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The Sustainability of Budget Deficits in an Inflationary Economy

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Abstract

This paper examines fiscal sustainability in an inflationary environment, particularly the interrelation between government debt and inflation. A model that explicitly incorporates the political utility/objective function of government is constructed. The government's borrowing behavior and inflation are determined through the simultaneous optimization of government and households. The sustainable fiscal debt in an inflationary environment was found to equal the present value of primary balances discounted by the time preference rate of government, not by the interest rate. This result raises the question of whether it is appropriate to apply the fiscal sustainability test of Hamilton and Flavin to high inflation countries.

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Keywords: Fiscal sustainability; Inflation; The present-value of primary balances; The fiscal theory of the price level; Leviathan

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I. INTRODUCTION

The argument that inflation will eventually accelerate if the government budget deficit increases greatly has an intuitive appeal. Many economists might accept the notion that unrestrained government borrowing will increase prices, a concept which implies that fiscal sustainability and inflation interact with one another. Hence, it appears that careful consideration must be given to the interrelation between government debt and inflation when analyzing fiscal sustainability. However, much of the literature on fiscal sustainability has not sufficiently considered the interrelation between them and has instead directed attention only to economic activities in the real term.[1] Hamilton and Flavin (1986) and Bohn (1995), two of the most prominent papers in this field, are not exceptions: they hardly mention the interrelation between government debt and inflation.

On the other hand, the fiscal theory of the price level (FTPL) directly examines the interrelation between government debt and inflation ― more correctly, the interrelation between government debt and the price level.[2] Although the focal point of the FTPL is not fiscal sustainability but price level, the FTPL also has an important implication on fiscal sustainability. According to the FTPL, fiscal sustainability can always be held because a government behaves so as to hold it in case of the Ricardian regime and households adjust prices so as to hold it in case of the non-Ricardian regime. Thus, the FTPL implies that any fiscal policy can be sustainable. Buiter (2002, 2004) criticizes the FTPL on this very point. He has denounced the FTPL as false because if default is ruled out, budget constraints must always be satisfied by any economic agent. This problem seems to be rooted in the very nature of the FTPL such that the concept of non-Ricardian fiscal policy is too general and allows many absurd fiscal policies.

[1] See, for example, Hamilton and Flavin (1986), Trehan and Walsh (1988), Wilcox (1989), Blanchard, Chouraqui, Hagemann, and Sartor (1990), Hakkio and Rush (1991), Haug (1991), Ahmed and Rogers (1995), and Bohn (1995).

[2] See, for example, Leeper (1991), Sims (1994, 1998, 2001), Woodford (1995, 2001), or Cochrane (1998a, 1998b, 2000).

The FTPL implicitly assumes that, in case of the non-Ricardian regime, households are totally passive and obey even apparently absurd fiscal policies. In other words, even though fiscal policies are unquestionably absurd, households will surely buy the bonds issued by the government that implements the unquestionably absurd fiscal policies and adjust prices accordingly. Because households are assumed to follow the government in any situation, any fiscal policy can be sustainable. In actuality, households do not appear so passive as to obey a government that implements apparently absurd fiscal policies, buy the bonds issued by such an absurd government, and adjust prices accordingly. As a result, the FTPL has been regarded as a useless gimmick which vaguely argues a curious possibility of fiscal sustainability.

However, many economists seem to agree that the central idea of the FTPL is still compelling although its arguments have not necessarily been sufficiently successful. To revitalize this compelling idea, it seems necessary to sort out non-absurd fiscal policies from absurd fiscal policies because an important drawback of the FTPL is that it allows many apparently absurd fiscal policies. It is necessary therefore to examine fiscal policies implemented by a government that considers household's responses when it pursues its objectives. The fiscal policies implemented by this kind of government will not be regarded as absurd because it considers households' rational responses carefully and thus households who know the careful behavior of government will obey the fiscal policies implemented by the government and adjust prices accordingly. That is, neither government nor households force the other to obey, but both government and households pursue their own objectives while simultaneously considering each other's responses. In case of these non-absurd fiscal policies, the mechanism argued in the FTPL may work to some extent and play an important role for fiscal sustainability as well as inflation. We therefore need a model of such a government to revitalize the compelling idea of the FTPL.

The purpose of my paper is to solve the aforementioned problems with the conventional theory of fiscal sustainability and the FTPL and to present an explanation for fiscal

sustainability in an inflationary environment. The drawbacks of both theories suggest that it is necessary to construct a model that incorporates the government's non-absurd borrowing behavior to analyze fiscal sustainability in an inflationary environment. I construct such a model in this paper.

The model assumes a Leviathan government.[3] As is known well, there are two extremely different views regarding government behavior—the Leviathan view and the benevolent view. In the Leviathan view, a government gives priority to pursuing its objectives. In the benevolent view, a government maximizes utility the same as a representative household does. Because the fiscal and monetary policies of a benevolent government are practically under the control of the representative household, the optimal behavior of a benevolent government is to supply money to the representative household's saturation point and keep the deflation rate equal to the real interest rate (the Friedman rule).[4] In the benevolent view, therefore, inflation is basically unrelated to government fiscal behavior. Hence, a model based on the benevolent view appears inappropriate for the purpose of an analysis of fiscal sustainability in an inflationary environment that focuses on the interrelation between a government's borrowing behavior and inflation. On the other hand, it is not necessarily guaranteed that the Leviathan government's behavior has no influence on the development of inflation because the fiscal and monetary policies of a Leviathan government are not perfectly under the control of the representative household. I therefore assume a Leviathan government in this model. A Leviathan government pursues its political objectives and does not necessarily pursue the economic objectives of the representative household. It should be noted, however, that people do not regard the government as absurd. Rather, the majority of people support the government because people choose a government not only from an economic point of view but also from a political point of view. In this sense, the Leviathan government argued here is an economically Leviathan government that

[3] The most prominent reference of Leviathan governments is Brennan and Buchanan (1980).

