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Incorporating Default Risk into Hamada's Equation for Application to Capital Structure

Ruben D. Cohen[‡]

Abstract

Implemented widely in the area of corporate finance, Hamada's Equation enables one to separate the financial risk of a levered firm from its business risk. The relationship, which results from combining the Modigliani-Miller capital structuring theorems with the Capital Asset Pricing Model, is used extensively in practice, as well as in academia, to help determine the levered beta and, through it, the optimal capital structure of corporate firms.

Despite its regular use in the industry, it is acknowledged that the equation does not incorporate the impact of default risk and, thus, credit spread - an inherent component within every levered institution. Several attempts have been made so far to correct this, but, for one reason or another, they all seem to have their faults. This, of course, presents a major setback, as there is a strong need, especially by practitioners, to have in place a solid methodology to enable them to assess a firm's capital structure in a consistent manner. This work addresses the issue and provides a robust modification to Hamada's Equation, which achieves this consistency.

1. Background and Introduction

Hamada's Equation (*HE*), which relates the beta of a levered firm to that of its unlevered counterpart, has proved useful in several areas of finance, including capital structuring, portfolio management and risk management, to name just a few. The equation is presented by

$$\beta_L = \beta_U \left[1 + (1 - T)\phi\right] \qquad (1)$$

where β_L and β_U are the levered and unlevered betas, respectively, T the tax rate and ϕ the leverage, defined here as the ratio of debt, D, to equity, E, of the firm.

The importance of this relationship is that it allows one to separate the risk of the business, reflected here by the beta of an unlevered firm, β_U, from that of its levered counterpart, β_L, which contains the financial risk of leverage. Hence, apart from the

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effect of the tax rate, which shall be taken here as constant, the discrepancy between the two betas can be attributed solely to how the business is financed.

Equation 1 displays two prominent features. One is that β_L increases linearly with ϕ and the other, but more subtle, is that the effect of default risk, which should generally present itself as a variable cost of debt [or credit spread] that rises with leverage, is absent. This work focuses on the latter and suggests a modification to *HE* that renders it consistent with the fundamentals of capital structuring in a risky environment.

2. The Flaw in Hamada's Equation

Hamada's Equation is derived by combining the *CAPM* with the first two propositions of Modigliani and Miller (*M&M*). However, since both *M&M* and *CAPM* rule out default risk, *HE* will then, by design, exclude it as well. Consequently, any application of *HE* becomes restricted to highly idealised scenarios, whereby interest rates remain constant and equal to the risk-free rate, irrespective of the degree of leverage.

This limitation poses significant problems, especially if one were to consider situations where debts are risky. For instance, any attempt to obtain the beta of a levered firm from *HE* is immediately doomed to fail, simply because the process becomes self-contradictory and, thus, intrinsically flawed. Therefore, with beta being an important feature in many areas of finance, it becomes imperative that one must seek to remove this constraint.

HE's failure to account for the risk of default has, never the less, been already identified and documented [Conine (1980) and subsequently others]. In this study, we focus on Conine's modification, as it seems to have dominated all related works that have followed thereafter.

Conine, in his work, highlighted *HE*'s flaw through a numerical example and, to get around it, proposed a modified version of the equation, which employs the notion of a *CAPM*-based "debt beta", β_{debt}.[1] This idea was followed through even though the merits of β_{debt} were seriously questioned earlier [Gonzales *et al*, 1977]. Moreover, Conine's modification contains another major setback, which shall be the focal point of our discussion in Section 4. Despite these, Conine still went ahead and devised various

[1] This is in analogy to the equity beta - i.e. β_L and β_U in Equation 1.

models for estimating the divisional cost of capital, all based on his original formulation [Conine and Tamarkin, 1985; among others].

Considering that these drawbacks have been known for almost three decades and the original relationship, Equation 1, is still being used frequently in practice[2], it is a surprise that until today practitioners have made little effort, if any, to either appropriately incorporate the impact of risky debt into *HE*, or to correct the more serious deficiencies that present themselves in Conine's modification [discussed Section 4]. Whether this is because today's practitioners are not aware of what actually underlies Equation 1, or they want to avoid the potential complications that might arise from using a modified form of *HE*, is not clear.

What is clear, however, is that there is a need for a more robust framework for modifying *HE*, one that would allow it to circumvent these faults in a non-contradictory and consistent manner. Also, just as important, especially to the practitioner, is that the new formulation must be simple and straightforward to use in practice. This, potentially, is why the original *HE*, despite its proven shortfalls, has remained so popular for such a long time.

With these in mind, therefore, this work focuses on re-deriving *HE* such that it achieves the above objectives. Beforehand, however, we illustrate, via numerical examples, the nature of the errors that underlie both Equation 1 and Conine's modification [Conine, 1980] and the misleading results they produce, should one apply them to situations where default risk comes into play.

3. A Numerical Demonstration of the Flaw in Hamada's Equation

Figure 1 depicts a simplified financial statement, consisting of the income statement, the balance sheet and some other parameters belonging to a hypothetical firm. Once again, the need for simplification is emphasised, so as to help better understand the problem.

