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Abstract

This study considers public education policy and its impact on growth and welfare across generations. In particular, the study compares two fiscal perspectives—tax finance and debt finance—and shows that in a competitive equilibrium context, the growth and utility in the debt-finance case could be higher than those in the tax-finance case in the long run. However, the opposite occurs when the policy is shaped by politics. When the degree of parents' altruism is low, they choose debt finance in their voting, despite its long-run worse performance because a current generation can pass the cost of debt repayment to future generations.

- Keywords: Economic growth, Human capital, Public debt, Political equilibrium
- JEL Classification: D70, E24, H63,

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1 Introduction

This paper considers the political determinants of fiscal policy and its impact on growth and welfare across generations with a focus on public education expenditures, which work as both an inter-generational transfer from parents to children and an engine of economic growth. Parents with an altruistic concern for their children are likely to support public education and bear the tax burden because they can benefit from highly educated children. However, when the government has access to debt finance, parents may prefer debt to taxes because debt enables the parents to shift the fiscal burden to future generations.

The discussion above leads to the following questions: how do debt and tax finance differ in terms of growth and welfare across generations and which fiscal stance do voters adopt? To consider these questions, this study presents a three-period overlapping-generations model with physical and human capital accumulation (see, e.g., Lambrecht, Michel, and Vidal, 2005; Kunze, 2014). Parents care about their children's wage income. Public education spending and parental human capital are inputs in the human capital formation process. Governments finance spending through tax on labor income and issue bonds if they have access to debt finance.

Within this framework, this study first investigates an economic equilibrium where fiscal policy is exogenously given. In particular, we assume a fixed education expenditure-GDP ratio and compare the case of 100% financing via labor income tax (henceforth, tax finance) to the case of financing through tax and public bond issues (henceforth, debt finance). The analysis shows that debt financing results in multiple equilibria with a negative correlation between capital and debt: one is a saddle point distinguished by low capital and high debt, and the other is a sink point distinguished by high capital and low debt. The analysis also shows that under certain conditions, the debt finance case has higher growth and utility than in the tax-finance case in the long run.

The multiple-equilibria story seems to fit well with the evidence in developed countries. Figure 1 illustrates this case. However, this approach has three limitations. First, the equilibrium converges to a sink point for most initial conditions. This makes it difficult to illustrate the variety patterns of capital and debt accumulation in Figure 1. Second, the multiple equilibria fail under empirically plausible parameter values. Finally, and most importantly, the analysis takes fiscal policy as given, and does not demonstrate the differences in growth and welfare between the two financing cases when fiscal policy choice has an effect.

[Figure 1 here.]

In order to overcome these issues, we investigate the politics of fiscal policy formation. In each period, three successive generations, the young, the middle-aged, and the elderly,

can participate in voting. However, the elderly are indifferent between any two policies because they have no tax burden and obtain no benefit from public education expenditure. The young may benefit from the public education in the future, but they are below the voting age. Therefore, the government's aim is to choose a fiscal policy that maximizes the utility of the middle-aged. Given this process, we characterize the political equilibrium in both the tax- and debt-finance cases, which yields three main findings.

First, the government's choice of fiscal policy depends on the parents's degree of altruism toward their children. Greater altruism encourages parents to leave higher wages to their children, and the parents will do so by decreasing public bond issues and thus weakening the crowding-out effect on capital accumulation. In particular, there is a threshold level of altruism such that the government finds it optimal (not) to issue public bonds when altruism is (above) below the threshold level. The result suggests that the degree of altruism is a key explanation for the cross-country difference in fiscal policy.

Second, the growth rate in the debt-finance case is lower than that in the tax-finance case. This result is in sharp contrast to that in the economic equilibrium case. The debt overrides capital accumulation and adds the cost of debt repayment, reducing the government's available resources, and thus decreasing public education expenditure as an engine of economic growth. The peculiar negative impact on the expenditure in the debt-finance case explains the result.

Third, debt finance has a trade-off between the present and future generations. When altruism is below the threshold level, the government in each period chooses debt finance for given physical and human capital levels. However, the debt finance discourages human capital accumulation, and thus bequeaths less physical and human capital to future generations. This implies that future governments must choose a fiscal policy subject to lower levels of capital than those expected from tax finance. In other words, the choice of the future governments is itself optimal for given capital levels, but they obtain less utility from the debt finance than tax finance.

The rest of the paper is structured as follows. We first present a literature survey in Subsection 1.1. Thereafter, Section 2 presents the model. Section 3 characterizes the economic equilibrium and Section 4 describes the political equilibrium and investigates the choice of fiscal policy and its impact on growth and welfare. Section 5 offers some extensions to the basic model, and Section 6 provides concluding remarks.

1.1 Literature Review

Our study is related to research into public education and economic growth in terms of parental altruism toward children. Examples include studies from Glomm and Kaganovich (2003, 2008) and Glomm and Ravikumar (1992, 2001, 2003) based in the competitive

equilibrium context and assuming that the central government can control policy processes and outcomes across periods. Under this assumption, these researchers investigate how changes in educational policy affect growth and welfare across generations and the optimal policy in terms of long-term growth and/or welfare.

Several studies attempted to relax the assumption by introducing voting into the policy-making process. These studies tend to focus on factors that affect policy, such as aging (Zhang, Zhang, and Lee, 2003; Gradstein and Kaganovich, 2004; Kunze, 2014), inequality (Saint-Paul and Verdier, 1993), expectations of future policy (Glomm and Ravikumar, 1995), private education as an alternative to public education (Gradstein and Justman, 1996), social cohesion (Gradstein and Justman, 2002), and social security (Kemnitz, 2000; Poutvaara, 2006; Soares, 2006; Gonzalez-Eiras and Niepelt, 2012; Iturbe-Ormaetxe and Valera, 2012; Kaganovich and Meier, 2012; Naito, 2012; Lancia and Russo, 2016; Ono, 2015; Ono and Uchida, 2016). However, all of these studies assume financing only from taxes and where the government budget constraint is balanced each period. In other words, debt financing of public education is abstracted away from their analyses.

One exception is Zhang's (2003) study that demonstrates the optimal policy for public education when the government can issue public bonds. Zhang (2003) is thus based in the competitive equilibrium context, and says nothing about the political determinants of debt finance policy. Our study instead demonstrates endogenous determinants of debt-financed public education expenditures and its impact on growth and welfare across generations. With this analysis, we can evaluate the relative performance of the two fiscal stances.

Our study is also related to the literature on the golden rule of public finance (see, for example, Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a, 2004b; Yakita, 2008; Minea and Villieu, 2009; Greiner, 2012). The rule allows the government to run a budget deficit as long as the government uses this to finance productive public capital expenditures. In the present framework, public education expenditures represent productive public capital expenditures as in Greiner (2008), who investigates the effect of different budgetary regimes on growth and welfare. The present study differs from Greiner (2008) in that it incorporates the effect of voting on fiscal policy, and considers its welfare effects across generations.

2 Model

The discrete time economy starts at period 0 and consists of overlapping generations. Individuals are identical within a generation, and live for three periods: youth, middle, and elderly ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for the period t is N_t, and the population grows at a constant rate of $n(>-1): N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, individuals make no economic decisions and receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their elderly years and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle aged and endowed with h_t units of human capital. The individual supplies them inelastically in the labor market, and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle age becomes:

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period $t+1$ budget constraint in the elderly age is

$$d_{t+1} \leq R_{t+1} s_t,$$

where d_{t+1} is consumption, $R_{t+1}(>0)$ is the gross return from investment in capital, and $R_{t+1}s_t$ is the return from savings.

Period t middle-aged individuals care about their children's income , $w_{t+1}h_{t+1}$. Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and the parent's human capital, h_t. In particular, h_{t+1} is formulated by the following equation:

$$h_{t+1} = D\,(x_t)^{\eta}\,(h_t)^{1-\eta},$$

where $D(>0)$ is a scale factor, and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to public education spending.

We note that private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In the present study, we abstract private education from the analysis to simplify the presentation of the model and to focus on the effect of public education on growth and utility.

We assume that parents are altruistic toward their children and are concerned about their income in middle age, $w_{t+1}h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function of the logarithmic form:

$$U_t = \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1} h_{t+1},$$

where $\beta \in (0,1)$ is a discount factor and $\gamma(>0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and human capital production function into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \beta \ln R_{t+1} s_t + \gamma \ln w_{t+1} D\left(x_t\right)^{\eta} \left(h_t\right)^{1-\eta}.$$

By solving the problem, we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta} \cdot (1-\tau_t) w_t h_t, \tag{1}$$

$$c_t = \frac{1}{1+\beta} \cdot (1-\tau_t) w_t h_t \text{ and } d_{t+1} = \frac{\beta R_{t+1}}{1+\beta} \cdot (1-\tau_t) w_t h_t. \tag{2}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t using two inputs, aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. The aggregate output is given by:

$$Y_t = A\left(K_t\right)^{\alpha} \left(H_t\right)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share.

Let $k_t \equiv K_t / H_t$ denote the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are:

$$w_t = (1-\alpha) A\left(k_t\right)^{\alpha}, \text{ and } \rho_t = \alpha A\left(k_t\right)^{\alpha-1}, \tag{3}$$

where w_t and ρ_t are labor wages and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices.

2.3 Government Budget Constraint

Public education expenditures are financed by both tax on labor income and public bond issues. Let B_t denote the aggregate inherited debt. A government budget constraint in period t is:

$$B_{t+1} + \tau_t w_t h_t N_t = N_{t+1} x_t + R_t B_t,$$

where B_{t+1} is newly issued public bonds, $\tau_t w_t h_t N_t$ is the aggregate labor income tax revenue, $N_{t+1} x_t$ is the aggregate expenditure for public education, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model, and assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the above expression, we obtain a per-capita form of the constraint:

$$(1+n)\hat{b}_{t+1} + \tau_t w_t h_t = (1+n)x_t + R_t \hat{b}_t, \tag{4}$$

where $\hat{b}_t \equiv B_t/N_t$ is the per-capita public debt. We use the notation $\hat{b}_t$ rather than b_t to distinguish the per-capita public debt, $\hat{b}_t \equiv B_t/N_t$, from the public debt per human capital, $b_t \equiv B_t/H_t$, which we introduce in the next section.

In an alternative case, the government imposes a rule of balancing the budget in each period. That is, the government satisfies the following constraint:

$$\begin{cases} \tau_0 w_0 h_0 = (1+n)x_0 + R_0 \hat{b}_0, \\ \tau_t w_t h_t = (1+n)x_t, t \geq 1. \end{cases} \tag{5}$$

This rule is stricter than the rule where debt cannot increase across periods (see, e.g., Azzimonti, Battaglini, and Coate, 2010). Our study thus adopts a stricter rule to demonstrate the impact of debt financing public expenditure simply.

3 Economic Equilibrium

Public bonds are traded in a domestic capital market. The market clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. Using $k_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function in (1), we can rewrite the condition as:

$$(1+n) \cdot \left(k_{t+1} h_{t+1} + \hat{b}_{t+1} \right) = \frac{\beta}{1+\beta} \cdot (1-\tau_t) w_t h_t. \tag{6}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\{\tau_t, x_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\left\{ c_t, d_t, s_t, k_{t+1}, \hat{b}_{t+1}, h_{t+1} \right\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0), \hat{b}_0(\geq 0)$ and $h_0(>0)$ such that (i) given $(w_t, R_{t+1}, \tau_t, x_t)$, $\left(c_t^y, c_{t+1}^o, s_t\right)$ solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves a firm's profit maximization problem; (iii) given $\left(w_t, h_t, k_t, \hat{b}_t \right)$, $\left(\tau_t, x_t, \hat{b}_{t+1} \right)$ satisfies the government budget constraint; (iv) $\rho_t = R_t$ holds; and (v) the capital market clears: $(1+n) \cdot \left(k_{t+1} h_{t+1} + \hat{b}_{t+1} \right) = s_t$.

In the following, we consider two cases: a "tax-finance" case where public education expenditures are financed solely through labor income tax, and a "debt-finance" case where the expenditures are financed by both labor income taxes and public bond issues. We characterize an economic equilibrium for each case, and then compare the two cases

in terms of steady-state growth rates and utility. In particular, we identify the condition in which shifting from tax to debt finance improves steady-state growth and utility.

3.1 Tax-finance Economic Equilibrium

In the tax-finance case, the government budget constraint is given by (5). Here, we impose the following expenditure rule:

$$\frac{N_{t+1}x_t}{Y_t} = \frac{(1+n)x_t}{A\left(k_t\right)^{\alpha}h_t} = X_{econ} \in (0, 1-\alpha), \tag{7}$$

where X_{econ} is a constant parameter. The rule indicates that the government fixes the ratio of public education spending to GDP at a constant rate. The upper limit of X_{econ}, $1-\alpha$ implies that the government can use a part of the labor income for public education expenditures through taxation. We introduce this rule to compare the tax- and debt-finance cases under the same expenditure structure.

Given X_{econ}, a tax rate is determined satisfying the constraint (5). Thus, the tax rate under the expenditure rule in (7) is given by:

$$\begin{cases} \tau_0 = \frac{X_{econ}}{1-\alpha} + \frac{\alpha}{1-\alpha}\cdot\frac{\hat{b}_0}{k_0h_0}, \\ \tau_t = \frac{X_{econ}}{1-\alpha} \text{ for } t \geq 1. \end{cases} \tag{8}$$

Substituting the tax rate and the human capital production function into the capital market clearing condition in (6) leads to the following law of the motion of capital:

$$\begin{cases} k_1 = \frac{\frac{\beta}{1+\beta}\left(1-\frac{X_{econ}}{1-\alpha}-\frac{\alpha}{1-\alpha}\cdot\frac{\hat{b}_0}{k_0h_0}\right)(1-\alpha)}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^{\eta}}\left(A\left(k_0\right)^{\alpha}\right)^{1-\eta}, \\ k_{t+1} = \frac{\frac{\beta}{1+\beta}\left(1-\frac{X_{econ}}{1-\alpha}\right)(1-\alpha)}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^{\eta}}\left(A\left(k_t\right)^{\alpha}\right)^{1-\eta} \text{ for } t \geq 1 \end{cases}$$

The expression suggests that there is a unique, stable, steady-state equilibrium described by:

$$k = k|_{tax} \equiv \left[\frac{\frac{\beta}{1+\beta}\left(1-\frac{X_{econ}}{1-\alpha}\right)(1-\alpha)}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^{\eta}}(A)^{1-\eta}\right]^{1/\{1-\alpha(1-\eta)\}}. \tag{9}$$

The following proposition summarizes the results up to this point.

Proposition 1. *Assume the expenditure rule in* (7)*. There is a unique, stable steady-state tax-finance economic equilibrium.*

3.2 Debt-finance Economic Equilibrium

We next consider the debt-finance case, where (4) represents the government's budget constraint. We follow the expenditure rule in the tax finance case to compare the two

cases using the same expenditure structure. We also assume that the tax rate is constant across periods, $\tau = \tau_{debt}$ for all t, and that τ_{debt} satisfies:

$$\tau_{debt} \leq \frac{X_{econ}}{1-\alpha}. \tag{10}$$

The assumption in (10) indicates that the tax revenue is not sufficient to finance public education expenditures, and the government covers the revenue shortfall by issuing public bonds.

Adopting the profit maximization conditions in (3), we can reformulate the government budget constraint in the debt-finance case as:

$$(1+n)\hat{b}_{t+1} + \tau_{debt}(1-\alpha)A\left(k_t\right)^{\alpha} h_t = (1+n)x_t + \alpha A\left(k_t\right)^{\alpha-1}\hat{b}_t.$$

Let $b_t \equiv \hat{b}_t/h_t = B_t/H_t$ denote public debt per human capital. The above expression is thus rewritten as:

$$(1+n)b_{t+1}\frac{h_{t+1}}{h_t} + \tau_{debt}(1-\alpha)A\left(k_t\right)^{\alpha} = X_{econ}A\left(k_t\right)^{\alpha} + \alpha A\left(k_t\right)^{\alpha-1} b_t. \tag{11}$$

Using the human capital production function, we reformulate the government budget constraint in (11) as:

$$(1+n)b_{t+1}D\left(\frac{X_{econ}}{1+n}A\left(k_t\right)^{\alpha}\right)^{\eta} = \left[X_{econ} - \tau_{debt}(1-\alpha)\right]A\left(k_t\right)^{\alpha} + \alpha A\left(k_t\right)^{\alpha-1} b_t. \tag{12}$$

This is the first key equation in the debt-finance economic equilibrium. The other key equation is the capital market clearing condition, which we can derive by substituting (12) into the capital market clearing condition in (6):

$$\begin{aligned} k_{t+1} = {} & \frac{1}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^{\eta}} \\ & \times \left[\left\{\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - \left(X_{econ} - \tau_{debt}(1-\alpha)\right)\right\}\left(A\left(k_t\right)^{\alpha}\right)^{1-\eta} - \alpha\left(A\left(k_t\right)^{\alpha}\right)^{1-\eta}\frac{b_t}{k_t}\right]. \end{aligned} \tag{13}$$

Therefore, given an initial condition, $k_0 > 0$ and $b_0 \geq 0$, the sequence of physical capital and public debt per human capital, $\{k_t, b_t\}$, in a debt-finance economic equilibrium is characterized by (12) and (13), respectively. The following proposition establishes the condition for the existence and stability of a steady-state debt-finance economic equilibrium.

