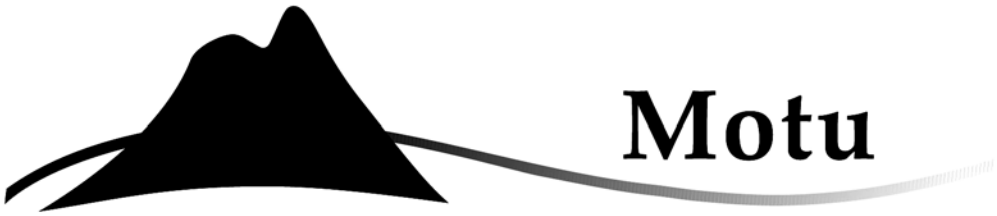

Tax, Credit Constraints, and the Big Costs of Small Inflation

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Abstract

This paper develops an overlapping generations model incorporating credit constraints, owner-occupier and rental sectors, and detailed tax regulations to examine how the interaction of inflation and the tax system affect the housing market. It shows that even modest rates of inflation can have very large effects on the home-ownership rates of young households, particularly at low real interest rates. This occurs even if there is a large supply response in the quantity of housing. The model suggests that the welfare costs of inflation could be ameliorated by exempting the inflation component of interest payments from income tax.

JEL classification
E40, E58

Keywords
Inflation, credit constraints, capital income taxes, housing markets, home-ownership rates, monetary policy

Contents

1 Introduction 1

2 An intergenerational model of housing demand 4
- 2.1 The basic framework 4
- 2.2 Agents 4
- 2.3 Taxes and the housing market 6
- 2.4 The lending market 8
- 2.5 Utility maximisation 10
- 2.6 Equilibrium conditions 11
- 2.7 Parameterisation 13
- 2.8 Solution technique 15
- 2.9 Comparison to other models 16

3 Results 16
- 3.1 Inelastic housing supply, all interest earnings taxed 16
- 3.2 Elastic housing supply, all interest earnings taxed 18
- 3.3 Exempting the inflation component of interest earnings from tax 18
- 3.4 Utility calculations 19
- 3.5 Parameter variations 21

4 Discussion and conclusions 23

5 References 26

1 Introduction

This paper develops a model that analyses the effect of inflation on housing markets. It has been sparked by concern among New Zealand policy makers that the home-ownership rates of young, low-income households are declining. The decline in home-ownership rates does not appear to be voluntary, for it has not been welcomed by younger, poorer, households. Rather, it has been accompanied by complaints that landlords have out-bid potential owner-occupiers because of tax laws that favour investment in property rather than interest earning assets when there is inflation.

Economists have long argued that the interaction of inflation with the tax system can cause substantial economic distortions (for example Viner (1923); Aaron (1976); Fischer and Summers (1989) or Feldstein (1996, 1997)). The primary issue concerns the taxation of capital income. Since most countries tax nominal interest earnings rather than real interest earnings, and since nominal interest rates typically rise in response to an increase in long-term inflation expectations, the real after-tax return on interest earning assets declines as inflation rises. This creates an incentive for owners of capital to seek out tax-sheltered assets as the inflation rate increases. A favoured investment class is residential housing, for in the long run prices tend to rise at the rate of inflation but capital gains are taxed lightly, if at all.

By itself, the interaction of inflation with the tax system is not enough to reduce home-ownership rates, for residential property acts as a tax shelter for owner-occupiers as well as landlords. Indeed, owner-occupiers with significant equity in their houses gain a larger tax advantage from housing than landlords, for imputed rent is typically not taxed. But inflation has a second effect on capital markets: it exacerbates the credit constraints facing borrowers. This is because bank imposed restrictions on the amount households can borrow are rarely adjusted for inflation, even though nominal interest rates increase when the inflation rate rises. If banks do not increase the amount credit-constrained households can borrow when nominal debt servicing payments increase, it becomes more difficult for these households to purchase houses (Modigliani (1976); Kearl (1979)).

The theoretical model developed in this paper attempts to unravel these competing effects. The model analyses how the interaction of households who differ by age and income determines house prices, and how these prices affect housing allocations. Because the tax and credit market effects of inflation depend on the wealth and income of households, and because these depend in turn on household age, a version of the Modigliani-Brumberg overlapping generations model is used. In the model, there is a plethora of households that differ by age, income, and marginal tax rate. These households choose to consume goods and services, to live in large or small houses, and to rent or buy. They save

for retirement, pay tax, borrow and lend, face realistic borrowing constraints, and choose whether or not to invest in housing. House prices are determined endogenously by matching the supply of houses with the collective demand for housing by owner-occupiers and landlords.

The model suggests that small changes in the inflation rate *can* have very large effects on home-ownership rates even when there is no uncertainty and inflation is perfectly anticipated. In the main parameterisation studied, in which real interest rates are 5 percent, a one percentage point increase in inflation leads to an 8–11 percent age point decline in home-ownership rates among young households. This occurs even if there is a large supply response in the quantity of housing. When inflation rises, young households find mortgage repayments more onerous, and landlords – attracted by tax-free capital gains – bid up property prices (or bid down rents if the housing supply is elastic). The result is a squeeze that induces households to delay house purchase.

The effect depends on the interaction of taxes and credit constraints, and largely disappears if there is only one distortion. When households can borrow as much as they like (but still repay their debt), moderate inflation rates have almost no effect on home-ownership rates irrespective of tax rates. Households simply borrow enough early in life to make the additional mortgage payments without reducing real consumption, knowing they will be able to repay the debt later on. If banks impose credit constraints but capital income is not taxed, a one percentage point rise in inflation leads to a 2–3 percentage point reduction in home-ownership rates among young households, rather than an 8–11 percentage point reduction. But taxation of capital income is not the primary reason why inflation has such a big effect on home-ownership rates when there are credit constraints. Rather, it is the taxation of the inflation component of interest payments. If real interest income is taxed at normal income tax rates but the inflation component of interest earnings is exempted from tax, a one percentage point increase in inflation again only leads to a 2 – 3 percentage point reduction in home-ownership rates. This reinforces economists' traditional view that taxing the inflation component of interest earnings is highly distortionary, even at what has been considered low inflation rates (Aaron 1976; Feldstein 1997).

While the size of the effect of inflation on home-ownership rates seems large, it should be noted that if real interest rates are 5 percent a one percentage point increase in the inflation rate increases nominal interest payments by approximately 20 percent. It is not implausible that a price change of this magnitude could induce a large number of young households to delay the purchase of a home and rent instead. In the model, the change in the number of households choosing to rent when the inflation rate increases is an increasing

function of the discount rate and a decreasing function of real interest rates. When real interest rates are above 6 percent (as they were in New Zealand between 1985 and 1998) moderate inflation has relatively little effect on the fraction of households choosing to rent.

The model is used to calculate how the inflation rate affects welfare. Under the assumption that households have a log-linear utility function, that interest income is taxed, and that the supply of housing is inelastic, an increase in the long-term inflation rate from 0 to 3 percent lowers welfare at all income levels by between 2 percent and 4 percent. This occurs for two reasons. First, inflation leads to higher house prices, forcing young people to rent for longer or to buy a small "starter" house rather than a large house. Secondly, it reduces real interest rates, distorting inter-temporal consumption and providing less incentive to accumulate capital to spend in retirement. If the supply of housing is elastic rather than inelastic, there are similar losses for middle and high income households, but in this case low income households gain as competition between landlords lowers equilibrium rents and leads to an overall increase in the housing stock. Nonetheless, inflation lowers welfare on average even in this case. In contrast, inflation has only small welfare effects if the inflation component of interest earnings is tax exempt. For this reason, this paper finds that when inflation is non-zero, welfare is enhanced if the inflation component of interest earnings is exempt from income tax.

The model highlights two issues relevant to current policy in New Zealand. First, given the structure of taxes in New Zealand, in which capital gains are exempt from tax, the recent deterioration in housing affordability may have been partly caused by the increase in the inflation target in 2001 and the subsequent increase in the average inflation rate. The analysis in this paper does not support the view that it is largely inconsequential whether inflation is in the top end or the bottom end of the 1–3 percent inflation target range. Secondly, if it is too difficult for the central bank to achieve very low rates of inflation in the medium term – say 0–2 percent – the adverse effects on home-ownership can be ameliorated through tax reform.

The model is related to several earlier papers. It is conceptually similar to Feldstein (1997) as it examines the welfare effects of low inflation, but it clarifies some of his insights by incorporating a rental housing sector and by allowing the after-tax interest rate to depend on whether an agent is a borrower or lender. It builds on the model used by Slemrod (1982) and Hayashi, Ito, and Slemrod (1988) to study individual optimisation over a lifecycle when there are taxes and credit constraints, but extends it by finding equilibrium house prices. It extends the equilibrium lifecycle model of housing markets analysed by Ortalo-

Magné and Rady (1998) and Coleman (2007) by incorporating taxes and by allowing the supply of housing to be elastic.

The paper is organised as follows. In section 2, the details of the model are presented and a sketch of the solution technique is offered. In section 3, key results of the model are shown for four different scenarios: when the housing supply is elastic or inelastic; and when the inflation component of interest is taxed or tax exempt. This is followed by an outline of the way the results of the model depend on key parameters such as interest rates and credit constraints. Finally, the results are discussed and conclusions are offered in section 4.

2 An intergenerational model of housing demand

2.1 The basic framework

The paper calculates equilibrium prices, saving patterns, consumption and housing allocations in an overlapping generations model of an economy that comprises a multitude of households that differ by age and income. The model is an extension of the model used by Coleman (2007) to analyse the effect of inflation and credit constraints on the housing market. In turn, it is based on the housing model of Ortalo-Magné and Rady (1998). In the model, there are four cohorts, each of which lives four periods and then dies. Agents have exogenously determined labour income and consume a single non-storable good. They also gain utility from renting or purchasing a single unit of housing. These housing units come in two sizes, small flats or large houses. Agents choose among different patterns of housing and consumption to maximise their utility. Agents can borrow or lend at exogenously determined interest rates, although young agents face credit constraints. Agents can also become landlords. They pay income tax on any interest earnings or on rental income. In the last period of life agents consume all wealth except their house, which is inherited by a younger generation.

The model is solved under two different assumptions about the supply of housing. In the first case, the supply of flats and houses is determined exogenously. In the second case, separate upward sloping supply curves for the quantity of flats and houses are specified, so that the equilibrium number and mix of properties is determined endogenously.

