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The end of the Bertrand Paradox?*

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Résumé

Cet article s'intéresse à un cas de concurrence en prix dans lequel deux firmes ont accès à une fonction de production à deux facteurs et à rendements d'échelle constants. Les facteurs sont choisis de manière séquentielle dans un jeu à deux étapes. Dans la première étape, les firmes adoptent le montant de facteur fixe. Dans la deuxième étape, elles fixent leur prix et servent la totalité de la demande qui s'adresse à elle. Nous montrons que le prix de collusion est la seule issue prévisible du jeu, i.e. l'unique équilibre de Nash en stratégies pures non Pareto-dominé. Ce papier permet d'établir un pont entre les approches « à la Bertrand-Edgeworth » avec contraintes de capacités et les approches sans rationnement de la demande avec coûts convexes.

Mots-clés : Concurrence en prix, collusion, coût convexe, Paradoxe de Bertrand, contraintes de capacité, rendements d'échelles constants.
Codes JEL : L13, D43

Abstract

This paper analyzes price competition in the case of two firms operating under constant returns to scale with more than one production factor. Factors are chosen sequentially in a two-stage game implying a convex short term cost function in the second stage of the game. We show that the collusive outcome is the only predictable issue of the whole game i.e. the unique non Pareto-dominated pure strategy Nash Equilibrium. Technically, this paper bridges the capacity constraint literature on price competition with the one of convex cost function, solving the Bertrand Paradox in the line of Edgeworth's research program.

JEL-code: L13, D43

Keywords: price competition, collusion, convex cost, Bertrand Paradox, capacity constraint, constant returns-to-scale

1 Introduction.

In his seminal model, Joseph Bertrand (1883) considered a nonrepeated interaction in which two firms have identical linear cost functions and simultaneously set their prices. According to this model, even if the number of competing firms is small, price competition leads to a perfectly competitive outcome in a market for a homogeneous good. The unique equilibrium price equals the firm's (constant and common) marginal cost and the profit of each firm is equal to zero. This result is referred as the Bertrand Paradox.

The literature on Industrial Organization theory proposes a resolution of the Bertrand Paradox by relaxing any one of the four crucial assumptions of the model.

The first assumption that can be relaxed is the *perfect substitutability of the firm's products*. Consumers are indifferent between goods at an equal price and they buy from the lowest-priced producer. In the price competition with differentiated products, the conclusions of the Bertrand model do not hold. The firms charge above the marginal cost and make a positive profit, and the Bertrand equilibrium is no longer welfare-optimal.

The second assumption that can be relaxed is the *timing of the game*. Repeated interactions can lead to implicit agreements that sustain prices above the marginal cost (see Vives, 1999, for a detailed discussion). This result is obtained in a repeated-game framework with a finite horizon (Benoît and Krishna, 1985) or with a infinite horizon (Friedman, 1971). Without conjectural variations (Bowley, 1924) or the threat of punishment in the case of noncompliance[1], all prices above the marginal cost cannot be sustained as Nash equilibria in a one-shot game.

The third assumption that can be relaxed is the *perfect information* (i) by consumers about each firm's price, or (ii) by rivals about costs. A well-known result, introduced by Diamond (1971), states that if consumers search sequentially and incur a positive cost for receiving a price quotation, the monopoly outcome is obtained, no matter how large the number of firms is. This result is viewed as a paradox, since a "small" search cost produces high prices, but a zero search cost would produce the usual Bertrand results. The imperfect information may also concern the fact that the marginal costs are not common knowledge among the competitors. Under this assumption, Spulber (1995) obtains a pure strategy price equilibrium substantially above marginal cost. Routledge (2010) considers a game of incomplete information in which each firm only knows its own cost type and the probability of dis-

[1] In the context of a repeated price game with perfect substitutes, maximal punishments correspond to the competitive Bertrand equilibrium, in which no firm makes a profit.

tribution over the possible cost types of their rivals. The main result is that there is a mixed strategy Bayesian Nash equilibrium.

The last assumption that can be relaxed concerns *returns to scale*. Since Edgeworth (1925), Bertrand's conclusion has been criticized for holding only in the case of a constant average cost. Francis Edgeworth (1925) pointed out that there are serious existence problems in Bertrand's model if marginal costs are not constant. He proposed, notably, a revisited Bertrand model in which firms have zero marginal costs and a fixed capacity: firms compete on price realizing that competitors may not be able or may not want to supply all the forthcoming demand at the set price. The existence of a capacity constraint is an extreme case of decreasing-returns-to-scale technology: a firm has a marginal cost that is equal to zero up to the capacity constraint and is then equal to infinity. In the modern literature, the Bertrand-Edgeworth debate has been treated primarily in two-stage games (Kreps and Scheinkman, 1983; Davidson and Deneckere, 1986; Allen et al., 2000). In this setting, one can guarantee the existence of a unique Nash equilibrium but most often in mixed, and not in pure strategies (Vives, 1980; Allen and Hellwig, 1986a; Maskin, 1986). However, mixed strategy equilibria do not appear very convincing when analyzing many real-life market interactions. Moreover, the results are sensitive to the choice of a rationing rule for the demand (see Vives, 1999, p.124, for details).

An other approach is to assume that firms supply all demand. For Vives (1999) this characterized Bertrand competition, to be distinguished from Bertrand-Edgeworth competition in which firms face rationing rules. The existence of a cost for the firms to turn customers away allows to justify the distinction between the two approaches (Dixon, 1990). In the Bertrand-Edgeworth approach, there is no cost for turning customers away; on the contrary, in Bertrand competition, firms never turn customers away.

In this last setting, Dastidar (1995) analyzes one-shot interaction game in which firms have to serve the whole market and compete on price under decreasing returns (i.e. strictly convex costs). He shows that positive price-cost margins are possible in pure-equilibrium. A whole range of prices can be sustained as pure strategies Nash equilibria with a minimum zero-profits equilibrium price below the competitive price and a maximum price above it[2]. Dastidar (2001) derives conditions for the joint-profit-maximizing price to fall within this interval. Unfortunately, if the existence of pure strategy equilibria is an appealing property of Dastidar's approach, the fact that there

[2]Dastidar (1995) results depend on the assumption that the revenue function are bounded. Kaplan and Wettstein (2000) show that, when revenues are unbounded and returns are constants, mixed-strategy equilibria yielding positive profit levels can arise.

is an infinity of such equilibria is a serious drawback, rising a coordination problem to be solved for the purpose of analyzing real life-market[3].

In the line of Dastidar (1995), recent papers propose many extensions considering decreasing returns to scale (i.e. convex costs). Weibull (2006) shows that there exists a whole interval of equilibrium prices in repeated price competition. Baye and Morgan (2002), Hoernig (2007) and Bagh (2010) examine the impact of the *market sharing rule*[4] on the existence of equilibria as well as on the determination of the profit's levels equilibrium. Other works study some limiting properties of a Bertrand competition model by considering entry of firms (Novshek and Chowdhury, 2003), or the possibility of limited cooperation among firms (Chowdhury and Sengupta, 2004). Notice that all the extensions considered above are characterized by a continuum of equilibria (in pure and/or mixed strategies).