This firm has an asset base of 130, which is funded by an equity, E, of 50, a debt, D, of 80 and, hence, a leverage, ϕ, of 1.60. In addition, it has earnings before interest and tax (*EBIT*) of 20, which is assumed to be equal to the expected *EBIT*, and a net profit of

[2] At present, practitioners, as well as independent consultants and financial advisors, who provide advice to corporate firms on managing debt capacity and improving their capital structure, among other things, use Hamada's Equation extensively.

8.9, after deducting an interest expense of 5.2 (= 80 x 6.50%) and tax of 5.9 (= 14.8 x 40%) from the *EBIT*. The return on equity, R_E, expressible by

$$R_E \equiv \frac{[\tilde{e}_b - R_D D] \times [1 - T]}{E} \qquad (2)$$

thus becomes 17.8%, as highlighted in Figure 1.

Another way to calculate the return, or rather the expected return, on equity is to use the following relationship:

$$R_E = R_D^* + \beta_L r_{pm} \qquad (3a)$$

where R_D^* is the risk-free rate, r_{pm} is the market equity risk premium and β_L is the levered beta of the firm, as defined earlier in relation to Equation 1. Similarly, one can write,

$$R_U = R_D^* + \beta_U r_{pm} \qquad (3b)$$

where R_U is the return on equity of the unlevered firm and β_U the beta of the unlevered firm. Thus, eliminating R_E from Equations 2 and 3a and solving for $\tilde{e}_b$, gives

$$\tilde{e}_b = \frac{[R_D^* + \beta_L r_{pm}] \times E}{1 - T} + R_D D \qquad (4)$$

If we now substitute the quantities 2.13, 5% and 6%, respectively, for β_L, R_D^* and r_{pm}, as taken from Figure 1, we obtain 20 for the expected *EBIT*, which is consistent with the *EBIT*, $\tilde{e}_b$, of the firm.[3]

To proceed, we also need the relationship for the unlevered value, V_U, of the firm [see, e.g. Cohen (2001) for a numerical derivation]. This value, which is supposed to be constant, as per *M&M*, and independent of changes in *D* and *E*, is given by

[3] For simplicity, it is assumed here that the expected *EBIT* is equal to the realised. This depends on the levered beta of the firm, where, for the hypothetical case in Figure 1, a β_L of 2.13 achieves this equality.

$$V_U = E + D[1-T] \qquad (5)$$

Although this expression has been proven incorrect when applied to risky debt[4] [Cohen, 2004], it has been used to derive *HE* (Equation 1) and, thus, presents one of the main obstacles here. In any event, for the scenario in Figure 1, where *E*, *D* and *T* are 50, 80 and 40%, respectively, one gets V_U = 98.

Finally, one needs the unlevered beta, β_U, of the firm. This reflects purely the business risk and, thus, should remain independent of leverage. Based on the numbers here, β_U is 1.09 after using Equation 1 and recognising that the firm is currently operating at a β_L of 2.13, leverage, ϕ, of 1.6 and tax rate, *T*, of 40% (see Figure 1).

With the aid of *M&M*'s theorems, Table 1 expands Figure 1 to cover different levels of leverage. There are 9 columns here, each explained in the notes provided underneath. The results here primarily illustrate how Hamada's Equation can be used to re-calculate the [expected] *EBIT* and how it fails to fulfil one of *M&M*'s basic assumptions – namely that *EBIT* should remain constant and independent of leverage along the path of constant V_U dictated by Equation 5. The breakdown is clearly depicted in Column 7, where *EBIT* is found to change with leverage.[5] As mentioned earlier, the problem has already been identified [Conine, 1980] and a modification to *HE* was subsequently proposed to resolve it. This is discussed next.

4. Conine's Equation and its Flaws

To resolve *HE*'s limitation, Conine (1980) began with Equation 2, utilised *M&M*'s definition for R_U [also equivalent to the *WACC* of the unlevered firm], which is given by

$$R_U \equiv \frac{\tilde{e}_b(1-T)}{V_U} = \frac{\tilde{e}_b(1-T)}{E + D(1-T)} \qquad (6)$$

and substituted in Equations 2, 3a and 3b to obtain:

[4] Where, owing to credit spread, interest rates vary with leverage.

[5] Another serious flaw in using *HE* in relation to the capital structure of a levered firm may be noted in Columns 8 and 9 of Table 1. The problem, which has also been documented [Cohen, 2004], is that the method produces a *WACC* that goes through a minimum [Column 8 in Table 1], while the levered firm's value, V_L, in Column 8 keeps increasing with leverage. The two are certainly inconsistent.

$$\beta_L = \beta_U\left[1+(1-T)\phi\right]-\beta_{debt}(1-T)\phi \qquad (7)$$

In the above, the modification to *HE* appears as an additional term that contains the "debt beta," β_{debt}, expressible as:

$$\beta_{debt} \equiv \frac{R_D - R_D^*}{r_{pm}} \qquad (8)$$

where R_D, R_D^* and r_{pm} are as defined above. Note the direct analogy between the debt beta presented Equation 8 and the equity beta in Equations 3a and 3b.

