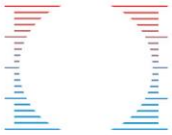

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September 2014

Centre for Climate Change Economics and Policy
Working Paper No. 186

Grantham Research Institute on Climate Change and the Environment
Working Paper No. 164

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Modelling land use, deforestation, and policy analysis: A hybrid optimization-ABM heterogeneous agent model with application to the Bolivian Amazon*

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September 9, 2014

Abstract: Policy interventions designed to simultaneously stem deforestation and reduce poverty in tropical countries entail complex socio-environmental trade-offs. A hybrid model, comprising an optimising, agricultural household model integrated into the 'shell' of an agent-based model, is developed in order to explore the trade-offs of alternative policy bundles and sequencing options. The model is calibrated to the initial conditions of a small forest village in rural Bolivia. Heterogeneous farmers make individually optimal land-use decisions based on factor endowments and market conditions. Endogenously determined wages and policy provided jobs link the agricultural labour market and rural-urban migration rates. Over a simulated 20-year period, the policymaker makes "real-time" public investments and public policy that in turn impact welfare, productivity, and migration. National and local land-use policy interventions include conservation payments, deforestation taxes and international REDD payments that both impact land use directly and affect the policymaker's budget. The results highlight trade-offs between reductions in deforestation and improvements in household welfare that can only be overcome either when international REDD payments are offered or when decentralized deforestation taxes are implemented. Yet, the sequencing of policies is also found to play a critical role in these results.

Key words: deforestation, REDD, ABM, Bolivia.

JEL codes: Q23, Q28, Q56, R14

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* This research was supported by the Ecosystem Services for Poverty Alleviation (ESPA) program (NERC grant no. NE/I003185/1), The SimPachamama software can be downloaded for free from www.inesad.edu.bo/simpachamama. We thank Tim Laing for his useful comments on an earlier draft.

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1. Introduction

For decades, deforestation and forest degradation in tropical nations have reduced supplies of forest ecosystem services (MA, 2005; FAO, 2010). These losses have had consequences at all scales, from local to global. Forest users with incomes and livelihoods dependent on, e.g. watershed services, have experienced adverse effects on their welfare. Emissions of carbon dioxide from deforestation and forest degradation influence the trajectory of anthropogenic climate change with welfare implications for future generations across the globe (Stern, 2006). Yet, policies which aim to conserve forests such as protected areas can also adversely affect the welfare of the forest-dependent poor (Barrett et al., 2011).

In response, policy makers have increasingly sought to design interventions which not only aim to conserve forests but also improve the incomes and livelihoods of forest users (e.g., see Merger et al., 2011). Targeted towards agents of deforestation, interventions such as payments for environmental services (PES) and the provision of off-farm labour opportunities could, under certain conditions, enhance their welfare (Groom and Palmer, 2010, 2014). But since this type of intervention necessitates public and/or private funding, there may be broader policy and welfare implications. Alternatively, the profitability of deforestation activities could be targeted, for example, by reducing agricultural subsidies or infrastructural investment (Angelsen, 2010). However, this type of intervention has the potential to reduce the welfare of agents unless accompanied by other policies, which can, in some way, compensate for welfare losses.

Policy design to conserve forests and improve welfare is thus a complex undertaking. In this paper, we examine potential trade-offs in policy outcomes with a focus on two design features that can help us to better understand dynamic policy interactions: 'policy bundles' and policy sequencing. The former refers to combinations of policies that all, in some way, impact on land-use decision making while the latter refers to the order in which policies are implemented. We incorporate these two features into a landscape- (or village-) scale model, which enable a local policymaker ('the mayor') the opportunity not only to implement policy bundles but also to react to the consequences of her policy choices over time. Thus, policy parameters can be changed and new policies can be implemented.

Our model is a novel hybrid, comprising on the one hand, an agent-based model (the ABM 'shell'), and an optimising, agricultural household model on the other. The latter allows households to make individually optimal land-use decisions according to their specific circumstances, e.g. landholdings, household size, as well as broader market conditions. The former allows us to define the landscape in which a community of heterogeneous households reside and make land-use decisions. Specifically, it allows for the endogenous determination of wages, which link the agricultural labour market to rural-urban migration rates, and adjustments of the state-space faced by households. This separation between the ABM shell and the optimising household allows the mayor to explore the interactions between her policy choices, the choices made by individual households, and important external drivers of change.

Using real-time information on community well-being, deforestation, macroeconomic conditions and the mayor's budget, the mayor can adjust a range of policies to try to reduce deforestation and improve welfare. Local policy interventions that can be adjusted throughout the simulated 20-year period of the model include public investments made from the mayor's budget that in turn impact welfare, productivity, and migration. National and local land use interventions include conservation payments and deforestation taxes that both impact land use and the mayor's budget.

The model is initialised and calibrated using rural household survey data from two small communities on the Bolivian Amazonian frontier. Bolivia provides an appropriate setting for our model. It loses an estimated 300,000 hectares of forest annually[1], mostly due to the expansion of the agricultural frontier (Andersen et al., 2012). Furthermore, as in many tropical countries, annual *per capita* income remains below $5,000. The government's approach has been to attempt to tackle both problems simultaneously, developing a programme for both reducing deforestation and rural poverty that relies on a broad set of interventions (INESAD, 2013).

Our hybrid model is designed to reflect both the realities of the forest frontier and existing knowledge of socio-environmental trade-offs in such a setting. In theory, the model allows us to explore policy outcomes across an infinite combination of policy choices; in practice, the mayor reacts by adjusting policy choices as these outcomes evolve in response to previous choices. Over repeated simulations, the relative degree of success of different strategies becomes apparent to the mayor. This allows for experimentation and active policy learning in a simulated yet 'real-world' setting that can be easily adjusted to other settings. For researchers, by recording and comparing these policy sequences and outcomes a number of potential lessons have emerged that are theoretically coherent and potentially empirically testable.

The remainder of the paper begins with a presentation of the Bolivian case study, followed by the model, in section 3. In section 4, we discuss some of these lessons, including the role of international incentives for reducing emissions from deforestation and forest degradation (REDD) and decentralized tax-raising powers. Section 5 concludes.

2. Bolivian Case Study

Bolivia is relatively early in its forest transition, with more than 50 percent forest cover remaining and medium rates of deforestation (FAO, 2010). The country's 1996 land tenure reform law formally recognises indigenous communal properties (*Tierra Comunitaria de Orígen*, TCOs), and a new forestry law promoting sustainable forest management recognises some rights of private and communal landowners to forest resources. Nevertheless, work remains to finalise reforms and consolidate new property rights.

Bolivia was one of the first countries to develop a national REDD strategy. Between 2006 and 2010 Bolivia's government advocated a strong role for forests in international climate change negotiations. There were more than 10 different,

[1] Killeen et al. (2007) and FAO (2010).

small-scale REDD projects and proposals in Bolivia, including some organised by local NGOs and indigenous groups. For example, the 'Subnational Indigenous REDD Programme in the Bolivian Amazon' was supposed to involve six million hectares in three TCOs, six municipal governments and national agencies responsible for forest monitoring.

However, in April of 2010 the political viability of REDD mechanisms in Bolivia was seriously challenged at the politically influential "World People's Conference on Climate Change and the Rights of Mother Earth:"

> *"We condemn market mechanisms such as REDD (Reducing Emissions from Deforestation and Forest Degradation) and its versions + and + +, which are violating the sovereignty of peoples and their right to prior free and informed consent as well as the sovereignty of national States, the customs of Peoples, and the Rights of Nature."*

Although political causality is unclear, after the Conference the REDD preparation process in Bolivia stalled and the political environment grew quite hostile, with the Bolivian Government writing to the UNFCCC: "in all actions related to forest, the integrity and multifunctionality of the ecological systems shall be preserved and no offsetting or market mechanisms shall be applied or developed."[2] (Andersen et al., 2012).

The Bolivian Government has instead started developing an alternative policy for reducing deforestation and rural poverty, called the *Joint Mitigation and Adaptation Mechanism for the Integral and Sustainable Management of Forests* (The Mechanism). While still in development, the Mechanism relies on a broad set of interventions, including both positive and negative incentives, as well as education and the active participation of local actors and policy makers (INESAD, 2013). In support of this effort UN-REDD has awarded Bolivia USD 1.1 million, and Denmark has also approved support in the amount of at least USD 26 million.