In this paper, we propose:

i) to solve endogenously the coordination problem (multiplicity of equilibria) arising in the Bertrand-competition literature in the line of Dastidar (1995); the structure of our model allows to achieve the solution's unicity in pure strategies,

ii) and, finally, to solve the paradox with keeping all Bertrand's assumptions (homogeneity, simultaneous interactions, perfect information, constant returns to scale).

Our model assumes that firms rely on a two-factor technology and sequentially choose the quantity of each input in a two-stage game[5]. In the first stage the firms invest *i.e.* they choose the quantity of the first factor, quantity that will be invariable over the second stage. In the second stage, firms compete on price and, incidentally, determine the quantity needed to satisfy the demand they will face at the equilibrium. The model is solved by backward induction.

The results are as follows. In the second stage, with given fixed factor chosen in the first stage (possibly different for each firm), there is a continuum of pure strategy Nash Equilibria. Using a Pareto domination criterium, the

[3]Laboratory experiments are well suited for investigating problems of multiplicity of equilibria. Abbink and Brandts (2008) provide an interesting example of the use of experiments in context of price competition under decreasing returns. They find a remarkable degree of coordination around the cartel price with two firms.

[4]In the original Bertrand's model, the firm which quotes the lowest price gets all the demand and must serve it. The higher price quoting firm gets nothing and hence sells zero. However, when the two firms quote the same price, they share the demand equally. As in the original Bertrand's model, Dastidar (1995) assumes that the market is split evenly in case of a tie; in recent works, other sharing rules have been considered explicitly.

[5]In his conclusion, Weibull (2006) suggests this relevant extension by considering a two stage interaction, where firms invest in the first stage.

set of predictable outcomes can be reduced. According to the geometry of the profit function, unicity prevails in some cases, and a reduced continuum in some others, raising a potential coordination problem. When firms have the same level of first factor (symmetric outcome), unicity always prevails. We then show that the only predictable outcome of the first stage of the game is symmetric. The two-stage structure of the game provides an endogenous way to select equilibria. Thus, the complete solution of the whole game is unique. Turning to welfare analysis, we prove that this result is equivalent to the collusive outcome, even if the returns to scale are constant. Thus, the contribution of this paper is altogether to solve the coordination problem arising in the Bertrand competition literature in the line of Dastidar (1995) and to provide a non ambiguous welfare prediction that is just the opposite of the one of the original Bertrand model.

The paper is organized as follows. Section 2 provides the model. Reasoning backward, the second stage of the game is resolved in Section 3 and the first one in Section 4. Section 5 discusses the results and the final section concludes.

2 The model.

Suppose there are two identical firms in a market for a homogeneous good. Consider a two-stage game where firms invest in the first stage and simultaneously choose the price in the second stage. We introduce the following assumptions:

1. Firms rely on a technology represented by a two-factor constant returns to scale production function. Classically, we will consider that the first factor is fixed in the short run, while the second one will vary to satisfy the demand faced by the firm. For the firm i the fixed factor will be denoted by z_i and a variable one by v_i. For simplicity, we will use a Cobb-Douglas production function $y_i = a{z_i}^{\alpha}v_i^{1-\alpha}$ with $i = 1, 2$ and $i \neq j$, where a is a positive constant, and α, the elasticity of the production according to the level of fixed factor, is a constant between 0 and 1. Hence the long-run total cost function can be written

$$C(y_i; w_1; w_2) = a^{-1}\left(\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\right) w_1^{\alpha} w_2^{1-\alpha} y_i$$

where w_1 is the unit price of the fixed input and w_2 is the unit price of the variable input. The long-run average cost is constant showing

constant returns to scale. In the short-run, the total cost function depending on y_i and the two factor inputs can be rewritten

$$\begin{aligned} C(y_i; z_i; v_i) &\equiv TFC + TVC \\ &= w_1 z_i + w_2 v_i \\ &= w_1 z_i + w_2 \frac{{y_i}^{\frac{1}{1-\alpha}}}{a^{\frac{1}{1-\alpha}} {z_i}^{\frac{\alpha}{1-\alpha}}} \end{aligned}$$

This short-run cost function is continuous, non-decreasing and convex[6].

2. The demand is continuous, twice differentiable and decreasing $D : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ with $D(p^{max}) = 0, D(0) = Q^{max}$. Classically, we denote the price elasticity of demand: $\mathcal{E}(p) = p\frac{D'(p)}{D(p)}$.

3. Firms have to supply all the demand they face. When both firms choose the same price p, they share the demand equally, each firm supplies $\dfrac{D(p)}{2}$. The firm with the lowest price gets all the demand and the one with the highest price gets nothing and sells zero. For each firm i with $i = 1, 2$ and $i \neq j$, let its demand function:

$$D_i(p_i; p_j) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2} D(p_i) & \text{if } p_i = p_j \\ D(p_i) & \text{if } p_i < p_j \end{cases}$$

We can now define the profit π_i for each firm i.

$$\pi_i(p_i, p_j, z_i) = pD_i(p_i, p_j) - C_i\left(D_i(p_i, p_j), z_i\right)$$

$$\pi_i(p_i, p_j, z_i) = \begin{cases} -w_1 z_i & \text{if } p_i > p_j \\ p\frac{D(p)}{2} - w_1 z_i - w_2 \frac{\left(\frac{D(p)}{2}\right)^{\frac{1}{1-\alpha}}}{a^{\frac{1}{1-\alpha}} {z_i}^{\frac{\alpha}{1-\alpha}}} \equiv \hat{\pi}(p, z_i) & \text{if } p_i = p_j = p \\ pD(p) - w_1 z_i - w_2 \frac{D(p)^{\frac{1}{1-\alpha}}}{a^{\frac{1}{1-\alpha}} {z_i}^{\frac{\alpha}{1-\alpha}}} \equiv \pi(p, z_i) & \text{if } p_j > p_i = p \end{cases}$$

4. The demand is such that $\hat{\pi}(p, z_i)$ and $\pi(p, z_i)$ are strictly concave in p and strictly concave in z, *i. e.* $\partial^2\hat{\pi}(p, z)/\partial p^2 < 0$, $\partial^2\hat{\pi}(p, z)\partial z^2 < 0$, $\partial^2\pi(p, z)/\partial p^2 < 0$, $\partial^2\pi(p, z)\partial z^2$. After trivial calculations, it can be shown that $\partial^2\hat{\pi}(p, z)/\partial p\partial z < 0$.

[6]Obviously, this will be the case for any constant returns to scale technology.

The function $\hat{\pi}(p, z_i)$ represents the profit of the firm i when both firms quote the same price and the function $\pi(p, z_i)$, represents the profit of firm i when it quotes the lowest price and supplies the market alone. These functions will be of special utility when solving the equilibria of the game. The comparison of these two functions indicates if there is a possibility of profitable deviation from a symmetric outcome. We define $\bar{p}_i(z_i)$ that solves $\hat{\pi}(p, z_i) = \pi(p, z_i)$. After calculation, we obtain

$$\bar{p}_i(z_i) = \frac{w_2}{a^{\frac{1}{1-\alpha}} z_i^{\frac{\alpha}{1-\alpha}}} \left(\frac{D(\bar{p}_i)}{2}\right)^{\frac{\alpha}{1-\alpha}} (2^{\frac{1}{1-\alpha}} - 1) \tag{1}$$

In the second period the fixed cost is sunken, and the firm will quote a price only if the variable part of the profit is positive *i. e.* $\hat{\pi}(p, z_i) \geq -TFC$. Thus we also define $\hat{p}_i$ that solves $\hat{\pi}(p, z_i) = -TFC$ for a given z_i.

