation in (4.5), is

[27] Due to the two-step procedure used to solve for the equilibrium of the model, the method described here to solve the central bank circularity problem is computationally faster than the approach taken in Svensson and Woodford (2004).

non-increasing. The statement follows directly from the well-known result that the variance of an optimal forecast is no-greater than the variance of the variable being forecasted (see Hamilton (1994)). As (4.5) makes clear, for $k \geq 1$, each element in the expectational state vector is a forecast of a previous element in the hierarchy. Repeatedly applying the optimal forecast result therefore establishes the statement. Second, as shown in Nimark (2011), the magnitude of the added elements in $A_{\bar{k}}$ and $D_{\bar{k}}$ decreases at a geometric rate as we increase the order of expectations $\bar{k}$.

Combined, these two results show that the added contribution to the dynamics of our system of each order of expectation decreases to zero. Hence, a truncated state-space solution can approximate, to an arbitrary degree of accuracy, the dynamics of our economy.

To solve for the approximate equilibrium, given by (4.6) to (4.8), I find the fix-point of the now finite-dimensional mapping, $\left\{M_{\bar{k}}^{j},\, N_{\bar{k}}^{j}\right\} \rightarrow \left\{M_{\bar{k}}^{j+1},\, N_{\bar{k}}^{j+1}\right\}$, using the two-step algorithm described above. In addition, I implement the approximation that expectations of order $\hat{k} > \bar{k}$ are redundant, and therefore set $X_t^{(\hat{k})} = \underline{0}_{2f_{\bar{k}} \times 1}$. Iterating until convergence on these two steps provides us with the truncated state-space solution (see Appendix B for the details of the algorithm).

4.3 Calibration

The model parameters can be split into three separate groups: (1) parameters from standard DSGE models; (2) parameters governing the degree of accuracy of private information; and (3) $\bar{k}$ determining how many orders of expectation are necessary to achieve a satisfactory approximation of the equilibrium dynamics.

To calibrate the first group of parameters, I set the quarterly discount rate such that: $\beta = 0.95^{1/4}$. The inverse Frisch elasticity of labor supply, σ_l, is set to one, while the parameter in the iso-elastic utility function with regards to consumption, σ_c, is set to 1.50. The parameter μ is set equal to 0.74, corresponding to an average price duration of c. $3\frac{3}{4}$ quarters. The degree of habit persistence and partial indexation are $h = 0.56$ and $\gamma_p = 0.46$, respectively. For the parameters governing the behavior of the central bank, I set $\rho_{mp} = 0.60$, $\phi_\pi = 1.50$ and $\phi_y = 0.25$. It remains to choose the variance parameters of the exogenous aggregate disturbances and the persistence parameter for TFP. I set the standard deviation of demand shocks, σ_d, to 1.40; the standard deviation of mark-up shocks, σ_p, to 1.40; the monetary policy shock, σ_{mp}, to 1.30; and last the standard deviation of innovations to aggregate productivity, σ_θ, to 2.00. The persistence parameter of TFP is set to 0.80. The above parameters are all within the range of those used in existing DSGE models, and, importantly, they are in accordance with the estimated dispersed information DSGE model in Nimark (2012).[28]

The remaining six model parameters $\{\sigma_{p\theta},\, \sigma_{cb\theta},\, \sigma_{pd},\, \sigma_{cbd},\, \sigma_{pp},\, \sigma_{cbp}\}$, comprising the second group, determine the signal-to-noise ratio of island and central bank specific information. The standard deviation of island-specific productivity, $\sigma_{p\theta}$, is set to 8.00, in accordance with the estimated value in Nimark (2012). Vavra (2013) finds a similar value using data on the cross-sectional standard deviation of firm level TFP. The remaining five parameters are calibrated to match five moments. The first two moments are the dispersion in one-period ahead private sector forecasts of inflation and output growth. The following two

[28] Appendix D examines the impact of changing the standard deviation of the non-invertibility shocks on the results.

are the difference in the mean-squared error of one-period ahead inflation and output growth forecasts between the central bank and the average private sector. The final moment matched is the standard deviation of the central bank's smoothed estimate of the output gap. To calibrate, I rely on private sector forecast data from the Survey of Professional Forecasters. Data on central bank estimates are taken from the Greenbook published by the Federal Reserve Bank of Philadelphia. I only use data from before 1994 as this corresponds to the year where Federal Reserve statements started to gradually become disseminated. Further details of the calibration procedure are in Appendix B. The resulting parameter vector is given by:

$$\{\sigma_{p\theta},\, \sigma_{cb\theta},\, \sigma_{pd},\, \sigma_{cbd},\, \sigma_{pp},\, \sigma_{cbp}\} = \{8,\, 5,\, 6,\, 4,\, 6,\, 1\}\,.$$

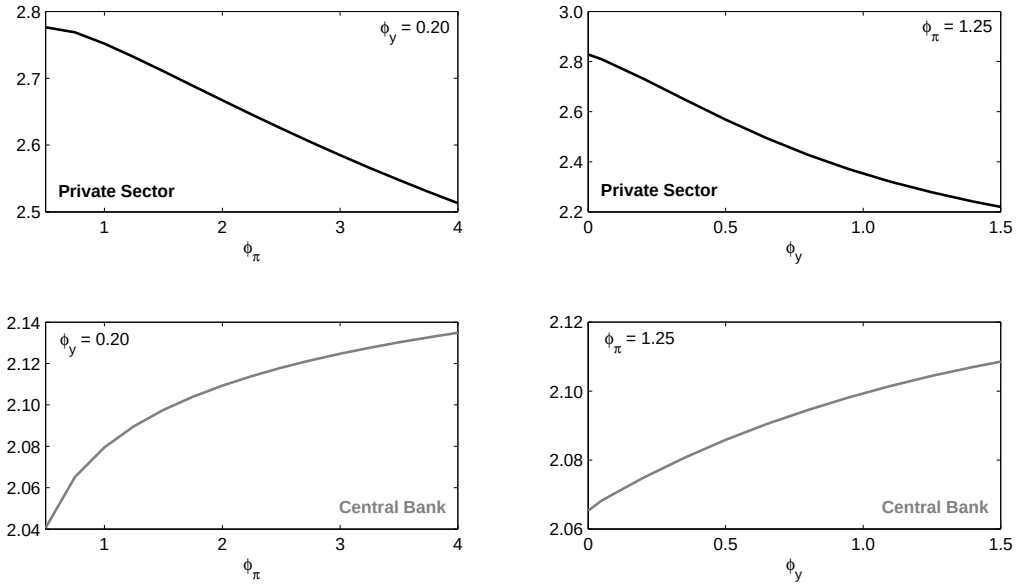
Using these parameters, the model captures almost the entire volatility of central bank estimates of the output gap (see Edge and Rudd (2012)). Furthermore, the model is able to fit both the superiority of central bank forecasts of output and the corresponding dispersion in private sector forecasts fairly well, explaining over 80% of the difference in the data. But as perhaps expected, it is difficult to generate enough dispersion in private sector forecasts of inflation. The combination of rational expectations with the observation of endogenous aggregates generates a strong incentive to base forecasts of the hierarchy of expectations, $X_t^{(0:\infty)}$, on the public signals, lowering the amount of forecast dispersion. As such, the model combined with the given parameter vector is only able to account for c. 40% of the amount of disagreement in private sector forecast of inflation observed in the data.

However, part of this discrepancy is presumably explained by the data used to estimate private sector forecast dispersion overstating the true amount of forecaster disagreement. For instance, several participants in the inflation survey often fail to update their expectation of previous period's value of the price level, despite new data releases, generating large forecast errors in the growth rates. Clearly, this effect would also, all else equal, exacerbate the relative superiority of central bank forecasts of inflation versus the private sector. The sensitivity of the results below to the above parameter vector is investigated in Appendix D.

Last, to calibrate the maximum amount of higher-order expectations necessary to achieve a satisfactory approximation to the equilibrium of our model, I set $\bar{k}$ such that the maximum contribution of an element in the $\bar{k}$th order to the dynamics of our model is close to zero. In addition, I verify the result by examining at which level of $\bar{k}$ the impulse response functions appear to stabilize. Both approaches indicate that orders of expectation higher than seven to eight do not seem to add meaningfully to the dynamics of our model, allowing for a relatively parsimonious state-space to approximate the equilibrium solution (see Appendix B). In the following, I therefore set $\bar{k} = 8$.[29]

[29] The exact value of $\bar{k}$ needed to achieve a reasonable approximation of the equilibrium solution will naturally depend on the entire parameter vector.

Figure 5.1: Current-Period Mean-Squared Error of y_t^{flex} I



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the representative agent on island i and the central bank. The charts are plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.25; 4.00]$. The charts are depicted as a function of one variable of monetary policy activism keeping the other constant. $\{\phi_y, \phi_\pi\} = \{0.20, 1.25\}$ is consistent with the values estimated in the dispersed information business cycle model by Nimark (2012).

5 Central Bank Policy and Uncertainty

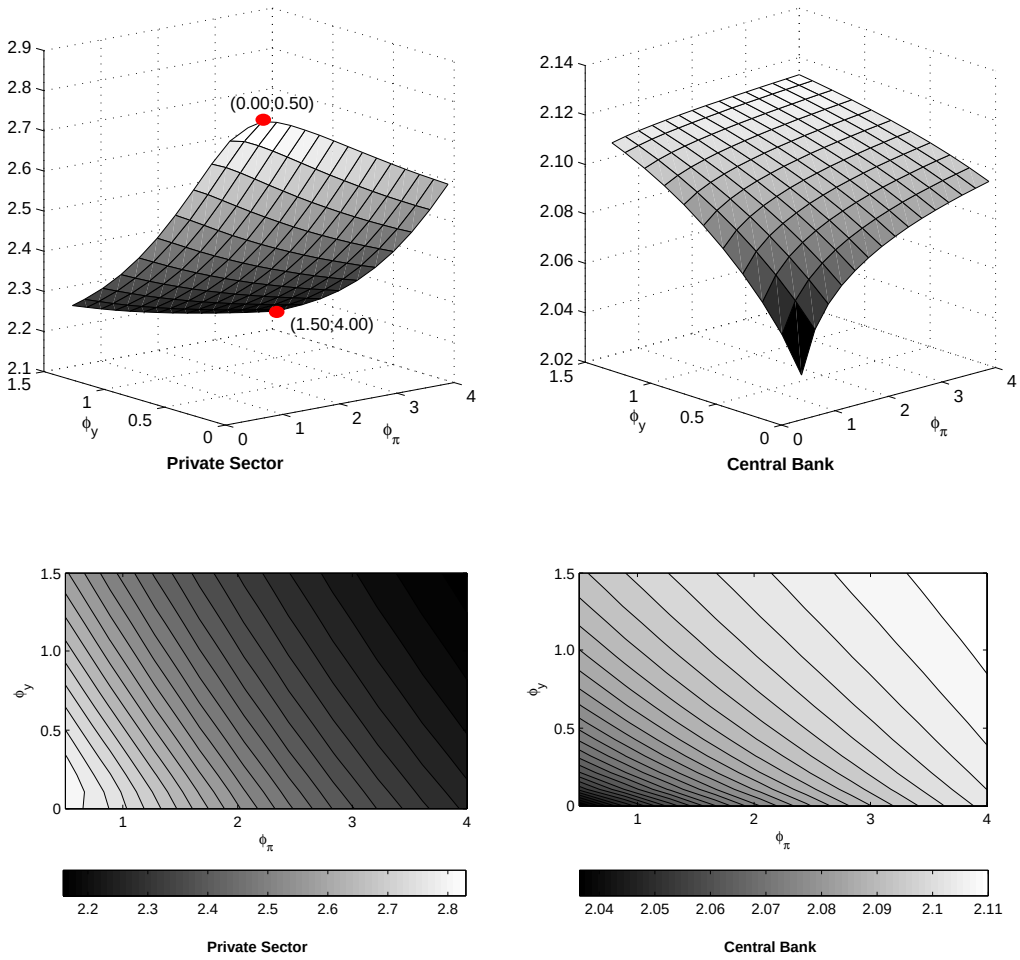
This Section uses the calibrated dispersed information business cycle model to analyze how central bank policy changes the information content of endogenous public signals, and hence uncertainty about economic fundamentals. Since the key equilibrium relationships of the business cycle model closely resemble the mathematical structure outlined in the baseline model in Section 2, the main insights from the previous analysis continue to hold. But unlike in the simple model, agents now observe both prices and quantities simultaneously. This is shown to be crucial for understanding the impact of central bank actions on the total amount of economic uncertainty.[30]

5.1 The Effects Under Central Bank Opacity

To illustrate how monetary policy without central bank information disclosure changes uncertainty about the underlying fundamentals, Figure 5.1 and 5.2 show the mean-squared error of current estimates of y_t^{flex} for both the private sector and the central bank. Because y_t^{flex} is driven by θ_t, the corresponding charts for total factor productivity resemble these closely (see Appendix C).

[30] How central bank policy affects higher-order belief uncertainty is analyzed in Appendix E.

Figure 5.2: Current-Period Mean-Squared Error of y_t^{flex} II

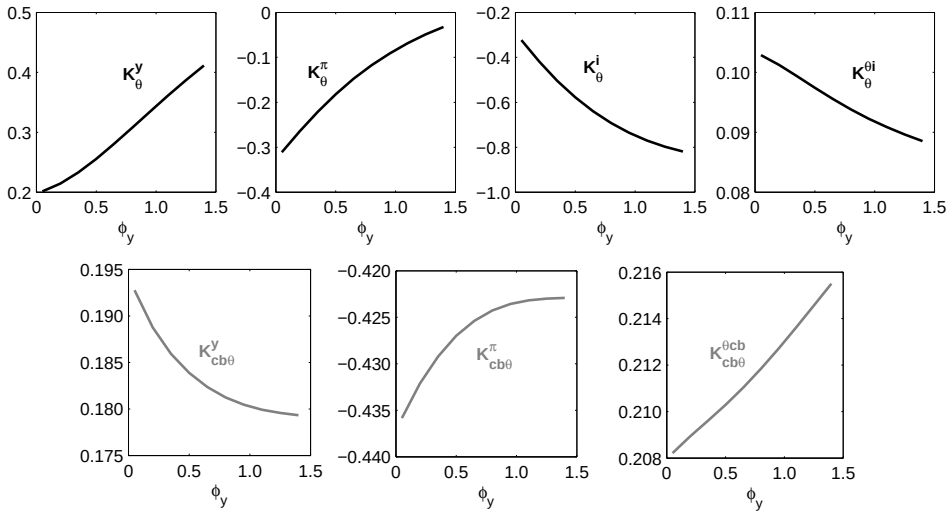


The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the representative agent on island i and the central bank. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$. The dots on the left-hand side chart represent the ϕ_y and ϕ_π coordinates for the max (0.00; 0.50) and min (1.50; 4.00) values.

Section 2 showed that monetary policy changes economic uncertainty along two channels: (1) a direct stabilization effect and (2) a two-sided learning externality. While the direct stabilization effect induces similar changes in uncertainty for the private sector and the central bank, the two-sided learning externality creates an informational trade-off between the two. Yet as Figure 5.1 and 5.2 illustrate, the two-sided learning externality appears to dominate as we increase the coefficients of monetary policy activism. Increasing ϕ_π and ϕ_y from the low-end of the range to the high-end implies a sizable drop in private sector uncertainty of y_t^{flex} (c. 22%) and non-negligible increase for the central bank (c. 5%).[31] In addition, the qualitative impact of changing ϕ_π and ϕ_y appears quite similar. The relatively smaller magnitude of the effect on the central bank is driven (to a large extent) by the superiority of central

[31] In terms of units of variance of total factor productivity, the drop for the private sector corresponds to a c. $\frac{1}{6}$ decrease while the rise for the central bank to a $\frac{1}{40}$ increase.

Figure 5.3: Signal Extraction Weights for θ_t



The figure plots island i inhabitants' and the central bank's signal extraction weights on the endogenous aggregates, y_t, π_t, i_t, as a function of ϕ_y. In addition, the chart shows the signal extraction weights on the private signals θ_{it} and θ_{cbt}. Black indicates private sector agents, while gray the central bank. Superscripts indicate variable names; for instance, $K_\theta^{\theta i}$ denotes the weight placed by island i inhabitants on their own private information, θ_{it}. Throughout, I keep $\phi_\pi = 1.50$.

bank forecasts – i.e. by the relatively more precise central bank private information.

Part of the reason for the relative strength of the two-sided learning externality is the offsetting direct stabilization effects inherent in our information structure: Increasing $\{\phi_\pi, \phi_y\}$ implies that future values of inflation respond less to current changes in productivity thus making current inflation less informative. But the flip-side of inflation becoming more stable is that output becomes more responsive to changes in the level of total factor productivity. Hence, the direct stabilization cost associated with increased stabilization of prices is offset (partially) by the direct stabilization benefit of more responsive output.[32] This offset will always exist when the economy is driven by innovations to total factor productivity and agents observe both quantities and prices simultaneously. And in this case, the offset contributes to the dominance of the two-sided externality (see also the Remark after Proposition 2).[33]

To take a closer look at the effect of central bank policy, Figure 5.3 plots the signal extraction weights on output, inflation, the interest rate and the two private signals, θ_{it} and θ_{cbt}, as a function of ϕ_y. The signal extraction weights as we vary ϕ_π look almost identical and are not reported below. I only depict the signal extraction weights on these five indicators as the remaining signals in agents' information sets do not *directly* contain information about total factor productivity; they are only direct signals of average and central bank beliefs of aggregate productivity, and so on.[34]

[32] The next section shows that the direct stabilization cost is somewhat greater than the direct stabilization benefit. The loss in informativeness of prices outweighs the benefits of more informative output.

[33] The more active the central bank is under opacity, the better the private sector is at estimating the central bank's private information. But, as discussed in Section 2, this also somewhat decreases the strength of the direct stabilization effect. The impact is, however, of second-order importance, and in either case, for the parameter values in question, not very sizable.

[34] For instance, knowledge of the monetary policy shock, ϵ_t^{mp}, does not directly contribute to better central bank information about θ_t. But it does help estimate, for instance, $\bar{\mathbb{E}}_t[\theta_t]$.

Consistent with the two-sided learning externality dominating, the upper part of Figure 5.3 shows that private sector signal extraction weights reassuringly behave as Lemma 1 would indicate. Increases in monetary policy activism cause island inhabitants to attach more weight to the interest rate as its signal-to-noise ratio improves. This, in turn, causes private sector agents to attach relatively less weight to their own private signal, θ_{it}, imparting less information into output and inflation. As a result, these economy-wide variables become less informative. The central bank therefore attaches less weight to them in its own signal extraction problem (see lower panel), and correspondingly more to its own private information, θ_{cbt}.[35]

Even though the two-sided learning externality dominates, Figure 5.3 also illustrates how the direct stabilization cost and benefit affect the signal extraction problems. As discussed, increasing ϕ_y makes output more responsive to changes in productivity and inflation less so.[36] Consistent with this, island i inhabitants actually attach more weight to y_t in their signal extraction problem, despite the decreased agglomeration of independent information.[37]

In sum, without central bank information disclosure, increases in monetary policy activism imply an informational trade-off between the private sector and the central bank: more active stabilization policy implies, on the one hand, more informed private sector decisions, but, on the other hand, less informed public policy. Both the (net-) direct stabilization effect and the two-sided learning externality imply that the more the central bank stabilizes output and inflation, the less it knows about the underlying shocks that hit the economy.

