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Dominik Groll

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JEL classification: F33, F41, E52.

Dominik Groll
Kiel Institute for the World Economy
24100 Kiel, Germany
Telephone: +49 431 8814 266
E-mail: dominik.groll@ifw-kiel.de

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Monetary Union and Macroeconomic Stabilization

Dominik Groll *

Kiel Institute for the World Economy, Hindenburgufer 66, 24105 Kiel, Germany.

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It is conventionally held that countries are worse off by forming a monetary union when it comes to macroeconomic stabilization. However, this conventional view relies on assuming that monetary policy is conducted optimally. Relaxing the assumption of optimal monetary policy not only uncovers that countries do benefit from forming a monetary union under fairly general conditions. More importantly, it also reveals that a monetary union entails the inherent benefit of stabilizing private-sector expectations about future inflation. As a result, inflation rates are more stable in a monetary union.

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* E-mail: dominik.groll@ifw-kiel.de. Phone: +49 431 8814 266.

1 Introduction

When do countries benefit from forming a monetary union? This question is at least as old as the Optimum Currency Area (OCA) literature initiated by Mundell (1961). One of the key insights of this literature is that for asymmetric countries to benefit from forming a monetary union, prices and wages have to be flexible and production factors have to be mobile. More recently, the New Keynesian literature, by using dynamic stochastic general equilibrium (DSGE) models, has come to the consensus that, from the perspective of macroeconomic stabilization, forming a monetary union makes countries generally worse off in terms of welfare.[1] This is because countries relinquish one of their most important policy instruments for macroeconomic stabilization, namely the short-term nominal interest rate controlled by national central banks.

However, this consensus is based on the assumption that the central bank conducts monetary policy optimally. While the concept of optimal monetary policy constitutes an important theoretical benchmark from a normative perspective, its practical limitations are widely acknowledged.[2] More importantly, the assumption of optimal monetary policy in the OCA context has led to the conclusion that the exchange rate regime is *irrelevant* for stabilization policy if optimal monetary policy is complemented by optimal fiscal policy (Adao, Correia, and Teles, 2009). Thus, it seems at least debatable whether optimal monetary policy is the best modeling choice to describe the behavior of central banks when one wants to know the conditions under which countries benefit from forming a monetary union.

Against this background, I take a different approach in this paper by assuming that monetary policy follows Taylor-type interest rate rules, according to which it responds only to macroeconomic variables that are observable, such as inflation or output. My main finding is that in the standard two-country New Keynesian DSGE model, in which monetary policy follows interest rate rules, countries may gain in welfare by forming a monetary union. The gain in welfare comes from a higher stability of inflation rates, which outweighs the costs of higher output-gap and terms-of-trade-gap instability. Whether countries gain in welfare depends strongly on the degree of price stickiness. When prices are relatively sticky, countries are better off forming a monetary union; when prices are relatively flexible, countries are better off maintaining a flexible exchange rate.

Two effects are responsible for this higher stability of inflation rates. First, the benefit of maintaining a flexible exchange rate diminishes as prices become stickier, since the nominal exchange rate inherits the stickiness of goods prices. As a result, an increasing degree of price stickiness reduces the effectiveness of the nominal exchange rate as a stabilization mechanism. Second, forming a monetary union entails an inherent benefit.[3] Since the nominal exchange rate is fixed, the terms of trade and, therefore, the inflation rates display an inertial or history-dependent behavior. This history dependence has the advantage of affecting the inflation expectations of price setters in such a way as to lower the responsiveness of inflation to changing economic conditions. The

[1] See surveys by Corsetti (2008), Dellas and Tavlas (2009), and Beetsma and Giuliodori (2010).

[2] For example, the central bank needs to be able to observe the flexible-price equilibrium. For more practical shortcomings of optimal monetary policy, see Gali (2008, Ch. 4.3.2).

[3] "Inherent" refers to the fact that the benefit is not modeled explicitly, like a reduction in transaction costs, but emerges from within the model.

higher the degree of price stickiness is, the stronger this effect is. As a result, inflation rates are more stable in a monetary union.

This second effect corresponds to an effect that is well-known from the analysis of optimal monetary policy in a closed economy. There, optimal monetary policy *under discretion* is inferior to optimal monetary policy *under commitment* because the former does not influence the inflation expectations of price setters in a favorable way. It suffers from the so-called stabilization bias. In contrast, optimal monetary policy under commitment induces history dependence into the economy and, therefore, exploits the fact that price setters are forward-looking. The intuition in this paper is completely analogous. Forming a monetary union may be superior to maintaining a flexible exchange rate because fixing the exchange rate induces history dependence.

This benefit, which manifests itself in a higher stability of inflation rates and which is related to the *stabilization bias*, obtains in addition to the benefit of eliminating a potential *inflation bias*, which is stressed by Alesina and Barro (2002) and Cooley and Quadrini (2003).[4] Whereas the latter benefit has been acknowledged in the literature (e.g., Dellas and Tavlas, 2009; Beetsma and Giuliodori, 2010), the former still seems to be unknown.

It is important to realize that both effects described above are endogenous to the model. The first effect is due to the presence of the uncovered interest parity condition on the one hand and monetary policy following Taylor-type interest rate rules on the other hand.[5] The second effect is due to the fact that price setters are forward-looking in the presence of nominal price rigidities. All these features belong to the core of new open economy macroeconomics (NOEM) models and, therefore, are present also in many medium-to-large-scale models that are built around this core.[6]

The other important finding of this paper is that whether forming a monetary union is beneficial or not depends heavily on the way monetary policy is conducted. When monetary policy responds to inflation aggressively or when it implements a high degree of interest rate smoothing, then maintaining a flexible exchange rate is superior. Thus, it is the quality of monetary policy that is crucial for the welfare ranking between the monetary union and the flexible exchange rate regime. Since monetary policy is more powerful under the flexible exchange rate regime, it is also more harmful when not conducted properly. This is because the quality of monetary policy is reinforced by the nominal exchange rate. In this sense, a monetary union provides a hedging mechanism against monetary policy mistakes.

Clearly, the finding that countries benefit from forming a monetary union when prices are relatively sticky but not when prices are relatively flexible stands in contrast to the predictions of the traditional OCA theory. Probably the most important reason for this discrepancy is the fact that expectations are treated as endogenous in New Keynesian models, unlike in the theoretical framework of the traditional OCA literature, in which expectations are treated as exogenous. Since the inherent benefit

[4] Giavazzi and Pagano (1988)'s "advantage of tying one's hands" follows the same logic, although they refer to the former European Monetary System (1979-1999).

[5] Notably, the uncovered interest parity condition need not hold exactly for this effect to exist. It suffices for the interest rates and the nominal exchange rate to be linked.

[6] The introduction of nominal price rigidities in the spirit of Calvo (1983) into NOEM models goes back to Kollmann (2001), Gali and Monacelli (2005), and Clarida, Gali, and Gertler (2002).

of monetary unions works through expectations, this channel is naturally missing in models without such an expectational feedback mechanism.[7]

This paper is related along several dimensions to the New Keynesian literature that analyzes the conditions under which countries benefit from forming a monetary union. In this literature, only a few studies have considered an environment without optimal monetary policy: Devereux (2004), Dellas and Tavlas (2005), Dellas (2006), and Ferreira-Lopes (2010). The models used in these studies, as well as their findings are, on the one hand, quite diverse. On the other hand, none of these studies addresses the inherent benefit of monetary unions, the role of the degree of price stickiness, nor the closely related issue of the inherited stickiness of the exchange rate, all of which are crucial for the welfare ranking between the monetary union and the flexible exchange rate regime.

Several studies have introduced explicit benefits of monetary unions to create a counterpart to the cost of giving up national monetary policy as a stabilization device. Such explicit benefits of monetary unions include the elimination of shocks to the uncovered interest parity condition (Kollmann, 2004), the gain in potential output (Ca'Zorzi, De Santis, and Zampolli, 2005), the gain in central bank credibility (Clerc, Dellas, and Loisel, 2011), and the possibility of higher consumption risk sharing across countries (Ching and Devereux, 2003). In contrast, no explicit benefits are introduced into the model employed in this paper. The benefit of stabilizing inflation expectations is inherent to monetary unions as a result of a fixed nominal exchange rate.

This paper is also related to Monacelli (2004).[8] Among other things, he finds that in a small open economy a fixed exchange rate regime induces inertia into the economy. On the one hand, I show that this benefit carries over to a two-country environment and is inherent to monetary union regimes as well.[9] On the other hand, I show that it does not hinge upon the stationarity of the price level, as stressed by Monacelli (2004). Stationarity of the price level is a special feature of the small open economy environment and does not carry over to a two-country setting employed here. Also, Monacelli (2004) does not address the role of the degree of price stickiness, the related issue of the inherited stickiness of the exchange rate, nor the role of monetary policy.

The rest of this paper is organized as follows. Section 2 outlines briefly the structure of the model. Section 3 provides important analytical results in the case of symmetric countries and presents the welfare results graphically. Section 4 presents the results in the case of asymmetric countries. Section 5 relates the results to the traditional OCA theory. Section 6 concludes.

[7] See King (1993) for a critical assessment of the Old-Keynesian, IS-LM models with respect to their treatment of expectations.

[8] For a small open economy, a fixed exchange rate regime may dominate a flexible exchange rate regime with optimal monetary policy under discretion. A flexible exchange rate regime with optimal monetary policy under commitment, however, always dominates the other two regimes. Comparing the same three regimes, Soffritti and Zanetti (2008) come to a different conclusion, namely that a fixed exchange rate regime fares worse than the two flexible exchange rate regimes. One possible explanation for the different finding could be the different weight attached to the output-gap variance relative to the weight attached to the inflation variance in the welfare loss function, which is ad hoc in both studies. Another explanation could be the different assumption about whether the rest of the world is also subject to shocks or not.