[4] See Friedman (1969).

reflects the political desire of the people.[5]

An important property of the model is that the utility/objective function of government is explicitly incorporated. The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related government activities in the utility/objective function of government.[6] The behavior of government therefore is determined through the optimization of its utility/objective function subject to the budget constraint and thus apparently absurd fiscal policies will be removed in the model. Both the Leviathan government and the representative household optimize expected utility simultaneously. Neither the Leviathan government that represents the political desire of people nor the representative household that represents the economic desire of people dominates because political and economic desires are balanced. Hence, unlike the conventional theory on fiscal sustainability, fiscal sustainability and inflation interact with one another in the model. In addition, it should be stressed that a Leviathan government maximizes its utility/objective function under the constraint of deficit financing. Even a Leviathan government must obey the budget constraint at any time. In this sense, the budget constraint is still a constraint in the optimization problem of government as the opponents of the FTPL contend it must be. As a whole, the nature of the model is fundamentally different from that of the models based on either the conventional theory on fiscal sustainability or the FTPL because the model considers both economic and political utility. This model, therefore, satisfies the aforementioned requirement that it explicitly incorporate the government's non-absurd borrowing behavior such that a government spends and borrows money considering households' responses.

Several important results are obtained by the model. First, the sustainable fiscal debt in an inflationary environment is equal to the present value of primary balances discounted by the time preference rate of government, less than the value discounted by the interest rate in

[5] See the literature on the public choice.

[6] See, for example, Edwards and Keen (1996).

Hamilton and Flavin (1986). The problem of the discount factor may be very important when this kind of test is applied to some developing countries where high inflation is endemic. Secondly, the model indicates that, because the level of government debt and the inflation acceleration rate commonly depend on the time preference rate of government, the relation between the level of government debt and the inflation rate is not linear. Rather, it is much more complex. Many empirical studies indicate that the relation between the level of government debt and the inflation rate is unclear and inconclusive.[7] This inconclusiveness may be due to the incorrect assumption that the relation between them is linear. In addition, the model indicates that a government gains by deliberately making inflation accelerate because steady state primary balance becomes smaller. This mechanism may tempt a government into accelerating inflation to lessen the burden of its debts.

The paper is organized as follows. A model that explicitly incorporates the government's borrowing behavior is constructed in section II. The model shows that the behavior of government is neither Ricardian nor non-Ricardian, but that the government behavior is optimal and consistent with both the budget constraint and the transversality condition. In section III, the model is used to show that the sustainable fiscal debt in an inflationary environment is equal to the present value of primary balances discounted by the time preference rate of government. In section IV, the appropriateness of the fiscal sustainability test developed by Hamilton and Flavin (1986) is questioned, particularly as it applies to some developing countries where high inflation is still endemic. Concluding remarks are offered in section V.

II. THE MODEL

1. An economically Leviathan government

A model that explicitly incorporates the government's non-absurd borrowing behavior is

[7] See, for example, Karras (1994), Darrat (2000), or Fischer, Sahay, and Végh (2002).

constructed so as to analyze fiscal sustainability in an inflationary environment. Since the government's borrowing activity is not isolated from various other activities of government, the overall nature of government behavior is examined. As Alesina and Cukierman (1990) argue, politicians are generally motivated by two desires: they want to hold office as long as possible and they have preferences on policy issues.[8] The former motive is the essence of the benevolent view of government and the latter motive is the essence of the Leviathan view of government, most prominently that of Brennan and Buchanan (1980). From the economic point of view, a benevolent government maximizes the expected utility of the representative household, but a Leviathan government does not. The expenditure of benevolent government is a tool to maximize the economic utility of the representative household. The expenditure of a Leviathan government is not a tool to maximize the economic utility of the representative household; rather, it is a tool to achieve its own objectives. Unlike a benevolent government, a Leviathan government is therefore not managed by politically neutral bureaucrats who are obligated to mechanically maximize the expected economic utility of the representative household in any time. It is instead managed by politicians who have strong wills to achieve their own objectives (e.g., strengthening national security, improving social welfare, or enhancing national prestige) by all means.[9]

Is it possible for such a Leviathan government to hold office for a long period? It is possible if economic and political points of view are considered. The majority of people will support a Leviathan government even though they know that the government does not

[8] Because of the latter motive, a complete policy convergence may not be the electoral equilibrium. See the literature on the policy convergence, most of which are based upon Downs (1957).

[9] The behavior of a government assumed in the FTPL reflects an aspect of the Leviathan government. Christiano and Fitzgerald (2000) argues that non-Ricardian policies correspond to the type of policies contemplated in the Ramsey literature, in which governments are viewed as selecting their policies and committing themselves to those policies in advance of prices being determined in markets.

necessarily pursue only the economic objective of the representative household because people choose a government from both economic and political points of view.[10] From the political point of view, households are not necessarily represented by the same the representative household usually presumed in the literature on economics. A government is generally chosen by the median of households under a proportional representation system, but the representative household that is usually presumed in the literature on economics is basically the mean household.[11] Thereby, a household represented from the economic point of view is not usually identical to a household represented from the political point of view. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people while the standard economically benevolent government maximizes the economic utility of people.

