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Luís Aguiar-Conraria
Manuel M. F. Martins
Maria Joana Soares

Centro de Economia e Finanças da UP
www.fep.up.pt

Analyzing the Taylor Rule with Wavelet Lenses

Luís Aguiar-Conraria* Manuel M. F. Martins† Maria Joana Soares‡

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Abstract

This paper analyses the Taylor Rule in the U.S. 1960-2014 with new lenses: continuous time partial wavelets tools. These allow us to assess the co-movement between the policy interest rate and the macroeconomic variables in the Rule, inflation and the output gap, both jointly and independently, for each frequency and at each moment of time.

Our results uncover some new stylized facts about U.S. monetary policy and add new insights to the record of U.S. monetary history since the early 1960s. Among other findings, we conclude that monetary policy has been forward-looking and aimed at stabilizing inflation throughout the whole sample, although with varying effectiveness both across time and frequencies. Monetary policy has lagged the output gap across most of the sample, but in recent times became more reactive. Volcker's disinflation, and the conquest of credibility in 1979-1986, were achieved with no extra costs in terms of output.

Keywords: Monetary Policy, Taylor Rule, Continuous Wavelet Transform, Partial Wavelet Coherency, Partial Phase-difference

JEL codes: C49, E43, E52

*NIPE and Department of Economics, University of Minho, E-mail address: lfaguiar@eeg.uminho.pt

†Cef.up and Faculty of Economics, University of Porto, E-mail address: mmfmartins@fep.up.pt

‡NIPE and Department of Mathematics and Applications, University of Minho, E-mail address: jsoares@math.uminho.pt

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1 Introduction

John B. Taylor first proposed what came to be known as the Taylor Rule (Taylor, 1993). The Taylor Rule describes the Fed's monetary policy as a simple relation between the federal funds rate, the interest rate at which banks trade federal funds with each other overnight,[1] and the inflation rate and the output gap (as a percent deviation of real GDP from potential GDP). Taylor parametrized the relation in the following way:

$$FFR_t = 2 + \pi_t + \frac{1}{2}y_t + \frac{1}{2}(\pi_t - 2), \tag{1}$$

where *FFR* is the (effective) federal funds rate, π is the inflation rate over the previous four quarters and y is the output gap. His calibration assumed a trend growth rate of real output of 2 percent and an inflation target of 2 percent. The positive coefficients associated with y_t and $(\pi_t - 2)$ mean that if GDP or the rate of inflation are above their targets, then interest rates should increase. When the inflation rate is 2 (the implicit target level) and output is at its potential, then the interest rate will be 4 and the real rate of interest will be 2.

Taylor's rule was thought out not only as a positive device — a simple way to describe U.S. monetary policy since the mid-1980s — but also as a normative prescription — a useful benchmark for monetary policy, highly valuable to inform and aid policymakers' decisions, even though not to be followed mechanically. Under a rules-based monetary policy, policymakers use a simple formula to gauge the policy instrument as a reaction to macroeconomic events, and conduct monetary policy in a more predictable, systematic, and effective way (Taylor, 2012). The simplicity of Taylor's rule makes it very easy to communicate and understand. Moreover, it is arguably more robust than a wide array of optimal policy rules derived in specific macroeconomic models (Taylor and Williams, 2010).

More than a guide for policy decisions, the Taylor Rule may become a measure of accountability for policymakers, in line with the predictions by Taylor and Williams (2010). In fact, the House Financial Service Committee voted to approve legislation requiring the Fed to report a Rule-based policy strategy for setting the instruments of monetary policy. According to this

[1] The Effective Federal Funds Rate is a weighted average rate of all these interest rates, and is manipulated by the Federal Reserve through open market operations.

bill,[2] the Federal Reserve should justify before the Congress whenever monetary policy results in interest rates that deviate from the reference policy rule, defined as:

> The term 'Reference Policy Rule' means a calculation of the nominal Federal funds rate as equal to the sum of the following:
>
> (A) The rate of inflation over the previous four quarters. (B) One-half of the percentage deviation of the real GDP from an estimate of potential GDP. (C) One-half of the difference between the rate of inflation over the previous four quarters and two. (D) Two.

— In *H.R.5018 - Federal Reserve Accountability and Transparency Act of 2014.*

Clearly, this 'Reference Policy Rule' corresponds to the classic Taylor rule in formula (1). Hence the enormous relevance of further research on the implementation and performance of U.S. monetary policy under the anchor of Taylor's basic formula, as we do in this paper.[3]

In Figure 1, we plot the (effective) Federal Funds Rate (FFR) and the Reference Policy Rule, or the Taylor (1993) rule interest rate, since 1960.[4] It is remarkable how such a simple linear formula mimics the overall path of interest rates, particularly if one bears in mind that it was proposed in 1993. Moreover, as documented, inter alia, by Kahn (2012), while there were extensive and recurrent references to Taylor-type rules in the Federal Open Market Committee discussions of U.S. monetary policy decisions since 1993, before such period policy discussions and decisions were much more discretionary, with no explicit reference to feed-back rules. In fact, the discussions by policymakers suggest that policy concentrated on fine-tuning real activity with no special focus on long-run price stability, as documented, for example, by Taylor (2012).

[2] https://beta.congress.gov/113/bills/hr5018/BILLS-113hr5018ih.pdf

[3] There is a vast set of variations to the original Taylor Rule in the literature. For example, following Clarida, Galí and Gertler (2000), many authors have replaced inflation with expected inflation, as policymakers need to be forward-looking because of the lags in the transmission of monetary policy; others have suggested replacing the output gap with alternative measures of real activity easier to observe and possibly allowing for some welfare gain, such as output growth (e.g. Sims, 2013); others have enhanced the Rule with a reaction to additional variables, such as asset prices (e.g. Sack and Rigobon, 2003), exchange rates (e.g. Lubik and Schorfheide, 2007), or long-term bond yields (e.g. Christensen and Nielsen, 2009). We focus on the original Taylor Rule because of its simplicity, robustness, and well-documented relevance for actual policymaking, as well as because we want to avoid econometric issues that are subsidiary to our purpose of conducting a time-frequency analysis of U.S. monetary policy — such as the problems of identification (see e.g. Cochrane, 2011) and of estimation of forward-looking rules (see e.g. Jondeau, Le Bihan and Gallès, 2004). Moreover, our wavelet tools are intrinsically able to detect lead-lag relations between the policy interest rate and the macroeconomic variables in the Taylor Rule, as explained below. For a recent example of research using the basic Taylor Rule formula, see Nikolsko-Rzhevskyy, Papell and Prodan (2014).

[4] In recent times, there are some observations in which $TR < 0$. In those situations we truncate at zero.

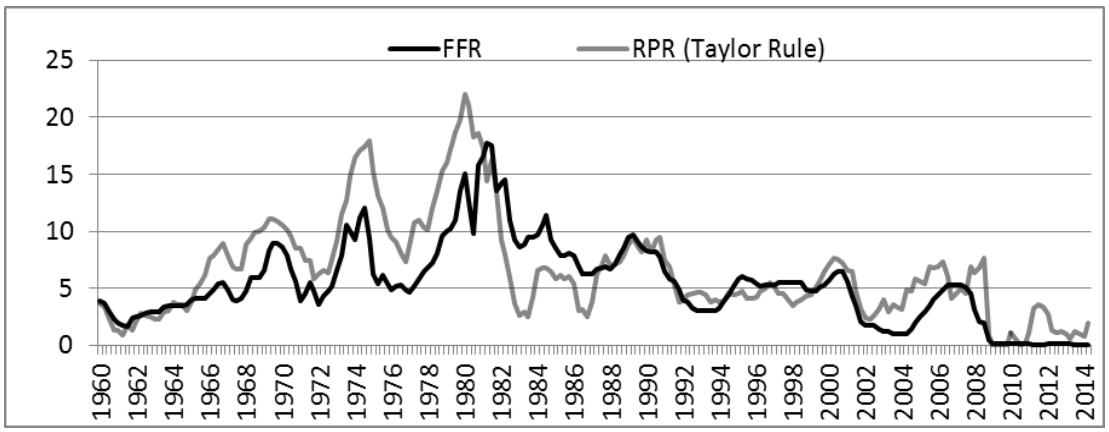

Figure 1: The Classic Taylor Rule — the proposed Reference Policy Rule — and the (effective) Federal Funds Rate since 1960.