[9] In general, fixed exchange rate regimes and monetary union regimes do not coincide. This depends on how the fixed exchange rate regime is implemented.

2 Model

The model I use is a completely standard two-country New Keynesian DSGE model and thus I keep the description very brief. It features two international monetary regimes:

1. A monetary union (MU) regime: Both countries share the same currency. A common monetary policy governs the common nominal interest.

2. A flexible exchange rate (FX) regime: Each country maintains its national currency and conducts its own, independent monetary policy. Nominal interest rates are country-specific. The nominal exchange rate between the two currencies is flexible.

The model is described in detail in Benigno (2004) and in Benigno and Benigno (2008) and it includes a microfounded, linear-quadratic welfare measure. Under both regimes, the model economy features two countries with trade in consumption goods (as opposed to trade in intermediate goods). Consumption preferences are of the Cobb-Douglas type and are, in addition, identical across countries, i.e., there is no home bias in consumption. These preferences imply that risk sharing is perfect in the sense that consumption is equal across countries at all times. Purchasing power parity holds, i.e., the real exchange rate is constant. While these assumptions are clearly restrictive, they greatly simplify the analysis, and relaxing them to allow for a home bias in consumption (i.e., no purchasing power parity and a variable real exchange rate) does not alter the findings significantly. The only factor of production is labor, which is immobile between countries. The only rigidity is the nominal price rigidity in the spirit of Calvo (1983).

Under the FX regime, prices are set in the currency of the producer's country ("producer currency pricing"), i.e., the producer does not discriminate the price between countries. The nominal exchange rate converts the price into foreign currency, i.e., the law of one price holds and exchange rate pass-through is complete. Given the same consumption preferences as under the MU regime, purchasing power parity holds as well. The nominal exchange rate is determined by the uncovered interest parity.

In both regimes, monetary policy is conducted via Taylor-type interest rate rules. Importantly, I assume that monetary policy is not able to observe the flexible-price equilibrium of the economy, in particular the flexible-price interest rate and flexible-price output. Thus, monetary policy reacts to inflation and to output (deviation from the steady state), not to the output gap (deviation from flexible-price output). The only shocks considered are country-specific productivity shocks. However, the findings are robust with respect to other shocks, such as government-spending shocks or cost-push shocks.[10]

2.1 Model equations

The equations of the complete log-linearized model are displayed below (for the full derivation, see Appendices B and C). Deviations of the logarithm of a variable X_t from its steady state are denoted by $\tilde{X}_t$ under flexible prices and by $\hat{X}_t$ under sticky prices.[11] Variables and parameters are defined in Table 1 and Table 2, respectively.

[10] In fact, under cost-push shocks the case for a monetary union becomes even stronger.
[11] Notation is adopted from Benigno (2004).

C_t	Consumption (identical across countries)
Y_t^i	Output of country $i = H, F$
Y_t^W	World output (weighted average of country-specific output)
π_t^i	Producer price inflation in country $i = H, F$
π_t^W	World inflation (weighted average of country-specific inflation)
π_t^R	Inflation differential between the two countries $\pi_t^F - \pi_t^H$
R_t^i	Nominal interest rate in country $i = H, F$
R_t	Nominal interest rate in monetary union
T_t	Terms of trade
S_t	Nominal exchange rate
$\overline{Y}_t^i$	Productivity shock in country $i = H, F$
ν_t^i	White noise process in country $i = H, F$

Table 1: Variables

ρ	Inverse of elasticity of intertemporal substitution in consumption
n	Country size measured by population
β	Discount factor
η	Inverse of elasticity of producing the differentiated good
σ	Elasticity of substitution between differentiated goods
α^i	Probability of not being able to reset the price in country $i = H, F$
ϕ_π	Inflation coefficient in interest rate rule
ϕ_Y	Output coefficient in interest rate rule
ϕ_R	Interest rate smoothing coefficient in interest rate rule
ρ_i	Persistence of productivity shock in country $i = H, F$
k_C^i	$k_C^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i}\frac{\rho+\eta}{1+\sigma\eta}$
k_T^i	$k_T^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i}\frac{1+\eta}{1+\sigma\eta}$

Table 2: Parameters

2.1.1 Flexible-price equilibrium under both regimes

Under completely flexible prices, the model equations are identical for both the FX and MU regime and are given by

$$\tilde{C}_t = \frac{\eta}{\rho+\eta}\overline{Y}_t^W \tag{2.1}$$

$$\tilde{T}_t = -\frac{\eta}{1+\eta}\overline{Y}_t^R \tag{2.2}$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho+\eta}\overline{Y}_t^W \tag{2.3}$$

$$\overline{Y}_t^W = n\overline{Y}_t^H + (1-n)\overline{Y}_t^F \tag{2.4}$$

$$\overline{Y}_t^R = \overline{Y}_t^F - \overline{Y}_t^H \tag{2.5}$$

$$\overline{Y}_t^i = \rho_i \overline{Y}_{t-1}^i + \nu_t^i, \tag{2.6}$$

with $i = H, F$.

2.1.2 Sticky-price equilibrium under the MU regime

Under sticky prices, the model equations for the MU regime are given by

$$E_t\hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}(\hat{R}_t - E_t\pi^W_{t+1}) \tag{2.7}$$
$$\hat{Y}^H_t = (1-n)\hat{T}_t + \hat{C}_t \tag{2.8}$$
$$\hat{Y}^F_t = -n\hat{T}_t + \hat{C}_t \tag{2.9}$$
$$\pi^H_t = (1-n)k^H_T(\hat{T}_t - \tilde{T}_t) + k^H_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^H_{t+1} \tag{2.10}$$
$$\pi^F_t = -nk^F_T(\hat{T}_t - \tilde{T}_t) + k^F_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^F_{t+1} \tag{2.11}$$
$$\hat{T}_t = \hat{T}_{t-1} + \pi^F_t - \pi^H_t \tag{2.12}$$
$$\hat{R}_t = \phi_R\hat{R}_{t-1} + (1-\phi_R)(\phi_\pi\pi^W_t + \phi_Y\hat{Y}^W_t) \tag{2.13}$$
$$\pi^W_t = n\pi^H_t + (1-n)\pi^F_t. \tag{2.14}$$

2.1.3 Sticky-price equilibrium under the FX regime

The model equations for the FX regime are given by

$$E_t\hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}\left(n(\hat{R}^H_t - E_t\pi^H_{t+1}) + (1-n)(\hat{R}^F_t - E_t\pi^F_{t+1})\right) \tag{2.15}$$
$$\hat{Y}^H_t = (1-n)\hat{T}_t + \hat{C}_t \tag{2.16}$$
$$\hat{Y}^F_t = -n\hat{T}_t + \hat{C}_t \tag{2.17}$$
$$\pi^H_t = (1-n)k^H_T(\hat{T}_t - \tilde{T}_t) + k^H_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^H_{t+1} \tag{2.18}$$
$$\pi^F_t = -nk^F_T(\hat{T}_t - \tilde{T}_t) + k^F_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^F_{t+1} \tag{2.19}$$
$$\hat{T}_t = \hat{T}_{t-1} + \pi^F_t - \pi^H_t + \Delta\hat{S}_t \tag{2.20}$$
$$E_t\Delta\hat{S}_{t+1} = \hat{R}^H_t - \hat{R}^F_t \tag{2.21}$$
$$\hat{R}^H_t = \phi_R\hat{R}^H_{t-1} + (1-\phi_R)(\phi_\pi\pi^H_t + \phi_Y\hat{Y}^H_t) \tag{2.22}$$
$$\hat{R}^F_t = \phi_R\hat{R}^F_{t-1} + (1-\phi_R)(\phi_\pi\pi^F_t + \phi_Y\hat{Y}^F_t). \tag{2.23}$$

2.2 Model description

Consumption is equal across countries at all times and is described by only one Euler equation, equation (2.7) under the MU regime and equation (2.15) under the FX regime. The only difference between the two Euler equations is that under the MU regime the nominal interest rate is common to both countries. The structure of aggregate demand is the same under both regimes and given by equations (2.8), (2.9), (2.16), and (2.17). Also, the country-specific New Keynesian Phillips curves are the same under both regimes and are given by (2.10), (2.11), (2.18), and (2.19). In contrast to a closed-economy framework, not only the consumption gap but also the terms of trade gap (difference between sticky-price and flexible-price terms of trade) matters for producer price inflation.[12] The terms-of-trade identity is given by (2.12) under the MU regime and by (2.20) under the FX regime, the difference between the two being the

[12]Note that the consumption gap is equal to the world output gap: $\hat{C}_t - \tilde{C}_t = \hat{Y}^W_t - \tilde{Y}^W_t$. Accordingly, the New Keynesian Phillips curves can be expressed in terms of the world output gap as well.

presence of the nominal exchange rate in the latter. Equation (2.21) is the uncovered interest parity condition. The expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, monetary policy is conducted via Taylor-type interest rate rules, given by equation (2.13) under the MU regime and by equations (2.22) and (2.23) under the FX regime.

Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only, given by equations (2.1), (2.2), and (2.3). Since money is neutral under flexible prices, the international monetary regime does not affect real variables, which therefore behave identically under both monetary regimes.

2.3 Welfare loss function

The welfare analysis follows the logic of the familiar linear-quadratic approach, according to which the log-linear model equations are used to evaluate a quadratic welfare loss measure (Woodford, 2003). The world welfare loss function is given by the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion.[13] Throughout the paper, it is assumed that the distortion induced by monopolistic competition is completely offset by an appropriate subsidy (see Appendix D for the full derivation). Thus,

$$\begin{aligned} W_t = -\frac{1}{2}\Bigg(& (\rho+\eta)\operatorname{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\operatorname{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\operatorname{var}\pi_t^H \\ & + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\operatorname{var}\pi_t^F\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{2.24}$$

As in the closed economy, the welfare loss depends on the inflation rate and the consumption gap.[14] In the open economy, the welfare loss depends additionally on the terms of trade gap. Intuitively, when the terms of trade deviate from the terms of trade that would prevail under flexible prices, the resulting allocation of production across countries is inefficient due to the presence of price stickiness.