An important difference between economically benevolent and Leviathan governments is that the fiscal and monetary policies of a benevolent government are practically under the control of the representative household but those of a Leviathan government are not. Because of this property, a benevolent government is obligated to supply money up to the representative household's saturation point and to keep the deflation rate equal to the real interest rate, which is the well-known Friedman rule.[12] Hence, a constant deflation rate continues in a model in which a government is assumed to be purely economically benevolent; thus, in the benevolent view, inflation is basically unrelated to a government's fiscal behavior. On the other hand, because the fiscal and monetary policies of a Leviathan government are not perfectly under the control of the representative household, it is not necessarily guaranteed that the Leviathan government's fiscal behavior has no influence on the development of inflation.

[10] See the literature on the public choice.

[11] See the literature on the median voter theorem (e.g., also Downs 1957), and also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

[12] See Friedman (1969).

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related government activities in the utility/objective function of government, unlike the benevolent view, which requires that the objective function of government is identical to the utility function of the representative household.[13] A Leviathan government derives political utility from government expenditure for its political purposes. Hence, the larger the expenditure is, the happier the Leviathan government is. For instance, if a government regards that the most important political issue is national security, the expenditure on defense will be increased greatly. If the improvement of social welfare is the top priority for a government, the expenditure on social welfare will be increased dramatically. There may be a government that gives priority to long-term economic development, and the expenditure on social infrastructure will be increased significantly. On the other hand, a Leviathan government knows that raising tax rates will provoke people's antipathy and reduce the probability of being reelected, which makes the government uncomfortable because it expects that if it loses power it cannot expend money for its political purposes anymore. Hence, a Leviathan government will regard taxes as necessary costs to obtain freedom of expenditure for its political purposes. In sum, a Leviathan government derives political utility from the expenditure that makes its ideological policies achievable and political disutility from the taxes that are necessary costs to achieve its ideological policies.

The above arguments about economically Leviathan government suggest that the political utility function that describes the political preference of a Leviathan government can be expressed as $u^G(g_t, x_t)$,[14] where $g_t = \frac{G_t}{p_t}$ is the real government expenditure, $x_t = \frac{X_t}{p_t}$ is

[13] See, for example, Edwards and Keen (1996).

[14] It may be possible to assume that a government is partially benevolent. In this case, the political utility function of government can be assumed to be $u^G(g_t, x_t, c_t, l_t)$ where c_t is the real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, government policies do not affect the

the real tax revenue of government in period t while G_t is the nominal government expenditure, X_t is the nominal tax revenue, and p_t is the price level in period t. In addition, it can be assumed by the abovementioned arguments that $\frac{\partial u^G}{\partial g_t} > 0$, $\frac{\partial^2 u^G}{\partial g_t^2} < 0$, $\frac{\partial u^G}{\partial x_t} < 0$ and $\frac{\partial^2 u^G}{\partial x_t^2} > 0$. The expenditure and taxes in the political utility function of the government is analogous to the consumption and labor hours in the economic utility function of the household. As the consumption and labor hours are both control variables, the government's expenditure and tax revenue are both control variables. A Leviathan government maximizes its political utility under the constraint of deficit financing. Even a Leviathan government must obey the budget constraint at any time. In this sense, the budget constraint is still a constraint in the optimization problem of government as the opponents of the FTPL contend it to be. As a whole, the problem an economically Leviathan government should solve is a maximization problem of its expected political utility subject to the budget constraint.

2. The model

It is first assumed in the model that a government is an economically Leviathan government because, as was argued previously, the benevolent view implies that inflation is unrelated to government fiscal behavior. On the basis of the above arguments, the political utility function of the Leviathan government is assumed to be $u^G(g_t, x_t)$, where $\frac{\partial u^G}{\partial g_t} > 0$, $\frac{\partial^2 u^G}{\partial g_t^2} < 0$, $\frac{\partial u^G}{\partial x_t} < 0$, and $\frac{\partial^2 u^G}{\partial x_t^2} > 0$. All variables are expressed in per capita terms. It is assumed that u^G is a constant relative risk aversion utility function. The government's rate of time preference is θ^G. The tax is assumed to be lump sum. The budget constraint of the

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steady state consumption and leisure hours. Hence, the political utility function can be assumed to be $u^G(g_t, x_t)$.

government is

$$\dot{B}_t = B_t R_t + G_t - X_t - S_t$$

where B_t is the accumulated nominal government bonds, R_t is the nominal interest rate for government bonds, and S_t is the nominal amount of seigniorage in period t. R_t is composed of the real interest rate r_t and the expected change of bonds' price by inflation $\pi^e_{b,t}$ such that $R_t = r_t + \pi^e_{b,t}$. Let $b_t = \frac{B_t}{p_t}$ and $s_t = \frac{S_t}{p_t}$; $\pi_t = \frac{\dot{p}_t}{p_t}$ is the inflation rate in period t. By dividing by p_t, the budget constraint is transformed to

$$\frac{\dot{B}_t}{p_t} = b_t R_t + g_t - x_t - s_t,$$

which is equivalent to

$$\dot{b}_t = b_t R_t + g_t - x_t - s_t - b_t \pi_t = b_t (R_t - \pi_t) + g_t - x_t - s_t.$$