2.2 Agents

The N agents in each cohort live for four periods labelled $i = \{0,1,2,3\}$. A period is T years long. Agents differ by income and while any pattern of income is possible, agents

are assumed to have a constant place in the within-cohort income distribution. Agent 1 has the lowest income. In period t, agent *j* born in period *t-i* has real labour income

$$Y_t^{i,j} = \omega_j g_i Y_{t-i}^0 \qquad (1)$$

where ω_j = idiosyncratic factor affecting agent j relative to average cohort earnings; g_i = factor reflecting the life-cycle earnings of the cohort in its i^{th} period; and Y_{t-i}^0 = average income of cohort at time of birth.

Nominal income is $P_t Y_t^{i,j}$, where P_t is the pre-tax price of the good. An indirect goods and services tax is applied to goods other than housing at rate τ^g, so the post tax price of the good is $(1+\tau^g)P_t$. Incomes and the prices of goods both increase at a constant inflation rate π, where $1+\pi = P_{t+1}/P_t$.

Agents obtain utility from the consumption of goods and housing. An agent chooses real consumption $c_t^{i,j}$, and has housing choices described by a vector of three indicator variables $\mathbf{I}_t^{i,j,h} = \left\{ I_t^{i,j,R}, I_t^{i,j,F}, I_t^{i,j,H} \right\}$ that equal one if the agent has housing tenure *h* in period *i* of his or her life at time *t*, and zero otherwise. There are three possible housing tenures: an agent can rent a flat (R), purchase a flat (F), or purchase a house (H). Age zero agents can live with their parents at zero cost, although they gain zero utility from doing so. In period t agents obtain utility

$$u(c_t^{i,j}, \mathbf{I}_t^{i,j,h}) = \ln(c_t^{i,j}) + \sum_h v^h I_t^{i,j,h} \qquad (2)$$

It is assumed $v^H > v^F$ as houses are bigger than flats, and $v^F > v^R$, as agents can shape an owned flat in their own image, whereas they cannot modify a rented flat. Agents can only live in one housing unit in any period. Agents born at time t choose consumption and housing paths to maximise discounted lifetime utility:

$$U = \sum_{i=0}^{3} \beta^i u(c_{t+i,}^{i,j} \mathbf{I}_{t+i}^{i,j,h}) \qquad (3)$$

Households are assumed to receive their income, purchase, rent, or sell property, borrow or lend, and consume at the start of each period, although they gain utility from housing by living in it throughout the period. In the last period, agents are assumed to sell or realise all assets except their last owned housing unit, repay any debts, and consume all of their wealth. They die at the end of period 3, at which point their housing unit is distributed

to younger cohorts. At time *t* a fraction κ_i is left to the cohort born at *t-i* for *i=0,1,2*; by assumption it is either equally distributed across all agents of that cohort, or left to the jth agent, thus preserving the intergenerational income ranking. The weights κ_i are chosen so that agents do not receive an inheritance until relatively late in life. This proves to be an important choice, for if agents inherit early, they have much less need to save for a housing unit. In this paper, $\kappa_2 = 1$. In the utility maximisation equation below, $Inherit_t$ is the average value of the inheritance left by the generation dying at the end of time *t-1* and inherited at time *t*.

2.3 Taxes and the housing market

Because the focus of the paper is the way capital income tax affects housing and consumption choices, the ways that the New Zealand tax system affects housing have been carefully modelled. Five features of the tax system have been modelled. First, interest and rent income is taxed at an agent's marginal tax rate. There are two marginal tax rates: τ_1 for agents with real income in period t less than τ^*; and $\tau_2 \geq \tau_1$ for agents with real income greater than or equal to τ^*. It is assumed that the tax threshold is automatically adjusted for inflation and thus constant in real terms. Secondly, the capital gains tax rate is zero. No property appreciation, either for an owner-occupied house or for a leased flat, is taxed. Thirdly, imputed rent is tax exempt. Fourthly, a landlord can deduct interest payments associated with a mortgage when calculating taxable income. Thus a landlord pays tax on rent net of interest payments, but no tax on any capital appreciation. Fifthly, there is a goods and services tax that is applied to consumption but not to rent or property. In the model, the goods and service tax rate is set endogenously at a rate that makes the total tax take (tax on capital income plus tax on goods and services) equal to a set fraction of labour income, in this case $\tau^{g^*} = 10$ percent. This ensures that any changes in the structure of capital incomes taxes do not have revenue implications for the Government. Agents do not receive utility from government expenditure.

Although the details of the tax system are closely based on the New Zealand tax regime, it is straightforward to change theses details. In the paper the effects of an alternative tax regime in which real interest earnings rather than nominal interest earnings are taxed are also examined.

Flats and houses cost P_t^F and P_t^H to purchase. Flats can also be leased, at price P_t^R that is paid in advance at the beginning of the lease. The rent is paid to a landlord, who, for convenience, is restricted to be an agent in period 2 of their lives. The number of

landlords is endogenous; an indicator variable $I_t^{i,j,R*}$ indicates whether or not the j[th] agent owns a rental property.[1] Because there is no uncertainty, the after-tax return from purchasing a flat in period *t*, leasing it, and selling it in period *t+1* is equal to the after-tax return from lending money. As such, the relationship between rent, tax rates, flat prices, and interest rates is

$$P_t^R(1-\tau_2)(1+r_t(1-\tau_2))^T + P_{t+1}^F = P_t^F(1+r_t(1-\tau_2))^T \qquad (4)$$

or

$$P_t^R = P_t^F\left(\frac{(1+r_t(1-\tau_2))^T - (1+\pi_t^F)}{(1-\tau_2)(1+r_t(1-\tau_2))^T}\right) \qquad (5)$$

where π_t^F is the rate of price appreciation for flats. The right hand side of equation 4 is the after-tax return in period *t+1* from investing P_t^F in interest earning bonds. The left hand side is the after-tax return at *t+1* from using the same sum to purchase a rental flat at time *t*. It comprises the after-tax rent paid at time *t* and reinvested at interest, plus the untaxed proceeds from selling the rental unit at time *t+1*. Since interest payments by landlords are fully tax deductible, the return to a landlord is independent of their level of gearing. It is assumed that the landlords are high income agents in period 2 of their lives, so after-tax returns are calculated using the top marginal tax rate τ_2.[2]

In each period, agents choose between one of the three housing options, or not having housing. Consequently, there are potentially 256 different housing patterns possible through an agent's lifetime. Rather than calculate the utility of each of these patterns, I only let agents choose from a much smaller set of patterns, $\mathcal{H}$. To reduce the number of possible patterns, I impose a series of restrictions on the lifetime housing options available to an agent. The three restrictions are: (i) only 0 period agents may choose no housing; (ii) only period 0 and period 1 agents may choose to rent; and (iii) except in the last period, agents' housing choices must not worsen through time. By this means, the set $\mathcal{H}$ is reduced to 23,

$\mathcal{H} = \{$0RFF, 0RHF, 0RHH, 0FFF, 0FHF, 0FHH, 0HHF, 0HHH, RRFF, RRHF, RRHH, RFFF, RFHF, RFHH, RHHF, RHHH, FFFF, FFHF, FFHH, FHHF, FHHH, HHHF,

[1] If there is demand for *f* flats, the *f* highest income individuals are assumed to own one flat each.

[2] If there is a high demand to rent property, it is possible that the last landlord in the model is on the low marginal tax rate. Nonetheless, it is assumed that competition between high income landlords determine rents, so the top marginal rate is used.

HHHH}. An agent's optimal discounted utility is calculated for each of these patterns, and the agent is assumed to choose the pattern that provides the greatest discounted utility.

The model is solved using two different assumptions about the stock of housing. In the first, the number of houses and flats is set exogenously and is completely inelastic. There are n^H houses and n^F flats. It is assumed that $n^H + n^F \leq 4N$. This means there are no vacant houses, so property prices and rents are positive in equilibrium. In the second, there is an elastic supply of flats and houses, and the quantity of each is determined in equilibrium along with rents and prices. Linear supply functions are specified:

$$\begin{aligned} P_t^F &= \alpha_0^F + \alpha_1^F \left(Q_t^F + Q_t^H \right) \\ P_t^H &= P_t^F + \alpha_0^H + \alpha_1^H Q_t^H \end{aligned} \tag{6}$$

In this specification the price of flats is an increasing function of the total number of properties (to reflect the possible scarcity of land), while the price of houses is determined as a variable premium over the price of flats (to reflect the additional building costs). In most of the simulations presented below, parameters are chosen so that a 1 percent increase in the number of properties leads to about a 1 percent increase in the price of flats.

2.4 The lending market

There is a non-profit financial intermediary that accepts deposits and issues mortgages at an interest rate r_t. Agents can lend or borrow as much as the bank allows them at the one period interest rate r_t, subject only to the restriction that they have a zero debt position at the end of their life. The economy can either be closed, in which case the interest rate is determined endogenously and aggregate deposits equal aggregate loans, or open, in which case real interest rates are determined exogenously and the net foreign asset position can be non-zero. There are no restrictions on the deposit contract, and interest on a deposit made at time *t* is paid at time *t+1*. Agents pay tax on this interest at their marginal tax rate, but do not get a tax deduction for interest paid on borrowed funds unless they borrow to fund a rental property.[3] An agent's positive funds are labelled $B_t^{i,j}$.

[3] To reduce computational complexity, the marginal tax rate is calculated on the basis of labour income, not total income. Otherwise the marginal tax rate is determined endogenously.

The mortgage contract is subject to three restrictions.[4]

i) The loan to value restriction.

The mortgage may not exceed a certain fraction of the value of the property. In particular, the gross amount borrowed $D_t^{i,j-}$ cannot exceed the value of property multiplied by the loan to value ratio θ: that is

$$D_t^{i,j-} \leq \sum_{h \in F,H} \theta P_t^h I_t^{i,j,h} \tag{7}$$

(Note $D_t^{i,j-} > 0$ if the agent borrows.) This restriction means that agents who rent cannot borrow to smooth consumption, although they can save.

ii) The regular cash payment restriction.