That is $\hat{p}_i \frac{D(\hat{p}_i)}{2} = TVC$, and thus

$$\hat{p}_i(z_i) = \frac{w_2}{a^{\frac{1}{1-\alpha}} z_i^{\frac{\alpha}{1-\alpha}}} \left(\frac{D(\hat{p}_i)}{2}\right)^{\frac{\alpha}{1-\alpha}} \tag{2}$$

Finally, we define p_i^*, the price that maximizes the profit of firm i when both firms operate the market.

$$p_i^*(z_i) \equiv \arg\max_p \{\hat{\pi}(p, z_i)\} = \frac{1}{1 + \frac{1}{\varepsilon(p_i^*)}} \frac{1}{1-\alpha} \frac{w_2}{a^{\frac{1}{1-\alpha}} z_i^{\frac{\alpha}{1-\alpha}}} \left(\frac{D(p_i^*)}{2}\right)^{\frac{\alpha}{1-\alpha}} \tag{3}$$

Notice that p^* is different from p^m, the monopoly price which maximizes the profit of a firm alone in the market.
In the rest of the paper, when reasoning with a given z, we will denote $\hat{\pi}(p, z_i) = \hat{\pi}_i(p)$ and $\pi(p, z_i) = \pi_i(p)$.

Lemma 1 (Geometry of profit functions for a given z_i)**.**

$$\hat{p}_i < \bar{p}_i \tag{4a}$$

$$\forall p \geqslant \hat{p}_i, \hat{\pi}_i(p) \geqslant -TFC \tag{4b}$$

$$\forall p \in [\hat{p}_i; \bar{p}_i], \forall \mu \in (0, p], \hat{\pi}_i(p) \geqslant \pi_i(p) > \pi_i(p - \mu) \tag{4c}$$

$$z_i > z_j \Rightarrow \begin{cases} \hat{p}_j > \hat{p}_i \\ \bar{p}_j > \bar{p}_i \\ p_j^* > p_i^* \end{cases} \tag{4d}$$

Proof: Following equations (1), (2) and (3), (4a),(4b),(4c) are obvious. Taking the total differential of expressions (1), (2) and (3), we can show that $\forall z$, $d\hat{p}(z)/dz < 0$, $d\bar{p}(z)/dz < 0$ and $dp^*(z)/dz < 0$, proving (4d). □

$\bar{p}(z)$ and $p^*(z)$ will play an important role in the resolution of the game and we have to settle the question of their relative position.

Lemma 2 (Comparison between $\bar{p}(z)$ and $p^*(z)$)**.**

$\forall \alpha \in (0,1), \exists! \tilde{z}$,

$$z = \tilde{z} \Leftrightarrow \bar{p}(z) = p^*(z) \equiv \tilde{p} \Leftrightarrow \frac{1}{1-\alpha}\frac{1}{2^{\frac{1}{1-\alpha}}-1} = 1 + \frac{1}{\mathcal{E}(\tilde{p})} \tag{5a}$$

$$z < \tilde{z} \Leftrightarrow \bar{p}(z) > p^*(z) \Leftrightarrow \frac{1}{1-\alpha}\frac{1}{2^{\frac{1}{1-\alpha}}-1} < 1 + \frac{1}{\mathcal{E}(\bar{p}(z))} \tag{5b}$$

Proof: Let us remark that p^* solves $\hat{\pi}'(p) = 0$. When $\bar{p}(z) = p^*(z) \equiv \tilde{p}$, $\hat{\pi}'(\bar{p}) = 0$. After calculations, we get the right part of (5-a). Due to the strict concavity of $\hat{\pi}$ according to p (assumption (4)), we have : $\bar{p}(z) > p^*(z) \Leftrightarrow \hat{\pi}'(\bar{p}) < 0$. After some easy calculations, we then get the right part of (5b). Let's now prove the unicity of $\tilde{p}$ and $\tilde{z}$. The left hand side of the expression $\frac{1}{1-\alpha}\frac{1}{2^{\frac{1}{1-\alpha}}-1} = 1 + \frac{1}{\mathcal{E}(p)}$ is a constant between 0 and 1. For $p \in (0, p_{max})$, the right hand side is strictly increasing, taking values between $-\infty$ and 1. That proves the unicity of $\tilde{p}$. Because $\bar{p}(z)$ is strictly decreasing on its definition set, the unicity of $\tilde{p}$ implies the unicity of $\tilde{z}$. □

The properties of $\bar{p}(z)$ and $p^*(z)$ can be summarized in Figure 1.

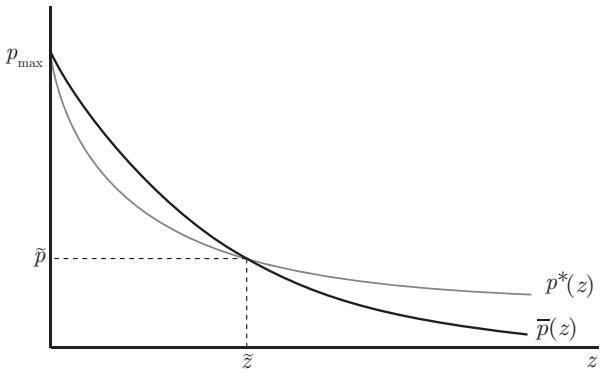

Figure 1

The first important point of Lemma 2 is that the choice of the fixed factor's level in the first stage has some qualitative implications for the geometry of the profit functions in the second stage. If the firm chooses a low level of z, it will have $\bar{p}(z) > p^*(z)$. In the contrary, if the firm chooses a high level of the fixed factor then, it will have $\bar{p}(z) < p^*(z)$. So when solving the

first stage of the game, z will be endogenous, and we will have to be careful with the qualitative implications for the resolution of the second stage. For a given demand function, α is the sole determinant of $\tilde{p}$ position. When α tends to 1, the expression $\frac{1}{1-\alpha}\frac{1}{2^{\frac{1}{1-\alpha}}-1}$ converges to 0 and the condition (5-b) is more easily verified.