Hence, the optimization problem of the government is

$$\text{Max} \quad E_0 \int_0^\infty u^G (g_t, x_t) \exp(-\theta^G t) dt$$

subject to

$$\dot{b}_t = b_t (R_t - \pi_t) + g_t - x_t - s_t.$$

On the other hand, a representative household maximizes the following expected economic utility:

$$\text{Max} \quad E_0 \int_0^\infty u^P (c_t) \exp(-\theta^P t) dt$$

where u^P and θ^P are the economic utility function and the rate of time preference of the representative household, subject to the following constraint:

$$\dot{k}_t = f(k_t) - c_t - g_t,$$

where $f(\bullet)$ is the production function, k_t is the real capital per capita, and c_t is the real

consumption per capita.[15] The constraint means that the output $f(k_t)$ in each period is demanded for private consumption c_t, private investment $\dot{k}_t$, and government expenditure g_t. Government expenditure g_t is an exogenous variable for the representative household because the government is Leviathan. It is assumed that $u^{P'} > 0$, $u^{P''} < 0$, and the population is constant.

An important property of this model is that neither the government nor the representative household dominates the other, but both equally pursue their own objectives while simultaneously considering each other's response. The government maximizes its expected political utility considering the response of the representative household reflected in R_t in its budget constraint, and the representative household maximizes its expected economic utility considering the response of government reflected in g_t in its budget constraint.

Note that the time preference rate of government θ^G is not necessarily identical to the time preference rate of the representative household θ^P. This property of heterogeneity plays an important role later in this study. The reasons why the rates of time preference are different between government and the representative household can be summed up as follows: (i) a government is chosen from among many political parties not only from an economic point of view but also from a political one while the time preference rate of the representative household is related only to economic activities and not to political activities; (ii) a government is usually chosen by the median of households under a proportional representation system and thus the converged policy reflects the median voter—not the mean voter—while a representative household is basically the mean household;[16] (iii) even though people want to choose a party that has the same time preference rate as the representative household, those of the chosen party

[15] The constraint is equivalent to $\dot{k}_t = f(k_t) - c_t - \dot{b}_t - x_t - s_t + b_t(R_t - \pi_t)$.

[16] See the literature on the median voter theorem (e.g., also Downs 1957), and also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

may differ from those of the representative household owing to errors in expectations;[17] and (iv) current voters cannot bind the choices of future voters and thus if current voters are aware of this possibility, they may vote more myopically compared to their own rates of impatience in private economic activities.[18] Hence, it seems that the rates of time preference of government and the representative household are usually heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, a Leviathan government behaves based only on its own time preference rate without hesitation although it behaves carefully considering households' responses.

3. Neither Ricardian nor non-Ricardian fiscal regime

Before examining fiscal sustainability with the model, an important aspect of the model must be examined to help understand the analyses on fiscal sustainability presented in the following sections. A unique feature of the model is that it explicitly includes the political utility function of government; this feature differentiates the model from other models on fiscal sustainability. The FTPL and the quantity theory of money on which the conventional theory of fiscal sustainability is based do not explicitly assume the political utility function of government. Nevertheless, it is easily shown that these theories implicitly assume a common special political utility function of government such that $E_0 \int_0^{\infty} u^G(g_t, x_t)\exp(-\theta^G t)dt$ = constant for any g_t and x_t; thus u^G = constant. Let Hamiltonian H_1 be $H_1 = u^G(g_t, x_t)\exp(-\theta^G t) + \lambda_{1t}[b_t(R_t - \pi_t) + g_t - x_t - s_t]$ where λ_{1t} is a costate variable. The optimality conditions are

(1) $\dfrac{\partial H_1}{\partial g_t} = 0$,

[17] See, for example, Alesina and Cukierman (1990).

[18] See, for example, Tabellini and Alesina (1990).

(2) $\frac{\partial H_1}{\partial x_t} = 0$,

(3) $\frac{d\lambda_{1t}}{dt} = -\frac{\partial H_1}{\partial b_t}$,

(4) $\frac{db_t}{dt} = -\frac{\partial H_1}{\partial \lambda_{1t}}$,

(5) $\lim_{t\to\infty} \lambda_{1t} b_t = 0$.

If the political utility function of the government is that u^G = constant, then conditions (1) and (2) are $\frac{\partial H_1}{\partial g_t} = -\lambda_{1t} = 0$ and $\frac{\partial H_1}{\partial x_t} = \lambda_{1t} = 0$ thus $\lambda_{1t} = 0$. Thereby, conditions (1) and (2) hold for any $\pi^e_{b,t}$, π_t, g_t, x_t, and s_t in any period. In addition, in case of $\lambda_{1t} = 0$, condition (3) $\frac{d\lambda_{1t}}{dt} = -\lambda_{1t}(R_t - \pi_t) = 0$ holds for any $\pi^e_{b,t}$, π_t, g_t, x_t, and s_t in any period. Hence, the optimality conditions are condition (4) and the transversality condition (5). Here, condition (4) is equivalent to the budget constraint $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$. As a result, if the political utility function of the government is a special one such that u^G = constant, then the optimality conditions are (i) the budget constraint $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$ and (ii) the transversality condition. Needless to say, both the FTPL and the quantity theory of money are commonly based upon (i) the budget constraint and (ii) the transversality condition.