Figure 1 — both the path of the policy rate and the actual interest rates — illustrates a number of key episodes in the recent history of U.S. monetary policy and macroeconomic performance that call for new evidence and analyses. Between the mid-1960s and the late 1970s, interest rates were systematically below the reference TR interest rate, with the Federal Reserve conducting monetary policy mostly to fine-tune real activity with little concern for long-run price stability which may have fueled the Great Inflation (Taylor, 2012). Then, since the start of the Volcker disinflation, it is apparent that interest rates were above the TR level until the mid-1980s, to achieve credibility and anchor inflation expectations, which is generally believed to have led to a prolonged recession. Between 1985 and 2003, the FFR followed quite closely the levels indicated by the Taylor Rule, a period that Taylor (2012) called a "rules-based era", further arguing that such predictable systematic approach to monetary policy has been key in the Great Moderation. Then, in 2003-2006, actual interest rates were substantially and persistently below the TR-prescribed interest rate, an episode that Taylor (2011) labeled the Great Deviation, which arguably fueled the financial and housing boom that led to the 2008 bust and the ensuing Great Recession (Taylor, 2012). More recently, since the outset of the current crisis, interest rates continue below the Taylor Rule level, having reached its zero lower bound since 2009, and the post-2003 "ad-hoc era" identified by Taylor (2012) persists, with unconventional monetary policy much more relevant than interest rate policy, and with macroeconomic instability enduring.

Deviations of the actual policy interest rate (FFR) from the rate prescribed by the original Taylor Rule may arise for two main reasons. First, they may come from deliberate episodes of discretionary policy; for example, in the same vein of Taylor (2011, 2012), but using formal structural breaks tests, Nikolsko-Rzhevskyy et al. (2014) find that U.S. policy interest rates followed quite closely the original Taylor Rule in 1965-1974 and in 1985-2000, but deviated significantly from the rule, in a discretionary way, in 1974-1984 and in 2001-2013. Second, they may result from changes in the specification and/or parameters of the Taylor-type Rule actually followed by U.S. policymakers; there is, indeed, a vast literature on policy rule shifts in the time domain, as well as on its possible causes and consequences, which we now briefly review.

Since Clarida et al. (2000) found that the U.S. interest rate policy has been more sensitive to inflation after 1979 than before, numerous studies have looked at the stability of the U.S. Taylor Rule using different data and different econometric approaches, such as threshold models (e.g. Bunzel and Wenders, 2010), time-varying parameters models (e.g. Trecroci and Vassalli, 2010), Markov-switching models (e.g. Assenmacher-Wesche, 2006), smooth-transition models (e.g. Alcidi et al., 2011), instrumental variables quantile regressions (e.g. Wolters, 2012), and Hamilton's (2001) flexible approach to nonlinear inference (e.g. Kim, Osborn and Sensier, 2005), among many others. The possible role that changes in U.S. monetary policy may have had in the Great Moderation then fed a prolific research program featuring a Taylor Rule in a variety of models, from structural time-varying coefficients VARs with stochastic volatility (e.g. Sims and Zha, 2006), to structural small-scale New Keynesian models (e.g. Canova, 2009), and to DSGE models with stochastic volatility and parameter drifting (e.g. Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez, 2010); in spite of disagreement about the magnitude of its impact on macroeconomic volatility, a common result in this literature is that the U.S. policy rule did change ahead of the Great Moderation. Changes in monetary policy regimes have also been associated to shifts in the persistence of inflation (e.g. Benati, 2008), and an extensive literature suggests that the persistence of U.S. inflation has been inversely associated with the coefficient of inflation in the policy rule, thus being higher during the Great Inflation of the 1970s; such result withstands across models that allow for regime switching

in the inflation target and so look at the persistence of inflation (e.g. Davig and Doh, 2013) as well as models that, in addition to changing regimes, allow for trend inflation and consider inflation-gap persistence (e.g. Cogley, Primiceri and Sargent, 2010).

As the Taylor Rule is a reduced-form relation, its functional form and coefficients depend on the policymaker's preferences as well as on the structure of the economy, and so it may change along time for many possible reasons. First, because of changes in the preferences of the monetary policymaker, i.e. shifts in the weights attributed to the targets or in the levels of the targets themselves in his loss function (e.g. Favero and Rovelli, 2003; Owyang and Ramey, 2004; Dennis, 2006; Aguiar and Martins, 2005a). Second, because there may be non-linearities in the policymakers' preferences, i.e. different reactions to outcomes below or above the targets (e.g. Nobay and Peel, 2003; Dolado, Maria-Dolores and Ruge-Murcia, 2004; Surico, 2007; Cukierman and Muscatelli, 2008). Third, the Rule may change because of non-linearities or breaks in the structure of the macroeconomy and, thus, in the transmission of monetary policy — for example, changes or non-linearities in the Phillips Curve (e.g. Dolado, Maria-Dolores and Naveira, 2005; Huh, Lee and Lee, 2009; Aguiar and Martins, 2005b). Finally, the policy rule may change, at some stages, because the policymaker is uncertain about the state and/or the functioning of the macroeconomy and uses additional information and/or judgement to design policy (e.g. Alcidi, Flamini and Fracasso, 2011; Tillmann, 2011; Billi, 2012). Moreover, the observed relation between the policy interest rate and the main macroeconomic variables may varybecause agents' or markets' perception of the policymaker's policy rule may be heterogeneous, uncertain and subject to important changes, thus modifying the transmission mechanisms of monetary policy. Indeed, a recent literature has found, using survey-based macroeconomic forecasts (e.g. Buraschi, Carnelli and Whelan, 2013) or measures of the effect of news on fundamentals' and policy forecasts (e.g. Hamilton, Pruitt and Borger, 2011), that there is considerable time- and state- dependence in the public's perception of the U.S. Taylor Rule.

While shifts in the specification and/or coefficients of the Taylor Rule — or of the perceived Taylor Rule — imply changes in the dynamic relation between the policy interest rate, output and inflation, a purely time-domain approach falls short of a thorough description of the nature and consequences of those changes, as they may occur differently at different frequencies. In fact,

given that monetary policy focuses on cyclical stabilization, one key concern of policymakers should be to understand and control which specific cyclical oscillations they want to, can, and do control at each period of time. For example, policymakers should care about the impact of policy in the frequency-domain, because oscillations at different frequencies may have different impacts on social welfare, or because controlling oscillations at some frequencies may imply larger variabilities at other frequencies (Yu, 2013). Moreover, it may be conjectured that policymakers would like to react differently to permanent and to short-lived fluctuations in the main macroeconomic variables (Ashley, Tsang and Verbrugge, 2013). Furthermore, it may be argued that specific changes in monetary policy regimes may be related to changes in the relative intensity of the policy reaction at different frequencies — say, a policymaker trying to conquer credibility may have to react very strongly to transitory changes in inflation, but once credibility is established, he may increase the focus on fluctuations of a more permanent nature. Yet, the analysis of shifts in the Taylor Rule at different frequencies is extremely scarce. The only study of the Taylor Rule in the frequency-domain is, to the best of our knowledge, Ashley et al. (2013), who compare the estimated coefficients of the U.S. Taylor Rule before and after 1979:8 for 19 separate frequencies; they find a significant frequency dependence of the Taylor Rule coefficients, and that monetary policy reacted more strongly to fluctuations with a lower frequency (longer period) after 1979, specially of inflation; overall, they conclude that ignoring frequency dependence leads to an underestimation of the break in the U.S. Taylor Rule.

In spite of the compelling arguments suggesting that the Taylor Rule may be unstable both in the time and in the frequency domains, the literature is silent about the assessment of changes in the Taylor Rule simultaneously in the time and in the frequency domains. Hence our contribution to the literature. Our wavelet tools — a combination of partial wavelet coherence and partial wavelet phase diagrams — allow for the assessment, in continuous time, of the direction and lags of co-movements between the policy interest rate and the two macroeconomic variables in the Taylor Rule, across the whole range of relevant cyclical frequencies. Also, our tools are particularly suited for the study of a simple multivariate equation such as the Taylor Rule, as we study the co-movement of the policy interest rate with each of the two involved macroeconomic variables — output gap and inflation — controlling for the effects of the co-

movement with the other macroeconomic variable. Overall, our paper provides a new set of stylized facts about the evolution of U.S. monetary policy between 1960 and 2014 that would have been difficult to find in purely time or frequency-domain analyses, thus allowing for a broader and deeper understanding of U.S monetary and macroeconomic history.

The paper proceeds as follows. In section 2, we describe our methodology and our data. In section 3, we apply our methodology to the data and provide an assessment of U.S. monetary policy under the perspective of the Taylor Rule in the time-frequency domain. Section 4 summarizes and concludes the paper.

2 Methodology and Data

2.1 Wavelet analysis

In this subsection, we present a necessarily brief and technical description of the wavelet tools that we employ in this paper. The reader familiar with these concepts may skip to section 2.2 without loss. The reader seeking an in-depth treatment of these tools is directed to Aguiar-Conraria and Soares (2014). On the other hand, a reader looking for a more intuitive explanation of most of these tools may consult Aguiar-Conraria, Magalhães and Soares (2012).