5.2 Solving the Model with Central Bank Information Disclosure

Letting the central bank disclose its private information changes the information structure of the model, which in turn affects the equilibrium. To solve for y_t, π_t and i_t under central bank transparency, we need to take into account both how the information structure changes and what impact this might have on the expectational state, $X_t^{(0:\infty)}$.

The information set of the inhabitants on island i under central bank information disclosure is given by:

$$\mathcal{F}_{it}^D = \{\theta_{i\tau},\, d_{i\tau},\, \lambda_{ip\tau},\, y_\tau,\, \pi_\tau,\, i_\tau,\, \theta_{cb\tau},\, d_{cb\tau},\, \lambda_{cbp\tau}, \epsilon_\tau^{mp}\}_{\tau=-\infty}^{\tau=t}$$

where superscript D denotes disclosure. The central bank's information set stays the same: $\mathcal{F}_{cbt} = \mathcal{F}_{cbt}^D$. Thus, as in Section 2, $\mathcal{F}_{cbt}^D \subset \mathcal{F}_{it}^D$.

[35] The extent of the increased reliance of the central bank on its private information is though not fully captured by the increase in $K_{\theta cb}^{cb\theta}$: As island inhabitants attach more weight to i_t in their signal extraction problem, they also impart more of the central bank's private information into output and inflation. Hence, even if the central bank did not adjust its signal extraction weights, it would, in fact, still attach more importance to its own private information. Put differently, Figure 5.3 considers non-orthogonalized signal extraction weights, including the inherent correlation between output, inflation, the interest rate and θ_{cbt}.

[36] Figure 5.3 shows that private sector signal extraction weights on inflation eventually become positive. This might initially seem counter-intuitive, but it is caused by the combination of the increased weight on y_t and a negative correlation between output and inflation.

[37] The weight on the signals omitted in Figure 5.3, d_{it}, λ_{ipt}, d_t^{cb}, λ_{pt}^{cb}, ϵ_t^{mp}, all behave intuitively. For both the private sector and the central bank, increasing monetary policy activism causes a relatively larger weight to be placed on output *vis-a-vis* inflation in the signal extraction problem. To counteract the larger weight thereby implicitly placed on ϵ_t^d both the private and public sector increase (in absolute terms) the relative weight placed on their signals of the demand innovation relative to the supply shock. The central bank also attaches an increasing (in absolute terms) weight on the monetary policy shock, due to the increased weight placed on the interest rate by the private sector.

The change in private sector agents' information set, and thus their signal extraction problem, implies that the equilibrium of the imperfect information business cycle model differs across transparency regimes. But the difference in the number of signals is not the only cause of changes to the equilibrium of the model: the hierarchy of expectations, driving y_t, π_t and i_t, will also change. In particular:

Proposition 7. *The expectational state vector under central bank transparency, $X_t^{D(0:\infty)}$, equals:*

$$X_t^{D(0:\infty)} \equiv \begin{bmatrix} x_t \\ \mathbb{E}_{cbt} x_t \\ \bar{\mathbb{E}}_t x_t \\ \bar{\mathbb{E}}_t^{(2)} x_t \\ \vdots \end{bmatrix}, \quad \bar{\mathbb{E}}_t^{(k)} [x_t] \equiv \bar{\mathbb{E}}_t \left\{ \bar{\mathbb{E}}_t^{(k-1)} [x_t] \right\}, \; k \geq 1, \tag{5.1}$$

where $x_t \equiv \begin{bmatrix} y_t^{flex} & \theta_t \end{bmatrix}'$.

Proof. See Appendix C. □

The reason the state changes is the same as in Section 2: information disclosure causes the central bank's information set to be nested within the private sector's. This, in turn, causes all "cross-expectations" in $X_t^{(0:\infty)}$ – i.e. average expectations of central bank expectations, and so on – to collapse into terms already defined higher-up in the hierarchy. For example, the average expectation of the central bank's expectation of x_t – one of the terms that drive the direct stabilization effect – now equals just the central bank's expectation, already defined higher-up in $X_t^{(0:\infty)}$. Thus, under central bank information disclosure, the expectational state will not contain "cross-beliefs" – only average beliefs will matter, in addition to the central bank's optimal estimate of x_t.[38]

To solve the model with central bank information disclosure, I proceed along the same lines as in Section 4, taking into account both the changes in the hierarchy of beliefs and in private sector agents' signal extraction problem (see Appendix C).

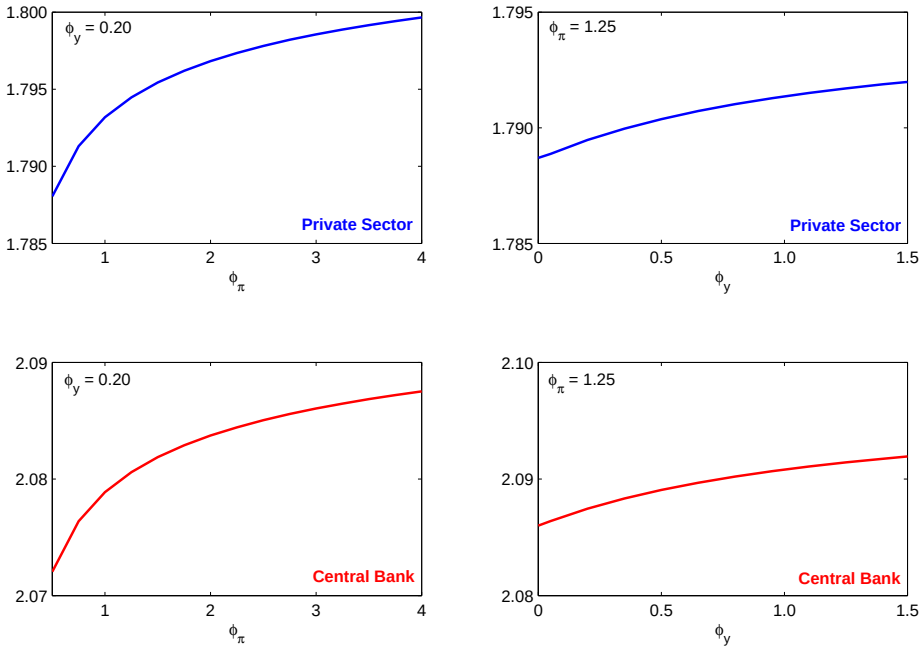
5.3 The Effects Under Central Bank Information Disclosure

The separation principle in Section 2 showed that central bank information disclosure nullifies the two-sided learning externality and substantially weakens the direct stabilization effect.[39] When the central bank's information set is a subset of the private sector's, individual agents have access to all the information they could learn from the setting of monetary policy. This eliminates the need for island inhabitants to learn from the interest rate equation, nullifying the two-sided learning externality. At the same time,

[38] The annihilation of "cross-expectations" in the expectational state vector also changes the dynamics of the model. First, all variables become more persistent: the increased persistence of endogenous variables caused by average higher-order beliefs are no-longer counteracted by the cross-expectation terms. Second, central bank transparency causes output to become less volatile while the opposite is true for inflation. The reason for this distinction between inflation and output is the same as for the direct stabilization benefit versus the direct stabilization cost. In response to a productivity shock, cross-expectations dampen inflation but make output more volatile. Information disclosure, by decreasing the correlation between average beliefs and the relevant cross-expectations (average beliefs of central bank beliefs, *ad infinitum*), counteracts this effect. The impact of both of these changes on the dynamics of the model is, however, small.

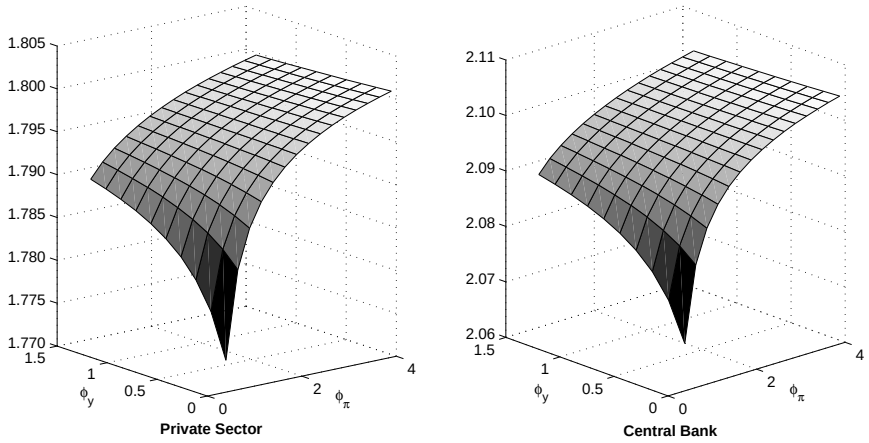
[39] In Section 2 information disclosure completely eliminated the direct stabilization. Here, however, it only weakens it due to the new information that the central bank receives in each period. See Section 2 for more on this.

Figure 5.4: Mean-Squared Error of y_t^{flex} under Central Bank Transparency



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the representative agent on island i and the central bank. The charts are plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.25; 4.00]$. The charts are depicted as a function of one variable of monetary policy activism keeping the other constant. $\{\phi_y, \phi_\pi\} = \{0.20, 1.25\}$ is consistent with the values estimated in the dispersed information business cycle model by Nimark (2012).

Figure 5.5: Mean-Squared Error of y_t^{flex} under Central Bank Transparency



The figure illustrates the mean-squared error of current-period estimates of y_t^{flex} for the representative agent on island i and the central bank. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$.

everyone now knows what everyone else (and so on) expects of the central bank's *current* beliefs. This weakens the direct stabilization effect. The arrival of new private information now only causes agents to update their expectation of the component of future interest rates that is due to the arrival of *future* central bank signals. Everyone knows the component that is due to current central bank beliefs.

Figure 5.4 and 5.5 illustrate these results. Because the direct stabilization cost is stronger than the direct stabilization benefit, increases in monetary policy activism are consistent with somewhat more uncertainty about the underlying fundamentals for both the private sector and the central bank. Correspondingly, as we increase monetary policy activism, island inhabitants and the monetary authority attach less weight to the endogenous public signals (output and inflation) in their respective signal extraction problems. That said, the direct stabilization effect is substantially muted and effects on economic uncertainty are therefore small (c. 1.5-2.0% from the low-end of the range to the high-end). To a rough first approximation, central bank transparency makes the choice of monetary policy activism largely independent of its implication for economic uncertainty. And thus, to a great extent, removes the need for the central bank to trade-off its stabilization motive against the informational consequences of its actions.

6 Central Bank Policy and the Informativeness of Public Signals

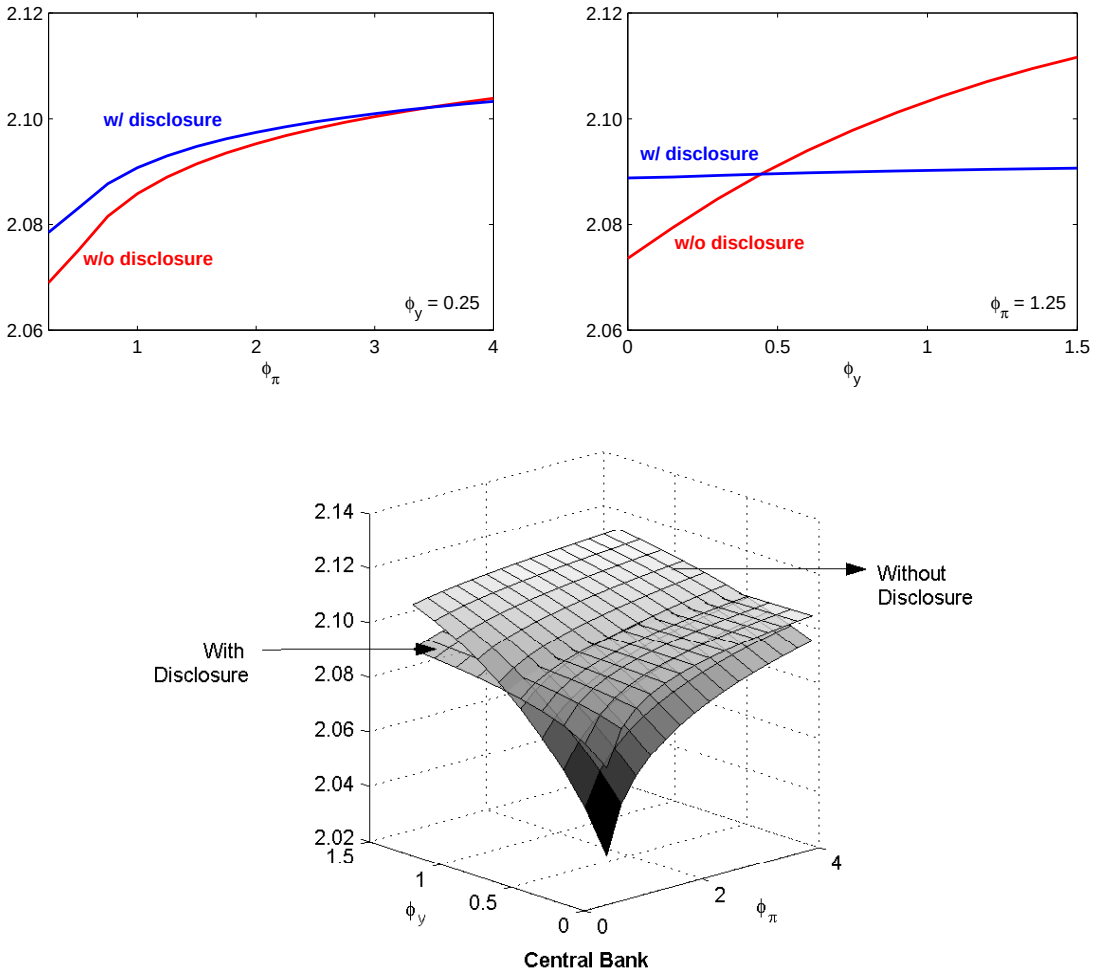
The previous Section showed how central bank information disclosure weakens the mechanisms by which monetary policy changes the information content of public signals. This, in turn, has implications for their overall informativeness. This Section illustrates that, on balance central, bank information disclosure has a relatively minor impact on the information content of public signals. The weakening of the direct stabilization effect more than compensates for the loss of informativeness due to the elimination of the two-sided learning externality. In fact, for plausible deviations from the baseline scenario, the weakening of the direct stabilization effect dominates. Central bank information disclosure is consistent, for plausible parameter values, with more informative public signals.

6.1 The Information Content of Public Signals

Figure 6.1 illustrates central bank uncertainty about the flex-price level of output for various coefficients of monetary policy activism. I focus exclusively on the central bank since it can only learn additional information from the endogenous public signals (output and inflation). Changes in the information content of these therefore translate one-for-one into movements in central bank uncertainty.

Consistent with the previous Section, for low values of monetary policy activism, the elimination of the two-sided learning externality dominates. Central bank information disclosure, like in Morris and Shin (2005) and Amador and Weill (2010), decreases the informational efficiency of public signals. However, for large values of monetary policy activism, that is not the case: the weakening of the direct stabilization effect more than offsets the removal of the two-sided learning externality.

Figure 6.1: Central Bank Uncertainty with/without Information Disclosure



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the central bank with/without information disclosure. The charts are plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.25; 4.00]$. The first set of charts are depicted as a function of one variable of monetary policy activism keeping the other constant. $\{\phi_y, \phi_\pi\} = \{0.25, 1.25\}$ is consistent with the values estimated in the dispersed information business cycle model by Nimark (2012).

Central bank information disclosure partially separates the expectation of future fundamentals from the expectation of future interest rates.[40] This leads central bank uncertainty, for large values of monetary policy activism, to be lower under central bank information disclosure than without.[41]

Because of the sparse literature estimating dispersed information business cycle models it is difficult to get a firm estimate of ϕ_y and ϕ_π. That said, if we take values consistent with those estimated in Nimark (2012) ($\phi_\pi = 1.50$ and $\phi_y = 0.25$) the results in Figure 6.1 show that central bank information disclosure has at most a small impact on the information content of endogenous public signals; we are almost exactly in the intersection between the two planes. A person with access to just the distribution of prices and economy-wide output has an estimate of total factor productivity that is only 0.4% worse (in a mean-squared error sense) under central bank information disclosure than without. The corresponding central bank estimate is 0.2% worse due to the existence of its additional signals (Figure 6.1). This demonstrates that the negative effects of information disclosure identified by Morris and Shin (2005) and Amador and Weill (2010) are, for plausible parameter values, almost completely offset by the weakening of the direct stabilization effect. Central bank information disclosure does not appear to come at the cost of substantially less informative public signals, and hence necessarily less informed public policy.

6.2 Alternative Specifications

The baseline calibration provides a useful benchmark for assessing the impact of central bank information disclosure on the information content of public signals. In general, however, the exact intersection, if any, between central bank uncertainty with and without information disclosure depends on two factors: (1) the extent to which there is a direct stabilization cost versus a direct stabilization benefit; and (2) the extent to which (cross) higher-order expectations influence the equilibrium of the model. The greater the direct stabilization cost, the more, all else equal, the additional imparting of private information through expectations of future interest rates deducts from the informativeness of output and inflation. Likewise, the more influential higher-order beliefs are the more future central bank expectations matter for these variables and the stronger the direct stabilization effect is.

The baseline calibration used so far is, however, a fairly conservative starting point for assessing these two factors. First, the persistent innovation in the business cycle model is total factor productivity. This per definition causes a relative small (net-) direct stabilization effect; there exists by assumption both a direct stabilization cost and a direct stabilization benefit. Demand shocks, typically thought to be a common driver of business cycles, however do not imply this type of inherent offset. With persistent innovation in demand (d_t) both output and inflation become less informative as feedback policy becomes more active. There is only a direct stabilization cost and no benefit. This, all else equal, increases the positive effects of central bank transparency.

[40] As detailed above, central bank transparency changes both private sector agents signal extraction problem and the expectational state. Disclosure implies that private sector agents can condition out the component of future interest rates that is due to current central bank beliefs, and thus the impact of all "cross-expectations" in Proposition 7 on the dynamics of inflation and output. The additional private information that agents impart into output and inflation under opacity in an attempt to second guess central bank beliefs can now be separated from the private information that is infused by average higher-order expectations. This leaves a (net-) cleaner signal of underlying total factor productivity.

[41] I also find that private sector uncertainty falls with the introduction of central bank information disclosure. The possibility, highlighted by Amador and Weill (2010), of private sector uncertainty increasing with an additional public signal does not appear consistent with the dispersed information business cycle for a range of plausible parameter values.

Second, I have above used a fairly low number for the persistence of total factor productivity ($\rho_\theta = 0.80$). Increasing ρ_θ to values consistent with those typically used in business cycle models, all else equal, increases the importance of higher-order expectations for the equilibrium of the model. This again increases the positive effects of central bank information disclosure when there is a (net-) direct stabilization cost.

Last, I have above included both habit persistence and partial indexation. Both features – like an increase in the coefficient of risk aversion – limit the effects of future interest rates, and hence future central bank beliefs, on island inhabitants' decisions today. Decreasing these coefficients therefore also increases the relative strength of the direct stabilization effect.

Figure 6.2 shows the impact of central bank information disclosure on the informativeness of public signals, output and inflation, across alternative specifications: Panel (a) introduces a persistent demand innovation whereas Panel (b) increases the persistence of total factor productivity to 0.90 and decreases the coefficient determining the intertemporal elasticity of substitution to one.[42] In Panel (c) I show the effect of information disclosure when the central bank does not have an informational advantage over the private sector.[43] Appendix D includes additional robustness checks.