Because both the FTPL and the quantity theory of money implicitly assume the common special political utility function of government such that u^G = constant, the difference between FTPL and the quantity theory of money is merely the difference between interpretations of (i) the budget constraint and (ii) the transversality condition. As is known well, two extremely different interpretations are possible. Because conditions (1) and (2) hold for any $\pi^e_{b,t}$, π_t, g_t, and x_t in any period and thus $\pi^e_{b,t}$, π_t, g_t, and x_t are indeterminate, exogenously setting either the values on prices $\pi^e_{b,t}$ and π_t or the values on government behavior g_t and x_t is necessary for

completing a model based on the FTPL or the quantity theory of money. The former option--prices $\pi^e_{b,t}$ and π_t are assumed to be exogenously given and the government adjusts g_t and x_t for b_t not to explode--is called Ricardian. The latter option--the government behavior g_t and x_t are exogenously given and the prices $\pi^e_{b,t}$ and π_t are adjusted for b_t not to explode--is called non-Ricardian.[19] Theoretically both options are equally possible, and it is difficult to judge *a priori* which option is more consistent with the real world.

The above result highlights the fundamental difference between the model in this paper and the models based on the FTPL or the quantity theory of money. In this model, neither fiscal policy nor inflation is indeterminate and must be given ad hoc and exogenously but, as will be shown in the following section, both are determined through the simultaneous optimization of the government and the representative household. This is in sharp contrast to the FTPL as well as the quantity theory of money, which presume that either the Ricardian or the non-Ricardian regime is given ad hoc and exogenously. The reason why the FTPL and the quantity theory of money need the assumption of an ad hoc and exogenously given fiscal regime is simple: a government has a special political utility function, the value of which is constant and does not change for any set of values of the government's control variables. As a result, the optimization of government's utility affects neither fiscal policies nor development of inflation, either of which therefore needs to be given exogenously (e.g., Ricardian or non-Ricardian regime). Contrarily, it does not matter whether the fiscal regime is Ricardian or non-Ricardian in my model because political utility $u^G(g_t, x_t)$ changes as the government maneuvers control variables g_t and x_t in its optimization. The behavior of the government in the model therefore indicates neither a Ricardian nor a non-Ricardian regime. Rather, the government behavior is

[19] Kocherlakota and Phelan (1999) argue that, in the Ricardian regime, the control of money supply on the assumption of the quantity theory of money is not sufficient to fix the time path of inflation rate. Traditionally a monetarist type rule (e.g., purely speculative time trends in velocity) has been often assumed implicitly.

merely optimal and consistent with both the budget constraint and the transversality condition.

III. FISCAL SUSTAINABILITY

1. Inflation

Because the purpose of this paper is to examine fiscal sustainability in an inflationary environment, the nature of inflation in the model is examined before analyzing fiscal sustainability. Let Hamiltonian H_2 be $H_2 = u^G(g_t,x_t)\exp(-\theta^G t)+\lambda_{2t}[b_t(R_t-\pi_t)+g_t-x_t-s_t]$ where λ_{2t} is a costate variable. The optimality conditions of the government's optimization problem shown in II. 2. are

(6) $\dfrac{\partial u^G(g_t,x_t)}{\partial g_t}\exp(-\theta^G t)=-\lambda_{2t}$,

(7) $\dfrac{\partial u^G(g_t,x_t)}{\partial x_t}\exp(-\theta^G t)=\lambda_{2t}$,

(8) $\dot{\lambda}_{2t}=-\lambda_{2t}(R_t-\pi_t)$,

(9) $\dot{b}_t = b_t(R_t-\pi_t)+g_t-x_t-s_t$,

(10) $\lim_{t\to\infty}\lambda_{2t}b_t=0$.

Combining conditions (6), (7), and (8) yields the following equations:

$$\frac{g_t\dfrac{\partial^2 u^G(g_t,x_t)}{\partial g_t^2}}{\dfrac{\partial u^G(g_t,x_t)}{\partial g_t}}\frac{\dot{g}_t}{g_t}+\theta^G=R_t-\pi_t=r_t+\pi^e_{b,t}-\pi_t \quad \text{and} \quad -\frac{x_t\dfrac{\partial^2 u^G(g_t,x_t)}{\partial x_t^2}}{\dfrac{\partial u^G(g_t,x_t)}{\partial x_t}}\frac{\dot{x}_t}{x_t}+\theta^G=R_t-\pi_t=r_t+\pi^e_{b,t}-\pi_t.$$

Because $\dfrac{g_t\dfrac{\partial^2 u^G(g_t,x_t)}{\partial g_t^2}}{\dfrac{\partial u^G(g_t,x_t)}{\partial g_t}}\dfrac{\dot{g}_t}{g_t}=0$ and $\dfrac{x_t\dfrac{\partial^2 u^G(g_t,x_t)}{\partial x_t^2}}{\dfrac{\partial u^G(g_t,x_t)}{\partial x_t}}\dfrac{\dot{x}_t}{x_t}=0$ at steady state such that $\dot{g}_t=0$ and $\dot{x}_t=0$, then $\theta^G=r_t+\pi^e_{b,t}-\pi_t$. Here, by the optimality conditions of the

representative household, $r_t = \theta^P$ at steady state such that $\dot{c}_t = 0$, $\dot{k}_t = 0$ and $\dot{g}_t = 0$. Hence $\theta^G = \theta^P + \pi^e_{b,t} - \pi_t$ and thus

(11) $\pi^e_{b,t} = \pi_t + \theta^G - \theta^P$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.