2.1.1 The wavelet

A function ψ is called a *wavelet* if it is square integrable (i.e. $\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty$) and satisfies the following technical condition, usually referred to as the *admissibility condition*: $\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$, where $\hat{\psi}$ is the Fourier transform of ψ, $\hat{\psi}(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt$.

To be of practical use, the wavelet ψ must satisfy more stringent decay conditions than just being square integrable, e.g., ψ must have compact support or exponential decay.[5] In this case, it can be shown that the admissibility condition is equivalent to requiring that ψ has zero mean, i.e. $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$ (Daubechies 1992). This means that the function ψ has to oscillate around the t-axis, thus behaving like a wave. Hence, a wavelet is simply a "small wave", justifying its

[5] Ideally, ψ must also be well-localized in the frequency-domain, i.e, its Fourier transform $\hat{\psi}$ must also be a fast decaying function; due to the Heisenberg uncertainty principle, however, there is always a trade-off between localization in time and localization in frequency.

name.

The specific wavelet we use in this paper is a complex-valued function selected from the so-called *Morlet wavelet* family, first introduced in Goupillaud et al. (1984),

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{2}$$

and corresponds to the particular choice of $\omega_0 = 6$.[6]

The popularity of the Morlet wavelets is due to their interesting characteristics. Since ψ_{ω_0} is simply a complex sinusoid of angular frequency ω_0 multiplied by a Gaussian window, it is natural to associate the angular frequency ω_0 — i.e. the usual Fourier frequency $f = \omega_0/(2\pi)$ — to this function; hence, the wavelets at scale s can be associated with frequencies $f_s = \frac{\omega_0}{2\pi s}$; for the value of $\omega_0 = 6$, used in this paper, we have $f_s \approx \frac{1}{s}$, which greatly facilitates the interpretation of the wavelet analysis — which, strictly speaking, is a time-scale analysis — as a time-frequency analysis. Also, this function has another important property: it has "optimal joint time-frequency concentration", in the following sense: the time-frequency localization of a function can be described by a rectangular region in the time-frequency plane — the so called Heisenberg box — whose area, according to the Heisenberg uncertainty principle, is bounded from below by a certain constant; the Morlet wavelets, because of their Gaussian envelopes, have Heisenberg boxes with areas attaining the minimum possible value prescribed by the Heisenberg principle.

The continuous wavelet transform Given a time-series $x(t)$, its *continuous wavelet transform* (CWT), with respect to a given wavelet ψ, is a function of two variables, $W_x(\tau, s)$, obtained by "comparing" $x(t)$ with a family of functions — the so-called *wavelet-daughters* — which are simply scaled and translated versions of ψ:

$$W_x(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t) \bar{\psi}\left(\frac{t-\tau}{s}\right) dt. \tag{3}$$

[6] Although, strictly speaking the above function is not a "true" wavelet, since it has no zero mean, the value of $\int_{-\infty}^{\infty} \psi_6(t) dt \approx 2.87 \times 10^{-}8$ is so small that, for all numerical purposes, it can be considered as a wavelet.

The scaling parameter s controls the width of the wavelet and the translation parameter τ controls its location along the t-axis; they both vary continuously over $\mathbb{R}$, with the constraint that $s \neq 0$. In the above formula and throughout, we use the bar to denote complex conjugation.

2.1.2 Univariate tools

All the quantities we are going to introduce are functions of time and scale. To simplify the notation, we will describe these quantities for a specific value of the argument, (τ, s), and this value of the argument will be omitted in the formulas.

The wavelet power and the wavelet phase In analogy with the terminology used in the Fourier case, the (local) *wavelet power spectrum* (sometimes called scalogram or wavelet periodogram) is defined as

$$(WPS)_x = |W_x|^2 . \tag{4}$$

This gives us a measure of the variance distribution of the time-series in the time-frequency plane.

When the wavelet ψ is chosen as a complex-valued function, as in our case, the wavelet transform W_x is also complex-valued. In this case, the transform can be separated into its real part, $\Re(W_x)$, and imaginary part, $\Im(W_x)$, or be expressed in polar form as $W_x = |W_x| \, e^{i\phi_x}$, $\phi_x \in (-\pi, \pi]$. The angle ϕ_x is known as the *(wavelet) phase.*[7]

2.1.3 Bivariate tools

In many applications, one is interested in detecting and quantifying relationships between two non-stationary time-series. The concepts of cross-wavelet power, cross-wavelet coherency and wavelet phase-difference are natural generalizations of the basic wavelet analysis tools that enable us to deal with the time-frequency dependencies between two time-series.

[7] Recall that the phase-angle ϕ_x of the complex number W_x can be obtained from the formula: $\tan(\phi_x) = \frac{\Im(W_x)}{\Re(W_x)}$, using the information on the signs of $\Re(W_x)$ and $\Im(W_x)$ to determine to which quadrant the angle belongs to.

The *cross-wavelet transform* of two time-series, $x(t)$ and $y(t)$, is defined as

$$W_{xy} = W_x \overline{W}_y \,, \tag{5}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. The absolute value of the cross-wavelet transform, $|W_{xy}|$, will be referred to as the *cross-wavelet power*. The cross-wavelet power of two time-series depicts the local covariance between two time-series at each time and frequency.

We can also define the *complex wavelet coherency* of x and y, ϱ_{xy}, by

$$\varrho_{xy} = \frac{S\left(W_{xy}\right)}{\left[S\left(|W_x|^2\right) S\left(|W_y|^2\right)\right]^{1/2}} \,, \tag{6}$$

where S denotes a smoothing operator in both time and scale. [8]

By analogy with the Fourier case, the wavelet coherency, R_{xy}, is defined simply as the absolute value of the complex wavelet coherency, i.e. is given by $R_{xy} = |\varrho_{xy}|$.

With a complex-valued wavelet, we can compute the phase of the wavelet transform of each series and, by computing their difference, we can then obtain information about the possible delays of the oscillations of the two series, as a function of time and frequency. It follows immediately from (5) that the *phase-difference*, which we will denote by ϕ_{xy}, can also be computed as the phase-angle of the cross-wavelet transform, i.e. by using the formula $\tan \phi_{x,y} = \frac{\Im(W_{xy})}{\Re(W_{xy})}$, with the information on the signs of each part to completely determine the value of $\phi_{xy} \in (-\pi, \pi]$. A phase-difference of zero indicates that the time-series move together at the specified frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series x leads y; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is y that is leading; a phase-difference of π indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y is leading; time-series x is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

Remark 1 *The wavelet-phase difference is sometimes defined as the phase-angle of the complex wavelet coherency, instead of the phase-angle of the cross-wavelet transform. Although this is not fully consistent with the difference between the individual phases, since it is affected by the*

[8] As in the Fourier case, smoothing is necessary, otherwise the magnitude of coherency would be identically one.

smoothing, the results obtained with the two definitions are not substantially different from each other; moreover, this alternative definition extends, in a more natural way, to the multivariate case, described below.

2.1.4 Multivariate tools

To deal with more than two series, extensions of the concepts of multiple and partial coherency from Fourier spectral analysis to the wavelet context have recently been introduced; see, e.g. Mihanovič, Orlič and Parsič (2009), for the case of three series, and Aguiar-Conraria and Soares (2014), for the more general case. Here, we only present the formulas for the (complex and absolute) partial coherency for case of three variables.

Given three series x, y, z, the *complex partial wavelet coherency* of x and y after controlling for z, denoted by $\varrho_{xy\,.z}$, is defined by

$$\varrho_{xy\,.z} = \frac{\varrho_{xy} - \varrho_{xz}\overline{\varrho}_{yz}}{\sqrt{(1 - R^2_{xz})(1 - R^2_{yz})}}, \tag{7}$$

and the *partial wavelet coherency*, $R_{xy\,.z}$, is defined as the absolute value of the complex partial wavelet coherency, i.e. $R_{xy\,.z} = |\varrho_{xy\,.z}|$.

Having defined the complex partial wavelet coherency $\varrho_{xy\,.z}$ between the series x and y, after removing the influence of z, the *partial phase-difference* of x over y, given z, denoted by $\varrho_{xy\,.z}$, is simply defined as the phase-angle of $\varrho_{xy\,.z}$.

2.1.5 Statistical significance

All our significance tests for coherency and partial coherency are obtained using Monte-Carlo simulations. To perform significance tests of wavelet measures, we fit an ARMA model to the series and construct new samples by drawing errors from a Gaussian distribution with a variance equal to that of the estimated error terms. For each time-series (or set of time-series), we perform the exercise 5000 times, and then extract the critical values at 5 and 10% significance. For the wavelet power spectrum we rely on a theoretical distribution for the null of a flat spectrum; see Torrence and Compo (1998) and Zhang and Moore (2012).