Proposition 2. *Assume the expenditure rule in (7) and that the following conditions hold:*

$$\begin{aligned} & \tau_{debt}(1-\alpha) < X_{econ} < \bar{X}_{econ}, \\ & 0 < \tau_{debt} \leq 1 - \frac{1+\beta}{\beta}\cdot\frac{\alpha}{1-\alpha}, \text{ and } \alpha < \frac{\beta}{1+2\beta}, \end{aligned}$$

where

$$\bar{X}_{econ} \equiv \alpha + \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) - 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2}.$$

There are two steady-state debt-finance economic equilibria with $b > 0$, denoted by e_l and e_h, where e_l (e_h) is distinguished by a lower (higher) k. Then, e_l is a saddle, and e_h is a sink.

Proof. See Appendix A.1.

[Figure 2 here.]

Figure 2 illustrates an example phase diagram for the planner system (12) and (13). The phase line denoted by KK is a set of points (k, b) for which there is no change in k in the capital market clearing condition in (13). The phase line denoted by BB is a set of points (k, b) for which there is no change in b in the government budget constraint in (12). The vertical line at $k = \hat{k}$ indicates that financial balance state varies around this point, where $\hat{k}$ is defined by:

$$\hat{k} \equiv \left[\frac{\alpha A}{(1+n)D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}}\right]^{1/(1-\alpha(1-\eta))}.$$

Below (above) $k = \hat{k}$, the government budget has a negative (positive) debt in the steady state. The two intersections in phase lines KK and BB correspond to the two steady-state equilibria with positive debt, which are common in the literature (see, e.g., Azariadis, 1993; de la Croix and Michel, 2002).

Proposition 2 demonstrates the three conditions for the existence of multiple steady-state equilibria with positive debt. The first condition determines the range of X_{econ} for the two steady-state equilibria with $b > 0$. In particular, the first inequality condition, $\tau_{debt}(1-\alpha) < X_{econ}$ implies that the government must finance part of the expenditure by borrowing in the capital market. If the condition fails, tax revenue is sufficient to cover the expenditure, so the steady-state equilibrium would have a negative public debt, which is out of scope for this study.

The second inequality condition, $X_{econ} < \bar{X}_{econ}$ ensures that the two steady-state equilibria exist. If the condition fails, the government must finance large expenditures for public education by issuing public bonds. However, this induces high debt repayment burdens on households, which creates a negative income effect on saving and thus implies a low debt level that clears the capital market. Therefore, given k, the government budget constraint is satisfied for a high level of public debt, while the capital market clears for a low level of public debt. That is, there is no debt level that satisfies both the government

budget constraint and the capital market clearing condition if X_{econ} is above the critical level.

The second condition requires a tax rate below the critical value of $1-\frac{1+\beta}{\beta}\cdot\frac{\alpha}{1-\alpha}$. This condition determines the upper bound of X_{econ} given in the first condition. The third condition ensures that the tax rate in the second condition is non-empty. If the third condition fails, there is no tax rate $\tau_{debt}\in(0,1)$ satisfying the second condition.

3.3 Growth and Utility in the Steady States

The previous two subsections describe the steady-state equilibrium for the tax- and debt-finance cases. To evaluate their relative performance, we here compare the two cases in terms of growth rate and utility in the steady state, and show the results.

Proposition 3. (i) *The steady-state growth rate in the debt-finance economic equilibrium is higher than that in the tax-finance economic equilibrium if* $X_{econ}>(1-\alpha)-\alpha(1+\beta)/\beta$; (ii) *the steady-state utility in the debt-finance economic equilibrium is higher than that in the tax-finance economic equilibrium if* $X_{econ}>(1-\alpha)-\alpha(1+\beta)/\beta$ *and* $\gamma>(1-\alpha)\beta/\alpha$.

Proof. See Appendix A.3.

To compare the two cases, we here denote by $h'/h|_{\text{tax}}$ and $U|_{\text{tax}}$ the steady-state growth rate and utility for the tax-finance case, respectively. $z|_{\text{tax}}$ and $h_{t,\text{tax}}$ denote the steady-state level of $z(=k,b)$ and the period-t human capital for the tax-finance case, respectively. The corresponding notations are $h'/h|_{\text{debt}}$, $U|_{\text{debt}}$, $z|_{\text{debt}}$ and $h_{t,\text{debt}}$ for the debt-finance case. The variable h' in the expressions $h'/h|_{\text{tax}}$ and $h'/h|_{\text{debt}}$ denotes the next-period h.

To understand the results in Proposition 3, let us first compare the steady-state growth rates. Given the expenditure rule in (7), the public education expenditures are $x=X_{econ}A(k)^{\alpha}h/(1+n)$ for both cases. We substitute this expression into the human capital production function, $h'=D(x)^{\eta}(h)^{1-\eta}$, to obtain:

$$\frac{h'}{h}=D\left(\frac{X_{econ}}{1+n}A(k)^{\alpha}\right)^{\eta}. \tag{14}$$

This is common for both cases.

Substituting the steady-state level of capital in (9) into (14) yields the steady-state growth rate in the tax-finance economic equilibrium:

$$\left.\frac{h'}{h}\right|_{\text{tax}}=\underbrace{\left[D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}}_{(\#1)}\cdot\underbrace{\left[\frac{\frac{\beta}{1+\beta}\left((1-\alpha)-X_{econ}\right)A}{1+n}\right]^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}}_{(\#2)}. \tag{15}$$

In the debt-finance case, the steady-state capital level is higher than $\hat{k}$, as illustrated in Figure 2. Therefore, the growth rate in the debt-finance economic equilibrium satisfies $h'/h|_{\text{debt}} > \left(X_{econ}A(\hat{k})^{\alpha}/(1+n)\right)^{\eta}$, that is,

$$\left.\frac{h'}{h}\right|_{\text{debt}} > \underbrace{\left[D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}}_{(\#1)} \cdot \underbrace{\left[\frac{\alpha A}{1+n}\right]^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}}_{(\#3)}. \tag{16}$$

The expressions in (15) and (16) suggest that the ratio of public education expenditure to GDP, X_{econ}, affects the steady-state growth rates in several ways. First, a higher expenditure ratio increases the growth rate for both cases, as shown by the term (#1) in (15) and (16). However, the tax-finance case also contains a negative effect of public education expenditures as shown by the term (#2) in (15). A higher ratio implies a larger household tax burden. This discourages savings and capital accumulation, thereby reducing public education expenditures and economic growth in the next period. This negative effect does not appear in the debt-finance case, as demonstrated by the term (#3) in (16) because the public bond issues in the debt-finance case compensates for the tax burden. Therefore, the growth rate in the debt-finance case is higher than that in the tax-finance case if the term (#3) outweighs the term (#2); that is, if $X_{econ} > (1-\alpha)\left(1-\alpha(1+\beta)/(1-\alpha)\beta\right)$.

Next, we compare the steady-state utility in both cases. Given k and b in the steady state, $U|_{\text{tax}}$ and $U|_{\text{debt}}$ are computed as follows:

$$\begin{aligned} U|_{\text{tax}} &= (1+\beta)\ln\left((1-\alpha)-X_{econ}\right)A\left(k|_{\text{tax}}\right)^{\alpha} + \left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln k|_{\text{tax}} \\ &\quad + \gamma\eta\ln\frac{X_{econ}}{1+n}A\left(k|_{\text{tax}}\right)^{\alpha} + (1+\beta+\gamma)\ln h_{t,\text{tax}} + C_0, \end{aligned} \tag{17}$$

$$\begin{aligned} U|_{\text{debt}} &= (1+\beta)\ln\left[\left((1-\alpha)-X_{econ}\right)A\left(k|_{\text{debt}}\right)^{\alpha} + \left\{-\alpha A\left(k|_{\text{tax}}\right)^{\alpha-1} + (1+n)\left.\frac{h'}{h}\right|_{\text{debt}}\right\}\cdot b|_{\text{debt}}\right] \\ &\quad + \left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln k|_{\text{debt}} + \gamma\eta\ln\frac{X_{econ}}{1+n}A\left(k|_{\text{debt}}\right)^{\alpha} + (1+\beta+\gamma)\ln h_{t,\text{debt}} + C_0, \end{aligned} \tag{18}$$

where C_0 includes constant terms. Appendix A.3 provides the derivation of (17) and (18) and the definition of C_0.

For both cases, the first term represents disposable income, the second term includes the rate of return from savings and the next generation's wage, the third term shows the public education expenditures, and the fourth term summarizes the human capital accumulation effect. By comparing (17) and (18), we find that $U|_{\text{debt}} > U|_{\text{tax}}$ holds if each term in Eq. (18) is larger than the corresponding term in Eq. (17).

After some calculation, we find that (i) in the debt-finance case, the debt repayment $-\alpha A\left(k|_{\text{tax}}\right)^{\alpha-1} b|_{\text{debt}}$ is equal to the debt issue revenue $(1+n)\cdot\left(h'/h|_{\text{debt}}\right)\cdot b|_{\text{debt}}$ in the

steady state, and (ii) if $h'/h|_{\text{debt}} > h'/h|_{\text{tax}}$ holds, then $k|_{\text{debt}} > k|_{\text{tax}}$ holds; that is, a higher growth rate results in higher steady-state capital. These observations suggest that the first, second, third, and fourth terms in Eq. (18) are larger than those in Eq. (17) if $h'/h|_{\text{debt}} > h'/h|_{\text{tax}}$ and $\beta(\alpha-1)+\gamma\alpha > 0$. Therefore, the debt-finance case attains a higher steady-state utility if $X_{econ} > (1-\alpha)(1-\alpha(1+\beta)/(1-\alpha)\beta)$ and $\gamma > (1-\alpha)\beta/\alpha$.

4 Political Equilibrium

The result of the multiple equilibria in Proposition 2 shows a negative correlation between capital and debt: one is distinguished by low capital and high debt, and the other is distinguished by high capital and low debt. The multiple-equilibria story may fit well with the evidence from developed countries in Figure 1. However, this explanation has three limitations.

First, the equilibrium converges to a higher steady-state capital level for most initial conditions. This makes it difficult to illustrate a variety growth and debt accumulation patterns. Second, the condition of $\alpha < \beta/(1+2\beta)$ in Proposition 2 is less likely to hold. For example, the condition fails to hold when $(\alpha,\beta) = \left(1/3,(0.98)^{30}\right)$. Finally, and most importantly, the economic equilibrium analysis in Section 3 takes the tax and the expenditure as given, and does not consider the influence of fiscal policy choices on cross-country differences in growth and public debt.

To overcome these limitations, we consider voting on fiscal policy. In particular, we demonstrate the politics of fiscal policy formation for the tax- and debt-finance cases, and compare them in terms of growth and utility as in the previous section. In the present framework, the young, the middle-aged, and the elderly can participate in voting. However, the elderly are indifferent between any two policies because they bear no tax burden and obtain no benefit. The young may benefit from the public education expenditure in the future, but we assume that they are unable to participate in voting because they are below the voting age.

Under the assumption described above, the government's aim in each period is to maximize the utility of the middle-aged, given by

$$V_t = (1+\beta)\ln\{(1-\alpha)A(k_t)^{\alpha}h_t - \tau_t w_t h_t\} + \{\beta(\alpha-1)+\gamma\alpha\}\ln k_{t+1} + \gamma\eta\ln x_t + \gamma(1-\eta)\ln h_t + C_0. \tag{19}$$

The term $\{(1-\alpha)A(k_t)^{\alpha}h_t - \tau_t w_t h_t\}$ shows the after-tax income, and thus represents utility from consumption. The second term includes the utility from the return on savings, $\beta(\alpha-1)\ln k_{t+1}$, and the utility from the next generation's wage income, $\gamma\alpha\ln k_{t+1}$. The third and fourth terms correspond to the utility from the next generation's human capital, h_{t+1}.

Given the state variables k_t, h_t and $\hat{b}_t$, the period$-t$ government chooses fiscal policy to maximize V_t subject to the capital-market-clearing condition and the government budget constraint. Formally, Definition 2 describes the political equilibrium in this framework.

Definition 2. A *political equilibrium* is a sequence of policies $\{\tau_t, x_t\}_{t=0}^{\infty}$, allocations $\left\{c_t, d_t, s_t, k_{t+1}, \hat{b}_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$, and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0), h_0(>0)$ and $\hat{b}_0(\geq 0)$ such that (i) the conditions in Definition 1 (Economic Equilibrium) are satisfied; and (ii) in period $t(\geq 0)$, the government chooses x_t and τ_t (or $\hat{b}_{t+1}$) to maximize V_t subject to the capital market clearing condition and the government budget constraint, given by k_t, h_t, and $\hat{b}_t$.

In the following analysis, we first characterize a political equilibrium in the tax-finance case in Subsection 4.1 and that in the debt-finance case in Subsection 4.2. Then, in Subsection 4.3, we compare the two cases in terms of government expenditure-GDP ratio, the tax rate, and the steady-state growth rate and welfare. In Subsection 4.4, we compare the economic and political equilibria in terms of growth and welfare.

4.1 Tax-finance Political Equilibrium

In the tax-finance case, (5) gives the government budget constraint. Substituting this into V_t in (19) leads to:

$$V_t = (1+\beta)\ln(i_t - (1+n)x_t) + \{\beta(\alpha-1)+\gamma\alpha\}\ln k_{t+1} + \gamma\eta\ln x_t + \gamma(1-\eta)\ln h_t + C_0, \quad (20)$$

where i_t is defined by

$$i_t \equiv (1-\alpha)A(k_t)^{\alpha} h_t - \alpha A(k_t)^{\alpha-1}\hat{b}_t.$$

The term $(1-\alpha)A(k_t)^{\alpha} h_t$ denotes labor income, and the term $\alpha A(k_t)^{\alpha-1}\hat{b}_t$ denotes debt repayment in period 0 only. Therefore, i_t represents the government's available resources for its expenditures. Using the definition of i_t and the human capital formation function, $h_{t+1} = D(x_t)^{\eta}(h_t)^{1-\eta}$, we can reformulate the capital-market-clearing condition in (6) as:

$$(1+n)k_{t+1}D(x_t)^{\eta}(h_t)^{1-\eta} = \frac{\beta}{1+\beta}(i_t - (1+n)x_t). \quad (21)$$

The problem of the period t government is to maximize V_t in (20) subject to (21) given k_t, h_t, and $\hat{b}_t$.

The first-order condition with respect to x_t is:

$$(1+n)\frac{\{1+(\beta+\gamma)\alpha\}}{i_t - (1+n)x_t} = \frac{\eta(\beta+\gamma)(1-\alpha)}{x_t},$$

where the left-hand side is the marginal cost of public education expenditure, and the right-hand side is its marginal benefit. The government chooses x_t to balance the marginal cost and benefit, so x_t is given by:

$$x_t = X_0 \cdot i_t, \tag{22}$$

where

$$X_0 \equiv \frac{\eta(\beta+\gamma)(1-\alpha)}{(1+n)\left[1+(\beta+\gamma)\left\{\alpha+(1-\alpha)\eta\right\}\right]}.$$

To understand the property of the policy function of x_t, let us focus on the numerator of X_0, $\eta(\beta+\gamma)(1-\alpha)$. This term is decomposed as $\eta(\beta+\gamma)(1-\alpha) = \eta\left[-\left\{\beta(\alpha-1)+\gamma\alpha\right\}+\gamma\right]$ where the terms $-\left\{\beta(\alpha-1)+\gamma\alpha\right\}$ and γ in the square brackets correspond to the coefficients of the second and third terms of V_t in (20), respectively. The term $-\left\{\beta(\alpha-1)+\gamma\alpha\right\}$ implies that an increase in public education expenditures reduces the physical to human capital ratio, and this in turn increases the return on saving but decreases the children's wage income. The term γ suggests that public education expenditures enhance the children's human capital formation and thus improves parents' utility. In sum, the net effect of these terms is positive.

Substituting the policy function in (22) into the government budget constraint, we obtain the corresponding tax rate:

$$\tau_t = \begin{cases} (1+n)X_0 + (1-(1+n)X_0)\,\frac{\alpha}{1-\alpha}\frac{b_0}{k_0} & \text{for } t=0. \\ (1+n)X_0 & \text{for } t \geq 1. \end{cases}$$

The term $(1+n)X_0$ indicates that an increase in education expenditures is financed by raising the tax rate. In addition, in the initial period, the government must further increase the tax rate for debt repayment, as represented by the term $1\cdot(\alpha/(1-\alpha))\cdot(b_0/k_0)$. However, increasing budget pressure is partially offset by a reduction in public education expenditures, as represented by the term $(-1)\cdot(1+n)X_0$.