On balance, the evidence in Figure 6.2 demonstrates that, if anything, the baseline calibration underestimates the importance of the weakening of the direct stabilization effect. When persistent innovations in demand are the fundamental driving force of the economy, central bank information disclosure improves the informational efficiency of endogenous public signals for all values of ϕ_π and ϕ_y in our range – i.e. for almost all empirical estimates of the Taylor Rule (Panel a). This holds true even for large variations in the calibrated parameter values. Central bank uncertainty is always lower under transparency than without. This demonstrates how with demand shocks the weakening of the direct stabilization effect dominates the elimination of the two-sided learning externality.

Increases in the persistence of total factor productivity combined with a decrease in the coefficient of relative risk-aversion likewise strengthen the importance of the direct stabilization effect (Panel b). Only for reasonably low values of monetary policy activism in our range does the elimination of the two-sided learning externality now dominate the direct stabilization effect. And even for those values, the effects are small. Panel (c) shows that the magnitude of the effects discussed so far do not depend crucially on the assumption of an informational advantage for the central bank.

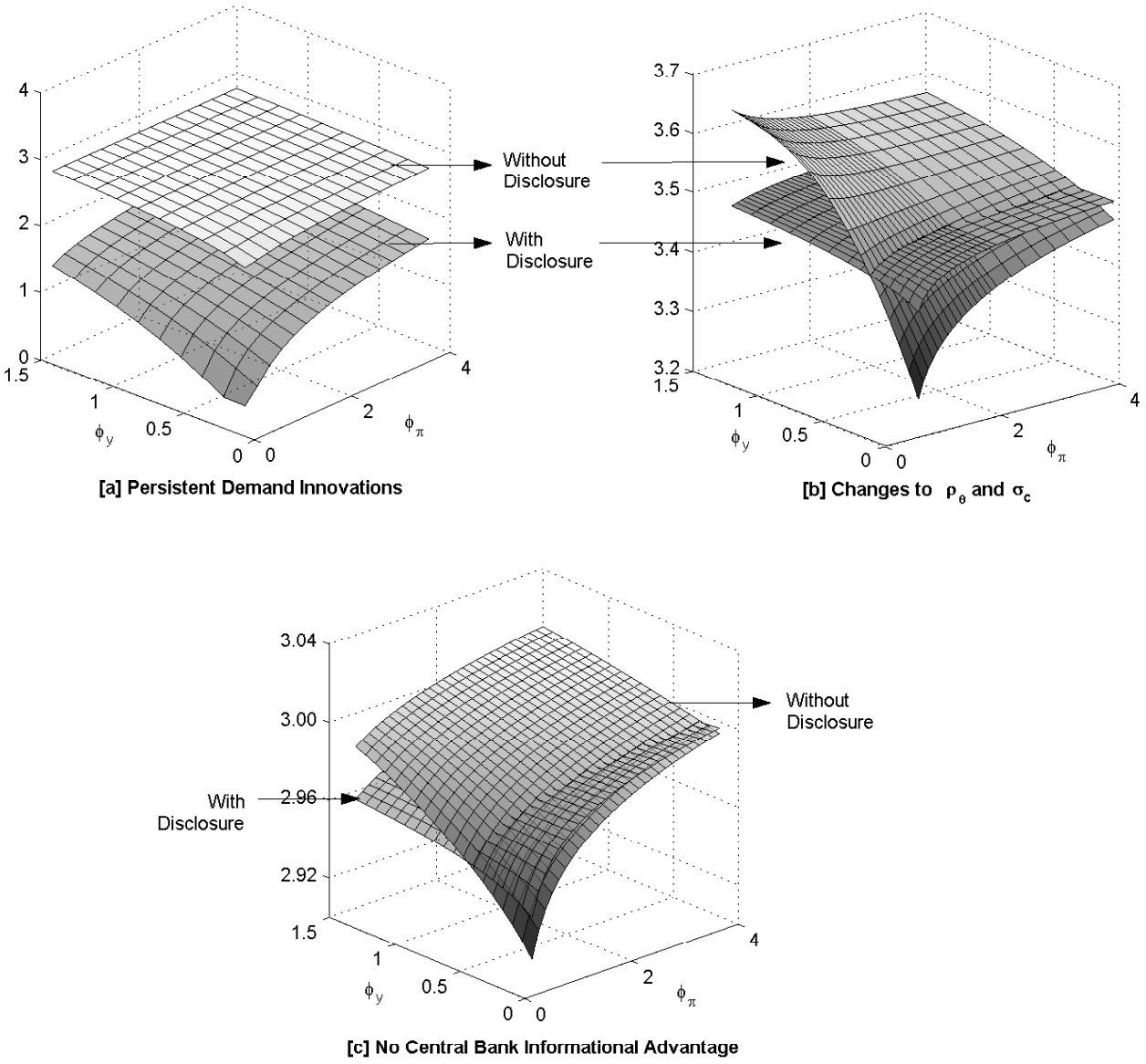
In sum, the alternative specifications show that the baseline calibration, if anything, perhaps underrates the importance of the direct stabilization effect. If demand shocks are an additional driver of the economy or if total factor productivity is very persistent (as in most of the DSGE literature), the informativeness of public signals tends to increase with central bank information disclosure. This, in turn, decreases the level of uncertainty for everyone in the economy – even the central bank itself.

[42] I set the persistence of demand shocks, d_t, to 0.80. The standard deviation of the innovation to demand is the same as in the baseline calibration. So too are the private information parameters determining the precision of private signals of d_t. The remaining private information parameters are set to uninformative values.

[43] The central bank's private information parameters are all set equal to the corresponding private sector ones.

Figure 6.2: Alternative Specifications:

Central Bank Uncertainty with/without Information Disclosure



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the central bank with/without information disclosure. The charts are plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.25; 4.00]$. Panel [a] demonstrates the effect of replacing the persistent productivity shock with a persistent innovation to demand; Panel [b] the effect of increasing the persistence of total factor productivity at the same time as decreasing the coefficient determining the intertemporal elasticity of substitution; and last Panel [c] the effect of removing the central bank's informational advantage. $\{\phi_y, \phi_\pi\} = \{0.25, 1.25\}$ is consistent with the values estimated in the dispersed information business cycle model by Nimark (2012).

7 Conclusion

This paper contributes to the discussion of the social value of central bank information. In essentially static settings, Morris and Shin (2005) and Amador and Weill (2010) show that central bank information disclosure decreases the information content of endogenous public signals. In this paper I have, however, argued that once one accounts for the effects of expected future central bank policy on people's choices, and hence how much information they convey via their decisions to others, information disclosure does not substantially alter the informativeness of public signals. In fact, for plausible parameter values in a dispersed information business cycle model, central bank transparency improves (on a net-basis) the informational efficiency of the public signals of output and inflation. By sharing its own view of the economy, even the central bank itself can learn more.

I show that what offsets the Morris and Shin (2005) and Amador and Weill (2010) "anti-disclosure result" is that central bank transparency alleviates an identification problem: People's choices – pricing and output decisions – are a function of both their beliefs about the future fundamentals of the economy and their beliefs about future central bank actions. But when the central bank attempts to stabilize the economy, the latter offsets the former: central bank stabilization policy makes people's decisions reflect less their expectations about the driving forces of the economy. This decreases their informativeness. Central bank information disclosure to a large extent alleviates this identification problem. Everyone knows what everyone else expects of the central bank under transparency. This makes people's decisions a cleaner signal of their expectations about the underlying fundamentals and hence improves the information content of endogenous public signals.

Last, this paper has addressed the issue of how central bank information disclosure affects the information content of public signals. I have, however, been mute on the welfare consequences, and thus the implications for the optimal design of policy. Lorenzoni (2010) and Angeletos and La'O (2013, 2014) provide important steps towards a benchmark framework capable of assessing optimal policy with dispersed information and learning from endogenous variables. Further steps in this direction present a natural avenue for future research.

References

Acharya, S. (2013). Dispersed beliefs and aggregate demand management. mimeo.

Allen, F., Morris, S. and Shin, H. S. (2006). Beauty contests and iterated expectations in asset markets. *Review of Financial Studies*, **19** (3), 719–752.

Amador, M. and Weill, P.-O. (2010). Learning from prices: Public communication and welfare. *Journal of Political Economy*, **118** (5), 866 – 907.

— and — (2012). Learning from private and public observations of others actions. *Journal of Economic Theory*, **147** (3), 910–940.

Angeletos, G.-M. and La'O, J. (2012). Optimal monetary policy with informational frictions.

— and — (2013). Sentiments. *Econometrica*, **Volume 81(2)**, 739–780.

— and — (2014). *Efficiency and Policy with Endogenous Learning*. mimeo.

— and Pavan, A. (2007). Efficient Use of Information and Social Value of Information. *Econometrica*, **75** (4), 1103–1142.

Baeriswyl, R. and Cornand, C. (2010). The signaling role of policy actions. *Journal of Monetary Economics*, **57** (6), 682–695.

Baxter, B., Graham, L. and Wright, S. (2011). Invertible and non-invertible information sets in linear rational expectations models. *Journal of Economic Dynamics and Control*, **35** (3), 295–311.

Bernanke, B. S. (2004). Central bank talk and monetary policy: Remarks at the japan society corporate luncheon.

Christiano, L. J., Eichenbaum, M. and Evans, C. (2001). *Nominal rigidities and the dynamic effects of a shock to monetary policy*. Working Paper 0107, Federal Reserve Bank of Cleveland.

Del Negro, M., Schorfheide, F., Smets, F. and Wouters, R. (2007). On the fit of new keynesian models. *Journal of Business & Economic Statistics*, **25**, 123–143.

Dincer, N. and Eichengreen, B. (2009). Central bank transparency: causes, consequences and updates.

Edge, R. M. and Rudd, J. B. (2012). Real-time properties of the federal reserve's output gap. mimeo.

Futia, C. (1981). Rational expectations in stationary linear models. *Econometrica*.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

George-Marios Angeletos, J. L., and Iovino, L. (2013). *Cycles, Gaps, and the Social Value of Information,*. mimeo.

— (2013). *Optimal Monetary Policy with Informational Frictions*. mimeo.

Gibbs, R. G. (2011). Square root modified bryson-frazier smoother. *IEEE Transactions on Aut*, **56**.

Gosselin, P., Lotz, A. and Wyplosz, C. (2007). *Interest Rate Signals and Central Bank Transparency*. CEPR Discussion Papers 6454, C.E.P.R. Discussion Papers.

Graham, L. and Wright, S. (2010). Information, heterogeneity and market incompleteness. *Journal of Monetary Economics*, **57** (2), 164–174.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton Univeristy Press.

Hayek, F. A. (1945). The use of knowledge in society. *American Economic Review*.

Hellwig, C. (2004). *Heterogeneous Information and the Benefits of Public Information Disclosures*. UCLA Economics Online Papers 283, UCLA Department of Economics.

— and Venkateswaran, V. (2012). *Hayek vs. Keynes: Dispersed Information and Market Prices in a Price-setting Model*. Working paper.

Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, **22** (3), 477–498.

Hubert, P. (2011). *Rivisiting the Greenbook's Relative Forecasting Performance*. Economics working papers, OFCE - SciencesPo.

James, J. G. and Lawler, P. (2011). Optimal policy intervention and the social value of public information. *The American Economic Review*, **101** (4), 1561–1574.

— and — (2012). Strategic complementarity, stabilization policy, and the optimal degree of publicity. *Journal of Money, Credit and Banking*, **44** (4), 551–572.

Kasa, K. (2000). Forecasting the forecasts of others in the frequency domain. *Review of Economic Dynamics*, **3** (4), 726–756.

—, Walker, T. and Whiteman, C. (2012). *Heterogenous Belifes and Tests of Present Value Models*. Economics working papers, Department of Economics, Simon Fraser University Working Paper.

King, R. G. (1982). Monetary policy and the information content of prices. *Journal of Political Economy*, **90** (2), 247–79.

Kohlhas, A. N. (2013). The informativeness of prices: Dispersed asymmetric information and higher-order beliefs.

Lorenzoni, G. (2009). A theory of demand shocks. *American Economic Review*, **99** (5), 2050–84.

— (2010). Optimal monetary policy with uncertain fundamentals and dispersed information. *Review of Economic Studies*, **77** (1), 305–338.

Lucas, R. J. (1972). Expectations and the neutrality of money. *Journal of Economic Theory*, **4** (2), 103–124.

Morris, S. and Shin, H. S. (2002). Social value of public information. *American Economic Review*, **92** (5), 1521–1534.

— and — (2005). Central bank transparency and the signal value of prices. *Brookings Papers on Economic Activity*, **36** (2), 1–66.

Nimark, K. (2011). *Dynamic Higher Order Expectations*. Economics Working Papers 1118, Department of Economics and Business, Universitat Pompeu Fabra.

— (2012). *Man-Bites-Dog Business Cycles*. Economics working papers, Department of Economics and Business, Universitat Pompeu Fabra.

Orphanides, A. (2001). Monetary policy rules based on real-time data. *American Economic Review*, **91** (4), 964–985.

Reifschneider, D. L., Stockton, D. J. and Wilcox, D. W. (1997). Econometric models and the monetary policy process. *Carnegie-Rochester Conference Series on Public Policy*, **47** (1), 1–37.

Romer, D. H. and Romer, C. D. (2000). Federal reserve information and the behavior of interest rates. *American Economic Review*, **90** (3), 429–457.

Rondina, G. and Walker, T. (2012). *Information Equilibria in Dynamic Economies with Dispersed Information*. Working paper, Department of Economics, UCSD.

Singleton, K. J. (1987). *Asset Prices in a Time Series Model with Disparately Informed, Competitive Traders,*. Proceedings of the second international symposium in economic theory and econometrics, Standford University.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, **97** (3), 586–606.

Svensson, L. E. O. (2006). Social value of public information: Comment: Morris and shin (2002) is actually pro-transparency, not con. *American Economic Review*, **96** (1), 448–452.

— and Woodford, M. (2004). Indicator variables for optimal policy under asymmetric information. *Journal of Economic Dynamics and Control*, **28** (4), 661–690.

Townsend, R. M. (1983). Forecasting the forecasts of others. *Journal of Political Economy*, **91** (4), 546–88.

Vavra, J. (2013). *Inflation Dynamics and Time-Varying Uncertainty: New Evidence and an Ss Interpretation*. mimeo.

Vives, X. (1997). Learning from others: A welfare analysis. *Games and Economic Behavior*, **20** (2), 177–200.

— (2013). Endogenous public information and welfare in market games.

Walsh, C. E. (2007). Optimal economic transparency. *International Journal of Central Banking*, **3** (1), 5–36.

Weiss, L. (1982). Information aggregation and policy. *Review of Economic Studies*, **49** (1), 31–42.

Woodford, M. (2002). *Imperfect Common Knowledge and the Effects of Monetary Policy*. Nber working papers, Department of Economics, Columbia University.

Appendix A: The Basic Model

This appendix details the proofs for the lemmas and propositions in Section 2 in the main text.

Model Solution

Proof of Proposition 1: To show that this is indeed the case, I use the above two equations (see (2.2) and (2.3) in the main text) as a conjecture for the solution. Using this conjecture, $\tilde{a}_t$ and i_t become:

$$\begin{aligned}\tilde{a}_t &= \beta\bar{\mathbb{E}}_t\left[k_0\theta_t + k_1\epsilon^{\theta}_{t+1} + k_2 i_{t+1} + k_3\epsilon^{a}_{t+1}\right] + \theta_t + \gamma i_t + \epsilon^a_t \\ &= \beta\left(k_0 + k_2\bar{k}_0\right)\bar{\mathbb{E}}_t\left[\theta_t\right] + \theta_t + \gamma i_t + \epsilon^a_t \qquad (7.1)\end{aligned}$$

$$\begin{aligned}i_t &= \phi\mathbb{E}_{cbt}\left[k_0\theta_t + k_1\epsilon^{\theta}_{t+1} + k_2 i_{t+1} + k_3\epsilon^{a}_{t+1} - \theta_{t+1}\right] + \epsilon^{mp}_t \\ &= \phi\left(k_0 + k_2\bar{k}_0 - 1\right)\mathbb{E}_{cbt}\left[\theta_t\right] + \epsilon^{mp}_t. \qquad (7.2)\end{aligned}$$

To verify the conjecture, we therefore need to solve both the private sector's and the central bank's signal extraction problem.

The Central Bank's Signal Extraction Problem: The goal is to find:

$$\mathbb{E}_{cbt}\left[\theta_t\right] = \theta_{t-1} + \mathbb{E}_{cbt}\left[\epsilon^{\theta}_t\right].$$

The policy authority has access to two signals of ϵ^{θ}_t: $\tilde{a}_t$ and θ_{cbt}. Due to dependence of $\tilde{a}_t$ on $\mathbb{E}_{cbt}\left[\theta_t\right]$ via i_t, there is, however, a simultaneity problem that needs to be solved within this signal extraction exercise. The simplest way to solve this fixed-point problem is to condition out the central bank's instrument, i_t, treating it as another, albeit uninformative, element of the central bank's information set. The monetary authority's signals can therefore be written as:

$$\begin{aligned}s^a_t &\equiv \frac{1}{k_1}\left(\tilde{a}_t - k_0\theta_{t-1} - k_2 i_t\right) = \epsilon^{\theta}_t + \frac{k_3}{k_1}\epsilon^a_t \\ \tilde{\theta}_{cbt} &= \epsilon^{\theta}_t + \epsilon^{\theta}_{cbt}.\end{aligned}$$

Since the error terms in the central bank's signal vector, comprised of s^a_t and θ_{cbt}, are uncorrelated it follows from standard linear projection that:

$$\begin{aligned}\mathbb{E}_{cbt}\left[\theta_t\right] &= \theta_{t-1} + w_{cba}\left(\epsilon^{\theta}_t + \frac{k_3}{k_1}\epsilon^a_t\right) + w_{cb\theta}\theta_{cbt} \\ &= \theta_{t-1} + w_{cba}\epsilon^{\theta}_t + w_{cb\theta}\tilde{\theta}_{cbt} + w_{cba}\frac{k_3}{k_1}\epsilon^a_t, \qquad (7.3)\end{aligned}$$

where:

$$w_{cba} \equiv \frac{\left(k_1/k_3\right)^2 p_a}{p_\theta + p_{cb\theta} + \left(k_1/k_3\right)^2 p_a}, \quad w_{cb\theta} \equiv \frac{p_{cb\theta}}{p_\theta + p_{cb\theta} + \left(k_1/k_3\right)^2 p_a},$$

and p_x denotes the precision (the inverse of the variance) of signal x.