[13] Computing country-specific welfare would complicate the calculations significantly because more accurate approximations of the non-linear model equations would be necessary (Benigno and Woodford, 2005). This is beyond the scope of this paper.

[14] In the basic closed-economy framework, consumption usually equals output. Note also that the welfare loss function (2.24) can be expressed alternatively in terms of the world output gap or the country-specific output gaps (see equation D.72). The specification in terms of the consumption gap was chosen for analytical convenience.

In the special case when prices are equally rigid in both countries ($\alpha^H = \alpha^F = \alpha$), the welfare loss function simplifies to

$$W_t = -\frac{1}{2}\Bigg((\rho+\eta)\operatorname{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\operatorname{var}(\hat{T}_t - \tilde{T}_t) + \sigma(1+\sigma\eta)\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\left[\operatorname{var}\pi_t^W + n(1-n)\operatorname{var}\pi_t^R\right]\Bigg) + t.i.p. + O(\|\xi\|^3). \tag{2.25}$$

2.4 Calibration

The values for the baseline calibration are taken from Benigno (2004), except for the interest rate rule coefficients (Table 3). A value of 0.99 for the discount factor β implies a steady state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods σ implies a steady state markup of prices over marginal costs of 15 percent. A value of 0.75 for the probability of not being able to reset the price α^i implies an average duration of price contracts of 4 quarters. Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good η is calculated as

$$\eta = \epsilon_{wy} - \rho + \frac{1-\gamma}{\gamma}, \tag{2.26}$$

where ϵ_{wy} denotes the elasticity of the average real wage with respect to production and γ denotes the labor income share.

ρ	1/6	Inv. of elasticity of intertemporal substitution in consumption
n	0.5	Country size measured by population
β	0.99	Discount factor
η	0.67	Inv. of elasticity of producing the differentiated good
ϵ_{wy}	0.5	Production elasticity of average real wage
γ	0.75	Labor income share
σ	7.66	Elasticity of substitution between differentiated goods
α^i	0.75	Probability of not being able to reset the price
ϕ_π	1.5	Inflation coefficient in interest rate rule
ϕ_Y	0	Output coefficient in interest rate rule
ϕ_R	0	Interest rate smoothing coefficient in interest rate rule
ρ_i	0.9	Persistence of productivity shock
$\operatorname{var}\nu_t^i$	1	Variance of white noise process
$\operatorname{corr}(\nu_t^H, \nu_t^F)$	0	Correlation between country-specific white noise processes

Table 3: Baseline calibration

Under the baseline calibration, monetary policy responds to inflation ($\phi_\pi = 1.5$), but it does not react to output ($\phi_Y = 0$) and does not engage in interest rate smoothing ($\phi_R = 0$). I assume throughout the paper that all interest rate rule coefficients are identical across countries and regimes.

I consider a broad range of values for the parameters of the model to check for the validity of the results (Table 4). In particular, the interest rate rules will also feature a reaction to output and interest rate smoothing.

ρ	[0.1,1.1]	Inv. of elasticity of intertemporal substitution in consumption
n	[0.05,0.95]	Country size measured by population
β	[0.9,1.0]	Discount factor
η	[0.2,3.0]	Inv. of elasticity of producing the differentiated good
ϵ_{wy}	[0.2,1.2]	Production elasticity of average real wage
γ	[0.5,0.9]	Labor income share
σ	[5,25]	Elasticity of substitution between differentiated goods
α^i	[0.05,0.95]	Probability of not being able to reset the price
ϕ_π	[1.1,3.5]	Inflation coefficient in interest rate rule
ϕ_Y	[0,3]	Output coefficient in interest rate rule
ϕ_R	[0,0.95]	Interest rate smoothing coefficient in interest rate rule

Table 4: Parameter range

3 Results under symmetry

First, I conduct the analysis under the assumption that the two countries are symmetric (except for country size n). In particular, the degree of price stickiness is equal across countries. The world welfare loss function under symmetry, equation (2.25), is repeated here for convenience:

$$\begin{aligned} W_t = -\frac{1}{2}\Bigg(& (\rho+\eta)\operatorname{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\operatorname{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\left[\operatorname{var}\pi_t^W + n(1-n)\operatorname{var}\pi_t^R\right]\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{3.1}$$

It contains four components: the variance of the consumption gap $(\hat{C}_t - \tilde{C}_t)$, the variance of the terms of trade gap $(\hat{T}_t - \tilde{T}_t)$, the variance of the world inflation rate (π_t^W), and the variance of the inflation differential (π_t^R).

3.1 Analytical results

The analytical results in this subsection are crucial to understanding the main finding of the paper. I derive the recursive law of motion (RLOM) of the model equations for each monetary regime using the method of undetermined coefficients to obtain the analytical expressions for the variances contained in the welfare loss function. The derivations are based on the assumption that the degree of price stickiness and the persistence of productivity shocks are identical across countries ($\alpha^H = \alpha^F$ and $\rho_H = \rho_F$) and that monetary policy does not engage in interest rate smoothing ($\phi_R = 0$).

Fortunately, it is not necessary to derive the RLOM for the variables consumption and world inflation, since they both behave identically across monetary regimes. To see this for the MU regime, substitute out the nominal interest rate $\hat{R}_t$ in the Euler equation (2.7) by inserting the interest rate rule (2.13) and the equations for aggregate demand (2.8) and (2.9):

$$\rho E_t \hat{C}_{t+1} = (\rho + \phi_Y)\hat{C}_t + \phi_\pi \pi_t^W - E_t \pi_{t+1}^W. \tag{3.2}$$

The same equation is obtained completely analogously for the FX regime.

For world inflation, inserting the New Keynesian Phillips curves, which are identical across regimes, into the definition of world inflation $\pi_t^W = n\pi_t^H + (1-n)\pi_t^F$, where $\alpha^H = \alpha^F = \alpha$ due to symmetry and therefore $k_T^H = k_T^F = k_T$ and $k_C^H = k_C^F = k_C$, yields

$$\pi_t^W = k_C(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^W. \tag{3.3}$$

The reason why world inflation is the same under both the MU and the FX regimes is that the terms of trade vanish from the equation when the degree of price stickiness is equal across countries. The fact that both consumption and world inflation behave identically across monetary regimes implies that the variance of consumption and the variance of world inflation are identical as well. As a result, they do not produce differences in welfare across the two regimes.

For the remaining two variables, the terms of trade and the inflation differential, the reduced system of equations under the MU regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \tag{3.4}$$
$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \tag{3.5}$$

The reduced system of equations under the FX regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \tag{3.6}$$
$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta\hat{S}_t \tag{3.7}$$
$$E_t \Delta\hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \tag{3.8}$$

Equations (3.4) and (3.6) are obtained by subtracting the New Keynesian Phillips curve of country H from that of country F. Equations (3.5) and (3.7) are the terms-of-trade identities. Equation (3.8) is obtained by inserting the interest rate rules (2.22) and (2.23) and the equations for aggregate demand (2.16) and (2.17) into the uncovered interest parity condition (2.21).

The RLOM under the MU regime is, then, given by (see Appendix A for the entire derivation)

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \tag{3.9}$$
$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t, \tag{3.10}$$

with coefficients

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$
$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$
$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}.$$

The RLOM under the FX regime is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \tag{3.11}$$
$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \tag{3.12}$$
$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t, \tag{3.13}$$

with coefficients

$$b_1 = 0$$
$$b_2 = 0$$
$$b_3 = -1$$
$$c_1 = \frac{(\phi_\pi - \rho_H)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$
$$c_2 = \frac{(1 - \rho_H + \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$
$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}.$$

Consequently, the variances of the terms of trade gap and the variances of the inflation differential under each regime are given by

$$\operatorname{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[\frac{(1 + \rho_H b_1)c^2}{(1 - b_1^2)(1 - \rho_H b_1)} - \frac{2c}{1 - \rho_H b_1} + 1\right] \operatorname{var} \tilde{T}_t \tag{3.14}$$
$$\operatorname{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \operatorname{var} \tilde{T}_t \tag{3.15}$$
$$\operatorname{var}_{MU} \pi_t^R = \frac{2c^2(1 - \rho_H)}{(1 + b_1)(1 - \rho_H b_1)} \operatorname{var} \tilde{T}_t \tag{3.16}$$
$$\operatorname{var}_{FX} \pi_t^R = c_2^2 \operatorname{var} \tilde{T}_t \tag{3.17}$$
$$\operatorname{var} \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left(\frac{\eta}{1 + \eta}\right)^2 \left[\operatorname{var} \nu_t^H + \operatorname{var} \nu_t^F - 2 \operatorname{cov}(\nu_t^H, \nu_t^F)\right]. \tag{3.18}$$

Two important differences exist between the MU and FX regime. First, in contrast to the MU regime, there is no persistence in the terms of trade nor in the inflation differential under the FX regime ($b_1 = b_2 = 0$). Hence, once the shock has vanished, both variables return immediately to their steady state. This is due to the nominal exchange rate. Intuitively, the coefficient $b_3 = -1$ implies that, if the terms of trade were, for example, one percent below the steady state in the previous period, the nominal ex-

change rate would increase by one percent in the current period, so that the terms of trade are at steady state. Naturally, this mechanism is absent under the MU regime, since the nominal exchange rate is fixed. Thus, both the terms of trade and the inflation differential are inertial or history-dependent in the sense that they depend on the realization of the terms of trade in the previous period. While the inertia of the terms of trade in the context of a monetary union has been recognized before (e.g., Benigno, 2004; Pappa, 2004), it was regarded solely as a source of inefficiency in the economy. However, as will be shown below, the inertia of the terms of trade will also prove to be beneficial, namely from the perspective of stabilizing inflation expectations.