Table 2, which was produced in exactly the same manner as Table 1, except that Equation 7 was implemented instead of 1 to calculate the levered beta, shows that Conine's modification rectifies the *EBIT*-related issue noted in Section 3 and Table 1. However, one could still argue that it has other serious issues.

In fact, there are two problems associated with Equation 7, one more subtle, but serious, than the other. First, Equation 7 relies on the debt beta, a debated concept that has long been deemed questionable [Gonzales *et al*, 1977][6]. Second, and definitely more serious, is that the methodology is <u>not</u> capable of generating a weighted average cost of capital [*WACC*], or a firm's value, V_L, curve, that passes though an optimal capital structure [i.e. minimum in *WACC* or maximum in V_L]. The reason for this is that if we were to force V_U, as defined in Equation 5, to remain constant along the *WACC* curve [as per *M&M*], then as one increases the debt level in increments to generate Table 2, the levered value of the firm, V_L, which is $V_U + DT$, would increase indefinitely with leverage. This is inconsistent with the principle that the firm's value should eventually fall at some point, owing to higher interest expense overtaking the benefits of the interest-related tax shield.

The absence of an optimal capital structure, clearly observed in Column 9 of Table 2, might also explain why this long-established modification to *HE*, which incorporates default risk, has not been so popular among practitioners. It seems that

[6] The seriousness of the debate behind the debt beta in Equation 8, which goes into Conine's formulation, could, nonetheless, be easily brushed aside by representing it in Equation 7 as the spread, $R_D - R_D^*$, divided by the market risk premium, rather than a credit market beta.

practitioners, in their effort to create a *WACC* curve that contains an optimal capital structure [a minimum], have always had to rely on the original *HE* because it is capable of producing *WACC* curves that contain this minimum – all this despite being fully aware of the self-contradictory and inconsistent nature of *HE* when used in conjunction with default risk.

5. Re-modifying Hamada's Equation to Incorporate Default Risk

In light of the discussions in Sections 3 and 4, we now move on to modify *HE* such that it incorporates the impact of default risk and, at the same time, stays clear of the above-mentioned problems. In the interest of space and to avoid repeating what is already documented in the literature, derivation of the modified *HE* will not be carried out here rigorously. Rather, we shall refer to the original paper [Cohen, 2004] when it becomes necessary.

The principal objection to using Equation 7 as modification to Equation 1 is that it is unable to generate a *WACC* curve that passes through a minimum, with a matching V_L curve that has a maximum, at some finite leverage. The underlying cause of this, as it will be argued below, is that the unlevered value of the firm, V_U, is represented by $E + D(1-T)$. For the case exemplified in Figure 1, this would be $50 + 80(1-0.4) = 98$, a value used subsequently in both Hamada's and Conine's approaches to produce Tables 1 and 2, respectively.

The source of the flaw in the above is that $D = 80$ represents the debt associated with the position of the firm as it currently is, which is at a leverage of $\phi = 1.6$ and paying interest at a rate of 6.5% [1.5% above the risk-free rate of 5%]. Thus, a debt of 80 is risky because it is associated with a risky interest rate of 6.5%, as opposed to a risk-free rate of 5%, and, therefore, cannot be employed, as is, to compute the unlevered firm's value. Recognising that the unlevered firm's marginal debt expenses must be evaluated at an interest rate of 5% rather than 6.5%, the current debt of 80, which was priced at an interest rate of 6.5%, should, consequently, be re-evaluated to reflect the 5% interest rate of the unlevered firm.

Re-pricing the debt of 80 can be achieved by dividing the current interest expense of 5.2 [$6.5\% \times 80$] by 5%, thereby yielding 104. Following Cohen [2004], this calculation may be generalised as

$$D^* = \frac{R_D D}{R_D^*} \tag{9}$$

where D^* was characterised as the "idealised debt". There, this formulation created the basis for generating the *WACC* or V_L, curves consistent with the "maximum value" methodology.

As a result of Equation 9 and the justification that led to it, the unlevered value of the firm must now be based on the new debt, which is the risky debt re-priced relative to the risk-free rate. Calling this V_U^*, we write

$$V_U^* = E + D^* \times (1 - T) \tag{10}$$

which will now be used to modify Hamada's Equation. For this, we essentially follow Conine's method in arriving at Equation 7, which is outlined in Section 4, but instead of utilising Equation 5 to get R_U, the return on equity for the unlevered firm, we employ Equation 10. The outcome, after combining and re-arranging, is

$$\beta_L = \beta_U \left[1 + (1 - T)\phi^*\right] \tag{11}$$

where ϕ^*, the "adjusted" leverage, which accounts for default risk and, hence, credit spread, is given as

$$\phi^* \equiv \frac{R_D}{R_D^*}\phi = \frac{R_D}{R_D^*}\frac{D}{E} \tag{12}$$

by virtue of the definition for ϕ provided earlier.