Thus:

$$\begin{aligned} i_t &= \phi\left(k_0 + k_2\bar{k}_0 - 1\right)\left[\theta_{t-1} + w_{cba}s_t^a + w_{cb\theta}\tilde{\theta}_{cbt}\right] + \epsilon_t^{mp} \\ &= \bar{k}_0\left[\theta_{t-1} + w_{cba}s_t^a + w_{cb\theta}\theta_{cbt}\right] + \epsilon_t^{mp}. \end{aligned}$$

The Private Sector's Signal Extraction Problem: The goal is (again) to find:

$$\mathbb{E}_{it}\left[\theta_t\right] = \theta_{t-1} + \mathbb{E}_{it}\left[\epsilon_t^{\theta}\right].$$

Private sector individual i has access to three signals: θ_{it}, $\tilde{a}_t$ and i_t. These can, alternatively, be written as:

$$\begin{aligned} \theta_{it} &= \epsilon_t^{\theta} + \epsilon_{it}^{\theta} \\ s_t^a &\equiv \frac{1}{k_1}\left(\tilde{a}_t - k_0\theta_{t-1} - k_2 i_t\right) = \epsilon_t^{\theta} + \frac{k_3}{k_1}\epsilon_t^a \\ s_t^i &\equiv \frac{1}{\bar{k}_0 w_{cb\theta}}\left(i_t - \bar{k}_0\theta_{t-1} - \bar{k}_0 w_{cba}s_t^a\right) = \epsilon_t^{\theta} + \epsilon_{cbt}^{\theta} + \frac{1}{\bar{k}_0 w_{cb\theta}}\epsilon_t^{mp} \end{aligned}$$

Agent i's estimate of θ_t therefore equals:

$$\mathbb{E}_{it}\left[\theta_t\right] = \theta_{t-1} + w_{i\theta}\theta_{it} + w_a s_t^a + w_i s_t^i,$$

where $w_{i\theta}$, w_a and w_i are given by:

$$w_{i\theta} = \frac{p_{i\theta}}{p_\theta + p_{i\theta} + \left(\frac{k_1}{k_3}\right)^2 p_a + \frac{p_{cb\theta}p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}{p_{cb\theta}+p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}}, \quad w_a \equiv \frac{\left(\frac{k_1}{k_3}\right)^2 p_a}{p_\theta + p_{i\theta} + \left(\frac{k_1}{k_3}\right)^2 p_a + \frac{p_{cb\theta}p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}{p_{cb\theta}+p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}}, \tag{7.4}$$

$$w_i \equiv \frac{\frac{p_{cb\theta}p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}{p_{cb\theta}+p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}}{p_\theta + p_{i\theta} + \left(\frac{k_1}{k_3}\right)^2 p_a + \frac{p_{cb\theta}p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}{p_{cb\theta}+p_{mp}\left(\bar{k}_0 w_{cb\theta}\right)^2}}.$$

The average private sector expectation of the underlying fundamental thus equals:

$$\begin{aligned} \bar{\mathbb{E}}\left[\theta_t\right] &= \theta_{t-1} + w_{i\theta}\epsilon_t^{\theta} + w_a s_t^a + w_i\frac{1}{\bar{k}_0 w_{cb\theta}}\left(i_t - \bar{k}_0\theta_{t-1} - \bar{k}_0 w_{cba}s_t^a\right) \\ &= \left(1 - \frac{w_i}{w_{cb\theta}}\right)\theta_{t-1} + \left(w_{i\theta} + w_a - w_i\frac{w_{cba}}{w_{cb\theta}}\right)\epsilon_t^{\theta} \\ &+ \left(w_a - w_i\frac{w_{cba}}{w_{cb\theta}}\right)\frac{k_3}{k_1}\epsilon_t^a + w_i\frac{1}{\bar{k}_0 w_{cb\theta}}i_t. \end{aligned} \tag{7.5}$$

Matching Coefficients: Inserting (7.3) and (7.5) into (7.1) and (7.2) verifies the claim:

$$\begin{aligned}\tilde{a}_t &= \left[1+\beta\left(k_0+k_2\bar{k}_0\right)\left(1-\frac{w_i}{w_{cb\theta}}\right)\right]\theta_{t-1}+\left[1+\beta\left(k_0+k_2\bar{k}_0\right)\left(w_{i\theta}+w_a-w_i\frac{w_{cba}}{w_{cb\theta}}\right)\right]\epsilon_t^{\theta}\\ &+ \left[\gamma+\beta\left(k_0+k_2\bar{k}_0\right)w_i\frac{1}{\bar{k}_0 w_{cb\theta}}\right]i_t+\left[1+\beta\left(k_0+k_2\bar{k}_0\right)\left(w_a-w_i\frac{w_{cba}}{w_{cb\theta}}\right)\frac{k_3}{k_1}\right]\epsilon_t^{a},\end{aligned}$$

$$\begin{aligned}i_t &= \left[\phi\left(k_0+k_2\bar{k}_0-1\right)\right]\theta_{t-1}+\left[\phi\left(k_0+k_2\bar{k}_0-1\right)w_{cba}\right]\epsilon_t^{\theta}\\ &+ \left[\phi\left(k_0+k_2\bar{k}_0-1\right)w_{cba}\frac{k_3}{k_1}\right]\epsilon_t^{a}+\epsilon_t^{mp}+\left[\phi\left(k_0+k_2\bar{k}_0-1\right)w_{cb\theta}\right]\tilde{\theta}_{cbt}.\end{aligned}$$

Matching coefficients implies that:

$$\begin{aligned}k_0 &= 1+\beta\left(k_0+k_2\bar{k}_0\right)\left(1-\frac{w_i}{w_{cb\theta}}\right)\\ k_1 &= 1+\beta\left(k_0+k_2\bar{k}_0\right)\left(w_{i\theta}+w_a-w_i\frac{w_{cba}}{w_{cb\theta}}\right)\\ k_2 &= \gamma+\beta\left(k_0+k_2\bar{k}_0\right)w_i\frac{1}{\bar{k}_0 w_{cb\theta}}\\ k_3 &= 1+\beta\left(k_0+k_2\bar{k}_0\right)\left(w_a-w_i\frac{w_{cba}}{w_{cb\theta}}\right)\frac{k_3}{k_1}\\ \bar{k}_0 &= \phi\left(k_0+k_2\bar{k}_0-1\right)\\ \bar{k}_1 &= \bar{k}_0 w_{cba}\\ \bar{k}_2 &= \bar{k}_0 w_{cba}\frac{k_3}{k_1}\\ \bar{k}_3 &= 1\\ \bar{k}_4 &= \bar{k}_0 w_{cb\theta}.\end{aligned}$$

The linear rational expectations equilibrium is given by the solution to this system of equations in $k_j \in \mathbb{R}$, $j \in \{0, 1, 2, 3\}$ and $\bar{k}_j \in \mathbb{R}$, $j \in \{0, 1, 2, 3, 4\}$.

To show that there exists a unique solution to this system of equations, combine the first, third and fifth equation to get:

$$\begin{aligned}\bar{k}_0 &= \frac{\phi\beta}{1-\beta-\phi\gamma}>0\\ k_0+\bar{k}_0 k_2 &= \frac{1-\phi\gamma}{1-\beta-\phi\gamma}>0.\end{aligned}$$

Given the solution to $\bar{k}_0$ and $k_0+\bar{k}_0 k_2$, there is a one-to-one mapping between the remaining coefficients above and the ratio k_1/k_3, determining the signal-to-noise ratio of the endogenous aggregate. This ratio is, in turn, determined by the solution to the equation:

$$\frac{k_1}{k_3}=1+\beta\frac{1-\phi\gamma}{1-\beta-\phi\gamma}w_{i\theta}, \tag{7.6}$$

where $w_{i\theta}$ is given by (7.4). From (7.6), we conclude that $k_1/k_3>0$ as $w_{i\theta}\geq 0$. Now from (7.4) we find

that:

$$\begin{aligned}\frac{\partial w_{i\theta}}{\partial(k_1/k_3)} &= -\frac{p_{i\theta}}{\left(p_\theta + p_{i\theta} + \left(\frac{k_1}{k_3}\right)^2 (p_a + p_{ai}) + \frac{p_{cb\theta} p_{mp} \bar{k}_4^2}{p_{cb\theta} + p_{mp}\bar{k}_4^2}\right)^2} \\ &\cdot \left[\frac{p_{ai}}{p_a} + 1 - \frac{2p_{mp}p_{cb\theta}\bar{k}_4^2}{\left(p_{cb\theta} + p_{mp}\bar{k}_4^2\right)^2} w_{cb\theta}\right] 2\frac{k_1}{k_3} p_a < 0\end{aligned}$$

since $0 < w_{cb\theta} \leq 1$ and all other terms are strictly positive. The right-hand side of (7.6) therefore decreases from a value above one towards zero as we increase k_1/k_3. Since the left-hand side increases linearly from zero, there will exist a unique solution to (7.6), and hence a unique equilibrium to our model. □

Model Solution under Central Bank Information Disclosure

Claim. The unique rational expectations equilibrium to the baseline model under central bank transparency can be written as:

$$\begin{aligned}\tilde{a}_t &= k_{0T}\theta_{t-1} + k_{1T}\epsilon_t^\theta + k_{2T} i_t + k_{3T}\tilde{\theta}_{cbt} + k_{4T}\epsilon_t^a \\ i_t &= \bar{k}_{0T}\theta_{t-1} + \bar{k}_{1T}\epsilon_t^\theta + \bar{k}_{2T}\tilde{\theta}_{cbt} + \bar{k}_{3T}\epsilon_t^a + \bar{k}_{4T}\epsilon_t^{mp},\end{aligned}$$

where $k_{jT} \in \mathbb{R}$, $j \in \{0, 1, 2, 3, 4\}$, $\bar{k}_{jT} \in \mathbb{R}$, $j \in \{0, 1, 2, 3, 4\}$ and $\tilde{\theta}_{cbt} \equiv \theta_{cbt} - \theta_{t-1}$.

Combining the *Claim* with (2.2) and (2.3) in the main text, $\tilde{a}_t$ and i_t become:

$$\tilde{a}_t = \beta\left(k_{0T} + k_{2T}\bar{k}_{0T}\right)\bar{\mathbb{E}}_t\left[\theta_t\right] + \theta_t + \gamma i_t + \epsilon_t^a \tag{7.7}$$

$$i_t = \phi\left(k_{0T} + k_{2T}\bar{k}_{0T} - 1\right)\mathbb{E}_{cbt}\left[\theta_t\right] + \epsilon_t^{mp}. \tag{7.8}$$

To verify the conjecture, we thus (again) need to solve both the private sector's and the central bank's signal extraction problem.

The Central Bank's Signal Extraction Problem

The policy authority still has access to two signals with independent error terms given by:

$$\begin{aligned}s_t^a &\equiv \frac{1}{k_{1T}}\left(\tilde{a}_t - k_{0T}\theta_{t-1} - k_{2T} i_t - k_{3T}\theta_{cbt}\right) = \epsilon_t^\theta + \frac{k_{4T}}{k_{1T}}\epsilon_t^a \\ \theta_{cbt} &= \epsilon_t^\theta + \epsilon_{cbt}^\theta.\end{aligned}$$

Hence:

$$\begin{aligned}\mathbb{E}_{cbt}\left[\theta_t\right] &= \theta_{t-1} + w_{aT}^{cb}\left(\epsilon_t^\theta + \frac{k_{4T}}{k_{1T}}\epsilon_t^a\right) + w_{\theta T}^{cb}\theta_{cbt} \\ &= \theta_{t-1} + w_{aT}^{cb}\epsilon_t^\theta + w_{\theta T}^{cb}\theta_{cbt} + w_{aT}^{cb}\frac{k_{4T}}{k_{1T}}\epsilon_t^a,\end{aligned} \tag{7.9}$$

where:

$$w_{aT}^{cb} \equiv \frac{\left(k_{1T}/k_{4T}\right)^2 p_a}{p_\theta + p_{cb\theta} + \left(k_{1T}/k_{4T}\right)^2 p_a}, \quad w_{\theta T}^{cb} \equiv \frac{p_{cb\theta}}{p_\theta + p_{cb\theta} + \left(k_{1T}/k_{4T}\right)^2 p_a}.$$

The Private Sector's Signal Extraction Problem

The private sector has access to three signals of θ_t. Analogously, we can view the private sector as only having access to two signals: its independent private signal, θ_{it}, and the policy authority's expectation, $\mathbb{E}_{cbt}\left[\theta_t\right]$. Either way, the expectation of individual i at time t of the underlying fundamental is given by:

$$\mathbb{E}_{it}\left[\theta_t\right] = \theta_{t-1} + w_{aT} s_t^a + w_{cb\theta T}\theta_{cbt} + w_{i\theta T}\theta_{it},$$

where:

$$\begin{aligned} w_{aT} &\equiv \frac{\left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a}{p_\theta + p_{cb\theta} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a + p_{i\theta}}, \qquad w_{cb\theta T} \equiv \frac{p_{cb\theta}}{p_\theta + p_{cb\theta} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a + p_{i\theta}}, \\ w_{i\theta T} &\equiv \frac{p_{i\theta}}{p_\theta + p_{cb\theta} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a + p_{i\theta}}. \end{aligned} \tag{7.10}$$

Hence:

$$\bar{\mathbb{E}}_t\left[\theta_t\right] = \theta_{t-1} + w_{aT}\left(\epsilon_t^\theta + \frac{k_{4T}}{k_{1T}}\epsilon_t^a\right) + w_{cb\theta T}\theta_{cbt} + w_{i\theta T}\epsilon_t^\theta. \tag{7.11}$$

Matching Coefficients

Now, inserting (7.11) and (7.9) into (7.8) and (7.7) verifies the *Claim*:

$$\begin{aligned} \tilde{a}_t &= \left[1 + \beta\left(k_{0T} + k_{2T}\bar{k}_{0T}\right)\right]\theta_{t-1} + \left[1 + \beta\left(k_{0T} + k_{2T}\bar{k}_{0T}\right)\left(w_{i\theta T} + w_{aT}\right)\right]\epsilon_t^\theta \\ &+ \left[\gamma\right] i_t + \left[\beta\left(k_{0T} + k_{2T}\bar{k}_{0T}\right) w_{cb\theta T}\right]\theta_{cbt} + \left[1 + \beta\left(k_{0T} + k_{2T}\bar{k}_{0T}\right) w_{aT}\frac{k_{4T}}{k_{1T}}\right]\epsilon_t^a, \end{aligned}$$

$$\begin{aligned} i_t &= \left[\phi\left(k_{0T} + k_{2T}\bar{k}_{0T} - 1\right)\right]\theta_{t-1} + \left[\phi\left(k_{0T} + k_{2T}\bar{k}_{0T} - 1\right) w_{aT}^{cb}\right]\epsilon_t^\theta \\ &+ \left[\phi\left(k_{0T} + k_{2T}\bar{k}_{0T} - 1\right) w_{\theta T}^{cb}\right]\theta_{cbt} + \left[\phi\left(k_{0T} + k_{2T}\bar{k}_{0T} - 1\right) w_{aT}^{cb}\frac{k_{4T}}{k_{1T}}\right]\epsilon_t^a + \epsilon_t^{mp}. \end{aligned}$$

Matching coefficients implies that:

$$\begin{aligned}
k_{0T} &= 1+\beta\left(k_{0T}+k_{2T}\bar{k}_{0T}\right)\\
k_{1T} &= 1+\beta\left(k_{0T}+k_{2T}\bar{k}_{0T}\right)\left(w_{i\theta T}+w_{aT}\right)\\
k_{2T} &= \gamma\\
k_{3T} &= \beta\left(k_{0T}+k_{2T}\bar{k}_{0T}\right)w_{cb\theta T}\\
k_{4T} &= 1+\beta\left(k_{0T}+k_{2T}\bar{k}_{0T}\right)w_{aT}\frac{k_{4T}}{k_{1T}}\\
\bar{k}_{0T} &= \phi\left(k_{0T}+k_{2T}\bar{k}_{0T}-1\right)\\
\bar{k}_{1T} &= \bar{k}_{0T}w_{aT}^{cb}\\
\bar{k}_{2T} &= \bar{k}_{0T}w_{\theta T}^{cb}\\
\bar{k}_{3T} &= \bar{k}_{0T}w_{aT}^{cb}\frac{k_{4T}}{k_{1T}}\\
\bar{k}_{4T} &= 1.
\end{aligned}$$

The rational expectations equilibrium is given by the solution to this system of equations in $k_{jT}\in\mathbb{R}$, $j\in\{0,\,1,\,2,\,3,\,4\}$ and $\bar{k}_{jT}\in\mathbb{R}$, $j\in\{0,\,1,\,2,\,3,\,4\}$. Following exactly the same steps as with the opacity case shows that there exists a unique solution to this system of equations.

The Direct Stabilization Effect

Proof of Proposition 2: The main text explains that if the central bank's information set is a subset of the private sector's, the two-sided learning externality no-longer affects the informativness of the public signal. As such, we can use the solution coefficients from the model under central bank transparency (see above) to show Proposition 2.

Using the equations for k_{0T} and $\bar{k}_{0T}$ allows us to find expressions for these coefficients only as a function of the parameters of the model:

$$\begin{aligned}
k_{0T} &= \frac{1-\gamma\phi-\beta\gamma\phi}{1-\beta-\gamma\phi}\\
\bar{k}_{0T} &= \frac{\beta\phi}{1-\beta-\gamma\phi}>0.
\end{aligned}$$

Since $0<\beta<1$, $\phi>0$ and $\gamma<0$, $\bar{k}_{0T}$ will always be positive. Thus $\gamma\bar{k}_{0T}$ will always be negative. Moreover, we can rewrite the condition:

$$-1<\gamma\bar{k}_0\Leftrightarrow(1-\beta)(1-\gamma\phi)>0,$$

which always holds under our maintained assumptions. This proves the first part of the proposition.[44]

To show the second part, it suffices to show that increases in monetary policy activism, as measured by ϕ, lead to a decrease in k_{1T}/k_{4T}. The reason being that the mean-squared error for both the private

[44] Identical expressions hold for the no-transparency case (see the proof of Proposition 1).

sector and the central bank depend negatively on k_{1T}/k_{4T}:

$$\begin{aligned} MSE^{D}_{cbt}(\theta_t) &\equiv \mathbb{E}_{cbt}\left[(\theta_t - \mathbb{E}_{cbt}\theta_t)^2\right] \\ &= \left[p_\theta + p_{cb\theta} + (k_{1T}/k_{4T})^2 p_a\right]^{-1} \qquad (7.12) \\ MSE^{D}_{it}(\theta_t) &\equiv \mathbb{E}_{it}\left[(\theta_t - \mathbb{E}_{it}\theta_t)^2\right] \\ &= \left[p_\theta + p_{cb\theta} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a + p_{i\theta}\right]^{-1}, \qquad (7.13) \end{aligned}$$

This is equivalent to showing that increases in monetary policy activism lead to a decrease in the signal-to-noise ratio of the endogenous aggregate. Since monetary policy under information disclosure only impacts the informativeness of $\tilde{a}_t$, decreases in the signal-to-noise ratio will transmit directly to increases in uncertainty about the fundamental θ_t.

To start with, define $x \equiv \left(k_{0T} + k_{2T}\bar{k}_{0T}\right)$. Hence:

$$x = \frac{1-\gamma\phi}{1-\beta-\gamma\phi} > 0.$$

Differentiating x wrt. ϕ gives:

$$\frac{dx}{d\phi} = \frac{\beta\gamma}{(1-\beta-\gamma\phi)^2} < 0. \qquad (7.14)$$

Increases in monetary policy activism therefore cause the variable x to monotonically decrease.

Now dividing the equation for k_{1T} with k_{4T} and re-arranging we get:

$$\frac{k_{1T}}{k_{4T}} = 1 + \beta x \frac{p_{i\theta}}{\tilde{k} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a},$$

where $\tilde{k} \equiv p_\theta + p_{cb\theta} + p_{i\theta}$. Since x is always greater than zero, this equation implies that $k_{1T}/k_{4T} \geq 1$. Totally differentiating the expression for k_{1T}/k_{4T} wrt. x gives:

$$\frac{d\left(k_{1T}/k_{4T}\right)}{dx} = \left[\tilde{k} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a\right] \frac{\beta p_{i\theta}}{\left[\tilde{k} + \left(\frac{k_{1T}}{k_{4T}}\right)^2 p_a\right]^2 + 2\beta x p_{i\theta}\frac{k_{1T}}{k_{4T}} p_a}.$$

Because all terms in this expression are strictly positive, we conclude that:

$$\frac{d\left(k_{1T}/k_{4T}\right)}{dx} > 0. \qquad (7.15)$$

Combining (7.14) and (7.15) completes the proof. □

The Two-sided Learning Externality.