We can use the policy functions of x_t and τ_t to reformulate the capital market clearing condition in (6) as follows:

$$k_{t+1} = \begin{cases} \tilde{\Psi}_K \cdot \left\{(1-\alpha)A\left(k_0\right)^{\alpha} - \alpha A\left(k_0\right)^{\alpha-1} b_0\right\}^{1-\eta} & \text{for } t=0, \\ \tilde{\Psi}_K \cdot \left\{(1-\alpha)A\left(k_t\right)^{\alpha}\right\}^{1-\eta} & \text{for } t \geq 1, \end{cases}$$

where

$$\tilde{\Psi}_K \equiv \frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0}{(1+n)D\left(X_0\right)^{\eta}}.$$

Given $k_0(>0)$ and $b_0(\geq 0)$, a unique sequence of $\{k_t\}$ satisfies the above condition. The condition for $t \geq 1$, $k_{t+1} = \tilde{\Psi}_K \cdot \left\{(1-\alpha)A\left(k_t\right)^{\alpha}\right\}^{1-\eta}$, suggests that the sequence stably converges to the unique steady state. The following proposition summarizes the result established thus far.

Proposition 4. *There is a unique, stable, steady-state tax-financed political equilibrium with* $k_{t+1} > 0$ *and* $\tau_t \in (0,1)$ *for all* $t \geq 0$ *if* $b_0/k_0 < (1-\alpha)/\alpha$.

The upper bound of b_0/k_0 guarantees that the period-0 tax rate is below 100%.

4.2 Debt-finance Political Equilibrium

In the debt-finance case, (4) gives the government budget constraint. Substituting this into V_t in (19) leads to:

$$V_t = (1+\beta)\ln\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) + \{\beta(\alpha-1)+\gamma\alpha\}\ln k_{t+1} + \gamma\eta\ln x_t + \gamma(1-\eta)\ln h_t + C_0. \tag{23}$$

A main departure from the tax-finance case is that the term $(1+n)\hat{b}_{t+1}$ appears in the debt-finance case. This term represents the positive income effect arising from public bond issues that enable the government to shift the burden of the expenditure from the current to future generations. Therefore, it produces a positive income effect on households.

The government's problem is to maximize V_t in (23) subject to the government budget constraint in (4) and the capital-market-clearing condition in (6). To more conveniently reformulate the problem, we substitute the government budget constraint and the human capital formation function into the capital-market-clearing condition and obtain:

$$k_{t+1} = \frac{1}{(1+n)D(x_t)^{\eta}(h_t)^{1-\eta}} \cdot \frac{\beta}{1+\beta} \cdot \left[i_t - (1+n)x_t - \frac{(1+n)\hat{b}_{t+1}}{\beta} \right]. \tag{24}$$

We substitute this into V_t in (23) and rearrange the terms to obtain

$$V_t = (1+\beta)\ln\left[i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right] + \{\beta(\alpha-1)+\gamma\alpha\}\ln\left[i_t - (1+n)x_t - \frac{(1+n)\hat{b}_{t+1}}{\beta}\right] \tag{25}$$

$$+ \eta(\beta+\gamma)(1-\alpha)\ln x_t + (1-\gamma)(\beta+\gamma)(1-\alpha)\ln h_t + C_1,$$

where C_1 includes the following constant terms:

$$C_1 \equiv C_0 + \{\beta(\alpha-1)+\gamma\alpha\}\ln\frac{\beta}{(1+\beta)(1+n)D}.$$

The first three terms in (25) relate to policy-making. The first term represents the utility of lifetime consumption. The second term shows the utility of the next generation's wage income affected by physical capital accumulation. The third term includes the three factors affected by public education: the utility of the return on savings, the utility of the next generation's wage income, and the utility of the next generation's human capital. The government's problem is to choose a pair of $(x_t, \hat{b}_{t+1})$ that maximizes V_t in (25). Solving the problem yields the following policy functions.

Lemma 1. *The policy functions of* $x_t, \hat{b}_{t+1}$, *and* τ_t *in the debt-finance political equilibrium are*

$$\begin{aligned} x_t &= X_0 \cdot i_t, \\ \hat{b}_{t+1} &= B_0 \cdot i_t, \\ \tau_t &= (1+n)(X_0 - B_0) + \frac{\alpha}{1-\alpha} \cdot \{1 - (1+n)(X_0 - B_0)\} \cdot \frac{b_t}{k_t}, \end{aligned}$$

where

$$B_0 \equiv \max\left\{0, \frac{(1+\beta)\beta - \{\beta(\alpha-1) + \gamma\alpha\}}{(1+n)[1 + (\beta+\gamma)\{\alpha + \eta(1-\alpha)\}]}\right\}$$

Proof. See Appendix A.4.

The policy function of x_t is identical to that in the tax-finance political equilibrium. Public debt produces debt repayment costs, and thus reduces the government's available resources, i_t. However, it does not affect the fraction of resources devoted to public education expenditures.

The policy function $\hat{b}_{t+1}$ indicates that $(1+\beta)\beta$ and $\{\beta(\alpha-1) + \gamma\alpha\}$ are crucial to the government's financial state. The term $(1+\beta)\beta$ corresponds to the coefficient of the first term of V_t in (25). A greater weight on the utility of consumption incentivizes the government to issue more public bonds to raise consumption levels in the middle and elderly ages. The term $\{\beta(\alpha-1) + \gamma\alpha\}$ corresponds to the coefficient of the second term of V_t in (25). An increase in public bond issues overwhelms physical capital accumulation, which in turn creates two opposing effects: an increase in the interest rate, $\beta(\alpha-1)$, and a decrease in the children's wage income, $\gamma\alpha$. The government chooses $\hat{b}_{t+1} > 0$ and thus borrows in the capital market if the effect of the first term, $(1+\beta)\beta$, outweighs the effect of the second term, $\{\beta(\alpha-1) + \gamma\alpha\}$; that is, if $B_0 > 0 \Leftrightarrow \gamma < \beta\{(1+\beta) + (1-\alpha)\}/\alpha$. However, if $B_0 = 0$, that is, $\gamma \geq \beta\{(1+\beta) + (1-\alpha)\}/\alpha$, the government finds it optimal to issue no public bonds.

The policy function τ_t suggests an influence from both public education expenditures and public bond issues. The first term $(1+n)(X_0 - B_0)$ indicates that the government attempts to increase the tax rate to finance public education expenditures, but can cut the tax rate and finance a part of the expenditure by issuing public bonds. The second term shows the effect of debt repayment on the tax rate. In particular, the term "1" of $\{1 - (1+n)(X_0 - B_0)\}$ indicates that the government raises the tax rate to finance debt repayment. However, debt repayment pressure incentivizes the government to cut public education expenditures and issue less public bonds. The former effect, represented by $(1+n)X_0$, works to decrease the tax rate, while the latter, represented by $(1+n)B_0$, works in the opposite direction.

Having established the policy functions, we are now ready to demonstrate the accumulation of physical and human capital and public debt. We substitute the policy functions

in Lemma 1 into the capital accumulation equation in (24), the government budget constraint in (4), and the human capital formation function given by $h_{t+1} = D\,(x_t)^{\eta}\,(h_t)^{1-\eta}$, and obtain:

$$\begin{aligned} k_{t+1} &= \Psi_K \cdot \left[(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right]^{1-\eta}, \\ b_{t+1} &= \Psi_B \cdot \left[(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right]^{1-\eta}, \\ \frac{h_{t+1}}{h_t} &= D\,(X_0)^{\eta} \cdot \left[(1-\alpha)A\,(k_t)^{\alpha} - \alpha A\,(k_t)^{\alpha-1}\,b_t\right]^{\eta}, \end{aligned}$$

where Ψ_K and Ψ_B are defined by:

$$\Psi_K \equiv \frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0 - \frac{1+n}{\beta}B_0}{(1+n)D\,(X_0)^{\eta}} \text{ and } \Psi_B \equiv \frac{B_0}{D\,(X_0)^{\eta}},$$

respectively.

Given $\{k_0, b_0, h_0\}$, the sequence $\{k_t, b_t, h_t\}$ contains the three equations above. In particular, if $B_0 = 0$, that is, $\gamma \geq \beta\,\{(1+\beta)+(1-\alpha)\}/\alpha$, then $b_{t+1} = 0$ for all t, and the laws of motion of physical and human capital match those in the tax-finance case. We use the three equations above to obtain Proposition 5.

Proposition 5. *Suppose that the government can issue public bonds to finance its expenditures. Given* $b_0/k_0 \in (0, (1-\alpha)/\alpha)$,

(i) *there is a unique, stable, steady-state equilibrium with* $k_{t+1} > 0, b_{t+1} = 0$, *and* $\tau_t \in (0,1)$ *if* $\gamma \geq \beta\,\{(1+\beta)+(1-\alpha)\}/\alpha$;

(ii) *there is a unique, stable, steady-state equilibrium with* $k_{t+1} > 0, b_{t+1} > 0$, *and* $\tau_t \in (0,1)$ *if*

$$\max\left\{\frac{\beta\,\{(1+\beta)+(1-\alpha)(1-\eta)\}}{\eta(1-\alpha)+\alpha}, \frac{\beta(1+\alpha\beta)}{\alpha}\right\} < \gamma < \frac{\beta\,\{(1+\beta)+(1-\alpha)\}}{\alpha}.$$

Proof. See Appendix A.5.

The upper bound of b_0/k_0 guarantees that the period-0 tax rate is below 100% as in the tax-finance case. A noteworthy feature of the result in Proposition 5 is that it provides a unique debt-finance political equilibrium different from the result in the previous section that demonstrates multiple economic equilibria: the two economies sharing the same structural parameter values but different initial conditions may converge to different steady states. In other words, the initial condition influences long-term outcomes when fiscal policy is exogenous (see, e.g., Azariadis, 1993; de la Croix and Michel, 2002). However, Proposition 5 shows that the economy stably converges to a unique steady state for any initial condition of $b_0/k_0 \in (0, (1-\alpha)/\alpha)$. Therefore, the initial condition is of little importance when fiscal policy is endogenous.

To gain the intuition for the γ assumption, first suppose that γ is high enough that $\gamma \geq \beta\{(1+\beta)+(1-\alpha)\}/\alpha$. Greater altruism encourages adults to leave higher wages rates to their children, and will do so by decreasing public bond issues and thus weakening its overpowering effect on capital accumulation. Given this adult preference, the government finds it optimal to issue no public bonds. The equilibrium sequence matches that in the tax-finance case in Subsection 4.1. However, if altruism is low enough that $\gamma < \beta\{(1+\beta)+(1-\alpha)\}/\alpha$, the government finds it optimal to finance part of its expenditures by borrowing in the capital market. Therefore, the degree of altruism is crucial in the choice of fiscal policies.

Next, consider the role of the lower bound of γ. When γ is below the first lower bound given by $\beta\{(1+\beta)+(1-\alpha)(1-\eta)\}/\{\eta(1-\alpha)+\alpha\}$, the degree of altruism is too low to incentivize adults to provide higher levels of education, and they would thus rather support more public bond issues to finance public education expenditures. In this case, the government gains a surplus in revenue, and can thus refund it by subsidizing households. We can rule out this possibility using the first lower bound of γ. In addition, more public bond issues increases the debt repayment cost, and thus results in tax rate higher than 100%. The second lower bound of γ guarantees that the tax rate is below 100% for period $t \geq 1$. The upper bound of b_0/k_0 guarantees that the period 0 tax rate is below 100%.

The effect of altruism on public bond issues differs from that found in Song, Storesletten, and Zilibotti (2012, 2014). In their framework, the government represents two generations, the young and the elderly, and chooses a moderate level of public debt despite low altruism. This is because the current young cohort wants to protect the future public goods provision that they enjoy in their old age by controlling public bond issues today. This fiscal disciplining effect does not arise in our framework due to the absence of public goods provision. In Subsection 5.2, we briefly investigate a case that includes public goods provision as an alternative mode of government spending, and show that public goods provision adds the fiscal disciplining effect into the present framework.

4.3 Fiscal Rule Comparison

Public bond issues enable the current generation to pass cost on to future generations. In fact, the public education expenditure, $(1+n)x_t$, plus the interest payment, $(R_t - 1)\hat{b}_t$, is greater than the tax revenue, $\tau_t w_t h_t$, along the steady-state debt-finance political equilibrium path with $h'/h > 1$, where h' denotes the next-period h. To confirm this statement, note that $h'/h > 1$ implies $(1+n)h'/h > 1$. We reformulate this as $(1+n)h'b > hb$, or $(1+n)\hat{b}' > \hat{b}$ in the steady state. The government budget constraint in (4) implies that $(1+n)\hat{b}' > \hat{b}$ leads to $(1+n)x < \tau wh + (R-1)\hat{b}$. Therefore, part of the fiscal burden

is passed on to future generations via public bond issues when adults have low altruism toward children, $\gamma < \beta\{(1+\beta)+(1-\alpha)\}/\alpha$.

To consider the impact of debt finance more in detail, we here assume $\gamma < \beta\{(1+\beta)+(1-\alpha)\}/\alpha$ to focus on a situation in which the government chooses debt finance from the viewpoint of maximizing its objective, and consider a constitutional rule that prohibits debt finance and requires tax finance as an alternative. We first compare the aggregate expenditure-GDP ratio and the tax rate in the debt-finance case with those in the tax-finance case, and obtain Proposition 6.

Proposition 6. *Consider the aggregate expenditure-GDP ratio, $N_{t+1}x_t/Y_t$ and the tax rate, τ_t.* (i) *For $t=0$, $N_1x_0/Y_0|_{\text{tax}} = N_1x_0/Y_0|_{\text{debt}}$. For $t \geq 1$, $N_{t+1}x_t/Y_t|_{\text{tax}} > N_{t+1}x_t/Y_t|_{\text{debt}}$.* (ii) *For $t=0$, $\tau_0|_{\text{tax}} > \tau_0|_{\text{debt}}$. For $t \geq 1$, $\tau_t|_{\text{tax}} \lesseqgtr \tau_t|_{\text{debt}}$ if and only if $\gamma \lesseqgtr \beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$.*

Proof. See Appendix A.6.

[Figure 3 about here.]

Figure 3 plots a numerical example of the evolution of the aggregate expenditure-GDP ratio (in Panel (a)) and the tax rate (in Panel (b)) from period 0. Appendix A.7 describes the choice of parameter values. The first part of Proposition 6 states that both cases have equal aggregate expenditure-GDP ratios in the initial period, but they differ from period 1. To understand this result, recall the policy function x_t, which indicates that the government uses a fraction, X_0, of its available resources, i_t, for public education expenditures in both cases. The government has the same resources available in period 0, given by $i_0 = \left\{(1-\alpha)A(k_0)^{\alpha} - \alpha A(k_0)^{\alpha-1} b_0\right\} h_0$ for both cases. Therefore, the expenditure-GDP ratios are equal in period 0. However, they differ from period 1. In the debt-finance case, the government must manage debt repayment, which reduces its available resources. Therefore, the government attains a lower expenditure-GDP ratio in the debt-finance case than in the tax-finance case.

The second part of Proposition 6 states that the tax rate in the tax-finance case is higher than that in the debt-finance case in the initial period, but this relationship reverses in the next period. In the debt-finance case, the government can implement a tax cut financed by public bond issues. Because of this tax cut, the period 0 tax rate is lower in the debt-finance case than in the tax-finance case. However, from period 1, the government has debt repayment costs in the debt-finance case, and must then raise the tax rate. The second part of Proposition 6 shows that this tax-hike effect is greater (less) than the tax-cut effect if γ is below (above) the critical value given by $\beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$. The next subsection explores the empirical plausibility of the $\gamma \lesseqgtr \beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$ condition.

We next compare the steady-state growth rates for the two cases.

Proposition 7. *The steady-state growth rate in the tax-finance political equilibrium is higher than in the debt-finance political equilibrium;* $h'/h|_{\text{tax}} > h'/h|_{\text{debt}}$.

Proof. See Appendix A.8.

The steady-state growth rates in the tax-finance and debt-finance cases are given by:

$$\left.\frac{h'}{h}\right|_{\text{tax}} = D \cdot \left[X_0 \left(\frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0}{(1+n)D\left(X_0\right)^{\eta}} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \left\{(1-\alpha)A\right\}^{\frac{1}{1-\alpha(1-\eta)}} \right]^{\eta},$$

$$\left.\frac{h'}{h}\right|_{\text{debt}} = D \cdot \left[X_0 \left(\frac{\beta}{1+\beta} \cdot \frac{1-(1+n)X_0 - \frac{1+n}{\beta}B_0}{(1+n)D\left(X_0\right)^{\eta}} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \left((1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K} \right)^{\frac{1}{1-\alpha(1-\eta)}} \right]^{\eta},$$

respectively. The equation $h'/h|_{\text{debt}}$ suggests that there are two negative effects on the growth rate peculiar to the debt-finance case. First, the cost of debt repayment reduces the government's resources, which in turn decreases public education expenditures as an engine of economic growth. The term $-(1+n)B_0/\beta$ demonstrates this effect. Second, the cost of debt repayment overwhelms capital accumulation and thus lowers the steady-state level of capital, resulting in a higher interest rate that then further increases the debt repayment cost. The term $\alpha A \Psi_B/\Psi_K$ illustrates this effect. These two negative effects from public debt result in a lower steady-state growth rate in the debt-finance case than the tax-finance case. Figure 4 plots a numerical example of the evolution of the growth rate.