At the same time, since 1996 Bolivia has actively pursued improved land tenure policies and as a result enjoys relatively strong and secure property rights, with a large proportion of plots officially entered in the land registry (INRA, 2008). For example, all of the households surveyed in this study (see below) either had clear legal title to their land, or were in the process of obtaining title. Thus, while insecure property rights has been a major obstacle to successful conservation policy in many developing countries (e.g. Streck, 2009; Sunderlin et al., 2009), the relative strength of land tenure in Bolivia should allow the impact of conservation policies themselves to be more observable.

The Bolivian case thus presents a good opportunity to explore the dynamic complementarities and trade-offs between policies designed both to reduce deforestation and alleviate poverty. Specifically, we make intensive use of a survey of 290 agricultural households from three communities in the region of Rurrenabaque and Buenaventura, on the Amazonian frontier (Leguia, Malky and Ledezma, 2011). The survey included information on property size, land use and

[2] FCCC/AWGLCA/2011/CRP.23, dated 4 October 2011.

deforestation, land tenure, labor force participation, household demographics, wealth, wages, cattle stocking and reproduction, and geographic and environmental variables. Summary statistics of the main variables of interest are presented in Table 1. In addition, the research team itself spent several weeks in the region interviewing local actors, validating parameters and predictions of the model, and conducting a participatory workshop in San Buenaventura (April 2012) with the participation of local farmers, cattle ranchers, loggers, teachers and the mayor of the municipality.

3. The Model

3a. Household optimization

Following the theoretical typology outlined in Angelsen (1999), an initial calibration exercise using the Leguia et al. data from Rurrenabaque and San Buenaventura rejected both the 'full belly' subsistence model and the Chayanovian model of joint consumption and production with limited off-farm opportunities. Instead, a model of 'modern' profit maximizing agents with full access to both labour and goods markets provided a much more plausible description of these communities, and was thus adopted as the primary framework for building the simulation model.

The main optimising 'engine' of the simulation is thus a model of the behaviour of household producer-consumers with varying access to an agricultural labour market and off-farm labour markets. The household model is outlined in Appendix 2. Households are heterogeneous in their initial endowment of land, land productivity, and family size. Total household time (T) is divided between on-farm labour (L), local off-farm labour (L^w, where *w* is the wage rate), city (out-migration) off-farm labour (L^{OFF}), and leisure (*l*). On-farm labour is in turn is divided between labour cultivating previously cleared land (L^f) and labour spent clearing (deforesting) and cultivating new land (L^D).

Off-farm labour may be constrained to some level so that $L^w<E$, where E is an upper bound that may be below the optimal level of L^w - in other words, there may be some involuntary unemployment with respect to off-farm labour. We assume all households value both consumption and leisure, and that these can be mapped to a welfare function $U(C,l)=C^{\alpha}l^{\beta}$. However household types differ by the internal and external constraints that they face, as explained above.

All households have an initial allocation of cleared land, H_{t-1}. This is a proportion of their overall land endowment, and they can clear more land if they choose. By supplying labour to work their land, or renting labour from the local market, they can produce output, which is translated into consumption at a given rate, *p* (the price of output). Households can also work for wages off-farm (at wage rate *w*), which also generates income that translates into consumption, or they can enjoy leisure, which also brings them well-being. The values of a and b reflect the substitutability of consumption and leisure. Diminishing marginal returns to consumption and leisure are deployed as working assumptions; allocate too much time to consumption-generating labour and the relative marginal well-

being from a time unit of leisure will increase until the rational household maximising well-being will reallocate time from labour to leisure.

Given their total household time budget, the wage at which households can earn in the off-farm labour market, *w*, the limit of off-farm labour they may supply (E), and the production function that maps their labour input and land use to output, households choose how much land to cultivate, how much new land to clear, how much labour to supply to off-farm activities for a wage and how much leisure time to enjoy in order to maximise their welfare function $U(C,l)$.

On the production side, we specify a parsimonious production function for cultivated land that approximates the Bolivian case for smallholders in the area of interest. In particular, we assume a linear production function in which labour and land are required in fixed proportion to produce output. However, diminishing productivity of labour and land is captured by a labour requirement that increases with the distance of land from the road. This could be interpreted as a travel cost associated with working far from the road. In addition to the travel cost of distance, the labour required for cultivating cleared land is different from the labour required for clearing (deforesting) and cultivating new land. Thus we have a linear, fixed proportion technology that varies with distance and discontinuously with type of land under production. Figure 1 illustrates the marginal cost for labour as a function of land cultivated.

We assume that each household's plot is of a fixed width. This reflects the way in which plots are organised along the roadside in Bolivia and also approximates the arrangement of farming more generally. With the width of the plot fixed, the area of land, *H*, is also a metric for distance from the roadside. Figure 1 shows how the marginal cost of labour varies with the area cultivated. The endowment of previously cleared land is given by H_{t-1}. The marginal cost of labour on this land is given by (*1+q*) and the total cost of cultivating this land is given by the blue area. If more land is cultivated then deforestation is required, and the marginal cost of labour on this land differs to reflect this: (*1+q+s*), where ***q*** captures travel costs of distance and ***s*** captures the differential labour required for cultivating new land that must be cleared first. Figure 1 shows the case where the maximising level of cultivation is given by *H**. In this case, deforestation is required at higher marginal cost: *s > 0*. Local interviews around Rurrenabaque and San Buenaventura indicated that common practice was to exchange forest clearing services for the wood extracted, suggesting that in their case it is likely that *s* is about zero. Thus, we assume *s=0*, although this parameter can be adjusted for other settings.

Given this discontinuous cost structure, household optimisation proceeds in two steps: first, households optimise over their converted land endowment; and second, households make a deforestation decision. The first step can be thought of as a constrained optimisation problem, with the converted land endowment as the constraint. The second step is only considered if the first stage solves as a corner solution (e.g. the household chooses to allocate all of its cleared land to agriculture) and the shadow price of land is positive (e.g. the marginal return of an additional unit of land, if they had it, would be positive). It is therefore

necessary, but not sufficient for deforestation to take place in the second stage. If the shadow price is sufficiently high to overcome the discontinuity in cost driven by $s > 0$, then deforestation will occur. If $s < 0$, then deforestation is more likely to occur in step 2 if there is a corner solution due to lower marginal costs.

In addition to using land for crops, clearing pasture for cattle is seen as a major cause of deforestation in the Amazon Basin (e.g. Palmer et al., 2012) and is an important component of land use around Rurrenabaque. In particular, the literature suggests (see Faminow, 1998; Birner, 1999) that current investment in cattle is not only an investment for future returns (which could be modelled at net present discounted value today), but also as a hedge against future risk (which would require additional assumptions about relative risk aversion and future expectations of shocks), and a source of social prestige (well-being in and of itself).

Cattle also have the unique property that, unlike crops, they reproduce themselves. We model the optimal livestock production decision in two steps; first, the intensive decision (step 1) in which a technological decision is taken about how intensively to undertake livestock farming (e.g. the stocking rate per hectare). This technological decision is conditioned on the biological/agronomic constraints of land and cattle. We assume that households are separable profit maximisers who understand the intertemporal nature of the stocking decision and undertake a dynamic optimisation. Diminishing returns to land are modelled by assuming that livestock follow a standard logistic growth function, and fixed investment costs make the potentially more productive intensive production inaccessible for certain households. Then, the household's optimal solution to the intensive problem provides the input to the second, extensive decision (step 2) in which a decision is taken on how many hectares to ranch conditional on the stocking rate. The full cattle model is described and solved in Appendix 3.