Proof of Lemma 1: Setting $\gamma = 0$ in (7.6) gives:

$$\frac{k_1}{k_3} = 1 + \frac{\beta}{1-\beta} w_{i\theta}, \qquad (7.16)$$

where $\partial w_{i\theta}/\partial\left(k_1/k_3\right) < 0$ (see the proof of Proposition 1). Moreover: $\bar{k}_0 = \phi\beta/(1-\beta)$.

Figure 7.1 illustrates the left-hand and right-hand side of (7.16), as well as their equilibrium intersection. From (7.16) and (7.4) we see that decreases in ϕ leave the left-hand side untouched, but induces an upward shift in the right-hand side for a given $k_1/k_3 > 0$. In equilibrium, decreases in ϕ therefore cause an increase in both k_1/k_3 and $w_{i\theta}$ (see Figure 7.1). I.e:

$$\frac{d(k_1/k_3)}{d\phi} < 0, \quad \frac{dw_{i\theta}}{d\phi} < 0. \tag{7.17}$$

Using the expressions for the signal extraction weights for both the central bank and the private sector, it is now straightforward to show that:

$$\begin{aligned} \frac{dw_a}{d\phi} < 0 \quad &, \quad \frac{dw_i}{d\phi} > 0, \\ \frac{dw_{cba}}{d\phi} < 0 \quad &, \quad \frac{dw_{cb\theta}}{d\phi} > 0, \end{aligned}$$

completing the proof. □

Proof of Proposition 3: The central bank's mean-squared error of the current value of the underlying fundamental, θ_t, is given by:

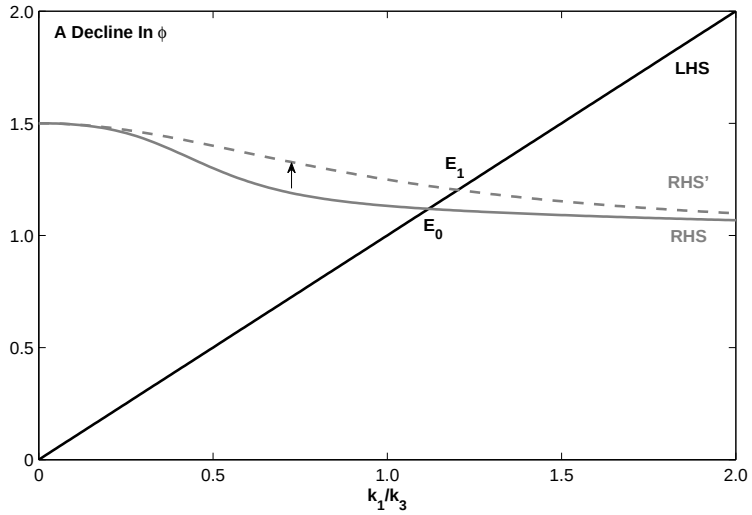
$$\begin{aligned} MSE_{cbt}\left(\theta_t\right) &\equiv \mathbb{E}_{cbt}\left[\left(\theta_t - \mathbb{E}_{cbt}\theta_t\right)^2\right] \\ &= \left[p_\theta + p_{cb\theta} + \left(k_1/k_3\right)^2 p_a\right]^{-1}. \end{aligned} \tag{7.18}$$

Individual $i's$ mean-squared error equals:

$$\begin{aligned} MSE_{it}\left(\theta_t\right) &\equiv \mathbb{E}_{it}\left[\left(\theta_t - \mathbb{E}_{it}\theta_t\right)^2\right] \\ &= \left[p_\theta + p_{i\theta} + \left(\frac{k_1}{k_3}\right)^2 p_a + \frac{p_{cb\theta}p_{mp}\bar{k}_4^2}{p_{cb\theta} + p_{mp}\bar{k}_4^2}\right]^{-1}. \end{aligned} \tag{7.19}$$

Using Lemma 1, in particular (7.17), on (7.18) and (7.19) shows Proposition 3. □

Figure 7.1: Relationship between k_1/k_3 and $w_{i\theta}$



The figure illustrates the left-hand and right-hand side of (7.16), determining the equilibrium value of the signal-to-noise of the endogenous aggregate. In equilibrium, the decrease in ϕ induces a move from E_0 to E_1; k_1/k_3 increases.

A Separation Principle

Proof of Proposition 4: Follows trivially from the equation determining the signal-to-noise ratio of the endogenous public signal under central bank information disclosure:

$$\left(k_1/k_3\right)^D = \beta^2 \left(w_\theta^D\right)^2, \qquad w_\theta^D = \frac{p_c}{p_\theta + 2p_c + \left(\left(k_1/k_3\right)^D\right)^2 p_a}. \quad \square$$

Appendix B: Model Solution and Calibration

This appendix details the algorithm used to solve the dispersed asymmetric information business cycle model, in addition to describing the particularities of the calibration. The appendix begins by outlining the two-step solution procedure – methodologically similar to Nimark (2012) and Kohlhas (2013) – used to solve for the approximate truncated state-space equilibrium. Subsequently, I define and prove that the elements of the true infinite-dimensional state vector are as given in the main text. The appendix ends with a discussion of the moments and data used in the calibration.

Model Solution

In this subsection, I focus on the two-step algorithm used to solve for the approximate truncated state-space equilibrium. The same two-step procedure, replacing $X_t^{(0:\bar{k})}$ with $X_t^{(0:\infty)}$ in all the calculations below, can be used to show that the true equilibrium of the model is given by (4.1) to (4.2) in the main text.

To summarize, the equations describing the evolution of the endogenous aggregates are:

$$\begin{aligned} y_t &= \frac{h}{1+h}y_{t-1} + \frac{1}{1+h}\bar{\mathbb{E}}_t\left[y_{t+1}\right] - \frac{1-h}{(1+h)\,\sigma_c}\left(i_t - \bar{\mathbb{E}}_t\left[\pi_{t+1}\right]\right) \qquad (7.20)\\ &+ \sigma_d \frac{1-h}{(1+h)\,\sigma_c}\epsilon_t^d, \\ \pi_t &= \frac{\gamma_p}{1+\gamma_p\beta}\pi_{t-1} + \frac{\beta}{1+\gamma_p\beta}\bar{\mathbb{E}}_t\left[\pi_{t+1}\right] \\ &+ \lambda\left[\left(\sigma_l + \frac{\sigma_c}{1-h}\right)(y_t - y_t^{flex}) - \frac{\sigma_c h}{1-h}(y_{t-1} - y_{t-1}^{flex})\right] + \sigma_p\epsilon_t^p, \qquad (7.21)\\ i_t &= \rho_{mp} i_{t-1} + \phi_\pi \mathbb{E}_{cbt}\left[\pi_{t+1}\right] + \phi_y \mathbb{E}_{cbt}\left[y_{t+1} - y_{t+1}^{flex}\right] + \sigma_{mp}\epsilon_t^{mp}, \qquad (7.22)\\ y_t^{flex} &= \frac{\sigma_c h}{\sigma_l(1-h)+\sigma_c} y_{t-1}^{flex} + \frac{1+\sigma_c}{\sigma_l + \sigma_c/(1-h)}\theta_t. \qquad (7.23) \end{aligned}$$

Claim. The approximate truncated state-space solution is given by:

$$\begin{aligned} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} &= J \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + AX_t^{(0:\bar{k})} + Bi_t + Cu_t, \qquad (7.24)\\ i_t &= T \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + DX_t^{(0:\bar{k})} + Eu_t, \qquad (7.25) \end{aligned}$$

where I have dropped the subscript $\bar{k}$ on the coefficient matrices for ease of notation.[45] u_t denotes the vector of aggregate shocks and equals: $u_t = \begin{bmatrix} \epsilon_t^\theta & \epsilon_t^{mp} & \epsilon_t^d & \epsilon_t^p & \epsilon_{cbt}^\theta & \epsilon_{cbt}^d & \epsilon_{cbt}^p \end{bmatrix}'$. The state is

[45] For proof that a truncated state-space solution exists see Nimark (2012).

conjectured to follow a VAR(1):

$$X_t^{(0:\bar{k})} = MX_{t-1}^{(0:\bar{k})} + Nu_t. \tag{7.26}$$

where the first segment, $X_t^{(0)} = x_t = \left[\begin{array}{cc} y_t^{flex} & \theta_t \end{array}\right]'$, follows:

$$\begin{aligned} x_t &= \left[\begin{array}{cc} \frac{h\sigma_c}{\sigma_l(1-h)+\sigma_c} & \frac{1+\sigma_l}{\sigma_l+\sigma_c/(1-h)}\rho_\theta \\ 0 & \rho_\theta \end{array}\right] x_{t-1} + \left[\begin{array}{c} \frac{1+\sigma_l}{\sigma_l+\sigma_c/(1-h)}\sigma_\theta \\ \sigma_\theta \end{array}\right] e_1' u_t \\ &\equiv \Phi x_{t-1} + \Sigma u_t. \end{aligned}$$

To verify the above conjecture, and find the equilibrium of the model, I use a two-part solution strategy.

Part-1-of-2: The Coefficient Matrices

The first part of the procedure consists of finding the coefficient matrices determining the evolution of the endogenous aggregates, y_t, π_t and i_t, given a law of motion for the state. To find these, I use the Method of Undetermined Coefficients. Stacking the Dynamic IS equation (7.20), the New Keynesian Phillips Curve (7.21) and the Taylor Rule (7.22), using the conjecture in (7.24) and (7.25), allows us to write these equations as a system given by:

$$\left[\begin{array}{c} y_t \\ \pi_t \end{array}\right] = G_L \left[\begin{array}{c} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{array}\right] + G_F \bar{\mathbb{E}}_t \left[\begin{array}{c} y_{t+1} \\ \pi_{t+1} \end{array}\right] + G_X X_t^{(0:\bar{k})} + G_i i_t + G_u u_t \tag{7.27}$$

$$i_t = F_L \left[\begin{array}{c} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{array}\right] + F_F \mathbb{E}_{cbt} \left[\begin{array}{c} y_{t+1} \\ \pi_{t+1} \end{array}\right] + F_X \mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right] + F_u u_t, \tag{7.28}$$

where the G matrices are defined as:[46]

$$G_L \equiv \left[\begin{array}{c} \left(\begin{array}{ccc} \frac{h}{1+h} & 0 & 0 \end{array}\right) \\ \left(\begin{array}{ccc} -\lambda\sigma_c\frac{h}{1-h} & \frac{\gamma_p}{1+\gamma_p\beta} & 0 \end{array}\right) + \lambda\left(\sigma_l + \frac{\sigma_c}{1-h}\right) J_{1.} \end{array}\right] \quad , \quad G_F \equiv \left[\begin{array}{cc} \frac{1}{1+h} & \frac{1-h}{(1+h)\sigma_c} \\ 0 & \frac{\beta}{1+\gamma_p\beta} \end{array}\right],$$

$$G_X \equiv \left[\begin{array}{c} \underline{0} \\ \lambda\left(\sigma_l + \frac{\sigma_c}{1-h}\right)(A_{1.} - e_1') + \lambda\left(\frac{\sigma_l(1-h)+\sigma_c}{1-h} e_1' - (1+\sigma_l)e_2'\right) \end{array}\right] \quad , \quad G_i \equiv \left[\begin{array}{c} -\frac{1-h}{(1+h)\sigma_c} \\ \lambda\left(\sigma_l + \frac{\sigma_c}{1-h}\right) B_{1.} \end{array}\right],$$

$$G_u \equiv \left[\begin{array}{c} e_3'\sigma_d \\ e_4'\sigma_p + \lambda\left(\sigma_l + \frac{\sigma_c}{1-h}\right) C_{1.} \end{array}\right],$$

while the F matrices are:

$$F_L \equiv \left[\begin{array}{ccc} 0 & 0 & \rho_i \end{array}\right] \quad , \quad F_F \equiv \left[\begin{array}{cc} \phi_y & \phi_\pi \end{array}\right]$$
$$F_X \equiv -\phi_y e_1' M \quad , \quad F_u \equiv e_2'\sigma_{mp}.$$

[46] To simplify notation, I denote the j'th row of matrix Γ as $\Gamma_{j.}$

Now, from (4.5) average and central bank expectations of the hierarchy of beliefs can be written as:

$$\bar{\mathbb{E}}_t\left[X_t^{(0:\bar{k})}\right] = \bar{H}X_t^{(0:\bar{k})}, \quad \mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right] = H^{cb}X_t^{(0:\bar{k})},$$

where the matrices H and H^{cb} select the correct sub-vectors and implement the approximation that all elements belonging to orders $k > \bar{k}$ are set equal to zero. This allows us to find expressions for $\bar{\mathbb{E}}_t\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix}$ and $\mathbb{E}_{cbt}\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix}$ only as functions of the vectors in our conjecture. Specifically,

$$\begin{aligned} \bar{\mathbb{E}}_t\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} &= (J+BT)\begin{bmatrix} J \\ T \end{bmatrix}\begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \\ &+ \left\{(J+BT)\begin{bmatrix} A \\ D \end{bmatrix} + (A+BD)M\bar{H}\right\}X_t^{(0:\bar{k})} \\ &+ (J+BT)\begin{bmatrix} B \\ 0 \end{bmatrix}i_t + (J+BT)\begin{bmatrix} C \\ E \end{bmatrix}u_t. \end{aligned} \tag{7.29}$$

The expression for $\mathbb{E}_{cbt}\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix}$ is analogous, just replacing $\bar{H}$ with H^{cb}. Inserting the expression for $\bar{\mathbb{E}}_t\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix}$ and $\mathbb{E}_{cbt}\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix}$ into (7.27) and (7.28) verifies the first part of our conjecture. Matching coefficients shows that the coefficient matrices in (7.24) and (7.25) equal the solution to the following set of matrix equations:

$$J = G_L + G_F (J + BT) \begin{bmatrix} J \\ T \end{bmatrix} \tag{7.30}$$

$$A = G_X + G_F \left\{ (J + BT) \begin{bmatrix} A \\ D \end{bmatrix} + (A + BD) M \bar{H} \right\} \tag{7.31}$$

$$B = G_i + G_F (J + BT) \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{7.32}$$

$$C = G_u + G_F (J + BT) \begin{bmatrix} C \\ E \end{bmatrix} \tag{7.33}$$

$$T = \left\{ I - F_F (J + BT) \begin{bmatrix} B \\ 0 \end{bmatrix} \right\}^{-1} \left\{ F_L + F_F (J + BT) \begin{bmatrix} J \\ T \end{bmatrix} \right\} \tag{7.34}$$

$$D = \left\{ I - F_F (J + BT) \begin{bmatrix} B \\ 0 \end{bmatrix} \right\}^{-1} \cdot \tag{7.35}$$

$$\left\{ F_X H^{cb} + F_F \left[(J + BT) \begin{bmatrix} A \\ D \end{bmatrix} + (A + BD) M H^{cb} \right] \right\}$$

$$E = \left\{ I - F_F (J + BT) \begin{bmatrix} B \\ 0 \end{bmatrix} \right\}^{-1} \left\{ F_u + F_F (J + BT) \begin{bmatrix} C \\ E \end{bmatrix} \right\}. \tag{7.36}$$

Part-1-of-2: The State Evolution

The second part of the solution procedure consists of finding expressions for M and N conditional on the above matrices for the solution of the endogenous variables. From (4.5) we can see that the expectational state is comprised of three components: (i) the underlying fundamentals, y_t^{flex} and θ_t; (ii) average private sector expectations of the hierarchy of beliefs; and last (iii) central bank expectations of certain elements of the expectation hierarchy. The strategy to find the coefficient matrices for the evolution of the state, M and N, is to find expressions for these three components as functions of last periods state, $X_{t-1}^{(0:\bar{k})}$ and the aggregate shock, u_t. Subsequently, these expressions can be stacked via (4.5) to achieve the conjectured VAR(1) form for $X_t^{(0:\bar{k})}$.

Before we start solving the private sector's and the central bank's signal extraction problem it is, however, necessary to define two more re-ordering matrices. Deleting the first two rows in (4.5) allows us to write $X_t^{(1:\bar{k}+1)}$ as:

$$X_t^{(1:\bar{k}+1)} = \tilde{H} \begin{bmatrix} \bar{\mathbb{E}}_t X_t^{(0:\bar{k})} \\ \mathbb{E}_{cbt} A_n X_t^{(0:\bar{k})} \end{bmatrix}, \tag{7.37}$$

where I have deleted the first two rows and columns in H to form $\tilde{H}$. The relationship between the central bank's expectation of the state, $\mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right]$, and $\mathbb{E}_{cbt}\left[A_n X_t^{(0:\bar{k})}\right]$ in (7.37) is given by yet

another re-ordering matrix, R:

$$\mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right] \equiv R\mathbb{E}_{cbt}\left[A_n X_t^{(0:\bar{k})}\right]. \tag{7.38}$$

While the matrix A annihilates the elements of $X_t^{(0:\bar{k})}$ that are superfluous, R reorders them back. In this way, $\mathbb{E}_{cbt}\left[A_n X_t^{(0:\bar{k})}\right]$ does not contain any redundant elements, helping to keep the state-space tractable. $\mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right]$, however, due to the Law of Iterated Expectations, for instance contains the central bank's expectation of x_t twice.[47]

To find expressions for $\bar{\mathbb{E}}_t\left[X_t^{(0:\bar{k})}\right]$ and $\mathbb{E}_{cbt}\left[A_n X_t^{(0:\bar{k})}\right]$, we need to solve agents' signal extraction problem. Starting with the private sectors, inhabitants on island $i \in [0, 1]$ observe the following two signal vectors:

$$\begin{aligned} z_{it} \equiv \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \\ \tilde{i}_t \\ \theta_{it} \end{bmatrix} &= \begin{bmatrix} A \\ D \\ e_2' \end{bmatrix} X_t^{(0:\bar{k})} + \begin{bmatrix} C \\ E \\ \underline{0} \end{bmatrix} u_t + \begin{bmatrix} \underline{0} \\ 0 \\ \sigma_{i\theta} \end{bmatrix} \epsilon_{it}^{\theta} \\ &\equiv LX_t^{(0:\bar{k})} + Qu_t + Q_i\epsilon_{it}^{\theta}, \end{aligned} \tag{7.39}$$

$$\begin{aligned} s_{it} \equiv \begin{bmatrix} d_{it} \\ \lambda_{ipt} \end{bmatrix} &= \begin{bmatrix} e_3'\sigma_d \\ e_4'\sigma_p \end{bmatrix} u_t + \begin{bmatrix} \sigma_{id} & 0 \\ 0 & \sigma_{ip} \end{bmatrix} \begin{bmatrix} \epsilon_{it}^d \\ \epsilon_{it}^p \end{bmatrix} \\ &\equiv Wu_t + R\epsilon_{it}, \end{aligned} \tag{7.40}$$

where tilde denotes a variable where I have deducted the impact of last periods observations and other elements of the information set. The structure of the signal extraction problem faced by the inhabitants on island i is (slightly) non-standard: they observe signals, z_{it}, of the aggregate state (as usual), but also directly signals, s_{it}, of the innovation to the state. To circumvent this issue, and use standard Kalman Filtering expressions, I stack the two observation vectors, forming a new signal vector $z_{it}^{\star}$:

$$\begin{aligned} z_{it}^{\star} \equiv \begin{bmatrix} z_{it} \\ s_{it} \end{bmatrix} &= \begin{bmatrix} L \\ \underline{0} \end{bmatrix} X_t^{(0:\bar{k})} + \begin{bmatrix} Q \\ W \end{bmatrix} u_t + \begin{bmatrix} Q_i & \underline{0} \\ \underline{0} & R \end{bmatrix} \begin{bmatrix} \epsilon_{it}^{\theta} \\ \epsilon_{it}^{d} \\ \epsilon_{it}^{p} \end{bmatrix} \\ &\equiv L^{\star}X_t^{(0:\bar{k})} + Q^{\star}u_t + R^{\star}\tilde{\epsilon}_{it}, \end{aligned}$$

where $\tilde{\epsilon}_{it} \equiv \begin{bmatrix} \epsilon_{it}^{\theta} & \epsilon_{it}' \end{bmatrix}'$. By the properties of linear projection, the expectation of agents on island i at time t of the underlying state, $X_t^{(0:\bar{k})}$, can be written as:

$$\mathbb{E}_{it}\left[X_t^{(0:\bar{k})}\right] = M\mathbb{E}_{it-1}\left[X_{t-1}^{(0:\bar{k})}\right] + K\left(z_{it}^{\star} - \mathbb{E}_{it-1}\left[z_{it}^{\star}\right]\right),$$