[Figure 4 about here.]

Debt finance enables the government to cut the tax rate (Proposition 6(ii)). The tax cut increases households's disposable income and thus lifetime consumption for the period-0 middle-aged agents, while the public education expenditures remain equal in both cases (Proposition 6(i)). The period-0 middle-aged agents find it optimal to choose debt rather than tax financing to maximize their utility when $\gamma < \beta(1-\alpha)/\alpha + (1+\beta)^2/(1-\alpha)$. However, future generations would be worse off due to debt repayment and low growth rates. This suggests the possibility of inter-generational utility trade-offs.

To investigate the welfare consequences of debt finance across generations, we here compare the tax-finance and debt-finance cases in terms of utility, and obtain Proposition 8.

Proposition 8. *Assume the parametric condition that realizes* $b > 0$ *in the political equilibrium. Along the equilibrium path with* $h'/h > 1$*, the steady-state utility of the middle-aged is higher in the tax-finance case than in the debt-finance case:* $V_{tax} > V_{debt}$.

Proof. See Appendix A.9.

In each period, the government, which represents the middle-aged agents, finds it optimal to choose debt finance for the state variables k, b, and h. The choice of debt finance discourages human capital accumulation (Proposition 7), which in turn bequeaths less physical and human capital to future generations. This implies that future governments are forced to choose a fiscal policy subject to lower levels of capital. If the government introduces tax finance and continues this for long periods, then future governments can enjoy higher capital and realize a higher utility for the middle-aged, although they are constrained by the choice of fiscal policy. In other words, the choice of future governments is itself optimal for given capital levels, but they obtain less utility for the middle-aged from debt finance than from tax finance. Table 1 presents a numerical example of the utility of successive generations from period 0 to period 2.

[Table 1 here.]

4.4 Comparing Economic and Political Equilibria

The results in Propositions 7 and 8 suggest that the shift from tax- to debt-finance worsens the steady-state growth and utility in the political equilibrium. This result is in sharp contrast to what we obtain for the economic equilibrium (Proposition 3). However, it is unclear thus far whether these contrasting results hold under the same parametric conditions. We investigate this issue by assuming $\alpha < \beta/(1+2\beta)$ as in Proposition 2, and comparing the range of γ in the economic equilibrium to that in the political equilibrium, as illustrated in Figure 5.

[Figure 5 here.]

Figure 5 indicates that the two contrasting results are obtained within a common range of γ. In the political equilibrium, the government chooses debt finance if γ is set within a certain range (Proposition 5). Within that range of γ, the steady-state growth and utility are higher in the tax-finance case than in the debt-finance case in the political equilibrium. However, within the same range of γ, the economic equilibrium attains lower steady-state growth and utility in the tax-finance case than in the debt-finance case if $X_{econ} > (1-\alpha) - \alpha(1+\beta)/\beta$ (Proposition 3). Therefore, we could conclude that the contrasting results for the economic and political equilibria occur under certain parametric values.

5 Extensions and Further Analysis

The analysis thus far assumes (i) no private education; (ii) no alternative public spending, such as public goods provision; and (iii) no international lending and borrowing. This

section briefly considers how the analysis and results would change when we relax each of these assumptions.

5.1 Private Education

This subsection introduces private education, denoted by e_t. The budget constraint of the young is $c_t + s_t + e_t \leq (1-\tau_t)w_t h_t$, and human capital formation is $h_{t+1} = D\left(x_t\right)^{\eta}\left(e_t\right)^{\delta}\left(h_t\right)^{1-\eta-\delta}$, where δ is the elasticity of human capital with respect to e_t. Solving the utility maximization problem leads to the following indirect utility of the middle-aged:

$$V_t = \left\{1+\beta+\underbrace{\delta\left(\gamma+\beta\left(1-\alpha\right)-\gamma\alpha\right)}_{e.1}\right\}\ln\left(i_t-(1+n)x_t+(1+n)\hat{b}_{t+1}\right)$$
$$+\left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln\left(i_t-(1+n)x_t-\underbrace{\frac{1+\gamma\delta}{\beta}}_{e.2}\cdot(1+n)\hat{b}_{t+1}\right)+\eta(\gamma+\beta)(1-\alpha)\ln x_t,$$

where the terms (e.1) and (e.2) are relevant to private education. When $\delta = 0$, this expression matches the indirect utility function in the absence of private education.

The term (e.1) includes the following effects of private education. First, an increase in private education improves the human capital of the next generation, represented by the term γ in parentheses. However, an improvement in the human capital decreases the physical-to-human capital ratio, and thus lowers the next generation's wage, but raises the interest rate. These effects are represented by the terms $\beta\left(1-\alpha\right)$ and $-\gamma\alpha$ in parentheses, respectively. The net effect of these three effects on utility is positive since $\gamma+\beta\left(1-\alpha\right)-\gamma\alpha > 0$. Therefore, the term (e.1) implies that the middle-aged attach larger weights to the cost of public education as well as on the benefit of public bond issues when private education is available than in its absence.

The term (e.2) shows that the private education increases the cost of public bond issues because it lowers saving, which in turn strengthens the crowding out effect of public bond issues. Overall, private education definitely decreases the public education spending, while it has two opposing effects on public bond issues. Appendix A.10 shows that the net effect is negative: the availability of private education lowers public bond issues.

5.2 Public Goods Provision

This subsection introduces public goods provision as an alternative mode of government spending. We modify the government budget constraint in (4) as

$$(1+n)\hat{b}_{t+1} + \tau_t w_t h_t = g_t + (1+n)x_t + R_t \hat{b}_t,$$

where g_t denotes public goods provision. The elderly benefit from public goods provision and thus have an incentive to control it through voting. Therefore, we write down the indirect utility function of the elderly and the middle-aged, denoted by V_t^o and V_t^m, respectively, as follows:

$$\begin{aligned} V_t^o &= \ln R_t s_{t-1} + \gamma \ln w_t h_t + \mu \ln g_t, \\ V_t^m &= (1+\beta)\ln\left\{(1-\alpha)A\left(k_t\right)^{\alpha} h_t - \tau_t w_t h_t\right\} + \left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln k_{t+1} + \gamma\eta \ln x_t \\ &\quad + \mu \ln g_t + \beta\mu \ln g_{t+1} + \gamma(1-\eta)\ln h_t + C_0, \end{aligned}$$

where $\mu(>0)$ denotes the weight on the utility of public goods. We keep the assumption that the young are below voting age.

To reflect the preferences of the elderly and middle-aged in voting, we here employ the probabilistic voting mechanism a la Lindbeck and Weibull (1987). In particular, the period-t political objective function is a weighted average of the elderly and the middle-aged:

$$\Omega_t = \omega V_t^o + V_t^m,$$

where $\omega(>0)$ is the relative political weight of the elderly. The problem of the period-t government is to maximize Ω_t subject to the government budget constraint and the capital-market-clearing condition in (6).

We substitute the two constraints into Ω_t and obtain

$$\begin{aligned} \Omega_t &= (1+\beta)\ln\left[i_t - g_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right] \\ &\quad + \left\{\beta(\alpha-1)+\gamma\alpha\right\}\ln\left[i_t - g_t - (1+n)x_t - \frac{(1+n)\hat{b}_{t+1}}{\beta}\right] \\ &\quad + \eta(\beta+\gamma)(1-\alpha)\ln x_t + (\omega+1)\ln g_t + \beta\mu \ln g_{t+1}, \end{aligned}$$

where the irrelevant terms are omitted from the expression. The government's problem is to choose g_t, x_t, and $\hat{b}_{t+1}$ that maximize Ω_t. However, we should note that we include the next-period public goods provision, g_{t+1}, which is chosen by the next-period government, in the period-t objective function. Therefore, we need to speculate on the next-period government's choice, which could be affected by the current policy choice.

Following the analysis in Section 4, it is natural to conjecture that $g_{t+1} = G(i_{t+1})$, where $G : \Re_+ \to \Re_+$ is a function of i_{t+1}. We reformulate i_{t+1} as follows:

$$\begin{aligned} i_{t+1} &\equiv (1-\alpha)A\left(k_{t+1}\right)^{\alpha} h_{t+1} - \alpha A\left(k_{t+1}\right)^{\alpha-1} \hat{b}_{t+1} \\ &= (1-\alpha)A\underbrace{(k_{t+1})^{\alpha}}_{(*3)} D(h_t)^{1-\eta}(x_t)^{\eta} - \alpha A\underbrace{(k_{t+1})^{\alpha-1}}_{(*2)}\underbrace{\hat{b}_{t+1}}_{(*1)}, \end{aligned}$$

where the second line comes from the human capital production function. The expression suggests that the issue of public bonds increases the repayment cost, (*1). In addition, it crowds out capital accumulation and thus increases the interest rate, (*2), and lowers the wage income, (*3).

In summary, issuing public bonds reduces the available resources for the next-period government, which in turn decreases the next-period public goods provision. The present government considers this effect by restraining public bond issues. This is the disciplining effect as demonstrated in Song, Storeletten, and Zilibotti (2014). Therefore, we expect a lower public debt in the presence of public goods provision than in its absence.

5.3 Small Open Economy

In this subsection, we modify the model by assuming a small open economy, which allows for international lending and borrowing for a given world interest rate, R. Within this specification, we compare the tax- and debt-finance cases in terms of growth and utility.

The profit maximization conditions imply that for a given R, per capita capital k and wage w are uniquely determined and constant over time. This implies that there is no inter-temporal effect of current fiscal policy choice through capital accumulation. Thus, we maximize V_t in (20) with respect to x_t and obtain

$$x_t|_{tax} = \begin{cases} \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_0 - R\hat{b}_0\right) \\ \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} w\, h_t|_{tax} \end{cases} \quad \text{for } t \geq 1.$$

This is the public education expenditure in the tax-finance case.

In the debt-finance case, the government's objective function is given in (23). The expression suggests that since k is constant, there is no crowding-out effect of public bond issues. This incentivizes the government to borrow as much as possible in the capital market. The borrowing reaches its ceiling, that is, the natural debt limit, $\bar{b}_t$, which is defined by setting $\tau_t = 1$ and $x_t = 0$ in the government budget constraint. Thus, $\bar{b}_t$ satisfies $(1+n)\bar{b}_t + wh_t = 0 + R\bar{b}_t$, or,

$$\bar{b}_t = \frac{w}{R-(1+n)} h_t = \bar{b} \cdot h_t,$$

where $\bar{b} \equiv w/(R-(1+n))$. We assume $R > 1+n$ in the following.

Given $\hat{b}_{t+1} = \bar{b} \cdot h_t$, the solution maximizing V_t in (23) is

$$x_t| = \begin{cases} \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} \left(wh_0 - R\hat{b}_0 + (1+n)\bar{b}_0 \right) \\ 0 \end{cases} \quad \text{for } t \geq 1.$$

In period 0, the government can finance its public education expenditure by borrowing in the capital market. However, from period 1 onward, the government is unable to spend its revenue for public education because it must use the revenue to repay debt. Therefore, the choice of fiscal policy in the debt-finance case results in no human capital growth, $h_{t+1}/h_t|_{debt} = 0$. This in turn leads to $V_t|_{debt} = -\infty$ from period 1 onward because of the specification of the logarithmic utility function.

The result thus far is summarized as

$$\begin{aligned} x_0|_{tax} &< x_0|_{debt}\,, \\ x_t|_{tax} &> x_t|_{debt} = 0 \text{ for } t \geq 1. \end{aligned}$$

Given the property that growth and utility increase in response to an increase in public education expenditure, we can conclude that in period 0, the growth and utility are higher in the debt-finance case than in the tax-finance case. However, the result is reversed from period 1 onward because there is no growth and no available resources for consumption and public education in the debt-finance case. This result is qualitatively similar to that shown in the closed economy analysis of the main text. Appendix A.11 provides a formal proof of this statement.

6 Conclusion

This paper developed an overlapping-generations model with physical and human capital accumulation using public education and parental human capital as inputs in the process of human capital formation. Public education spending is financed through tax on labor income in the tax-finance case, and by both tax and public bond issues in the debt-finance case. Within this framework, we compared and evaluated the two fiscal choices in terms of growth and utility, and showed that the debt-finance case could be superior to the tax-finance case when the education expenditure-GDP ratio is fixed.

Studies examining growth often assume a fixed expenditure-GDP ratio, which enables us to evaluate the effect of a change in the ratio on growth and welfare. However, in the real world, the ratio is endogenous since the political process of voting by agents who recognize the impact of their actions determines the expenditure itself. Given this background, we demonstrated the endogenous policy formation and showed that the current generation will prefer debt finance since it passes the debt repayment cost on to future

generations. However, the debt repayment cost induces successive generations to cut the expenditure, making them worse off than in the tax-finance case. Our result provides one possible explanation for why debt finance continues to dominate policy in many developed countries, despite its expected worse performance.

A Proofs

A.1 Proof of Proposition 2

We first show the existence of two steady-state equilibria with $b > 0$. Recall that given an initial condition (k_0, b_0), the equilibrium sequence of $\{k_t, b_t\}$ has the government budget constraint in (12) and the capital-market-clearing condition in (13). In the steady state, they are:

$$b = M \cdot \left[(X_{econ} - \tau_{debt}(1-\alpha)) + \alpha \frac{b}{k} \right] \cdot (A(k)^{\alpha})^{1-\eta},$$
$$k = M \cdot \left[\frac{\beta}{1+\beta} (1-\tau_{debt})(1-\alpha) - (X_{econ} - \tau_{debt}(1-\alpha)) - \alpha \frac{b}{k} \right] \cdot (A(k)^{\alpha})^{1-\eta},$$

where M is defined by:

$$M \equiv \frac{1}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^{\eta}}.$$

These expressions are summarized as:

$$\frac{b}{k} = \frac{(X_{econ} - \tau_{debt}(1-\alpha)) + \alpha \frac{b}{k}}{\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - (X_{econ} - \tau_{debt}(1-\alpha)) - \alpha \frac{b}{k}},$$

or

$$f\left(\frac{b}{k}\right) = \alpha \cdot \left(\frac{b}{k}\right)^2 - x \cdot \frac{b}{k} + (X_{econ} - \tau_{debt}(1-\alpha)) = 0, \tag{26}$$

where $f(0) = X_{econ} - \tau_{debt}(1-\alpha) > 0$ under the assumption in Proposition 2, and x is defined by:

$$x \equiv \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - (X_{econ} - \tau_{debt}(1-\alpha)) - \alpha.$$

Our task is to show that (26) has two distinct solutions with $b/k > 0$.

There are two distinct solutions for $f(b/k) = 0$ if $x^2 - 4\alpha(X_{econ} - \tau_{debt}(1-\alpha)) > 0$, i.e.,

$$\left[\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - (X_{econ} - \tau_{debt}(1-\alpha)) + \alpha \right]^2 > 4(\alpha)^2 \frac{\beta}{1+\beta}(1-\tau_{debt}) \frac{1-\alpha}{\alpha}. \tag{27}$$

In addition, given that $f(0) > 0$, the two distinct solutions are both positive if $f'(0) < 0$, that is:

$$X_{econ} < \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) - \alpha. \tag{28}$$

Under the condition in (28), we have $\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - (X_{econ} - \tau_{debt}(1-\alpha)) + \alpha > 0$. (27) is reformulated as

$$X_{econ} < \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) + \alpha - 2\alpha \left(\frac{\beta}{1+\beta}(1-\tau_{debt}) \frac{(1-\alpha)}{\alpha} \right)^{1/2}. \tag{29}$$

Therefore, there are two distinct solutions with $b > 0$ if (28) and (29) hold assuming $\tau_{debt} < 1-\alpha$ in (7) and $\tau_{debt}(1-\alpha) \leq X_{econ}$ in (10). That is:

$$\tau_{debt}(1-\alpha) \leq X_{econ} < \min\left\{1-\alpha, \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) - \alpha, \right.$$
$$\left. \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) + \alpha - 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2}\right\}. \tag{30}$$

To determine the upper limit of X_{econ} in (30), we first compare the first two terms in parentheses in (30), and find:

$$1-\alpha > \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) - \alpha.$$

We next compare the last two terms and find that the upper limit of X_{econ} is given by:

$$\text{(i) } \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha)) - \alpha \text{ if } \tau_{debt} \geq 1 - \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha},$$
$$\text{(ii) } \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha)) + \alpha$$
$$- 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2} \text{ if } \tau_{debt} < 1 - \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}.$$

In case (i), the range of X_{econ} is given by:

$$\tau_{debt}(1-\alpha) \leq X_{econ} < \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) - \alpha.$$

The set of X_{econ} is nonempty if $\tau_{debt} < 1 - \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}$, which contradicts the presumption in case (i).