Equation (11) is a natural consequence of the simultaneous optimization by a Leviathan government and the representative household. What should be stressed is that $\pi^e_{b,t} \neq \pi_t$ if the rates of time preference are heterogeneous between the government and the representative household. Some economists may be surprised by the possibility of $\pi^e_{b,t} \neq \pi_t$ because it has been naturally conjectured that $\pi^e_{b,t} = \pi_t$ under rational expectations. However, this conjecture is a simple misunderstanding because, by definition, $\pi^e_{b,t}$ indicates a total price change by inflation during a period. On the other hand π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{p}_t}{p_t} = \frac{\lim_{h \to 0} \frac{p_{t+h} - p_t}{h}}{p_t}$ and does not indicate a total general price change by inflation during a period. Hence, if π_t = constant, the equation $\pi^e_{b,t} = \pi_t$ holds, but if $\pi_t \neq$ constant, the equation $\pi^e_{b,t} = \pi_t$ does not necessarily hold. Equation (11) indicates that the equation $\pi^e_{b,t} = \pi_t$ holds only in a special case such that $\theta^G = \theta^P$ (i.e., a homogeneous rate of time preference). Because a homogeneous rate of time preference such that $\theta^G = \theta^P$ has been regarded as naturally prevailing, equation $\pi^e_{b,t} = \pi_t$ has not been questioned. However, as argued above, a homogeneous rate of time preference is not usually guaranteed. If there are heterogeneous rates of time preference between the government and the representative household such that $\theta^G \neq \theta^P$, equation (11) indicates that equation $\pi^e_{b,t} = \pi_t$ cannot hold anymore.

What does equation (11) indicate? It indicates that inflation accelerates/decelerates when

the rates of time preference are heterogeneous. The reason for the acceleration/deceleration of inflation is simple: by definition, $\pi^e_{b,t}$ indicates a total price change by inflation during a period and π_t indicates the instantaneous rate of inflation at a point. Thereby if π_t = constant, the equation $\pi^e_{b,t} = \pi_t$ holds and conversely if $\pi^e_{b,t} \neq \pi_t$, then $\pi_t \neq$ constant. Without the acceleration/deceleration of inflation, therefore, equation (11) cannot hold in an economy with $\theta^G \neq \theta^P$. That is, inflation accelerates/decelerates as a result of reconciling the contradiction in heterogeneous rates of time preference.[20]

2. The sustainable fiscal debt

Much of the sustainability literature since Hamilton and Flavin (1986) defines fiscal sustainability as the implementation of a fiscal policy by which the transversality condition is satisfied. As in the literature, this paper defines fiscal sustainability also as implementing a fiscal policy by which the transversality condition (10) is satisfied. Hamilton and Flavin (1986) show that the sustainable fiscal debt is equal to the present value of primary balances discounted by the interest rate. On the other hand, Bohn (1995) argues that, in a stochastic environment, the sustainable fiscal debt is equal to the present value of primary balances discounted by the marginal rate of substitution. In this subsection, I examine the sustainable fiscal debt in an inflationary environment.

First, the return on government bonds is examined. By equation (11) and assumptions (A1) and (A2), $\int_t^{t+1} \pi_v dv - \pi_t = \pi^e_{b,t} - \pi_t = R_t - r_t - \pi_t = \theta^G - \theta^P$ at steady state. Hence,

(12) $R_t - \pi_t = \theta^G$

at steady state because $r_t = \theta^P$. Equation (12) indicates that the real return on government bonds $r^G_t = R_t - \pi_t$ is equal to the time preference rate of government θ^G at steady state, (i.e.,

[20] The model therefore can be used for the analysis of inflation. See, for example, Harashima (2004, 2005).

$r_t^G = \theta^G$). Intuitively the equation $r_t^G = \theta^G$ appears quite reasonable because the equation $r_t^G = \theta^G$ is analogous to the well-known steady state condition $r_t = \theta^P$ in the private sector in the Ramsey model.

By equations (11) and (12), the requirement for satisfying the transversality condition (10) is obtained. Substituting equations (11) and (12) into conditions (8) and (9) and solving both differential equations yields the equation: $\lambda_{2t} b_t = -\exp\left[\left(g_t - x_t - s_t \right) \int \frac{1}{b_t} dt + C^{\#} \right]$ at steady state where $C^{\#}$ is a certain constant. Thereby, it is necessary to satisfy $g_t - x_t - s_t < 0$ and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$ for the transversality condition (10) to be held. Here, by condition (9), $\frac{\dot{b}_t}{b_t} = \theta^G + \frac{g_t - x_t - s_t}{b_t}$ at steady state. Hence if $\frac{\dot{b}_t}{b_t} = \theta^G + \frac{g_t - x_t - s_t}{b_t} = 0$ at steady state, then b_t is constant and thus $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$. Thereby, the transversality condition holds. However, if $\frac{\dot{b}_t}{b_t} = \theta^G + \frac{g_t - x_t - s_t}{b_t} < 0$ at steady state, then b_t diminishes to zero and the transversality condition (10) cannot hold because $g_t - x_t - s_t < 0$. If $\frac{\dot{b}_t}{b_t} = \theta^G + \frac{g_t - x_t - s_t}{b_t} > 0$ at steady state, then $\lim_{t \to \infty} \frac{\dot{b}_t}{b_t} = \theta^G$ and thus b_t increases as time passes and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \frac{C^{\#\#}}{\theta^G}$ where $C^{\#\#}$ is a certain constant. The transversality condition (10) therefore also cannot hold and thus, if and only if $\theta^G = -\frac{g_t - x_t - s_t}{b_t}$ at steady state can the transversality condition (10) $\lim_{t \to \infty} \lambda_{2t} b_t = 0$ hold. The requirement $\theta^G = -\frac{g_t - x_t - s_t}{b_t}$ indicates that the increase of government debt $\theta^G b_t$ (i.e., the real return on government bonds θ^G times accumulated debts b_t) should be equal to the primary surplus

$-\left(g_t - x_t - s_t\right)$ at steady state.

