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Sustaining collusion in markets with a general evolution of demand

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Abstract. This paper proposes a general framework to study the sustainability of collusion in markets where demand growth (although deterministic) is not restricted to occur at a constant rate and may trigger future entry. It is shown that, typically, entry occurs later along the collusive path than along the punishment path (since profits are lower in the latter case). The possibility of delaying entry, therefore, constitutes an additional incentive for deviating just before entry is supposed to occur along the collusive path. In case discontinuing collusion does not delay entry, collusion is shown to be typically more difficult to sustain after than before entry. The proposed model encompasses and explains conflicting results derived in the extant literature under more restrictive settings, and derives some additional results. In particular, it is shown that whether collusion is more difficult before or after entry crucially depends on the magnitude of the entry costs and on the speed at which demand converges to its long-term evolution.

Keywords: Collusion; Demand evolution; Entry.

JEL Classification Numbers: K21, L11, L13.

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1 Introduction

The impact of demand growth on collusion sustainability is not straightforward. On the one hand, demand growth makes the market more profitable in the future, which reduces the incentives for firms to disrupt the collusive agreement in the present. On the other hand, demand growth is likely to attract new firms to the market.[1] The prospect of entry makes collusion harder to sustain because it reduces the future benefits of complying with the collusive agreement, without affecting the present benefits of deviating.[2]

Some recent theoretical contributions have studied the effects of demand growth and entry on collusion sustainability (Capuano, 2002; Vasconcelos, 2008; Brandão et al., 2013; Correia-da-Silva et al., 2013).[3] These works aim at investigating whether collusion is sustainable, and whether it is easier to sustain before or after entry. The answer to these questions has been shown to crucially depend on the type of punishment strategies adopted by cartel members.

Capuano (2002), Vasconcelos (2008) and Brandão et al. (2013) concluded that, if firms permanently revert to the Cournot equilibrium after a deviation (grim trigger strategies), collusion is typically more difficult to sustain before entry than after entry. This result is surprising because, in stationary models, sustaining collusion becomes more difficult as the number of firms in the market increases. Two characteristics of their frameworks generate this result: (i) entry occurs later under Cournot competition than under collusion (because the profits of the entrant are lower in the Cournot-Nash equilibrium than under collusion);

[1] In their survey, Siegfried and Evans (1994) reported several empirical studies finding that market growth (measured by the past growth rate of industry sales revenue) positively effects entry.

[2] For theoretical discussions on the impact of entry and demand growth on collusion sustainability, see Ivaldi et al. (2007) and Motta (2004). Empirical studies diverge on their conclusions about the impact of demand growth on collusion sustainability. For instance, Dick (1996) concluded that the Webb-Pomerene cartels (i.e., associations of exporters that are exempt from some antitrust laws) were more frequent in growing industries. Contrariwise, Asch and Seneca (1975) found that if sales growth is low, firms have more incentives to collude than if sales growth is high. Finally, Symeonidis (2003) arrived at a non-monotonic relation between collusion sustainability and demand growth: collusion is easier to sustain when the market grows at a moderate rate than when the market declines or grows at a fast rate.

[3] The work of Vasconcelos (2008) underpins the contributions of Brandão et al. (2013) and Correia-da-Silva et al. (2013). Brandão et al. (2013) considered asymmetric cartel members, while Correia-da-Silva et al. (2013) studied the impact of considering alternative punishment strategies and reactions to entry.

and (*ii*) the Cournot profit before entry is higher than the collusive profit after entry. As a result, delaying entry constitutes an additional incentive to break the collusive agreement before entry.

In contrast, Capuano (2002) and Correia-da-Silva et al. (2013) found that, if firms adopt optimal penal codes, the collusive agreement is more difficult to sustain after entry than before entry. With optimal penal codes, the incentive for deviating before entry in order to delay entry disappears, because the potential benefit does not exist when the punishment scheme drives firms profits to their minmax payoff.

Despite providing important conclusions, all these works rely on quite specific assumptions: linear demand, constant rate of market growth and constant marginal costs. Our aim is to relax these assumptions and provide a general framework to study the sustainability of collusion in markets with entry driven by deterministic demand growth. Apart from encompassing the above mentioned models as special cases, the proposed framework also enables us to obtain some novel results.

We consider a market with two incumbents and one potential entrant that produce homogeneous goods and have the same cost function.[4] Firms interact during an infinite number of periods, with the objective of maximizing the discounted sum of their profits. At the beginning of the game, the two incumbents combine to maximize the industry profit in all periods,[5] and to accommodate the entrant in the collusive agreement immediately after entry.[6] We consider that firms abide by the collusive agreement as long as the others do the same; and irreversibly revert to a punishment equilibrium after a deviation.[7]

As stated before, we contribute to the literature on collusion with entry driven by demand growth by generalizing the existing framework. Instead of considering a linear demand

[4]The number of incumbents is not crucial for our results and could be easily relaxed. In contrast, the consideration of more than one entrant would complicate the analysis.

[5]Vasconcelos (2008) considered that, when perfect collusion is not sustainable, firms may establish less demanding agreements, according to which output is less restricted along the collusive path.

[6]For a discussion on cartel reactions to entry, see Correia-da-Silva et al. (2013) and the references therein.

[7]The punishment equilibrium can be, for example, the Cournot equilibrium or a zero-profit equilibrium. The only restriction that we make is that the profit flow along the punishment path is proportional to the profit flow under collusion.

function and constant marginal costs, we only require that market growth has the same relative impact on profits in the different market regimes (collusion, unilateral deviation and punishment). Furthermore, instead of considering a constant market growth rate, we only restrict the evolution of market profitability to be quasi-concave. We are able to recover most of the existing results and to obtain new results.

When deciding when to enter the market, the entrant faces the following trade-off: on the one hand, it wants to enter as soon an possible, in order to start receiving profits; on the other hand, delaying entry decreases the (discounted) value of the entry cost. The entrant chooses the period that maximizes the discounted value of its flow of profits.[8] We find that entry occurs when the profit hits a certain threshold, regardless of its past or future evolution. Typically, entry occurs later along the punishment path than along the collusive path (because profits are lower). However, the entry periods may coincide or it may be the case that entry is not even profitable along the punishment path.

When studying sustainability of collusion with entry, we analyze the incentives of cartel members to comply with the agreement before and after entry, and try to understand which is the most critical moment for collusion sustainability.

We find that the effect of market growth on the condition for collusion sustainability in a given period after entry is given by an adjustment term that is multiplied to the discount factor. This term is the ratio between the value of the market in the next period and the value of the market in the current period.[9] The infimum of this ratio across all periods after entry is the relevant adjustment term in the condition for collusion sustainability after entry. Thus, the critical period for collusion sustainability after entry is the one in which the value of the market grows the least (or declines the most). If the value of the market is non-decreasing, collusion is sustainable after entry as long as the discount factor is sufficiently high. If, at some point, the value of the market decreases by more than a certain factor, collusion cannot be sustained.

[8]Vasconcelos (2008) made the same assumption, which is reasonable when there is only one potential entrant. If there are several potential entrants, as in the model of Capuano (2002), entry occurs when the discounted flow of the profits becomes positive. The two alternatives lead to the same qualitative results regarding collusion sustainability.

[9]By value of the market, we mean the discounted sum of present and future profits.

Regarding the sustainability of collusion before entry, under a mild condition (which is weaker than sustainability of collusion without entry), we conclude, as Vasconcelos (2008), that the period that immediately precedes entry is the most critical. The reason is easy to understand. In all periods before entry, the incumbents are tempted to defect in order to enjoy the deviation profit and to delay entry (by lowering the profits of the entrant from the collusive level to the punishment path level). Deviating in the period that immediately precedes entry allows an incumbent to get these benefits without forgoing the benefit of colluding in the previous periods. We derive a necessary and sufficient condition for collusion to be sustainable before entry, and show that collusion becomes harder to sustain as the entry delay that results from the cartel breakdown increases. We obtain more clear-cut results in two extreme cases: (i) when cartel breakdown does not delay entry; and (ii) when cartel breakdown prevents entry. In the latter scenario, if the collusive profit after entry is lower than the punishment profits before entry, collusion before entry is never sustainable.

To illustrate our main conclusions, we consider an example of a market with linear demand and quantity-setting firms that have no production costs.[10] Unsurprisingly, we conclude that collusion without entry is always easier to sustain than with entry. We also show that if a deviation before entry has no impact on the entry period, collusion is harder to sustain after entry than before entry. In contrast, if a deviation delays entry by more than two periods, collusion is surely more difficult to sustain before entry. For this result to hold, it is actually enough that the entry delay is strictly positive and that entry does not occur too soon.

We also conclude that the comparison of collusion sustainability before and after entry crucially depends on magnitude of the entry cost and on the speed at which demand converges to its long-term trend. These two variables determine whether entry occurs at an early stage, in which demand is growing very fast; or at a later stage, in which demand is growing at a lower rate. The faster is the growth of demand when entry occurs, the lower is the entry delay that results from breaking the cartel (because it takes a lower number of periods for demand to go from the threshold for entry under collusion to the threshold

[10]This example captures the models of Capuano (2002) and Vasconcelos (2008) as particular cases and also goes beyond their assumption of constant demand growth.

for entry along the punishment path). Therefore, it may happen that: if the entry cost is sufficiently low, collusion is more difficult to sustain after entry; while, if it is sufficiently high, collusion is more difficult to sustain before entry.

Finally, we analyze the following two variations of the benchmark scenario: (i) firms adopt severe punishment strategies that drive profits to zero after a deviation;[11] and (ii) firms are price-setters (rather than quantity-setters). In these variations, the cartel breakdown permanently deters entry, but, since profits are null after a deviation, such deterrence is irrelevant. As a result, collusion is more difficult to sustain after entry than before entry (because there are more firms in the market). It is clear that collusion is easier to sustain if firms use optimal penal codes (Abreu, 1986) than if they revert to competition (because punishment profits are lower with Abreu-type punishments whereas everything else is equal). But it is not clear whether collusion is easier to sustain if firms set prices instead of quantities: on the one hand, the deviation profit is higher; on the other hand, the punishment is harsher. However, in the case of constant market growth, we conclude that collusion is harder to sustain when firms set quantities if and only if the cartel breakdown implies a strictly positive entry delay.

The remainder of the paper is organized as follows. In Section 2, we briefly relate our work to the existing literature. In Section 3, we present the general model, derive conditions for entry profitability (under collusion and along the punishment path), and characterize the optimal entry periods. In Section 4, we study collusion sustainability before and after entry. In Section 5, we consider markets with linear demand and quantity-setting firms. In Section 6, we consider two variations of the benchmark case (price-setting firms and optimal pneal codes). In Section 7, we offer some concluding remarks. Finally, in Section 8, we present most proofs and auxiliary calculations.

[11] As suggested by Correia-da-Silva et al. (2013), this scenario is possible even in the absence of production costs, as long as firms can engage in observable dissipative costs. Alternatively, if firms support production costs that are strictly positive but not too high, these punishment strategies consist in overproducing to flood the market so that the price is null in all periods that follow a deviation.

2 Related literature

Several authors have analyzed the sustainability of collusion in markets with varying demand, but assuming different specifications for demand evolution. Essentially, the existing specifications differ in the deterministic or stochastic nature of the evolution of demand; and in the consideration of a fixed demand trend (expansion or contraction forever) or fluctuations. We consider that demand evolves in a deterministic way, but do not either restrict to a fixed trend or allow for fluctuations. Our assumption is that demand may increase forever or have one single peak.[12]

Let us start by briefly describing the literature in which a deterministic evolution of demand is assumed. Using simple theoretical models, Motta (2004) and Ivaldi et al. (2007) analyzed the impact of a constant (and perfectly observed) demand growth rate on collusion sustainability. They concluded that the critical discount factor is inversely related to the demand growth rate. More precisely, a positive (resp. negative) demand growth rate makes collusion easier (resp. more difficult) to sustain. Interestingly, in their experimental work, Abbink and Brandts (2009) obtained the opposite result. In particular, they found that collusion was more easily established when demand was shrinking than when it was expanding.[13] Abbink and Brandts (2009, p. 48) suggested the following explanation for their unexpected result: *"it is not the promise of growing profit opportunities, but rather the pressure from rapidly declining profits that exerts a disciplining effect on firms. If the prize shrinks at a dramatic rate, high profits need to be made early."*

Assuming that demand is subject to (deterministic and perfectly observed) cyclical fluctuations, Haltiwanger and Harrington (1991) reached to a similar conclusion. More precisely, they found that the deviation gains are the highest at the peak of the cycle, but collusion is most difficult to sustain when demand is declining. The reason is that the punishment is less severe during recessions, since demand will shrink in periods adjacent to the deviation.