Second, in contrast to the MU regime, monetary policy is able to influence the terms of trade gap and the inflation differential under the FX regime. Technically, the variance of the terms of trade gap and of the inflation differential depend on the interest rate rule coefficients ϕ_π and ϕ_Y. Moreover, if monetary policy is extremely aggressive towards inflation under the FX regime ($\phi_\pi \to \infty$), the variance of the terms of trade gap and of the inflation differential converge towards zero (since $c_1 \to 1$ and $c_2 \to 0$). Thus, the efficient equilibrium can be approximated arbitrarily well, reducing the welfare loss to zero. In contrast, the variance of the terms of trade gap and of the inflation differential under the MU regime cannot be zero, and therefore the efficient equilibrium is not feasible.[15]

3.2 Price stickiness

The analytical expressions for the variances can be used to derive the condition under which world welfare is larger in one or the other monetary regime. Unfortunately, the resulting condition is a complex inequality that provides hardly any intuition. In the following, I thus compute the welfare losses numerically and display the results graphically. The deep parameters are calibrated according to the baseline calibration (Table 3), except for the parameters of interest, which take on a broad range of values (Table 4).

Whether the world welfare loss is higher in one than in the other monetary regime depends crucially on the Calvo parameter α, i.e., the degree of price stickiness in both economies (Figure 1). In both regimes, the world welfare loss is increasing in the degree of price stickiness.[16] When the degree of price stickiness is rather low, the world welfare loss is higher under the MU regime than under the FX regime. The countries are better off with their own currency and their own independent monetary policy. However, beyond a certain threshold ($\alpha \approx 0.5$), where the degree of price stickiness is rather high, the world welfare loss is higher under the FX regime than under the MU regime. The countries are better off forming a monetary union with one currency and one common monetary authority. Quantitatively, the difference in welfare between the two monetary regimes can be substantial. Under the baseline calibration, the welfare loss under the MU regime is roughly 40 percent lower than under the FX regime (0.8/1.3).

As described above, two components of the world welfare loss function (3.1) behave identically across monetary regimes and, therefore, cannot create welfare differences

[15] Thus, only the FX regime features the so-called divine coincidence (see Corsetti, Dedola, and Leduc (2011) for details on the divine coincidence in open economies).

[16] This feature is common to the closed-economy setup of the basic New Keynesian model, as in Gali (2008).

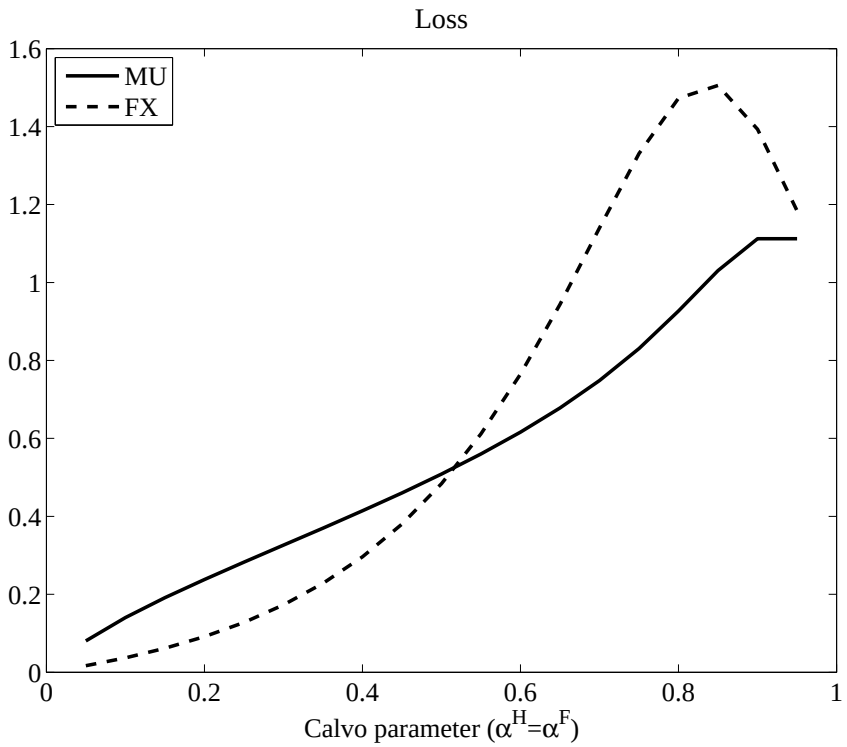

Figure 1: World welfare loss under various degrees of price stickiness ($\alpha^H = \alpha^F$)

across regimes: the consumption gap and the world inflation rate (Figure 2, upper and lower left panel). However, this does not hold for the terms of trade gap and the inflation differential (Figure 2, upper and lower right panel). The contribution of the terms of trade gap to the world welfare loss is higher under the MU regime than under the FX regime regardless of the degree of price stickiness.[17] This indicates that the MU regime entails costs. However, the contribution of the terms of trade gap is much smaller than the contribution of the inflation differential. This is due to the fact that agents attach by far the highest weight to inflation, which is traditionally the case in microfounded welfare measures derived from New Keynesian models. Therefore, the inflation differential is the key to understanding the above finding that the MU regime yields higher world welfare when prices are relatively sticky. In fact, the pattern in the lower right panel of Figure 2 closely resembles the pattern in Figure 1, with a similar threshold value of $\alpha \approx 0.5$. This indicates that the MU regime entails benefits.

The contribution of a component to the world welfare loss is the product of the variance of that component and its weight. The weight and variance of the inflation differential show opposite patterns with respect to price stickiness. Whereas the variance decreases with a rising degree of price stickiness (Figure 3), the weight increases (Figure 4). Thus, although the variance decreases with the degree of price stickiness, which per se enhances the agents' welfare, the agents attach a higher weight to inflation as prices become stickier.[18]

[17] This holds for the contribution of the country-specific output gaps as well. The corresponding graphs are available upon request.

[18] The agents attach a higher weight to inflation as prices become stickier because the degree of inefficient price dispersion between differentiated goods is increasing in the degree of price stickiness for a given level of aggregate inflation (Woodford, 2003).

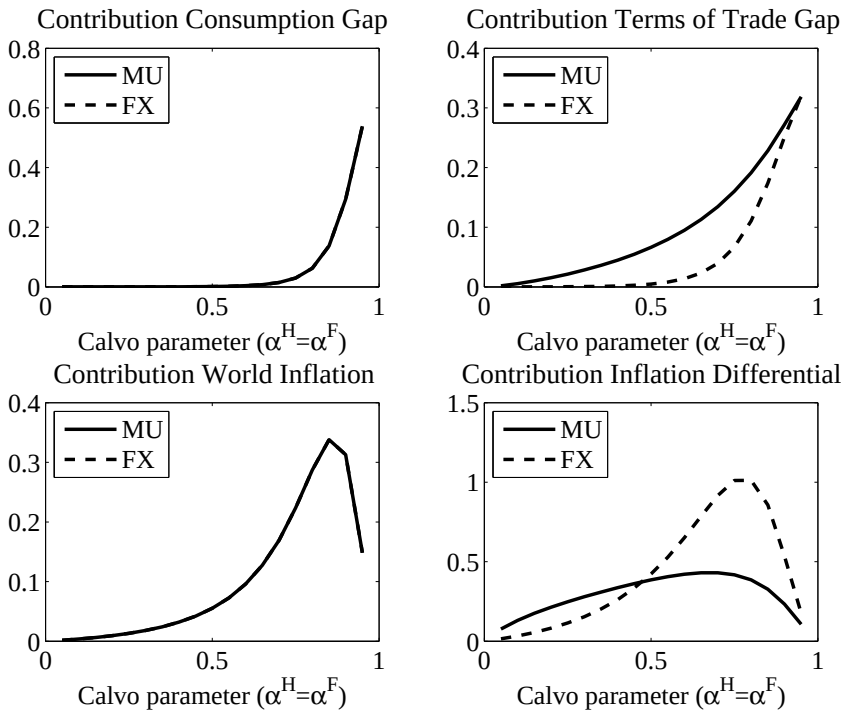

Figure 2: Contributions to world welfare loss in Figure 1

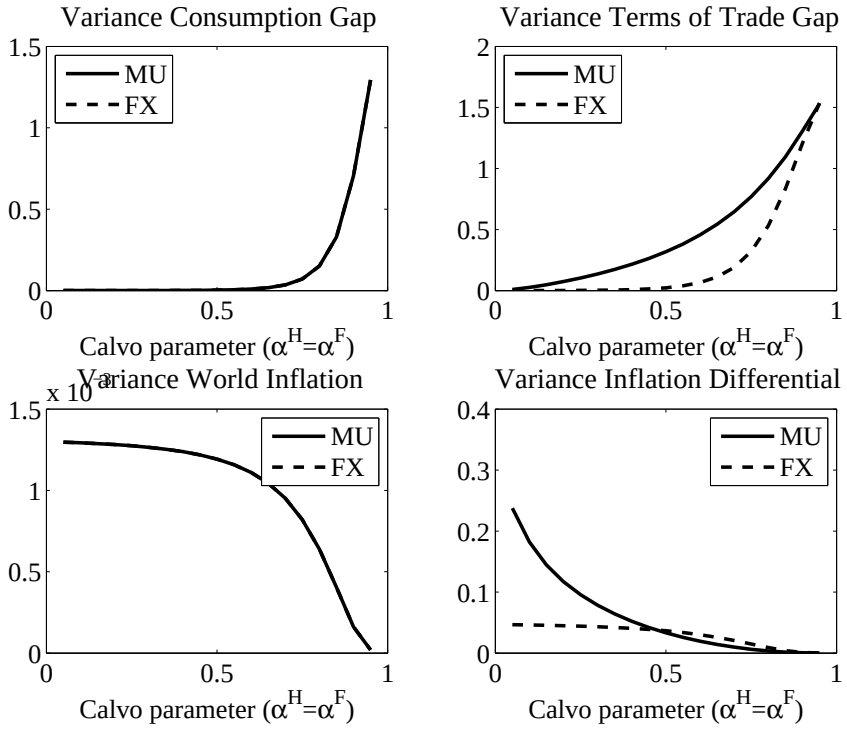

Figure 3: Variances underlying the contributions in Figure 2

Since the weight of the inflation differential is identical across the two monetary regimes (this holds for all components), it is the variance that causes the difference in welfare. Whereas the variance of the inflation differential is higher under the MU regime for lower degrees of price stickiness, it is higher under the FX regime for higher degrees of price stickiness. That is, inflation rates are more stable under the MU regime

when prices are relatively sticky.