In case (ii), the range of X_{econ} is given by:

$$\tau_{debt}(1-\alpha) \leq X_{econ}$$
$$< \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) + \alpha - 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2}. \tag{31}$$

The set of X_{econ} is nonempty for any α, β, and τ_{debt} because

$$\tau_{debt}(1-\alpha) \leq \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) + \alpha - 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2}$$
$$\Leftrightarrow 0 \leq \left[\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2} - 1\right]^2.$$

Therefore, there are two distinct solutions with $b/k > 0$ if (31) and $0 < \tau_{debt} \leq 1 - \frac{1+\beta}{\beta}\frac{\alpha}{1-\alpha}$ hold. The range of τ_{debt} is nonempty if $\alpha < \beta/(1+2\beta)$.

Next, we show the stability of the two steady-state equilibria. We differentiate (12) and (13) with respect to k and b and evaluate them at the steady state to obtain:

$$dk = M \cdot \left[\mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 \right] dk - M\mu_2 db, \tag{32}$$

$$db = M\mu_1 dk + M\mu_2 db, \tag{33}$$

where

$$\mu_1 \equiv \left[(X_{econ} - \tau_{debt}(1-\alpha))\alpha(1-\eta) + \alpha \left\{ \alpha(1-\eta) - 1 \right\} \frac{b}{k} \right] \cdot (A(k)^\alpha)^{1-\eta} \cdot \frac{1}{k},$$

$$\mu_2 \equiv \alpha \cdot (A(k)^\alpha)^{1-\eta} \cdot \frac{1}{k}. \tag{34}$$

Let $b = \Psi_k(k)$ denote the phase line of $\Delta k = 0$ in Eq. (13), and $b = \Psi_b(k)$ the phase line of $\Delta b = 0$ in Eq. (12). Using (32) and (33), we can compute the slope of $\Psi_k(k)$ and $\Psi_b(k)$ as follows:

$$\frac{\partial \Psi_k(k)}{\partial k} = \frac{1}{M\mu_2} \cdot \left[M \left\{ \mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 \right\} - 1 \right], \tag{35}$$

$$\frac{\partial \Psi_b(k)}{\partial k} = \frac{M\mu_1}{1 - M\mu_2}. \tag{36}$$

The Jacobian matrix at any steady state of the planner system in (12) and (13) is given by:

$$J = \begin{bmatrix} M \left\{ \mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 \right\} & -M\mu_2 \\ M\mu_1 & M\mu_2 \end{bmatrix}.$$

The term $M\mu_2$ satisfies

$$0 < M\mu_2 < 1. \tag{37}$$

The first inequality condition holds because $M > 0$ and $\mu_2 > 0$. To show that the second inequality condition holds, rewrite $M\mu_2 < 1$ as:

$$M\mu_2 < 1 \Leftrightarrow \frac{1}{(1+n)D\left(\frac{X_{econ}}{1+n}\right)^\eta} \cdot \alpha \cdot (A(k)^\alpha)^{1-\eta} \cdot \frac{1}{k} < 1$$

$$\Leftrightarrow \hat{k} \equiv \left[\frac{\alpha A}{(1+n)D\left(\frac{X_{econ}}{1+n}A\right)^\eta} \right]^{1/(1-\alpha(1-\eta))} < k.$$

The last condition, $\hat{k} < k$, holds, as illustrated in Figure 1.

For any $(k, b) \gg 0$, the trace and the determinant of J are

$$\mathrm{tr}J = M \cdot \left[\mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 + \mu_2 \right],$$

$$\det J = (M\mu_2)^2 \cdot \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) \in (0,1),$$

where $\det J \in (0,1)$ holds because $M\mu_2 \in (0,1)$ holds from (37). Moreover,

$$\begin{aligned}
\Delta &\equiv (\mathrm{tr}J)^2 - 4\det J \\
&= (M)^2 \cdot \left[\left\{ \mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 + \mu_2 \right\}^2 - 4\mu_2\mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) \right] \\
&= (M)^2 \cdot \left[\left\{ \mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 - \mu_2 \right\}^2 - 4\mu_2\mu_1 \right].
\end{aligned}$$

Appendix A.2 shows that $\mu_1 < 0$ if the condition in Proposition 2 applies. Therefore, $\Delta > 0$ holds and thus J has two distinct eigenvalues at each steady state if the condition in Proposition 2 holds.

We derive the eigenvalues of J by solving the following equation:

$$p(\lambda) = (\lambda)^2 - \mathrm{tr}J \cdot \lambda + \det J = 0,$$

where λ denotes an eigenvalue. At the $e = e_l$ steady state, the slope of Ψ_k is larger than that of $\Psi_b : \partial\Psi_k/\partial k > \partial\Psi_b/\partial k$. Applying (35) and (36), we obtain:

$$\begin{aligned}
&\frac{\partial \Psi_k(k)}{\partial k} > \frac{\partial \Psi_b(k)}{\partial k} \\
&\quad \Leftrightarrow M\mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - M\mu_1 + M\mu_2 \\
&\quad - (M\mu_2)^2 \cdot \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - 1 > 0 \\
&\quad \Leftrightarrow \mathrm{tr}J - \det J - 1 > 0 \\
&\quad \Leftrightarrow p(1) = 1 - \mathrm{tr}J + \det J < 0. \qquad (38)
\end{aligned}$$

Moreover,

$$\begin{aligned}
p(-1) &= 1 + \mathrm{tr}J + \det J \\
&> 1 + (1 + \det J) + \det J \\
&> 0,
\end{aligned}$$

where we obtain the second line from $\mathrm{tr}J > 1 + \det J$ in (38) and the third line from $\det J > 0$. Therefore, $p(1) < 0$ and $0 < p(-1)$ imply that the $e = e_l$ steady-state equilibrium exhibits saddle point stability.

At the $e = e_h$ steady state, $\partial\Psi_k/\partial k < \partial\Psi_b/\partial k$ holds. Using (35) and (36), we obtain:

$$\frac{\partial \Psi_k(k)}{\partial k} < \frac{\partial \Psi_b(k)}{\partial k} \Leftrightarrow p(1) = 1 - \mathrm{tr}J + \det J > 0. \qquad (39)$$

In addition,

$$\begin{aligned}
&p(0) = \det J > 0, \\
&p(\lambda) = \left(\lambda - \frac{\mathrm{tr}J}{2} \right)^2 - \left(\frac{\mathrm{tr}J}{2} \right)^2 + \det J.
\end{aligned}$$

Therefore, if $0 < \text{tr}J < 2$, then there are two distinct eigenvalues, λ_1 and λ_2, satisfying λ_1, $\lambda_2 \in (0,1)$.

To show that the condition $0 < \text{tr}J < 2$ holds, we first use the condition in (39), which implies

$$\text{tr}J < 1 + \text{det}J < 2,$$

where we obtain the second inequality from $\text{det}J < 1$ in (37). To show $\text{tr}J > 0$, recall $\text{tr}J = M \cdot \left(\mu_2 \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)(1-\eta) - \mu_1 + \mu_2\right)$, which is rewritten as:

$$\text{tr}J = M \cdot \Bigg[\mu_2 \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha)(1-\eta) - \underbrace{\left\{(X_{econ} - \tau_{debt}(1-\alpha))(1-\eta) + \{\alpha(1-\eta)-1\}\frac{b}{k}\right\}\mu_2}_{=\mu_1} + \mu_2\Bigg],$$

or

$$\frac{\text{tr}J}{M\mu_2} = \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha)(1-\eta) - (X_{econ} - \tau_{debt}(1-\alpha))(1-\eta) - \{\alpha(1-\eta)-1\}\frac{b}{k} + 1.$$

Applying the upper limit of X_{econ} in (31), we can reformulate the above expression as:

$$\begin{aligned}
\frac{\text{tr}J}{M\mu_2} &> \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha)(1-\eta) \\
&\quad - \left\{\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) + \tau_{debt}(1-\alpha) + \alpha - 2\alpha\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2}\right\}\cdot(1-\eta) \\
&\quad + \tau_{debt}(1-\alpha)(1-\eta) + \{1-\alpha(1-\eta)\}\frac{b}{k} + 1 \\
&= \alpha(1-\eta)\cdot 2\left(\frac{\beta}{1+\beta}(1-\tau_{debt})\frac{(1-\alpha)}{\alpha}\right)^{1/2} + \{1-\alpha(1-\eta)\}\frac{b}{k} + \{1-\alpha(1-\eta)\} \\
&> 0,
\end{aligned}$$

indicating that $\text{tr}J > 0$. Therefore, there are two distinct eigenvalues, λ_1 and λ_2, with λ_1, $\lambda_2 \in (0,1)$, implying that the $e = e_h$ steady-state equilibrium exhibits a sink.

■

A.2 Proof of $\mu_1 < 0$

Recall the definition of μ_1 in (34):

$$\mu_1 \equiv \left[(X_{econ} - \tau_{debt}(1-\alpha))(1-\eta) + \{\alpha(1-\eta)-1\}\frac{b}{k}\right]\cdot\mu_2.$$

Given that $\mu_2 > 0$, we have

$$\mu_1 < 0 \Leftrightarrow (X_{econ} - \tau_{debt}(1-\alpha))(1-\eta) < \{1-\alpha(1-\eta)\}\frac{b}{k}. \tag{40}$$

Let denote by $\underline{b/k}$ a smaller solution of $f(b/k) = 0$ in (26). (40) indicates that $\mu_1 < 0$ if

$$(X_{econ} - \tau_{debt}(1-\alpha))(1-\eta) < \{1-\alpha(1-\eta)\} \cdot \left(\underline{b/k}\right). \tag{41}$$

Our task is to show that (41) holds under the condition provided in Proposition 2.

Solving $f(b/k) = 0$, we obtain:

$$\frac{b}{\underline{k}} = \frac{x - \sqrt{(x)^2 - 4\alpha \cdot (X_{econ} - \tau_{debt}(1-\alpha))}}{2\alpha}.$$

We substitute this solution into (41) and obtain:

$$\sqrt{(x)^2 - 4\alpha \cdot (X_{econ} - \tau_{debt}(1-\alpha))} < x - \frac{2\alpha(1-\eta) \cdot (X_{econ} - \tau_{debt}(1-\alpha))}{1-\alpha(1-\eta)}. \tag{42}$$

The right-hand side in (42) is positive if

$$X_{econ} < \frac{1-\alpha(1-\eta)}{1+\alpha(1-\eta)} \cdot \left\{ \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) - \alpha \right\} + \tau_{debt}(1-\alpha).$$

This is satisfied since the right-hand side in the above expression is greater than the upper limit of X_{econ} provided in Proposition 2.

Given that both sides of (42) are positive, (42) is reformulated as follows:

$$\begin{aligned}(x)^2 - 4\alpha \cdot (X_{econ} - \tau_{debt}(1-\alpha)) < (x)^2 - 2x \cdot \frac{2\alpha(1-\eta) \cdot (X_{econ} - \tau_{debt}(1-\alpha))}{1-\alpha(1-\eta)} \\ + \frac{4(\alpha)^2(1-\eta)^2 \left(X_{econ} - \tau_{debt}(1-\alpha)\right)^2}{\{1-\alpha(1-\eta)\}^2},\end{aligned}$$

or

$$0 < \{1-\alpha(1-\eta)\}^2 - x(1-\eta)\{1-\alpha(1-\eta)\} + \alpha(1-\eta)^2 \left(X_{econ} - \tau_{debt}(1-\alpha)\right).$$

Using the definition of x, we reformulate the above expression as:

$$\tau_{debt}(1-\alpha) + \{1-\alpha(1-\eta)\}\left[-\frac{1}{1-\eta} + \frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha)\right] < X_{econ}. \tag{43}$$

The left-hand side is less than $\tau_{debt}(1-\alpha)$ because $-\frac{1}{1-\eta} < -1$ and $\frac{\beta}{1+\beta}(1-\tau_{debt})(1-\alpha) \in (0,1)$. This implies that (43) holds if $\tau_{debt}(1-\alpha) \leq X_{econ}$: this is the condition for X_{econ} provided in Proposition 2. Therefore, $\mu_1 < 0$ holds under the condition in Proposition 2.

■

A.3 Proof of Proposition 3

(i) The proof of the first part is given in the text.

(ii) The utility function for the middle-aged in period-t is:

$$\begin{aligned}U_t &= \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1} h_{t+1} \\ &= \ln \frac{1}{1+\beta}(1-\tau_t) w_t h_t + \beta \ln \frac{\beta R_{t+1}}{1+\beta}(1-\tau_t) w_t h_t + \gamma \ln w_{t+1} + \gamma \ln D \left(x_t\right)^{\eta} \left(h_t\right)^{1-\eta} \\ &= (1+\beta) \ln (1-\tau_t) w_t h_t + \beta \ln R_{t+1} + \gamma \ln w_{t+1} + \gamma\eta \ln x_t + \gamma(1-\eta) \ln h_t \\ &+ \left(\ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta} + \gamma \ln D \right),\end{aligned}$$

where we derive the second line by substituting the consumption and human capital production functions.

We substitute the profit maximization conditions in (3) into the above expression and obtain:

$$U_t = (1+\beta) \ln \left\{(1-\alpha) A \left(k_t\right)^{\alpha} h_t - \tau_t w_t h_t\right\} + \left\{\beta(\alpha-1) + \gamma\alpha\right\} \ln k_{t+1} + \gamma\eta \ln x_t + \gamma(1-\eta) \ln h_t + C_0,$$

where

$$C_0 \equiv \ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta} + \gamma \ln D + \beta \ln \alpha A + \gamma \ln (1-\alpha) A.$$

We rewrite this expression using the expenditure rule in (7), as follows:

$$\begin{aligned}U_t &= (1+\beta) \ln \left\{(1-\alpha) A \left(k_t\right)^{\alpha} h_t - \tau_t w_t h_t\right\} + \left\{\beta(\alpha-1) + \gamma\alpha\right\} \ln k_{t+1} \\ &+ \gamma\eta \ln \frac{X_{econ}}{1+n} A \left(k_t\right)^{\alpha} h_t + \gamma(1-\eta) \ln h_t + C_0. \qquad (44)\end{aligned}$$

In the tax-finance case, the government budget constraint is reduced to $\tau_t = X_{econ}/(1-\alpha)$. Plugging this into (44), we obtain:

$$\begin{aligned}U_{t,tax} &= (1+\beta) \ln \left\{(1-\alpha) - X_{econ}\right\} A \left(k_t\right)^{\alpha} + \left\{\beta(\alpha-1) + \gamma\alpha\right\} \ln k_{t+1} + \gamma\eta \ln \frac{X_{econ}}{1+n} A \left(k_t\right)^{\alpha} \\ &+ (1+\beta+\gamma) \ln h_t + C_0. \qquad (45)\end{aligned}$$

In the debt-finance case, the government budget constraint is:

$$(1+n)\hat{b}_{t+1} + \tau_{debt}(1-\alpha) A \left(k_t\right)^{\alpha} h_t = (1+n) x_t + \alpha A \left(k_t\right)^{\alpha-1} \hat{b}_t,$$

that is,

$$(1+n) b_{t+1} \frac{h_{t+1}}{h_t} h_t + \tau_{debt}(1-\alpha) A \left(k_t\right)^{\alpha} h_t = X_{econ} A \left(k_t\right)^{\alpha} h_t + \alpha A \left(k_t\right)^{\alpha-1} b_t h_t.$$

We substitute this expression into (44) and obtain:

$$U_{t,debt} = (1+\beta)\ln\left[(1-\alpha)A\left(k_t\right)^{\alpha}h_t - X_{econ}A\left(k_t\right)^{\alpha}h_t - \alpha A\left(k_t\right)^{\alpha-1}b_t h_t + (1+n)b_{t+1}\frac{h_{t+1}}{h_t}h_t\right]$$
$$+\left\{\beta\left(\alpha-1\right)+\gamma\alpha\right\}\ln k_{t+1} + \gamma\eta\ln\frac{X_{econ}}{1+n}A\left(k_t\right)^{\alpha}h_t + \gamma(1-\eta)\ln h_t + C_0,$$

that is,

$$U_{t,debt} = (1+\beta)\ln\left[\left\{(1-\alpha)-X_{econ}\right\}A\left(k_t\right)^{\alpha} + \left\{-\alpha A\left(k_t\right)^{\alpha-1}b_t + (1+n)b_{t+1}\frac{h_{t+1}}{h_t}\right\}\right]$$
$$+\left\{\beta\left(\alpha-1\right)+\gamma\alpha\right\}\ln k_{t+1} + \gamma\eta\ln\frac{X_{econ}}{1+n}A\left(k_t\right)^{\alpha} + (1+\beta+\gamma)\ln h_t + C_0. \tag{46}$$

In the steady state, k and b are constant. Therefore, by comparing (45) and (46), we find that $U_{t,debt} > U_{t,tax}$ holds if each term in (46) is larger than the corresponding term in (45), that is, if the following four conditions hold in the steady state:

$$\begin{cases} -\alpha A\left(k|_{debt}\right)^{\alpha-1} + (1+n)\left(\frac{h'}{h}\big|_{debt}\right) \geq 0, \\ k|_{debt} > k|_{tax}\,, \\ h_{t,debt} > h_{t,tax}, \\ \beta\left(\alpha-1\right)+\gamma\alpha \Leftrightarrow \gamma > \frac{(1-\alpha)\beta}{\alpha}. \end{cases} \tag{47}$$

To show the first condition in (47) holds, we rewrite it as:

$$(1+n)\left(\frac{h'}{h}\bigg|_{debt}\right) \geq \alpha A\frac{1}{\left(k|_{debt}\right)^{1-\alpha}}.$$

Given that $k_{debt} > \hat{k}$, the above condition holds if

$$(1+n)\left(\frac{h'}{h}\bigg|_{debt}\right) \geq \alpha A\frac{1}{\left(\hat{k}\right)^{1-\alpha}},$$

that is,

$$(1+n)\cdot\left[D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}\cdot\left(\frac{\alpha A}{1+n}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}$$
$$\geq \alpha A\cdot\frac{1}{\left[\frac{\alpha A}{(1+n)D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}}.$$

This expression is rewritten as:

$$\left[D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}\cdot\left(\frac{\alpha A}{1+n}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}} \geq \left[D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}\cdot\left(\frac{\alpha A}{1+n}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}},$$

which holds with an equality. Therefore, the first condition in (47) is satisfied in the steady state.