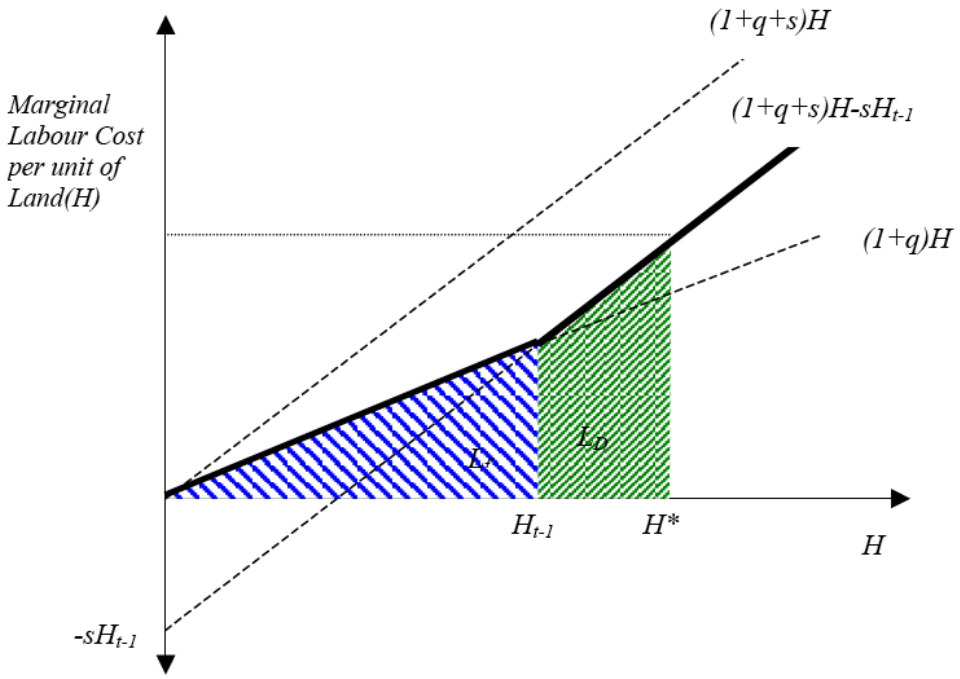


Figure 1. The Marginal Cost of Labour as a Function of Distance from the Road

3b. The ABM 'shell' of heuristic dynamics and general equilibrium effects

So far our household optimisation model is mostly static (except for the quasi-dynamic decision in cattle stocking); given the initial conditions and parameter values, households make a (myopic) optimal land use decision. In theory it would be feasible to allow households to dynamically optimise over a given time horizon, but in practice this is computationally much more demanding, especially since we allow users the possibility to continually adjust policies throughout the 20-year runtime of the simulation.

Instead, we use the ABM 'shell' to adjust the state-space faced by each household at the beginning of each period, as a function of the decisions taken by the household in the previous period, the decisions taken by other households (that will affect the labour supply and wage), and other changes in macro-economic conditions. For example, if more households choose to supply labour to the local market than choose to hire in labour, households are constrained and the ABM shell finds a new market-clearing (or near-clearing) wage so that in the next period the market wage faced by all households will be lower. In addition, each period a certain amount of land must be left fallow. Rather than build this in as choice variable (difficult in the absence of dynamic optimisation), households are required in the ABM shell to leave land fallow on a regular schedule. Thus although we forgo the opportunity of explaining fallow (we take this as given), we do incorporate the constraint via the ABM in a simple, straightforward fashion.

The ABM shell also allows us to incorporate migration and population growth into the model. Both are important both for economic and environmental outcomes and for the dynamics in the model. Population growth is assumed to increase by 2% per year, with households adding the requisite number of new members every 20 years to achieve this. As households begin the simulation with varying lengths of residency, this population growth in practice is achieved with some subset of households increasing in size each period. In addition, following the standard of the Bolivian settlement policy in this area (INRA, 2008), each additional new person is allocated 50 hectares of forested land, which is appended to the household plot.

As in real life, in- and out- migration are also important determinants of how the simulated settlement evolves. Based on interviews in the region, we assume that migration into the community is mediated by a government assisted settlement programme (INRA 2008). The number of new families arriving each year is endogenously determined within the ABM shell and increases the local population by between 0-5%, depending on the availability of land and the well-being of existing households. Specifically, we assume that households will not migrate into the community if the community "score" (broadly a weighted average of economic prosperity and environmental health, explained in more detail below in section 3d) falls below a particular threshold, nor will there be any in-migration if land is unavailable. However if land is available and community well-being is sufficiently high, then in-migration increases the local population by 0.1% for each one point increase in the community well-being score, up to a maximum of 5%. The families that arrive are very poor (4 persons in each family, 0 savings) and are allocated a 50 hectare plot on the next empty spot along the road, along with one cow. They then start farming according to the small-farmer model, and the cattle grow according to the cattle model.

Reflecting local realities in the region of study, migration out of the community is dominated by young people who would rather work in non-agricultural jobs. In the simulation out-migration varies between 0 and 5% of the total population per year, and based on the findings from interviews with the local communities, we assume that it depends both on the level of education (positively) and the availability of non-agricultural jobs (negatively) in the community. Specifically, out-migration increases by 0.1 percentage points for every $2000 in Public Investment (which increases education) up to the maximum of 5% per year, while for every non-agricultural job created, out-migration is reduced by one person.

Thus the ABM shell adjusts the state-space characteristics each year based on the outcome of the previous year's decision of optimising households. New market-clearing prices and wages are determined, some land is set aside for fallow, people arrive – either naturally or through in-migration, and leave. The households 'wake up' anew, face their new initial conditions and constraints as generated by the ABM shell, and repeat the optimisation exercise.

3c. Policy Levers

The ABM shell is also the component of the model which allows for 'real time' policy adjustments by the user. The shell provides users with a host of information about the current state-space of the simulation, including the average well-being of the households (explained in detail below in section 3d), the extent of deforestation, the cattle herd, the wage level, and the local government's budget. Based on this information the user/policy maker can make adjustments across five different types of policy levers that are included in the model:

1. Public investment
2. Investment in local, non-agricultural jobs (*Green Jobs*)
3. Deforestation Tax
4. Conservation payments
5. International compensation for reduced emissions from deforestation

Public Investment combines investment in education, health and public infrastructure. Such investments tend to increase human well-being but are also costly. The default value of *Public Investment* is set at $15,000 per year, which is approximately the amount the community receives in transfers from the central government every year (in Bolivia this money comes mainly from the Direct Tax on the extraction of oil and gas and amounts to approximately $50 per person depending on the price of oil). As the local government spends this down or brings in additional funds (from, say, a tax on deforestation), this budget may increase or decrease.

The second type of policy intervention is to provide alternative off-farm employment opportunities which cause less deforestation and at the same time higher incomes. This not only has favorable direct effects on the people who are employed, but by reducing the supply of agricultural labor these initiatives also lead to increases in agricultural wages, which in turn tends to both reduce economic inequality as well as raise the costs of agriculture and deforestation. We dub these *Green Jobs* policies as they have a series of attractive effects, but they are also extremely expensive. For example, if the local government wants to stimulate jobs in the tourism sector, it has to invest in good tourism facilities, such as roads, airports, water, sanitation, communication, etc. We assume the cost of one green job at $6000, about half the estimated country-wide average investment needed to create a job in Bolivia (Muriel and Jemio 2010).

The *Deforestation Tax*, between 0 and US$500 per hectare, will directly affect the decision to deforest. If very high, farmers will find it more profitable to cultivate already cleared land instead of deforesting new areas, and cattle ranchers will chose to sell more of their cattle instead of letting the stock increase every year. At the same time, the tax reduces household net incomes and thus their level of well-being. As the big deforesters tend to be relatively higher income large-scale cattle ranchers, the *Deforestation Tax* will also tend to reduce inequality.

The last policy lever, conservation payments, represents a scheme where households are paid a compensation for any land that they promise to keep forested for at least 20 years. The scheme is similar to SocioBosque in Ecuador and COMSERBO in Pando, Bolivia, with payments varying from 0 to US$100 per year per hectare. When offered the option of participating in such a scheme, each household will calculate how much land it is optimal for them to dedicate to conservation, and how much it should make available for its agricultural needs for the following 20 years. They will always inscribe the marginal land farthest away from the road, as that is the least profitable to cultivate. The poorest households tend to benefit disproportionately from this scheme, as they will often not have the financial resources to cultivate their entire plot anyway. In addition, while all policies can be changed at any time during the 20-year simulation period, we assume that if the Conservation Payment is changed, households who have already signed a contract will be liberated and are free to re-optimize their decision under the new conditions.

Finally, in addition to the four local policies, we include the possibility of accepting an international mechanism of compensation for reduced deforestation (e.g. REDD payment). When this option is active the community will receive a reward for every hectare of reduced deforestation, with a default price of $5000 per hectare (corresponding to $10 per ton of avoided CO_2 emissions from deforestation) that can be changed. The "Reduction" here is calculated as the difference from the 'business-as-usual' scenario obtained by letting the model run for 20 years with only default Public Investment.