The requirement $\theta^G = -\frac{g_t - x_t - s_t}{b_t}$ also implies that the sustainable fiscal debt in an inflationary environment is different from that in a non-inflationary environment that is argued in Hamilton and Flavin (1986). The sustainable fiscal debt in Hamilton and Flavin (1986) is equal to the present value of primary balances discounted by the interest rate. The present value of primary balances at steady state in Hamilton and Flavin (1986) is $-\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj =$

$-\left(g_t - x_t - s_t\right)\int_0^\infty (1+r)^{-j}dj = -\frac{g_t - x_t - s_t}{r} = -\frac{g_t - x_t - s_t}{\theta^P}$; this is the value discounted by the interest rate–that is, the value discounted by the time preference rate of the representative household. However, the requirement $\theta^G = -\frac{g_t - x_t - s_t}{b_t}$ indicates that the sustainable fiscal debt b_t^* must satisfy the condition

(13) $b_t^* = -\frac{g_t - x_t - s_t}{\theta^G}$.

Hence, if $\theta^G > \theta^P$, then $b_t^* = -\frac{g_t - x_t - s_t}{\theta^G} < -\frac{g_t - x_t - s_t}{\theta^P}$ and the sustainable fiscal debt b_t^* is less than the present value of primary balances discounted by the interest rate $-\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$, i.e., $b_t^* < -\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$.

IV. DISCUSSION

1. The problem of discount factor

Equation (13) indicates that, in a deterministic but inflationary environment (i.e., $\theta^G > \theta^P$), the sustainable fiscal debt is equal to the present value of primary balances

discounted by the time preference rate of government and needs to be less than the one discounted by the interest rate.[21] The sustainable fiscal debt is therefore quite different from that in Hamilton and Flavin (1986). An intuitive explanation of this result is that, because the inequality $\theta^G > \theta^P$ means that the real return on government bonds is larger than the interest rate, government debts grow more rapidly and thus the sustainable fiscal debt must be smaller. As equation (12) indicates, the real return on government bonds $r_t^G = R_t - \pi_t$ is equal to the time preference rate of government at steady state (i.e., $r_t^G = \theta^G$), which is analogous to the well-known steady state condition $r_t = \theta^P$ in the private sector in the Ramsey model, and thus the real return on government bonds $r_t^G = \theta^G$ is larger than the interest rate $r_t = \theta^P$ if $\theta^G > \theta^P$.

Nevertheless, if $\theta^G = \theta^P$ (i.e, if in a non-inflationary environment), then equation (13) also indicates that $b_t^* = -\frac{g_t - x_t - s_t}{\theta^G} = -\frac{g_t - x_t - s_t}{\theta^P} = -\frac{g_t - x_t - s_t}{r} = -\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$ and thus the sustainable fiscal debt is equal to the present value of primary balances discounted by the interest rate, as in the model in Hamilton and Flavin (1986). Hence, the argument in Hamilton and Flavin (1986) describes a special case of my model such that $\theta^G = \theta^P$. In other words, the conventional model implicitly assumes a non-inflationary environment such that $\theta^G = \theta^P$.

Equation (13) questions the appropriateness of Hamilton and Flavin's (1986) fiscal sustainability test of the hypothesis $b_t^* = -\int_0^\infty (1+r)^{-j}\left(g_{t+j} - x_{t+j} - s_{t+j}\right)dj$. This kind of test may be valid if $\theta^G = \theta^P$ (i.e., if in a non-inflationary environment), but equation (13) indicates

[21] In a deflationary environment (i.e., $\theta^G < \theta^P$), the sustainable fiscal debt is more than the present value of primary balances discounted by the interest rate such that $b_t^* = -\frac{g_t - x_t - s_t}{\theta^G} > -\frac{g_t - x_t - s_t}{\theta^P}$.

that in an inflationary environment such that $\theta^G > \theta^P$, satisfying the equation $b_t^* = -\int_0^\infty (1+r)^{-j} (g_{t+j} - x_{t+j} - s_{t+j})\, dj$ does not guarantee fiscal sustainability. To claim fiscal sustainability in an inflationary environment, the equation $b_t^* = -\int_0^\infty (1+\theta^G)^{-j} (g_{t+j} - x_{t+j} - s_{t+j})\, dj$ instead needs to be satisfied and thereby we need information on the time preference rate of government rather than the real interest rate to judge fiscal sustainability. This discount factor problem may not be serious when this kind of test is applied to most developed countries where inflation is currently very low, but it may be more important when this kind of test is applied to some developing countries where even now high inflation is endemic. In those countries, even if fiscal sustainability is validated by Hamilton and Flavin's (1986) test, debts may not be sustainable in reality.

The discount factor problem in Hamilton and Flavin's test has also been raised from another point of view. Bohn (1995) criticizes it for not considering stochastic environments and argues that, in a stochastic environment, the discount factor cannot be represented by the real interest rate but rather by the marginal rate of substitution. Tests using arbitrarily selected real interest rates are therefore inappropriate. This paper raises another important problem regarding the choice of discount factor. Even in a deterministic environment, the real interest rate is not the appropriate discount factor if the environment is deterministic and inflationary.