The second condition in (47) holds if $\hat{k} > k|_{tax}$ because $k|_{debt} > \hat{k}$ holds in the steady state. Given the definition of $\hat{k}$ and k_{tax} in (9), we can reformulate the second condition in (47) as:

$$\left[\frac{\alpha A}{(1+n)D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}}\right]^{\frac{1}{1-\alpha(1-\eta)}} > \left[\frac{\frac{\beta}{1+\beta}\left(1-\frac{X_{econ}}{1-\alpha}\right)(1-\alpha)A}{(1+n)D\left(\frac{X_{econ}}{1+n}A\right)^{\eta}}\right]^{\frac{1}{1-\alpha(1-\eta)}},$$

that is,

$$X_{econ} > (1-\alpha) - \alpha\frac{1+\beta}{\beta}.$$

The third condition in (47) holds if $h'/h|_{debt} > h'/h|_{tax}$, i.e., $X_{econ} > (1-\alpha) - \alpha\frac{1+\beta}{\beta}$ as shown in the first part of Proposition 3. Therefore, the steady-state utility in the debt-finance case is higher than that in the tax-finance case if $X_{econ} > (1-\alpha) - \alpha\frac{1+\beta}{\beta}$ and $\gamma > \frac{(1-\alpha)\beta}{\alpha}$.

■

A.4 Proof of Lemma 1

The first-order conditions with respect to x_t and $\hat{b}_{t+1}$ are:

$$x_t : \frac{(1+\beta)(1+n)}{i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}} + \frac{\{\beta(\alpha-1)+\gamma\alpha\}(1+n)}{i_t - (1+n)x_t - \frac{1+n}{\beta}\hat{b}_{t+1}} = \frac{\eta(\beta+\gamma)(1-\alpha)}{x_t}, \quad (48)$$

$$\hat{b}_{t+1} : \frac{(1+\beta)(1+n)}{i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}} = \frac{\{\beta(\alpha-1)+\gamma\alpha\}\frac{1+n}{\beta}}{i_t - (1+n)x_t - \frac{1+n}{\beta}\hat{b}_{t+1}}, \quad (49)$$

where an equality holds in (49) if $\hat{b}_{t+1} > 0$, while an inequality holds if $\hat{b}_{t+1} = 0$. We substitute (49) into (48), and obtain:

$$\frac{\{\beta(\alpha-1)+\gamma\alpha\}\frac{1+n}{\beta}}{i_t - (1+n)x_t - \frac{1+n}{\beta}\hat{b}_{t+1}} + \frac{\{\beta(\alpha-1)+\gamma\alpha\}(1+n)}{i_t - (1+n)x_t - \frac{1+n}{\beta}\hat{b}_{t+1}} = \frac{\eta(\beta+\gamma)(1-\alpha)}{x_t},$$

or

$$x_t = \frac{\eta(\beta+\gamma)(1-\alpha)}{(1+n)\left[\{\beta(\alpha-1)+\gamma\alpha\}(1+\beta)+\eta\beta(\beta+\gamma)(1-\alpha)\right]} \cdot \left[\beta i_t - (1+n)\hat{b}_{t+1}\right]. \quad (50)$$

We substitute (50) into (49). After rearranging the terms, we obtain the policy function of $\hat{b}_{t+1}$ as

$$\hat{b}_{t+1} = \frac{(1+\beta)\beta - \{\beta(\alpha-1)+\gamma\alpha\}}{(1+n)\left[1+(\beta+\gamma)(\alpha+\eta(1-\alpha))\right]} i_t.$$

We then substitute this into (50) to obtain the policy function of x_t as $x_t = X_0 \cdot i_t$. Therefore, we obtain

$$x_t = X_0 \cdot i_t,$$
$$\hat{b}_{t+1} = B_0 \cdot i_t,$$

where

$$B_0 \equiv \max\left\{0, \frac{(1+\beta)\,\beta - \{\beta(\alpha-1)+\gamma\alpha\}}{(1+n)\,[1+(\beta+\gamma)\,\{\alpha+\eta(1-\alpha)\}]}\right\}.$$

To obtain the policy function of τ_t, recall the government budget constraint in the debt-finance case, which is reformulated as $(1+n)\hat{b}_{t+1} + \tau_t(1-\alpha)A(k_t)^\alpha h_t = (1+n)x_t + \alpha A(k_t)^{\alpha-1}\hat{b}_t$. Plugging the policy function of x_t and $\hat{b}_{t+1}$ into this constraint and using $b_t = \hat{b}_t h_t$, we have:

$$\begin{aligned}
\tau_t(1-\alpha)A(k_t)^\alpha h_t &= (1+n)X_0 i_t + \alpha A(k_t)^{\alpha-1} b_t h_t - (1+n)B_0 i_t \\
&= (1+n)(X_0 - B_0)\left\{(1-\alpha)A(k_t)^\alpha h_t - \alpha A(k_t)^{\alpha-1} b_t h_t\right\} + \alpha A(k_t)^{\alpha-1} b_t h_t \\
&= (1+n)(X_0 - B_0)(1-\alpha)A(k_t)^\alpha h_t + \{1-(1+n)(X_0-B_0)\}\alpha A(k_t)^{\alpha-1} b_t h_t.
\end{aligned}$$

Dividing both sides by $(1-\alpha)A(k_t)^\alpha h_t$, we obtain:

$$\tau_t = (1+n)(X_0 - B_0) + \frac{\alpha}{1-\alpha}\{1-(1+n)(X_0-B_0)\}\frac{b_t}{k_t}.$$

■

A.5 Proof of Proposition 5

First, suppose that $\gamma > \beta\left\{(1+\beta)+(1-\alpha)\right\}/\alpha$ (i.e., $B_0 = 0$) holds. We obtain $\hat{b}_{t+1} = 0$ from Lemma 1. The policy functions in Lemma 1 match those in the tax-finance case in Subsection 4.1. Thus, we can apply the result in Proposition 4.

Next, suppose that $\gamma \leq \beta\left\{(1+\beta)+(1-\alpha)\right\}/\alpha$ (i.e., $B_0 > 0$) holds. We first derive the conditions for which $k_{t+1} > 0, b_{t+1} > 0$, and $\tau_t \in (0,1)$ hold along the equilibrium path. Second, we show the existence and uniqueness of the steady-state equilibrium. Finally, we show that the unique steady-state equilibrium is a sink.

Step 1.

Recall the capital and debt accumulation equations given by:

$$k_{t+1} = \Psi_K \cdot \left[(1-\alpha)A\left(k_t\right)^\alpha - \alpha A\left(k_t\right)^{\alpha-1} b_t\right]^{1-\eta} = \Psi_K \cdot \left(\frac{i_t}{h_t}\right)^{1-\eta}, \tag{51}$$

$$b_{t+1} = \Psi_B \cdot \left[(1-\alpha)A\left(k_t\right)^\alpha - \alpha A\left(k_t\right)^{\alpha-1} b_t\right]^{1-\eta} = \Psi_B \cdot \left(\frac{i_t}{h_t}\right)^{1-\eta}, \tag{52}$$

respectively. $k_{t+1} > 0$ and $b_{t+1} > 0$ hold along the equilibrium path if $i_t/h_t > 0$ for all $t, \Psi_K > 0$, and $\Psi_B > 0$.

In period 0, we reformulate the term i_0/h_0 as:

$$\frac{i_0}{h_0} = (1-\alpha)A\left(k_0\right)^\alpha \cdot \left[1 - \frac{\alpha}{1-\alpha}\cdot\frac{b_0}{k_0}\right].$$

Thus, $i_0/h_0 > 0$ holds if

$$\frac{b_0}{k_0} < \frac{1-\alpha}{\alpha}. \tag{53}$$

In period $t \geq 1$, the term i_t/h_t is rewritten as

$$\begin{aligned}
\frac{i_t}{h_t} &= (1-\alpha)A\left(k_t\right)^{\alpha} \cdot \left[1 - \frac{\alpha}{1-\alpha} \cdot \frac{b_t}{k_t}\right] \\
&= (1-\alpha)A\left(k_t\right)^{\alpha} \cdot \left[1 - \frac{\alpha}{1-\alpha} \cdot \frac{\Psi_B}{\Psi_K}\right] \\
&= (1-\alpha)A\left(k_t\right)^{\alpha} \cdot \frac{\left[\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0\right]}{1-(1+n)X_0 - \frac{1+n}{\beta}B_0},
\end{aligned}$$

where we obtain the second line using (51) and (52), and the third line using the definition of Ψ_K and Ψ_B.

The expression above suggests that $i_t/h_t > 0$ holds for $t \geq 1$ if the following condition holds:

$$B_0 > 0 \text{ and } \left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0 > 0. \tag{54}$$

In addition, the definitions of Ψ_K and Ψ_B suggest that $\Psi_K > 0$ and $\Psi_B > 0$ hold if (54) holds. After some manipulation, we have:

$$B_0 > 0 \Leftrightarrow \gamma < \frac{\beta}{\alpha}\left\{(1+\beta)+(1-\alpha)\right\}, \tag{55}$$

$$\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} - (1+n)\frac{\alpha(1+\beta)}{(1-\alpha)\beta}B_0 > 0 \Leftrightarrow \frac{\beta}{\alpha}(1+\alpha\beta) < \gamma. \tag{56}$$

Therefore, $k_{t+1} > 0$ and $b_{t+1} > 0$ for $t \geq 0$ if (53), (55), and (56) hold.

Next, consider the tax rate:

$$\tau_t = (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha} \cdot \left\{1-(1+n)\left(X_0 - B_0\right)\right\} \cdot \frac{b_t}{k_t},$$

where

$$1-(1+n)\left(X_0 - B_0\right) > 0.$$

Given $b_0/k_0 > 0$, $\tau_0 > 0$ holds if $X_0 - B_0 > 0$, that is, if

$$\frac{\beta\left\{(1+\beta)+(1-\eta)(1-\alpha)\right\}}{\eta(1-\alpha)+\alpha} < \gamma. \tag{57}$$

For $t \geq 1$, $\tau_t > 0$ holds if $X_0 - B_0 > 0$ and $b_t/k_t > 0$, that is, if (55), (56), and (57) hold.

To find the condition for $\tau_t < 1$, let us first consider the period-0 tax rate. By direct calculation, we have:

$$\tau_0 < 1 \Leftrightarrow \frac{b_0}{k_0} < \frac{1-\alpha}{\alpha} \Leftrightarrow (53).$$

For period $t \geq 1$, $b_t/k_t = \Psi_B/\Psi_K$ holds. We thus have:

$$\tau_t < 1 \Leftrightarrow \frac{\alpha}{1-\alpha} \cdot \frac{\Psi_B}{\Psi_K} < 1 \Leftrightarrow \frac{\beta(1+\alpha\beta)}{\alpha} < \gamma \Leftrightarrow (56).$$

The results established thus far suggest that $k_{t+1} > 0$, $b_{t+1} > 0$, and $\tau_t \in (0,1)$ hold if (53), (55), (56), and (57) hold, that is, if:

$$\max\left\{\frac{\beta\left\{(1+\beta)+(1-\alpha)(1-\eta)\right\}}{\eta(1-\alpha)+\alpha}, \frac{\beta(1+\alpha\beta)}{\alpha}\right\} < \gamma < \frac{\beta\left\{(1+\beta)+(1-\alpha)\right\}}{\alpha},$$

and

$$\frac{b_0}{k_0} \in \left(0, \frac{1-\alpha}{\alpha}\right).$$

Step 2.

The steady-state pair of (k,b) satisfies:

$$k = \Psi_K \cdot \left[(1-\alpha)A\left(k\right)^{\alpha} - \alpha A\left(k\right)^{\alpha-1} b\right]^{1-\eta},$$
$$b = \Psi_B \cdot \left[(1-\alpha)A\left(k\right)^{\alpha} - \alpha A\left(k\right)^{\alpha-1} b\right]^{1-\eta}.$$

These equations lead to $k/b = \Psi_K/\Psi_B$, that is, $b = (\Psi_B/\Psi_K) \cdot k$. Plugging this into $k = \Psi_K \cdot \left[(1-\alpha)A\left(k\right)^{\alpha} - \alpha A\left(k\right)^{\alpha-1} b\right]^{1-\eta}$, and rearranging the terms, we obtain a unique value of k given by:

$$k = \left[\Psi_K \cdot \left\{(1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}\right\}^{1-\eta}\right]^{1/(1-\alpha(1-\eta))}. \tag{58}$$

The corresponding value of b is also uniquely determined using $b = \left(\Psi_B/\Psi_K\right) k$.

Step 3.

Recall the law of motions of capital and debt:

$$k_{t+1} = \Psi_K \cdot \left[(1-\alpha)A\left(k_t\right)^{\alpha} - \alpha A\left(k_t\right)^{\alpha-1} b_t\right]^{1-\eta},$$
$$b_{t+1} = \Psi_B \cdot \left[(1-\alpha)A\left(k_t\right)^{\alpha} - \alpha A\left(k_t\right)^{\alpha-1} b_t\right]^{1-\eta}.$$

Differentiating these with respect to k and b and evaluating them at the steady state, we obtain:

$$\begin{bmatrix} dk_{t+1} \\ db_{t+1} \end{bmatrix} = J \begin{bmatrix} dk_t \\ db_t \end{bmatrix},$$

where

$$J \equiv \begin{bmatrix} \Psi_K \cdot \hat{\imath}(k,b) \cdot \alpha(1-\alpha)A(k)^{\alpha-2}(k+b) & -\Psi_K \cdot \hat{\imath}(k,b) \cdot \alpha A(k)^{\alpha-1} \\ \Psi_B \cdot \hat{\imath}(k,b) \cdot \alpha(1-\alpha)A(k)^{\alpha-2}(k+b) & -\Psi_B \cdot \hat{\imath}(k,b) \cdot \alpha A(k)^{\alpha-1} \end{bmatrix},$$

and

$$\hat{\imath}(k,b) \equiv \left[(1-\alpha)A(k)^{\alpha} - \alpha A(k)^{\alpha-1} b\right]^{-\eta} \cdot (1-\eta).$$

For any $(k, b) \gg 0$, the trace and determinant of J are

$$\begin{cases} \mathrm{tr}J = \alpha A(k)^{\alpha-1} \cdot \hat{\imath}(k,b) \cdot [(1-\alpha)\Psi_K - \alpha\Psi_B] > 0, \\ \quad \det J = 0. \end{cases}$$

Here, the sign of the term $(1-\alpha)\Psi_K - \alpha\Psi_B$ is

$$(1-\alpha)\Psi_K - \alpha\Psi_B > 0 \Leftrightarrow \gamma > \frac{\beta}{\alpha}(1+\alpha\beta),$$

where $\gamma > \frac{\beta}{\alpha}(1+\alpha\beta)$ is satisfied with the assumption in Proposition 5. Moreover, it is true that:

$$\Delta \equiv (\mathrm{tr}J)^2 - 4\det J = (\mathrm{tr}J)^2 > 0.$$

Therefore, J has two distinct positive eigenvalues, denoted by λ_1 and $\lambda_2 (> \lambda_1)$, at the steady state.