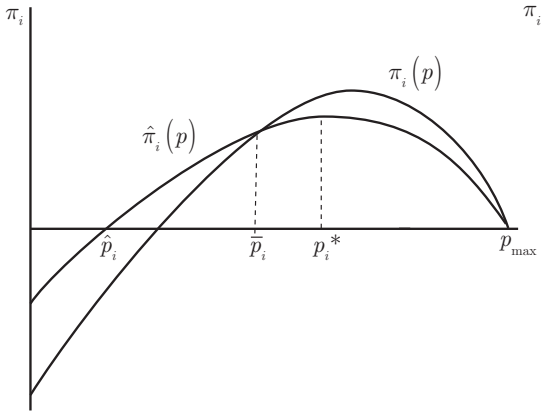

Figure 2: $\tilde{z} > z$

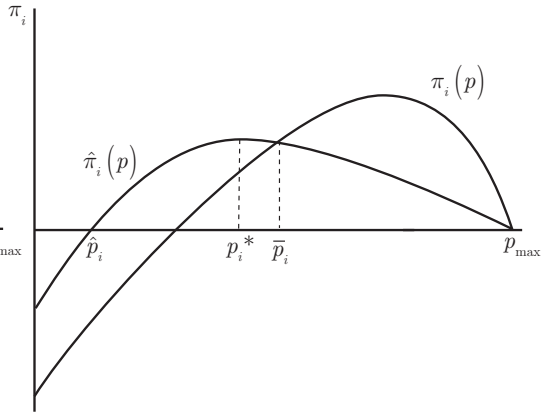

Figure 3: $\tilde{z} < z$

Figure 2 and Figure 3 illustrate the geometry of the profit functions. Considering just one firm with a definite level of the fixed factor, (4a), (4b) and (4c) allow us to draw the functions $\hat{\pi}_i(p)$ and $\pi_i(p)$.

These functions are parameterized by the level of the fixed factor. What happens if this level increases? (4d) shows that the curves will be transformed with $\hat{p}_j$, $\bar{p}_j$ and p^* moving to the left.

The two-stage game is solved by backward induction. First, we analyze the price competition in the second stage of the game, for a given fixed input at levels z_1 and z_2. Secondly, the firms optimally choose their fixed input levels in the first stage of the game. We derive our main results and provide some possible intuitions behind them.

3 The second stage of the game: price competition

In this section, we take the firms' fixed input levels as given and look for the Nash equilibrium in prices. Thus, for a better readability, we omit the z variable when denoting the price. For reasons that will become clearer when we resolve the first stage of the game in the next section, we will consider the possibility that z_1 and z_2, chosen in the first stage, can be different but

"not too much". More precisely, we will assume that z_1 and z_2 are such that $[\hat{p}_1; \bar{p}_1] \cap [\hat{p}_2; \bar{p}_2] \neq \emptyset$.

3.1 Nash Equilibria

Proposition 1. *In the second stage,* (p_1, p_2) *is a pure strategy Nash equilibrium* if and only if $p_1 = p_2 = p^N$, *with* $p^N \in [\hat{p}_1; \bar{p}_1] \cap [\hat{p}_2; \bar{p}_2] \neq \emptyset$.

Proof: When we consider price competition, we can no longer resolve the game using the reaction functions, because they are strongly discontinuous. Thus, we have to check for each possible strategy profile whether it is a Nash equilibrium or not.

Let's first investigate symmetric strategy profiles belonging to the interval. When the competitor charges any price $p \in [\hat{p}_i; \bar{p}_i]$, the best response for the firm i is to quote the same price. When the firm i quotes the same price, it gets $\hat{\pi}_i(p)$. We know that for all $p \geqslant \hat{p}_i$, $\hat{\pi}_i(p) \geqslant -TFC$ (see eq. (4b)). If the firm deviates (by quoting $p - \mu$), it gets $\pi_i(p - \mu)$. We also know that for all $p \in [\hat{p}_i; \bar{p}_i]$, $\hat{\pi}_i(p) \geqslant \pi_i(p) > \pi_i(p - \mu)$ (from eq (4c)). Since the firm must supply all the demand it faces, the increase in additional revenue (from higher sales) is less than the increase in costs: the firm must sell additional units at excessive marginal costs. By quoting $p + \mu$, the firm i obtains no demand and gets zero variable profit. Hence it is optimal for each firm to quote the same price. There are no incentives to deviate, which proves the implication in Proposition 1. It also proves that all asymmetrical strategy profiles with at least one firm quoting a price in the interval are not Nash equilibria. We now have to investigate all the other strategy profiles, symmetric and asymmetric, in which none of the firms quote a price within the interval. It is easy to check that for all symmetric or asymmetric strategy profiles such that $p < \hat{p}$, the firm has interest to increase its price. The firm has interest to lower its price for symmetric profiles with $p > \bar{p}$. Finally, for asymmetric profiles with $p > \bar{p}$, the firm with the highest price will improve its profit in matching the other's price. Thus, asymmetric strategy profiles cannot be Nash Equilibria.□

At $p^N = \max(\hat{p}_1; \hat{p}_2)$, the price equals the marginal cost of the firm with the lowest level of fixed factors (the price is also equal to the short term average costs for this firm). In this sense, and from the point of view of this firm, we can say that the competitive outcome is an equilibrium of the second stage of the game. That corresponds to the conventional wisdom about Bertrand competition. But the important point is that this outcome is only one equilibrium among an infinity. All the other equilibria are characterized by positive short term profits for each firm. In spite of the fact that returns

to scale are constant in our model, the variable cost in the second stage is nevertheless a convex function. Thus, the Nash equilibrium prediction in the second stage game are basically the one of Dastidar (1995), with the same drawback. Because of the infinite number of equilibria, it is strongly indeterminate with a minimum zero short-term profit equilibrium price and a maximum above the competitive price.

3.2 Equilibrium selection

The large set of equilibria leads to coordination problems since firms find it difficult to anticipate which equilibrium the other firm will attempt to implement (Ochs, 1995; Van Huyck et al., 1990; Cooper et al., 1990). How is the equilibrium chosen? When multiple equilibrium points can be ranked, it is possible to use a Pareto criterion to select one equilibrium or a subset of equilibria (for example Harsanyi and Selten, 1988; Luce and Raiffa, 1957; Schelling, 1960). An equilibrium point is said to be payoff-dominant if it is not strictly Pareto-dominated by an other equilibrium point, i.e. when there exists no other equilibrium in which payoffs are higher for all players. Considerations of efficiency may induce decision-makers to focus on, and hence select, the non Pareto-dominated equilibrium point. Proposition 2 allows us to reduce the set of equilibria according to the Pareto criterion but does not necessarily imply unicity. In the following corollary, we discuss the unicity problem.

Proposition 2. *In the second stage,* (p_1, p_2) *is a non Pareto-dominated pure strategy Nash equilibrium* if and only if:

$$p_1 = p_2 = p^{NPDN}, \text{ with } p^{NPDN} \in [min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2); min(max(p_1^*; p_2^*); \bar{p}_1; \bar{p}_2)]$$

Proof: As shown in Proposition 1, all the Nash equilibria of the second stage are symmetric. So we can, without loss of generality, consider only symmetric strategy profiles (p, p) and the associated gains for both firms $(\hat{\pi}_1(p), \hat{\pi}_2(p))$.