Note, firstly, the similarity between Equation 11 and *HE*, Equation 1, and, secondly, *HE*'s tendency to always under-estimate the ratio β_L/β_U for levered firms because $\phi^* > \phi$, owing to default risk. With β_L derived empirically from the definition in 3a, one cannot but conclude here that Hamada's formulation would always lead to an over-stated measure of the business risk, β_U.

We now use Equation 11 to populate Table 3 the same way we did both Tables 1 and 2. For the firm in Figure 1, we get 112.4 [$= 50 + 104(1 - 40\%)$] as the value of the unlevered firm, V_U^*. This then allows us to go on to calculate the equity, *E*, leverage and adjusted leverage, ϕ and ϕ^*, respectively, and subsequently the levered beta, firm's value and *WACC* at every increment of debt.

The results here are twofold. First, they portray Hamada's over-estimation of β_U. For instance, for the firm represented in Figure 1, while *HE* produces a β_U of 1.09, the proposed modification leads to 0.95 – e.g. compare the betas in Tables 1 and 3, corresponding to debt $D = 0$. Second, in contrast to Table 1 [based on *HE*], the *EBIT* in Table 3 remains constant in relation to leverage and, contrary to Table 2 [based on Conine's formulation], Table 3 depicts a distinct optimal capital structure, consistent with a minimum *WACC* and maximum V_L, of 9.1% and 131.4, respectively, occurring at a leverage of $\phi =$ 1.14. Therefore, the problems associated with both *HE* and Conine's modification of it are now gone.

6. Summary and Conclusions

This paper addresses a major weakness in Hamada's Equation, namely its failure to account for default risk and the shortcomings this can create in practice. It also explores the failure of a later attempt to integrate default risk into it [Conine, 1980]. To correct these, we apply here a more robust framework to modify *HE*; one that would help it gain better acceptance among practitioners, as well as academics.

The main results of this work are outlined in Tables 1-3 and further elucidated in Figures 2-5. The figures, derived directly from the tables, compare the *WACC*, V_L and R_E curves, as well as the *E-vs-D* relationship, associated with the three approaches. Where all the curves intersect in Figures 2-5 is the current position of the firm, as depicted in Figure 1.

Table 1 portrays *HE*'s shortfall [for not accounting for default risk] by an *EBIT* that varies with leverage along the *WACC* curve. This, obviously, conflicts with one of *M&M*'s main assumptions that went into deriving *HE*, namely *EBIT* remains constant along the curve. Another flaw observed in Table 1, as well as in Figures 2 and 3, again relates to employing *HE* to generate the *WACC* curve. Here, although the *WACC* seems

to pass through a distinct optimum in the capital structure[7] [see Figure 2], the firm's value does not, as shown in Figure 3.[8] This, by itself, contradicts the very definition, as well as notion, that a minimum in a firm's *WACC* must correspond to a maximum in its value.

Table 2 follows the same numerical procedure as in Table 1 to arrive at the relevant parameters and examine the validity of Conine's modification [Equation 7] to *HE*. It is noted here that, even though Conine's solution rectifies the issue with the variable *EBIT* in Table 1, it fails to generate a *WACC* curve, as well as a corresponding V_L curve, that contain an optimum[9], as shown in Figures 2 and 3, respectively. As a consequence, Equation 7 has fallen out of favour with practitioners, whose job is to justify the existence of the optimal capital structure to corporate firms.

To circumvent the above complications, an alternative modification to *HE* is proposed here. It uses the notion of the re-priced debt, initially suggested by Cohen (2004) to plot the *WACC* curve of a corporate firm. The methodology does indeed amend *HE* in a way that it not only resolves all the above-mentioned problems, as depicted in Table 3 and Figures 2 and 3, but it also remains as straightforward in form and simple to implement in practice as the original equation itself.

Figure 4 presents a comparison of the return on equity, R_E, generated by the three methods considered. Most notable here is that, when plotted against leverage, R_E follows a fundamentally different pattern in each case. For instance, while *HE* produces a straight line[10], Conine's is concave down and the one based on the present approach is concave up.

The upward concavity associated with the last approach does not mean that higher leverage improves the situation for the equity holder and, hence, it would serve him best if the firm levered up to buy back its own shares. Rather, with R_E defined as the net profit divided by equity capital, it is caused by the steeper-than-linear decline in the *E-vs-D* curve in Figure 5, which contrasts sharply with the linear and more gradual drop stemming from the first two methods [a result of using Equation 5]. What is happening here is that, owing to the negative impact of default risk and credit spread, the equity holder suffers as well, as his equity share in the firm falls faster with increasing debt. This variation in the patterns is remarkable because it

[7] The optimum is a minimum in the case of *WACC* and a maximum in the case of the firm's value.
[8] Moreover, the two curves are not only distinctly different in shape from each other, but, also, the locations of the minima are so far apart that one could not be used to approximate the other.
[9] This is apart from its use of the highly debated notion of β_{debt}.

means that, depending on which one to believe, each R_E-*vs*-ϕ curve will have an entirely different story to tell - a striking conclusion given the practical significance of this parameter in the area of equity valuation.