As for the literature assuming stochastic demand, Rotemberg and Saloner (1986) are

[12] In the latter case, demand starts by expanding, reaches the peak and declines thenceforth.

[13] Abbink and Brandts (2009) analyzed the evolution of prices set by two agents (hypothetically) supplying homogeneous goods to a perfectly inelastic demand.

considered a benchmark study. In their setting, demand is subject to i.i.d. shocks and firms observe the actual state of demand before making decisions. They found that collusion is harder to sustain in periods of high demand. Since the (expected) punishment does not depend on the level of demand in the deviation period, firms feel most tempted to deviate when the deviation gain is the highest, i.e., when demand is high.[14] As a result, the optimal collusive mechanism implies lower prices when demand is high than when it is low. In the current paper, we obtain the opposite relation between the level of demand and the price, since the collusive price evolves proportionally to the market size.

Despite studying the sustainability of collusion in markets with changing demand, all these works have assumed a fixed number of active firms over time. However, favorable market conditions may encourage entry, while adverse conditions may induce exit. In addition, the number of firms in the market is usually perceived as a key determinant in the likelihood of collusion (Ivaldi et al., 2007). The focus of the present paper is precisely the impact of the possibility of entry on collusion sustainability.[15]

An independent branch of the literature has focused on collusion sustainability in markets where entry is possible. Pashigian (1968) took the first steps in this direction, analyzing whether a monopolist should set the limit price since the beginning of the game (and block entry of potential competitors); or admit entry of some competitors and only then charge the limit price. The trade-off of the monopolist is the following: a later entry block allows the firm to charge the monopoly price during some periods; but leads to a lower long-run price.[16] More recent contributions have analyzed alternative cartel reactions to entry (Harrington, 1989; Stenbacka, 1990; Friedman and Thisse, 1994; Vasconcelos, 2004; Correia-da-Silva et al., 2013), and tried to understand what is the optimal cartel reaction to entry. The answer to this question depends on the credibility of the threats made by the incumbent firms. As Vasconcelos (2008), we assume that there is one single entrant, which is accommodated in the collusive agreement immediately after entry.

[14]Posterior contributions tried to understand whether the main conclusions of Rotemberg and Saloner (1986) remained valid under the existence of serial correlation between demand shocks or capacity constraints. See, for example, Haltiwanger and Harrington (1991), Kandori (1991), Staiger and Wolak (1992), Bagwell and Staiger (1997), Fabra (2006), Knittel and Lepore (2010) and Montero and Guzman (2010).

[15]Exit or hit-and-run strategies are never optimal in our setting.

[16]See also Wenders (1971).

On the one hand, most studies on collusion sustainability with varying demand have ignored the possible changes in the number of active firms over time. On the other hand, most contributions on collusion sustainability with entry have assumed constant demand. Capuano (2002) and Vasconcelos (2008) were the first to tie these loose ends.[17] They developed theoretical models to study the sustainability of collusion in markets where entry of a new firm is triggered by demand growth. These works took insightful steps toward the understanding of collusion sustainability when two opposite forces operate in the market (market expansion and entry). However, these frameworks can still be enriched to better fit some market characteristics. In particular, the assumption that the market expands forever may not be realistic, especially if taken together with the hypothesis of a single entry. Thus, taking these two works as a starting point, we consider a more general framework, by not relying on a specif demand function or permanent demand growth. We also allow for different punishment mechanisms, capturing the standard grim trigger strategies and optimal penal codes as particular cases. Finally, we do not restrict marginal costs to be null. Although relaxing these assumptions of Capuano (2002) and Vasconcelos (2008), we are able to recover their main results and even put forward some new findings.

3 Model

Consider a market with two incumbents (firms 1 and 2) and one potential entrant (firm 3) that produce homogeneous goods and have the same cost function. The objective of each firm, $i \in \{1, 2, 3\}$, is to maximize the discounted value of its flow of profits, $V_i \equiv \sum_{t=0}^{+\infty} \delta^t \pi_{it}$, where $\delta \in (0, 1)$ is the common discount factor and π_{it} denotes the profit of firm i in period $t \in \{0, 1, ...\}$. As a function of own output, the profit of firm i in period t is given by:

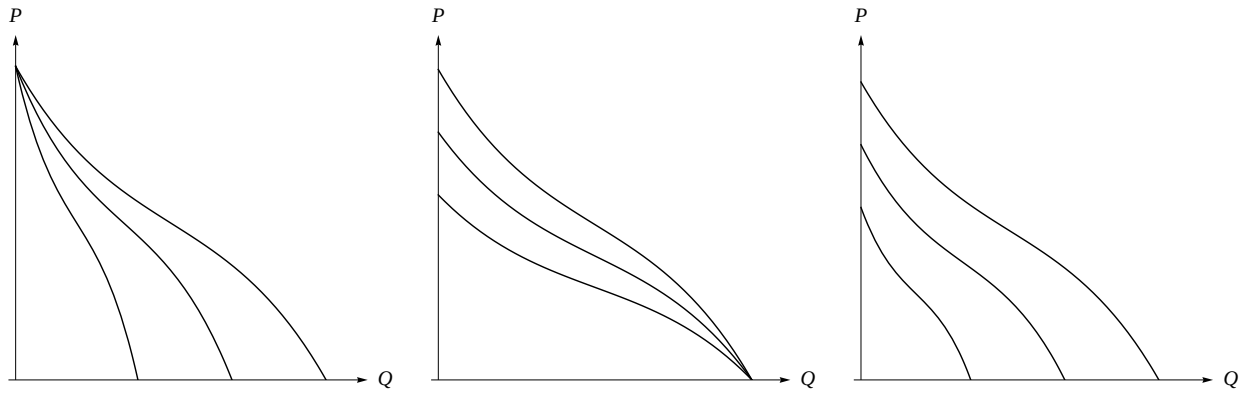
$$\pi_{it}(q_{it}) = P_t(Q_t)q_{it} - C_t(q_{it}),$$

[17] Brandão et al. (2013) and Correia-da-Silva et al. (2013) have analyzed whether the main results of Vasconcelos (2008) were robust, respectively, to: asymmetries in production costs between incumbents and entrant; and different punishment strategies or cartel reactions to entry.

where $P_t(\cdot)$ is the inverse demand function, $C_t(\cdot)$ is the cost function, q_{it} is the quantity produced by firm i, and $Q_t = \sum_{j=1}^{n} q_{jt}$ is the total output of the industry. We assume that the inverse demand function and the cost function can be written as:[18]

$$P_t(Q_t) = P\left(\frac{Q_t}{g_t}\right) h_t \quad \text{and} \quad C_t(q_t) = C\left(\frac{q_t}{g_t}\right) g_t h_t,$$

where g_t and h_t are strictly positive market size parameters and $P(\cdot)$ and $C(\cdot)$ are functions that do not change over time. The normalized inverse demand function, $P : \mathbb{R}_+ \to \mathbb{R}_+$, is assumed to be such that: $P(0) = \bar{P}$; $P'(Q)$ is negative and continuous, $\forall Q \in \left(0, \bar{Q}\right)$; and $P(Q) = 0$, $\forall Q \geq \bar{Q}$. The normalized cost function, $C : \mathbb{R}_+ \to \mathbb{R}_+$, is assumed to be such that: $C(0) = 0$ and $C'(q)$ is non-negative and continuous. The effect of changes in g_t (extensive growth) and h_t (intensive growth) on market demand is illustrated in Figure 1.



(a) Changing g. (b) Changing h. (c) Changing g and h.

Figure 1: Allowed forms of demand growth

In this setting, market growth (increase of g_t and h_t) corresponds to a transformation of scale. As a result, it has the same proportional impact on profits in the three market regimes that we will consider: collusion (m), unilateral deviation from the collusive agreement (d), and continuation equilibrium after a deviation (c).[19] In period t, the profit of firm i can be

[18] At the first glance, it may seem very restrictive to assume that the market size parameters also impact the cost function. However, this specification captures the existing formulations in the literature as particular cases. More precisely, Capuano (2002), Vasconcelos (2008) and Correia-da-Silva et al. (2013) consider null production costs. Except for the asymmetry among firms, Brandão et al. (2013) is also captured by our formulation, with $P(Q) = 1 - Q$, $g_t = h_t = \mu^t$ and $C(q) = aq^2$, where a is a firm-specific parameter.

[19] This is shown in Appendix 8.1 for the scenario in which firms set quantities.

written as:

$$\pi_{it}^{jn} = \pi^{jn} f_t,$$

where $j \in \{m, d, c\}$ denotes the market regime, $n \in \{2, 3\}$ is the number of active firms, and $f_t \equiv g_t h_t$.[20] Profits in the different market regimes are assumed to satisfy the usual ordering: $\pi^{cn} \leq \pi^{mn} \leq \pi^{dn}$.

The function $f : \mathbb{N}_0 \to \mathbb{R}_+$, therefore, captures the evolution of market profitability over time. We assume that f is quasi-concave and that $\sum_{t=0}^{+\infty} \delta^t f_t < +\infty$. It is useful to define $F : \mathbb{N}_0 \to \mathbb{R}_+$ as $F_t \equiv \sum_{s=t}^{+\infty} f_s \delta^{s-t}$. While f_t measures the current profitability of the market, F_t measures the profitability of the market from t onwards.

Our framework generalizes the models of Capuano (2002) and Vasconcelos (2008), where profits grow at a constant rate (i.e., $f_t = \beta^t$ with $\beta > 1$) and single-period profits (π^{cn}, π^{mn} and π^{dn}) are derived for quantity-setting firms that face a linear market demand.

To enter the market, firm 3 must support a fixed entry cost, $K > 0$. Exiting the market is assumed to entail no costs or revenues. As a result, given the assumption that $C(0) = 0$, it is never profitable to exit the market. Firms remain in the market (even if inactive) forever after entering.

Since we are assuming the existence of a single potential entrant, entry occurs in the period that maximizes the discounted value of the entrant's flow of profits:[21]

$$V_3(T) = \sum_{t=T}^{+\infty} \delta^{t-T} \pi_{3t} - \delta^T K.$$

The following result describes if and when entry occurs.

Lemma 1. *If firm* 3 *expects the post-entry market regime to be* $j \in \{m, c\}$*, it enters the*

[20] The fact that collusive, deviation and continuation profits vary in the same proportion greatly simplifies the study of collusion sustainability, because (as we will see) it implies that the incentive compatibility constraints in the different periods differ by term that only depends on the shape of market growth.

[21] With more than one potential entrant, entry would occur in the period at which the discounted value of the flow of profits of the entrant becomes positive. This variant of the model, considered by Capuano (2002), yields very similar results.

market if and only if the entry cost is sufficiently low:

$$K < \pi^{j3} \sup_t \{F_t\}. \tag{1}$$

If the above condition is satisfied, entry occurs at the lowest T^j such that:[22]

$$f_{T^j} \geq \frac{(1-\delta)K}{\pi^{j3}}. \tag{2}$$

Proof. See Appendix 8.2. □

This result shows that: it is possible that entry occurs under collusion but not in the continuation equilibrium after a deviation; and that entry occurs earlier under collusion than in the continuation equilibrium ($T^m \leq T^c$).

Our aim is to analyze the impact of entry on collusion sustainability. Therefore, we assume that the entry cost is sufficiently high so that firm 3 does not enter in period $t = 0$ (even if firms are colluding), i.e., $f_0 < \frac{(1-\delta)K}{\pi^{m3}}$.

4 Collusion

Before period $t = 0$, the incumbents make a collusive agreement to maximize the industry profit (perfect collusion) in all periods. They also combine to accommodate the entrant in a more inclusive agreement immediately after it enters the market (full collusion).[23]

Firms adopt grim trigger strategies: as long as no firm deviates, they maximize the joint profit; if one firm disrupts the collusive agreement, they permanently revert to a continuation equilibrium.[24]

[22] The timing of entry, given by (2), results from the comparison between the current period's profit with the gain associated with supporting the entry cost one period later.

[23] Alternative reactions to entry have been studied by Harrington (1989), Stenbacka (1990), Friedman and Thisse (1994), Vasconcelos (2004), and Correia-da-Silva et al. (2013).

[24] In Section 6, we consider more severe punishment strategies.