If we consider an open interval (p_a, p_b) such that, $\forall p \in (p_a, p_b), \forall i \in \{1; 2\}$, $\partial\hat{\pi}_i(p)/\partial p > 0$, then the symmetric strategy profile $(p_b; p_b)$ dominates all other profiles corresponding to a price in the interval. Considering the geometry of the profit function, the biggest open price interval with the profit of both firms being strictly increasing is $(0, \min(p_1^*; p_2^*))$. If we restrict ourselves to the set of Nash equilibria the biggest such interval is:

$$(0, (\min(p_1^*; p_2^*))) \bigcap \{[\hat{p}_1; \bar{p}_1] \cap [\hat{p}_2; \bar{p}_2]\} = (\max(\hat{p}_1, \hat{p}_2), \min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2))$$

Thus, all symmetric profiles (p, p) such that $p < \min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2))$ cannot be non Pareto-dominated pure strategy Nash equilibria. Reasoning the same way, we can define the biggest open price interval with the profit of both firms being strictly decreasing and such that the symmetric strategy profiles corresponding to those prices are Nash equilibria, $(\min(\max(p_1^*; p_2^*), \bar{p}_1; \bar{p}_2), \min(\bar{p}_1; \bar{p}_2))$. Thus all symmetric profiles (p, p) such that $p > \min(\max((p_1^*; p_2^*); \bar{p}_1; \bar{p}_2)$ cannot be non Pareto-dominated pure strategy Nash equilibria.

Finally, over the closed interval $[\min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2), \min(\max(p_1^*; p_2^*); \bar{p}_1; \bar{p}_2)]$ all symmetric profiles corresponding to those prices are Nash equilibria, with the profit of both firms varying in the opposite way. □

Apparently Proposition 2 reduced the set of predictable outcome of the game but does not imply unicity. However, the following corollary will prove that unicity prevails in some important cases.

Corollary 2-A. *When firms have chosen the same level of factor z then $p_1^* = p_2^* \equiv p^*$, $\bar{p}_1 = \bar{p}_2 \equiv \bar{p}$ and the symmetric profile $(min(p^*, \bar{p}), min(p^*, \bar{p}))$ is the unique non Pareto-dominated pure strategy Nash equilibrium in the second stage of the game.*

Corollary 2-B. *When firms have not chosen the same level of factor z and $max(z_1; z_2) \geqslant \tilde{z}$, then there exists $i \in \{1, 2\}$ such that $\bar{p}_i = min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2)$ and the symmetric profile $(\bar{p}_i, \bar{p}_i)$ is the unique non Pareto-dominated pure strategy Nash equilibrium at the second stage of the game.*

Corollary 2-C. *When firms have not chosen the same level of factor z and $max(z_1; z_2) < \tilde{z}$, then there exists $i \in \{1, 2\}$ such that $p_i^* = min(p_1^*; p_2^*; \bar{p}_1; \bar{p}_2)$ with $p_i^* \neq \bar{p}_i$ and there exist an infinity of non Pareto-dominated pure strategy Nash equilibrium at the second stage of the game. This set of Nash equilibria is $[min(p_1^*; p_2^*), min(max(p_1^*; p_2^*); \bar{p}_1; \bar{p}_2)]$.*

The following Table (1) presents the results of the price competition in the different configurations.

	$\min(\bar{p}_1;\bar{p}_2) \leqslant \min(p_1^*;p_2^*)$ $\Leftrightarrow \max(z_1;z_2) \geqslant \tilde{z}$	$\min(\bar{p}_1;\bar{p}_2) > \min(p_1^*;p_2^*)$ $\Leftrightarrow \max(z_1;z_2) < \tilde{z}$
$z_1 = z_2$	2A Stage 2: Unicity $p^{NPDN} = \bar{p}_1 = \bar{p}_2 \equiv \bar{p}$	2A Stage 2: Unicity $p^{NPDN} = p_1^* = p_2^* \equiv p^*$
$z_1 \neq z_2$	2B Stage 2: Unicity $p^{NPDN} = \min(\bar{p}_1;\bar{p}_2)$	2C Stage 2: Multiplicity $p^{NPDN} \in$ $[\min(p_1^*;p_2^*), \min(\max(p_1^*;p_2^*);\bar{p}_1;\bar{p}_2)]$

Table 1: The different cases of price competition.

Figures 4 and 5 illustrate respectively Case 2B and 2C.