Banks only issue η-year table mortgages, and require a "cash payment" in the period the mortgage is issued. This restriction is imposed to mimic a standard condition of a table mortgage, namely that a customer is required to make regular cash repayments *CP* of equal size throughout the life of the mortgage rather than a large repayment at its terminal date. The payment size *CP* is chosen to ensure the mortgage is retired at the end of the term: if D^{0-} is initially borrowed, the annual payment is

$$CP = D^{0-} r \left[\frac{(1+r)^{\eta}}{(1+r)^{\eta} - 1} \right] \tag{8}$$

and η is assumed to be 25 years.[5]

It is not possible to exactly replicate this feature of a standard mortgage contract in the model. However, a close approximation is achieved by requiring the customer to make a payment that pays off some of the interest and principal in any period he or she has debt. In particular, a customer with gross debt of $D_t^{i,j-}$ is required to open up a separate account with the bank and make a deposit of size

$$D_t^{i,j*} = D_t^{i,j-} \frac{r_t}{1+r_t} \left[\frac{\left(1+r_t\right)^{\eta/T}}{\left(1+r_t\right)^{\eta/T} - 1} \right] \tag{9}$$

[4] Note that banks impose these restrictions even though there is no uncertainty in the model

[5] Until recently, this has been the standard term for a table mortgage in Australia and New Zealand.

into this account. This deposit earns (untaxed) interest at rate r_t. This means the net borrowing position of a borrowing agent, $D_t^{i,j} = D_t^{i,j-} - D_t^{i,j*}$, is less than the gross borrowing position. Without this "cash payment" feature, many agents would prefer to purchase rather than rent simply because the interest payment occurs a period later than the rental payment. When the "cash payment" requirement is imposed, purchasing a house requires a larger payment to the bank in period t than the cost of renting a house.

iii) The mortgage-repayment-to-income restriction.

The maximum amount an agent can borrow is restricted to ensure the mortgage repayment given by equation 8 is smaller than a fraction δ of income:

$$D_t^{i,j-} \frac{r_t}{1+r_t}\left[\frac{(1+r_t)^{\eta/T}}{(1+r_t)^{\eta/T}-1}\right] \le \delta P_t Y_t^{ij} \qquad (10)$$

Note that this constraint is expressed in terms of nominal interest rates.

The mortgage conditions are only imposed on agents in periods 0 and 1 of their lives in order to simplify the solution algorithm. In period 2 agents can borrow unrestricted amounts. The absence of a restriction in period 2 has little effect because agents are in their peak earning years, receive their inheritance at this time, and are actively saving or reducing debt to finance their retirement.

2.5 Utility maximisation

An agent born at time t solves the following constrained maximisation problem (the jth superscript is omitted):

$$Max_{\{c_{t+i},\mathbf{I}_{t+i}^{i,h}\}} U = \sum_{i=0}^{3} \beta^i u(c_{t+i,}^i \mathbf{I}_{t+i}^{i,h}) \qquad (11)$$

$$-\lambda_0 \left(P_t Y_t^0 - B_t^0 + D_0^t - (1+\tau^g) P_t c_t^0 - \sum_h P_t^h I_t^{0,h} \right)$$

$$-\sum_{i=1}^{3} \lambda_i \begin{pmatrix} (1+\pi)^i P_t Y_{t+i}^i + B_{t+i-1}^i (1+r_{t+i-1}(1-\tau^i)) - D_{t+i-1}^i (1+r_{t+i-1}) - B_{t+i}^i + D_{t+i}^i \\ -(1+\pi)^i (1+\tau^g) P_t c_t^i - \sum_h P_{t+i}^h I_{t+i}^{i,h} + \sum_{h=F,H} P_{t+i}^h I_{t+i-1}^{i-1,h} + \kappa_i Inherit_{t+i} \\ +\left[P_{t+i}^R (1-\tau^i) I_{t+i}^{i,R*} + (P_{t+i}^F - P_{t+i-1}^F (1+r_{t+i-1}(1-\tau^i)) I_{t+i-1}^{i-1,R*} \right] \end{pmatrix}$$

$$-\sum_{i=0}^{1} \chi_i \left(D_{t+i}^{i,-} - \sum_h \theta P_{t+i}^h I_{t+i}^{i,h} \right)$$

$$-\sum_{i=0}^{1}\phi_i\left(D_{t+i}^{i,-}\frac{r_{t+i}}{1+r_{t+i}}\left[\frac{(1+r_{t+i})^{\tau/T}}{(1+r_{t+i})^{\tau/T}-1}\right]-\delta Y_{t+i}^{i}\right)$$
$$-\sum_{i=0}^{3}\varsigma_i\left(B_{t+i}^{i}\right)-\sum_{i=0}^{3}\nu_i\left(D_{t+i}^{i}\right)$$

Lines 2 and 3 of equation (11) are the budget constraints facing the agent in the four periods. Lending and borrowing are entered separately as there are different after tax interest rates, and there are terms to reflect inheritance and rental income. Lending and borrowing in period 3 are restricted to equal zero, and τ^i is the marginal tax rate applying in period *i* of the agent's life. The Kuhn-Tucker conditions in lines 4 and 5 reflect the loan-to-value ratio constraints and the mortgage-repayment-to-income ratio constraints respectively. The Kuhn-Tucker conditions in line 6 reflect the requirement that non-negative amounts are lent and borrowed. The agent solves the problem by calculating the maximum utility for each housing pattern in the set $\mathcal{H}$, and then selecting the housing pattern with the highest utility. The use of log-linear utility functions means it is relatively straightforward to calculate an analytical solution for the optimal consumption path given a particular housing pattern, even though each solution has 48 parts corresponding to the 48 possible combinations of Kuhn-Tucker conditions.[6]

2.6 Equilibrium conditions

In the simulations, the steady state equilibrium is found for an open economy in which agents borrow or lend at the world interest rate. In the steady state, the following price relationships hold:

$$(1+r_t)/(1+\pi_t)=r \tag{12a}$$

$$\frac{P_{t+1}^{F}}{P_t^{F}}=1+\pi^{F} \tag{12b}$$

$$\frac{P_t^{H}}{P_t^{F}}=\rho^{H} \tag{12c}$$

$$\frac{P_t^{R}}{P_t^{F}}=\left(\frac{(1+r_t(1-\tau_2))^{T}-(1+\pi_t^{F})}{(1-\tau_2)(1+r_t(1-\tau_2))^{T}}\right)=\rho^{R} \tag{12d}$$

[6] In the periods 0 and 1, the financial asset position can be positive, zero, negative, or equal to the borrowing constraint; in period 2, the financial asset position can be positive, zero or negative; and in period 3 it is zero.

Equation (12a) states that real interest rates are constant. In the open economy model, the rate *r* is the foreign real interest rate. Equation (12b) states that flat prices appreciate at a constant rate.[7] Equation (12c) states that the ratio of house prices to flat prices is constant. Equation (12d) is a restatement of equation 5, linking rents to interest rates and the flat price appreciation rate.

For a set of parameters $\left\{N, T, Y_t^0, \omega_j, g_i, \pi, \beta, \nu_h, \kappa_i, \mathcal{H}, n^H, n^F, \eta, \theta, \delta, \tau^{g*}, \tau_1, \tau_2, \tau^*\right\}$ and housing parameters either $\left\{n^F, n^H\right\}$ or $\left\{\alpha_0^F, \alpha_1^F, \alpha_0^H, \alpha_1^H\right\}$ the steady state equilibrium is described by a set of prices $\left\{r, \pi^F, \rho^H, \rho^R\right\}$, a GST rate τ^g, a set of housing and consumption demands $\{c_{t-i+s,}^{s,j}, \mathbf{I}_{t-i+s}^{s,j,h}\}_{s=0,\ldots,3}$ for each agent j in each cohort born in period *t-i*, and a net foreign asset position B_t^{net} such that all agents have maximal utility and

$$\sum_{i=0}^{3}\sum_{j=1}^{N} c_t^{i,j} = (1-\tau^{g*})\sum_{i=0}^{3}\sum_{j=1}^{N} y_t^{i,j} - \left(\frac{r-\pi}{1+\pi}\right) B_t^{net} \tag{13a}$$

$$\sum_{i=0}^{3}\sum_{j=1}^{N} (B_t^{i,j} - D_t^{i,j}) - P_t^F \sum_{j=1}^{N} I_t^{2,j,R*} = B_t^{net} \tag{13b}$$

$$\tau^{g*}\sum_{i=0}^{3}\sum_{j=1}^{N} y_t^{i,j} = \tau^{g}\sum_{i=0}^{3}\sum_{j=1}^{N} c_t^{i,j} + \sum_{i=0}^{3}\sum_{j=1}^{N} B_{t-1}^{i,j} r \tau_t^{ij} + \sum_{j=1}^{N} P_t^R \tau_t^{2,j} I_t^{2,j,R*} - \sum_{j=1}^{N} P_{t-1}^F r \tau_t^{3,j} I_{t-1}^{2,j,R*} \tag{13c}$$

and either

$$\sum_{i=0}^{3}\sum_{j=1}^{N} (I_t^{i,j,R} + I_t^{i,j,F}) = n^F \tag{13d'}$$

$$\sum_{i=0}^{3}\sum_{j=1}^{N} I_t^{i,j,H} = n^H \tag{13e'}$$

or

$$\sum_{i=0}^{3}\sum_{j=1}^{N} (I_t^{i,j,R} + I_t^{i,j,F}) = Q^F \tag{13d''}$$

[7] If the number of flats and houses is determined exogenously, an equilibrium can be found in which incomes in the economy grow at a constant rate, and in this case the steady state equilibrium will have property prices growing at a faster rate than the rate of inflation. If the number of properties is determined endogenously and the income growth rate is positive, the only possible steady states occur

$$\sum_{i=0}^{3}\sum_{j=1}^{N} I_t^{i,j,H} = Q^H \qquad (13e'')$$

where Q^F and Q^H are the number of houses produced when the supply of properties is elastic,

$$Q^H = \frac{P^H - P^F - \alpha_0^H}{\alpha_1^H} \quad \text{and } Q^F = \frac{P^F - \alpha_0^F}{\alpha_1^F} - Q^H .$$

Equation (13a) requires that total consumption plus tax plus real earnings on the net bond position in each period equals total production. Equation (13b) is the net supply of foreign bonds, given that landlords are assumed to borrow 100 percent of the price of a flat. This will change through time if there is economic growth or inflation. Equation 13c says that the total tax take is equal to total GST revenue plus tax on interest and rent minus the tax deduction for landlords. Note that while it has been assumed landlords borrow 100 percent of the value of the property, tax revenue would not change if landlords had different gearing as the tax rate on positive balances is the same as the tax deduction they get when they borrow. Equations (13d') and (13e') require that the total demand for flats equals the supply of flats, and that the total demand for houses equals the supply of houses, when the supply of properties is determined exogenously. Equations (13d'') and (13e'') are the same equations for the case when the supply of property is elastic.