4.1 Without entry

As a benchmark, we start by considering the case in which there is no entry. This may be due to an exogenous barrier to entry or to entry not being profitable.[25]

In the absence of entry, the incumbents abide by the collusive agreement if the following incentive compatibility condition (ICC) is satisfied:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_{is}^{m2} \geq \pi_{it}^{d2} + \sum_{s=t+1}^{+\infty} \delta^{s-t} \pi_{is}^{c2}, \quad \forall t \geq 0. \tag{3}$$

Proposition 1. *Collusion is sustainable in the absence of entry if and only if:*

$$\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{c2}} \sup_{t \geq 0} \left\{ \frac{F_t}{F_{t+1}} \right\}. \tag{4}$$

Proof. Firms comply with the collusive agreement in period t if and only if:

$$\begin{aligned} & \pi^{m2} \sum_{s=t}^{+\infty} \delta^{s-t} f_s \geq \pi^{d2} f_t + \pi^{c2} \delta \sum_{s=t+1}^{+\infty} \delta^{s-(t+1)} f_s \\ \Leftrightarrow \quad & \pi^{m2} F_t \geq \pi^{d2} F_t - \pi^{d2} \delta F_{t+1} + \pi^{c2} \delta F_{t+1} \quad \Leftrightarrow \quad \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_t}{\left(\pi^{d2} - \pi^{c2}\right) F_{t+1}}. \end{aligned}$$

□

The assumption that the profits in the different market regimes are proportional is crucial to obtain this result. Notice that we have obtained the standard critical discount factor (without market fluctuations) multiplied by a constant that only depends on how the market evolves over time.

[25] From Lemma 1, entry is not profitable if $K \geq \pi^{m3} \sup_t \{F_t\}$.

4.2 After entry

Suppose, now, that the entry cost is sufficiently low for firm 3 to enter the market at some point (at least under collusion). The conditions for collusion sustainability before and after entry are different. We start by analyzing collusion sustainability after entry.

The three firms abide by the collusive agreement in period $t \geq T^m$ (after entry) if the following ICC is satisfied:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_{is}^{m3} \geq \pi_{it}^{d3} + \sum_{s=t+1}^{+\infty} \delta^{s-t} \pi_{is}^{c3}. \tag{5}$$

Proposition 2. *Collusion is sustainable after entry if and only if:*

$$\delta \geq \frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}} \sup_{t \geq T^m} \left\{ \frac{F_t}{F_{t+1}} \right\}.$$

Proof. Analogous to the proof of Proposition 1. □

4.3 Before entry

The incumbents comply with the collusive agreement in period $t < T^m$ (before entry) if the following ICC is satisfied:

$$\sum_{s=t}^{T^m-1} \delta^{s-t} \pi_{is}^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-t} \pi_{is}^{m3} \geq \pi_{it}^{d2} + \sum_{s=t+1}^{T^c-1} \delta^{s-t} \pi_{is}^{c2} + \sum_{s=T^c}^{+\infty} \delta^{s-t} \pi_{is}^{c3},$$

which is equivalent to:

$$\begin{aligned} -\left(\pi^{d2} - \pi^{m2}\right) F_t + \left(\pi^{d2} - \pi^{c2}\right) \delta F_{t+1} - \left(\pi^{m2} - \pi^{m3}\right) \delta^{T^m - t} F_{T^m} \\ + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c - t} F_{T^c} \geq 0. \end{aligned} \tag{6}$$

Assumption 1. *Firms are not too impatient:*

$$\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{c2}} \sup_{t \leq T^m - 1} \left\{ \frac{f_{t-1}}{f_t} \right\}.$$

Since f is quasi-concave, we surely have $f_{t-1} \leq f_t$ in periods before entry (i.e., for $t \leq T^m$). Therefore, Assumption 1 is weaker than $\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{c2}}$, which is the condition for collusion sustainability in a duopoly with constant demand.

Lemma 2. *Under Assumption 1, if the ICC (6) is satisfied in the period that immediately precedes entry, it is satisfied in all previous periods.*

Proof. See Appendix 8.2. □

According to Lemma 2, if Assumption 1 is satisfied, we only need to check the ICC (6) in period $t = T^m - 1$.

Proposition 3. *Under Assumption 1, collusion is sustainable before entry if and only if:*

$$-\left(\pi^{d2} - \pi^{m2}\right) F_{T^m - 1} + \left(\pi^{d2} - \pi^{c2} - \pi^{m2} + \pi^{m3}\right) \delta F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c - T^m + 1} F_{T^c} \geq 0. \quad (7)$$

Proof. Using Lemma 2, write condition (6) at $t = T^m - 1$. □

From the ICC (7), we conclude that the greater is the entry delay that results from a cartel breakdown before entry, $T^c - T^m$, the greater are the incentives for the incumbents to deviate from the collusive agreement before entry.

Lemma 3. *Under Assumption 1, for given T^m, collusion before entry becomes more difficult to sustain as T^c increases.*

Proof. Consider a given T^m. The left-hand side of (7) with $T^c + 1$ instead of T^c is lower than the left-hand side of (7) with T^c if and only if:

$$\left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c+1-T^m+1} F_{T^c+1} \leq \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c-T^m+1} F_{T^c}$$
$$\Leftrightarrow \ \delta F_{T^c+1} \leq F_{T^c} \ \Leftrightarrow \ 0 \leq f_{T^c},$$

which is always true. □

To better understand the importance of the entry delay as an incentive to defect, we will analyze two extreme scenarios: (*i*) cartel breakdown does not delay entry; and (*ii*) cartel breakdown deters entry (i.e., entry is not profitable along the punishment path).

4.3.1 When cartel breakdown does not delay entry

Suppose that entry is profitable in the continuation equilibrium after a deviation, i.e., that $K < \pi^{c3} \sup_t \{F_t\}$. From Lemma 1, the cartel breakdown does not delay entry (i.e., $T^c = T^m$) if and only if the thresholds of demand for firm 3 to enter under collusion or along the punishment path are attained in the same period. This is the case if there exists a period T such that:[26]

$$\pi^{m3} f_{T-1} < (1-\delta)K \leq \pi^{c3} f_T.$$

Proposition 4. *Under Assumption 1, if entry occurs at the same period under collusion and along the punishment path ($T^m = T^c$), collusion is sustainable before entry if and only if:*[27]

$$\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) F_{T^m}}. \tag{8}$$

[26] On the one hand, f is quasi-concave, which implies that it is increasing until T^c (i.e., $f_{T-1} < f_T$). On the other hand, the collusive profit is greater than the punishment profit (i.e., $\pi^{m3} > \pi^{c3}$). Thus, the interval $\left(\pi^{m3} f_{T-1}, \pi^{c3} f_T\right]$ may be empty or not.

[27] An equivalent condition would be $\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) f_{T^m-1}}{(\pi^{m3} - \pi^{c3}) F_{T^m}}$.

Proof. From Lemma 2, we know that the binding ICC for collusion sustainability before entry is in period $T^m - 1$. Substituting $T^c = T^m$ in ICC (7), we obtain:

$$\left(\pi^{m2} - \pi^{d2}\right) F_{T^m-1} + \left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) \delta F_{T^m} \geq 0$$
$$\Leftrightarrow \quad \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) F_{T^m}}.$$

□

4.3.2 When cartel breakdown deters entry

The other extreme case, in which the cartel breakdown effectively deters entry, occurs if and only if $K \in [\pi^{c3} \sup_t \{F_t\}, \pi^{m3} \sup_t \{F_t\})$.

Proposition 5. *Under Assumption 1, if the incumbents deter entry by disrupting the collusive agreement before entry, collusion is sustainable before entry if and only if:*[28]

$$\pi^{c2} < \pi^{m3} \qquad \textit{and} \qquad \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c2}\right) F_{T^m}}.$$

Proof. See Appendix 8.2. □

5 Linear demand and no production costs

The remainder of the analysis focuses on the case in which firms have no production costs and face a linear market demand function. More precisely, in each period $t \in \{0, 1, ...\}$, the inverse demand function is given by:

$$P_t = \left(1 - \frac{Q_t}{g_t}\right) h_t,$$

[28] An equivalent condition would be $\pi^{c2} < \pi^{m3} \ \wedge \ \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) f_{T^m-1}}{\left(\pi^{m3} - \pi^{c2}\right) F_{T^m}}$.

where g_t and h_t are strictly positive parameters that change over time. The evolution of market size is described by $f_t \equiv g_t h_t$.

We start by considering that firms set quantities and permanently revert to Cournot equilibrium if there is a deviation from the collusive agreement.[29]

The profits that are relevant for our analysis are the collusive profits, the deviation profits and the Cournot profits.[30] If, in period t, the $n \in \{2, 3\}$ active firms collude to maximize their joint profit and divide it in equal parts, the profit of each firm is:

$$\pi_{it}^{mn} = \frac{1}{4n} f_t.$$

If a firm deviates from the collusive agreement in period t and there are $n \in \{2, 3\}$ active firms in the industry, the profit of the deviating firm is:

$$\pi_{it}^{dn} = \left(\frac{n+1}{4n}\right)^2 f_t.$$

Finally, if the $n \in \{2, 3\}$ active firms compete *à la* Cournot, the profit of each firm is:

$$\pi_{it}^{cn} = \frac{1}{(n+1)^2} f_t.$$

Using these expressions for profits, we can obtain, from Proposition 1, the condition for collusion sustainability without entry:

$$\delta \geq \frac{9}{17} \sup_{t \geq 0} \left\{ \frac{F_t}{F_{t+1}} \right\}.$$

Similarly, from Proposition 2, collusion is sustainable after entry if and only if:

$$\delta \geq \frac{4}{7} \sup_{t \geq T^m} \left\{ \frac{F_t}{F_{t+1}} \right\}$$

[29] In Section 5, we study two variations: the case in which firms permanently revert to a zero profit equilibrium (Section 5.1); and the case in which firms set prices instead of quantities (Section 5.2).

[30] The expressions for π_{it}^{mn}, π_{it}^{dn} and π_{it}^{cn}, presented below, are obtained in Appendix 8.3.

Finally, from Proposition 3 and under Assumption 1, collusion is sustainable before entry if and only if:[31]

$$-9F_{T^m-1} - 7\delta F_{T^m} + 28\delta^{T^c-T^m+1}F_{T^c} \geq 0. \tag{9}$$

Result 1. *If the cartel breakdown permanently deters entry, collusion is not sustainable before entry.*

Proof. Observe that, in this example, $\pi^{c2} > \pi^{m3}$. From Proposition 5, this implies that collusion cannot be sustained before entry. □

Result 2. *Under Assumption 1, if $\frac{F_t}{F_{t+1}}$ is non-decreasing and the cartel breakdown does not delay entry ($T^c - T^m = 0$), collusion is harder to sustain after entry than before entry.*

Proof. If $T^c = T^m$, the ICC (9) becomes:

$$-9F_{T^m-1} + 21\delta F_{T^m} \geq 0 \;\Leftrightarrow\; \delta \geq \frac{3F_{T^m-1}}{7F_{T^m}}.$$

If $\frac{F_{T^m-1}}{F_{T^m}} \leq \frac{F_{T^m}}{F_{T^m+1}}$, the above condition is weaker than $\delta \geq \frac{3}{7}\sup_{t\geq T^m}\left\{\frac{F_t}{F_{t+1}}\right\}$. Therefore, it is implied by collusion sustainability after entry. □

5.1 Constant rate of market growth

It is instructive to focus on the case in which demand grows at a constant rate, as assumed in the models of Capuano (2002) and Vasconcelos (2008):[32]

$$f_t = \beta^t, \quad \text{with} \;\; 1 < \beta < \delta^{-1}. \tag{10}$$

[31] Recall that Assumption 1 is weaker than the condition for sustainability of collusion in a duopoly without market growth.

[32] If $\beta \leq 1$, demand would not grow over time. This case is not interesting for this analysis, because entry would either occur at $t = 0$ or never.