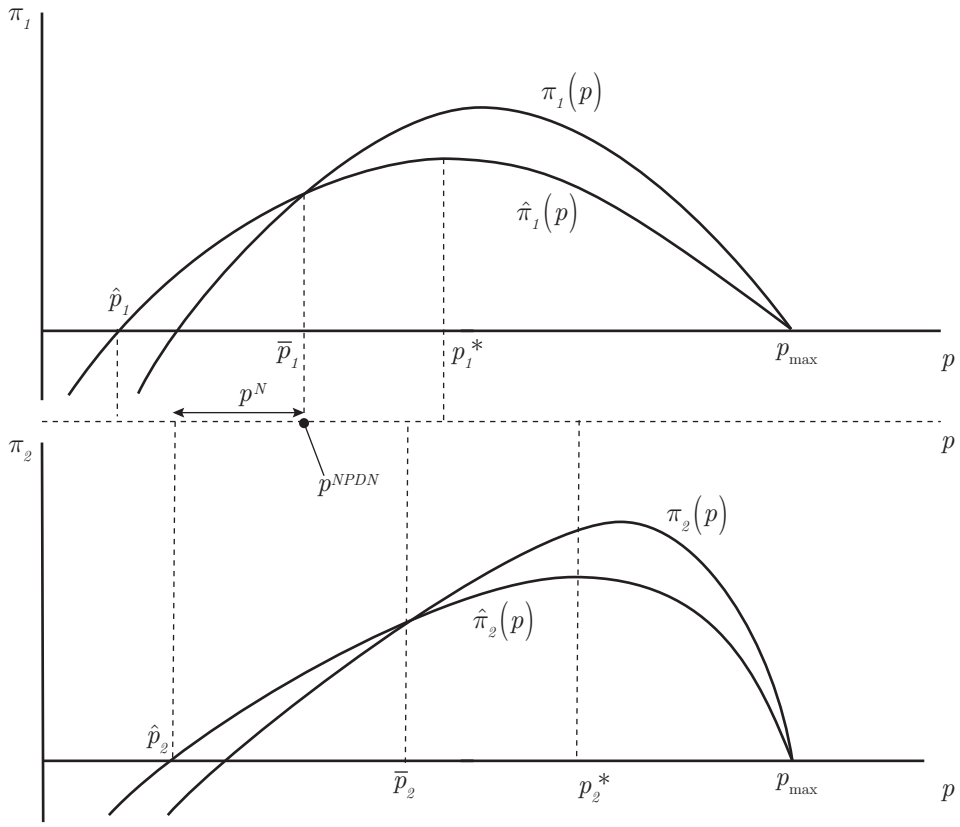

Figure 4: Case 2B

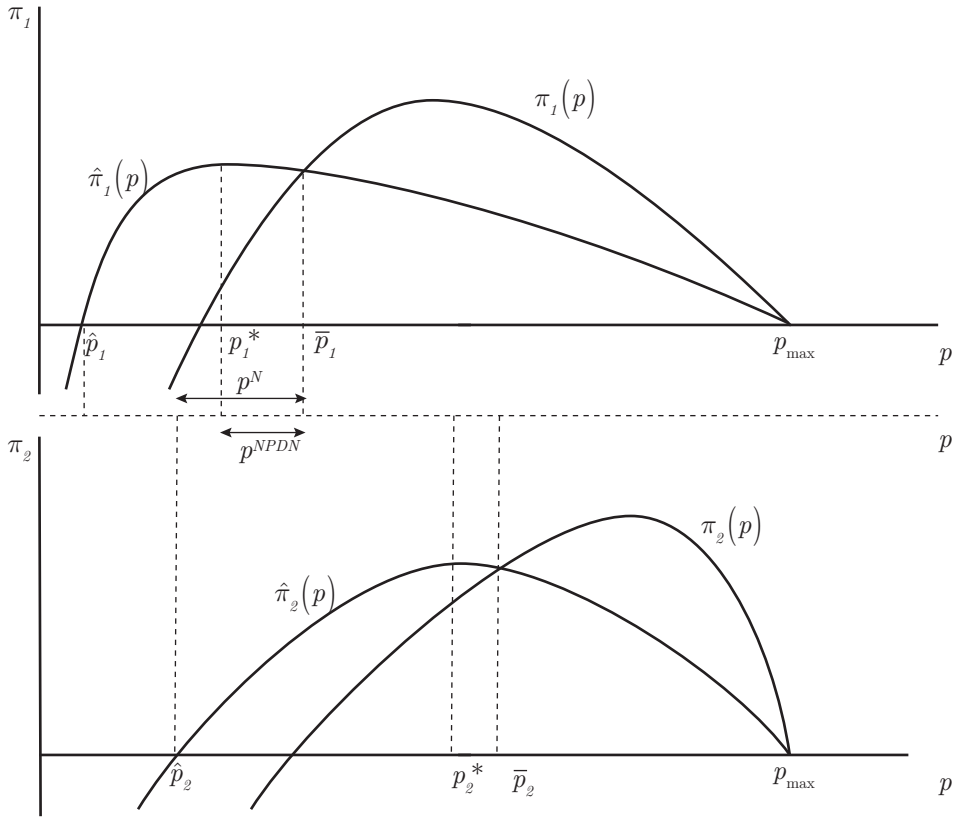

Figure 5: Case 2C

The Pareto criterion seems insufficient to achieve the solution's unicity. The corollary 2-C shows that the coordination problem (multiplicity of equilibria) follows from the profits geometry. But the corollary 2-A shows that the coordination problem can be solved in the endogenous way by the agents' decisions upstream, whatever the special values of the parameters.

4 The first stage of the game

In this section, firms determine their level of fixed factor anticipating the effect on the price equilibria at the second stage of the game. As we are in Bertrand competition, the profit function of the first stage of the game inherits the potential discontinuity of the profit function in the second stage of the game. For this reason, as in the second stage, we can not use reaction functions, and we have to check directly for every strategy profiles, wether they are Nash Equilibria or not. Thus, for each outcome of the game, we will

have to look for the consequences of unilateral deviation in z on the profit function. Two problems may arise. First, we have to consider the potential asymmetry between the effect of a deviation in z on the market price, *i.e.* it is the fixed factor level of the firm with the highest one that determines the market price alone. Second, a deviation in the level of the fixed factor can also modify the nature of the equilibrium because, as we have shown in the Lemma 2, the relative position of $\bar{p}(z)$ and $p^*(z)$ depends on the relative position of z according to $\tilde{z}$. So, before explicitly resolving the equilibrium in the first stage of the game, we have to go back to the geometry of the profit function.

4.1 The geometry of the profit function, once again

Let's define: $\Pi^*(z) \equiv \hat{\pi}(\bar{p}(z), z)$ and $\Pi^{**}(z) \equiv \hat{\pi}(p^*(z), z)$. We define, $z^* = \arg\max\{\Pi^*(z)\}$ and $z^{**} = \arg\max\{\Pi^{**}(z)\}$. Finally, we define the function $\Pi(z)$ satisfying:

$$\begin{cases} \Pi(z) = \Pi^{**}, \forall z \in (0, \tilde{z}] \\ \Pi(z) = \Pi^{*}, \forall z \in (\tilde{z}, +\infty) \end{cases}$$

$\Pi(z)$ characterizes the profit function of both firm when they choose the same level of fixed factor. When $z \geqslant \tilde{z}$, it also characterizes the profit of the firm with the highest level of fixed factor.

Lemma 3 (Geometry of the profit function)**.**

$$\frac{d\Pi^*}{dz}(z) = \frac{\partial \hat{\pi}}{\partial p}(\bar{p}(z), z)\frac{d\bar{p}}{dz}(z) + \frac{\partial \hat{\pi}}{\partial z}(\bar{p}(z), z) \tag{6a}$$

$$\frac{d\Pi^{**}}{dz}(z) = \frac{\partial \hat{\pi}}{\partial z}(p^*(z), z) \tag{6b}$$

$$\forall z \neq \tilde{z}, \Pi^{**}(z) > \Pi^*(z) \tag{6c}$$

$$\Pi^{**}(\tilde{z}) = \Pi^*(\tilde{z}) and \ \frac{d\Pi^{**}}{dz}(\tilde{z}) = \frac{d\Pi^*}{dz}(\tilde{z}) = \frac{\partial \hat{\pi}}{\partial z}(p^*(\tilde{z}), \tilde{z}) \tag{6d}$$

Proof: (6a) is obtained by taking the total differential of the definition. For (6b) and (6c), we use the fact that $p^*(z)$ maximize $\hat{\pi}(p, z)$ for a given z. (6d) is straightforward. □

The last important point is to discuss the relative position of z^* and z^{**} on the one hand with $\tilde{z}$ on the other hand. It will depend on the property of the demand function and the value of the parameter α. We can have two cases: $z^* > \tilde{z}$ (case a) (it will also hold for z^{**}) or $z^* \leqslant \tilde{z}$ (case b). Both case are possible and are illustrated respectively in Figures 6-a and 6-b. For example, with a linear demand function, case 6-a occurs when α is low. Those figures also illustrate the general property enunciated in Lemma 3.

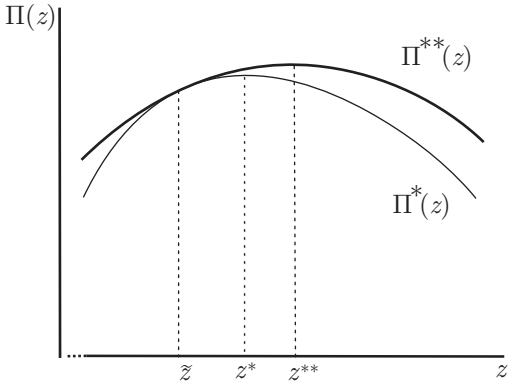

Figure 6-a: $z^* > \tilde{z}$

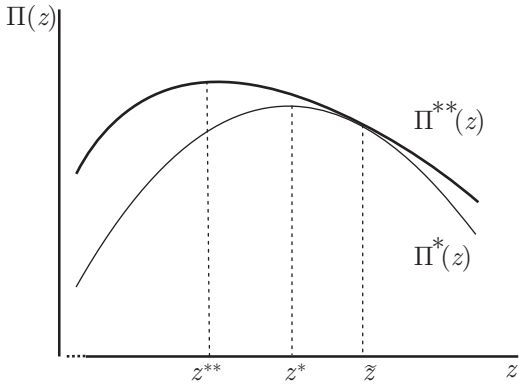

Figure 6-b: $z^* \leqslant \tilde{z}$

We can now solve the first stage of the game.