2.7 Parameterisation

The set of baseline parameters $\left\{N, T, Y_t^0, \omega_j, g_i, \pi, \beta, \nu_h, \kappa_i, \mathcal{H}, n^H, n^F, \eta, \theta, \delta, \tau^{g^*}, \tau_1, \tau_2, \tau^*\right\}$ and housing parameters $\left\{n^F, n^H\right\}$ are nearly the same as those used by Coleman (2007) and have been chosen to approximate features of the New Zealand economy.[8] These are listed in table 1. Except for income distribution, the income parameters approximately match the basic lifecycle and cohort income patterns of New Zealanders reported in census documents, 1966–2001, under the assumption that the basic agent is a household comprised of a male and female of the same age. For simplicity, income is assumed to be uniformly distributed over the range \$20,000 to \$80,000.

when all people live in large houses, or when the quality of flats and houses steadily improves. This paper does not analyse these cases although the model is set up to do so.

[8] Coleman (2007) uses 5 cohorts, not 4, and the parameters have been slightly modified.

In the baseline model, the discount rate is 3 percent, the real interest rate is 5 percent (assumed equal to the world rate), and banks impose borrowing restrictions that limit households to borrow up to 90 percent of the value of a property and to pay no more than 30 percent of their income in debt servicing. The banking sector parameters are changed in some of the simulations, but these reflect the conditions facing New Zealand borrowers since the year 2000.

The tax rates also reflect New Zealand tax settings in 2000. In the baseline model, the marginal tax is 20 percent for households with incomes less than $50,000, and 33 percent for households with incomes above that level. To demonstrate how taxes and inflation interact, the model was solved under a variety of alternative tax regimes: (τ_1,τ_2) = (0%,0%,), (10%, 10%), (20%, 20%), and (20%, 39%).[9] The model is also solved for a set of tax rules that exclude the inflation component of interest income from tax, and which only allow landlords to deduct real interest payments from their taxable income.[10] The GST rate was chosen to ensure that capital income taxes and consumption taxes total to 10 percent of labour income.

When the housing supply is inelastic, 57 percent of the properties are large houses, and 42 percent of the properties are small houses. The total number of properties is 1 percent less than the number of agents, to ensure rents are positive in equilibrium. The parameters $(\upsilon^R, \upsilon^F, \upsilon^H) = (0.33, 0.35, 0.45)$ mean (approximately) that at the margin a household would be prepared to spend a third of their income on rent rather than have no accommodation; the benefit from living in an owner-occupied flat rather than a rented flat is 2 percent, and the additional benefit from living in a large house a further 10 percent. Housing supply parameters were chosen so that that the quantity of flats would increase by approximately one percent for a one percent increase in prices, but that the number of houses and flats would be approximately the same in the elastic and inelastic cases.

The model was solved for inflation rates ranging from 0 to 3 percent, reflecting the legal requirement that the Reserve Bank of New Zealand achieve stability in the general level of prices. This requirement currently requires the Bank to keep inflation between 1 and 3 percent. The effect of higher inflation rates were not investigated as the purpose of the

[9] The top income tax rate was increased from 33 percent to 39 percent in 2000. However, many landlords can use property trusts to lower their marginal tax rate to 33 percent. This is the reason why the main simulations have been done using a top tax rate of 33 percent.

[10] In this case the constraints in equation 11 and the aggregation condition (13c) are modified accordingly.

paper was to ascertain whether low rates of inflation have an appreciable effect on economic welfare.

2.8 Solution technique

The solution is found numerically. The algorithm searches for a set of prices $\left\{\tau^g, P_t^R, P_t^F, P_t^H\right\}_{t=-3,..0,...3}$ so that when each agent *j* born in period *t-i, i= 0,…3* is consuming a sequence of goods and tenure options $\{c_{t-i+s,}^{s,j}\mathbf{I}_{t-i+s}^{s,j,h}\}_{s=0,...,3}$ that solves their constrained utility problem given by equation (11), the aggregation conditions 13a–13e applied at time *t* are satisfied. In the steady state, the vector $\left\{\tau^g, P_t^R, P_t^F, P_t^H\right\}_{t=-3,..0,...3}$ can be calculated from the vector $P^* = \{\tau^g, P_0^F, \pi^F, \rho^H\}$ and the parameters $\left\{r, \tau_2\right\}$.

The basic structure of the algorithm is as follows.

a) Let the vector $P^{*,k} = \{\tau^g, P_0^F, \pi^F, \rho^H\}^k$ be the *k*th estimate of the steady state solution P^*. Given $P^{*,k}$, calculate the optimal consumption and housing tenure paths for each of the N households who are born at *t=0* by searching over the different possible tenure paths in the set $\mathcal{H}$.

b) Use these results to calculate the demand for consumption goods and housing at time *t=0* for all households in the economy.

c) Use these results to calculate aggregate consumption, the aggregate demand for flats, and the aggregate demand for houses at time *t=0*. Then calculate the excess demand functions given by 13a–13e.

d) If the excess demand functions are not sufficiently close to zero, a new estimate of the equilibrium prices $P^*, P^{*,k+1}$, is calculated. This is done using a discrete approximation to the Newton-Rhapson method. A set of quasi-derivatives is calculated by recalculating the set of excess demand functions at the prices $\{\tau^g + \Delta_1, P^F, \pi^F, \rho^H\}$, $\{\tau^g, P^F + \Delta_2, \pi^F, \rho^H\}$, $\{\tau^g, P^F, \pi^F + \Delta_3, \rho^H\}$ and $\{\tau^g, P^F, \pi^F, \rho^H + \Delta_4\}$. These quasi derivatives are used to calculate the updated

price vector using Broyden's method. The process is continued until the sequence of estimates $P^{*,k}$ converges.

2.9 Comparison to other models

The model is related to earlier models by Slemrod (1982), Hayashi, Ito and Slemrod (1988), Feldstein (1997), Ortalo-Magné and Rady (1998, 2006), and Coleman (2007). Slemrod (1982) solves a similar model of the household to explore how taxes and credit constraints affect inter-temporal housing and saving choices. His model has a similar asymmetric treatment of capital taxes on positive and negative asset positions, and imposes a maximum loan-to-value constraint. The price of housing is determined exogenously, but agents can choose the size of their house. Hayashi, Ito, and Slemrod solve a version of this model that uses log-linear utility of the form used in this paper. The basic framework including the housing model is adapted from Ortalo-Magné and Rady, although they only use the version with inelastic supply. Several of the modifications were first made in Coleman (2007) including the use of log-linear preferences rather than linear preferences, the incorporation of the mortgage-repayment-to-income constraint, and a wider range of housing options. This model extends the 2007 model by incorporating taxes and an elastic housing supply.

3 Results

The focus of the paper is the way that the inflation rate changes the effect of the tax system on the housing market. The main results are established by finding the equilibrium prices and GST rate that correspond to a set of parameters as the inflation rate is varied from 0 to 3 percent. In tables 2–5, various outcomes of interest are presented when the tax rates are $(\tau_1, \tau_2) =$ {(0%, 0%), (20%, 20%), (20%, 33%)}. In tables 2 and 3, the housing supply is either inelastic or elastic and all interest earnings are taxed; in tables 4 and 5 the housing supply is either inelastic or elastic and the inflation component of interest earning are tax exempt. The tables show how rents, flat prices, house prices, the number of flats and houses, the number of people renting, and the GST rate vary with the inflation rate for each different tax combination.

3.1 Inelastic housing supply, all interest earnings taxed

Table 2 and figures 1 and 2 indicate how inflation and tax affect the housing market when the supply of housing is determined exogenously. When the income tax rate is

zero, flat prices, house prices and rents are little affected by the inflation rate, falling by approximately 1 percent as the inflation rate increases from 0 to 3 percent.[11] However, the fraction of the youngest cohort renting increases as the inflation rate increases from 0 to 3 percent, from 21 percent to 31 percent. These results follows from the way rents are determined. When the number of flats and houses is fixed, the price of a flat is determined by its value to the marginal resident, which depends on the additional benefit of living independently rather than in the parental home. For the assumed values of ν^R and ν^F, this is approximately equal to 30 percent of the income of the marginal occupier; this sum varies only slightly with the inflation rate as lifetime utility and thus the amount desired to be spent on housing changes only slightly.[12] As rents vary little with inflation, flat prices also vary little. The housing tenure choice is affected by the inflation rate, however, because more and more low income households find it preferable to rent rather than purchase as nominal interest rates increase.

When the tax rates are positive, there are three competing tendencies. When the inflation rate is zero, the fraction of the young cohort renting is decreasing in the tax rate. This is because imputed rent is tax exempt, providing households with an incentive to own their own home rather than to rent and accumulate financial assets. The effect is sufficiently large that the fraction renting falls to zero when (τ^1, τ^2)=(20%, 33%). As the inflation rate increases, however, there are two other effects. First, landlords are attracted to the housing market to take advantage of tax free capital gains. Because they are on a higher marginal tax rate than low income tenants, they value the tax concession more highly than tenants and thus the amount they are prepared to pay for rental property is a steeply increasing function of the inflation rate (Litzenberg and Sosin (1979)). When both the high and low marginal tax rates are 20 percent, the equilibrium flat price increases by 8 percent when the inflation rate increases from 0 to 3 percent; when the top marginal tax rate is 33 percent, there is a 17 percent increase. Secondly, nominal interest rates increase as the inflation rate increases and low income, credit constrained households find it increasingly difficult to make mortgage payments. The result is a squeeze that sees landlords replace young households as the owners of flats. For (τ^1, τ^2)=(20%, 20%), the fraction of the young cohort renting increases from 6 to 32 percent as the inflation rate increases from 0 to 3 percent; for (τ^1, τ^2)=(20%, 33%), the increase is larger, from 0 to 42 percent.

[11] Coleman (2007) discusses the situation at length.

[12] Assuming that a low income young person spends all their income, the amount they are willing to spend on rent is $y_t^{0,j}(1-e^{-\nu^R})$.