In this case, we have:

$$F_t = \delta^{-t} \sum_{s=t}^{+\infty} (\beta\delta)^s = \frac{\beta^t}{1-\beta\delta},$$

which is strictly increasing in t. Since $\lim_{t\to\infty} F_t = +\infty$, firm 3 always enters the market, regardless of the market regime (collusion or competition). The optimal entry periods under competition and under collusion are, respectively:

$$T^c = int\left\{\frac{\ln\left[16(1-\delta)K\right]}{\ln\beta}\right\} + 1 \quad \text{and} \quad T^m = int\left\{\frac{\ln\left[12(1-\delta)K\right]}{\ln\beta}\right\} + 1,$$

where $int\{x\}$ denotes the integer part of x. Hence, the entry delay that results from breaking the cartel can be:[33]

$$T^c - T^m = int\left[\frac{\ln(4/3)}{\ln\beta}\right] \quad \text{or} \quad T^c - T^m = int\left[\frac{\ln(4/3)}{\ln\beta}\right] + 1. \tag{11}$$

Observe that the ratio $\frac{F_t}{F_{t+1}}$ is constant and equal to β^{-1}. Using Proposition 1, we find that, in the absence of entry, the collusive agreement is sustainable if and only if:

$$\beta\delta \geq \frac{9}{17}.$$

From Proposition 2, collusion is sustainable after entry if and only if:

$$\beta\delta \geq \frac{4}{7},$$

From Proposition 3, under the hypothesis that $\beta\delta \geq \frac{9}{17}$, collusion is sustainable before entry if and only if:

$$-9 - 7(\beta\delta) + 28(\beta\delta)^{T^c - T^m + 1} \geq 0. \tag{12}$$

Figure 2 illustrates the fact that: (i) collusion is harder to sustain after entry than in the

[33] To obtain the two possible values for the entry delay, note that $\frac{\ln[16(1-\delta)K]}{\ln\beta} - \frac{\ln[12(1-\delta)K]}{\ln\beta} = \frac{\ln(4/3)}{\ln\beta}$. This ratio corresponds to the number of periods that are necessary for the market to grow from the threshold for entry under collusion, $f_t = 12(1-\delta)K$, to the threshold for entry under competition, $f_t = 16(1-\delta)K$.

absence of entry; (ii) collusion may be harder to sustain before entry or after entry.

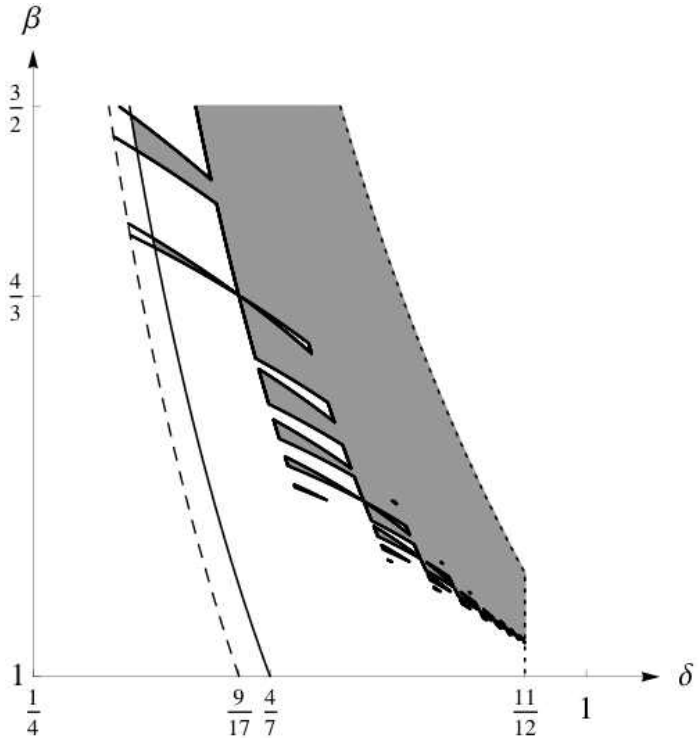


Figure 2: Collusion sustainability with a constant rate of market growth (K=1).

The thin solid line represents the ICC for collusion sustainability after entry ($\beta\delta = \frac{4}{7}$), while the dashed line on the left represents the ICC for collusion sustainability in the absence of entry ($\beta\delta = \frac{9}{17}$). To the right of this curve, if the ICC at $T^m - 1$ holds, then the ICC at any $t \in \{0, ..., T^m - 2\}$ also holds. The thicker and erratic solid line represents the ICC for collusion sustainability at $T^m - 1$, given by (12). The dotted line on the right represents the limit of the domain of analysis: we exclude the region in which $\beta\delta \geq 1$, for the discounted sum of profits to be finite; and the region in which entry occurs at $t = 0$, which corresponds to $\delta \geq \frac{11}{12}$ when $K = 1$. Finally, the painted area represents the combinations of parameters for which collusion is sustainable before and after entry.

Result 3. *Collusion is harder to sustain before entry than after entry if and only if the cartel breakdown delays entry ($T^c - T^m \geq 1$). A sufficient condition for that is $\beta \leq \frac{4}{3}$.*

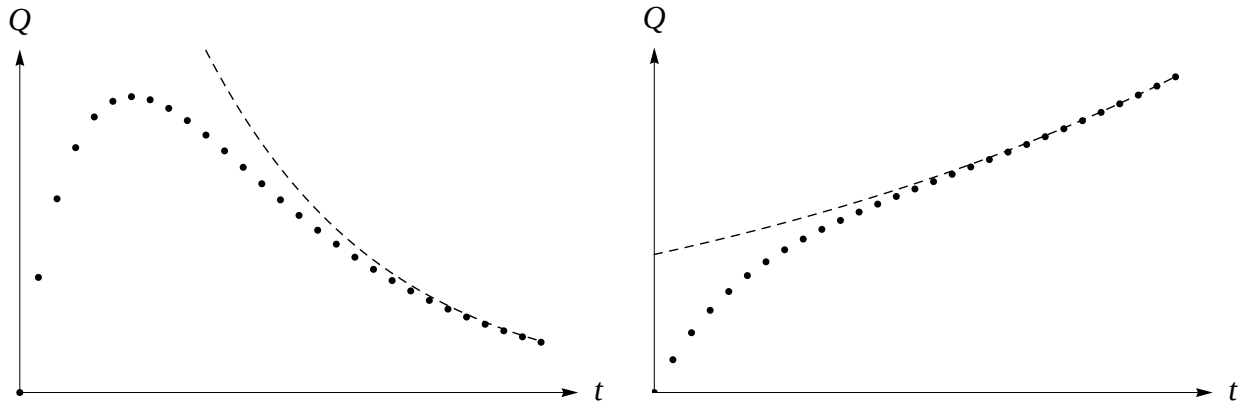
Proof. See Appendix 8.4. □

If the cartel breakdown effectively delays entry, collusion is harder to sustain before entry than after entry. As explained by Vasconcelos (2008), before entry, there is an additional incentive to defect, because the breakdown of the cartel delays entry, and competition between two firms is more profitable than collusion with three firms.

5.2 Decreasing rate of market growth

Consider, now, that demand grows at a decreasing and convergent rate, according to:[34]

$$f_t = (1 - \alpha^{-t})\beta^t, \quad \alpha > 1, \quad 0 < \beta < \delta^{-1}. \tag{13}$$



(a) $\beta < 1$. (b) $\beta > 1$.

Figure 3: Market growth over time.[35]

In this case:

$$F_t = \sum_{s=t}^{+\infty}(1 - \alpha^{-s})\beta^s\delta^{s-t} = \frac{\beta^t}{1-\beta\delta} - \frac{\beta^t\alpha^{1-t}}{\alpha-\beta\delta} = \beta^t\frac{\alpha - \beta\delta - \alpha^{1-t}(1-\beta\delta)}{(1-\beta\delta)(\alpha-\beta\delta)}; \tag{14}$$

and:

$$\frac{F_t}{F_{t+1}} = \frac{\alpha - \beta\delta - \alpha^{1-t}(1-\beta\delta)}{\beta\left[\alpha - \beta\delta - \alpha^{-t}(1-\beta\delta)\right]}.$$

[34] A constant growth rate ($f_t = \beta^t$) is the pointwise limit of (13) when $\alpha \to +\infty$, except for $t = 0$.

[35] The dashed line represents the long-term tendency, β^t, while the dots represent $f_t = (1 - \alpha^{-t})\beta^t$.

Since the ratio $\frac{F_t}{F_{t+1}}$ is strictly increasing over time, we obtain:

$$\sup_t \left\{ \frac{F_t}{F_{t+1}} \right\} = \lim_{t \to +\infty} \left\{ \frac{F_t}{F_{t+1}} \right\} = \beta^{-1}.$$

This allows us to conclude that, in the absence of entry, the collusive agreement is sustainable if and only if (Proposition 1):

$$\beta\delta \geq \frac{9}{17}.$$

After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{4}{7},$$

The critical discount factors for collusion sustainability without entry and after entry are independent of α, because, since $\frac{F_t}{F_{t+1}}$ is increasing over time, what is relevant for collusion sustainability is the value of this ratio in the long-run, and, when $t \to +\infty$, market demand is approximately given by $f_t = \beta^t$. We obtain, therefore, exactly the same conditions as in the case of constant growth rate.

Analysing sustainability of collusion before entry is more complicated. Under the assumption that collusion without entry is sustainable ($\beta\delta \geq \frac{9}{17}$), Assumption 1 is satisfied. Therefore, we only need to consider the ICC at $T^m - 1$:

$$\begin{aligned} -9\left[\alpha - \beta\delta - \alpha^{2-T^m}(1-\beta\delta)\right] - 7\beta\delta\left[\alpha - \beta\delta - \alpha^{1-T^m}(1-\beta\delta)\right] \\ +28\left(\beta\delta\right)^{T^c-T^m+1}\left[\alpha - \beta\delta - \alpha^{1-T^c}(1-\beta\delta)\right] \geq 0. \end{aligned} \tag{15}$$

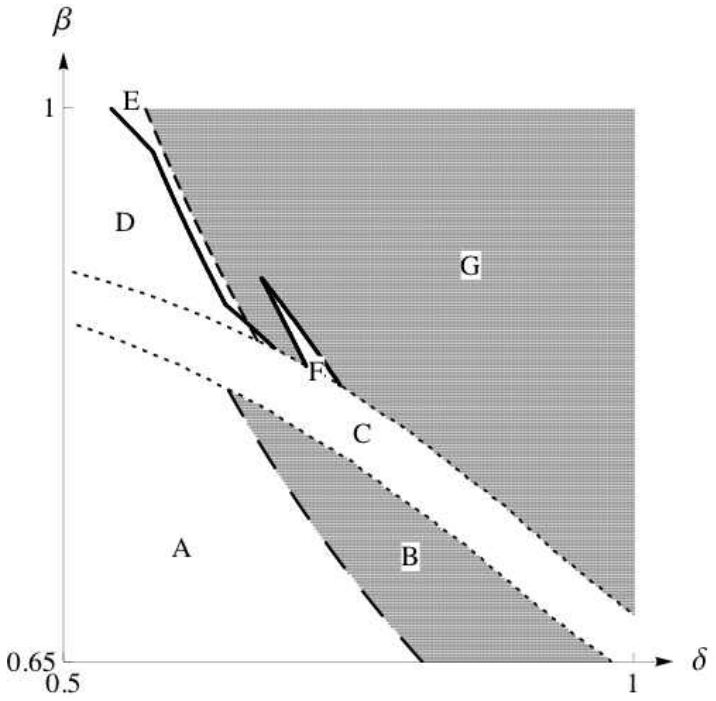
Result 4. *Collusion is harder to sustain before entry than after entry when: $T^c - T^m \geq 3$; $T^c - T^m = 2$ and $T^m \geq 2$; or $T^c - T^m = 1$ and $T^m \geq 4$.*

Proof. See Appendix 8.4. □

With this form of market evolution, the comparison between the sustainability of collusion

before and after entry originates a similar result to that obtained for a constant market growth rate. Result 4 is only more mitigate than Result 3 because it concerns a scenario in which the market is relatively smaller in early periods, implying that the short-term gain from a deviation is lower. This is why collusion may be easier to sustain before entry than after entry even if the entry delay is not null ($T^c - T^m \geq 1$). However, this can only be the case if entry occurs at a very early stage ($T^m \leq 3$).

Figure 4 illustrates the various conditions involved in the analysis of collusion sustainability: the solid line is the ICC before entry (its shape is explained in Appendix 8.5); the dashed line between regions E and G is the ICC after entry; the dashed line between regions A and B is the ICC after entry; and the dotted lines are the conditions for entry to be profitable (under collusion and under competition). The painted area is the parameter region in which collusion is sustainable.



[A] There is no entry. Collusion is not sustainable.

[B] There is no entry. Collusion is sustainable.

[C] Cartel breakdown deters entry, thus, collusion is not sustainable.

[D] Collusion is neither sustainable before nor after entry.

[E] Collusion would be sustainable before entry, but it is not after entry.

[F] Collusion would be sustainable after entry, but it is not before entry.

[G] Collusion is sustainable before and after entry.

Figure 4: Sustainability of collusion with market growth and decline ($\alpha = 1.01$, $K = 0.004$).