Computing the beta of a firm, whether it's used for risk assessment, capital structure analysis or other purposes, is certainly far more complicated than what has been presented here. Obviously, a lot more work remains to be done to come up with a model that would please the majority of users - academics and practitioners alike. But, never the less, regardless of the methodology that goes into obtaining this parameter, it is crucial that, at least, some effort goes into ensuring that the approach does not contradict itself and, also, stays consistent with the fundamentals of finance. This, we can claim, is the direction we closely followed here.

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[10] This is consistent with *M&M*'s proposition II, because it is derived directly from it.

Income Statement	
EBIT (1)	***20.0***
Interest expense (2)	-5.2
EBT (3)	***14.8***
Tax (4)	-5.9
Net profit	***8.9***
Balance Sheet	
Assets	***130***
Shareholders' equity	50
Interest-bearing debt	80
Total liab. & equity	***130***
Parameters	
Risk-free rate (5)	5.00%
Current cost of debt (6)	6.50%
Current stock beta (7)	2.13
Market risk premium	6%
Tax rate	40%
Leverage	1.60
Cost of/return on equity (8)	***17.8%***

(1) Earnings before interest and tax.
(2) Calculated as current cost of debt multiplied by debt
(3) Earnings before tax.
(4) Calculated as tax rate [40%] multiplied by the EBT.
(5) This is the current cost of debt of 6.5% minus a credit spread of 1.5%.
(6) This takes into account the credit spread due to leverage.
(7) This is that beta that leads to an EBIT of 20, consistent with the income statement.
(8) Calculated as net profit of 8.9 divided by Shareholders' equity of 50.

Figure 1 – Simplified financial statement, including the income statement, balance sheet and a number of other relevant parameters.

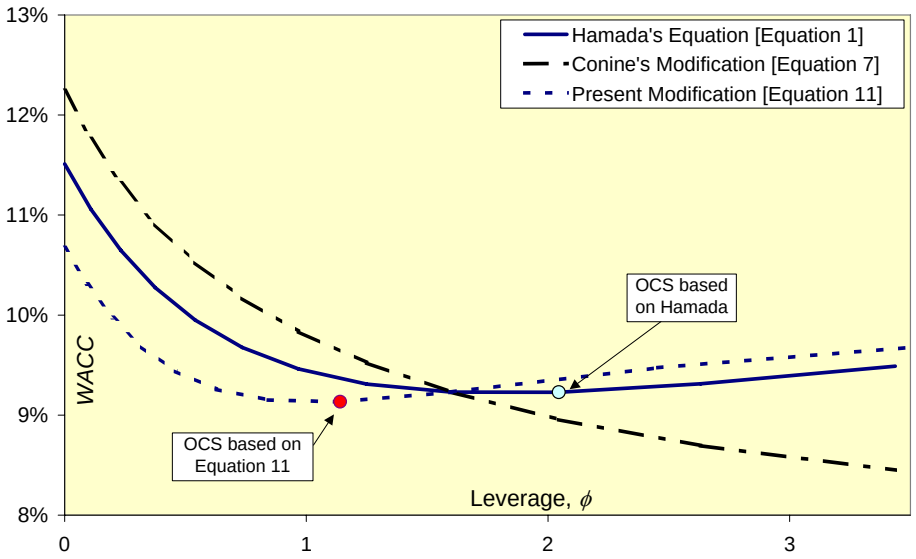

Figure 2 – The *WACC* plotted against leverage, ϕ, comparing the outcome for the three approaches, Hamada's [Equation 1], Conine's [Equation 7] and the present [Equation 11]. Note that while both Hamada and the present approach produce a minimum, Conine's doesn't.