Figure 3 shows the fraction of the young cohort that rents as a function of tax rates and inflation rates. When the inflation rate is less than 1 percent, the fraction of the young cohort that rents declines as tax rates increase, due to the rising value of the imputed rent tax concession. When the inflation rate is 2 or 3 percent, the effect of higher mortgage rates and the increasing attractiveness of property to high marginal rate investors means that the fraction that rents is an increasing function of the top marginal tax rate. Thus even though landlords and owner-occupiers derive benefits from residential housing tax concessions, the effect of taxes on ownership patterns depends crucially on the inflation rate.

3.2 Elastic housing supply, all interest earnings taxed

Table 3 and figures 4–6 indicate how inflation and tax affect the housing market when the supply of housing is elastic. Again, flat prices, house prices, rents, and the total number of houses are little affected by the inflation rate when the income tax rate is zero. Moreover, as before, the fraction of the youngest cohort renting increases as the inflation rate increases from 0 to 3 percent, from 23 to 28 percent. There is a difference from the inelastic case, however: as the inflation rate increases, the composition of the housing stock changes, with more flats and fewer large houses. This is because fewer young households can afford the nominal mortgage payment on large houses when the inflation rate increases, so they purchase a small house as a temporary measure.

When tax rates are positive, the fraction of the young cohort that rents increases with the inflation rate by a similar amount as when the housing supply is inelastic: in the case that (τ^1, τ^2) =(20%, 33%), the fraction renting increases from 0 to 35 percent as inflation increases from 0 to 3 percent. The mechanism is quite different, however. In the inelastic case, flat prices increased as inflation increased because landlords bid up prices to take advantage of the tax concession. In the elastic case, prices of houses do not increase by very much, but landlords bid rents down. The combination of slightly higher flat prices and significantly lower rents results in a big increase in the number of low income households choosing to rent, but it also leads to a larger stock of flats and a reduction in the number of households living at home with their parents.

3.3 Exempting the inflation component of interest earnings from tax

It has long been argued that many of the economic distortions caused by the interaction of the inflation and income tax could be avoided by changing the tax code so that only real interest income was taxed (Aaron 1976; Fischer and Summers 1989; Feldstein 1996). In practice, this would mean two adjustments: the inflation component of interest earnings would be exempt from income tax; and borrowers would only be allowed to deduct

the real interest component of interest payments from taxable income.[13] The effects of these two changes in the tax code on the housing market are shown in tables 4 and 5, and in figures 1–2 and 4–6. The GST rate is adjusted so that total nominal taxes are the same.

The results are dramatically different. When the supply of housing is inelastic, changes in the inflation rate only have small effects on house prices and home-ownership rates. Indeed the effects are very similar to the case when taxes are zero: as the inflation rate increases from 0 to 3 percent, the fraction of the youngest cohort renting increases because nominal mortgage rates rise, but by 8 percent rather than by 42 percent. The difference with the "normal" tax case is that wealthy households have little incentive to become landlords as they no longer pay tax on the inflation component of interest earnings if they accumulate financial assets.[14] Consequently, landlords do not bid up the price of flats as the inflation rate increases. A similar result holds when the supply of housing is elastic. In this case, wealthy households do not compete to become landlords, rents are not bid down as the inflation rate increases, fewer houses are built, and home-ownership rates amongst the young scarcely change.

It is worth noting that exempting the inflation component of interest earnings from tax has little effect on tax revenue. Even when the inflation rate is 3 percent, the GST rate needs to be increased by less than 0.2 percent to raise the same amount of revenue. This is because little revenue is raised by taxing the inflation component of interest even when inflation is moderate, for high income households rearrange their finances to avoid it, in this case by becoming landlords. Even though this generates tax revenue on rental income, it is offset by a decrease in tax paid on interest. This offset occurs partly because the mortgage payments made by landlords are tax deductible.

3.4 Utility calculations

The maximum lifetime utility levels of each agent can be calculated as a function of equilibrium prices. These can be used to calculate how welfare changes across the income distribution as the inflation rate changes. Figures 7 and 8 show how utility levels change

[13] Economists have also recommended changing depreciation allowances and exempting the inflation component of capital gains from tax.

14 The tax on the inflation component of interest earnings is often called the widows' tax because most lenders in an economy are over 65.

when the inflation rate increases from 0 to 3 percent. In each case the change in lifetime utility is shown as a function of income in the first period.[15]

Figure 7 shows how utility changes when the supply of houses is inelastic and the tax rates are $(\tau_1, \tau_2) = (20\%, 33\%)$. When nominal interest earnings are taxed, the increase in the inflation rate reduces welfare across the board, by between 2 and 4 percent. The effect is largest on middle income households. These households experience a reduction in welfare because of changes to their housing consumption patterns, as they rent rather than purchase or delay their purchase of large houses in response to higher nominal interest rates. High income households, whose housing arrangements are typically unaffected by the inflation rate, experience a welfare reduction because they change the timing of their consumption and face higher real capital income taxes.

The figure also shows how welfare changes when the inflation component of interest earnings is exempt from tax. In this case there are few welfare changes. Middle income agents increase utility a little; low income agents experience a small welfare reduction, as they rent rather than own their homes in response to higher nominal interest rates; but for both groups the welfare changes are smaller than 1 percent, and typically less than 0.5 percent. The middle income group experiences an improvement in welfare because the GST rate falls when inflation rises. This occurs because the Government derives revenue from the tax on rent.

Figure 8 shows the change in utility that occurs when the housing supply is elastic. In the case that nominal interest earnings are taxed, the welfare changes are very different than when the housing supply is inelastic. In particular, inflation improves the welfare of low income households, by up to 4 percent. There are two reasons for this improvement. First, rents fall, and this fall more than offsets the decline in welfare that comes from being a tenant rather than an owner-occupier. Secondly, more flats are built, and the lowest income households gain from the opportunity to live independently rather than in the parental home. (The peak welfare improvement accrues to the highest income person that lived at home when the inflation rate was zero.) These gains are offset by welfare losses of middle income and high income households, the latter of approximately 2 percent, as

[15] As the utility function is log linear, the change in utility has an approximate interpretation as the percentage change in consumption that would make an agent indifferent between the two inflation rates.

these households pay higher taxes on their interest income. For the economy as a whole, average welfare is reduced.[16]

When the inflation component of interest is exempt from the tax, the welfare changes are almost the same irrespective of whether the housing supply is elastic or inelastic. This is because neither rent nor the total quantity of housing change by much in the elastic case, and thus outcomes are similar to the situation when the housing supply is elastic. Note that this means that if the inflation component of interest earnings were tax exempt and the housing supply was sufficiently elastic, welfare would decline for some low income households. This decline would take place as high income households would no longer compete with each other to provide low rent accommodation to low income households in order to avoid the tax on the inflation component of interest earnings.

3.5 Parameter variations

The above results show how various facets of the economy are affected by the inflation rate or tax rates. These results were calculated for a single set of the remaining parameters. In this section, the effect of changing some of these parameters is outlined. In general, with two exceptions, the results are qualitatively robust to changes in other parameters such as the tightness of credit constraints, the mix of houses and flats, or the utility households gain from different tenure arrangements. The exceptions, discussed below, concern the discount rate and the real interest rate.

Tables 6 and 7 show the effects of different parameter variations in the inelastic supply and elastic supply cases. Each row of table 6 shows the outcome when a single parameter is changed. The table shows the rent, flat price, house price and the fraction of the young cohort that rents when the inflation rate is 2 percent, and also the fraction of the young cohort that rents when the inflation rate is 0. The difference between the last two numbers is used to calculate the "slope" column: how the number of young households that rent changes as the inflation rate increases by 1 percentage point. This statistic is used as a summary statistic for measuring how inflation affects home-ownership rates. In table 7 this information is presented as well as the equilibrium number of flats and houses.

Table 6 indicates that neither the tightness of credit constraints, the mix of housing types, or the relative utility of different tenure options has much effect on the extent to which inflation affects the home-ownership rate of young households. In all cases, a one percent increase in the inflation rate led to a 9–14 percent increase in the number of young

[16] In particular, the equally weighted average change in utility is negative.

households renting. However, the table indicates that the effect of inflation on home-ownership rates is very sensitive to both the real interest rate and the discount rate. When real interest rates were 6 percent or larger, inflation had very little effect on home-ownership rates; in fact the model predicts that property prices would fall sufficiently far that almost all households would purchase flats or houses rather than rent. In contrast, when real interest rates fall to 4 percent, inflation had a very large effect on home-ownership rates: each percentage point increase in the inflation rate is associated with a 24 percent increase in the fraction of young households that rent. At even lower real interest rates, almost all young households rent rather than purchase, as the price of flats and houses becomes very high.

Home-ownership rates are sensitive to real interest rates when the supply of housing is inelastic for two reasons. First, house prices are sensitive to real interest rates, falling as real interest rates increase and rising as real interest rates decrease. By itself, this would not lead to changes in home-ownership rates, as the real financing cost (real interest rate multiplied by the flat price) changes little as interest rates change. However, the ratio of the annual loan repayment amount to the annual interest payment amount increases as real interest rates fall, as the fraction of the payment that is loan repayment rises (equation 8). This means that credit constrained households that rent have the option of consuming much more than households that purchase and are obliged to make large loan repayments. The model suggests that when interest rates are low this option is sufficiently attractive that a large number of young households are prepared to lower their lifetime consumption in order to have more of this consumption at a young age.[17]

A very similar pattern of results holds in the elastic supply case. The effect of inflation on home-ownership rates is not particularly sensitive to the tightness of credit constraints, building costs, or the utility value of different tenure options, but is sensitive to the discount rate and real interest rates. However, the mechanism is quite different. In this case, there is little effect on property prices as parameters change, but rents and the equilibrium number of flats and houses are affected. When real interest rates decline, rents fall, more flats are built, and the home-ownership rates of young households decline sharply.

In the main parameterisation, with real interest rates equal to 5 percent and the discount rate β equal to 0.97, a one percent increase in the inflation rate reduces home-ownership rates among young households by 11 percentage points in either the elastic or inelastic cases. When households are more patient, represented by an increase in the discount rate to 0.98, the reduction in the home-ownership rate is only 5 percent as fewer households

choose to rent in order to increase current consumption at the expense of future consumption. This suggests that overall effect of inflation on the housing market will depend not only on the distribution of income but also on the distribution of discount rates. In an attempt to ascertain the effect of having households that differ by discount rate and income, the model was reconfigured to have half the households with a discount rate of 0.98 and half with 0.97.[18] In this simulation, the effect of inflation on home-ownership rates was midway between 5 and 11 percent. From this evidence, it would appear that inflation is likely to have smaller effects on home-ownership rates in societies where households are more patient.