Land use and human well-being outcomes in the simulation are therefore simultaneously affected by both the decisions of the households, the evolving external economic environment, and the dynamic trajectory of choices of the user for both local and national policies. Agricultural subsidies and taxes, as well as conservation payments or REDD+ payments for avoided deforestation will affect decision making essentially through the price of land. This allows us to make predictions about the likely effects of REDD+ at the individual household level as well as providing the simulation with a means of evaluating different policies at the level of the geographical region of interest. Households in turn will adjust their supply of labour to the agricultural and off-farm market, effecting wages and land use in the next period. Local policies that generate Green Jobs or that increase education, for example, will similarly affect wages and labour supply choices.

3d. Human well-being, environmental health and calculation of the 'Score'

The objective of the simulation is to explore dynamic trade-offs faced when simultaneously trying to reduce deforestation and increase human well-being. Furthermore, in addition to the average outcome we are also interested in the *distribution* of overall well-being across the community. While the simulation is potentially able to output all the variables of interest each 'year,' in practice it is more practical to provide a summary statistic of overall policy success that incorporates the objectives parsimoniously. Thus in each year the ABM shell evaluates human well-being and deforestation per capita for each quintile of the

community wealth distribution, and the community receives five 'Scores' corresponding to the relative well-being and deforestation intensity (e.g. deforestation per capita) of each quintile. These five quintile scores sum to the Community Score for that year, and the objective of a policy-maker user is to maximize the average community score over the 20-year run of the simulation model.

Calculating deforestation per capita is straightforward. To calculate the human well-being of each household the ABM shell evaluates a five-argument Cobb-Douglas utility function:

(1) $U(c, l, s, ES, PI) = c^{\alpha} l^{\beta} s^{\gamma} ES^{\delta} PI^{\varepsilon}$

Where:

c = private consumption per capita (average consumption within the family, measured in tons of rice equivalents).
l = private leisure per capita (average leisure within the family, measured as a fraction of total time available in a year)
s = private cattle stock per capita (average number of cattle per person within the family)
ES = ecosystem services (total forest area in community measured in square kilometers)
PI = public infrastructure (stock of public infrastructure measured in millions of dollars).

The parameters α, β, γ, δ and ε are set to reflect how much time households would typically dedicate to/benefit from each component in an average 24-hour-day. For example, people generally want at least 10 hours of leisure per day, so β has been set to 0.4. They would dedicate about a third of the day to production for consumption so α has been set to 0.3. Since cattle constitute their main savings vehicle, and people would like to save about 5% of their potential income (corresponding to about 10% of realized income), we have set γ to 0.05. Ecosystem services from the forest surrounding the community provide services that we assess to be roughly equal to a couple of hours of work per day, so δ is set to 0.1. The same kind of logic holds for public infrastructure, like roads, schools, health clinics, telephone networks etc. and we have set ε to 0.15.

For households in each quintile in the wealth distribution, the average well-being as calculated by equation (1) and average deforestation per capita are plotted by the ABM shell in "well-being – deforestation" space, which in turn is divided into four ringed 'zones,' each which corresponds to a 'score' indicating how well the two objectives have been achieved. The approach is illustrated below in Figure 2, with each of the four rings earning a 'score' of 25, 10, 5 and 0, respectively. A total community score is then the sum of the individual quintile scores

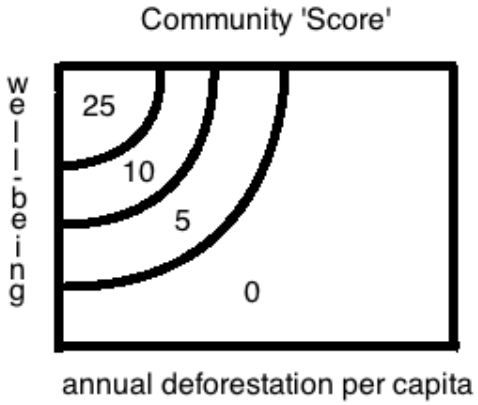


Figure 2: Calculation of community 'Scores' in Well-being and Deforestation space

In sum, the hybrid optimization-ABM simulation model features an array of heterogeneous households, calibrated to the conditions of a small agricultural community on the Bolivian Amazonian frontier. An ABM 'shell' sets the state-space characteristics at the start of each period, determining equilibrium market-clearing wages and prices (including taxes and conservation payments, if any) and household endowments, based on the outcomes from previous household decisions and policy choices by the user. Households make constrained optimization choices about land use, deforestation, and labour supply based on their endowments and the prevailing macro-environment. The policy-maker user can then adjust any of the five policy levers based on real-time information on the community score, average well-being, total deforestation, the government's ('mayor's') budget, and wages.

4. Discussion and lessons learned

As discussed in section 3, the hybrid optimization-ABM approach allows for a rich quasi-general equilibrium, quasi-dynamic modeling that is based largely on micro-fundamentals, while also permitting us to explore the implications of highly heterogeneous households and general equilibrium feedback effects. The ABM shell plays the role here of producing the latter effects that would not have been apparent from a simple partial equilibrium analysis of the household, but in a more feasible manner and at much lower computational cost than a true dynamic general equilibrium optimisation model.

The hybrid nature of the simulation allows us to explore outcomes produced by different combinations and dynamic sequences of policies, and in theory there are an almost infinite number of these possible combinations and alternative sequences of policies that could be tried. However, an important difference between our simulation and more conventional policy analysis tools is that the policy-maker/user receives feedback on a range of economic and environmental state-space characteristics from the ABM shell in real time over the run of the simulation, and can adjust any of the policy levers in response to this feedback to try to improve the community outcome. As such, the approach more closely

approximates real world policy making, with the exception that the user/policy-maker of the simulation can make multiple attempts to improve the outcome. Over repeated attempts to maximise the overall average community score, users/policy-makers experience a learning process about the dynamic complementarities and trade-offs among policy choices and over time develop certain dynamic strategies. The simulation model has been run now thousands of times both by ourselves and by students and participants in several workshops held around the world. Thus rather than run Monte-Carlo simulations on random combinations of dynamic policy choices, in our analysis we make intensive use of this learning process to draw out the main policy conclusions derived to date.

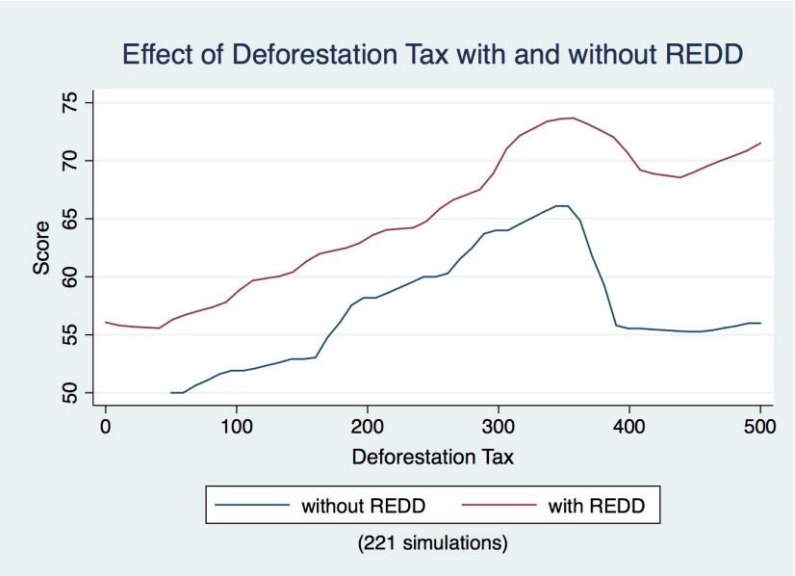
Lesson 1: Implementing a tax on deforestation early on is important to generate revenue to spend on the other policies, but in practice may only be effective if the revenue generated remains with the community.

We assume that the local government has a hard budget constraint and cannot borrow, which is fairly realistic for the case of Bolivia. As a result we find that implementing a local tax on deforestation as soon as possible in the simulation has three major advantages. First, it generates local tax revenues that can be used for public investment, off-farm jobs, and conservation payments. Second, a tax on deforestation reduces deforestation directly, and thus, if there is international compensation for reduced deforestation, this will quickly generate large amounts of supplementary revenues. Third, a local deforestation tax is mainly paid by the wealthier (large) cattle ranchers who need much more cleared land than a subsistence farmer, so it serves as a form of "Robin Hood" tax that takes from the wealthiest and distributes the revenue to the poor, through public spending.