2.2 The Data

Our data are quarterly time-series for the effective federal funds rate (FFR), consumer price index (CPI) inflation and the output gap, for the United States (U.S.) 1960:I-2014:II. The output gap is the log deviation of real gross domestic product (GDP) from real potential GDP. All data, including potential GDP, are taken from the Federal Reserve of St. Louis and are measured in percentage points.[9] In Figure 2 we plot the three time-series, on the left-hand side charts, and their wavelet power spectra, on the right-hand side charts; these are a measure of the variance of the series at each time-frequency locus, and provide a first time-frequency description of the data.

A first overall conclusion from Figure 2 is that the variability of all of the three time-series occurs at cycles of period larger than 4 years; hence, very short-run and noisy variability is not important in the time-frequency history of any of the three time-series involved in the Taylor Rule.

The chart of the inflation series shows its gradual rise since the mid-1960s, the disinflation between 1980 and 1986, the ensuing period of low and stable inflation and, finally, the rise and fall of inflation in the 2008-10 financial and economic crisis. The wavelet power spectrum of inflation is statistically significant for a cycle with a period around 12 years since the mid-1960s, and then, after the late 1960s, also for frequencies corresponding to cycles of period between 4 and 8 years, consistently with the inflationary period. The episode with a larger power occurs for cycles of a period around 6 years, between 1974 and 1980, which clearly corresponds to the disinflationary period. While the significance of the power spectrum endures until the mid-2000s for cycles of period around 12 years, for those in the range of $4 \sim 8$ years it disappears after the completion of the disinflation, i.e. in the mid-1980s. Moreover, after 1986, the areas of statistically significant power spectrum of inflation become gradually thinner, meaning that the variability of inflation occurred at gradually longer cycles; from the mid-1990s onward, the only significant variability of inflation occurred at cycles very narrowly concentrated in the 12 period; such pattern of the wavelet power spectrum of inflation illustrates the impressive success of the U.S. disinflation, with a solid anchoring of inflation (and surely its expectations) and a

[9] http://research.stlouisfed.org/fred2/.

prolonged period of very low inflation variance, during the Great Moderation.

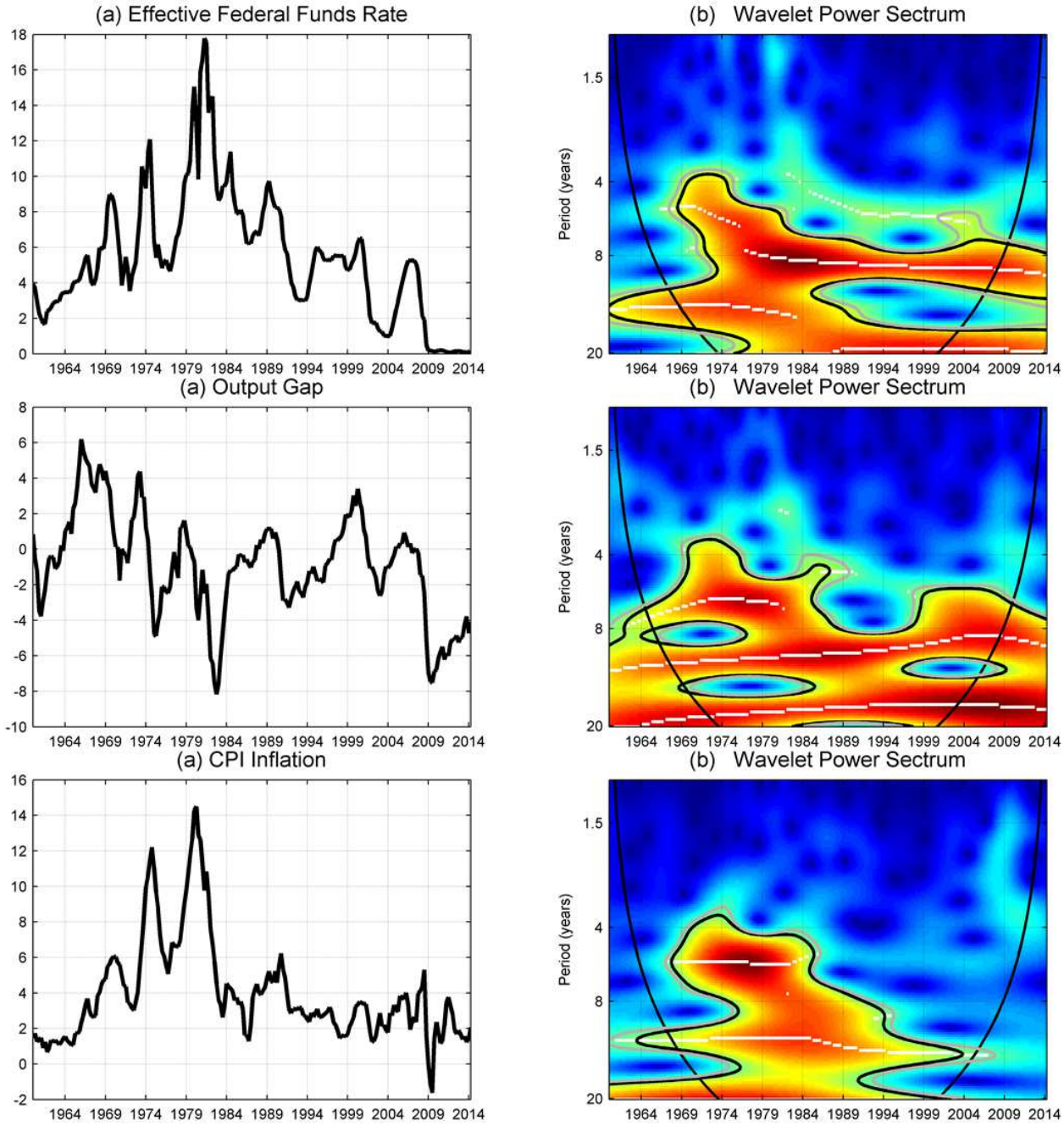

Figure 2: (a) Plot of each time-series. (b) The corresponding wavelet power spectrum. The black/gray contour designates the 5%/10% signi cance level. The cone of influence, which indicates the region a ected by edge e ects when computing the power, is shown with a black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum.

The chart of the output gap series shows the strong recession associated with the first oil shock in the mid-1970s, the recession of the late 1970s, typically associated with the second oil shock, the disinflation of 1980-1986, the Great Moderation between 1984 and 2008, and the Great Recession of 2008-09. The wavelet power spectrum of the gap is statistically significant since the mid-1960s, for a wide range of frequencies, with a stronger variance of the gap at three main cycles: one with a period close to 20 years, another with a period around 12 years, and a third with a period around 8 years. During the 1970s, all these three cycles gradually become shorter, and in the second half of the 1970s and early 1980s there is a very strong variance of the gap at cycles of $6 \sim 7$ years, corresponding to the recession associated with the second oil shock and the disinflation. Then, the Great Moderation is apparent, as after 1984 there is no significant variability of the gap at cycles of period below 4 years, the areas of significant power spectrum become narrower, and the larger variance of the gap occurs at cycles with a gradually longer period: in the second half of the 1980s, the power spectrum is higher for cycles of a period around $10 \sim 12$ years, and, since the second half of the 1990s, the larger power occurs at cycles of a period of $16 \sim 18$ years. The Great Recession explains an increase in the wavelet power spectrum activity at cycles of a period around 8 years since the mid-2000s.

The chart of the effective federal funds rate (FFR) shows that nominal interest rates increased with inflation since the mid-1960s, and indicates that real interest rates increased since the late-1970s and were kept at a high level until the end of the 1980s, then fell since the early 2000s and even more so since the Great Recession. The power spectrum of the FFR is statistically significant since the beginning of the sample for a cycle with a period around 12 years, in line with a cycle of a similar frequency observed in the inflation rate. Since the late-1960s, there was an increase in the width of frequencies at which the variance of the FFR is strong and significant, then comprising the whole range of cycles with period larger than 4 years, with a particular incidence on cycles of period around 6 years. Between 1979 and 1987 there is an extremely high and significant level of the wavelet power spectrum at cycles of period around 8 years, clearly corresponding to the period of very restrictive monetary policy conducted by Paul Volcker in order to disinflate the U.S. economy.[10] After completion of the disinflationary

[10] Paul Volcker was the 12^{th} Chairman of the Federal Reserve and he remained in office between August of 1979 and August of 1987.

process, in the mid-1980s, the wavelet power spectrum of the FFR is significant only for a rather small range of cyclical frequencies, specifically cycles of period around 8 years (say, $7 \sim 9$ years) and cycles of period close to 20 years. Interestingly, these are the cyclical frequencies for which the wavelet power spectrum of the output gap is most significant, and not frequencies with a significant power spectrum of inflation, which is prima facie evidence that, in the Greespan period, policy interest rates effectively dealt with fluctuations in real activity, in the context of an apparently controlled inflation.