4 Discussion and conclusions

This paper extends a long literature that has examined how the tax system affects the economy when the inflation rate is non-zero. Like many of these papers, its focus is the way that the tax system distorts the housing market and inter-temporal consumption choices. This aspect of the economy is the focus because when inflation is at low or moderate levels it has its greatest proportional effects on nominal interest rates. Consequently, the effects of inflation are likely to be largest in sectors of the economy where changes in nominal interest rates have the biggest effect.

In the model inflation has real effects because of two different nominal rigidities. The first is the Government imposed requirement that households pay income tax on the inflation component of interest earnings – the widows' tax. The second is financial sector imposed restrictions on the amount households can borrow, restrictions that are increasingly onerous as the inflation rate increases because they are expressed in nominal not real terms. These restrictions mean inflation has welfare effects even when it is constant and perfectly anticipated.

The paper's key contribution to the existing literature is to endogenise property prices in a model that simultaneously incorporates owner-occupied and rental sectors. This means the effects of changes in tax rates and the inflation rate that take into account the interaction of households that differ by age and income can be calculated. These interactions are important, because tax rules and credit constraints affect households in quite different ways. Indeed, the model suggests that endogenous changes in house prices or rents are a key mechanism by which the effects of inflation on the economy are transmitted.

[17] The reduction in lifetime consumption occurs because households save less early in life. In addition, they receive less utility from living in a rented house than an owned house.
[18] The income distribution spanned the same range as before.

Four results of the model should be emphasised. Firstly, when all nominal interest earnings are taxed, small increases in the inflation rate can cause large reductions in the home-ownership rates of young cohorts. This can occur because young households find the higher nominal mortgage payments associated with higher inflation onerous as they cannot borrow additional amounts to maintain desired consumption patterns. When the model is parameterised to mimic New Zealand tax rates, and real interest rates are 5 percent, a one percent increase in the inflation rate is associated with an 8 - 10 percent decrease in the home-ownership rates of young households, irrespective of whether the supply of housing is elastic or inelastic.

Secondly, the welfare effects of inflation depend on the elasticity of the housing supply. When the supply of housing is inelastic, inflation causes a significant reduction in the welfare of all households. In this case, inflation leads to higher house prices as landlords bid up prices to avoid the tax on the inflation component of interest income and take advantage of the concessionary tax status of residential housing. Inflation reduces the welfare of low income households because they rent rather than purchase a home in order to maintain consumption levels early in life; inflation reduces the welfare of middle and high income because they delay purchasing large houses and earn lower real after tax returns from their investments. In contrast, when the supply of housing is elastic, competition between landlords leads to lower rents and the construction of additional houses. In this case inflation raises the welfare of the lowest income groups, for even though a large fraction choose to rent rather than purchase a home, the lower rent more than offsets the welfare losses stemming from being a tenant rather than an owner-occupier. Middle and high income households still lose from inflation, however, partly because they delay purchasing large houses, and these losses are on average much bigger than the gains to low income households.

Thirdly, the effects of inflation on home-ownership rates depend on real interest rates, and are much greater when real interest rates are low rather than when they are high. This is because the ratio of mortgage repayments (including capital repayments) to rents reduces as real interest rates increase, reducing the disincentive for credit constrained households to purchase a house. In the model simulations, moderate inflation had little effect on home-ownership rates when real interest rates were 6 percent or more. This feature of the model suggests that the effect of low inflation on the housing market may have been minimal in New Zealand prior to 2000, for real interest rates between 1985 and 2000 typically exceeded six percent.

Fourthly, the effects of inflation on the housing market, and on home-ownership patterns, would largely be avoided if the inflation component of nominal interest earnings were exempt from income. The main changes in home-ownership patterns stem from the incentives that induce high-income households to become landlords when the inflation rate increases, to take advantage of tax free capital gains from property appreciation. If the inflation component of interest earnings were exempt from income tax, this incentive would disappear and inflation would have only modest effects on the housing market due to the effect on credit constraints.

These findings are consistent with Feldstein's (1997) argument that even low inflation can cause significant welfare losses because of its effect on housing markets and inter-temporal consumption choices. The paper adds sophistication to his analysis, however, by simultaneously considering the effects of inflation on housing markets and inter-temporal consumption patterns, by allowing for the different effects of income tax on households with positive and negative debt positions, by allowing for a rental market as well as home-ownership, and by endogenising house prices. It proves that none of these factors overturn his essential insights into the harmful consequences of low inflation.

This paper also adds to a long line of papers that suggests welfare could be improved if the inflation component of interest income were not taxed. This paper suggests that the extent of the welfare gains depends on the elasticity of housing supply, but the gains are substantial – not surprising, perhaps, since the inflation component of interest earnings is not income. The paper further suggests that if the inflation component of interest were exempt from tax (and only real interest payments were tax deductible) home-ownership rates would increase. Given the policy concern expressed by the New Zealand Government about falling home-ownership rates, it may be time to consider changing the tax system to reduce the harm caused by taxing the inflation component of interest earnings. Claims by Feldstein (1996) notwithstanding, it would not be difficult to change tax laws so that approved financial instruments were taxed only on the real component of interest earnings; nor would it be difficult to only allow the deduction of real rather than nominal interest payments from taxable income.

5 References

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Table 1. Parameterisation of the model.

Parameter	Description	Value	Source/Rationale
T	Length of period	10 years	To approximate work history from age 25–75
Y_t^0	Average income of 25–35 cohort	50,000	NZ Census 2001: average male and female earnings, 25–35 year olds, are $32,800 and $23,300 respectively
ω_j	Income distribution	Uniform on [20000,80000]	
g_i	Lifecycle income pattern	{1, 1.5, 1.5, 0.75+20000}	NZ Census, 1966–2001. Based on real lifecycle earnings of cohort turning 20 in 1946, 1961.
B	Discount factor	0.97 annualised	Arbitrary
$\{v^R, v^F, v^H\}$	Utility from housing	$\{0.33, 0.35, 0.45\}$	Arbitrary
κ_i	Inheritance timing	{0,0,1,0}	Arbitrary
n^H/N	Fraction of houses	0.57	Arbitrary
n^F/N	Fraction of flats	0.42	Arbitrary
H	Mortgage term	25 years	Standard mortgage term in 1990s
δ	Maximum debt service-income ratio	30%	Reflects NZ banking conditions
Θ	Maximum loan to value ratio	90%	Reflects NZ banking conditions
τ^{g*}	GST rate	0.10	Tax take equals 10% of labour income; arbitrary, but close to NZ rate.
τ_1, τ_2, τ^*	Income tax rates and threshold	20%, 33% $50,000	Reflects NZ rates in 2000.
α_0^F, α_1^F α_0^H, α_1^H	Housing supply parameters	10, -1250 1, 9300	Arbitrary, generates approximately 1% price elasticity for flats.

Table 2: Output of Model with inelastic supply.

Normal taxes

Tax rates	Inflation rate			
	0	1	2	3
	Flat prices			
(0,0)	$166,286	$166,397	$165,788	$165,034
(20,20)	$154,558	$158,922	$162,615	$166,516
(20,33)	$147,207	$154,534	$163,247	$171,990
	House prices			
(0,0)	$269,364	$268,470	$267,434	$266,331
(20,20)	$256,960	$260,735	$262,004	$264,058
(20,33)	$249,712	$255,027	$260,838	$266,629
	Rents			
(0,0)	$6,425	$6,425	$6,400	$6,375
(20,20)	$6,469	$6,475	$6,475	$6,475
(20,33)	$6,521	$6,467	$6,482	$6,468
	Number people renting			
(0,0)	86	98	113	126
(20,20)	24	64	83	135
(20,33)	0	40	87	180
	Fraction of young cohort renting			
(0,0)	21.5%	24.5%	28.3%	31.5%
(20,20)	5.8%	16.0%	20.8%	32.3%
(20,33)	0.0%	10.0%	21.8%	42.3%
	Number of flats and houses			
(0,0)	580 + 900	580 + 900	580 + 900	580 + 900
(20,20)	580 + 900	580 + 900	580 + 900	580 + 900
(20,33)	580 + 900	580 + 900	580 + 900	580 + 900
	GST rate			
(0,0)	9.5%	9.5%	9.4%	9.4%
(20,20)	8.2%	8.1%	7.9%	7.9%
(20,33)	8.3%	8.1%	8.0%	8.0%

Table 3: Output of Model with elastic supply.

Normal taxes

Tax rates	Inflation rate			
	0	1	2	3
	Flat prices			
(0,0)	$147,266	$147,279	$147,242	$147,263
(20,20)	$146,388	$146,716	$147,043	$147,275
(20,33)	$145,634	$146,393	$147,059	$147,535
	House Prices			
(0,0)	$249,551	$249,482	$249,343	$249,297
(20,20)	$248,461	$248,747	$248,922	$248,979
(20,33)	$247,695	$248,238	$248,763	$249,028
	Rents			
(0,0)	$5,681	$5,683	$5,681	$5,682
(20,20)	$6,123	$5,983	$5,857	$5,735
(20,33)	$6,442	$6,130	$5,842	$5,551
	Number of people renting			
(0,0)	92	98	102	114
(20,20)	25	61	90	111
(20,33)	0	38	90	145
	Fraction of young cohort renting			
(0,0)	23.0%	24.5%	25.5%	28.5%
(20,20)	6.3%	15.3%	22.5%	27.8%
(20,33)	0.0%	9.5%	22.5%	35.3%
	Number of flats and houses			
(0,0)	669 + 929	678 + 920	687 + 910	694 + 903
(20,20)	682 + 907	689 + 903	708 + 888	727 + 870
(20,33)	675 + 906	704 + 884	725 + 870	751 + 849
	GST rate			
(0,0)	9.4%	9.4%	9.4%	9.4%
(20,20)	8.2%	8.0%	7.9%	7.8%
(20,33)	8.3%	8.1%	8.0%	8.0%

Table 4: Output of Model with inelastic supply.

The inflation component of interest is tax exempt.