Notice that there are two painted regions in Figure 4, labeled B and G. The white region between the two, labeled C, corresponds to the combinations of parameters for which entry only occurs under collusion, i.e., for which the cartel breakdown before entry permanently

deters entry. Therefore, according to Result 1, the collusive agreement is not sustainable (before entry). In region B, there is no entry and collusion (between the two incumbents) is sustainable. In region G, there is entry and collusion is sustainable (before and after entry). This description also applies to Figure 5.

We conclude that collusion may be harder to sustain before entry or after entry. As illustrated in Figure 5, the entry cost may be relevant for this comparison. A low entry cost implies that entry occurs early, when market growth is fast. In this case, the entry delay caused by the cartel breakdown is small, implying that the ICC after entry is binding. A high entry cost implies that entry occurs late, when market growth is slower and, therefore, the entry delay caused by the cartel breakdown is greater. In that case, it is the ICC before entry that binds.

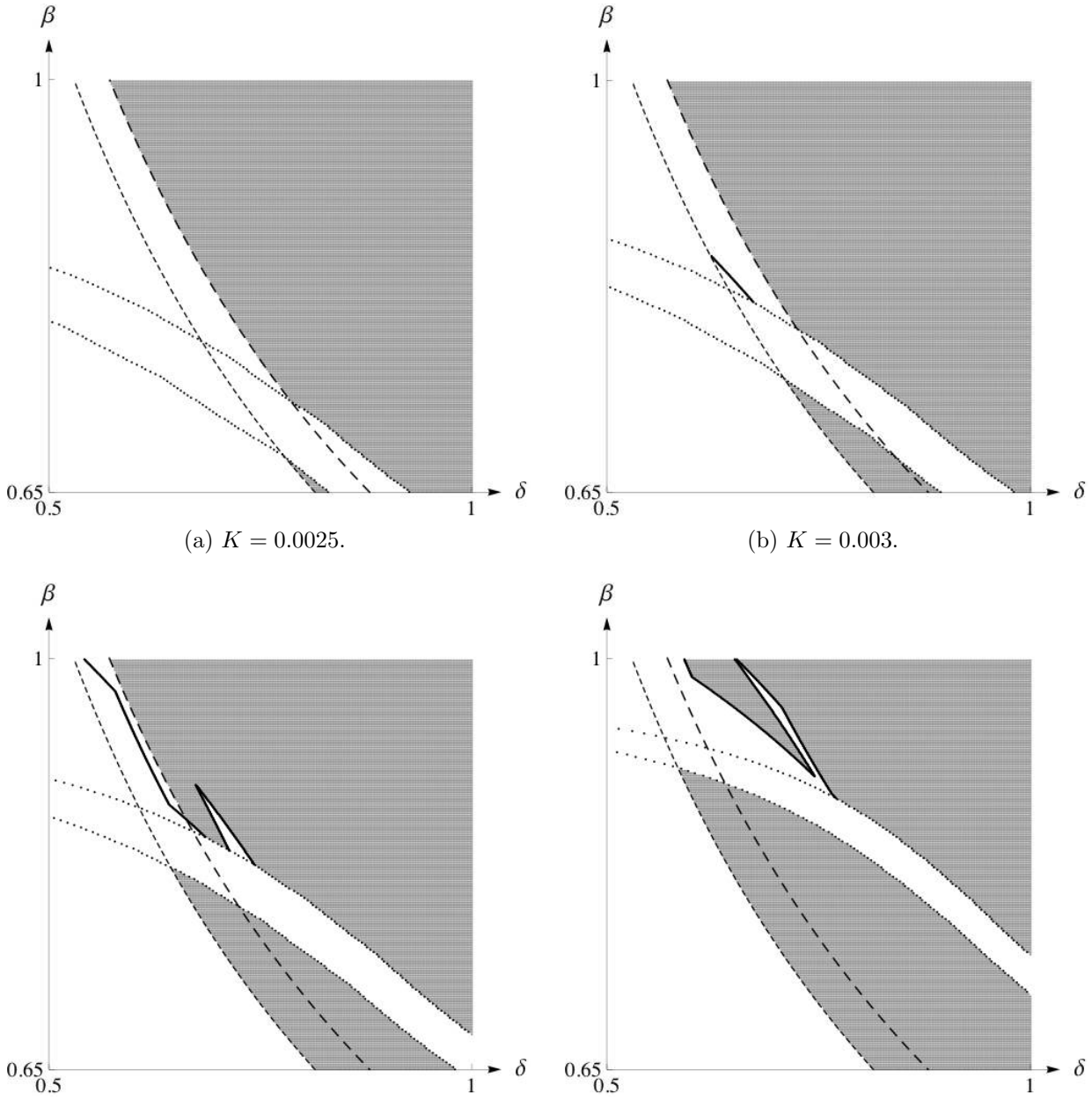
6 Extensions

Let us now study two variations of the benchmark model: a scenario in which firms use an optimal penal code (Section 6.1); and a scenario in which firms set prices instead of quantities (Section 6.2).

If firms set quantities, there exists an important distinction between grim trigger strategies and optimal penal codes. With grim trigger strategies, firms permanently revert to the single-period Cournot equilibrium. With an optimal penal code, firms permanently revert to a zero profit equilibrium.

In the case of a price-setting game, as firms sell homogeneous goods, grim trigger strategies actually correspond to the maximal punishment that firms can inflict, since firms permanently revert to a zero profit equilibrium after a deviation. With a harsher punishment, firms would exit the market or deviate from the punishment (by becoming inactive) and guarantee a zero profit. Further, there is no deviation from the punishment, since firms set prices that are best-responses to the prices set by others.

The distinction between quantity competition and price competition also concerns the



(a) $K = 0.0025$.

(b) $K = 0.003$.

(c) $K = 0.004$.

(d) $K = 0.007$.

Figure 5: Collusion sustainability ($\alpha = 1.01$).

deviation profit. In the case of quantity competition, the deviating firm chooses a profit-maximizing output taking as given that the rivals produce the collusive output. In the case of price competition, the deviating firm slightly undercuts the collusive price (set by the others). Therefore, in the price-setting scenario, the deviation profit coincides with the monopoly profit.

6.1 Optimal penal code

Studying the case in which punishment strategies consist of a permanent reversion to a zero profit equilibrium is much simpler than the benchmark case, because deviations from the collusive agreement permanently deter entry.[36]

Suppose that demand grows at a decreasing and convergent rate:

$$f_t = (1 - \alpha^{-t})\beta^t, \quad \alpha > 1, \quad 0 < \beta < \delta^{-1}.$$

Without entry, the collusive agreement is sustainable if and only if (Proposition 1):

$$\beta\delta \geq \frac{1}{9}.$$

After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{1}{4}.$$

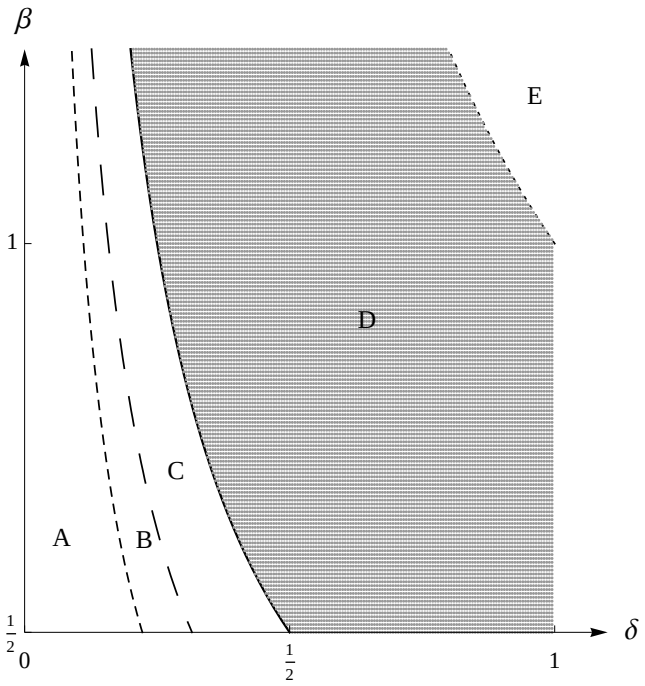
Result 5. *Collusion is easier to sustain before entry than after entry.*

Proof. See Appendix 8.4. □

Assuming that collusion is sustainable after entry, we find that collusion is sustainable before entry if and only if $\beta\delta \geq \frac{3}{19}$ (see the proof of Result 5). However, since collusion sustainability after entry is a pre-requisite for collusion sustainability before entry, we conclude that collusion is only sustainable before entry if $\beta\delta \geq \frac{1}{4}$.[37] In other words, the condition for collusion sustainability after entry is the one that binds.

[36] Since profits along the punishment path are null, the potential entrant is not willing to support any strictly positive entry cost.

[37] If firms expected the cartel to breakdown at some t, they would have no incentives to comply with the collusive agreement at $t - 1$.



[A] Collusion is not sustainable.

[B] Collusion is sustainable without entry, but not with entry.

[C] Collusion would be sustainable before entry, but it is not sustainable after entry.

[D] Collusion is sustainable.

[E] Excluded area: $\beta\delta > 1$.

Figure 6: Sustainability of collusion with reversion to zero-profit equilibrium after deviation.

Unsurprisingly, collusion is more easily sustained if firms adopt these much more severe punishment strategies than if they revert to Cournot equilibrium after a deviation.

6.2 Price-setting firms

In a scenario in which firms set prices instead of quantities, the analysis is again simplified by the fact that continuation profits after a deviation are null. The only difference with respect to the case studied in Section 6.1 is the fact that the deviation profits coincide with the monopoly profits. Formally:

$$\pi_{it}^{mn} = \frac{1}{4n} f_t, \quad \pi_{it}^{dn} = \frac{1}{4} f_t \quad \text{and} \quad \pi_{it}^{cn} = 0.$$

As before, suppose that demand grows at a decreasing and convergent rate:

$$f_t = (1 - \alpha^{-t})\beta^t, \quad \alpha > 1, \quad 0 < \beta < \delta^{-1}.$$

Without entry, the collusive agreement is sustainable if and only if (Proposition 1):

$$\beta\delta \geq \frac{1}{2}.$$

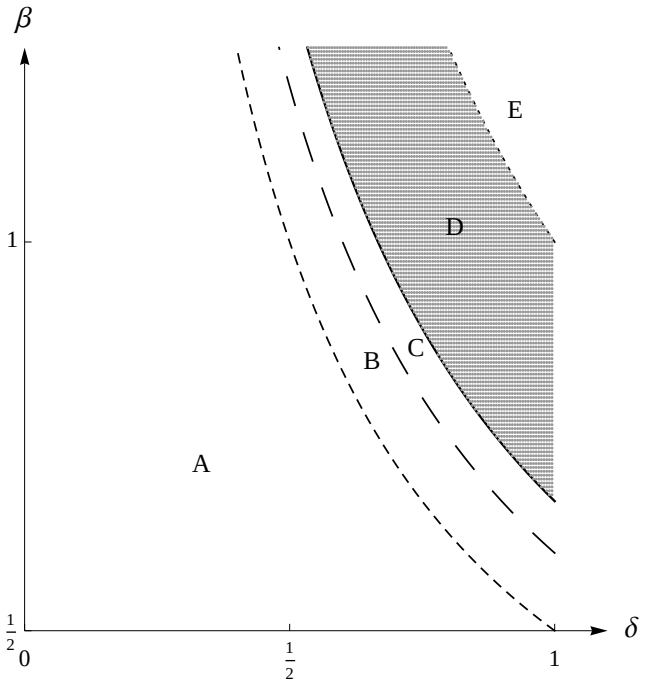
After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{2}{3}.$$

Result 6. *Collusion is easier to sustain before entry than after entry.*

Proof. See Appendix 8.4. □

The ICC for collusion sustainability at $T^m - 1$ is satisfied if and only if $\beta\delta \geq \frac{3}{5}$ (see the proof of Result 6). Under Assumption 1, which is implied by $\beta\delta \geq \frac{1}{2}$, the ICC is also satisfied at any $t \in \{0, ..., T^m - 2\}$. However, since collusion sustainability after entry is a pre-requisite for collusion sustainability before entry, collusion is only sustainable before entry if $\beta\delta \geq \frac{2}{3}$.



[A] Collusion is not sustainable.

[B] Collusion is sustainable without entry, but not with entry.

[C] Collusion would be sustainable before entry, but it is not sustainable after entry.

[D] Collusion is sustainable.

[E] Excluded area: $\beta\delta > 1$.

Figure 7: Collusion sustainability in a price-setting game.

The two variations of the benchmark model only differ in the deviation profit, which is higher if firms are price-setters than if they are quantity-setters. As a result, collusion is easier to sustain if firms are quantity-setters and adopt the severest possible punishment than if firms are price-setters.