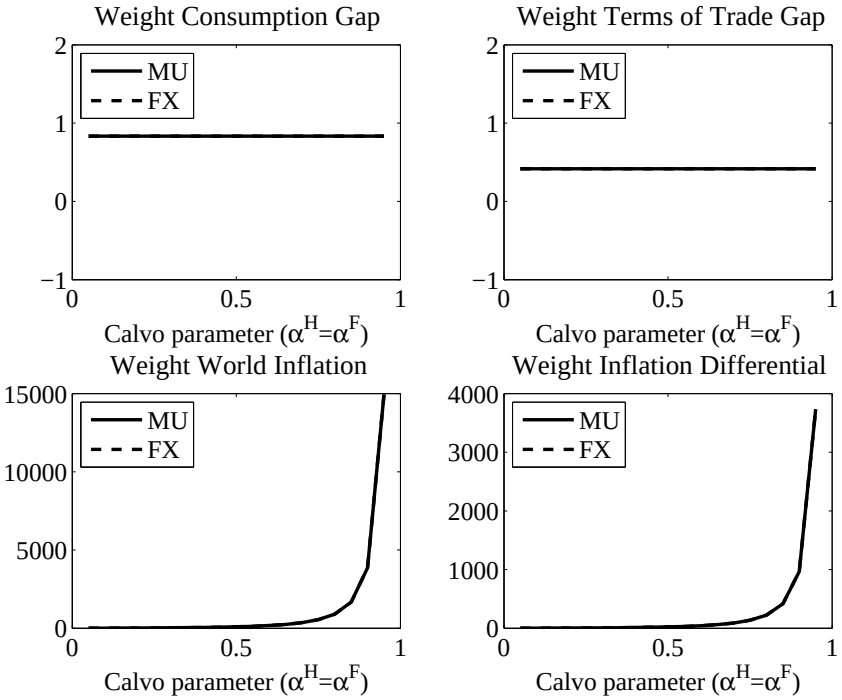

Figure 4: Weights underlying the contributions in Figure 2

The inflation differential in period t can be expressed as the sum of current and discounted expected future terms of trade gaps. Solving equation (3.4) forward, which holds under both regimes, gives

$$\pi_t^R = -k_T E_t \sum_{k=0}^{\infty} \beta^k (\hat{T}_{t+k} - \tilde{T}_{t+k}). \tag{3.19}$$

Accordingly, current and expected future terms of trade gaps are the only determinant for the current inflation differential. As shown next, two endogenous effects that both influence the terms of trade gap explain why inflation rates are more stable under the MU regime when prices are relatively sticky. First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier ("inherited stickiness of the nominal exchange rate"). Second, fixing the exchange rate entails the benefit of affecting inflation expectations in a favorable way by inducing history dependence into the economy ("inherent benefit of monetary unions").

3.2.1 Inherited stickiness of the nominal exchange rate

Whether the nominal exchange rate stabilizes or destabilizes the terms of trade gap, thereby facilitating an efficient allocation across countries, depends on the way monetary policy is conducted. Recall the recursive law of motion for the nominal exchange rate, (3.13):

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t,$$

with

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y) k_T}{(\phi_\pi - \rho_H) k_T + (1 - \rho_H + \phi_Y)(1 - \beta \rho_H)}.$$

Under the baseline calibration, where monetary policy reacts to inflation, but not to output ($\phi_Y = 0$), the coefficient c_3 is unambiguously positive (since $\phi_\pi > 1$) and smaller than one. Accordingly, in response to a shock that leads to an increase in the flexible-price terms of trade, the nominal exchange rate will increase as well, pushing up the sticky-price terms of trade closer to the flexible-price terms of trade. Thus, the nominal exchange rate stabilizes the terms of trade gap.

However, the stabilizing effect of the nominal exchange rate weakens as prices become stickier. The size of the response of the nominal exchange rate to a productivity shock decreases with the degree of price stickiness. Analytically, as the degree of price stickiness α increases, k_T decreases and c_3 decreases. In the limit, when prices become fixed ($\alpha \to 1$), the nominal exchange rate is fixed as well ($k_T \to 0, c_3 \to 0$).

The reason for this is that the expected change in the nominal exchange rate depends on the interest rate differential across countries according to the uncovered interest parity condition, (2.21):

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F.$$

Interest rates, in turn, are set by monetary policy in response to inflation according to the interest rate rules. Therefore, an increase in price stickiness, which reduces inflation variability, reduces interest rate variability and, ultimately, reduces the variability of the nominal exchange rate.

Thus, the nominal exchange rate inherits the stickiness of goods prices. This, in turn, hampers the stabilization of the terms of trade gap. Therefore, the stabilizing property of the nominal exchange rate of facilitating an efficient allocation across countries declines with the degree of price stickiness. Notably, for this effect to be effective, the uncovered interest parity condition need not hold exactly. It suffices for the interest rates and the nominal exchange rate to be linked.

3.2.2 Inherent benefit of monetary unions

The fact that the benefit of a flexible nominal exchange rate declines with the degree of price stickiness cannot alone explain the finding that the MU regime is welfare-improving over the FX regime. For even under relatively sticky prices, the nominal exchange rate stabilizes the terms of trade gap at least to some extent compared to a situation with a completely fixed nominal exchange rate, as under the MU regime. This is also the reason why the variance of the terms of trade gap is lower under the FX regime regardless of the degree of price stickiness (Figure 3, upper right panel). Therefore, the MU regime must also provide a benefit.

The MU regime differs from the FX regime in one important respect, as the analytical results from Section 3.1 have shown. In contrast to the FX regime, the economy under the MU regime is intrinsically inertial. So, even in the presence of a one-off shock, the inflation differential and the terms of trade gap are persistent. As shown next, this inertia will result in a higher stability of inflation rates.

The qualitative difference between the two monetary regimes can be seen clearly by looking at the impulse response of the terms of trade gap to a positive one-off productivity shock in country H (Figure 5).[19] On impact, the terms of trade gap decreases under both regimes because the sticky-price terms of trade do not increase as much as

[19] The degree of price stickiness was chosen to be low ($\alpha = 0.2$), so as to make the differences in the impulse responses clearly visible. The differences are much smaller for higher degrees of price stickiness, but are qualitatively the same.

the flexible-price terms of trade due to the stickiness of prices. However, in the following period, when the shock has vanished, the terms of trade gap has returned to the steady state under the FX regime, but not under the MU regime. Under the FX regime, it is the nominal exchange rate that brings the terms of trade gap automatically back to the steady state in the absence of shocks.[20] Under the MU regime, this mechanism is absent, since the nominal exchange rate is fixed. As a result, the terms of trade gap is intrinsically inertial or history-dependent.

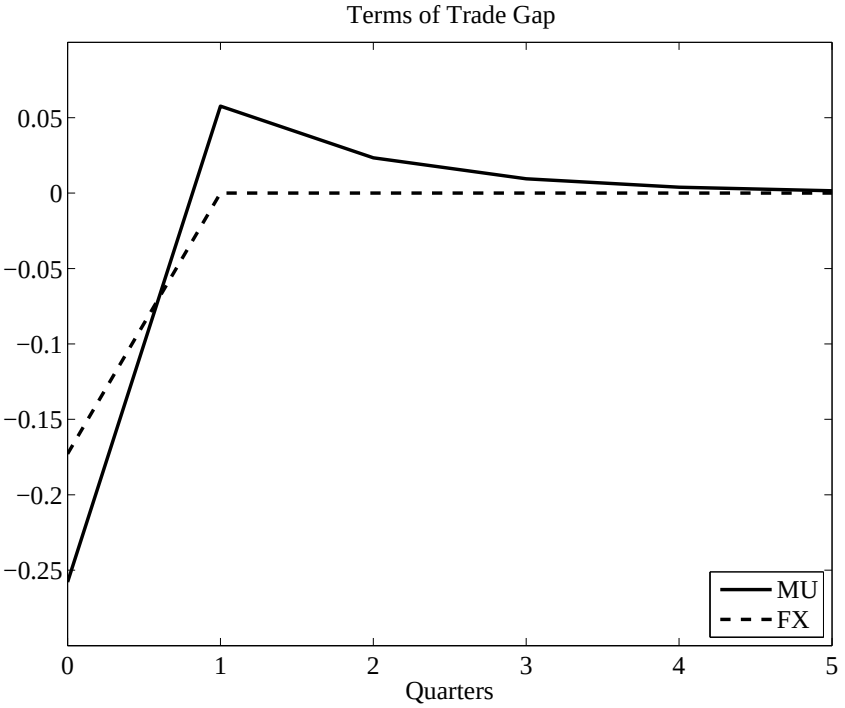

Figure 5: Impulse response of the terms of trade gap to a positive one-off productivity shock in country H ($\rho_H = 0$), with $\alpha = 0.2$

Importantly, the history dependence of the terms of trade gap manifests itself in an overshooting pattern. The terms of trade gap overshoots because the sticky-price terms of trade are still elevated above the steady state after the shock has vanished, whereas the flexible-price terms of trade are back at the steady state. In subsequent periods, the terms of trade gap converges back to the steady state.