Tax rates	Inflation rate			
	0	1	2	3
	Flat prices			
(0,0)	$166,286	$166,397	$165,788	$165,034
(20,20)	$154,419	$154,638	$154,613	$154,811
(20,33)	$146,588	$146,507	$146,270	$146,917
	House Prices			
(0,0)	$269,364	$268,470	$267,434	$266,331
(20,20)	$256,835	$256,847	$256,684	$256,404
(20,33)	$248,971	$247,772	$247,330	$248,066
	Rents			
(0,0)	$6,425	$6,425	$6,400	$6,375
(20,20)	$6,467	$6,475	$6,475	$6,475
(20,33)	$6,485	$6,489	$6,476	$6,502
	Number of people renting			
(0,0)	86	98	113	126
(20,20)	23	40	56	69
(20,33)	0	8	19	30
	Fraction of young cohort renting			
(0,0)	21.5%	24.5%	28.3%	31.5%
(20,20)	5.5%	10.0%	14.0%	17.3%
(20,33)	0.0%	2.0%	4.8%	7.5%
	Number of flats and houses			
(0,0)	580 + 900	580 + 900	580 + 900	580 + 900
(20,20)	580 + 900	580 + 900	580 + 900	580 + 900
(20,33)	580 + 900	580 + 900	580 + 900	580 + 900
	GST rate			
(0,0)	9.5%	9.5%	9.4%	9.4%
(20,20)	8.2%	8.1%	8.1%	8.1%
(20,33)	8.3%	8.2%	8.1%	8.0%

Table 5: Output of Model with elastic supply.

The inflation component of interest is tax exempt.

Tax rates	Inflation rate			
	0	1	2	3
	Flat prices			
(0,0)	$147,266	$147,279	$147,242	$147,263
(20,20)	$146,388	$146,377	$146,404	$146,295
(20,33)	$145,585	$145,637	$145,571	$145,645
	House Prices			
(0,0)	$249,551	$249,482	$249,343	$249,297
(20,20)	$248,461	$248,430	$248,448	$248,327
(20,33)	$247,639	$247,553	$247,461	$247,548
	Rents			
(0,0)	$5,681	$5,683	$5,681	$5,682
(20,20)	$6,123	$6,123	$6,124	$6,121
(20,33)	$6,438	$6,439	$6,436	$6,443
	Number of people renting			
(0,0)	92	98	102	114
(20,20)	25	41	55	70
(20,33)	0	9	19	29
	Fraction of young cohort renting			
(0,0)	23.0%	24.5%	25.5%	28.5%
(20,20)	6.3%	10.3%	13.8%	17.5%
(20,33)	0.0%	2.3%	4.8%	7.3%
	Number of flats and houses			
(0,0)	669 + 929	678 + 920	687 + 910	694 + 903
(20,20)	682 + 907	684 + 905	685 + 904	685 + 903
(20,33)	675 + 905	690 + 892	692 + 889	691 + 890
	GST rate			
(0,0)	9.4%	9.4%	9.4%	9.4%
(20,20)	8.2%	8.1%	8.1%	8.1%
(20,33)	8.3%	8.2%	8.1%	8.1%

Table 6: Parameter variations with inelastic supply.

Parameter change	Rent	Flat price	House price	% young renting π= 0%	% young renting π= 2%	Slope
Base Parameters	6480	163200	260800	0%	22%	11
	Changing credit constraints (loan to value ratio)					
θ = 0.8	6470	163100	259300	2	29	14
θ = 0.9	6480	163200	260800	0%	22%	11
θ = 1.0	6500	163500	262700	0	18	9
	Changing number of flats and houses					
N_h=600,N_f=980	6450	162700	298300	0%	29%	15
N_h=900,N_f=680	6480	163200	260800	0%	22%	11
N_h=900,N_f=660	7357	185600	281600	0%	26%	13
	Changing the valuation of houses					
value= 32 35 45	6300	158800	258000	0%	17%	9
value= 33 35 45	6480	163200	260800	0%	22%	11
value= 33 35 46	6500	163500	270600	0%	20%	10
	Changing real interest rates, discount β = 0.97					
β = 0.97 r = 4	6440	207600	298400	15%	63%	24
β = 0.97 r = 4.5	6490	183400	278400	7%	51%	22
β = 0.97 r = 5	6480	163200	260800	0%	22%	11
β = 0.97 r = 5.5	6510	148500	247200	0%	9%	5
β = 0.97 r = 6	6850	133300	234300	0%	0%	0
	Changing real interest rates, discount β = 0.98					
β = 0.98 r = 4	6870	220900	324100	6%	50%	22
β = 0.98 r = 4.5	6410	180900	284400	1%	21%	10
β = 0.98 r = 5	6390	161800	267300	0%	10%	5
β = 0.98 r = 5.5	6320	145600	252300	0%	2%	1
β = 0.98 r = 6	6990	135600	242300	0%	0%	0

Table 7: Parameter variations with elastic supply.

Parameter change	Rent	Flat price	House price	% young renting π= 0%	% young renting π= 2%	Slope	N_h	N_f	Total
Base Parameters	5840	147100	248800	0%	23%	11	870	725	1595
	Changing credit constraints (loan to value ratio)								
θ = 0.8	5830	147000	248500	3%	25%	11	853	741	1594
θ = 0.9	5840	147100	248800	0%	23%	11	870	725	1595
θ = 1.0	5840	147000	248800	0%	19%	10	880	715	1595
	Changing additional cost of constructing a large house								
a_{0h} = 93000	5840	147100	248800	0%	23%	11	870	725	1595
a_{0h} = 143000	6440	145600	293800	0	5%	3	528	1053	1581
	Changing the valuation of houses								
vv= 32 35 45	5820	146600	248400	0%	16%	8	877	715	1592
vv= 33 35 45	5840	147100	248800	0%	23%	11	870	725	1595
vv= 33 35 46	5840	147100	249700	0%	20%	10	964	632	1596
	Changing real interest rates, discount rate β = 0.97								
β= 0.97 r= 4	4580	147500	249200	15%	59%	22	876	724	1600
β= 0.97 r= 4.5	5230	147500	249200	7%	39%	16	866	734	1600
β= 0.97 r= 5	5840	147100	248800	0%	23%	11	870	725	1595
β= 0.97 r= 5.5	6380	145700	247400	0%	9%	4	863	719	1582
β= 0.97 r= 6	6850	144500	246200	0%	2%	1	866	705	1571
	Changing real interest rates, discount rate β = 0.98								
β= 0.98 r= 4	4580	147500	250000	7%	31%	11	945	655	1600
β= 0.98 r= 4.5	5220	147500	250000	2%	21%	10	951	649	1600
β= 0.98 r= 5	5830	146900	249400	0%	10%	5	948	646	1594
β= 0.98 r= 5.5	6330	145500	247900	0%	2%	1	942	638	1580
β= 0.98 r= 6	6890	144600	247000	0%	0%	0	942	629	1571

Figure 1: Price of small houses as inflation increases.

Inelastic supply, r= 5%.

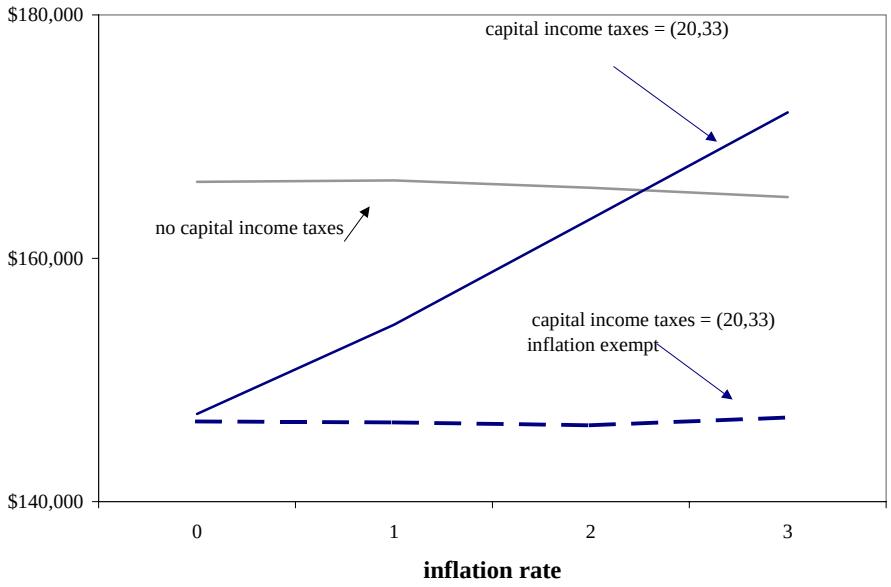


Figure 2: Fraction of young cohort renting as inflation increases.

Inelastic supply, r= 5%.

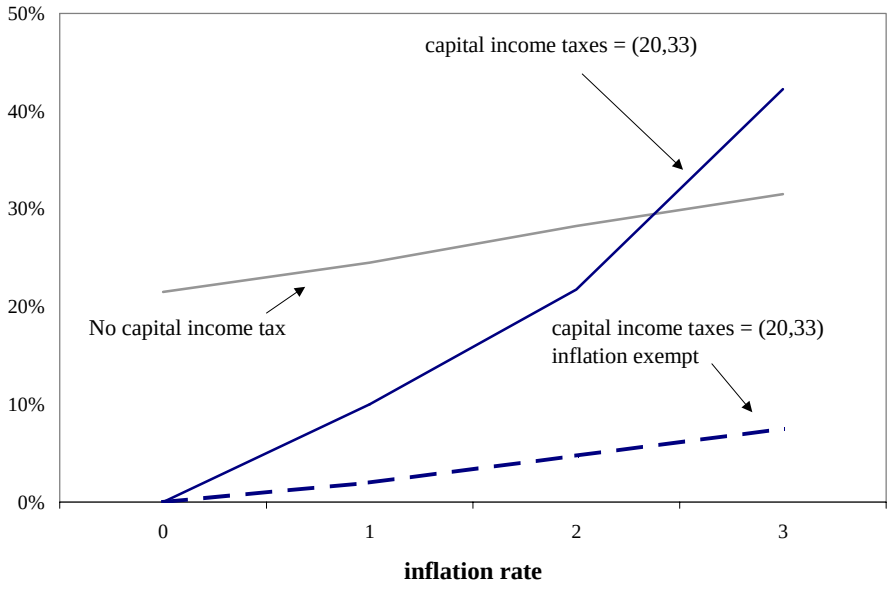


Figure 3: Fraction of young cohort renting as the tax rate and inflation rate vary.