Now, comparing the sustainability of collusion between price-setting firms and quantity-setting firms (in a context wherein there is reversion to competition after a deviation) is not obvious: on the one hand, the punishment is more severe in the price-setting scenario (which favors collusion sustainability); on the other hand, the deviation profit is higher in the price-setting scenario (which hinders collusion sustainability). We conclude that it is harder to sustain collusion when firms set quantities if and only if $T^c - T^m > 0$, i.e., if and only if discontinuance of collusion strictly delays entry. This comparison is made in Figure 8 for the case in which demand grows at a constant rate ($f_t = \beta^t$) and $K = 1$.

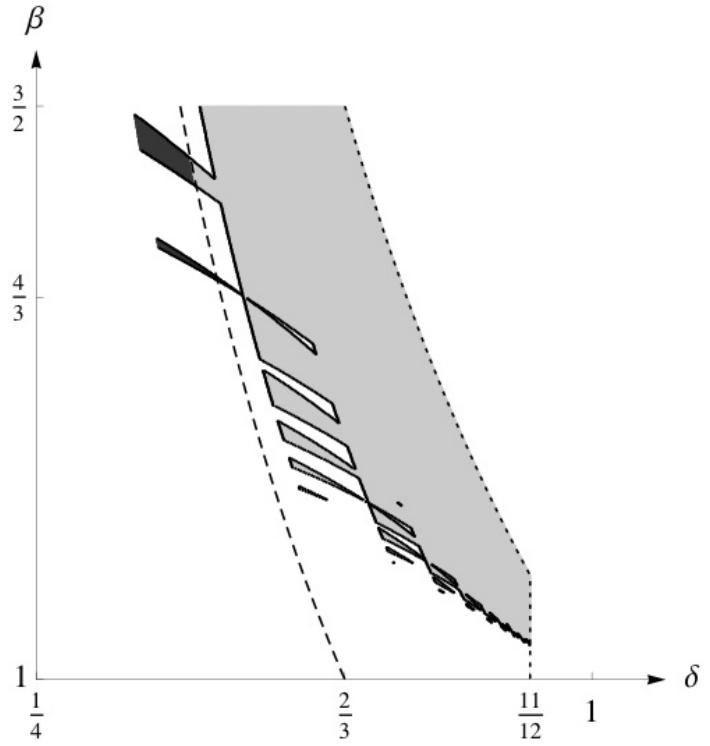


Figure 8: Collusion may be harder to sustain when firms set prices (dark grey area) or quantities (light grey area), given a constant demand growth and $K = 1$.

Result 7. *Let $f_t = \beta^t$ and consider reversion to competition after a deviation. Collusion is harder to sustain when firms set prices rather than quantities if and only if $T^c - T^m = 0$. A necessary condition for this is $\beta > \frac{4}{3}$.*

Proof. See Appendix 8.4. □

7 Conclusions

We have built a general framework to study the sustainability of collusion in markets where the evolution of demand may induce the entry of a new firm. In the baseline model, we have only assumed that market growth does not affect the ratios between profits in the three market regimes (collusion, deviation and punishment), and that the punishment lasts forever. We have concluded that the entry delay resulting from the cartel breakdown before entry is a key determinant of collusion sustainability before entry. For example, if the cartel breakdown deters entry and the collusive profit after entry is lower than the punishment profit before entry, collusion is impossible to sustain.

We have generalized the models and conclusions of Capuano (2002) and Vasconcelos (2008), by removing the assumptions of linear demand and constant market growth rate. The shape of the demand function only impacts collusion sustainability through its influence on the relative magnitudes of the collusive, deviation and punishment profits.

The impact of considering a more general form of market evolution is that the critical period for collusion sustainability after entry is the one in which the value of the market, i.e., the discounted sum of present and future profits grows the least (or declines the most). The growth of the value of the market in this period is all that matters for collusion sustainability after entry. Collusion sustainability before entry is impacted not only through this mechanism, but also through the timing of entry. The slower is the market growth when entry occurs, the greater is the entry delay that results from a cartel breakdown before entry, and, therefore, the greater is the incentive for deviating before entry.

In a scenario in which the market starts by expanding and then declines, the results are qualitatively different from those obtained by Capuano (2002) and Vasconcelos (2008). The reason is the following. After reversion to Cournot competition, entry is surely profitable if the rate of market growth is constant, but it may not be profitable if the market initially

grows and then declines. Consequently, in our scenario, the incumbents may be able to permanently deter entry by disrupting the collusive agreement before entry. If this is the case, and if collusion after entry is less profitable than Cournot competition before entry, collusion is never sustainable.

We have also considered two variations of the benchmark scenario: one in which firms adopt extremely severe punishment strategies; and another in which firms set prices instead of quantities. In both scenarios, profits along the punishment path are null. This simultaneously implies that the cartel breakdown permanently deters entry and that this deterrence is irrelevant. As a result, collusion is more difficult to sustain after entry then before entry. We have also compared the sustainability of collusion in these variants with respect to the benchmark scenario. As expected, the collusive agreement becomes easier to sustain if firms adopt optimal penal codes. If firms set prices instead of quantities, collusion becomes harder to sustain after entry, but, globally, it may become easier to sustain. In the case of constant market growth it becomes easier to sustain unless the entry delay caused by the cartel breakdown is null in the benchmark scenario.

8 Appendix

8.1 Proof that profits are proportional to market size parameters

As a function of its output, the profit of firm $i \in \{1, ..., n\}$ in period $t \in \{0, 1, ...\}$ is:

$$\pi_{it}(q_{it}) = P_t(Q_t)q_{it} - C_t(q_{it}),$$

where, in period t, $P_t(\cdot)$ is the inverse demand function, $C_t(\cdot)$ is the cost function, q_{it} is the quantity produced by firm i and $Q_t = \sum_{i=1}^{n} q_{it}$ is the total output of the industry.

Suppose that the market demand function in period t can be written as:

$$Q_t(P_t) = Q\left(\frac{P_t}{h_t}\right) g_t \;\Leftrightarrow\; P_t(Q_t) = Q^{-1}\left(\frac{Q_t}{g_t}\right) h_t,$$

where g_t and h_t are strictly positive parameters, $P(\cdot)$ is the inverse of $Q(\cdot)$, $Q(0) = \bar{Q} > 0$, $P(0) = \bar{P} > 0$ and Q is continuously differentiable with $Q'(x) < 0$, $\forall x \in \left(0, \bar{P}\right)$.

Suppose also that the cost function in period t can be written as:

$$C_t(q_t) = C\left(\frac{q_t}{g_t}\right) g_t h_t,$$

where $C(0) \geq 0$ and $C(\cdot)$ is continuously differentiable with $C'(x) \geq 0$, $\forall x > 0$.

Under these assumptions, the profit function of firm $i \in \{1, ..., n\}$ can be written as:

$$\pi_{it}(q_{it}) = \left[P\left(\frac{Q_t}{g_t}\right) q_{it} - C\left(\frac{q_{it}}{g_t}\right) g_t\right] h_t. \tag{16}$$

8.1.1 Cournot profits

Under Cournot competition, each firm produces the quantity that maximizes its individual profit, taking as given the quantities produced by the rivals. Hence, the equilibrium quantity

of firm i satisfies the following first-order condition:[38]

$$\frac{d\pi_{it}}{dq_{it}} = 0 \;\Leftrightarrow\; \left[P'\left(\frac{Q_t}{g_t}\right)\frac{q_{it}}{g_t} + P\left(\frac{Q_t}{g_t}\right) - C'\left(\frac{q_{it}}{g_t}\right)\right] h_t = 0. \tag{17}$$

As firms are symmetric, they produce the same quantity in equilibrium. Replacing $Q_t = nq_{it}$ in (17), we obtain:

$$P'\left(\frac{n\,q_{it}}{g_t}\right)\frac{q_{it}}{g_t} + P\left(\frac{n\,q_{it}}{g_t}\right) - C'\left(\frac{q_{it}}{g_t}\right) = 0, \tag{18}$$

which can also be written as:

$$P'(nx)\,x + P(nx) - C'(x) = 0.$$

The solution of this equation, x^c, is independent of g_t and h_t.

The Cournot profit of firm i, from (16), is:

$$\pi_{it}^{cn} = \left[P\left(nx^c\right)x^c - C\left(x^c\right)\right] g_t h_t,$$

which is proportional to $g_t h_t$.

8.1.2 Collusive profits

Suppose that firms combine to produce quantities that maximize their joint profit. Since the marginal cost is non-decreasing, firms divide the output in equal parts. Thus, the joint profit of the industry is:

$$\Pi_t(Q_t) = P_t(Q_t)Q_t - nC_t\left(\frac{Q_t}{n}\right) = \left[P\left(\frac{Q_t}{g_t}\right)Q_t - nC\left(\frac{Q_t}{n\,g_t}\right)g_t\right] h_t. \tag{19}$$

[38] The second-order condition that must be satisfied for this solution to be a maximum of π_{it} is: $P''\left(\frac{Q_t}{g_t}\right)\frac{q_{it}}{g_t} + 2P'\left(\frac{Q_t}{g_t}\right) - C''\left(\frac{q_{it}}{g_t}\right) < 0$. If $P(\cdot)$ is concave and $C(\cdot)$ is convex, it is surely satisfied.

The first-order condition for joint profit maximization is:[39]

$$\Pi_t'(Q_t) = 0 \;\Leftrightarrow\; \left[P'\left(\frac{Q_t}{g_t}\right)\frac{Q_t}{g_t} + P\left(\frac{Q_t}{g_t}\right) - C'\left(\frac{Q_t}{n\,g_t}\right)\right] h_t = 0. \tag{20}$$

This condition can be written as:

$$P'\left(n\,x\right) n\,x + P\left(n\,x\right) - C'\left(x\right) = 0.$$

The solution, x^m, does not depend on g_t or h_t. Replacing it in (16), we obtain the collusive profit of firm i:

$$\pi_{it}^{mn} = \left[P\left(n\,x^m\right) x^m - C\left(x^m\right)\right] g_t h_t,$$

which is proportional to $g_t h_t$.

8.1.3 Deviation profits

Finally, suppose that firms have combined to produce the quantities that maximize the industry profit, but firm i decides to deviate from the collusive agreement. In this case, firm i produces the quantity that maximizes:

$$\pi_{it}(q_{it}) = P\left[\frac{q_{it}}{g_t} + (n-1)\,x^m\right] q_{it} h_t - C\left(\frac{q_{it}}{g_t}\right) g_t h_t. \tag{21}$$

The corresponding first-order condition is:[40]

$$P'\left[\frac{q_{it}}{g_t} + (n-1)\,x^m\right]\frac{q_{it}}{g_t} h_t + P\left[\frac{q_{it}}{g_t} + (n-1)\,x^m\right] h_t - C'\left(\frac{q_{it}}{g_t}\right) h_t = 0.$$

[39] The second-order condition is: $P''\left(\frac{Q_t}{g_t}\right)\frac{Q_t}{g_t} + 2P'\left(\frac{Q_t}{g_t}\right) - C''\left(\frac{Q_t}{n\,g_t}\right) < 0$. Again, this condition is satisfied if $P(\cdot)$ is concave and $C(\cdot)$ is convex.

[40] The second-order condition, $P''\left[\frac{q_{it}}{g_t} + (n-1)\,x^m\right]\frac{q_{it}}{g_t} + 2P'\left[\frac{q_{it}}{g_t} + (n-1)\,x^m\right] - C''\left(\frac{q_{it}}{g_t}\right) < 0$, is satisfied if $P(\cdot)$ is concave and $C(\cdot)$ is convex.

Equivalently:

$$P'\left[x^d + (n-1)\,x^m\right]x^d + P\left[x^d + (n-1)\,x^m\right] - C'\left(x^d\right) = 0.$$

The equation above determines x^d independently of g_t and h_t. Replacing in (21), we obtain the deviation profit of firm i:

$$\pi_{it}^{dn} = \left\{P\left[x^d + (n-1)\,x^m\right]x^d - C\left(x^d\right)\right\} g_t h_t,$$

which is proportional to $g_t h_t$.