The optimal tax is not necessarily the highest tax, though. When students at the Catholic University of Bolivia were asked to maximize their scores for different levels of deforestation tax, they found that a tax of $350/hectare was the optimal tax, both with and without international compensation for reduced deforestation (see Figure 3).

Note that for this result to hold, however, it is necessary that the tax revenues raised locally be available for local public investment. In Bolivia, although the government has recently implemented a tax on deforestation, the revenues go directly to the central government. Local communities do not currently perceive any benefits from a tax on deforestation. Instead, it is viewed as a drain on community revenues, reducing both incomes and jobs. Thus, one of the key policy recommendations arising from this study is that deforestation taxes should be as decentralized as possible. Such taxes not only have the potential to raise revenues for other policies, which could help build and maintain local support for policy goals, but may also allow greater flexibility in local policy-making .

Figure 3:



Note: kernel estimator of mean Score across 12 teams at different set levels of deforestation tax.

Lesson 2: International finance makes a big difference

As can be observed in Figure 3 above, the score achieved is much higher when international compensation for reduced deforestation (REDD) is available. In the above simulations, the price of reduced CO_2 emissions was set at \$10/tCO_2. Since Bolivian forests have the potential to release, on average, about 500 tons of CO_2 per ha, if burned, this implies a payment of \$5000/ha of reduced deforestation. These payments are very large compared to the community's regular revenues. Indeed, if the mayor is successful at halting deforestation, he will receive so much international compensation that he can spend the maximum amount on all the other policies that increase human well-being.

In contrast, when there is no international financing available, the mayor's budget is severely constrained, and even with high levels of deforestation taxes, the mayor has to watch public spending closely. Indeed,the simulation stops if the mayor runs out of money. Over many repeated runs of the simulation users have found that it is very difficult to simultaneously increase human well-being and reduce deforestation without international compensation. Thus, the REDD payments help overcome the apparent trade-off between well-being and deforestation even at relatively modest carbon prices.

Lesson 3: Timing and sequencing of policies is important

Users of the simulation tool quickly figure out that the first policy to be implemented has to be the deforestation tax, because all the other policies are costly and cause the mayor to run out of money quickly.

But it is clear from our results that policies have to be continuously adjusted and fine-tuned to obtain the best results, and that correct sequencing is critical. Revenue must be generated to fund public policies that both raise welfare and in

turn generate more revenue. Reduced deforestation, induced either by taxation or by positive conservation payments or REDD, while providing some positive effects for household well-being, by itself does not improve welfare enough to compensate for lost agricultural output. Thus complementary spending on public policies to increase education and access to relatively well paid jobs is essential to achieve a good overall outcome. As the sequence of policies is critical, the user/mayor cannot achieve a good outcome with a single set of fixed policies at the start of the simulation. Expensive investments, such as the creation of green jobs, have to be introduced gradually, as the mayors' revenues increase. In fact, the Green Job policy is really only an option after having received substantial international compensation for reducing deforestation. If only local funds are available, a possible strategy is for the user/mayor to implement a maximum tax on deforestation and save for 10 years, in order to create 5 green jobs during the second decade. While this strategy successfully reduces deforestation, community well-being suffers and the final score tends to be quite low.

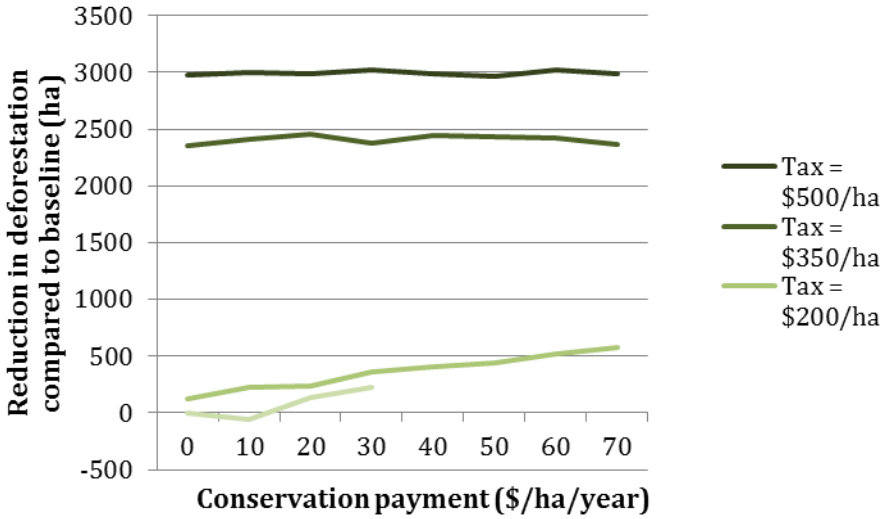
Lesson 4: Policies interact in complex ways

The effects of the different policies are not only non-linear, but they also interact in complex ways, generating outcomes that are not *ex ante* obvious, but that *ex post* make sound economic sense. For example, Figure 4 shows that increasing conservation payments will contribute to further reductions in deforestation, but only if the deforestation tax is low. And the deforestation tax cannot be too low, because then there is no money to finance the conservation payments. For example, if the deforestation tax is $100/ha, then the conservation payment cannot increase to more than $30/ha/year without the mayor running out of money. In contrast, with high deforestation taxes (above $350/ha), adding a conservation payment does nothing to reduce deforestation, since marginal land has already been removed from production due to the tax - in that case quite a high payment would be required to even further reduce deforestation.

In comparison to the deforestation tax, a conservation payment is clearly not very effective at reducing deforestation. This is not surprising, since farmers and ranchers in the model calculate how much land they are going to need for the next 20 years, and only enter into a conservation agreement with the most remote land that they do not plan on using anyway.

Nevertheless the conservation payments could potentially increase human well-being, as participants are basically receiving windfall income. The simulations do not confirm this, however. Indeed, Figure 5 shows that for each given level of deforestation taxes, the SCORE decreases with increasing conservation payments, and this decrease is exclusively due to reductions in human well-being.

Figure 4: Reductions in deforestation can be achieved for different combinations of deforestation taxes and conservation payments

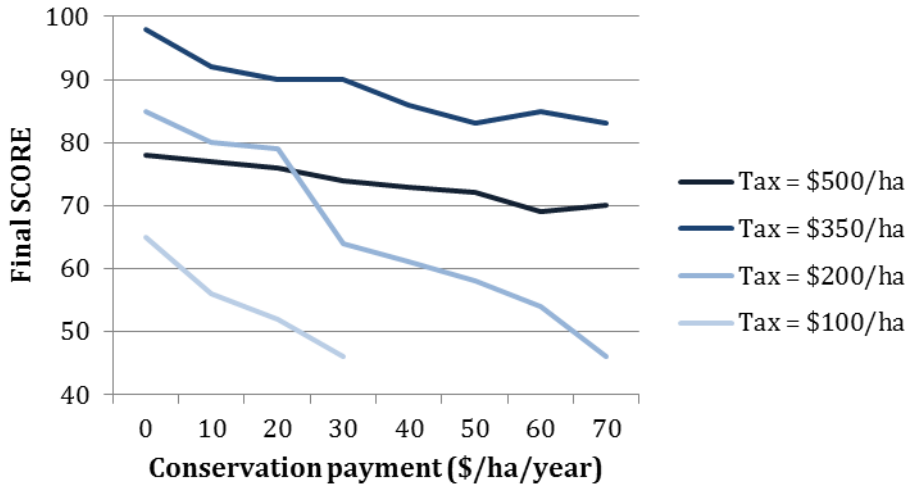


Note: Highest score achieved by "expert mayor" for each combination of deforestation tax and conservation payment.

The decrease is particularly steep for low levels of the deforestation tax. This is because the mayor's budget constraint is more binding when he has low tax revenues, and each dollar that is spent on conservation payments then cannot be spent on health, education or non-agricultural (e.g. green) jobs.

For higher deforestation taxes, this is less of a problem, but with the "utility" parameters chosen in the model, it is only optimal to make conservation payments if you have money left over after investing the maximum amount in public investments and the creation of non-agricultural jobs.