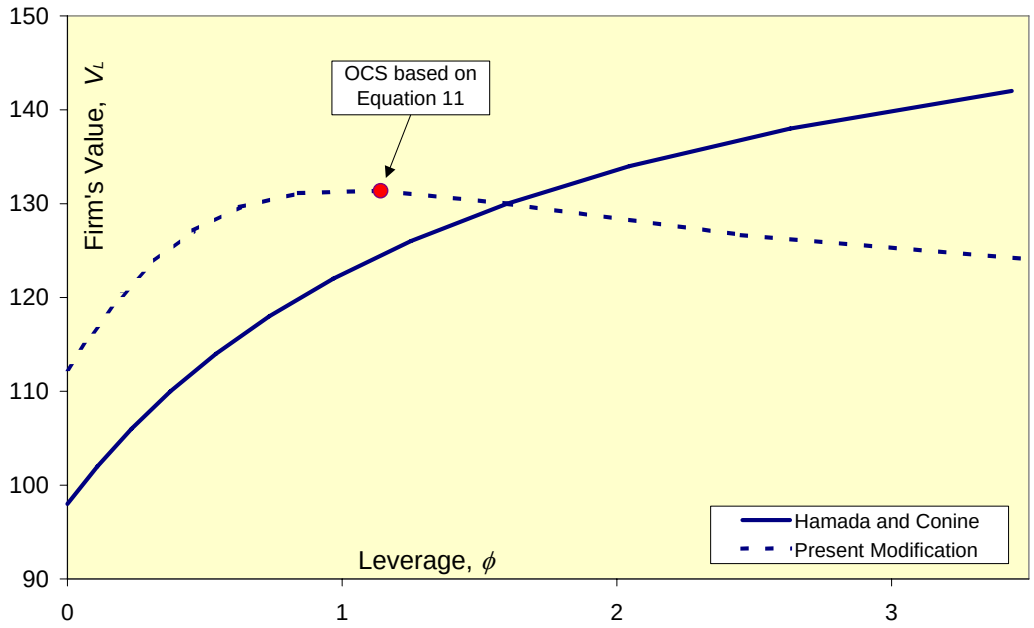

Figure 3 - The levered firm's value, V_L, plotted against leverage, ϕ, comparing the outcome of the three approaches - Hamada's [Equation 1], Conine's [Equation 7] and the present [Equation 11]. Note that only the present approach produces an optimal [a maximum] and the other two don't. The location of the maximum in the firm's value coincides exactly with that found in the *WACC* curve in Figure 2.

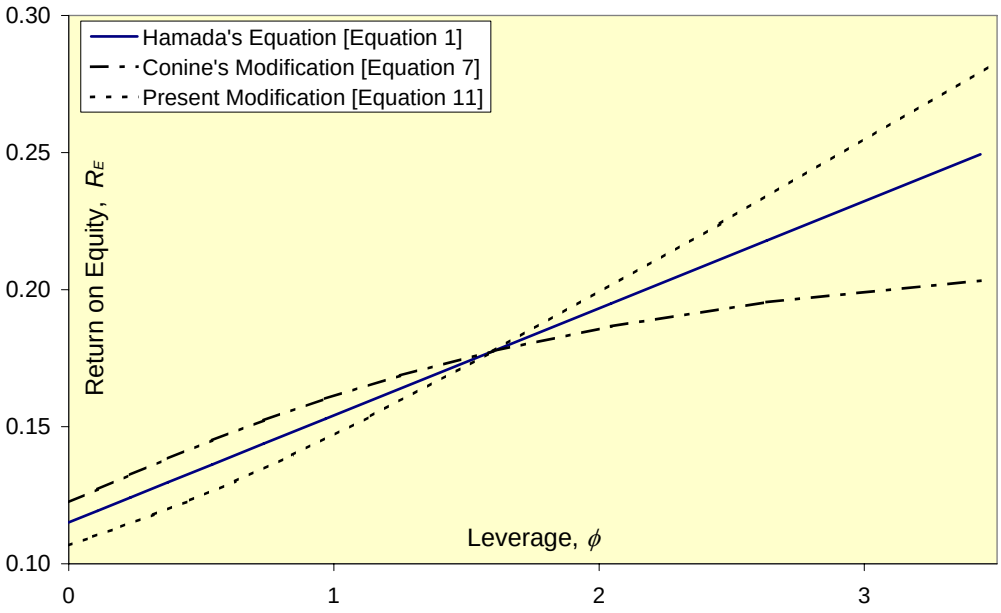

Figure 4 - The return on equity, R_E, plotted against leverage, ϕ, comparing the outcome of the three approaches - Hamada's [Equation 1], Conine's [Equation 7] and the present [Equation 11]. Note the significant difference between the three. While Hamada's produces a straight line, Conine's is concave down and the one based on the present approach is concave up. Where they cross is the current position of the firm portrayed in Figure 1.