We obtain the eigenvalues of J by solving the following equation:

$$p(\lambda) \equiv (\lambda)^2 - (\mathrm{tr}J)\,\lambda + \det J = 0.$$

The solution of $p(\lambda) = 0$ is

$$\lambda_1 = 0 \text{ and } \lambda_2 = \mathrm{tr}J (> 0).$$

The remaining task is to show $\lambda_2 < 1$. For this purpose, recall the law of motion of capital in the steady state, $k = \Psi_K \cdot \left[(1-\alpha)A\,(k)^{\alpha} - \alpha A\,(k)^{\alpha-1}\, b\right]^{1-\eta}$. This is reformulated as:

$$\hat{\imath}(k,b) = (1-\eta)\left(\frac{k}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}. \tag{59}$$

Using (58) and (59), we rewrite $\lambda_2 = \mathrm{tr}J$ as follows:

$$\begin{aligned} \lambda_2 &= \mathrm{tr}J \\ &= \alpha A(k)^{\alpha-1}(1-\eta)\left(\frac{k}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}[(1-\alpha)\Psi_K - \alpha\Psi_B] \\ &= \alpha A\,(\Psi_K)^{\frac{-1}{1-\eta}}\,\frac{1}{(1-\alpha)A - \alpha A\frac{\Psi_B}{\Psi_K}}(1-\eta)\left(\frac{1}{\Psi_K}\right)^{\frac{-\eta}{1-\eta}}[(1-\alpha)\Psi_K - \alpha\Psi_B] \\ &= \alpha(1-\eta) \in (0,1), \end{aligned}$$

where the first line comes from (59) and the second line is derived using (58).

■

A.6 Proof of Proposition 6.

(i) In both cases, the policy function of x is $x_t = X_0 \cdot i_t$, where $i_t \equiv \{(1-\alpha)A(k_t)^{\alpha} - \alpha A(k_t)^{\alpha-1}\} h_t$. Given that $Y_t = A(k_t)^{\alpha} h_t N_t$, we compute the ratio $N_{t+1}x_t/Y_t$ as follows:

$$\begin{aligned}\left.\frac{N_{t+1}x_t}{Y_t}\right|_{\text{tax}} &= \frac{(1+n)X_0\left\{(1-\alpha)A\left(k_t\right)^{\alpha} - \alpha A\left(k_t\right)^{\alpha-1} b_t\right\} h_t}{A\left(k_t\right)^{\alpha} h_t} \\ &= \begin{cases} (1+n)X_0\left\{(1-\alpha) - \alpha b_0/k_0\right\} & \text{for } t=0, \\ (1+n)X_0(1-\alpha) & \text{for } t \geq 1, \end{cases}\end{aligned}$$

and

$$\begin{aligned}\left.\frac{N_{t+1}x_t}{Y_t}\right|_{\text{debt}} &= \frac{(1+n)X_0\left\{(1-\alpha)A\left(k_t\right)^{\alpha} - \alpha A\left(k_t\right)^{\alpha-1} b_t\right\} h_t}{A\left(k_t\right)^{\alpha} h_t} \\ &= (1+n)X_0\left\{(1-\alpha) - \alpha A\, b_t/k_t|_{\text{debt}}\right\},\end{aligned}$$

where $b_t > k_t$ holds for any equilibrium path. Therefore, we obtain the first part of Proposition 6.

(ii) Recall that the tax rates in the tax-finance and debt-finance cases are given by:

$$\tau_t|_{\text{tax}} = \begin{cases} (1+n)X_0 + (1-(1+n)X_0)\,\frac{\alpha}{1-\alpha}\frac{b_0}{k_0} & \text{for } t=0, \\ (1+n)X_0 & \text{for } t \geq 1, \end{cases}$$

and

$$\tau_t|_{\text{debt}} = \begin{cases} (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha} \cdot \left\{1-(1+n)\left(X_0 - B_0\right)\right\} \cdot \frac{b_0}{k_0} & \text{for } t=0, \\ (1+n)\left(X_0 - B_0\right) + \frac{\alpha}{1-\alpha} \cdot \left\{1-(1+n)\left(X_0 - B_0\right)\right\} \cdot \frac{\Psi_B}{\Psi_K} & \text{for } t \geq 1. \end{cases}$$

For $t=0$, $\tau_0|_{\text{tax}}$ and $\tau_0|_{\text{debt}}$ are compared as follows:

$$\tau_0|_{\text{tax}} \gtrless \tau_0|_{\text{debt}} \Leftrightarrow \frac{b_0}{k_0} \lessgtr \frac{1-\alpha}{\alpha}.$$

Assuming $\frac{b_0}{k_0} < \frac{1-\alpha}{\alpha}$ in Proposition 5, we obtain $\tau_0|_{\text{tax}} > \tau_0|_{\text{debt}}$.

For $t \geq 1$, $\tau_t|_{\text{tax}}$ and $\tau_t|_{\text{debt}}$ are compared as follows:

$$\begin{aligned}\tau_t|_{\text{tax}} \gtrless \tau_t|_{\text{debt}} &\Leftrightarrow 1 \gtrless \frac{\alpha}{1-\alpha}\left\{1-(1+n)\left(X_0 - B_0\right)\right\} \frac{1+\beta}{\beta\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\}} \\ &\Leftrightarrow \beta\left\{1-(1+n)X_0 - \frac{1+n}{\beta}B_0\right\} \gtrless \frac{\alpha}{1-\alpha}(1+\beta)\left\{1-(1+n)\left(X_0 - B_0\right)\right\} \\ &\Leftrightarrow \gamma \gtrless \frac{\beta(1-\alpha)}{\alpha} + \frac{(1+\beta)^2}{1-\alpha},\end{aligned}$$

where the second line comes from $1-(1+n)X_0 - \frac{1+n}{\beta}B_0 > 0$, and the third line comes from $(\beta+\gamma)\,\alpha/\beta - 1 > 0$, which holds under the assumption in Proposition 5. Therefore, we obtain the second part of Proposition 6.

■

A.7 Calibration of Parameters

In Figures 2, 3, and 4, we fix the share of capital at $\alpha = 1/3$, following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two-period overlapping-generations model (see, for example, Gonzaelz-Eiras and Niepelt, 2009; Lanica and Russo, 2016). Our selection of β is 0.98, which is also standard in the literature. Since the agents in the present model plan over generations that span 30 years, we discount the future by $(0.98)^{30}$.

We assume an annual gross population growth rate of 1.007, which is the OECD average rate during $2000-2010$.[1] This assumption implies that the net population growth rate for 30 years is $(1.007)^{30} - 1 \simeq 0.233$. For η, the estimate in Card and Krueger (1992) implies an elasticity of school quality of 0.12. In addition, recent simulation studies suggest that η is in the rage of $0.1 - 0.3$ (Cardak, 2004) and $0.05 - 0.15$ (Glomm and Ravikumar, 1998). Following these earlier studies, we here set at $\eta = 0.12$.

For γ, we focus on the income tax revenue-to-GDP ratio in the steady state, $\tau whN/Y = (1-\alpha)\tau$, or

$$\frac{\tau whN}{Y} = (1-\alpha) \cdot \left[(1+n)(X_0 - B_0) + \frac{\alpha}{1-\alpha} \left\{ 1 - (1+n)(X_0 - B_0) \right\} \frac{\Psi_B}{\Psi_K} \right].$$

Given $\alpha = 1/3$, $\beta = (0.98)^{30}$, $1+n = (1.007)^{30}$, and $\eta = 0.12$, we can solve this expression for γ by using the ratio $\tau whN/Y$ observed in OECD countries. In particular, we use the data on "Tax on personal income," defined as the taxes levied on the net income (gross income minus allowable tax reliefs) and capital gains of individuals.[2] The OECD average during 2000–2010 is 0.08665. We can determine γ by solving the above expression and obtain $\gamma = 3.47$.

The productivity of final goods, A, is normalized at $A = 1$. For the productivity of human capital D, we use the data on the per capita GDP gross growth ratio of 1.072, which is the OECD average during $2000 - 2010$.[3] We substitute this data and the values of α, β, n, η, γ, and A into the following equation that expresses the per capita growth rate in the debt-finance case:

$$\frac{h'}{h} = D \cdot (X_0)^{\eta} \cdot (\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}} \cdot \left\{ (1-\alpha) - \alpha A \frac{\Psi_B}{\Psi_K} \right\}^{\frac{\eta}{1-\alpha(1-\eta)}}.$$

We solve this expression for D and obtain $D = 14.21$.

■

[1] Source: World Development Indicators (http://data.worldbank.org/products/wdi) (Accessed on May 24, 2016).

[2] Source: OECD, Tax on personal income (indicator). doi:10.1787/94af18d7-en (Accessed on June 6, 2016).

[3] Source: World Development Indicators (http://data.worldbank.org/products/wdi) (Accessed on May 24, 2016).

A.8 Proof of Proposition 7

Recall the policy function of x and the capital accumulation equation in the steady state presented in Subsection 4.1:

$$\begin{aligned} x &= X_0 \cdot (1-\alpha) A (k)^{\alpha} h, \\ k &= \tilde{\Psi}_K \cdot \{(1-\alpha) A (k)^{\alpha}\}^{1-\eta}. \end{aligned}$$

Combining these equations, we obtain:

$$x = X_0 (1-\alpha) A \left(\tilde{\Psi}_K\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \{(1-\alpha)A\}^{\frac{\alpha(1-\eta)}{1-\alpha(1-\eta)}} h.$$

Substituting this into $h' = D(x)^{\eta}(h)^{1-\eta}$ leads to:

$$h' = D \cdot \left[X_0 (1-\alpha) A \left(\tilde{\Psi}_K\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \{(1-\alpha)A\}^{\frac{\alpha(1-\eta)}{1-\alpha(1-\eta)}} h \right]^{\eta} (h)^{1-\eta},$$

or

$$\left.\frac{h'}{h}\right|_{\text{tax}} = D \cdot \left[X_0 \{(1-\alpha)A\}^{\frac{1}{1-\alpha(1-\eta)}} \left(\tilde{\Psi}_K\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \right]^{\eta}. \tag{60}$$

Next, recall the set of the three equations presented in Subsection 4.2 and evaluate them at the steady state:

$$\begin{aligned} k &= \Psi_K \cdot \left[(1-\alpha) A (k)^{\alpha} - \alpha A (k)^{\alpha-1} b\right]^{1-\eta}, \\ b &= \Psi_B \cdot \left[(1-\alpha) A (k)^{\alpha} - \alpha A (k)^{\alpha-1} b\right]^{1-\eta}, \\ \frac{h'}{h} &= D (X_0)^{\eta} \cdot \left[(1-\alpha) A (k)^{\alpha} - \alpha A (k)^{\alpha-1} b\right]^{\eta}. \end{aligned}$$

The first two equations imply

$$k = (\Psi_K)^{\frac{1}{1-\alpha(1-\eta)}} \left[(1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K} \right]^{\frac{1-\eta}{1-\alpha(1-\eta)}}.$$

Plugging this into the first equation, we obtain:

$$\left[(1-\alpha) A (k)^{\alpha} - \alpha A (k)^{\alpha-1} b\right]^{1-\eta} = (\Psi_K)^{\frac{\alpha(1-\eta)}{1-\alpha(1-\eta)}} \left[(1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K} \right]^{\frac{1-\eta}{1-\alpha(1-\eta)}},$$

or

$$\left[(1-\alpha) A (k)^{\alpha} - \alpha A (k)^{\alpha-1} b\right]^{\eta} = (\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}} \left[(1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K} \right]^{\frac{\eta}{1-\alpha(1-\eta)}}.$$

We substitute this into the third equation and obtain:

$$\left.\frac{h'}{h}\right|_{\text{debt}} = D (X_0)^{\eta} (\Psi_K)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}} \left[(1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K} \right]^{\frac{\eta}{1-\alpha(1-\eta)}}. \tag{61}$$

A direct comparison of (60) and (61) leads to:

$$\begin{aligned}
\left.\frac{h'}{h}\right|_{\text{tax}} \gtrless \left.\frac{h'}{h}\right|_{\text{debt}} &\Leftrightarrow \left(\tilde{\Psi}_K\right)^{\alpha}(1-\alpha)A \gtrless (\Psi_K)^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right] \\
&\Leftrightarrow \left[\frac{\beta}{1+\beta}\cdot\frac{1-(1+n)X_0}{(1+n)D\left(X_0\right)^{\eta}}\right]^{\alpha}(1-\alpha)A \\
&\gtrless \left[\frac{\beta}{1+\beta}\cdot\frac{1-(1+n)X_0-\frac{1+n}{\beta}B_0}{(1+n)D\left(X_0\right)^{\eta}}\right]^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right] \\
&\Leftrightarrow \left[1-(1+n)X_0\right]^{\alpha}(1-\alpha)A \gtrless \left[1-(1+n)X_0-\frac{1+n}{\beta}B_0\right]^{\alpha}\left[(1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}\right],
\end{aligned}$$

where we obtain the second line from the definitions of $\tilde{\Psi}_K$ and Ψ_K.

Given that $B_0 > 0$ and $X_0 > 0$, we have:

$$\begin{aligned}
1-(1+n)X_0 &> 1-(1+n)X_0-\frac{1+n}{\beta}B_0, \\
(1-\alpha)A &> (1-\alpha)A-\alpha A\frac{\Psi_B}{\Psi_K}.
\end{aligned}$$

Therefore, we find that $\left.\frac{h'}{h}\right|_{\text{tax}} > \left.\frac{h'}{h}\right|_{\text{debt}}$.

■

A.9 Proof of Proposition 8

Recall the indirect utility function of the period-t middle-aged in the tax-finance case, (20), and that in the debt-finance case, (23). Given k_t, b_t, and h_t, we substitute the corresponding policy functions into (20) and (23) and obtain

$$\begin{aligned}
V_{t,\text{tax}} &= \{1+(\beta+\gamma)(\alpha+\eta(1-\alpha))\}\ln\left\{(1-\alpha)A(k_t)^{\alpha}-\alpha A(k_t)^{\alpha-1}b_t\right\} \\
&\quad +(1-\eta)(\beta+\gamma)(1-\alpha)\ln h_t + C_{tax} \\
V_{t,\text{debt}} &= \{1+(\beta+\gamma)(\alpha+\eta(1-\alpha))\}\ln\left\{(1-\alpha)A(k_t)^{\alpha}-\alpha A(k_t)^{\alpha-1}b_t\right\} \\
&\quad +(1-\eta)(\beta+\gamma)(1-\alpha)\ln h_t + C_{debt},
\end{aligned}$$

where

$$\begin{aligned}
C_{tax} &\equiv \{1+(\beta+\gamma)\alpha\}\ln\left(1-(1+n)X_0\right) + +\eta(\beta+\gamma)(1-\alpha)\ln X_0 + C_1, \\
C_{debt} &\equiv (1+\beta)\ln\left[1-(1+n)X_0+(1+n)B_0\right] + +\{\beta(\alpha-1)+\gamma\alpha\}\ln\left\{1-(1+n)X_0-\frac{1+n}{\beta}B_0\right\} \\
&\quad + +\eta(\beta+\gamma)(1-\alpha)\ln X_0 + C_1,
\end{aligned}$$

and

$$V_{t,\text{tax}} \lesseqgtr V_{t,\text{debt}} \Leftrightarrow C_{tax} \lesseqgtr C_{debt}.$$

Notice that $C_{tax} < C_{debt}$ holds because we assume the parametric condition that realizes $b > 0$, that is, $V_{t,\text{tax}} < V_{t,\text{debt}}$.

In the steady state, the expressions of $V_{t,\text{tax}}$ and $V_{t,\text{debt}}$ are reformulated as

$$\begin{aligned}
V_{tax} &= \{1 + (\beta+\gamma)(\alpha + \eta(1-\alpha))\} \ln \underbrace{(1-\alpha)A(k_{tax})^{\alpha}}_{(\#1)} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha) \ln \underbrace{\left(\lim_{t\to\infty} h_{t,tax}\right)}_{(\#2)} + C_{tax}, \\
V_{debt} &= \{1 + (\beta+\gamma)(\alpha + \eta(1-\alpha))\} \ln \underbrace{\left\{(1-\alpha)A(k_{debt})^{\alpha} - \alpha A(k_{debt})^{\alpha-1} b\right\}}_{(\#3)} \\
&\quad + (1-\eta)(\beta+\gamma)(1-\alpha) \ln \underbrace{\left(\lim_{t\to\infty} h_{t,debt}\right)}_{(\#4)} + C_{debt},
\end{aligned}$$

where k_{tax} and k_{debt} denote the steady-state capital in the tax-finance and the debt-finance cases, respectively. While $C_{tax} < C_{debt}$ holds under the present parametric condition, the term (#2) outweighs the term (#4) as demonstrated in Proposition 7. In addition, in the steady state, we obtain

$$\begin{aligned}
&(1-\eta)(\beta+\gamma)(1-\alpha) \ln \left(\lim_{t\to\infty} h_{t,tax}\right) + C_{tax} \\
&> (1-\eta)(\beta+\gamma)(1-\alpha) \ln \left(\lim_{t\to\infty} h_{t,debt}\right) + C_{debt},
\end{aligned}$$

because the difference between $h_{t,tax}$ and $h_{t,debt}$ becomes larger as time passes. Therefore, $V_{tax} > V_{debt}$ holds in the steady state if (#1) is larger than (#3), that is, if $k_{tax} > k_{debt}$.