8.2 Proofs of Lemmas and Propositions

Proof of Lemma 1

Under the market regime $j \in \{m, c\}$, the discounted value of the profits of firm 3 if it enters in period T^j is given by:

$$V_3^j(T^j) = \pi_3^{j3} \sum_{s=T^j}^{+\infty} \delta^s f(s) - K\delta^{T^j} = \delta^{T^j}\left(\pi_3^{j3} F_{T^j} - K\right).$$

It is profitable for firm 3 to enter the market if and only if:

$$K < \pi_3^{j3} F_{T^j}.$$

If the above condition is satisfied, entry occurs in the first period, T^j, in which the following condition holds:

$$\begin{aligned} V^j(T^j) \geq V^j(T^j+1) \;&\Leftrightarrow\; \pi_3^{j3}\left(f_{T^j} + \delta F_{T^j+1}\right) - K \geq \delta\left(\pi_3^{j3} F_{T^j+1} - K\right) \;\Leftrightarrow \\ &\Leftrightarrow\; f_{T^j} \geq \frac{(1-\delta)\,K}{\pi_3^{j3}}. \end{aligned}$$

□

Proof of Lemma 2

We start by proving that if $\delta > \frac{\pi^{d2}-\pi^{m2}}{\pi^{d2}-\pi^{c2}}$ and the ICC (6) is satisfied at $t+1$, it is also satisfied at t, for $t \in \{1, ..., T^m - 2\}$.

If condition (6) is satisfied at $t+1$, the following variable is positive:

$$\Delta \equiv -\left(\pi^{d2} - \pi^{m2}\right) F_{t+1} + \left(\pi^{d2} - \pi^{c2}\right) \delta F_{t+2} - \left(\pi^{m2} - \pi^{m3}\right) \delta^{T^m - t - 1} F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c - t - 1} F_{T^c} \geq 0.$$

Condition (6) at t can be written as:

$$\begin{aligned} -\left(\pi^{d2} - \pi^{m2}\right)\left(f_t + \delta F_{t+1}\right) + \left(\pi^{d2} - \pi^{c2}\right)\delta\left(f_{t+1} + \delta F_{t+2}\right) \\ - \left(\pi^{m2} - \pi^{m3}\right)\delta^{T^m - t} F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right)\delta^{T^c - t} F_{T^c} \geq 0 \\ \Leftrightarrow \ \delta\Delta - \left(\pi^{d2} - \pi^{m2}\right) f_t + \left(\pi^{d2} - \pi^{c2}\right)\delta f_{t+1} \geq 0. \end{aligned}$$

Since $\Delta \geq 0$, for (6) to be satisfied at t, it is sufficient that:

$$\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) f_t}{\left(\pi^{d2} - \pi^{c2}\right) f_{t+1}},$$

which is true under Assumption 1. □

Proof of Proposition 5

Assume that the entrant prefers not to enter the market if incumbents deviate from the collusive agreement before entry. In this case, from (7), the ICC for collusion sustainability at $T^m - 1$ becomes:

$$\begin{aligned} -\left(\pi^{d2} - \pi^{m2}\right)\left(f_{T_m - 1} + \delta F_{T^m}\right) + \left(\pi^{d2} - \pi^{c2} - \pi^{m2} + \pi^{m3}\right)\delta F_{T^m} \geq 0 \\ \Leftrightarrow \ -\left(\pi^{c2} - \pi^{m3}\right)\delta F_{T^m} \geq \left(\pi^{d2} - \pi^{m2}\right) f_{T^m - 1}, \end{aligned}$$

which cannot be satisfied if $\pi^{c2} \geq \pi^{m3}$.

For $\pi^{c2} < \pi^{m3}$, the ICC becomes:

$$\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) f_{T^m - 1}}{\left(\pi^{m3} - \pi^{c2}\right) F_{T^m}}.$$

□

8.3 Profits with linear demand and zero costs

8.3.1 Perfect collusion

Suppose that, in each period t, the $n \in \{2, 3\}$ active firms maximize their joint profit:

$$\Pi_t^{mn}(Q_t) = \left(1 - \frac{Q_t}{f_t}\right) Q_t.$$

From the first-order condition for profit-maximization, assuming that firms divide the industry profit in equal parts, we obtain the collusive output and profit of each firm:

$$q_{it}^{mn} = \frac{f_t}{2n} \quad \text{and} \quad \pi_{it}^{mn} = \frac{f_t}{4n}.$$

8.3.2 Deviation profits

Suppose that firm i deviates from the collusive agreement in period t and there are $n \in \{2, 3\}$ firms in the market. This firm produces the quantity that maximizes its individual profit, assuming that the rival firms produce the collusive quantities:

$$\Pi_{it}^{dn}(q_{it}) = \left(1 - \frac{n-1}{2n} - \frac{q_{it}}{f_t}\right) q_{it}.$$

Solving the corresponding first-order condition, we obtain:

$$q_{it}^{dn} = \frac{n+1}{4n} f_t \quad \text{and} \quad \pi_{it}^{dn} = \left(\frac{n+1}{4n}\right)^2 f_t.$$

8.3.3 Punishment profits

When the $n \in \{2,3\}$ active firms compete *à la* Cournot, each firm i produces the quantity that maximizes its individual profit:

$$\Pi_{it}^{cn}(q_{it}) = \left(1 - \frac{Q_{-it}}{f_t} - \frac{q_{it}}{f_t}\right) q_{it},$$

where Q_{-it} denotes the quantity produced by the rivals of firm i.

From the first-order condition for profit-maximization, we obtain the output and profit of each firm:

$$q_{it}^{cn} = \frac{f_t}{n+1} \quad \text{and} \quad \pi_{it}^{cn} = \frac{f_t}{(n+1)^2}.$$

8.4 Proofs of the results with linear demand and zero costs

Proof of Result 3

Replacing $\beta\delta = \frac{4}{7}$, which is the critical value for sustainability of collusion after entry, in condition (12), we obtain:

$$T^c - T^m \geq \frac{\ln(13/28)}{\ln(4/7)} - 1 \;\Leftrightarrow\; T^c - T^m \geq 1.$$

From the expression for the lower entry delay in (11), it is clear that $T^c - T^m \geq 1$ is implied by:

$$\frac{\ln(4/3)}{\ln\beta} \geq 1 \;\Leftrightarrow\; \beta \leq \frac{4}{3}.$$

□

The following result is instrumental for the proof of Result 4.

Result 8. *Let $\beta\delta = \frac{4}{7}$. If collusion before entry is not sustainable for given values of T^m and T^c, with $T^c > T^m$, it is also not sustainable if T^m and T^c increase by the same amount.*

Proof. The left-hand side of (15) with T^m+1 and T^c+1 instead of T^m and T^c is lower than the left-hand side of (15) with T^m and T^c if and only if:

$$\begin{aligned}-9\left(-\alpha^{-T^m}+\alpha^{1-T^m}\right)(1-\beta\delta)(\alpha-\beta\delta)-16\,\beta\delta\left(-\alpha^{-T^m}+\alpha^{1-T^m}\right)(1-\beta\delta)\\ +28\,(\beta\delta)^{T^c-T^m+1}\left(-\alpha^{-T^c}+\alpha^{1-T^c}\right)(1-\beta\delta)\leq 0\\ \Leftrightarrow\ -9-7\left(\frac{\beta\delta}{\alpha}\right)+28\left(\frac{\beta\delta}{\alpha}\right)^{T^c-T^m+1}\leq 0.\end{aligned}$$

The worst case for the above inequality to be satisfied is when $T^c-T^m=1$. In that case, it becomes:

$$-9-7\left(\frac{\beta\delta}{\alpha}\right)+28\left(\frac{\beta\delta}{\alpha}\right)^{2}\leq 0,$$

which is satisfied for $\frac{\beta\delta}{\alpha}\leq 0.7056$ (approximately). Thus, it is satisfied if $\beta\delta=\frac{4}{7}$. □

Profitability of entry

Here, we study whether entry is profitable in the example of Section 5.2. Recall that entry is profitable if and only if condition (1) is satisfied.

Entry is surely profitable (under collusion and under competition) if $\beta>1$. For $\beta\leq 1$, we need to calculate $\sup_t\{F_t\}$. The function F_t is (strictly) increasing if and only if:

$$\begin{aligned}F_t<F_{t+1}&\Leftrightarrow\frac{\alpha-\beta\delta-\alpha^{1-t}(1-\beta\delta)}{\beta\left[\alpha-\beta\delta-\alpha^{-t}(1-\beta\delta)\right]}<1\\ &\Leftrightarrow(\alpha-\beta\delta)(1-\beta)-(1-\beta\delta)(\alpha-\beta)\alpha^{-t}<0\\ &\Leftrightarrow t<\frac{1}{ln\alpha}ln\left[\frac{(1-\beta\delta)(\alpha-\beta)}{(\alpha-\beta\delta)(1-\beta)}\right].\end{aligned}$$

Thus, there is entry under market regime $j\in\{m,c\}$ if:

$$K<\pi_{j3}\,F_{\hat{t}},\quad\text{where}\ \ \hat{t}=int\left\{\frac{1}{ln\alpha}ln\left[\frac{(1-\beta\delta)(\alpha-\beta)}{(\alpha-\beta\delta)(1-\beta)}\right]\right\}+1. \tag{22}$$

Proof of Result 4

The strategy of the proof is to consider the critical discount factor for sustainability of collusion after entry ($\beta\delta = \frac{4}{7}$) and check whether the ICC (15) for collusion sustainability before entry is satisfied for: (i) $T^c - T^m = 1$; (ii) $T^c - T^m = 2$; (iii) $T^c - T^m = 3$.

Replacing the critical discount factor for sustainability of collusion after entry ($\beta\delta = \frac{4}{7}$), the ICC (15) becomes:

$$-27\left(1-\alpha^{1-T^m}\right)\left(\alpha-\frac{4}{7}\right)-64\left(\alpha-\frac{4}{7}-\frac{3}{7}\alpha^{1-T^m}\right) +112\left(\frac{4}{7}\right)^{T^c-T^m}\left(\alpha-\frac{4}{7}-\frac{3}{7}\alpha^{1-T^c}\right)\geq 0. \tag{23}$$

(i) Sustainability of collusion before entry when $T^c - T^m = 1$

Replacing $T^c = T^m + 1$ in (23), we obtain:

$$\begin{aligned} &-63\left(1-\alpha^{1-T^m}\right)\left(\alpha-\frac{4}{7}\right)+64\left(\alpha^{1-T^m}-\alpha^{-T^m}\right)\geq 0 \\ \Leftrightarrow\ &-63\alpha^{T^m+1}+36\alpha^{T^m}+63\alpha^2+28\alpha-64\geq 0. \end{aligned} \tag{24}$$

The derivative of the left-hand side of (24) with respect to T^m is:

$$-63\ln(\alpha)\alpha^{T^m+1}+36\ln(\alpha)\alpha^{T^m},$$

which is always negative, as it has the same sign of $-63\alpha + 36$.

Hence, the most favorable case for the ICC (24) to be satisfied is when $T^m = 1$. Replacing $T^m = 1$, we obtain:

$$64\alpha - 64 \geq 0,$$

which is true.

The second most favorable case is $T^m = 2$. Replacing $T^m = 2$, we obtain:

$$-63\alpha^3 + 99\alpha^2 + 28\alpha - 64 \geq 0,$$

which is satisfied for $\alpha \leq \frac{4}{3}$.

Replacing $T^m = 3$, we obtain:

$$-63\alpha^4 + 36\alpha^3 + 63\alpha^2 + 28\alpha - 64 \geq 0,$$

which is satisfied for $\alpha \leq 1.046$ (approximately).

Replacing $T^m = 4$, and dividing by $\alpha - 1$, we obtain:

$$-63\alpha^4 - 27\alpha^3 - 27\alpha^2 + 36\alpha + 64 \geq 0,$$

which is never satisfied. It is not satisfied, therefore, for any $T^m \geq 4$.

(ii) Sustainability of collusion before entry when $T^c - T^m = 2$

Replacing $T^c = T^m + 2$ in (23), and expanding, we obtain:

$$-889\alpha^{2+T^m} + 508\alpha^{1+T^m} + 441\alpha^3 + 196\alpha^2 - 256 \geq 0.$$

The left-hand side is decreasing in T^m. The most favorable case is when $T^m = 1$:

$$-448\alpha^3 + 704\alpha^2 - 256 \geq 0,$$

which is satisfied for $\alpha \leq 1.094$ (approximately).

When $T^m = 2$, the ICC becomes:

$$-889\alpha^4 + 949\alpha^3 + 196\alpha^2 - 256 \geq 0.$$

One of the roots is 1. Dividing by $\alpha - 1$, we obtain:

$$-889\alpha^3 + 60\alpha^2 + 256\alpha + 256 \geq 0,$$

which is always false. Therefore, the ICC is not satisfied for any $T^m \geq 2$.