The qualitative difference in the dynamics between the two monetary regimes prevails in situations in which the productivity shock itself is persistent (Figure 6, left panel).[21] Whereas the terms of trade gap converges monotonically back to the steady state under the FX regime, it overshoots the steady state under the MU regime.

Since the inflation differential is determined by the terms of trade gap and its expected future path (recall equation 3.19), it exhibits the same qualitative difference. Accordingly, under the FX regime the inflation differential increases on impact and converges monotonically back to the steady state (Figure 6, right panel). In contrast,

[20] Interestingly, this mechanism is independent of the interest rate rule coefficients ϕ_π and ϕ_Y (recall the RLOM coefficient $b_3 = -1$).

[21] The calibration underlying the impulse responses in Figure 6 is now identical to the calibration underlying the welfare results in Figure 1 through 4.

under the MU regime, price setters adjust their prices less in the initial period despite the stronger initial change in the terms of trade gap because they anticipate the future overshooting of the terms of trade gap. In subsequent periods, inflation approaches the steady state faster than under the FX regime and eventually overshoots the steady state as well.[22] As a result, the variance of the inflation differential, i.e., the sum of squared deviations of the inflation differential from zero, is lower under the MU regime than under the FX regime.

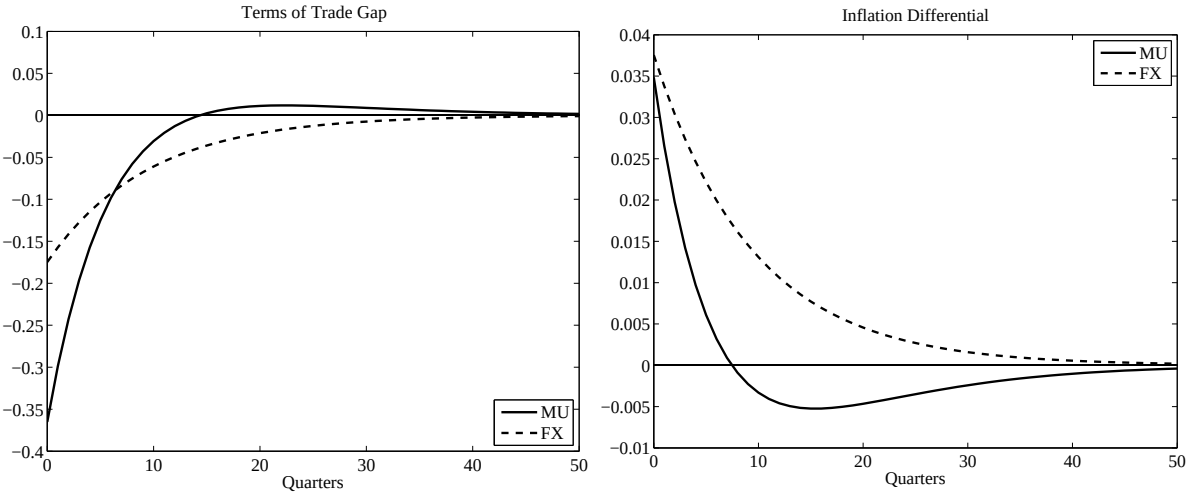

Figure 6: Impulse responses to a positive productivity shock in country H with $\rho_H = 0.9$ and $\alpha = 0.75$

To sum up: Since price setters are forward-looking, not only present, but also expected future terms of trade gaps matter for current inflation. Since the nominal exchange rate is fixed under the MU regime, the terms of trade gap overshoots in response to a shock at some point in time, which would then call for the opposite price adjustments as in the present. In anticipation of this, current price responses are smaller in magnitude than under the FX regime. As a result, inflation is more stable under the MU regime. Thus, the inherent benefit of monetary union is that it affects inflation expectations in such a way as to lower the welfare-relevant variance of inflation by inducing history dependence into the economy. The strength of this benefit increases as prices become stickier, since price setters attach higher weights to future terms of trade gaps as the probability of being able to reset prices decreases.

The benefit of history dependence is well-known from the analysis of optimal monetary policy in a closed-economy environment. Optimal monetary policy *under discretion* is inferior from a welfare perspective to optimal policy *under commitment* because the former does not influence the inflation expectations of price setters in a favorable way. It suffers from the so-called stabilization bias.[23] In contrast, optimal monetary policy under commitment induces history dependence into the economy, therefore taking advantage of the fact that price setters are forward-looking. This results in a higher stability of inflation. In exactly the same sense, forming a monetary union may be superior to maintaining a flexible exchange rate under certain conditions because

[22] Throughout the paper, I use the term "overshooting" to describe both "overshooting" and "undershooting".

[23] For details on the stabilization bias, see, e.g., Woodford (2003, Ch. 7), Gali (2008, Ch. 5), or Walsh (2010, Ch. 8).

fixing the nominal exchange rate affects inflation expectations in a favorable way by inducing history dependence into the economy.

Notably, this benefit exists despite the fact that price levels are not stationary.[24] Thus, and in contrast to Monacelli (2004), the benefit does not hinge upon stationarity of price levels. This is a particular feature of the small open economy assumption and does not generally carry over to a two-country environment. Instead, the benefit hinges upon the overshooting pattern of the terms of trade, the anticipation of which reduces the magnitude of price changes, rendering the inflation rates more stable.

3.3 Monetary policy

The finding that forming a monetary union is beneficial when prices are relatively sticky is very robust to the range of parameter values considered in Table 4.[25] The important exception is the parameters that govern the behavior of monetary policy, i.e., the coefficients of the interest rate rules. These will be considered next.

3.3.1 Inflation coefficient

Whether forming a monetary union turns out to be beneficial depends crucially on the inflation coefficient ϕ_π in the interest rate rules, i.e., on the aggressiveness of monetary policy towards inflation (Figure 7).[26] Starting out at a very low response of monetary policy to inflation (ϕ_π above, but close to, one), the MU regime yields a lower world welfare loss for every degree of price stickiness. Increasing the aggressiveness of monetary policy a little bit results in the FX regime being superior for very low degrees of price stickiness, but inferior for higher degrees of price stickiness. As the aggressiveness of monetary policy increases further, the threshold value for α increases, beyond which the MU regime yields a lower world welfare loss. Eventually, beyond a certain aggressiveness of monetary policy towards inflation ($\phi_\pi \approx 2.5$), the MU regime is inferior to the FX regime regardless of the degree of price stickiness.

The intuition for this is as follows. Under the FX regime, when monetary policy reacts to inflation only, the nominal exchange rate stabilizes the terms of trade gap in response to shocks (see Section 3.2.1). The strength of this stabilizing property increases with the aggressiveness of monetary policy towards inflation, since monetary policy directly influences the nominal exchange rate via the uncovered interest parity condition. As a result, even when prices are relatively sticky, monetary policy can counteract by being more aggressive towards inflation. In the limit ($\phi_\pi \to \infty$), monetary policy perfectly stabilizes all welfare-relevant variables, reducing the welfare loss to zero (divine coincidence). This is not the case under the MU regime because the common monetary policy has no influence on the terms of trade gap and the inflation differential when prices are equally sticky across countries (see Section 3.1).

[24] Impulse responses for price levels are available upon request.

[25] The corresponding graphs are available upon request.

[26] The graph in the right panel of Figure 7 is a rotation of the graph in the left panel, in order to be able to see behind the steep surface area. Cutting through the two surface areas along $\phi_\pi = 1.5$ produces Figure 1.

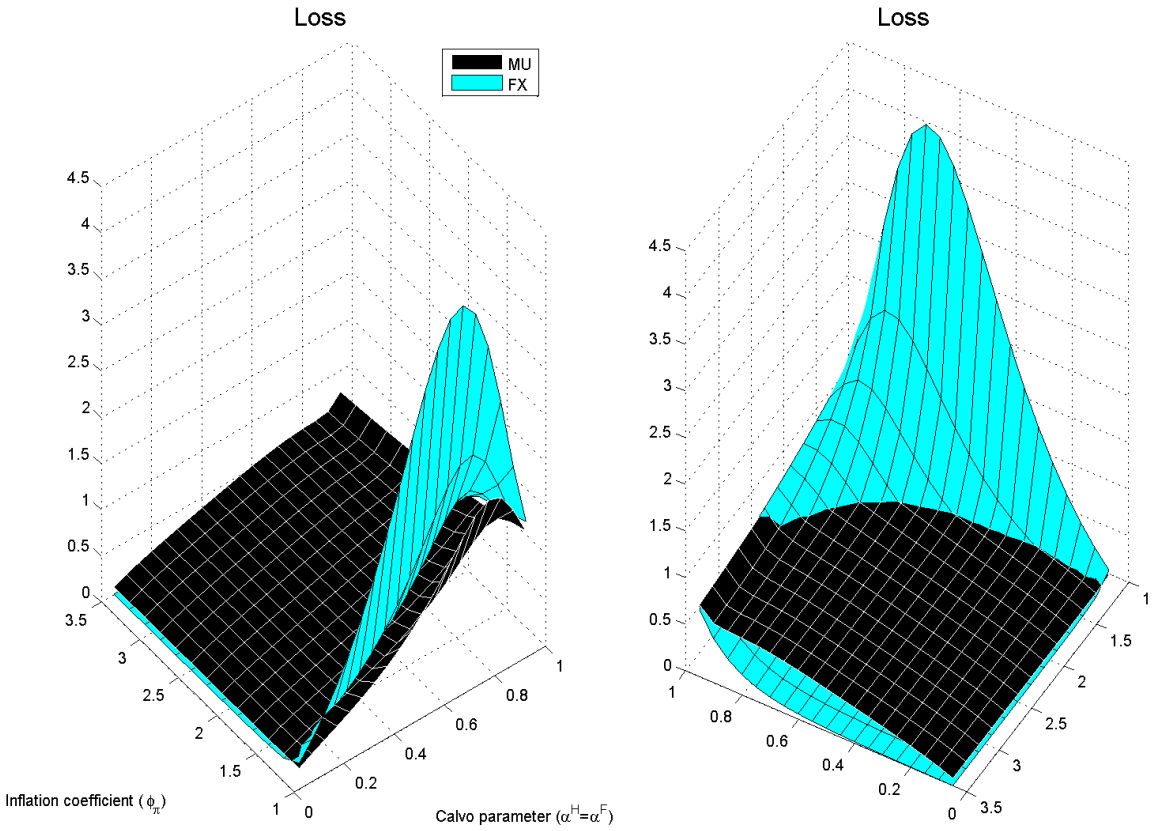

Figure 7: World welfare loss under various degrees of price stickiness ($\alpha^H = \alpha^F$) and various values for the inflation coefficient (ϕ_π), from two different angles