Figure 5: Final SCORE achieved for different combinations of deforestation tax and conservation payment



Note: Highest score achieved by "expert mayor" for each combination of deforestation tax and conservation payment.

5. Conclusions

Policy interventions designed to simultaneously stem deforestation and reduce poverty in tropical countries entail complex socio-environmental trade-offs. In order to explore these trade-offs we develop a model of land use change and human well-being using a parsimonious representation of the essential features of agricultural and economic decisions among smallholders in the Amazon Basin in Bolivia. While our hybrid dynamic optimizing- heterogeneous- agent based model (which we call *Sim Pachamama,* Quechua for Sim MotherEarth) is calibrated to the initial conditions of a small village on the Amazonian frontier in rural Bolivia, the optimization problems solved by the agents across a number of dimensions are broadly generalizable. In particular, heterogeneous households endogenously choose how much land to cultivate, how large a cattle herd to maintain, and whether to expand at the intensive margin through input choice, or the extensive margin by deforesting. Labour allocation is also an important determinant of land use (e.g. Groom et al. 2010, Pascual and Barbier 2008, Shively and Pagiola 2008) and the optimization problem of households takes into account wage differentials and the availability of local non-agricultural jobs to determine how much labour to rent out or rent in agricultural labour markets, and how much labour to supply outside of farming activities. A village mayor (the model user) makes policy choices in 'real time' over public investment and taxes subject to its budget constraint that in turn impacts household's opportunities; the net effect of these decisions is transmitted to the next period through stock, wage and price adjustments via the ABM 'shell' program, and the continuously variable policy adjustments by the village mayor (user) alter the trajectory across both economic and environmental outcomes.

The model is methodologically innovative and highly interactive. To date, after many thousands of 'runs' where users (primarily students thus far) choose and adjust policy levers throughout the 20-year simulation, a number of interesting and consistent predictions and implications have emerged, especially in relation to policy bundling and sequencing. For example, we find that international finance can relax critical credit constraints, reducing the local tax burden and bringing the deforestation reduction forward to the extent that a surplus need not be generated in the first few years. Otherwise a key finding of the simulation is that in the absence of outside finance there are significant and virtually unavoidable trade-offs between human well-being and reduced deforestation. Deforestation taxes can reduce land clearing and raise critical revenue for local public policy initiatives (such as education and non-agricultural job opportunities), but only if the revenue from the tax remains in local hands and is used appropriately; otherwise, environmental taxes will unambiguously reduce well-being. In the simulation, the sequencing of policies is critical for long run successful outcomes, and policies can have unexpected nonlinear effects both independently and in combination. Policy complementarities emerge, such as that between conservation payments and deforestation taxes, for example, as these policies have different effects on the government's budget and thus indirectly on household decisions and overall well-being.

We have made *Sim Pachamama* publicly available and open-source, so parameters can be adjusted or re-calibrated to other environments (see information in Appendix 1). In addition the hybrid ABM structure allows users the opportunity for adding new elements at relatively low cost in the future; for example in its current form we have not yet introduced any explicitly spatial interactions, but through the ABM shell this is a straightforward extension of the model.

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Tables

Table 1: summary statistics from the 2010-2011 Bolivian household survey

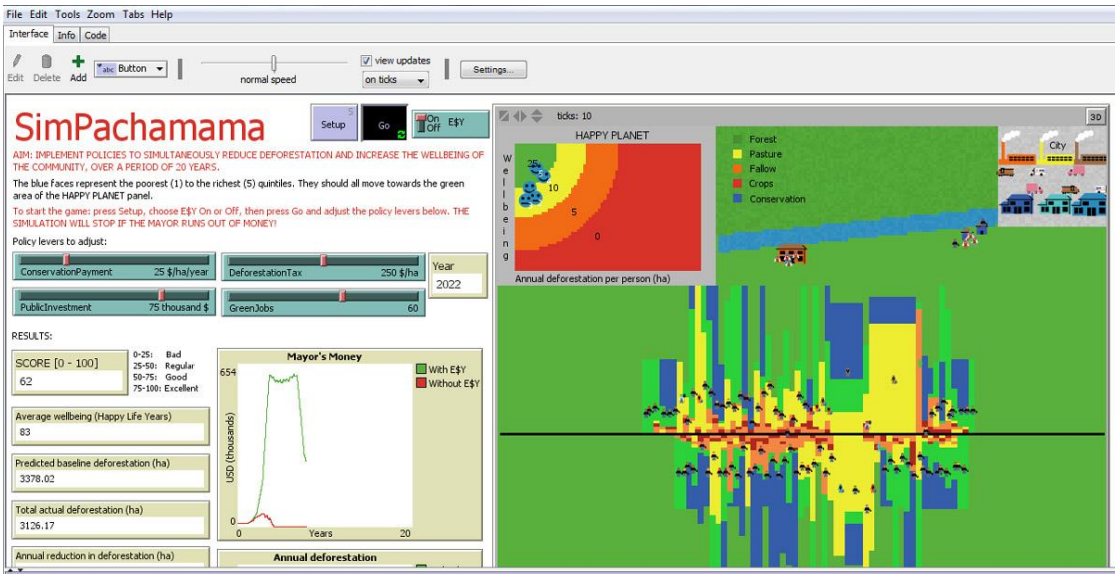
	Rurrenabaque - Reyes			Rurrenabaque-Yucumo			San Buenaventura - Ixiamas		
Variable	**Obs**	**Mean**	**Std.Dev.**	**Obs**	**Mean**	**Std.Dev.**	**Obs**	**Mean**	**Std.Dev.**
Property Size (ha)	45	302.14	986.34	149	53.16	102.65	96	88.76	175.92
Forest (ha)	45	185.77	639.48	149	25.94	62.24	96	47.27	102.60
Agriculture (ha)	45	2.00	2.61	149	2.94	3.02	96	3.48	3.36
Pasture (ha)	45	86.95	416.24	149	17.31	46.57	96	25.20	104.51
Fallow (ha)	45	27.43	79.87	149	6.98	9.90	96	12.81	23.67
Distance to Community (km)	45	2.63	3.92	149	1.68	2.49	96	3.64	4.69
Family Size	45	5.67	2.89	149	5.65	2.53	96	5.59	2.10
Share of land deforested	45	0.64	0.37	149	0.60	0.28	96	0.42	0.27
Sells Produce (1=Yes, 0=No)	45	1.00	0.00	149	1.00	0.00	96	0.96	0.20
Share Income from Agriculture	45	0.50	0.45	149	0.52	0.45	96	0.44	0.42
Hires Labour (1=Yes, 0=No)	45	0.53	0.50	149	0.62	0.49	96	0.76	0.43
Years in community	45	16.64	17.39	149	15.87	9.52	96	21.52	14.57
Net Income (Bs)	45	305.82	481.60	149	309.63	568.00	96	285.62	342.79
Annual Deforestation (ha)	45	3.14	5.31	149	1.92	1.63	96	2.64	2.39
Communal land title (1=Yes, 0=No)	45	0.49	0.51	149	0.26	0.44	96	0.53	0.50
Private land title (1=Yes, 0=No)	45	0.40	0.50	149	0.36	0.48	96	0.43	0.50
Land titling in process (1=Yes, 0=No)	45	0.11	0.32	149	0.38	0.49	96	0.04	0.20