[47] Because of the re-ordering of elements: $RA \neq I$.

where the matrix of Kalman Gains is determined by the linear projection formula:

$$K \equiv \mathbb{E}\left\{\left(X_t^{(0:\bar{k})} - \mathbb{E}_{it-1}\left[X_t^{(0:\bar{k})}\right]\right)\left(z_{it}^\star - \mathbb{E}_{it-1}\left[z_{it}^\star\right]\right)'\right\}\mathbb{E}\left\{\left(z_{it}^\star - \mathbb{E}_{it-1}\left[z_{it}^\star\right]\right)\left(z_{it}^\star - \mathbb{E}_{it-1}\left[z_{it}^\star\right]\right)'\right\}^{-1}$$

and is equal to:

$$K = \left(PL^{\star\prime} + NQ^{\star\prime}\right)\left(L^\star PL^{\star\prime} + Q^\star Q^{\star\prime} + R^\star R^{\star\prime} + L^\star NQ^{\star\prime} + Q^\star N'L^{\star\prime}\right)^{-1}.$$

The one-step-ahead mean-squared error conditional on $\{z_{i\tau}^\star\}_{\tau=\infty}^{\tau=t}$ is given by the solution to the Ricatti equation:

$$P = M\left[P - K\left(PL^{\star\prime} + NQ^{\star\prime}\right)'\right]M' + NN'.$$

Integrating across all islands allows us to write the average private sector expectation of $X_t^{(0:\bar{k})}$ at time t as:

$$\bar{\mathbb{E}}_t\left[X_t^{(0:\bar{k})}\right] = \left(I - KL^\star\right)M\bar{\mathbb{E}}_{t-1}\left[X_{t-1}^{(0:\bar{k})}\right] + KL^\star MX_{t-1}^{(0:\bar{k})} + K\left(L^\star N + Q^\star\right)u_t.$$

Finally, using that from (7.37): $\bar{\mathbb{E}}_{t-1}\left[X_{t-1}^{(0:\bar{k})}\right] = \begin{pmatrix} I & \underline{0} \end{pmatrix} H^{-1}X_{t-1}^{(1:\bar{k}+1)} \equiv \bar{S}X_{t-1}^{(1:\bar{k}+1)}$, implies that the average expectation of the state is:

$$\bar{\mathbb{E}}_t\left[X_t^{(0:\bar{k})}\right] = \left(I - KL^\star\right)M\bar{S}X_{t-1}^{(1:\bar{k}+1)} + KL^\star MX_{t-1}^{(0:\bar{k})} + K\left(L^\star N + Q^\star\right)u_t, \tag{7.41}$$

completing the description of the private sector's signal extraction problem.

Moving on to the central bank's expectation of the state, we (again) start by defining two observation vectors, z_t^{cb} and s_t^{cb}:

$$\begin{aligned} z_t^{cb} \equiv \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \\ \theta_t^{cb} \end{bmatrix} &= \begin{bmatrix} A \\ e_2' \end{bmatrix} X_t^{(0:\bar{k})} + \begin{bmatrix} C \\ e_5'\sigma_{cb\theta} \end{bmatrix} u_t \\ &\equiv L^{cb}X_t^{(0:\bar{k})} + Q^{cb}u_t, \end{aligned} \tag{7.42}$$

$$\begin{aligned} s_t^{cb} \equiv \begin{bmatrix} \epsilon_t^{mp} \\ d_t^{cb} \\ \lambda_{pt}^{cb} \end{bmatrix} &= \begin{bmatrix} e_2'\sigma_{mp} \\ e_3'\sigma_d + e_6'\sigma_{cbd} \\ e_4'\sigma_p + e_7'\sigma_{cbp} \end{bmatrix} u_t \\ &\equiv W^{cb}u_t. \end{aligned} \tag{7.43}$$

Now, proceeding as with the private sector – stacking z_t^{cb} and s_t^{cb}, and denoting the new observation vector, $z_t^{cb\star}$ – implies that we can write the central bank's expectation of the state, $X_t^{(0:\bar{k})}$, as:

$$\begin{aligned} \mathbb{E}_{cbt}\left[X_t^{(0:\bar{k})}\right] &= \left(I - K^{cb}L^{cb\star}\right)M\mathbb{E}_{cbt-1}\left[X_{t-1}^{(1:\bar{k}+1)}\right] + K^{cb}L^{cb\star}MX_{t-1}^{(0:\bar{k})} \\ &+ K^{cb}\left(L^{cb\star}N + Q^{cb\star}\right)u_t, \end{aligned} \tag{7.44}$$

where the matrix of Kalman Gains equals:

$$K^{cb} = \left(P^{cb}L^{cb\star\prime} + NQ^{cb\star\prime}\right)\left(L^{cb\star}P^{cb}L^{cb\star\prime} + Q^{cb\star}Q^{cb\star\prime} + L^{cb\star}NQ^{cb\star\prime} + Q^{cb\star}N'L^{cb\star\prime}\right)^{-1}$$

and P^{cb} solves another Ricatti equation:

$$P^{cb} = M\left[P^{cb} - K^{cb}\left(P^{cb}L^{cb\star\prime} + NQ^{cb\star\prime}\right)'\right]M' + NN'.$$

To rewrite the above expression for the central bank's expectation of the state in terms of elements of $X_t^{(1:\bar{k}+1)}$, we can use the definition of R to get:

$$\begin{aligned}\mathbb{E}_{cbt}\left[A_n X_t^{(0:\bar{k})}\right] &= R_{left}^{-1}\left(I - K^{cb}L^{cb\star}\right)MRS^{cb}X_{t-1}^{(1:\bar{k}+1)} + \\ &+ R_{left}^{-1}K^{cb}L^{cb\star}MX_{t-1}^{(0:\bar{k})} + R_{left}^{-1}K_z^{cb}\left(L^{cb\star}N + Q^{cb\star}\right)u_t,\end{aligned} \tag{7.45}$$

where $S^{cb} \equiv \begin{pmatrix} \underline{0} & I \end{pmatrix} H^{-1}$ and R_{left}^{-1} denotes the left-inverse of R. This completes the description of the central bank's signal extraction problem.

The last steps to get the conjectured form (7.26) are to stack (7.41) and (7.45), collect terms and append the underlying fundamental, x_t. Stacking via (7.37) gives:

$$\begin{aligned}X_t^{(1:\bar{k}+1)} &= \tilde{H}\begin{bmatrix}\bar{\mathbb{E}}_t X_t^{(0:\bar{k})} \\ \mathbb{E}_{cbt}A_n X_t^{(0:\bar{k})}\end{bmatrix} \\ &= \tilde{H}\begin{bmatrix}\bar{M}_1 \\ M_1^{cb}\end{bmatrix}X_{t-1}^{(1:\bar{k}+1)} + \tilde{H}\begin{bmatrix}\bar{M}_2 \\ M_2^{cb}\end{bmatrix}X_{t-1}^{(0:\bar{k})} + \tilde{H}\begin{bmatrix}\bar{N} \\ N^{cb}\end{bmatrix}u_t,\end{aligned}$$

where the matrices $\bar{M}_j$, M_j^{cb}, $j = \{1, 2\}$, $\bar{N}$ and N^{cb} are implicitly defined. Appending the process for the fundamental, x_t, verifies the conjectured VAR(1) form:

$$\begin{aligned}X_t^{(0:\bar{k}+1)} &= \begin{bmatrix}x_t \\ X_t^{(1:\bar{k}+1)}\end{bmatrix} \\ &= \begin{bmatrix}\Phi & \underline{0} \\ \underline{0} & \underline{0}\end{bmatrix}\begin{bmatrix}x_{t-1} \\ X_{t-1}^{(1:\bar{k}+1)}\end{bmatrix} + \begin{bmatrix}0 & \underline{0} \\ \underline{0} & \tilde{H}\begin{pmatrix}\bar{M}_1 \\ M_1^{cb}\end{pmatrix}\end{bmatrix}\begin{bmatrix}x_{t-1} \\ X_{t-1}^{(1:\bar{k}+1)}\end{bmatrix} \\ &+ \begin{bmatrix}\underline{0} \\ \tilde{H}\begin{pmatrix}\bar{M}_2 \\ M_2^{cb}\end{pmatrix}\end{bmatrix}\begin{bmatrix}x_{t-1} \\ X_{t-1}^{(1:\bar{k}+1)}\end{bmatrix} + \begin{bmatrix}\Sigma \\ \tilde{H}\begin{pmatrix}\bar{N} \\ N^{cb}\end{pmatrix}\end{bmatrix}u_t.\end{aligned}$$

And last, equating coefficients with the conjecture in (7.26) gives the solution to the coefficient matrices in the law of motion for the state:

$$M = \begin{bmatrix} \Phi & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \tilde{H} \begin{pmatrix} \bar{M}_1 \\ M_1^{cb} \end{pmatrix} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \tilde{H} \begin{pmatrix} \bar{M}_2 \\ M_2^{cb} \end{pmatrix} \end{bmatrix} \tag{7.46}$$

$$N = \begin{bmatrix} \Sigma \\ \tilde{H} \begin{pmatrix} \bar{N} \\ N^{cb} \end{pmatrix} \end{bmatrix}, \tag{7.47}$$

where the last two rows/columns have been cropped to make the matrices conformable; i.e. implementing the approximation that for all orders of expectation where $k > \bar{k}$, $X_t^{(k)} = \underline{0}_{2f_k \times 1}$.

Equations (7.30) to (7.36), (7.46) and (7.47) present a mapping from $\{M, N\} \to \{M, N\}$. The fixed-point of which provides the approximate rational expectations equilibrium to our dispersed asymmetric information business cycle model. To find the coefficient matrices, I iterate on the described two steps until convergence. To initialize the algorithm, I use the full-information solution.

The State Vector

In this subsection, I show that the contemporaneous expectation hierarchy, $X_t^{(0:\bar{k})}$, defined in Proposition 6, and used in the subsection directly above, is indeed part of our economy's state vector.

Proof of Proposition 6: The proof is by induction. First, conjecture that:

$$X_t^{(0)} = x_t = \Phi x_{t-1} + \Sigma \epsilon_t^{\theta}.$$

From the MUC solution described above, we know that the dynamics of the endogenous aggregates, y_t, π_t and i_t, depend upon $\bar{\mathbb{E}}_t[X_t]$ and $\mathbb{E}_{cbt}[X_t]$. Hence, if we can find expressions for these two expectations purely as functions of x_{t-1} and ϵ_t^{θ}, we can verify the claim. That said, from the signal extraction exercise above, we know that:

$$\bar{\mathbb{E}}_t\left[X_t^{(0)}\right] = \tilde{M}_0 \bar{\mathbb{E}}_{t-1}\left[X_{t-1}^{(0)}\right] + \tilde{M}_1 X_{t-1}^{(0)} + \tilde{N} u_t,$$

where $\tilde{M}_0$, $\tilde{M}_1$ and $\tilde{N}$ denote matrices. And analogously for the central bank. Thus, unless we can write $\bar{\mathbb{E}}_{t-1}\left[X_{t-1}^{(0)}\right]$ and $\mathbb{E}_{cbt-1}\left[X_{t-1}^{(0)}\right]$ as a linear combination of x_{t-1} and ϵ_t^{θ} the conjecture cannot be true. Unfortunately, this is not possible. Assuming only that the underlying fundamental matters is wrong, as with dispersed information and no-common prior assumption the average and central bank expectation of the fundamental will also matter, and thus also the associated "noise" in these expectations.

Now, conjecture that:

$$X_t^{(0:1)} = \begin{bmatrix} x_t \\ \bar{\mathbb{E}}_t x_t \\ \mathbb{E}_{cbt} x_t \end{bmatrix} = M X_{t-1}^{(0:1)} + N u_t.$$

If we continue along the same lines as before, we note that $\bar{\mathbb{E}}_{t-1}\left[X_{t-1}^{(0:1)}\right]$ and $\mathbb{E}_{cbt-1}\left[X_{t-1}^{(0:1)}\right]$ will appear in our expressions. Again, one potential solution could exist: the second order expectations, $\bar{\mathbb{E}}_t^{(2)} x_t$,

$\bar{\mathbb{E}}_t\mathbb{E}_{cbt}x_t$ and $\mathbb{E}_{cbt}\bar{\mathbb{E}}_t x_t$, might be some linear combination of the elements in $X_t^{(0:1)}$. That said, due to the Kalman Filtering this cannot be the case. We could therefore try including $\bar{\mathbb{E}}_t^{(2)}x_t$, $\bar{\mathbb{E}}_t\mathbb{E}_{cbt}x_t$ and $\mathbb{E}_{cbt}\bar{\mathbb{E}}_t x_t$ into our state-space. But, if we do this, and repeat the same exercise as above, we end up with third order expectations showing up, implying an even larger state-space, and so on. More generally, if we conjecture a state-space with k segments of higher-order expectations, the rational expectations equilibrium that arises using this conjecture implies that the relevant state-space has $k+1$ segments of higher-order expectations. By induction, the relevant state-space is of infinite order and is given by the expression in Proposition 6. □

Calibration

This subsection discusses the details behind the calibration of the parameters determining the signal-to-noise ratio of island- and central bank specific information. In addition, this subsection shows how to choose the maximum order of expectations, $\bar{k}$, necessary for the algorithm to achieve a satisfactory solution to the model.

Data and Model Moments

The six parameters $\{\sigma_{i\theta},\, \sigma_{cb\theta},\, \sigma_{id},\, \sigma_{cbd},\, \sigma_{ip},\, \sigma_{cbp}\}$ are calibrated to match five moments subject to the constraint $\sigma_{i\theta} = 8$. The first two moments are the dispersion in one-period ahead private sector forecasts of inflation and output growth. To measure forecast dispersion in the data, I use the empirical analog of the mean-squared error of one-period ahead forecasts around the average private sector estimate, given by the diagonal elements of:

$$\Omega_{y,\pi} \equiv \mathbb{E}\left\{\left[\mathbb{E}_{it}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \int_0^1 \mathbb{E}_{it}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} di\right]\left[\mathbb{E}_{it}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \int_0^1 \mathbb{E}_{it}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} di\right]'\right\}.$$

The data used comes from the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia. The sample period stretches from 1980Q1 to 1993Q4. In February 1994 the Federal Reserve Open Market Committee begins a long process of increased transparency (altering the informational assumption in our model) by releasing statements announcing moves in the overnight federal funds rate. To calculate forecast dispersion in the model, I rewrite $\Omega_{y,\pi}$ as:

$$\begin{aligned}\Omega_{y,\pi} &= \mathbb{E}\left\{\left[\mathbb{E}_{it}\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \bar{\mathbb{E}}_t\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]\left[\mathbb{E}_{it}\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \bar{\mathbb{E}}_t\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]'\right\}\\ &= (A+BD)\,M\mathbb{E}\left\{\left[\mathbb{E}_{it}X_t^{(0:\bar{k})} - \bar{\mathbb{E}}_t X_t^{(0:\bar{k})}\right]\left[\mathbb{E}_{it}X_t^{(0:\bar{k})} - \bar{\mathbb{E}}_t X_t^{(0:\bar{k})}\right]'\right\}M'\,(A+BD)'.\end{aligned}$$

The dispersion in beliefs about the the current estimate of the hierarchy can be found from the Kalman Filtering expressions above, and equals:

$$\begin{aligned}\Omega_X &\equiv \mathbb{E}\left\{\left[\mathbb{E}_{it}X_t^{(0:\bar{k})} - \bar{\mathbb{E}}_t X_t^{(0:\bar{k})}\right]\left[\mathbb{E}_{it}X_t^{(0:\bar{k})} - \bar{\mathbb{E}}_t X_t^{(0:\bar{k})}\right]'\right\} \\ &= (I - KL^\star) M\Omega_X M' (I - KL^\star)' + KR^\star R^{\star\prime} K'.\end{aligned}$$

Inserting the solution to this Ricatti equation, providing the value of Ω_X, into the above expression gives us $\Omega_{y,\pi}$.

The next two moments matched are the difference in mean-squared error of one-period ahead inflation and output growth forecasts between the central bank and the average private sector. The central bank's mean-squared error of one-period ahead forecasts is given by the diagonal of:

$$P_{y,\pi}^{cb} \equiv \mathbb{E}\left\{\left[\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \mathbb{E}_{cbt}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]\left[\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \mathbb{E}_{cbt}\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]'\right\}, \tag{7.48}$$

while the corresponding moment for the (average) private sector is:

$$\bar{P}_{y,\pi} \equiv \mathbb{E}\left\{\left[\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \bar{\mathbb{E}}_t\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]\left[\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \bar{\mathbb{E}}_t\begin{pmatrix}\Delta y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]'\right\}. \tag{7.49}$$

To measure these quantities in the data, I again use the SPF survey to provide me with data on private sector forecasts. Central bank forecasts are taken from the Greenbook data set. The Greenbook contains forecasts computed by the Staff of the Federal Reserve. These forecasts are published a few days before the FOMC meeting and collected with a five year lag in the Greenbook data set (see Reifschneider et al (1997) and Orphanides (2001)). To calculate the corresponding moments in the model, I rewrite (7.48) using (7.24) to (7.26) as:

$$\begin{aligned}P_{y,\pi}^{cb} &= \mathbb{E}\left\{\left[\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \mathbb{E}_{cbt}\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]\left[\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix} - \mathbb{E}_{cbt}\begin{pmatrix}y_{t+1}\\ \pi_{t+1}\end{pmatrix}\right]'\right\} \\ &= (A+BD) P^{cb} (A+BD)' + (A+BD) N (C+BE)' \\ &+ (C+BE) N' (A+BD)' + (C+BE)(C+BE)',\end{aligned}$$

where P^{cb} denotes the central bank's one-period mean-squared error of the hierarchy of beliefs from further above. Inserting P^{cb} into our expression for $P_{y,\pi}^{cb}$ provides us with the central bank's one-period ahead mean-squared error for inflation and output growth in the model. The mean-squared error for the private sector (7.49) can be calculated along similar lines. The expressions are:

$$\begin{aligned}\bar{P}_{y,\pi} &= (A+BD) \bar{P} (A+BD)' + (A+BD) N (C+BE)' \\ &+ (C+BE) N' (A+BD)' + (C+BE)(C+BE)',\end{aligned}$$

and

Table 1: Calibration

Moments Targeted	Data	% Model Fit
Dispersion y_t forecast	0.14	84.4
Dispersion π_t forecast	0.24	37.2
Relative accuracy y_t forecast	0.26	80.3
Relative accuracy π_t forecast	0.17	50.6
Std. of $\mathbb{E}_{cbt\|t^\star}\left[y_t - y_t^{flex}\right]$	1.49	99.6

$$\begin{aligned}\bar{P} &= M(I-KL^\star)\bar{P}(I-KL^\star)'M' + M[KQ^\star Q^{\star\prime}K' - (I-KL^\star)N\left(KQ^\star\right)' \\ &- \left(KQ^\star\right)N'(I-KL^\star)']M' + NN'.\end{aligned}$$

In the calculation of the above four moments in the data I do not explicitly take into account changes in the trend growth of output or changes in the (implicit) inflation target. This amounts to assuming that changes in these from one quarter to the next are common knowledge.