3 The Taylor rule in the time-frequency domain

We now study the relation between the FFR and the macroeconomic variables present in the Taylor Rule, simultaneously in the time and frequency domains. Our tools are the bivariate and the partial wavelet coherency, as well as the bivariate and partial phase-difference.

The interpretation of our econometric results proceeds along the standard approach in similar literature (see e.g. Aguiar-Conraria, Martins and Soares, 2012) and may be summarized as follows. First, we check the time-frequency regions in which the coherency is statistically significant, meaning that, in those episodes, we may confidently say that there has been a significant co-movement between the two series for cycles of the indicated period. Then, for the statistically significant time-frequency locations, we analyze the phase-differences, to detect whether the co-movement has been positive or negative, and which variables were leading and lagging.

We start by describing the relation between the FFR and the interest rate that would have been described by the Taylor Rule, in terms of coherency and phase-differences. Then, we analyze the causes and effects of the effective monetary policy in terms of inflation and the output gap. To do so, we begin by analyzing simple coherencies and then move to the analysis of partial wavelet coherencies, thus providing an effective account of the co-movement between the FFR and each macroeconomic variable, controlling for the co-movement with the other variable also present in the Taylor Rule.

In Figure 3, we describe the relation between the FFR and the Taylor Rule in terms of

coherency and phase-differences. Figure 1 had already described the magnitude and timing of the deviations between the FFR and the interest rate prescribed by the Taylor Rule; Figure 3 further informs at which frequencies those deviations occur.

In this figure, as throughout the paper, we present phase-diagrams for three frequency intervals, namely for cycles of period $1.5 \sim 4$ years (the short end of business cycles), cycles of period $4 \sim 8$ years (the bulk of business cycles fluctuations) and cycles of period $8 \sim 20$ years (which aim at capturing long run relations).

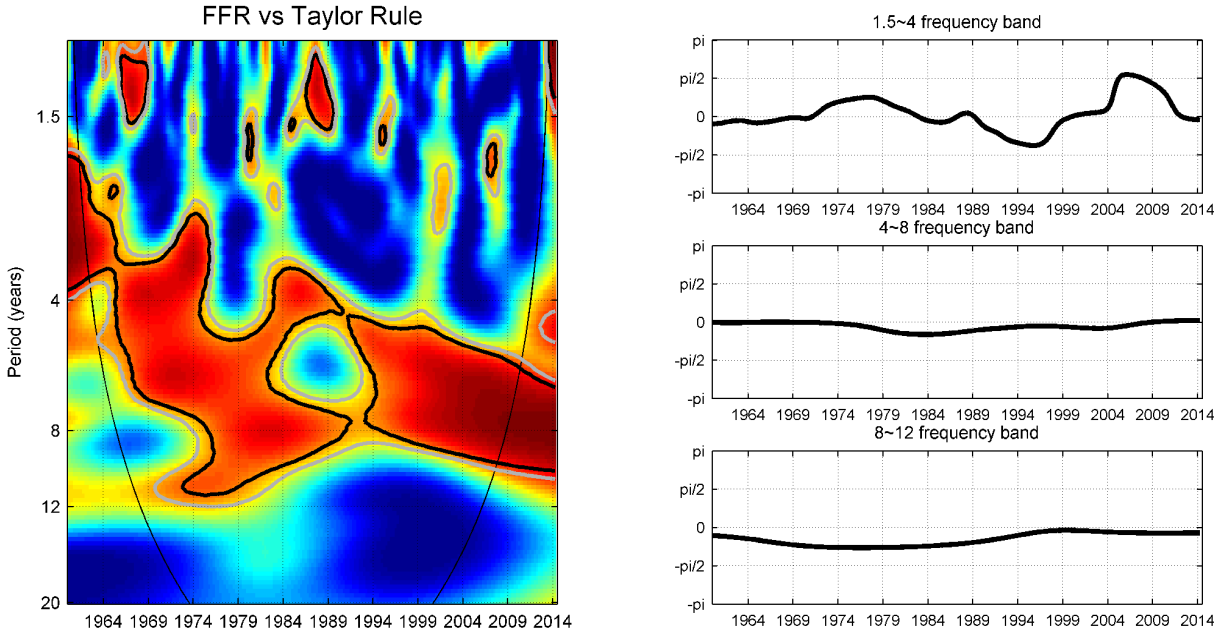

Figure 3: on the left – wavelet coherency between the effective policy interest rate and the interest rare prescribed by the Taylor rule. The black/grey contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). On the right – phase-differences between FFR and TR.

Figure 3 displays a very stable coherent region in the $4 \sim 8$ year frequency-band. The only exception is between 1984 and 1994, where we can observe a small blue lagoon of insignificant coherency. At these frequencies, for most of the time, the phase-difference is essentially zero (at most, slightly negative), suggesting that the Taylor rule is an excellent descriptor of monetary policy at business cycle frequencies. The only exception to this rule happens during the Volcker mandate, during which the phase-difference is negative, i.e. the interest rate prescribed by the

Taylor Rule leads the observed FFR.

At longer run frequencies, namely in the frequency band of $8 \sim 12$ years, there are also regions of high coherency, between 1974 and 1989, and again after 1999. However, while in the latter the phase-difference is very close to zero, in the former period the phase-difference is located between $-\pi/2$ and 0. Altogether, this means that from mid-1970s to late 1980s, monetary policy deviated from the Taylor rule, but the Taylor rule anticipated the changes in the policy interest rate. In sum, the Taylor rule behaved like an advanced indicator of the U.S. monetary policy.

For a first look at the time-frequency relation between the FFR and inflation, in Figure 4 we present the coherency and phase-diagrams between these variables, without and with control for the output gap, respectively in the first and second chart. Figure 4 is consistent with the expected result that the policy interest rate co-moves positively with the rate of inflation. The first chart shows a period of significant coherency at the $1.5 \sim 4$ years cycles between 1965 and 1975, in which the FFR leads the rate of inflation. A similar relation is detected between the mid-1960s and the late 1970s at cycles of period $4 \sim 8$ years. Taken together, these results indicate that nominal interest rates have accommodated the gradual rise of inflation that developed in the U.S. since the mid-1960s and 1979. Interestingly, there is not much statistically significant coherency between the nominal FFR and inflation during the Volcker disinflation period, and, after the early 1990s, our wavelet tools detect again a positive co-movement, virtually contemporaneous, between the nominal FFR and the rate of inflation. The second chart has, overall, yet less episodes of significant coherency between the nominal FFR and inflation: at the shorter period cycles ($1.5 \sim 4$ and $4 \sim 8$ years) the coherencies and phase-diagrams are quite erratic, and the chart essentially shows the positive co-movement between the nominal policy interest rate and inflation at longer cycles ($8 \sim 20$ years), with inflation leading movements in the nominal policy rate.