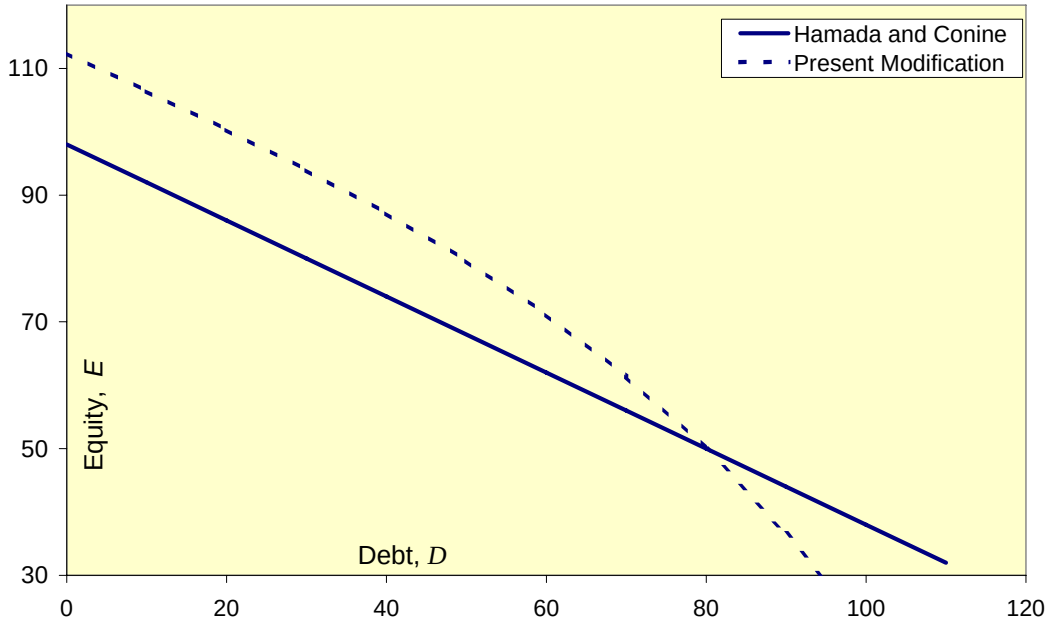

Figure 5 - The equity, *E*, plotted against debt, *D*, comparing the outcome of the three approaches - Hamada's [Equation 1], Conine's [Equation 7] and the present [Equation 11]. Note that while the first two lead to a straight line [a consequence of using Equation 5], the latter, which is based on re-priced debt, leads to a steeper-than-linear decline. This, essentially, is the reason behind the differences in the R_E curves plotted in Figure 4.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Debt, *D*	**Interest rate, R_D**	**Equity, *E***	**Leverage, ϕ**	**Levered beta, β_L**	**R_E based on β_L**	**Re-calculated *EBIT(1-T)***	**Firm's value, V_L**	***WACC***
0	5.00%	98.0	0.00	1.09	0.115	11.3	98.0	11.5%
10	5.01%	92.0	0.11	1.16	0.119	11.3	102.0	11.1%
20	5.05%	86.0	0.23	1.24	0.124	11.3	106.0	10.6%
30	5.13%	80.0	0.38	1.33	0.130	11.3	110.0	10.3%
40	5.27%	74.0	0.54	1.44	0.136	11.3	114.0	10.0%
50	5.46%	68.0	0.74	1.56	0.144	11.4	118.0	9.7%
60	5.73%	62.0	0.97	1.72	0.153	11.5	122.0	9.5%
70	6.07%	56.0	1.25	1.90	0.164	11.7	126.0	9.3%
80	**6.50%**	**50.0**	**1.60**	**2.13**	**0.178**	**12.0**	**130.0**	**9.2%**
90	7.01%	44.0	2.05	2.42	0.195	12.4	134.0	**9.2%**
100	7.62%	38.0	2.63	2.80	0.218	12.9	138.0	9.3%
110	8.33%	32.0	3.44	3.32	0.249	13.5	142.0	9.5%

Current → (80 | 6.50% | 50.0 | 1.60 | 2.13 | 0.178 | 12.0 | 130.0 | 9.2%)

OCS → (90 | 7.01% | 44.0 | 2.05 | 2.42 | 0.195 | 12.4 | 134.0 | 9.2%)

(1) Debt, D, increasing in increments of 10.

(2) Interest rate increasing due to rising credit spread (owing to higher debt).

(3) With D given in Column 1 and Vu and tax held constant at 98 and 40%, respectively, E is calculated from Equation 5.

(4) Leverage defined as E/D - i.e. Column 1 divided by Column 3.

(5) Levered beta calculated using an unlevered beta of 1.09, together with Equations 3b and 6.

(6) Return on equity calculated from the beta (Column 5) based on Equation 3a.

(7) Bottom-up re-calculation of EBIT(1-T) using Equation 4.

(8) The firm's value calculated as E+D, sum of Columns 1 and 3.

(9) The WACC calculated by dividing Column 7 by Column 8.

Table 1 - Application of Hamada's Equation to extend the financial statement in Figure 1 to account for different levels of leverage. The row corresponding the Figure 1 is highlighted and marked "current." Also highlighted is the location of the optimal capital structure (*OCS*) in relation to the *WACC*. The procedure for computing the values in each column is outlined in the footnotes, as well as in the text.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Debt, *D*	**Interest rate, R_D**	**Equity, *E***	**Leverage, ϕ**	**Debt beta, β_{debt}**	**Levered beta, β_L**	**R_E based on β_L**	**Re-calculated *EBIT(1-T)***	**Firm's value, V_L**	***WACC***
	0	5.00%	98.0	0.00	0.00	1.21	0.122	12.0	98.0	12.2%
	10	5.01%	92.0	0.11	0.00	1.29	0.127	12.0	102.0	11.8%
	20	5.05%	86.0	0.23	0.01	1.37	0.132	12.0	106.0	11.3%
	30	5.13%	80.0	0.38	0.02	1.47	0.138	12.0	110.0	10.9%
	40	5.27%	74.0	0.54	0.04	1.58	0.145	12.0	114.0	10.5%
	50	5.46%	68.0	0.74	0.08	1.71	0.152	12.0	118.0	10.2%
	60	5.73%	62.0	0.97	0.12	1.84	0.160	12.0	122.0	9.8%
	70	6.07%	56.0	1.25	0.18	1.98	0.169	12.0	126.0	9.5%
→ Current	**80**	**6.50%**	**50.0**	**1.60**	**0.25**	**2.13**	**0.178**	**12.0**	**130.0**	**9.2%**
	90	7.01%	44.0	2.05	0.34	2.28	0.187	12.0	134.0	9.0%
	100	7.62%	38.0	2.63	0.44	2.42	0.195	12.0	138.0	8.7%
	110	8.33%	32.0	3.44	0.55	2.55	0.203	12.0	142.0	8.5%