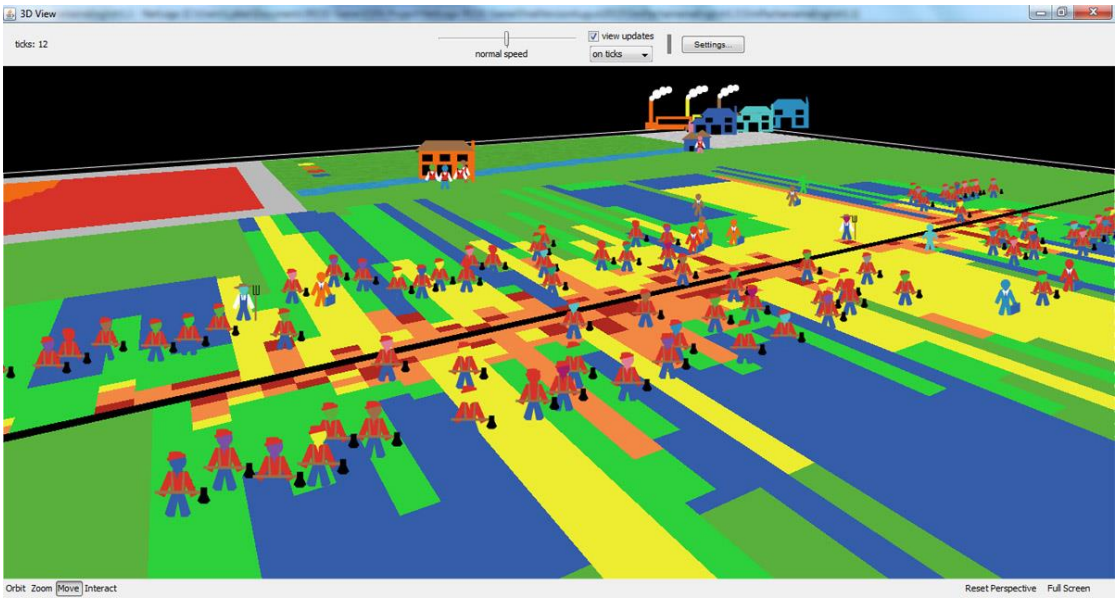
Source: Leguia, D., A. Malky & J. C. Ledezma (2011)

Appendices

Appendix 1

2D and 3D screen shot from the model, called 'SimPachamama'





Download Sim Pachamama at:
http://www.inesad.edu.bo/simpachamama/download/

Appendix 2: The Household Model

For a separable household the problem is simple profit maximisation. With output price, *p,* normalised to 1, and labour receiving the market wage *w*, the problem can be described as follows. Households at time *t* have some already converted land from the previous period: $H_{t-1} \geq 0$. Cultivation in excess of this requires deforestation to take place. Labour and land are assumed to be the only inputs to production. Labour used on previously cultivated land is given by:

$$L^f = L^{fo} + L^{fh}$$

Labour used on deforested land is given by:

$$L^D = L^{Do} + L^{Dh}$$

Labour is applied to converted and unconverted land in a fixed relationship depending on distance from the road. Labour can be provided from the households own family $\left(L^{fo}\right)$ or hired $\left(L^{fh}\right)$. Note that if L^{fo*} is greater than the constraint H_{t-1} allows, then it is possible that $L^{fh} = 0$, and hired labour is then only used for deforestation. Production, *X*, is linear in already converted land used, H , and deforested land, *D*, and a technology parameter *A:*

$$X = AH + AD$$

Profit is therefore written as follows:

$$\Pi = p\left(AH + AD\right) - w\left(L^f + L^D\right),$$

Labour and land have a fixed relationship such that the amount of labour required for each additional hectare is given by:

$$\frac{\partial L^f}{\partial H} = \left(1 + q\right)H$$

Given this, an expression for *H* as a function of labour can be obtained via integration. With the width of the plot fixed, *H* is a measure of distance and so the parameter *q* can be interpreted as a distance cost of labour (see also Angelson, 1999). As discussed in the text, this distance cost changes when deforestation is privately optimal and becomes *q + s*. Profit maximisation proceeds in two steps:

Step 1: solve for H^*

$$\max_{L^f} \mathrm{P}$$

subject to the constraint that labour used on converted land is limited by the amount of land converted in the previous period (H_{t-1}):

$$H_{t-1} \geq \left(\frac{2L^f}{1+q} \right)^{\frac{1}{2}}$$

With the functional forms given the first order conditions obtain L^{f*} :

$$L^{f*} = \left(\frac{(1+q)w}{Ap} \right)^2 \frac{(1+q)}{2}$$

The split of L^f between L^{fo} and L^{fh} is determined by the utility maximisation problem and the choice of 'leisure'. The first order conditions of this problem obtain that the MRS of labour and consumption (the shadow price of labour) should equal the wage rate (given that the price of output =1) that is l^* : $U_l / U_c = w$. If the solution in step 1 is interior then the household stops there. If the land constraint is binding the household moves to step 2. Otherwise it is easy to show that with preferences given by $u(c,l) = c^\alpha l^\beta$ the solutions for l and c become:

$$l^* = \frac{\beta\left(\frac{(pA)^2}{2(1+q)w} + wT\right)}{(\alpha+\beta)w}, \quad c^* = \frac{\alpha\left(\frac{(pA)^2}{2(1+q)w} + wT\right)}{(\alpha+\beta)p},$$

Where $\Pi^* = \frac{(pA)^2}{2(1+q)w}$ is the profit function.

Step 2:

If $\frac{A}{(1+q)}\left(\frac{2L^f}{1+q}\right)^{-\frac{1}{2}} - w > 0$: move to step 2 and consider deforestation. This means that $L^{f0} = L^f = L^f_{t-1}$, i.e. the labour associated with the constraint on H_{t-1}. If not then the solutions become:

$$l^* = \frac{\beta\left(\Pi^*(p,w) + wT\right)}{(\alpha+\beta)w}, \quad c^* = \frac{\alpha\left(\Pi^*(p,w) + wT\right)}{(\alpha+\beta)p}$$

$\Pi^*(p,w,)$ is once again the profit function. Given the leisure decision, we can now define the labour allocations using the constraints.

Households are endowed with overall time T . Time is divided between on farm work, L^{fo} , 'leisure', l , and off-farm labour L^m . The constraint can be written: $T = L^{fo} + L^m + l$. L^{fo*} can be calculated as the residual:

$$L^{fo*} = T - l - L^m$$

and:

$$L^{f*} = T - l - L^m + L^{fh*}$$

or more intuitively:

$$T - L^{f*} - l^* = L^m - L^{fh*}$$

where the right hand side is the net off-farm labour supply (the difference between what is rented in and what is rented out).

Step 2: The Deforestation Decision

If it turns out that there is no internal solution $(m > 0)$, the deforestation decision is evaluated as in previous models. Where L^f_{t-1} is the labour requirement for production on the previously cultivated land H_{t-1}, and since $\frac{\partial L^D}{\partial D} = \frac{1}{2}(1+q+s)D + (1+q)H_{t-1}$ the first order conditions for an interior solution are:

$$D^* : \frac{\partial \Pi}{\partial L^D} = 0$$

Which leads to D^*:

$$D^* = \frac{\frac{Ap}{w} - (1+q)H_{t-1}}{(1+q+s)}$$

These are the analytical solutions. This defines the total amount of labour applied to land, L as follows:

$$\begin{aligned} L^* &= L^{f*} + L^{D*} \\ &= L^f_{t-1} + L^{D*} \\ &= L^o + L^h \end{aligned}$$

The last line shows that labour is split between own and hired labour. It is possible to identify each of these allocations once the utility maximisation problem has solved for the leisure decision.

Appendix 3: The Cattle Model

Specifically, assume the instantaneous stock of cattle is given by $X(t)$, the 'harvest' (the amount of cattle sold) is given by $R(t)$, which can be sold at price p_C. Labour costs for harvesting cattle are given by $c(R, X) = cRX$, and are determined by the size of the herd X as well as the amount that is harvested, R. The cost parameter is c.

Step 1: The intensive decision
In the first step the technological decision is conditioned on the biological/agronomic constraints of land and cattle. We assume that households are separable profit maximisers who understand the intertemporal nature of the stocking decision and undertake a dynamic optimisation.

The intensive decision is a per-hectare decision. Diminishing returns to land are modelled by assuming that livestock follow a standard logistic growth function of the form:

$$G(X, H) = \alpha X + \beta X^2,$$

with growth parameters $\alpha > 0$ and $\beta < 0$ where α is the "intrinsic growth rate" and $-a/b$ is the carrying capacity of a hectare of land. This represents the production function of livestock. Different technologies would be reflected by different values for these growth parameters, leading to different reproductive growth and carrying capacities.