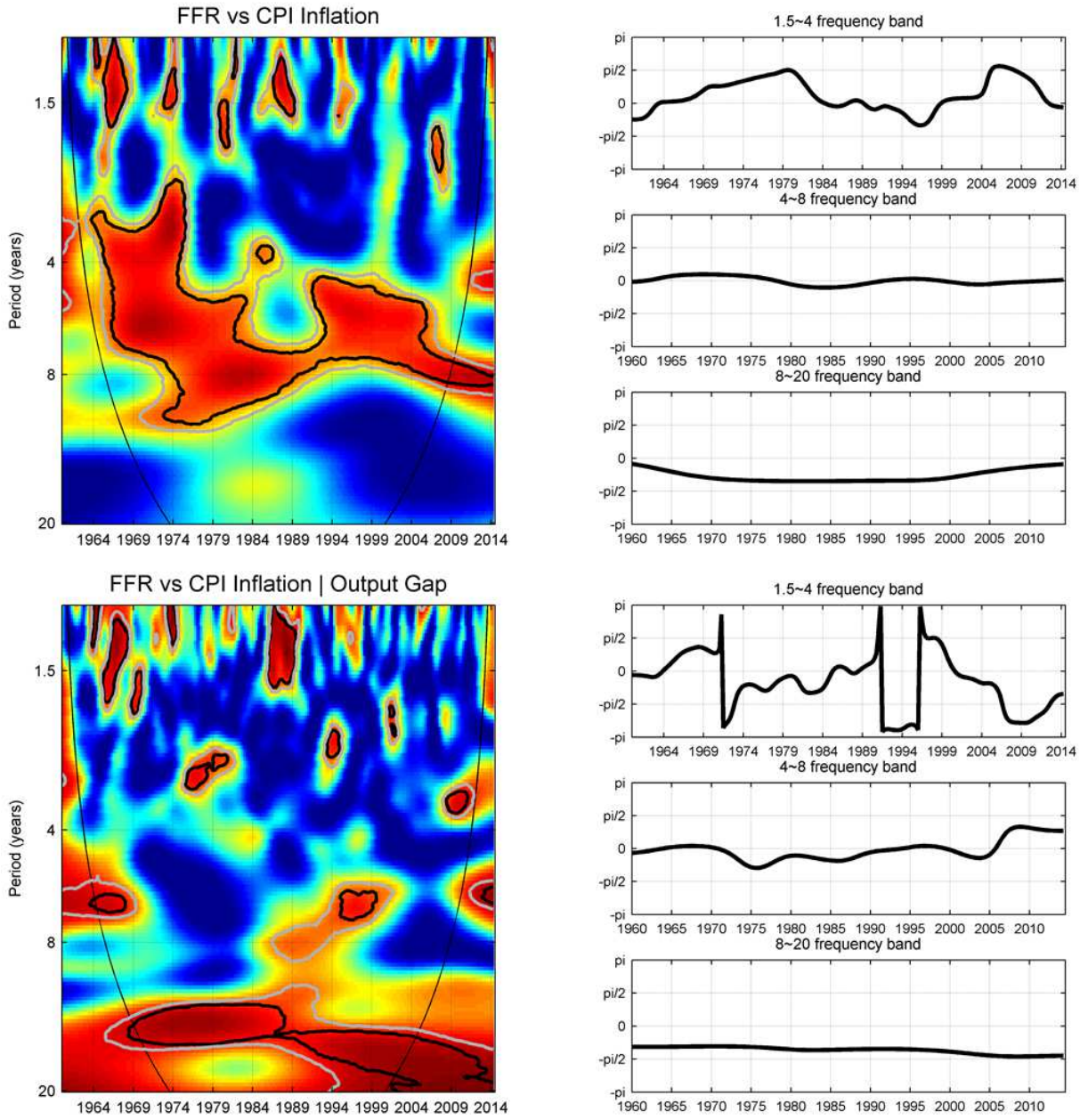

Figure 4: on the left – wavelet coherency (top) and partial wavelet coherency, after controlling for the output gap (bottom) between interest rate and inflation. The black/grey contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). On the right – phase-differences (top) and partial phase-differences (bottom) between interest rate and inflation.

Figure 4 is not of much help to describe U.S. monetary policy, because what matters, as regards monetary policy, are not the movements in nominal interest rates but those of real interest rates. Hence, we move to Figure 5, which replicates the wavelet computations of

Figure 4, but uses the real FFR (RFFR) instead of the nominal FFR; as will be made clear in our detailed interpretation of its charts, Figure 5 is entirely consistent with the expected result that inflation reacts to changes in real interest rates, with increases in the real interest rate leading to decreases in inflation.

Focusing on Figure 5, the apparent difference between its first and second charts highlights the worth of our partial wavelets methodology: while in the first chart there are no clear patterns of co-movement between the RFFR and inflation, in the second chart, after controlling for the effect of the other variable in the Taylor Rule (the output gap), a number of relevant patterns arise: these are quite in line with the main episodes in the recent history of U.S. monetary policy as well as with results elsewhere in the literature, with the advantage of complementing those with information simultaneously in the time and frequency domains.

Focusing on the second chart of Figure 5, a first result is that there is a stable co-movement between the RFFR and inflation (controlling for the output gap effect) at cycles of long period, i.e. in the frequency band of $8 \sim 20$ years. The phase-difference, which is located between $-\pi$ and $-\pi/2$, informs that the RFFR leads movements in inflation in the opposite way. Taken together, these results show that U.S. monetary policy has effectively stabilized inflation as regards cycles of long duration, in the whole sample period. Moreover, as oscillations of the RFFR have anticipated the oscillations of inflation, our evidence indicates that monetary policy has successfully been forward-looking; such result is also found for cycles of shorter periods, as will be made clear in the following paragraphs.

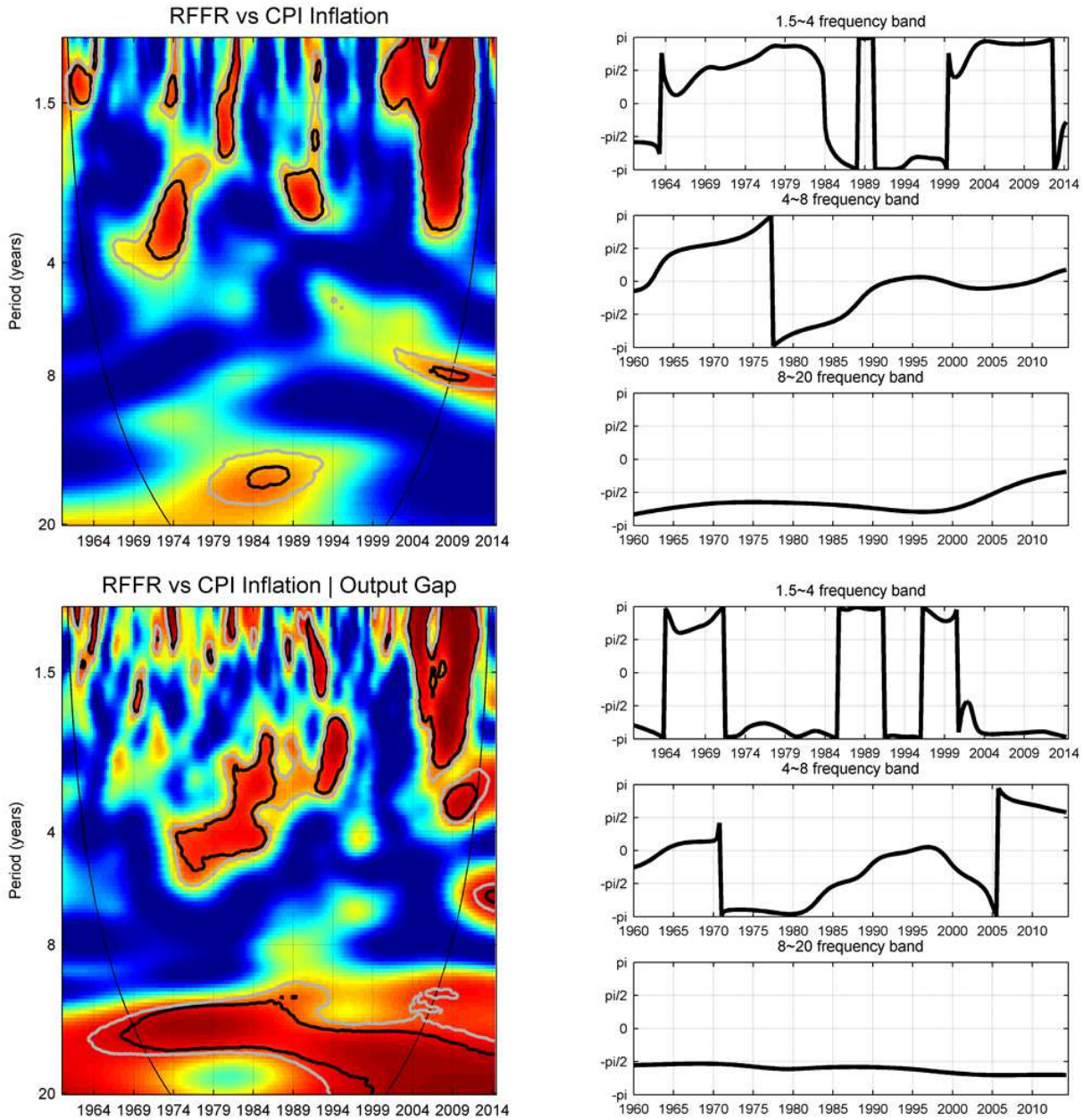

Figure 5: on the left – wavelet coherency (top) and partial wavelet coherency , after controlling for the output gap (bottom) between real interest rate and inflation. The black/grey contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). On the right – phase-differences (top) and partial phase-differences (bottom) between interest real rate and inflation.

For cycles of shorter period, a statistically significant coherency starts around 1974, for cyles in the $4 \sim 6$ years period, which then gradually spreads to cycles in the frequency range of $1.5 \sim 4$ years, from 1980 onward. Such episode of statistically significant co-movement

endures until around 1986, and is characterized by phase-differences in the interval $(-\pi, -\pi/2)$, indicating that the RFFR and inflation move in opposite directions. This episode clearly includes the Volcker disinflation, when interest rates were aggressively used to disinflate the U.S. economy, and, quite interestingly, closely corresponds to one of the periods of discretionary policy – i.e. significant deviations from the Taylor Rule – identified by Nikolsko-Rzhevskyy et al. (2014), namely 1974-1984. It further comprises the date at which many studies find a structural break in the U.S. monetary policy (e.g. Clarida et al., 2000), providing additional information that purely time or frequency-domain studies can not offer: on the one hand, our results show that the break disappeared around 1986, when policy essentially re-converged to the interest rate path prescribed by the original Taylor Rule; on the other hand, they show that the 1974-1986 regime was related to controlling inflation in short-run cycles; regarding the latter, our evidence effectively complements the frequency domain findings by Ashley et al. (2013) that, after 1979, U.S. policy interest rates reacted more intensely to inflation at cycles of period not below 3 years.