The final moment matched in the calibration is the standard deviation of the central bank's smoothed estimates of the output gap. In the model, the smoothed estimate of the output gap, $\mathbb{E}_{cbt|t^\star}\left[y_t - y_t^{flex}\right]$, where $t^\star \geq t$, is found using the Modified Bryson–Frazier smoother algorithm on the central bank's signal extraction problem (see Gibbs (2011)). The standard deviation of the estimates are matched to the historical Federal Reserve values used in Edge and Rudd (2012). The data used in Edge and Rudd (2012) are compilations of different vintages of Greenbook data sets. $t^\star$ is chosen in accordance with the available data.

Table 1 illustrates the fit of the model.[48]

Choosing the Maximum Order of Expectations

Section 4 in the main text discusses the existence of an approximate truncated state-space solutions. To use the two-step algorithm highlighted in the previous subsection, we nonetheless still need to choose the maximum order of expectations. This subsection discusses how to select $\bar{k}$. Throughout, I use the calibration from the main text.

The left-hand side chart in Figure 7.2 illustrates the maximum value of:

$$\left[vec\left\{abs\left(A_{\bar{k}}\right)\right\}', vec\left\{abs\left(D_{\bar{k}}\right)\right\}'\right]'$$

for $\bar{k} = \{1, 2, \ldots, 11\}$. As we can see from Figure 7.2, the maximum absolute value of the coefficients on $X_t^{(0:\bar{k})}$ in the solution decrease relatively fast towards zero. Already at $\bar{k} = 8$, the maximum absolute value of the added order is small, and the values for the orders below $\bar{k}$ appear to have stabilized. Concurrently, the right-hand side in Figure 7.2 indicates that the $\max\left\{diag\left[\mathbb{V}\left(X_t^{(\bar{k})}\right)\right]\right\}$ is also decreasing in k, and appears to stabilize well before $\bar{k} = 8$.[49] Thus, orders of expectation above seven to eight do

[48] All moments targeted are standard deviations.

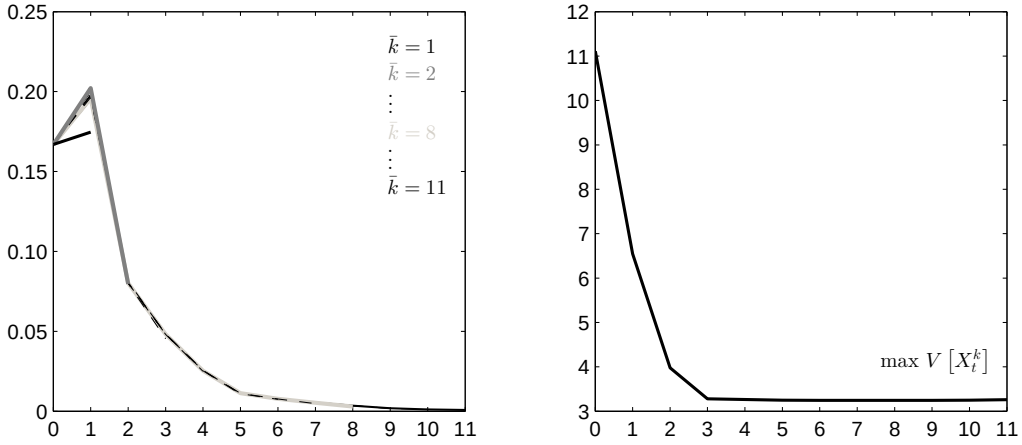
[49] The small increase visible for very high values of $\bar{k}$ in $\max\left\{\mathbb{V}\left[X_t^{\bar{k}}\right]\right\}$ is an artifact of the truncation based approach.

not seem to add meaningfully to the dynamics of the endogenous variables.

To verify the choice of $\bar{k}$, Figure 7 shows the changes in the impulse responses to the four aggregate shocks ϵ_t^{θ}, ϵ_t^{mp}, ϵ_t^{d} and ϵ_t^{p}. Similar to before, orders of expectation above seven to eight are consistent with no change in the impulse response functions for output, inflation and the interest rate.

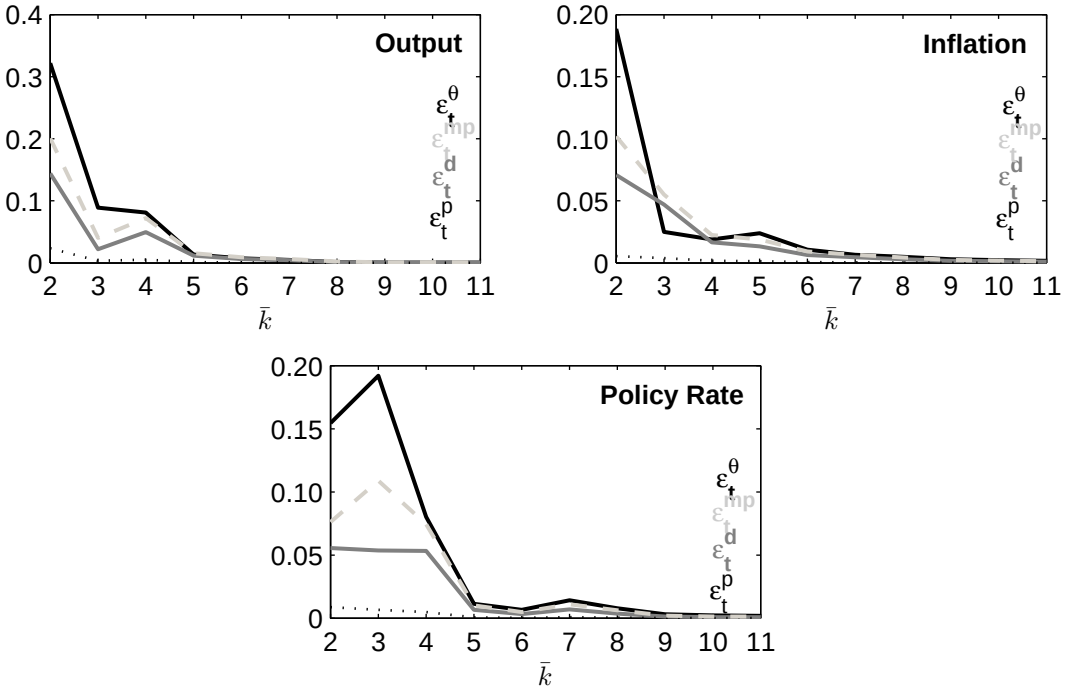
In sum, for values of $\bar{k}$ above eight, the two-step algorithm, discussed in the previous subsection, appears to approximate well the equilibrium dynamics of y_t, π_t and i_t, allowing for a relatively parsimonious representation of the state-space. Throughout the rest of this paper, I therefore set $\bar{k} = 8$.

Figure 7.2: Coefficients and the Variance of the Hierarchy



The left-hand side chart illustrates the maximum value of $\left[vec\left\{abs\left(A_{\bar{k}}\right)\right\}', vec\left\{abs\left(D_{\bar{k}}\right)\right\}'\right]'$ for each k and for all $\bar{k} = \{1, 2, \ldots, 11\}$. The right-hand side chart shows $\max\left\{diag\left\{\mathbb{V}\left[X_t^{(k)}\right]\right\}\right\}$ as a function of k.

Figure 7.3: Changes in Impulse Responses

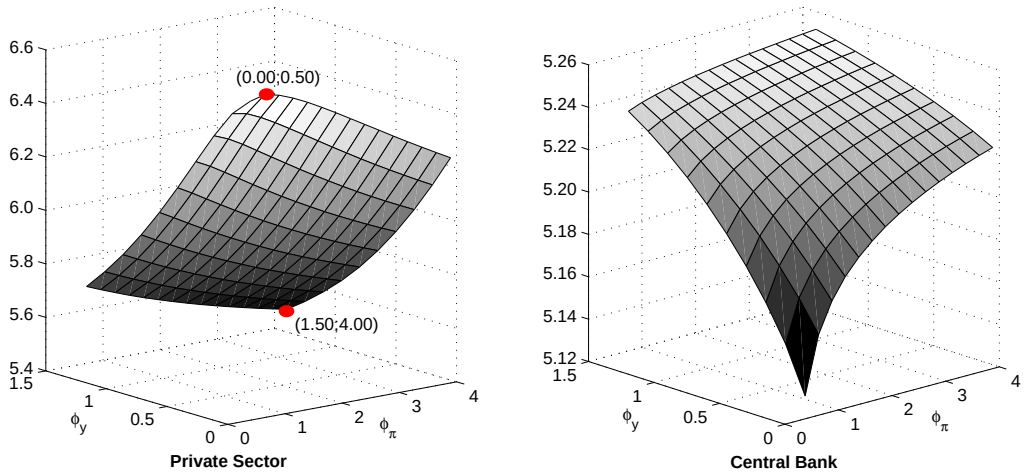


The chart shows the euclidean norm of the change in the impulse response of y_t, π_t and i_t from a shock to ϵ_t^{θ}, ϵ_t^{mp}, ϵ_t^{d} and ϵ_t^{p}, respectively, over $T = 20$ periods as a function of $\bar{k}$.

Appendix C: Central Bank Policy and Uncertainty

Results Without Central Bank Information Disclosure

Figure 7.4: Current-Period Mean-Squared Error of θ_t



The figure illustrates the mean-squared error of current estimates of θ_t for the representative agent on island i and the central bank. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$. The dots on the left-hand side chart represent the ϕ_y and ϕ_π coordinates for the max (0.00; 0.50) and min (1.50; 4.00) values.

The State Vector With Central Bank Information Disclosure

Proof of Proposition 7: From the main text, we know that:

$$\mathcal{F}_{cbt}^{D} \subset \mathcal{F}_{it}^{D}.$$

Thus, from the Law of Iterated Expectations, the central bank's expectation of the average expectation of x_t is:

$$\mathbb{E}_{cbt}\bar{\mathbb{E}}_t\left[x_t\right] = \mathbb{E}_{cbt}\int_0^1 \mathbb{E}_{it}\left[x_{it}\right]di = \mathbb{E}_{cbt}\left[x_t\right]. \tag{7.50}$$

Now, since private sector agents know all of the central bank's information, they can compute the central bank's expectation. Hence:

$$\bar{\mathbb{E}}_t\mathbb{E}_{cbt}\left[x_t\right] = \mathbb{E}_{cbt}\left[x_t\right]. \tag{7.51}$$

Repeatedly applying (7.50) and (7.51) to the state vector given in Proposition 6 shows the result. □

Solving the Model With Central Bank Information Disclosure

To find the approximate truncated state-space solution to the model with transparency, I first rewrite the state vector as (see Proposition 7):

$$X_t^{D(0:\bar{k})} = \begin{bmatrix} x_t \\ S_{(1:2)}\mathbb{E}_{cbt}\left[X_t^{D(0:\bar{k})}\right] \\ A_{n(3:4)}\bar{\mathbb{E}}_t\left[X_t^{D(0:\bar{k})}\right] \end{bmatrix}, \tag{7.52}$$

where $S_{(1:2)}$ denotes a matrix that selects the first two elements of the corresponding vector. $A_{n(3:4)}$ is an annihilation matrix that deletes the third and fourth element as well as implementing the approximation that orders of expectation above $\bar{k}$ are redundant. The expectational state vector is (again) conjectured to follow a VAR(1):

$$X_t^{D(0:\infty)} = M^D X_{t-1}^{D(0:\infty)} + N^D u_t^D, \tag{7.53}$$

where $u_t^D = \begin{bmatrix} \epsilon_t^\theta & \epsilon_t^d & \epsilon_t^p & \epsilon_t^{cb,\theta} & \epsilon_t^{cb,d} & \epsilon_t^{cb,p} \end{bmatrix}'$. Since the monetary policy innovation is no-longer a shock to a private sector signal, ϵ_t^{mp} is not a driver of the hierarchy of beliefs under central bank transparency.

To solve the model with central bank transparency, I use the two part procedure outlined in Appendix B. The first step is identical to the one in that appendix; the only change being that the expectational state is now driven by u_t^D rather than u_t, and that the selection matrices, H^{cb} and $\bar{H}$, change due to the different expectational state vector.[50] Adjusting the second step requires slightly more explaining: First, similar to before, we need to find expressions for $\mathbb{E}_{cbt}\left[X_t^{D(0:\bar{k})}\right]$ and $\bar{\mathbb{E}}_t\left[X_t^{D(0:\bar{k})}\right]$ as functions of last period's state, $X_{t-1}^{D(0:\infty)}$, and innovation, u_t^D. However, the subsequent stacking of these expressions can now be done directly using (7.52) rather than via (7.37). Second, since central bank forecast disclosure changes island i inhabitants' information set it will also change their associated signal extraction problem. Specifically, island i inhabitants' two signal vectors are now given by:

$$\begin{aligned} z_{it}^D \equiv \begin{bmatrix} \tilde{y}_t^D \\ \tilde{\pi}_t^D \\ \theta_{it} \\ \theta_t^{cb} \end{bmatrix} &= \begin{bmatrix} A^D \\ e_2' \\ e_2' \end{bmatrix} X_t^{D(0:\bar{k})} + \begin{bmatrix} C^D \\ \underline{0} \\ e_5'\sigma_{cb\theta} \end{bmatrix} u_t + \begin{bmatrix} \underline{0} \\ \sigma_{i\theta} \\ 0 \end{bmatrix} \epsilon_{it}^\theta \\ &\equiv L^D X_t^{D(0:\bar{k})} + Q^D u_t + Q_i^D \epsilon_{it}^\theta, \end{aligned} \tag{7.54}$$

[50] The change in the information structure, for instance, causes $\mathbb{E}_{cbt}\left[X_t^{D(0:\bar{k})}\right]$ to collapse to a vector only comprised of $\mathbb{E}_{cbt}\left[x_t\right]$. See also the proof for the elements included in $X_t^{D(0:\bar{k})}$ directly above.

$$s_{it}^D \equiv \begin{bmatrix} d_{it} \\ \lambda_{ipt} \\ \epsilon_t^{mp} \\ d_t^{cb} \\ \lambda_t^{cb} \end{bmatrix} = \begin{bmatrix} W \\ W^{cb} \end{bmatrix} u_t + \begin{bmatrix} R \\ \underline{0} \end{bmatrix} \begin{bmatrix} \epsilon_{it}^d \\ \epsilon_{it}^p \end{bmatrix}$$

$$\equiv W^D u_t + R^D \epsilon_{it}. \tag{7.55}$$

For the central bank, disclosing its own forecasts clearly does not alter its information set. So its signal extraction problem stays the same. To solve the island inhabitants' and the central bank's signal extraction problem, I proceed as in the no-transparency case. Last, in accordance with the change of expectational state vector, the selection matrices $\bar{S}$ and S^{cb} in (7.41) and (7.45) also change. The rest of the model is solved as in Appendix B.

Appendix D: Robustness

In this Appendix, I investigate the sensitivity of the quantitative results presented in the main text along two dimensions: (1) changes to the standard deviation of the non-invertibility shocks; and (2) changes to the extent of habit persistence and partial indexation. Both set of changes affect how informative output and inflation are, and hence change the effect of monetary policy on economic uncertainty. I demonstrate that, in each case, the basic insights from the baseline parameter configuration remain intact.

Mark-up and Preference Shocks

The baseline parameter configuration uses a slightly larger value for the standard deviation of mark-up shocks than estimated in certain DSGE models (see, instance, Justiano et al (2010)). That said, the markup shock, in essence, captures everything which makes inflation an imperfect gauge of the change in the output gap. Extending the model to a more realistic setting with wage and commodity price shocks, acknowledging also the noise in the preliminary estimates of the GDP deflator, would clearly generate a value of σ_p greater than the one used – even when taking the Justiano et al (2010) estimates as the base case. Figure 7.5 illustrates the impact of decreasing the standard deviation of mark-up shocks from the 1.40 used in the baseline scenario to the 0.50 estimated in Justiano et al (2010).

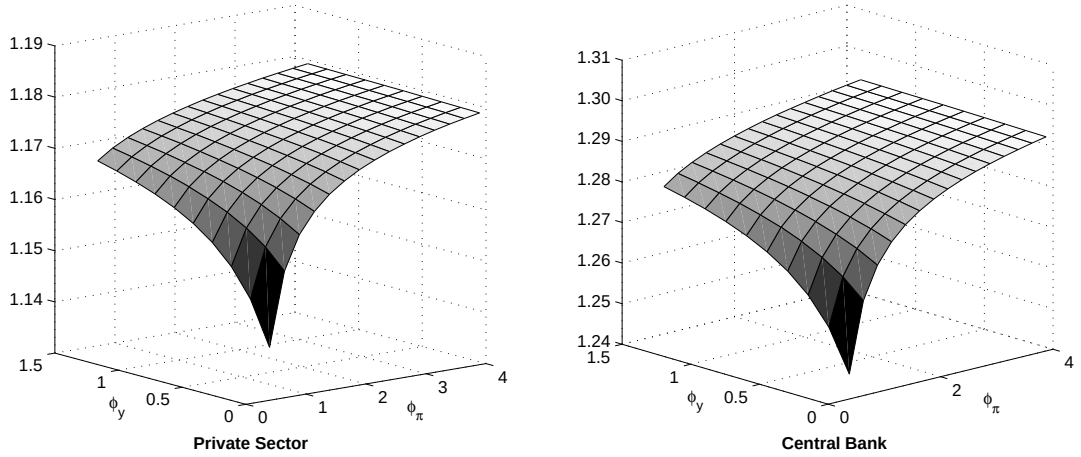
Decreasing the variance of mark-up shocks, all else equal, make inflation a more accurate signal of total factor productivity. This decreases the level of uncertainty for both the private and public sector. However, the decrease in σ_p not only changes the level of uncertainty but also the relative strength of the (net-) direct stabilization effect. The added weight put on inflation in agents' signal extraction problem cause changes to the informativeness of prices, driven by more active monetary policy, to have a greater impact on agents' beliefs. The direct stabilization concern becomes more potent relative to the direct stabilization benefit (contrast Figure 7.5 with the equivalent in Section 5 in the main text).

Nevertheless, decreasing the standard deviation of mark-up shocks by almost a third does not change the key quantitative result from the baseline case. Central bank information disclosure still, for realistic parameter values for the coefficients of monetary policy activism, does not have a substantial effect on the information content of public signals. The larger direct stabilization concern is roughly balanced out with a decrease in higher-order uncertainty (everyone attaches more weight to inflation in their signal extraction problem), such that the cross-over between central bank uncertainty with and without information disclosure does not substantially change.

Lowering instead the standard deviation of demand shocks, also from the current 1.40 to 0.50, likewise does not significantly alter the results. There is a decrease in overall uncertainty, and a now relatively more powerful direct stabilization benefit. But as with the mark-up shock, the cross-over between central bank uncertainty with and without information disclosure does not move materially.