(iii) Sustainability of collusion before entry when $T^c - T^m = 3$

Replacing $T^m = 1$ and $T^c = 4$ in (23), we obtain:

$$\begin{aligned} -64(\alpha - 1) + 112\left(\frac{4}{7}\right)^3\left(\alpha - \frac{4}{7} - \frac{3}{7}\alpha^{-3}\right) &\geq 0 \\ -3136(\alpha - 1) + 1024\left(\alpha - \frac{4}{7} - \frac{3}{7}\alpha^{-3}\right) &\geq 0 \\ -14784\alpha + 17856 - 3072\alpha^{-3} &\geq 0, \end{aligned}$$

which is exactly satisfied for $\alpha = 1$.

The derivative of the left-hand side with respect to α is:

$$-14784 + 9216\alpha^{-4},$$

which is always negative.

Therefore, since $\alpha > 1$, the ICC is never satisfied if $T^c - T^m = 3$. From Lemma 3, we conclude that it is never satisfied if $T^c - T^m \geq 3$. □

Proof of Result 5

Given that a deviation from the collusive agreement permanently deters entry, the condition for sustainability of collusion before entry is (Proposition 5):

$$\delta \geq \frac{3}{16}\frac{f_{T^m-1}}{F_{T^m}}.$$

Collusion is surely sustainable before entry if $T^m = 1$ (because $f_0 = 0$).

For $T^m \geq 2$, the ICC is:

$$\delta \geq \frac{3}{16} \frac{\beta^{T^m-1}\left(1-\alpha^{1-T^m}\right)}{\beta^{T^m} \frac{\alpha-\beta\delta-\alpha^{1-T^m}(1-\beta\delta)}{(1-\beta\delta)(\alpha-\beta\delta)}} \Leftrightarrow \beta\delta \geq \frac{3}{16} \frac{\left(1-\alpha^{1-T^m}\right)(1-\beta\delta)(\alpha-\beta\delta)}{\alpha-\beta\delta-\alpha^{1-T^m}(1-\beta\delta)}$$

$$\Leftrightarrow \beta\delta(\alpha-1)+\beta\delta\left(1-\alpha^{1-T^m}\right)(1-\beta\delta) \geq \frac{3}{16}\left(1-\alpha^{1-T^m}\right)\left[\alpha-\beta\delta-\alpha\beta\delta+(\beta\delta)^2\right]$$

$$\Leftrightarrow \beta\delta\frac{\alpha-1}{1-\alpha^{1-T^m}}+\beta\delta-(\beta\delta)^2 \geq \frac{3}{16}(\beta\delta)^2-\frac{3}{16}(\alpha+1)\beta\delta+\frac{3}{16}\alpha$$

$$\Leftrightarrow \frac{19}{16}(\beta\delta)^2-\left[1+\frac{\alpha-1}{1-\alpha^{1-T^m}}+\frac{3(\alpha+1)}{16}\right]\beta\delta+\frac{3}{16}\alpha \leq 0.$$

The above condition becomes more restrictive as T^m increases. Therefore, a sufficient condition for the ICC to be satisfied can be obtained by considering $T^m \to +\infty$:

$$19(\beta\delta)^2-(19\alpha+3)\beta\delta+3\alpha \leq 0$$
$$\Leftrightarrow \beta\delta \geq \frac{3}{19}.$$

□

Proof of Result 6

From Proposition 5, the ICC for collusion sustainability before entry is:

$$\delta \geq \frac{3}{2}\frac{f_{T^m-1}}{F_{T^m}}.$$

If $T^m = 1$, we have $f_{T^m-1} = 0$, which implies that collusion before entry is surely sustainable.

For $T^m \geq 2$, the ICC is:

$$\delta \geq \frac{3}{2} \frac{\beta^{T^m-1}\left(1-\alpha^{1-T^m}\right)}{\beta^{T^m} \frac{\alpha-\beta\delta-\alpha^{1-T^m}(1-\beta\delta)}{(1-\beta\delta)(\alpha-\beta\delta)}}$$

$$\Leftrightarrow \beta\delta(\alpha-1)+\beta\delta\left(1-\alpha^{1-T^m}\right)(1-\beta\delta) \geq \frac{3}{2}\left(1-\alpha^{1-T^m}\right)\left[\alpha-\beta\delta-\alpha\beta\delta+(\beta\delta)^2\right]$$

$$\Leftrightarrow \beta\delta\frac{\alpha-1}{1-\alpha^{1-T^m}}+\beta\delta-(\beta\delta)^2 \geq \frac{3}{2}(\beta\delta)^2-\frac{3}{2}(\alpha+1)\beta\delta+\frac{3}{2}\alpha$$

$$\Leftrightarrow \frac{5}{2}(\beta\delta)^2-\left[1+\frac{\alpha-1}{1-\alpha^{1-T^m}}+\frac{3(\alpha+1)}{2}\right]\beta\delta+\frac{3}{2}\alpha \leq 0.$$

A sufficient condition for the ICC to be satisfied can be obtained by considering $T^m \to +\infty$:

$$\begin{aligned} 5(\beta\delta)^2 - (5\alpha + 3)\,\beta\delta + 3\alpha \leq 0 \\ \Leftrightarrow \;\; \beta\delta \geq \frac{3}{5}. \end{aligned}$$

□

Proof of Result 7

When firms are price-setters, collusion is sustainable if and only if $\beta\delta \geq \frac{2}{3}$. When firms are quantity-setters, we must have $\beta\delta \geq \frac{4}{7}$ for firms to comply with the collusive agreement after entry. However, this is not a sufficient condition for collusion sustainability with quantity-setting firms, since the ICC before entry may be binding.

Replacing $\beta\delta = \frac{2}{3}$ in the ICC (12) for collusion sustainability before entry, when demand grows at a constant rate, we obtain:

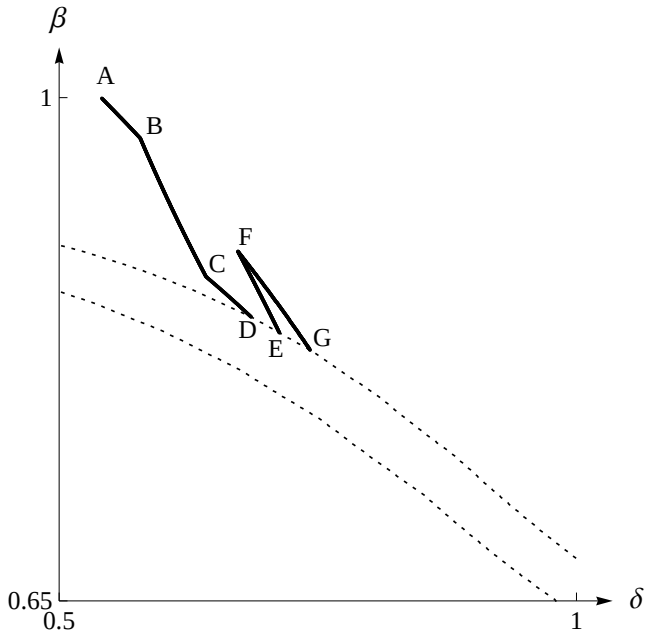
$$-9 - \frac{14}{3} + 28\left(\frac{2}{3}\right)^{T^c - T^m + 1} \geq 0 \;\Leftrightarrow\; T^c - T^m \leq \frac{ln\,(41/56)}{ln\,(2/3)} \approx 0.77.$$

Hence, collusion is harder to sustain when firms are price-setters if and only if $T^c - T^m = 0$.

Since $\frac{\pi^{m3}}{\pi^{c3}} = \frac{4}{3}$, the thresholds for entry under collusion and under competition can only be attained in the same period if $\beta > \frac{4}{3}$. □

8.5 Illustrations

In Figure 9, the dashed lines correspond to the conditions under which firm 3 enters the market, under collusion and under Cournot competition, given by (22); while the solid line corresponds to the ICC for sustainability of collusion before entry, given by (15).



[AB] T^c jumps from 3 (to the right of the line segment) to 4 (to the left).

[BC] ICC before entry with $T^c = 4$ and $T^m = 3$ (satisfied in equality).

[CD] T^c jumps from 4 (below) to 5 (above).

[EF] T^m jumps from 2 (below) to 3 (above).

[FG] T^c jumps from 3 (to the right of the line segment) to 4 (to the left).

Figure 9: ICC for collusion sustainability before entry (solid line) and conditions for entry profitability under collusion and competition (dashed lines), with $\alpha = 1.01$ and $K = 0.004$.

The erratic shape of the ICC is due to the discrete nature of time. More precisely, a slight change in the parameters may be enough for T^c or T^m to jump by one period, which leads to kinks in the ICC.

References

Abbink, K. and Brandts, J. (2009). Collusion in growing and shrinking markets: empirical evidence from experimental duopolies. In Hinloopen, J. and Normann, H.-T., editors, *Experiments and Competition Policy*, pages 34–60. Cambridge University Press.

Abreu, D. (1986). Extremal equilibria of oligopolistic supergames. *Journal of Economic Theory*, 39(1):191–225.

Asch, P. and Seneca, J. J. (1975). Characteristics of collusive firms. *Journal of Industrial Economics*, 23(3):223–237.

Bagwell, K. and Staiger, R. (1997). Collusion over the business cycle. *RAND Journal of Economics*, 28(1):82–106.

Brandão, A., Pinho, J., and Vasconcelos, H. (2013). Asymmetric collusion with growing demand. *Journal of Industry, Competition and Trade*, forthcoming.

Capuano, C. (2002). Demand growth, entry and collusion sustainability. *Fondazione Eni Enrico Mattei Working Paper*, 62.

Correia-da-Silva, J., Pinho, J., and Vasconcelos, H. (2013). How should cartels react to entry triggered by demand growth? *FEP Working Papers*, 487.

Dick, A. R. (1996). Identifying contracts, combinations and conspiracies in restraint of trade. *Managerial and Decision Economics*, 17(2):203–216.

Fabra, N. (2006). Collusion with capacity constraints over the business cycle. *International Journal of Industrial Organization*, 24(1):69–81.

Friedman, J. W. and Thisse, J.-F. (1994). Sustainable collusion in oligopoly with free entry. *European Economic Review*, 38(2):271–283.

Haltiwanger, J. and Harrington, J. E. (1991). The impact of cyclical demand movements on collusive behavior. *RAND Journal of Economics*, 22(1):89–106.

Harrington, J. E. (1989). Collusion and predation under (almost) free entry. *International Journal of Industrial Organization*, 7(3):381–401.

Ivaldi, M., Jullien, B., Rey, P., Seabright, P., and Tirole, J. (2007). The economics of tacit collusion: implications for merger control. In Ghosal, V. and Stennek, J., editors, *The Political Economy of Antitrust*. Elsevier.

Kandori, M. (1991). Correlated demand shocks and price wars during booms. *Review of Economic Studies*, 58(1):171–180.

Knittel, C. R. and Lepore, J. J. (2010). Tacit collusion in the presence of cyclical demand and endogenous capacity levels. *International Journal of Industrial Organization*, 28(2):131–144.

Montero, J.-P. and Guzman, J. I. (2010). Output-expanding collusion in the presence of a competitive fringe. *Journal of Industrial Economics*, 58(1):106–126.

Motta, M. (2004). *Competition policy: theory and practice*. Cambridge University Press.

Pashigian, B. P. (1968). Limit price and the market share of the leading firm. *Journal of Industrial Economics*, 16(3):165–177.

Rotemberg, J. J. and Saloner, G. (1986). A supergame-theoretic model of price wars during booms. *American Economic Review*, 76(3):390–407.

Siegfried, J. J. and Evans, L. B. (1994). Empirical studies of entry and exit: a survey of the evidence. *Review of Industrial Organization*, 9(2):121–155.

Staiger, R. W. and Wolak, F. A. (1992). Collusive pricing with capacity constraints in the presence of demand uncertainty. *RAND Journal of Economics*, 23(2):203–220.

Stenbacka, L. R. (1990). Collusion in dynamic oligopolies in the presence of entry threats. *Journal of Industrial Economics*, 39(2):147–154.

Symeonidis, G. (2003). In which industries is collusion more likely? Evidence from the UK. *Journal of Industrial Economics*, 51(1):45–74.

Vasconcelos, H. (2004). Entry effects on cartel stability and the joint executive committee. *Review of Industrial Organization*, 24(3):219–241.

Vasconcelos, H. (2008). Sustaining collusion in growing markets. *Journal of Economics & Management Strategy*, 17(4):973–1010.

Wenders, J. T. (1971). Collusion and entry. *Journal of Political Economy*, 79(6):1258–1277.

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