(1) Debt, D, increasing in increments of 10
(2) Interest rate increasing due to rising credit spread (owing to higher debt)
(3) With D given in Column 1 and Vu and tax held constant at 98 and 40%, respectively, E is calculated from Equation 5.
(4) Leverage defined as E/D - i.e. Column 1 divided by Column 3
(5) Debt beta calculated from Equation 8.
(6) Conine's levered beta calculated from Equation 7.
(7) Return on equity calculated from the beta (Column 5) based on Equation 3a.
(8) Bottom-up re-calculation of EBIT(1-T) using Equation 4.
(9) The firm's value calculated as E+D, sum of Columns 1 and 3.
(10) The WACC calculated by dividing Column 8 by Column 9.

Table 2 – Conine's approach (Section 4) has been applied here to extend the financial statement in Figure 1 to account for different levels of leverage. The row corresponding the Figure 1 is highlighted and marked "current." Note that there is no optimal capital structure here, either in relation to V_L, or to *WACC*. The procedure for computing the values in each column is outlined in the footnotes, as well as in the text.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	
	Debt, D	**Interest rate, R_D**	**Adjusted debt, D^***	**Equity, E**	**Leverage, ϕ**	**Adjusted leverage, ϕ^***	**Levered beta, β_L**	**R_E based on β_L**	**Re-calculated *EBIT(1-T)***	**Firm's value, V_L**	***WACC***	
	0	5.00%	0.0	112.4	0.00	0.00	0.95	0.107	12.0	112.4	10.7%	
	10	5.01%	10.0	106.4	0.09	0.09	1.00	0.110	12.0	116.4	10.3%	
	20	5.05%	20.2	100.3	0.20	0.20	1.06	0.114	12.0	120.3	10.0%	
	30	5.13%	30.8	93.9	0.32	0.33	1.13	0.118	12.0	123.9	9.7%	
	40	5.27%	42.1	87.1	0.46	0.48	1.22	0.123	12.0	127.1	9.4%	
	50	5.46%	54.6	79.6	0.63	0.69	1.34	0.130	12.0	129.6	9.3%	
	60	5.73%	68.8	71.1	0.84	0.97	1.49	0.140	12.0	131.1	9.2%	
	70	6.07%	85.0	61.4	1.14	1.39	1.73	0.154	12.0	**131.4**	**9.1%**	OCS
Current	**80**	**6.50%**	**104.0**	**50.0**	**1.60**	**2.08**	**2.13**	**0.178**	**12.0**	**130.0**	**9.2%**	
	90	7.01%	126.2	36.7	2.46	3.44	2.90	0.224	12.0	126.7	9.5%	
	100	7.62%	152.4	21.0	4.77	7.27	5.07	0.354	12.0	121.0	9.9%	
	110	8.33%	183.2	2.5	43.92	73.14	42.46	2.598	12.0	112.5	10.7%	

(1) Debt, D, increasing in increments of 10.
(2) Interest rate increasing due to rising credit spread (owing to higher debt).
(3) Re-pricing the debt, D, on the risk-free rate, based on Equation 9.*
(4) Calculation of equity using Equation 10, based on the re-priced or idealisd debt defined in Equation 9.
(5) Leverage defined as E/D - i.e. Column 1 divided by Column 4.
(6) Adjusted leverage calculated from the re-priced debt - i.e. Column 3 divided by Column 4 (see Equation 12).
(7) Levered beta calculated from the modified Hamada's Equation, Equation 11.
(8) Return on equity calculated from the beta (Column 7) based on Equation 3a.
(9) Bottom-up re-calculation of EBIT(1-T) from beta (in Column 8) using Equation 4.
(10) The firm's value calculated as E+D, sum of Columns 1 and 4.
(11) The WACC calculated by dividing Column 9 by Column 10.

Table 3 – The proposed modification to *HE* [Equation 11] used here to extend the financial statement in Figure 1 to account for different levels of leverage. The row corresponding the Figure 1 is highlighted and marked "current." Also highlighted is the location of the optimal capital structure (*OCS*) in relation to both the *WACC* and the Firm's value, V_L. The procedure for computing the values in each column is outlined in the footnotes, as well as in the text.