3.3.2 Output coefficient

The welfare ranking between the two monetary regimes depends also on the output coefficient in the interest rate rules ϕ_Y, i.e., on the aggressiveness of monetary policy towards output (Figure 8). For almost all the combinations of ϕ_Y and ϕ_π considered, the FX regime yields a higher world welfare loss than the MU regime, although the degree of price stickiness was deliberately chosen to favor the FX regime ($\alpha = 0.2$). Increasing the degree of price stickiness would favor the MU regime further. In general, the stronger monetary policy reacts to output, the stronger it needs to react to inflation for the FX regime to remain superior. This relationship is very steep; a small increase in ϕ_Y (e.g. from 0 to 0.25) requires a strong increase in ϕ_π (from roughly 1.5 to 2.3).

Unlike in the case of the response to inflation, the more aggressive monetary policy reacts to output, the smaller the impact response of the nominal exchange rate to shocks becomes (see coefficient c_3 of the RLOM). When the aggressiveness of monetary policy towards output relative to inflation exceeds a certain degree ($\phi_Y > \phi_\pi - 1$), the nominal exchange rate destabilizes the terms of trade gap in response to shocks ($c_3 < 0$).

The reason for this is that from a welfare perspective, a response to output by monetary policy is detrimental. It is not the deviation of output from the steady state that is

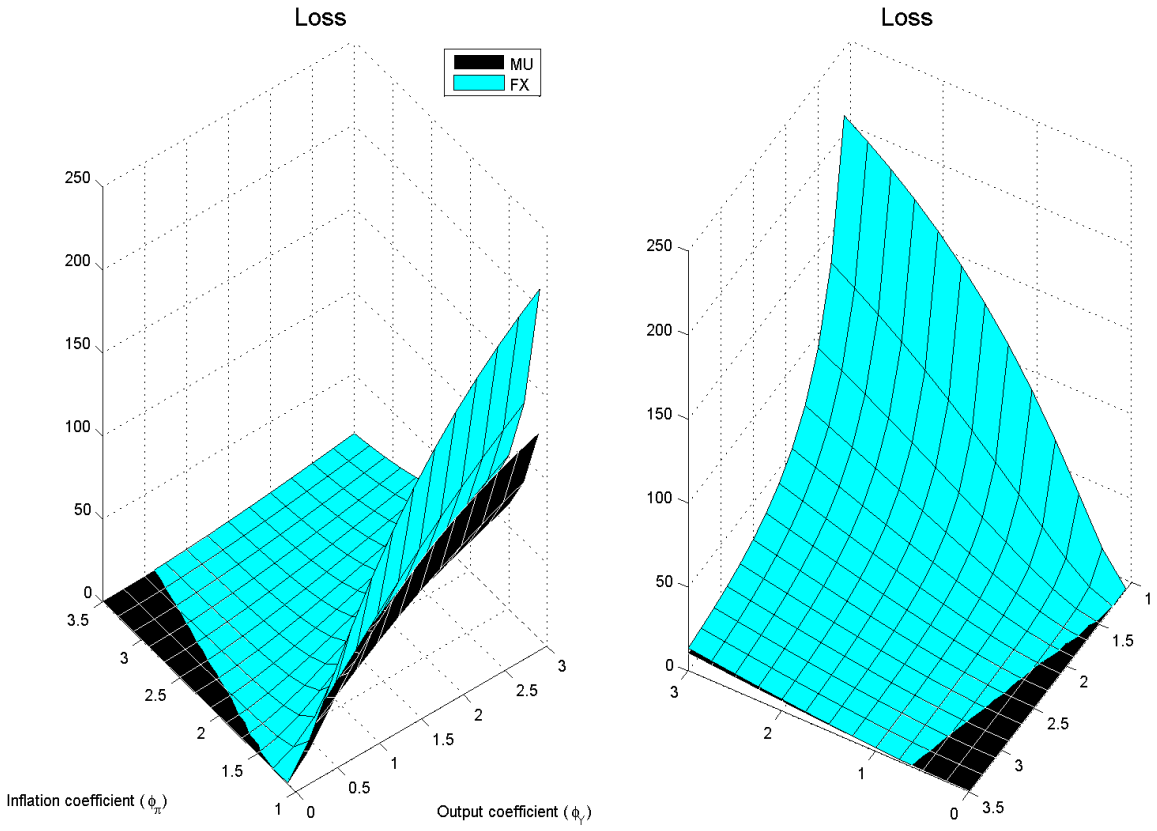

Figure 8: World welfare loss under various values for the output coefficient (ϕ_Y) and for the inflation coefficient (ϕ_π), with $\alpha = 0.2$, from two different angles

welfare-relevant, it is the deviation from the flexible-price counterpart (output gap).[27] For example, a positive productivity shock in country H induces an increase in output, but a decrease in the output gap, since the increase in output is lower than the increase in flexible-price output. A welfare-oriented reaction of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output. As a result, the variance of the inflation differential is higher when monetary policy reacts to output ($\phi_Y > 0$) than when it does not react to output ($\phi_Y = 0$).[28]

While a reaction to output is detrimental under both regimes (in Figure 8, the welfare loss is increasing in the output coefficient ϕ_Y under both regimes), the damage in terms of welfare is greater under the FX regime. The reason for this is that, in contrast to the MU regime, monetary policy under the FX regime affects every component of the welfare loss function (see Section 3.1). Thus, conducting "bad" monetary policy is more harmful under the FX regime because monetary policy is more powerful in this regime. Essentially, the nominal exchange rate does not compensate for monetary

[27] Recall that the welfare loss function can be expressed alternatively in terms of country-specific output gaps instead of the consumption gap (see equation D.72).

[28] This is common to the closed-economy setup of the basic New Keynesian model, as in Gali (2008).

policy mistakes; instead, it reinforces the quality of monetary policy. In this sense, a monetary union provides a hedging mechanism against monetary policy mistakes.

3.3.3 Interest rate smoothing coefficient

The welfare ranking between the two monetary regimes depends on the degree of interest rate smoothing ϕ_R as well (Figure 9).[29] When monetary policy does not engage in interest rate smoothing ($\phi_R = 0$), the MU regime yields a lower welfare loss for relatively sticky prices. As the degree of interest rate smoothing increases, the threshold value for α, beyond which the MU regime is superior, increases as well. For very high degrees of interest rate smoothing, the MU regime is welfare-improving only for extremely high degrees of price stickiness. Thus, interest rate smoothing makes a beneficial monetary union less likely.

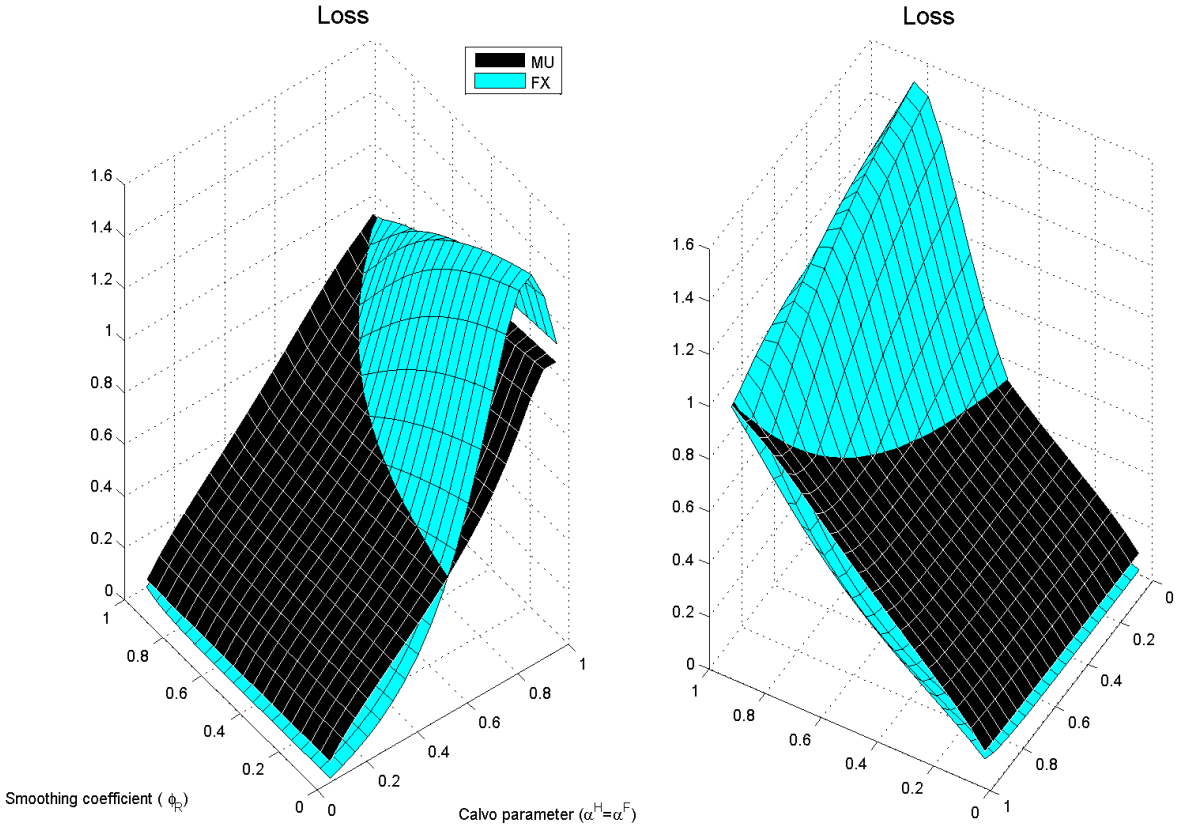

Figure 9: World welfare loss under various degrees of price stickiness ($\alpha^H = \alpha^F$) and various values for the interest rate smoothing coefficient (ϕ_R), from two different angles