Closer to the end of the sample, our results identify a new episode of statistically significant co-movement between the RFFR and inflation, with movements in the real policy interest rate leading movements in inflation in the opposite direction, since 2003, for cycles in the frequency range of $1.5 \sim 4$ years. That episode closely matches the final period of discretionary policy identified by Nikolsko-Rzhevskyyet et al. (2014) — 2001-2013 — and the period of ad-hoc monetary policy that Taylor (2011, 2012) called the Great Deviation. Our evidence clarifies that, while interest rates may have deviated from those prescribed by the original Taylor rule (with interest rates systematically below the Rule, see Figure 1), policy maintained the usual stance of controlling inflation and the usual forward-lookingness; thus, while the Great Deviation may have fueled the Great Recession (Taylor, 2011, 2012), apparently it has not done so through transmission mechanisms directly involving the inflation rate (rather, as has been suggested elsewhere in the literature, through asset prices and financial transmission mechanisms not covered by the Taylor Rule); moreover, the figure shows that around 2012 such co-movement essentially disappeared, which is rather posterior than the usual timing of the recent loss of effectiveness of monetary policy associated to the zero lower bound of interest

rates (2009).

Another episode worth discussing occurs between 1992 and 1996. Figure 2 suggests that the FFR peaked in 1989, triggering a recession (with the trough in the output gap around 1992) and a fall in inflation, after a peak in 1991. Figure 5 shows that between 1991 and 1996 the RFFR led movements in inflation, in the usual opposite direction, for cycles of period in the range of $1.5 \sim 4$ years. Such episode highlights the standard stabilizing role of monetary policy at business cycles frequencies, when conducted according to the Taylor Rule (note that Figure 1 indicates that the actual policy rate was in line with the path prescribed by the Taylor Rule, and the literature typically classifies this period as a rules-based policy era). Interestingly, as further discussed in the analysis of Figure 6, in this episode, our partial wavelet coherency and phase-diagram between the FFR and the output gap (controlling for inflation) only capture as statistically significant the cyclical recovery, 1993-1996, indicating that oscillations of period 1.5-4 in the gap led oscillations in the FFR in the same direction.

A key overall conclusion motivated by the results in the second chart of Figure 5 is that whenever there has been a statistically significant relation between the real policy interest rate and inflation (controlling for oscillations in the output gap), it has featured a sign and a leading pattern consistent with a monetary policy that has effectively controlled inflation and, given the lags in the transmission of policy, has necessarily been forward-looking: in such episodes, cyclical oscillations in the real FFR anticipate cyclical oscillations of the inflation rate in the opposite direction. The effectiveness of monetary policy in controlling inflation is apparent for longer cycles ($8 \sim 20$ years) throughout most of the sample, and appears in cycles of short period ($1.5 \sim 4$ years) in the Volcker disinflation, but also in the early 1990s recession and in the post-2003 discretionary policy period; at cycles of period $4 \sim 8$ years, it appears about five years before the standard dating of the U.S. disinflation (i.e. around 1974) — which is a novel result uncovered by our tools — and then disappears when credibility of the new regime is generally considered to have been achieved (around 1986).

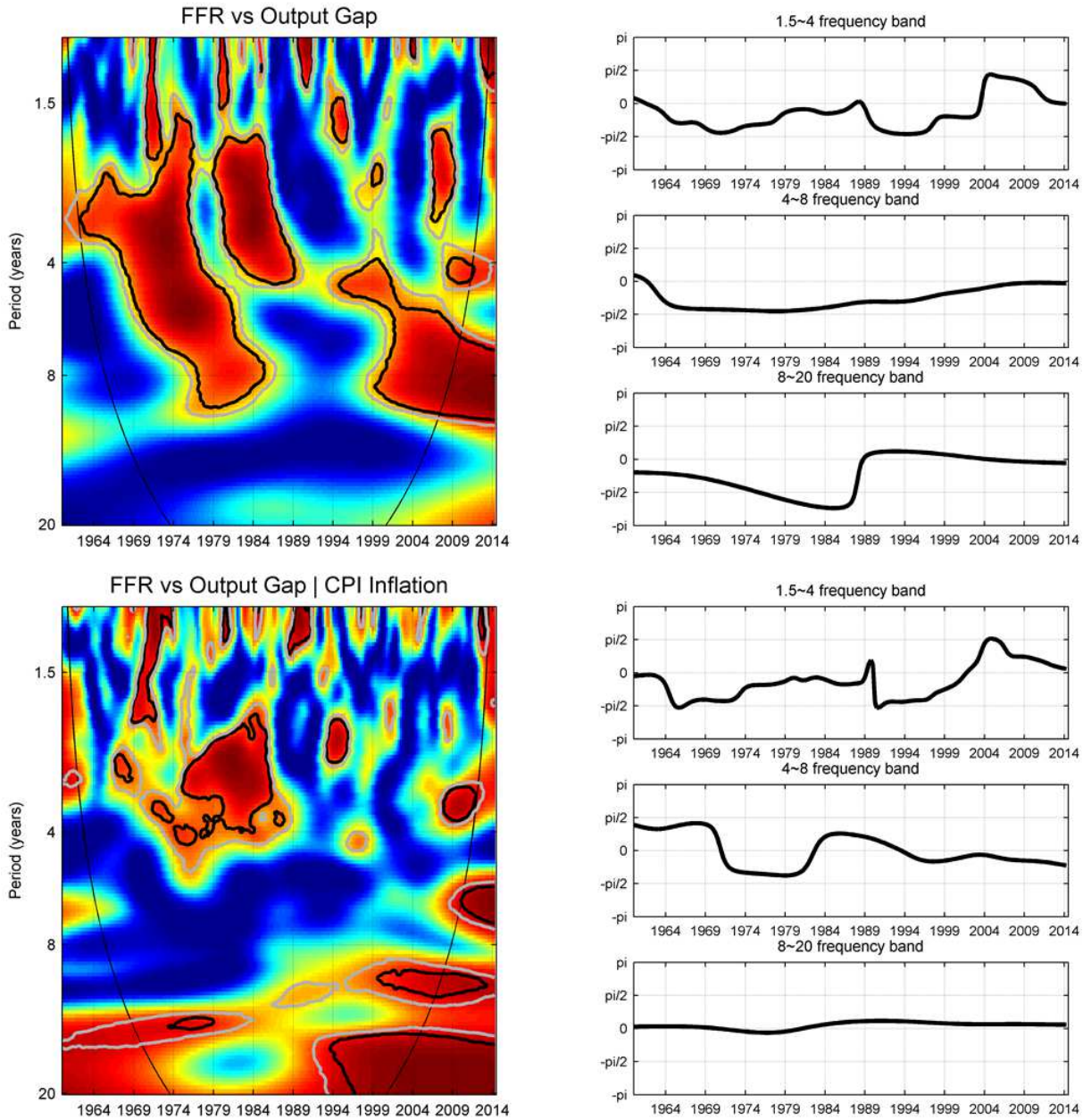

Figure 6: on the left — wavelet coherency (top) and partial wavelet coherency (bottom), after controlling for inflation, between interest rate and the output gap. The black/grey contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency — close to zero) to red (high coherency — close to one). On the right — phase-differences (top) and partial phase-differences (bottom) between interest rate and the output gap.

Figure 6 shows the coherency and phase-diagram between the FFR and the output gap. As in the previous figures, the first charts display simple bivariate measures, and the second charts show partial coherency and phase-diagrams, i.e. the relation between the FFR and the

output gap in the time-frequency domain, after controlling for the time-frequency effects of inflation over the FFR; the difference between the results from simple and partial wavelet tools confirms, again, the adequacy of our methodology to study a multivariate relation such as the Taylor Rule.