To show $k_{tax} > k_{debt}$, recall that k_{tax} satisfies the capital-market-clearing condition, $k_{tax} = \tilde{\Psi}_k \cdot \{(1-\alpha)A\,(k_{tax})^{\alpha}\}^{1-\eta}$, that is,

$$k_{tax} = \left(\tilde{\Psi}_K\right)^{\frac{1}{1-\alpha(1-\eta)}} \cdot [(1-\alpha)A]^{\frac{1-\eta}{1-\alpha(1-\eta)}},$$

and k_{debt} is given by

$$k_{debt} = (\Psi_K)^{\frac{1}{1-\alpha(1-\eta)}} \cdot \left[(1-\alpha)A - \alpha A \frac{\Psi_B}{\Psi_K}\right]^{\frac{1-\eta}{1-\alpha(1-\eta)}},$$

from Eq. (58). Because $\Psi_B > 0$, $\Psi_K > 0$, and $\tilde{\Psi}_K > \Psi_K$ hold under the present parametric condition, we obtain $k_{tax} > k_{debt}$.

■

A.10 Supplementary Explanation for Subsection 5.1

The utility maximization problem of the middle in the presence of private education is

$$\begin{aligned}
&\max \ln c_t + \beta \ln d_{t+1} + \gamma \ln w_{t+1} h_{t+1} \\
&\text{s.t. } c_t + s_t + e_t \leq (1-\tau_t) w_t h_t, \\
&d_{t+1} \leq R_{t+1} s_t, \\
&h_{t+1} = D\left(x_t\right)^{\eta} \left(e_t\right)^{\delta} \left(h_t\right)^{1-\eta-\delta}.
\end{aligned}$$

Solving this problem leads to

$$\begin{aligned}
s_t &= \frac{\beta}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t, c_t = \frac{1}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t, \text{ and} \\
e_t &= \frac{\gamma\delta}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t. \qquad (62)
\end{aligned}$$

The capital-market-clearing condition is

$$(1+n) \cdot \left(k_{t+1} h_{t+1} + \hat{b}_{t+1}\right) = \frac{\beta}{1+\beta+\gamma\delta} \cdot (1-\tau_t) w_t h_t.$$

This is reformulated as

$$\begin{aligned}
k_{t+1} = &\frac{1}{(1+n) D\left(x_t\right)^{\eta} \left(\frac{\gamma\delta}{1+\beta+\gamma\delta} \cdot \left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right)\right)^{\delta} \left(h_t\right)^{1-\eta-\delta}} \\
&\times \left[\frac{\beta}{1+\beta+\gamma\delta}\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) - (1+n)\hat{b}_{t+1}\right]. \qquad (63)
\end{aligned}$$

We substitute (62) and (63) into the utility function and obtain the indirect utility function of the middle as follows:

$$\begin{aligned}
V_t = &\left\{1+\beta+\delta\left(\gamma+\beta\left(1-\alpha\right)-\gamma\alpha\right)\right\} \ln\left(i_t - (1+n)x_t + (1+n)\hat{b}_{t+1}\right) \\
&+ \left\{\beta(\alpha-1)+\gamma\alpha\right\} \ln\left(i_t - (1+n)x_t - \frac{1+\gamma\delta}{\beta}(1+n)\hat{b}_{t+1}\right) + \eta(\beta+\gamma)(1-\alpha)\ln x_t,
\end{aligned}$$

where irrelevant terms are omitted from the expression.

We solve the maximization problem of V_t and obtain the following policy functions:

$$\begin{aligned}
\hat{b}_{t+1} &= \tilde{B}_0 \cdot i_t, \\
x_t &= \tilde{X}_0 \cdot i_t,
\end{aligned}$$

where

$$\begin{aligned}
\tilde{B}_0 &\equiv \frac{\{(1+\beta)\beta - (\beta(\alpha-1)+\gamma\alpha)\} + \delta\cdot\{\beta(\beta+\gamma)(1-\alpha)+\beta\gamma(1-\alpha)-\gamma\gamma\alpha\}}{(1+n)(1+\gamma\delta)\cdot[1+(\beta+\gamma)(\alpha+\eta(1-\alpha))+\delta(\beta+\gamma)(1-\alpha)]}, \\
\tilde{X}_0 &\equiv \frac{\eta(\beta+\gamma)(1-\alpha)\left[1+(\beta+\gamma)(\alpha+\eta(1-\alpha)) - \frac{1}{\beta}\{(1+\beta)\beta - (\beta(\alpha-1)+\gamma\alpha)\} + \frac{\gamma\delta}{\beta}(\beta(\alpha-1)+\gamma\alpha)\right]}{(1+n)\cdot\left[(\beta(\alpha-1)+\gamma\alpha)\frac{1+\beta+\gamma\delta}{\beta} + \eta(\beta+\gamma)(1-\alpha)\right]\cdot[1+(\beta+\gamma)(\alpha+\eta(1-\alpha))+\delta(\beta+\gamma)(1-\alpha)]}.
\end{aligned}$$

We compare $\tilde{X}_0$ and X_0 and obtain

$$\tilde{X}_0 < X_0 \Leftrightarrow 0 < \delta(\beta+\gamma)(1-\alpha),$$

which holds for any positive parameter values. We also compare $\tilde{B}_0$ and B_0 and obtain

$$\begin{aligned}\tilde{B}_0 \gtrless B_0 \Leftrightarrow \beta\underbrace{\{(\beta+\gamma)(1-\alpha)-\gamma(1+\beta)\}}_{(*1)} \cdot [1+(\beta+\gamma)(\alpha+\eta(1-\alpha))] \\ \gtrless (1+\gamma\delta)(\beta+\gamma)(1-\alpha)\underbrace{[(1+\beta)\beta-(\beta(\alpha-1)+\gamma\alpha)]}_{(*2)}.\end{aligned}$$

The term marked by (*1) is negative while the term marked by (*2) is positive under the assumption in Proposition 5. Therefore, we obtain $\tilde{B}_0 < B_0$.

■

A.11 Supplementary Explanation for Subsection 5.3

In the tax-finance case, the indirect utility function is given by (20). Given $k_{t+1} = k$ for all t, the solution of maximizing V_t is

$$x_t = \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_t - R\hat{b}_t\right).$$

Therefore, the growth rate is

$$\left.\frac{h_{t+1}}{h_t}\right|_{tax} = D \cdot \left[\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(w\, h_t|_{tax} - R\hat{b}_t\right)\frac{1}{h_t|_{tax}}\right]^{\eta},$$

and the indirect utility is

$$\begin{aligned}V_t|_{tax} = (1+\beta+\gamma\eta)\ln i_t|_{tax} + \{\beta(\alpha-1)+\gamma\alpha\}\ln k + \gamma(1-\eta)\ln h_t|_{tax} \\ + (1+\beta)\ln\frac{1+\beta}{1+\beta+\gamma\eta} + \gamma\eta\ln\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} + C_0.\end{aligned}$$

In the debt-finance case, the indirect utility function is given by (23). Given $k_{t+1} = k$ for all t, the solution of maximizing V_t is

$$\begin{aligned}\hat{b}_{t+1} &= \bar{b}\cdot h_t, \\ x_t &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(i_t + (1+n)\bar{b}_t\right),\end{aligned}$$

where $\bar{b} \equiv w/(R-(1+n))$. Therefore, the growth rate is

$$\left.\frac{h_{t+1}}{h_t}\right|_{debt} = D \cdot \left[\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(i_t|_{tax} + (1+n)\bar{b}_t\right)\frac{1}{h_t|_{debt}}\right]^{\eta},$$

and the indirect utility function is

$$V_t|_{debt} = (1+\beta+\gamma\eta)\ln\left(i_t|_{debt} + (1+n)\bar{b}_t\right) + \{\beta(\alpha-1)+\gamma\alpha\}\ln k + \gamma(1-\eta)\ln h_t|_{debt}$$
$$+ (1+\beta)\ln\frac{1+\beta}{1+\beta+\gamma\eta} + \gamma\eta\ln\frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)} + C_0.$$

These expressions suggest that in period 0, we have

$$\left.\frac{h_1}{h_0}\right|_{tax} < \left.\frac{h_1}{h_0}\right|_{debt} \text{ and } V_0|_{tax} < V_0|_{debt}\,.$$

From period 1 onward, we have

$$\begin{aligned} x_t|_{debt} &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_t - R\bar{b}h_t + (1+n)\bar{b}h_t\right) \\ &= \frac{\gamma\eta}{(1+n)(1+\beta+\gamma\eta)}\left(wh_t - \frac{R-(1+n)}{R-(1+n)}wh_t\right) \\ &= 0, \end{aligned}$$

implying that $h_{t+1}/h_t|_{debt} = 0$ and $V_t|_{debt} = -\infty$ for $t \geq 1$.

■

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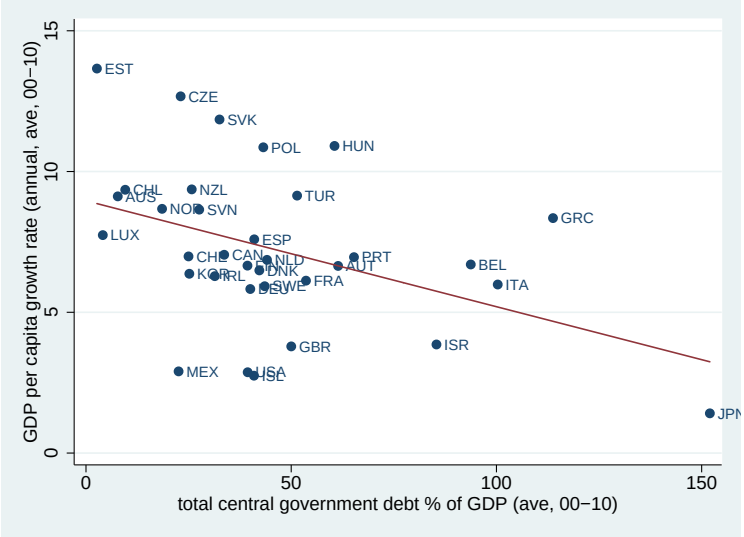


Figure 1: Average central government debt to GDP ratios and average GDP per capita growth rates during 2000–2010. Source: OECD Statistics (http://stats.oecd.org/); World Development Indicators (http://worldbank.org/indicator)

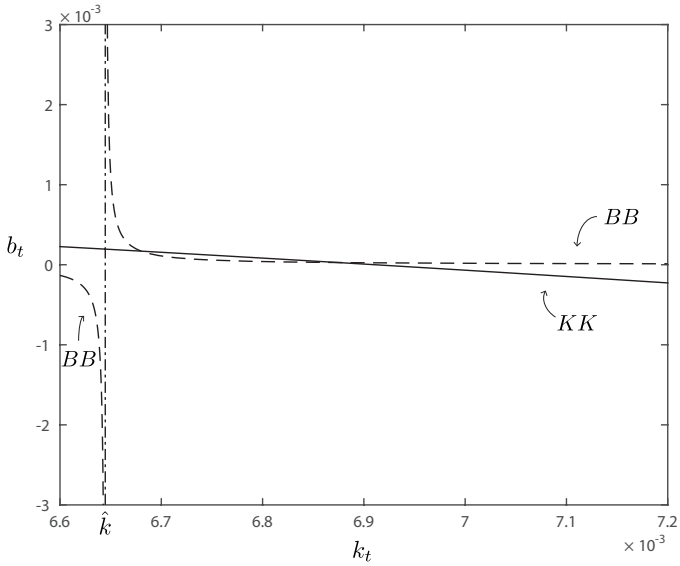
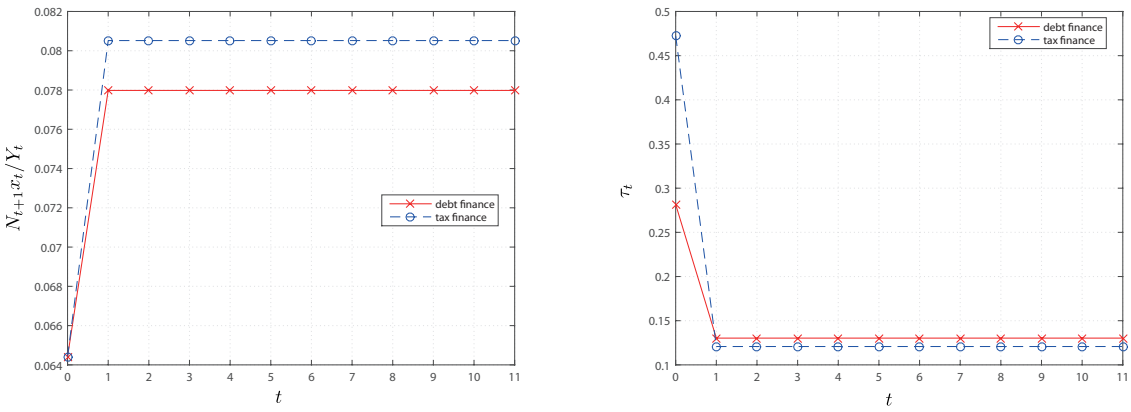


Figure 2: Numerical example of the phase diagram for the planner system, (12) and (13). The choice of parameter values, η, n, A, and D is explained in Appendix A.7. We set α at 0.3 and β at $(0.997)^{30}$ rather than $\alpha = 1/3$ and $\beta = (0.98)^{30}$ as in Appendix A.7 in order to satisfy the condition $\alpha < \beta/(1+2\beta)$ in Proposition 2. In addition, we set $X_{econ} = 0.05323$ and $\tau_{debt} = 0.076$ to satisfy the conditions for X_{econ} and τ_{debt} in Proposition 2.



(a) (b)

Figure 3: Numerical example of the evolution of the aggregate expenditure-GDP ratio (Panel (a)) and the tax rate (Panel (b)) for the tax-finance and the debt cases from period 0. The choice of parameter values is explained in Appendix A.7. The initial conditions are $k_0 = 0.001, h_0 = 0.0005$, and $b_0 = 0.0004$. All values apply in the following examples.

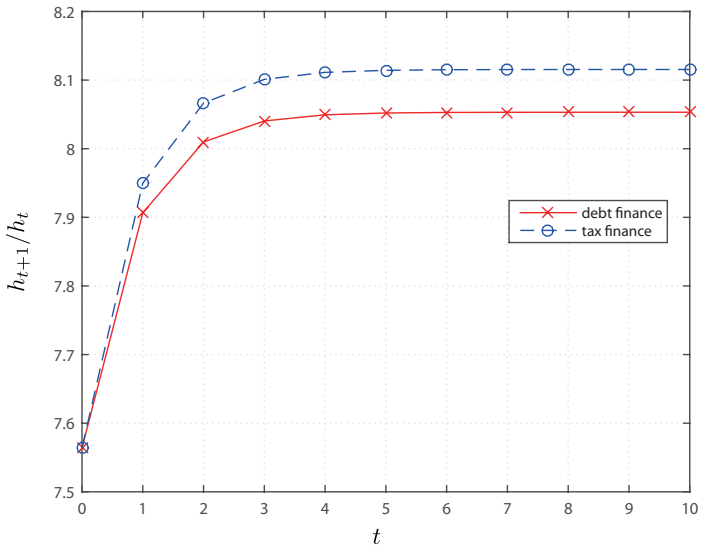


Figure 4: Numerical example of the evolution of the growth rate for the tax-finance and the debt-finance cases.

Table 1: Summary: Numerical results from period 0 to period 2.

	Tax-finance case	Debt-finance case
Period-0 utility	-43.8627	-43.8618
Period-1 utility	-32.6123	-32.7321
Period-2 utility	-21.8913	-22.0738

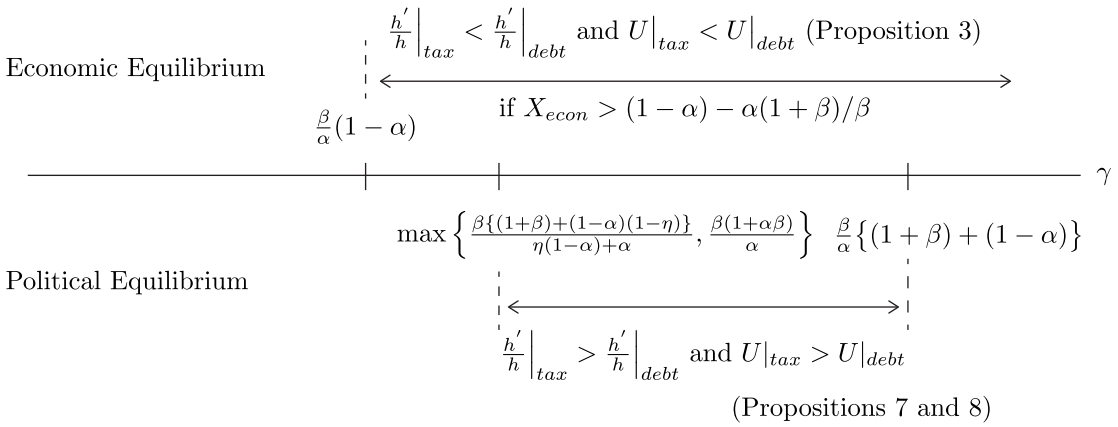


Figure 5: Comparing economic and political equilibria in a common range of γ.