In sum, the key insight provided by our baseline parameter values – that central bank information disclosure does not substantially change the information content of endogenous public signals – is fairly robust to changes in the standard deviation of "non-invertibility shocks".

Figure 7.5: The Impact of ϵ_t^p



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the representative agent on island i and the central bank. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$. σ_p is decreased from its baseline values of 1.40 to 0.50.

Habit Persistence and Partial Indexation

I next explore the stability of the results with regards to habit persistence and partial indexation. Both features are commonly used in full-information medium-scale DSGE models (see e.g. Smets and Wouters (2007)). But in a dispersed information setting, they also change the magnitude of the direct stabilization effect.

Figure 7.6 illustrates the impact of increasing h, governing the extent of habit persistence, and γ_p, controlling the amount of partial indexation, from their baseline values to 0.80, in line with the estimates in Del Negro et al (2010). All else equal, increasing h and γ_p decreases the relative strength of the direct stabilization effect versus the two-sided learning externality. The less output and inflation depend on future values, the less prospective feedback policy will be able to change their responsiveness to new information – similar to the effect of a decrease in β in Section 2. Increases in h and γ_p will thus correspond to a situation where the informativeness of output and inflation are relative more insulated from the direct stabilization effect (compare Figure 7.6 with the comparable one in the main text).

This results in the two-sided learning externality now dominating the (net-)direct stabilization effect somewhat more: private sector uncertainty about θ_t decreases by c. 25% when moving from the low-end of the range of monetary policy activism to the high-end. This compares with c. 20% previously. The cut-off between central bank fundamental uncertainty with and without information disclosure correspondingly moves outside what would be considered "typical" Taylor rule parameter values.[51] That said, the effects of information disclosure on the information content of public signals for plausible Taylor Rule parameter values is still quite small (c. 0.75%).

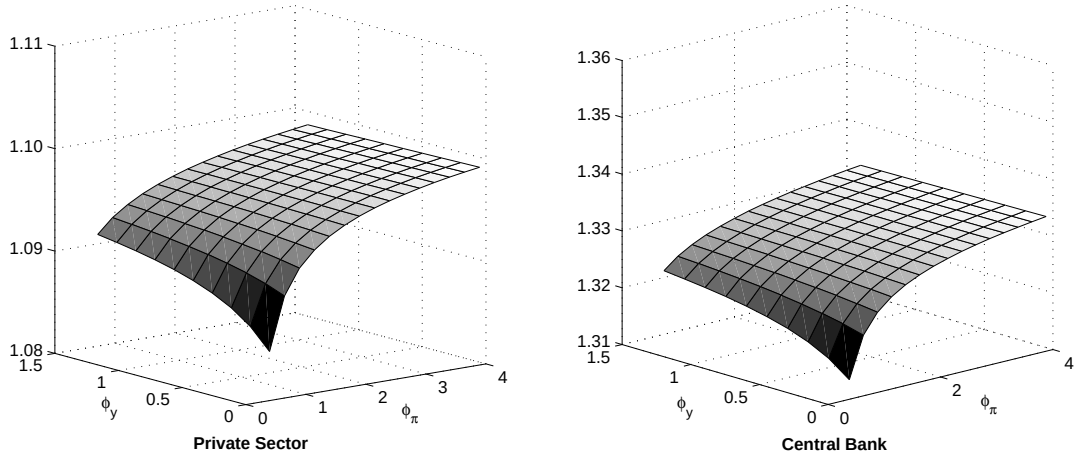
I also re-run the model using low values for habit persistence and partial indexation ($h = \gamma_p = 0.15$).

[51] Changing one of the parameters, for instance, γ_p, while keeping the other, h, constant tilts the relative contribution from the two sub-components comprising the total direct stabilization effect. Decreases in γ_p, for instance, increase the relative contribution from the direct stabilization concern by making inflation more responsive to future policy. This, else equal, increases the informational benefit of central bank information disclosure.

This results in, for almost all values of $\{\phi_\pi, \phi_y\} \in [0.5, 4.0] \times [0.0, 1.5]$, the central bank now having less uncertainty about total factor productivity under transparency than under opacity. This increase in the strength of the direct stabilization offset, in turn, is driven by the increase in the responsiveness of output and – in particular – inflation to future policy.

In sum, the key quantitative results presented in the baseline case are robust to changes in the extent of habit persistence and partial indexation.

Figure 7.6: The Impact of h and γ_p



The figure illustrates the mean-squared error of current estimates of y_t^{flex} for the representative agent on island i and the central bank. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$. h and γ_p are increased from their baseline values of 0.57 and 0.46, respectively, to 0.80.

Appendix E: Higher-Order Uncertainty

Section 2 and Section 5 discussed how changes in monetary policy activism and central bank information disclosure have important effects on higher-order belief uncertainty. This appendix takes a closer look at these effects. I demonstrate that increases in monetary policy activism decrease higher-order belief uncertainty. This partially offsets the strengthening of the direct stabilization effect stemming from larger expected increases in interest rates. Central bank information disclosure, as expected, decreases higher-order belief uncertainty substantially.

Knowledge of Higher-Order Beliefs w/o Central Bank Information Disclosure

To illustrate how changes in monetary policy activism affect higher-order uncertainty, Figure 7.7 shows the euclidean norm of the mean-squared errors of current estimates of $X_t^{(1:\bar{k})}$. The norm is chosen as a metric to summarize the changes in uncertainty across all $\bar{k}$ orders of expectation.[52]

Interestingly, more active monetary policy induces a substantial decrease in our measure of higher-order uncertainty for both the private and public sector. The decrease in private sector uncertainty is nonetheless still larger. Unlike the Figure in Section 5, showing the strength of the two-sided learning externality, the effect of monetary policy on higher-order uncertainty across the private sector and the central bank has the same direction.

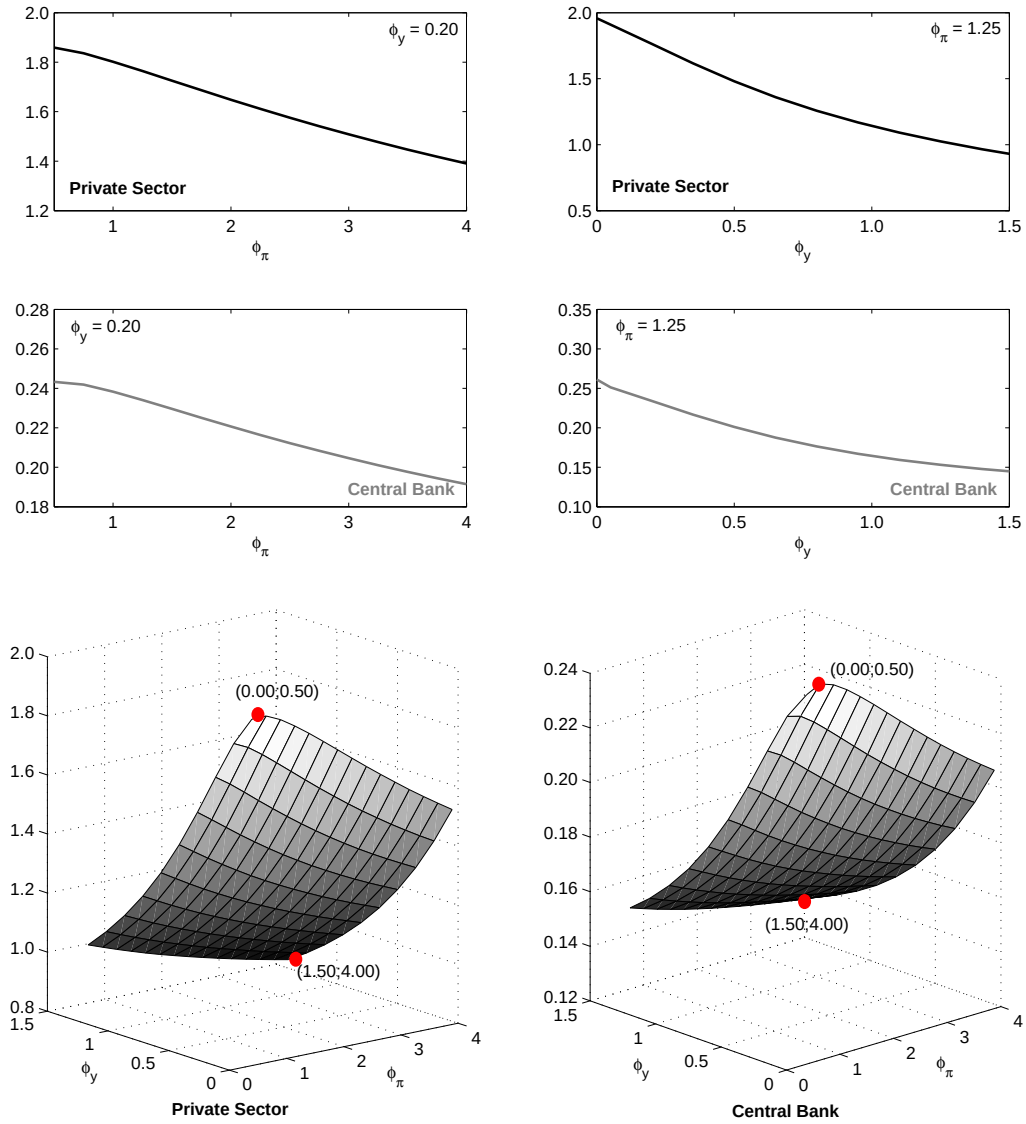
To further investigate the impact of central bank policy on higher-order uncertainty, Figure 7.8 plots the mean-squared errors of current estimates of elements in $X_t^{(1:\bar{k})}$ pertaining to θ_t. The chart is plotted as a function of ϕ_y. The uncertainty surrounding elements of the hierarchy connected to y_t^{flex} looks almost identical and is therefore not reported below.

Starting with the private sector, the two-sided learning externality affects higher-order uncertainty in three ways. First, the two-sided learning externality makes it easier for island inhabitants to discern the central bank's expectation from the observation of the interest rate. As such, the two-sided learning externality decreases private sector uncertainty about central bank beliefs. Second, the accompanying decrease in the weight placed by island inhabitants on their private information, all else equal, makes it easier to also estimate average private sector beliefs. Counteracting these two, however, is a third effect: the decrease in private sector uncertainty about θ_t. When there is little information about the underlying fundamentals, average beliefs respond in a sluggish manner to new information, making it relatively easy to estimate them. The more precise private sector expectations of θ_t thus, paradoxically, can make it relatively more difficult to infer average higher-order beliefs.

The left-hand panel of Figure 7.8 illustrates that the two former effects outweigh the latter (the only exception being $\theta_t^{cb(1)}$). Similarly, the second effect of the two-sided learning externality on higher-order uncertainty also drives down central bank uncertainty about higher-order elements (see right-hand panel of Figure 7.8). An increase in monetary policy activism is therefore consistent with a general decrease in both private sector and central bank higher-order uncertainty. This partially offsets the strengthening of the direct stabilization effect stemming from larger expected increases in interest rates.

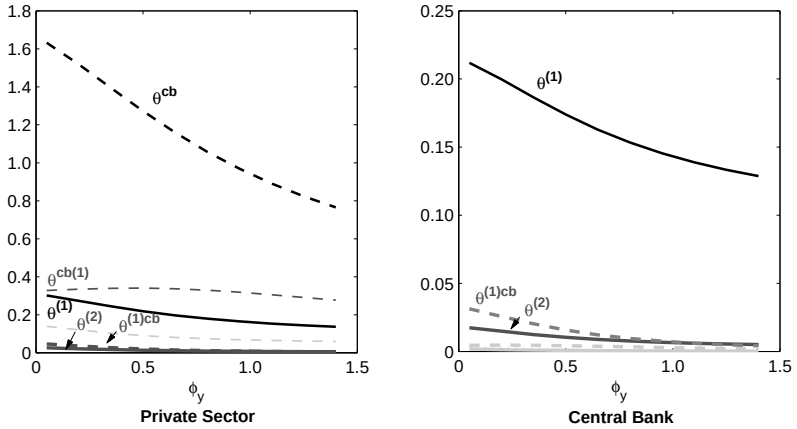
[52] Admittedly, this choice of metric is arbitrary. See the more detailed analysis below.

Figure 7.7: Total Higher-Order Uncertainty w/o Transparency



The figure illustrates the euclidean norm of the mean-squared error of current estimates of $X_t^{(1:\bar{k})}$. The chart is plotted for values of monetary policy activism with regards to inflation and output in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$. The first set of charts are depicted as a function of one variable of monetary policy activism, keeping the other constant. The dots on second set of charts represent the ϕ_y and ϕ_π coordinates for the max (0.00; 0.50) and min (1.50; 4.00) values.

Figure 7.8: Higher-Order Uncertainty of θ_t w/o Transparency



The figure shows the mean-squared error of current estimates of the elements of $X_t^{(1:\bar{k})}$ pertaining to θ_t. The left-hand side chart depicts private sector mean-squared errors, whereas the right-hand chart illustrates central bank mean-squared errors. The chart is plotted as a function of ϕ_y. Throughout, I keep $\phi_\pi = 1.50$. Superscripts in brackets indicates the order of average expectations, while superscript cb denotes central bank expectations. Thus, the line for, for instance, $\theta^{(2)}$ denotes the mean-squared error of the estimate (either the central bank's or the private sector's) of the second order average expectation of the fundamental, θ_t, at time t. Likewise, the line for, for instance, $\theta^{cb(1)}$ denotes the mean-squared error of the estimate of the central bank's belief of the average expectation of θ_t.

Knowledge of Higher-Order Beliefs w/ Central Bank Information Disclosure

To examine how central bank information disclosure changes the relationship between monetary policy activism and higher-order belief uncertainty, Figure 7.9 illustrates the euclidean norm of the mean-squared errors of current estimates of $X_t^{D(1:\bar{k})}$. Figure 7.10 shows the associated change in higher-order uncertainty pertaining to θ_t, as we increase ϕ_y, broken down into individual higher-order beliefs.

The flip-side of central bank information disclosure nullifying the two-sided learning externality – and thus only the weakened net-direct stabilization cost affecting how monetary policy changes the information content of public signals – is that increases in monetary policy activism now increase higher-order uncertainty. The loss in informational efficiency of public signals, caused by the direct stabilization effect, implies that island inhabitants attach relatively more weight to their private information as we increase $\{\phi_\pi, \phi_y\}$. This, all else equal, makes it more difficult for individual agents to estimate higher-order average beliefs, driving up higher-order uncertainty for both the private and public sector.

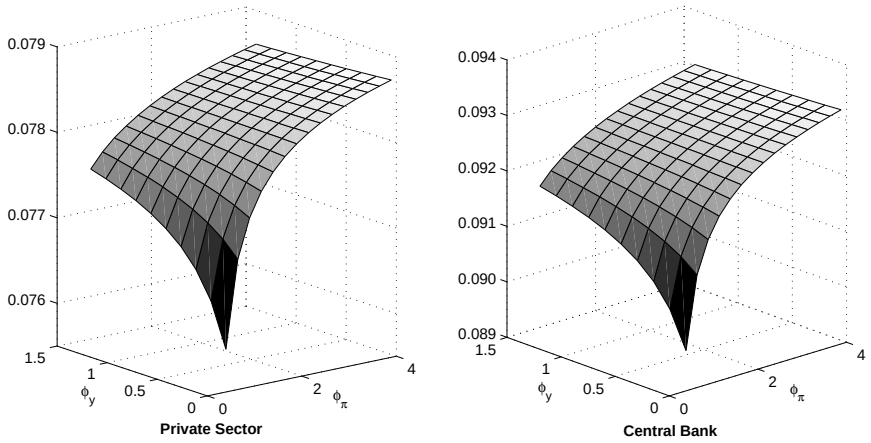
Nevertheless, the effect of changes in monetary policy activism on higher-order uncertainty is overall quite small. This is consistent with the relatively small effect found on first-order uncertainty above. The conclusion from the previous Section therefore appears to hold. To a rough first approximation, central bank information disclosure appears to make the choice of monetary policy activism independent of its implication for economic uncertainty.

Comparing the *level* of higher-order uncertainty under central bank information disclosure in Figure 7.9 with the equivalent figure under no-disclosure directly above, shows that transparency causes both private and public sector higher-order uncertainty to fall for all values of $\{\phi_\pi, \phi_y\}$. Central bank information disclosure causes the cross-expectations in the belief vector, $X_t^{(0:\bar{k})}$, to collapse to elements

defined higher-up in the hierarchy. Fewer higher-order beliefs therefore matter for any given $\bar{k}$, decreasing total higher-order uncertainty. Likewise, the decreased weight placed on private information, driven by the release of the central bank's signals, also makes it, all else equal, easier for agents to estimate higher-order beliefs.

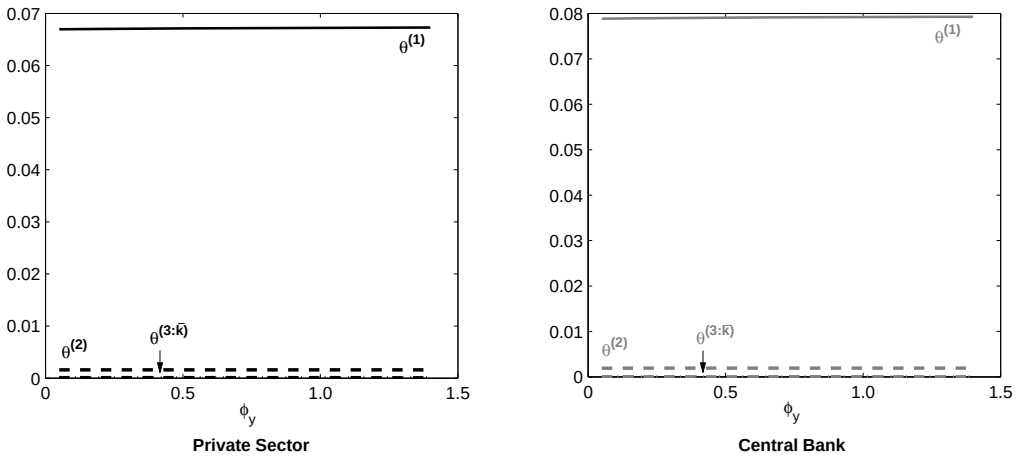
In sum, central bank information disclosure decreases the level of both private sector and central bank higher-order uncertainty by eliminating the need to form expectations about central bank beliefs, in addition to decreasing the weight placed on private information.

Figure 7.9: Total Higher-Order Uncertainty w/ Transparency



The figure illustrates the euclidean norm of the mean-squared error of current estimates of $X_t^{(1:\bar{k})}$. The chart is plotted for values of monetary policy activism in the range: $\{\phi_y, \phi_\pi\} \in [0.00; 1.50] \times [0.50; 4.00]$.

Figure 7.10: Higher-Order Uncertainty of θ_t w/ Transparency



The figure shows the mean-squared error of current estimates of the elements of $X_t^{(1:\bar{k})}$ pertaining to θ_t. The left-hand side chart depicts private sector mean-squared errors, whereas the right-hand chart illustrates central bank mean-squared errors. The chart is plotted as a function of ϕ_y. Throughout, I keep $\phi_\pi = 1.50$. Superscripts in brackets indicates the order of average expectations, while superscript cb denotes central bank expectations. Thus, the line for, for instance, $\theta^{(2)}$ denotes the mean-squared error of the estimate (either the central bank's or the private sector's) of the second order average expectation of the fundamental, θ_t, at time t.