The simple bivariate coherency features extensive areas of statistically significant co-movement between the FFR and the output gap. For cycles of shorter period ($1.5 \sim 4$ years) the coherency is statistically significant essentially until the end of the 1980s, while for cycles in the range of $4 \sim 8$ years the coherency is significant from the late 1960s until 1985, and then from 1995 until the end of the sample. The evidence of statistically significant coherency for longer cycles ($8 \sim 20$ years) is scarce and limited to cycles with period not longer than 9 to 10 years, and to 2 episodes (1974-1984 and 1999 onwards). Overall, the FFR and the output gap co-move positively (phase-differences in the interval $(-\pi/2, 0)$, and fluctuations of the FFR lag behind fluctuations of the output gap; while the direction of the co-movement is consistent with a monetary policy that aims at stabilizing real economic activity, the delay in the response of the policy interest rate to oscillations of the output gap requires further interpretation; given that the output gap anticipates inflation and that policy has effectively controlled inflation anticipating its oscillations (as shown in Figure 5 and discussed above), it could be expected that the FFR and the output gap would have an approximately contemporaneous co-movement; the results indicating that the policy interest rate lagged behind such an approximately contemporaneous reaction are consistent with the difficulties that policy-makers have faced in estimating the output gap in real time (see e.g. Orphanides, 2001); with this regard, our phase-diagrams show interesting evidence that in the more recent episodes (after 1994, for cycles of period $4 \sim 8$ years, and after 1999, for cycles of period above 8 years) the magnitude of the lead of the gap over the FFR decreased and actually vanished by the end of our sample period, which we may interpret meaning that policy-makers did a better job in assessing economic slack in real time in recent times compared with the previous decades; ultimately, that implies that the efficacy of monetary policy has improved in the last 20 years.

We now consider the second charts in the figure, in which the coherency and the phase-diagram are computed after controlling for inflation. Note that in addition to describing the

time-frequency relation between the FFR and the output gap once the effects of fluctuations in inflation on the interest rate are removed, these charts describe the relation between the *real* FFR (RFFR) and the output gap, which is the relevant analysis for the assessment of monetary policy.

A first result apparent in the charts is that, after controlling for inflation, the co-movement between the RFFR and the output gap at the most typical business cycles frequencies, i.e. cycles of period $4 \sim 8$ years, essentially disappears. Hence, we focus our analysis on cycles of $1.5 \sim 4$ years on the one hand, and on cycles of $8 \sim 20$ years, on the other hand.

Regarding co-movement at higher frequencies, i.e. cycles of period $1.5 \sim 4$ years, there is also a visible decrease of the areas with statistically significant coherency, but still important episodes of significant co-movement persist. First, there are a few small islands of significant coherency for different frequencies, including very short cycles, between the late 1960s and 1974, and then between 1992 and 1996 (in this case, for cycles around a range of $2 \sim 3$ years). Second, and most importantly, there is a large area of statistically significant coherency for a wide range of frequencies ($2 \sim 4$ years cycles) between 1974 and 1987 (which, interestingly, matches an episode of significant coherency between the RFFR and inflation for cycles of period $1.5 \sim 6$ years, discussed above, and corresponds to a period of discretionary policy according to Nikolsko-Rzhevskyy et al. 2014). In all these episodes, the phase diagram is consistently located in the interval $(-\pi/2, 0)$, meaning that fluctuations in the RFFR have lagged behind those in the output gap and that these series have co-moved positively, which is, again, consistent with a monetary policy effectively stabilizing real activity but responding with a lag to the ex-post output gap.

A particularly important implication of our results is that, during the 1979-1986 period, in which the FED disinflated the U.S. economy and conquered credibility, the changes in the real policy interest rate do not appear to have caused a recession beyond the one that would have been needed to achieve disinflation and credibility. Indeed, our time-frequency results indicate that, at the higher business cycles frequencies, once the FFR-inflation co-movements are controlled for, RFFR increased whenever the economy was expanding and decreased following economic downturns. To summarize, combining these results with those in the second charts of

Figure 5, during the Volcker disinflation, monetary policy — measured as changes in the real policy interest rate — was deliberately and effectively used to reduce inflation (for cycles of period between 2 and 6 years) but, beyond such control of inflation, aimed at stabilizing real economic activity following its upturns or downturns, albeit with a lag (for cycles of period between 2 and 4 years). These findings — which would not have been detected without our partial wavelet tools — are new evidence, provided by a new perspective of analysis, about the real effects of the Volcker disinflation (on the old debate about the Volcker disinflation see e.g. Goodfriend and King, 2005).

Regarding cycles of period longer than 8 years, there is a very brief episode of statistically significant coherency in 1974-79 for cycles of period around 12 years, in which the phase diagram indicates that, as happens for the short cycles, the output gap leads changes in the RFFR. For long cycles, the most important result, however, occurs from 1994 onward, when there is a statistically significant coherency between the RFFR and the output gap for a wide range of cyclical frequencies; initially, the significant coherency occurs at cycles of very long period, but after 1999 there is also a significant coherency for cycles of period around $8 \sim 12$ years. For both, the phase diagram is consistently located in the interval $(0, \pi/2)$ indicating that changes in the RFFR anticipated changes in the output gap (once the co-movement between the FFR and inflation is controlled for) in the same direction. A significant coherency with a similar lead-lag pattern is also detected for cycles of period $3 \sim 4$ years between 2006 and 2010. These results are reminiscent of the structural break found by Ashley, Tsang and Verbrugge (2013) at 1995 in their frequency-dependent Taylor Rule, which they related to the New Economy issue addressed by Ball and Tchaidze (2002), among others. Our evidence indicates that since 1994 (i.e. ahead of the Great Recession) monetary policy has been accommodative: real interest rates have not moved strongly enough to prevent the ups and downs of real activity and so monetary policy has fueled expansions and aggravated downturns, especially at cycles of long period (larger than 8 years) but also for shorter cycles ($3 \sim 4$ years) since 2006.

4 Conclusions

In this paper we assess the relation between the federal funds rate (FFR), inflation and the output gap — the Taylor Rule — in the U.S. for 1960:I-2014:II, in the time-frequency domain. The analysis is motivated by the pervasiveness of Taylor's (1993) rule as a simple description, a robust prescription, and an eventual measure of accountability of monetary policy in the U.S.. The time-frequency framework is motivated by arguments and evidence of instability, nonlinearity and deviations of actual policy from the original rule, on the one hand, and by arguments and evidence of frequency-dependence of monetary policy-makers' decisions, on the other hand.

We use continuous time wavelet tools that detect statistically significant co-movements — coherency — and identify synchronization, leads or lags in such co-movements — phase diagram. We begin with simple bivariate tools, but then focus on partial wavelet tools, which are particularly well suited for the analysis of a simple multivariate relation such as the Taylor Rule: the partial coherency and partial phase diagrams assess, across time and the relevant frequencies, the co-movement between the monetary policy interest rate (FFR) and each of the main macroeconomic variables, properly controlling for the effect of the co-movement of the FFR with the other.

The paper uncovers a wide set of stylized facts about the last five and a half decades of monetary policy in the U.S. . Many of our results are entirely new and could only be detected with a time-frequency approach. In this concluding remarks we highlight only a few, focusing on results obtained with the partial wavelet tools (and, therefore, references to a co-movement between the interest rate and one macroeconomic variable should be interpreted as a co-movement with the real interest rate, controlling for the effects of the other macroeconomic variable in the Rule).

Overall, we find that the U.S. monetary policy has been forward-looking with regard to the inflation rate, at least since the mid-1970s, with the real FFR anticipating changes in inflation in the opposite direction. The Federal Reserve (FED) interest rate policy has effectively stabilized the low-frequency cyclical fluctuations of inflation since the mid-1960s, with a particular strength in 1970-1986 and from 1992 onward. Roughly coinciding with a period described in

some literature as one of discretionary policy, in 1974-1986 the real FFR has co-moved significantly with inflation at cycles of an intermediate duration, first accommodating the inflationary pressures (for cycles of period of $4 \sim 6$ years) and then, after 1979, achieving disinflation and the FED's credibility (for cycles of period $2 \sim 5$ years). During most of the Great Moderation there was not much significant co-movement between the policy interest rate and inflation (with the exception of 1992-96), but after 2003, when policy appears to have deviated considerably from the original Taylor Rule, there was a new episode of significant inverse co-movement of the FFR and inflation for shorter cycles ($1.5 \sim 4$ years). In contrast, U.S. monetary policy has been forward-looking with regard to the ex-post output gap only after the mid-1990s: initially for cycles of very long period, after 1999 for cycles of period $8 \sim 12$ years, and between 2006 and 2010 for cycles of period $3 \sim 4$ years. Yet, during these episodes, the policy rate co-moved positively with the output gap and so monetary policy appears to have been accommodative, rather than anti-cyclical. Before 1994, whenever the co-movement between the RFFR and the output gap was significant, policy reacted with a lag – probably due to problems of real-time information – but was stabilizing, as positive changes in the gap were followed by positive changes in the RFFR. In addition to the 1992-96 period, there was an important era of significant co-movement in 1974-1987, for cycles of period $2 \sim 4$ years – interestingly, both episodes with a significant co-movement between the FFR and inflation and, in the latter, corresponding to a period of deviation from the Taylor Rule. A particularly important implication of our results is that between 1979 and 1986, when the FED disinflated the U.S. economy and conquered credibility, monetary policy does not appear to have caused a recession beyond the one that would have been needed to achieve disinflation and credibility.

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