The dynamic profit maximisation problem for the household/farm is therefore:

$$\max_R \int_0^T p_c R(t) - cR(t)X(t)\exp(-\delta t)dt,$$

subject to constraint on the initial stock and the livestock growth dynamics:

$$X(0) = X_0$$
$$\dot{X} = G(X) - R$$

The current value Hamiltonian of this dynamic problem:

$$H(R; X) = p_c R - cRX + \lambda(\alpha X + \beta X^2 - R)$$

Appendix 3b below shows that the steady state solution to this problem for R and X is given by the positive root of the quadratic $aX + bX^2 + d = 0$:

$$X^* = \frac{a \pm \sqrt{a^2 - 4bd}}{2b}$$

And:

$$R^* = \alpha X^* + \beta X^{*2}$$

Where $a = (p_c 2b + cr - 2a)$, $b = -3\beta$ and $d = p_c(\alpha - r)$, are collections of the parameters. The numerical examples below illustrate the solution to this problem for households with differing livestock technology: an intensive farm and an extensive farm.

Step 2: The extensive decision

The Intensive decision determines the cattle per hectare, and the harvest rate. What remains is the extensive decision, that is, how much land (H) is used in the cattle operation. Step 2 of the model proceeds as follows.

For simplicity we assume linear relationships between land (H), labour used in livestock (L_C) and livestock harvest (R) of the form: $H = \theta X$, $L^C = \gamma X$, and $R = \phi L_C$. This assumption means that the problem effectively involves one decision variable. The value of θ, γ and ϕ can be determined by the solution to step 1: the intensive problem. E.g. the intensity of cattle (cattle per hectare) is q^{-1} which was determined in step 1.

Profit is derived from the revenue from harvest $(p_C R)$ minus the costs of variable inputs: land (H) and labour (L_C)[3], and fixed costs, F_C. The marginal costs of harvesting are assumed to be increasing, reflecting some diminishing returns to extensive production. This could be motivated by monitoring costs, costs of disease, etc., akin to the costs of distance in the agricultural model. We assume a cost curve for harvesting of the form $c(R) = cR^k$ where $k > 1$, and c is a constant, ensuring that $c'(.) > 0, c''(.) > 0$. Given the linear relationships above, the instantaneous profit maximisation problem can be written as:[4]

$$\max_H \Pi = p_C \frac{\phi\gamma}{\theta} H - c\left(\frac{\phi\gamma}{\theta} H\right)^k + p_H(\bar{H} - H) - w\frac{\gamma}{\theta} H - F_C$$

where the price p_H reflects the opportunity cost of land. The general solution to this problem is:

$$H^* = \frac{\theta}{\phi\gamma}\left(\frac{p_C - w\phi - p_H \frac{\phi\gamma}{\theta}}{kc}\right)^{\frac{1}{k-1}}, R^* = \frac{\phi\gamma}{\theta} H^*, L_C^* = \frac{\gamma}{\theta} H^*$$

This shows that H^* is inversely related to: i) the cattle per hectare (θ^{-1}) ; ii) the

[3]Other variables inputs can also be considered, such as feed.

[4] Note: $R = \frac{\phi\gamma}{\theta} H$, so as cattle per hectare increases (θ^{-1}) , so does the harvest. Also, $L_C = \frac{\gamma}{\theta} H$.

marginal cost parameters, c and k; iii) the harvest per unit of labour (γ); iv) the labour requirement per head of cattle; and lastly, v) H^* is negatively related to p_H, and this relationship is stronger if the cattle intensity is higher. Wages (w) and the price of land (p_H) affect land in slightly different ways. The effects on land translate linearly into aggregate harvest (R), labour requirements (L_C) and the total stock of cattle (X). The resource constraints associated with the dynamic intensity/harvest decision, when land is constrained, are embodied in the parameters q, g, and ϕ.

Appendix 3b: Solution to the Livestock Problem Step 1

The first order necessary conditions for an optimum are:

$$R^*: p_c - cX - \lambda = 0$$
$$\lambda^*: -\frac{\partial H}{\partial X} = \dot{\lambda} - r\lambda = cR - \lambda(\alpha + 2\beta X)$$

The general solution becomes:

$$\frac{\dot{\lambda}}{\lambda} = r - (\alpha + 2\beta X) + \frac{cR}{\lambda}$$
$$= r - (\alpha + 2\beta X) + \frac{cR}{p_c - cX}$$

In the steady state where $\dot{\lambda} = 0$ and $\dot{X} = 0$, the steady state stock and harvest rate are determined by the following equations. In this is a quadratic equation of the following form:

$$aX + bX^2 + d = 0$$

where:

$$a = (p_c 2b + cr - 2a)$$
$$b = -3b$$
$$d = p_c(a - r)$$

The solution is then:

$$X^* = \frac{a \pm \sqrt{a^2 - 4bd}}{2b}$$

The steady state solution for R^* is then simply:

$$R^* = aX^* + bX^{*2}$$

Numerical example 1 (The Extensive Farmer): For parameter values: $\alpha = 0.25$, $\beta = -0.1$, $r = 0.05$, $p = 600$, $c = 0.1$, the carrying capacity per hectare 2.5, the maximum sustainable yield is 1.25 per hectare and the optimal

stock and harvest are $X^* = 1.198$ $R^* = 0.156$

Numerical Example 2 (The Intensive Farmer): For parameter values: $\alpha = 0.5, \beta = -0.1,\ r = 0.05,\ p = 500,\ c = 0.10$. Here the parameters of the growth function differ now, as do the costs of harvest/management. In short intrinsic growth is higher $(\alpha = 0.5)$, and hence MSY and Carrying Capacity are respectively 2.5 and 5, that is they have doubled. The cost parameter for harvest has increased to reflect greater cost of intensive activities: $c = 10$. Here the solution is: $X^* = 2.243$ $R^* = 0.618$

Appendix 3c: Solution to Livestock Problem Step 2

The maximisation problem is:

$$\max_H \Pi = p_C \frac{\gamma}{q} H - c\left(\frac{\gamma}{q} H\right)^k + p_H\left(\bar{H} - H\right) - w\frac{\gamma}{q} H - F_C$$

The Solution is:

$$H^* = p_C \frac{\gamma}{q} - \frac{\gamma}{q} kc\left(\frac{\gamma}{q} H\right)^{k-1} - \frac{w\gamma}{q} = 0$$

From which it is easy to derive the remaining solutions.

Numerical Example 3 (The extensive decision):

The solution to the intensive problem provides the input to the extensive problem. From the linear relationships between labour, harvest, land and stock we have: $H = \theta X \Rightarrow \theta^{-1} = X / H$, that is, the number of cattle per hectare. The solution for the intensive and extensive farmers respectively are: $1/\theta^I = 2.24,\quad 1/\theta^E = 1.2$. Furthermore, we know that $R = \phi\gamma X \Rightarrow \phi\gamma = R / X$, which in the intensive and extensive cases are given by: $\phi^I \gamma^I = 0.276,\quad \phi^E \gamma^E = 0.130$. So the aggregate harvest is lower for the extensive rancher. The unknown parameters are γ and γ which indicate the labour requirement per head of cattle, and the labour requirement per unit of harvest. The choice of one automatically defines the other. Defining $\gamma^I = 0.02$, $\gamma^E = 0.04$ which means that on average 1 person can look after 50 cattle in the intensive case and 25 cattle in the extensive case. This means that: $\phi^I = 13.8,\ \phi^E = 3$. This implies that 1 person can harvest nearly 14 cattle in the intensive case, and 3 cattle in the extensive case. This leads to the following solutions.

Extensive Cattle farming:
For parameter values:

$c = 10. k = 1.5. \gamma = 0.04, \phi = 3\ \theta = 0.833, p_H = 0, p_C = 300\ w = 30$.

The solutions are: $H^* = \frac{q}{jg}\left(\frac{p_C - wj - p_H \frac{jg}{q}}{kc}\right)^{\frac{1}{k-1}} = 1360.6\ \ R^* = \frac{\phi\gamma}{\theta} H^* = 196$

$L_C^* = \frac{\gamma}{\theta} H^* = 65.35\ \ X^* = \frac{1}{\theta} H^* = 1633$. This is a big farm, with over 1000 cattle.

Intensive Cattle farming:

With all parameters identical except $\gamma = 0.02, \phi = 13.8, \theta = 0.45, p_C = 700$,

the solutions are:[5] $H^* = 592.7\ R^* = 363.5\ L_C^* = 26.32\ X^* = 1317.1$

This is a big farm, with over 1000 cattle, but more intensively farmed.

[5] For a maximum it must be the case that whatever is raised to the power $\frac{1}{k-1}$, must be positive. This is satisfied if $p_C - w\phi - p_H \frac{\phi\gamma}{\theta} > 0$