2. The interrelation between debt and inflation

Many empirical studies analyzing the relation between government debt and inflation assume a simple linear relation between them.[22] My model indicates, however, that the relation between the level of government debt and the inflation rate is not linear and is much more complex because the level of government debt and the acceleration of inflation depend

[22] Particularly, the FTPL predicts that fiscal deficits cause inflation.

commonly on θ^G. For example, equations (11) and (13) indicate that a situation such that $\dot{b}_t = 0$ while $\dot{\pi}_t \neq 0$ is possible. Many empirical studies indicate that the relation between the level of government debt and the inflation rate is unclear and inconclusive.[23] This inconclusiveness may be due to the incorrect assumption that the relation between the two is linear.

Equation (13) also suggests an interesting aspect of the interrelation between government debt and inflation. Assume that initially $\theta^G = \theta^P$ but θ^G is unexpectedly raised to be $\theta^{G\#}$ at a time and thus, after that time, $\theta^{G\#} > \theta^P$. This unexpected surprise upward shift of the time preference rate of government has interesting consequences. First, inflation starts to accelerate by equation (11) because, as was shown in the section III, equation (11) cannot be held without the acceleration of inflation if $\theta^{G\#} > \theta^P$. Secondly, the real value of sustainable government bonds $b_t^* = -\frac{g_t - x_t - s_t}{\theta^G}$ is shifted to be $b_t^{*\#} = -\frac{g_t^{\#} - x_t^{\#} - s_t^{\#}}{\theta^{G\#}}$ where both the sustainable fiscal debt and steady state primary balance are smaller than before such that $b_t^{*\#} < b_t^*$ and $-\left(g_t^{\#} - x_t^{\#} - s_t^{\#}\right) < -\left(g_t - x_t - s_t\right)$. The downward shifts of the sustainable fiscal debt and steady state primary balance are analogous to those of capital stock and consumption in the Ramsey model on the private economy. Government debt b_t^* corresponds to the capital stock in the Ramsey model. The primary balance $-\left(g_t - x_t - s_t\right)$ corresponds to consumption in the Ramsey model. Finally, the time preference rate of government θ^G that equals the real return on government bonds at steady state as was shown in equation (12) corresponds to the time preference rate of the representative household that equals the real interest rate at steady state in the Ramsey model. As both steady state capital and consumption shift downwards in the Ramsey model if the time preference rate of the representative household shifts upwards, both

[23] See, for example, Karras (1994), Darrat (2000), or Fischer, Sahay, and Végh (2002).

steady state government debt and primary balance shift downwards if the time preference rate of government shifts upwards. As a result, $b_t^{*\#} < b_t^*$ and $-\left(g_t^{\#} - x_t^{\#} - s_t^{\#}\right) < -\left(g_t - x_t - s_t\right)$ when θ^G shifts upwards such that $\theta^{G\#} > \theta^P$. Because market participants know this mechanism and thus nobody buy these bonds in markets unless the real value of government bonds has sufficiently fallen in this environment of accelerating inflation, the real value of already issued government bonds soon falls from b_t^* to $b_t^{*\#}$.

If the time preference rate of government is unexpectedly raised, therefore, households will experience double suffering, namely, from accelerating inflation and from the loss of the value of government bonds they hold. On the other hand, the government gains by the unexpected upward shift of θ^G because steady state primary balance that the government is obligated to achieve in the future becomes smaller. This mechanism may tempt a government into raising θ^G to lessen the burden of debts, although this action also accelerates inflation.

3. The concept of fiscal sustainability in an inflationary environment

Another important contribution of this paper is that the concept of fiscal sustainability is extended to an inflationary environment. Although the deficit financing of government has been regarded as interacting with the development of inflation, the conventional theory on fiscal sustainability has not considered this aspect but rather limited analyses to economic activities in the real term. On the other hand, the FTPL argues this point explicitly and contends that the price level is closely related to fiscal factors, but the FTPL has been criticized for merely arguing a curious possibility of fiscal sustainability. The model presented here, however, is fundamentally different from both of these models. Neither government (which represents the political desire of people) nor the representative household (which represents the economic desire of people) dominates the other because political and economic desires are balanced. The government determines fiscal policies and the representative household adjusts prices in the

simultaneous optimization of government and the representative household. As a result, fiscal policy and inflation are determined simultaneously in the model, which makes detailed analyses on the interrelation between them possible, and the concept of fiscal sustainability can be extended to an inflationary environment.

V. CONCLUDING REMARKS

A model that incorporates government's non-absurd borrowing behavior is constructed by explicitly including the political utility function of an economically Leviathan government in order to analyze fiscal sustainability in an inflationary environment. In the model, both behavior of government and development of inflation are determined through the simultaneous optimization of government and the representative household. The concept of fiscal sustainability is extended to an inflationary environment by the model. The model is therefore fundamentally different from models based on the FTPL or the quantity theory of money because both the economic and political utility of people are considered.

The main finding of the paper is that the sustainable fiscal debt is equal to the present value of primary balances discounted by the time preference rate of government. The sustainable fiscal debt is therefore less than the present value of primary balances discounted by the interest rate in an inflationary environment. This result questions the appropriateness of the fiscal sustainability test developed by Hamilton and Flavin (1986). The problem of the fiscal sustainability test appears very important, particularly when studying fiscal sustainability in developing countries where high inflation is still endemic. The model also indicates that the relation between the level of government debt and the inflation rate is not linear. The relation between them is unclear and inconclusive in empirical studies, possibly because the relation is wrongly assumed to be linear. In addition, the model indicates that a government gains by deliberately making inflation accelerate because the steady state primary balance decreases.

This mechanism may tempt a government into accelerating inflation to lessen the burden of its debts.

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