4.2 Equilibrium prediction for the first stage game

The cases 6-a and 6-b determine the equilibrium prediction of the first stage game.

Proposition 3. *There exists a unique payoff dominant pure strategy Nash equilibrium in the first-stage of the game.*
i) If $z^* > \tilde{z}$ (Fig. 6-a), there exists an infinity of pure strategy Nash equilibria in the first-stage of the game, with (z^*, z^*) the unique payoff dominant one.
ii) If $z^* \leqslant \tilde{z}$ (Fig. 6-b), there exists an infinity of pure strategy Nash equilibria in the first-stage of the game with (z^{**}, z^{**}) the unique payoff dominant one.

Proof:
For a purpose of readability, we will denote by the subscript h the firm with the highest level of fixed factor, by the subscript l the one with the lowest level (*i.e.* $z_h = \max(z_1, z_2) \geqslant z_l = \min(z_1, z_2)$). Let's start by demonstrating Proposition 3 i).
Step 1: Asymmetric strategy profiles cannot be Nash equilibria of the first stage game.
a) If we are in the case $(z_l < z_h < \tilde{z})$ then the equilibria of the second stage are characterized by the Corollary 2-C. Firms have a coordination problem. They are not able to predict the price in the second stage. But, by choosing to match the other firm's fixed factor level, they can solve endogenously the coordination problem. So, for asymmetric profiles of that kind, firms have an incentive to deviate. Those strategy profiles cannot be Nash equilibria of the first stage game. b) If we are in the case $(\tilde{z} \leqslant z_h \neq z^*)$, the equilibrium of the second stage is characterized by the Corollary 2-B. The profit of the

firm h is characterized by the function $\Pi^*(z_h)$. Thus, the firm h has an incentive to deviate getting closer to z^*, the fixed factor level that maximizes $\Pi^*(z)$. Those strategy profiles cannot be Nash equilibria of the first stage game. c) If we are in the case $(z_l < z_h = z^*)$, the firm h has no incentive to deviate. Because of the definition of p^*, we have $\partial\hat{\pi}(p^*(z), z)/\partial p = 0$. Because $z^* \geqslant \tilde{z}$, we have $p^*(z^*) \geqslant \bar{p}(z^*)$ and thus because $\partial^2\hat{\pi}(p, z)/\partial p^2 < 0$ we have $\partial\hat{\pi}(\bar{p}(z^*), z^*)/\partial p > \partial\hat{\pi}(p^*(z^*), z^*)/\partial p = 0$. According to the definition of z^*, we have $d\Pi(z^*)/dz = 0$. Using equation (6b), and the fact that $d\bar{p}(z)/dz < 0$, we deduce that $\partial\hat{\pi}(\bar{p}(z^*), z^*)/\partial z > 0$. Finally, because $z_l < z^*$ and $\partial^2\hat{\pi}(p, z)/\partial z^2 < 0$, we have $\partial\hat{\pi}(\bar{p}(z^*), z_l)/\partial z > 0$. This last expression characterizes the effect of an increase in z on the profit of l (i.e. there is no price effect, only the cost effect). It implies that the firm l has always an incentive to deviate by increasing its level of fixed factor. Those strategy profiles cannot be Nash equilibria of the first stage game. a), b) and c) characterize all the possible asymmetric profiles of the first stage game and proves step 1.

Step 2: (z^*, z^*) is a Nash Equilibrium of the first stage game.

If a firm unilaterally decide to increase its level of fixed factor to $z_+ > z^*$, it will become the only firm with the high level of fixed factor. Its profit will be $\Pi^*(z_+)$. But, because $\Pi^*(z)$ reach it's maximum in z^*, we necessarily have $\Pi^*(z^*) > \Pi^*(z_+)$. Firms have no interest to unilaterally increase the level of the fixed factor. If a firm unilaterally decides to decrease its level of fixed factor to $z_- < z^*$, its profit will be $\hat{\pi}(\bar{p}(z^*), z_-)$. We have demonstrate in step 1c that $\forall z < z^*, \partial\hat{\pi}(\bar{p}(z^*), z)/\partial z > 0$. Thus, we have $\hat{\pi}(\bar{p}(z^*), z_-) < \hat{\pi}(\bar{p}(z^*), z^*)$. That proves step 2. Firms have no interest to unilaterally decrease the level of the fixed factor. That ends the proof of step 2.

Step 3: (z^*, z^*) dominates all the other symmetric strategy profiles of the first stage game.

With the same reasoning as in the preceding steps, it can easily be demonstrated that for "small" ϵ, $(z^* + \epsilon, z^* + \epsilon)$ are also Nash equilibria of the game. But because all the Nash equilibria belong to the set of the symmetric strategy profiles and, because, in the symmetric cases, the firms earn $\Pi(z)$, when $z^* > \tilde{z}$, z^* maximize $\Pi(z)$ and thus (z^*, z^*) is payoff dominant.

Let's turn now to Proposition 3 ii).

Step 1': Asymmetric strategy profiles cannot be Nash equilibria of the first stage game. Case a) If we are in the case $(z_l < z_h < \tilde{z})$, the reasoning is exactly same as in step 1 a). b) If we are in the case $(\tilde{z} < z_h)$, the equilibrium of the second stage is characterized by the Corollary 2-B. The profit of the firm h is characterized by the function $\Pi^*(z_h)$. Thus, the firm h has an incentive to deviate getting closer to $\tilde{z}$ ($\Pi^*(z)$ is strictly decreasing for $z > \tilde{z}$). Those profiles cannot be Nash equilibria of the first stage game. c)