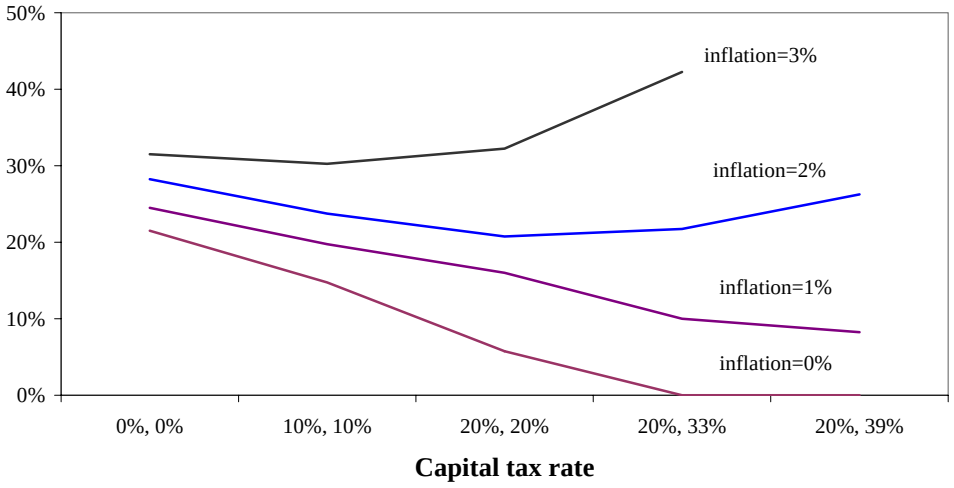


Figure 4: Price of small houses as inflation increases.

Elastic supply, r = 5%.

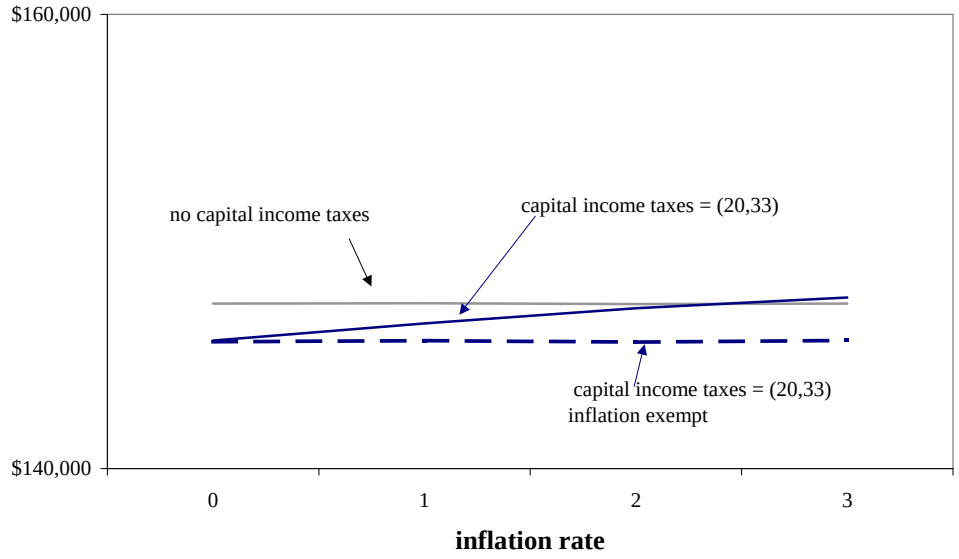


Figure 5: Rent as inflation increases.

Elastic supply, r = 5%.

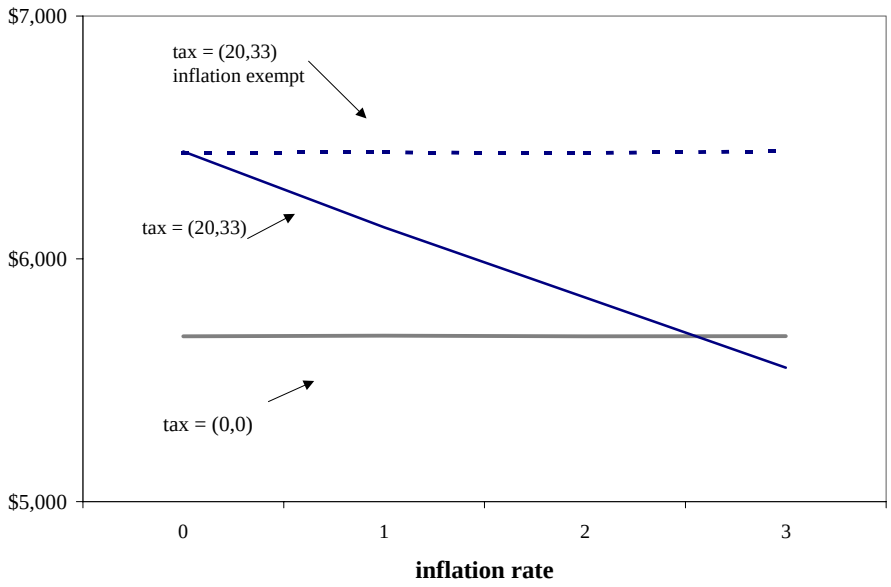


Figure 6: Fraction of young cohort renting as inflation increases

Elastic supply, r = 5%.

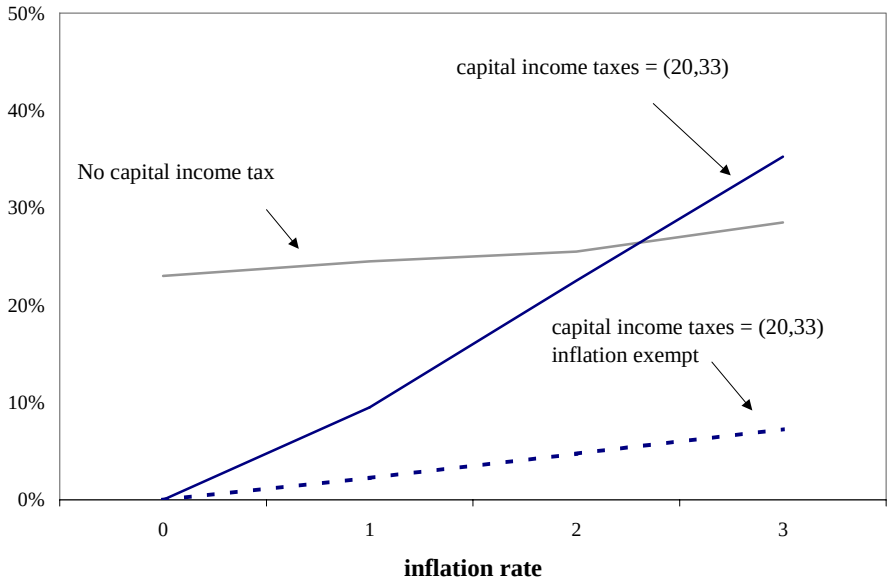


Figure 7: Welfare change when inflation increases from 0% to 3%

Inelastic supply, tax = (20%, 33%) r= 5%

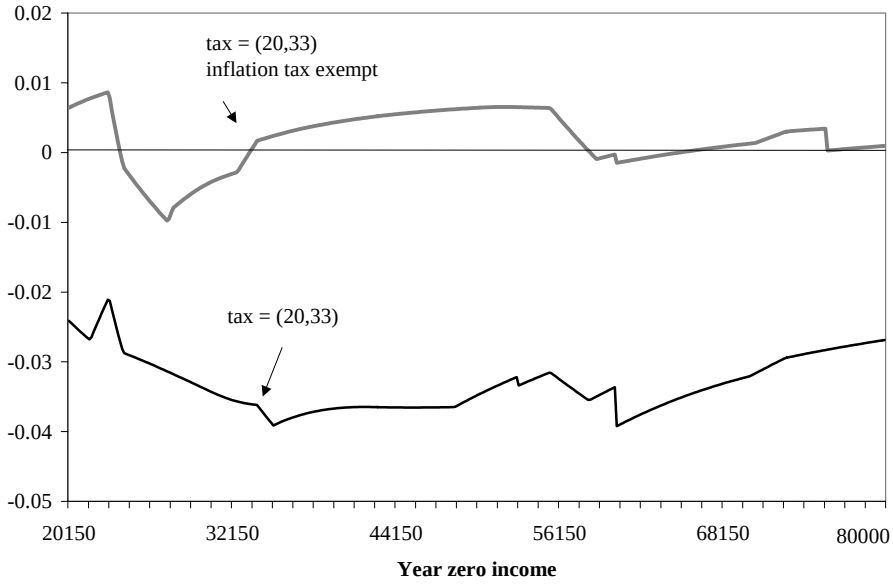
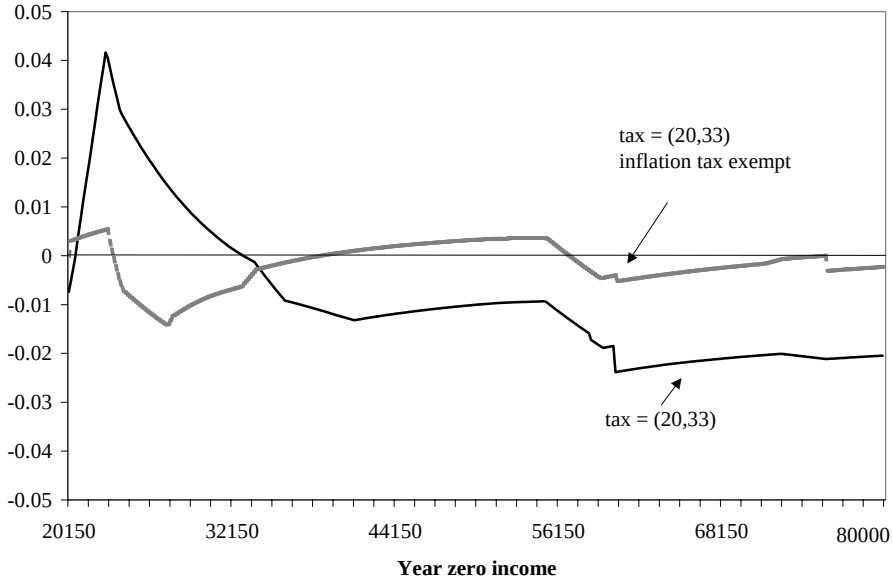


Figure 8: Welfare change when inflation increases from 0% to 3%

Elastic and inelastic supply, tax = (20%, 33%) r= 5%.