If we are in the case ($z_l < z_h = \tilde{z}$). Let's define $z^{\#} = \arg\max\{\hat{\pi}(\bar{p}(\tilde{z}), z)\}$, the optimal level of the fixed factor for the firm l,when $z_h = \tilde{z}$. Because of the concavity according to z of the function $\hat{\pi}$ and the fact that, when $z^* < \tilde{z}$, $\partial\hat{\pi}(\bar{p}(\tilde{z}), \tilde{z})/\partial z < 0$, we have $z^{\#} < \tilde{z}$. We can distinguish two subcases. c1) If $z_l \neq z^{\#}$, then the firm l has an incentive to deviate, choosing $z^{\#}$. c2) If $z_l = z^{\#}$, we can show that the firm h has an incentive to deviate, choosing $z^{\#}$, because $\Pi^{**}(z^{\#}) > \Pi^{**}(\tilde{z}) = \Pi^{*}(\tilde{z})$. For proving that, we have to notice that by definition of $z^{\#}$, $\partial\hat{\pi}(\bar{p}(\tilde{z}), z^{\#})/\partial z = 0$. $z^{\#} < \tilde{z} \Rightarrow p^*(z^{\#}) > p^*(\tilde{z})$, and thus because $\partial^2\hat{\pi}(p, z)/\partial p\partial z < 0$, we get $\partial\hat{\pi}(p^*(z^{\#}), z^{\#})/\partial z < \partial\hat{\pi}(p^*(\tilde{z}), z^{\#})/\partial z = \partial\hat{\pi}(\bar{p}(\tilde{z}), z^{\#})/\partial z = 0$. It implies that, due to the strict concavity of the profit function Π^{**} is strictly decreasing between $z^{\#}$ and $\tilde{z}$ and thus, $\Pi^{**}(z^{\#}) > \Pi^{**}(\tilde{z})$. Those profiles cannot be Nash equilibria of the first stage game. a), b) c1) and c2) characterize all the possible asymmetric profiles of the first stage game and proves step 1' [7].
Step 2': (z^{**}, z^{**}) is a Nash Equilibrium of the first stage game.
If one of the two firms choose to unilaterally deviate, two cases are possible. If it chooses $z < \tilde{z}$, the equilibria of the second stage are characterized by the Corollary 2-C. We go back to the coordination problem and the firm has no incentive to do that. If the firm chooses $z \geqslant \tilde{z}$, it will become the only firm with the high level of fixed factor. Its profit will be $\Pi(z)$. But, because $\Pi(z)$ reaches its maximum in z^{**}, the firm has no incentive to do that.
Step 3': (z^{**}, z^{**}) dominates all the other symmetric strategy profiles of the first stage game.
If we define $\tilde{z}^-$ such that $\tilde{z}^- < z^{**}$ and $\Pi(\tilde{z}^-) = \Pi(\tilde{z})$, then all symmetric profiles (z, z) such that $z \in [\tilde{z}^-, \tilde{z}]$ are Nash equilibria of the game. In those cases, the firms have no incentive to unilaterally deviate choosing $z' < \tilde{z}$, because the equilibrium of the second stage is characterized by corollary 2-C, and we are back to the coordination problem. Firms have also no incentive to unilaterally deviate choosing $z' \geqslant \tilde{z}$ because $\Pi(z) \geqslant \Pi(z')$.
Because all the Nash Equilibria belongs to the set of symmetric strategy profiles, (z^{**}, z^{**}) is the only payoff dominant Nash Equilibrium of the game. □

We can characterize the complete solution of the game. For a given demand function and for a given α (possibly low), both firms choose z^* in the first stage and quote $\bar{p}(z^*)$. For other values of α (possibly high), both firms choose z^{**} in the first stage and quote $p^*(z^{**})$.

[7] For a specific value of α, we have $z^* = z^{**} = z^{\#} = \tilde{z}$. In this case $z_l = z^{\#} = \tilde{z} = z_h$ is not an asymmetric outcome.

5 Discussion and welfare analysis

The present paper proposes a model of price competition which mainly preserves Bertrand's assumptions. The only departure from the textbook case is just to assume that there is more than one production factors. Some of them are chosen in the first stage and the remaining ones are chosen in the second stage in which firms compete on price. We show that, even if two identical firms produce homogeneous products under constant returns to scale, marginal cost pricing cannot be an equilibrium outcome of the game. The equilibrium (z, p) is symmetric and corresponds to the collusive outcome in the sense that it satisfies the joint-profit maximization of two firms sharing equally the production under the constraint of non-profitable deviation. This joint-profit maximization problem can be written as follows:

$$\begin{cases} \max\limits_{p,z} 2\hat{\pi}(p, z) \\ \text{s.t. } p \leq \bar{p}(z) \end{cases}$$

When the constraint is binding, the solution of joint-profit maximization problem is z^*, which corresponds to a second best. When the constraint is slack, z^{**} is the solution.

By lowering its price and serving the whole demand in the second stage, a firm increases its revenue by higher sales. But, since short-term costs are convex in the second stage, this leads to a disproportionate increase in cost. The firm must sell the additional units at an increasing marginal cost. For this reason, prices above marginal costs can be sustained in a Nash equilibrium, which is not possible under constant marginal costs. Using the Pareto criterion, the continuum of the Nash equilibria can be ranked, and hence firms select the non-Pareto dominated equilibrium point. When the Pareto criterion is not sufficient to achieve unicity of the solution, the resulting coordination problem in the second period can be solved endogenously by the investment decision of the firms in the first stage of the game. In the long run, the choice of the fixed factor (or level of investment) allows the firms to focus on the outcome corresponding to the collusive solution defined above, even if the returns to scale are constant.

6 Conclusion

How does our model differ from the standard analysis of price competition? Usually the idea that Bertrand competition with homogeneous products implies competitive pricing holds only when marginal costs are constant and are identical for two firms. Edgeworth's criticism of Bertrand is based on demand

rationing that may emerge in the case of a variable marginal cost. When firms face capacity constraints (because of decreasing returns), they compete on price realizing that competitors may not be able or may not want to supply all the forthcoming demand at the set price. The problem is that, as pointed out by Edgeworth, an equilibrium in pure strategies in such a game may not exist[8], but most often it can exist in mixed strategies (Maskin, 1986; Osborne and Pitchik, 1986). In the modern literature, the Bertrand-Edgeworth debate has been settled in a two-stage game, with capacity competition followed by price competition. In this framework, rationing rules are critical for characterizing the regions of existence of equilibria in pure strategies.

In a quite similar manner, this paper proposes a model with two-stages where the firms choose the quantity of the fixed factor in the first stage and compete on price in the second stage. However, our model differs from the standard Bertrand-Edgeworth literature on two points:

(i) We have assumed that each firm has to supply all the forthcoming demand at the set price. In our model, firms determine in the second stage the quantity of variable factor needed to satisfy the demand they will face at the equilibrium. Consequently, our results are no longer sensitive to the rationing rule being used (as is usually the case in the Bertrand-Edgeworth competition models).

(ii) Even if costs are strictly convex in the second stage of the game, we keep the constant marginal cost assumption of Bertrand's model. We have considered that firms rely on a technology represented by a two-factor constant returns to scale production function. The first factor is fixed in the short run (first stage), while the second one will vary to satisfy the demand faced by the firm (second stage). Hence the long-run average cost is constant, showing constant returns to scale. The production function choice is not only a textbook case[9] but also a realistic one. For example, in the retailing sector, the fixed factor refers to the surface needed to sell and the length of the shelves used to display the products, whereas the variable factor can be interpreted as the employees needed to fill up the shelves.

The conventional wisdom, based on Bertrand's result, is to believe that price competition is a much more drastic form of competition than quantity competition. Even if there are few firms on the market, Bertrand's result shows

[8]Under some assumptions, the existence of an equilibrium in pure strategies may be restored (Allen and Hellwig, 1986b).

[9]We renew the mashallian tradition of distinguishing between short and long term cost functions.

that some form of imperfect competition can lead to marginal cost pricing and thus to social optimality. Bertrand competition offers a case in which price regulation or anti-trust policies are not necessary, even when there is no free entry to the market. Thus, the possibility of Bertrand competition is a logical argument in favor of more deregulation. However, as we have surveyed in the introduction, the plausibility of Bertrand competition has been under heavy criticism since at least Edgeworth. Our paper weakens Bertrand's results even more. Because it reduces their domain of validity. But also because, in our model, price competition leads not only to a socially inefficient outcome, but to the worst possible one, collusive outcomes as defined above. Maybe it's time to have a closer look at sectors where price competition seems to be the rule, and to finally forget the conventional wisdom created by the very special case of Bertrand price competition.

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