The reason for this becomes clear by looking again at the impulse response of the terms of trade gap to a positive one-off productivity shock in country H, but now with a relatively high degree of interest rate smoothing (Figure 10). The impulse response

[29] Cutting through the two surface areas along $\phi_R = 0$ produces Figure 1

under the MU regime is identical to the situation without interest rate smoothing (Figure 5) because monetary policy continues to exert no influence on the terms of trade when prices are equally sticky across countries. In contrast, the impulse response under the FX regime now resembles the response under the MU regime. Although the productivity shock is one-off, the terms of trade gap displays inertia in the form of overshooting.

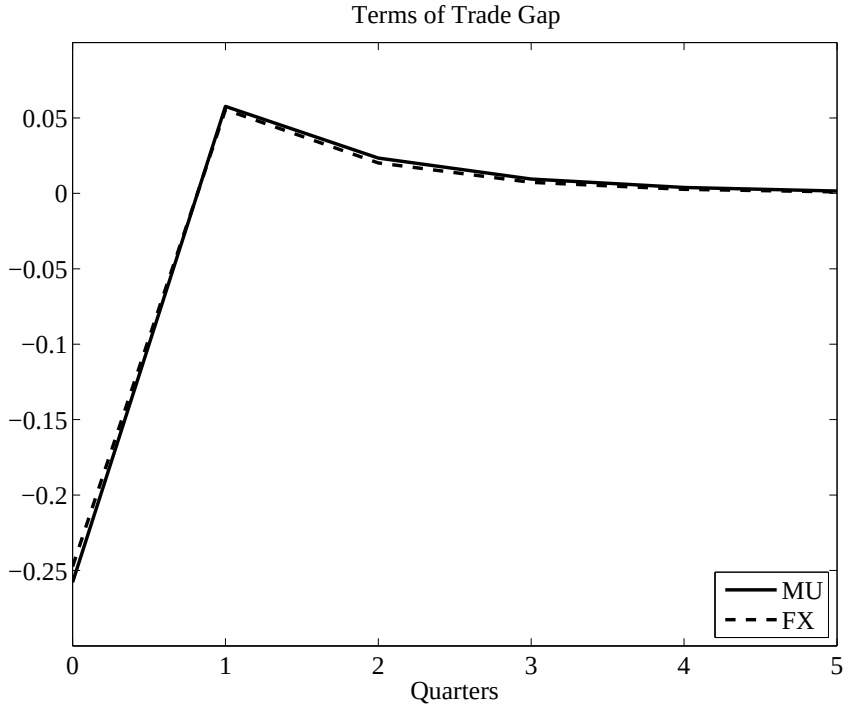

Figure 10: Impulse response of the terms of trade gap to a positive one-off productivity shock in country H ($\rho_H = 0$), with $\phi_R = 0.9$ and $\alpha = 0.2$

As a result, inflation expectations are affected in the same favorable way as under the MU regime, namely by inducing history dependence into the economy. Only the source of history dependence is different. Under the FX regime, monetary policy has to engage in interest rate smoothing to induce history dependence. Under the MU regime, history dependence is induced automatically by the fact that the nominal exchange rate is fixed. For the FX regime to be welfare-improving over the MU regime under relatively sticky prices, monetary policy has to implement a sufficiently high degree of interest rate smoothing. This renders the inflation differential more stable under the FX regime.

The fact that interest rate smoothing under the FX regime reduces the welfare loss by stabilizing inflation comes as no surprise. As shown by Woodford (1999), one way for monetary policy to implement the kind of history dependence that is desirable from the perspective of optimal monetary policy is to engage in interest rate smoothing by including a feedback of the current nominal interest rate to past realizations of the nominal interest rate, as is the case in the interest rate rules given by equations (2.22) and (2.23).

4 Results under asymmetry

In the following, I check whether asymmetries in country size or in the degree of price stickiness matter for the welfare ranking between the MU and FX regime.

4.1 Country size

If the two countries differ only in population size, the analytical results from Section 3.1 carry over. Accordingly, the RLOM under both monetary regimes, equations (3.9) through (3.13), are valid in this case. As one can see, the RLOMs are independent of the country size n. Thus, the welfare-relevant inflation differential and the terms of trade gap are independent of n. As a consequence, the threshold value for α, beyond which the MU regime yields a lower world welfare loss, is completely insensitive with respect to n. Therefore, the welfare ranking between the MU and FX regime does not depend on country size.

4.2 Price stickiness

The world welfare loss function under different degrees of price stickiness across the two countries is given by equation (2.24) and repeated here for convenience:

$$\begin{aligned} W_t = -\frac{1}{2}\Bigg(& (\rho+\eta)\operatorname{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\operatorname{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\operatorname{var}\pi_t^H \\ & + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\operatorname{var}\pi_t^F\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{4.1}$$

It contains four components: the variance of the consumption gap $(\hat{C}_t - \tilde{C}_t)$, the variance of the terms of trade gap $(\hat{T}_t - \tilde{T}_t)$, the variance of inflation in country H (π_t^H), and the variance of inflation in country F (π_t^F).

Unless the degree of price stickiness is extremely high ($\alpha \geq 0.9$, which corresponds to an average duration of price contracts of at least 10 quarters), asymmetry in the degree of price stickiness does not matter for the welfare ranking between the MU and FX regime (Figure 11). Drawing from the analysis above, the intuition is the following. First, the inherent benefit of monetary unions of inducing history dependence is independent of country characteristics. It depends only on the fact that the nominal exchange rate is fixed and that price setters are forward-looking. Second, the nominal exchange rate inherits the stickiness of goods prices from both countries. It does not matter if the stickiness is equally present in both countries or if the stickiness comes primarily from one country. Thus, as long as the aggregate degree of price stickiness in the world as a whole is sufficiently high, the MU regime continues to be beneficial.

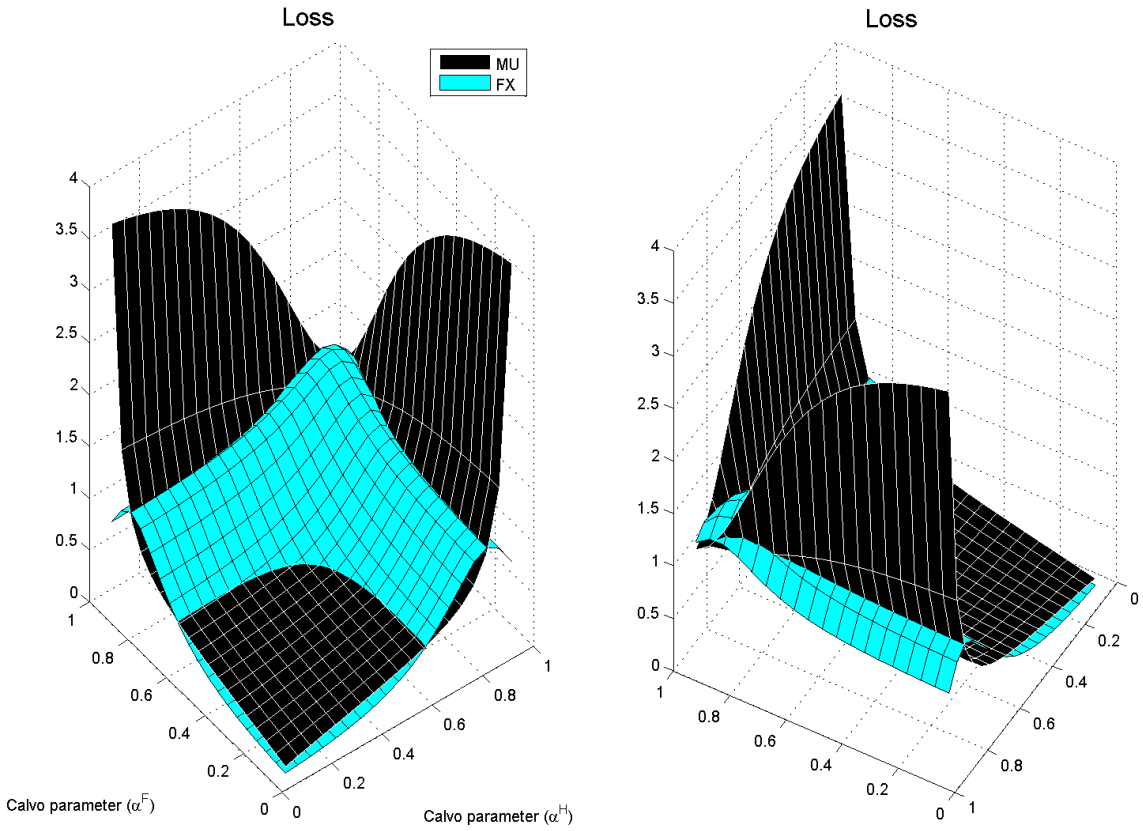

Figure 11: World welfare loss under different degrees of price stickiness across countries ($\alpha^H \neq \alpha^F$), from two different angles

5 Contrast to traditional OCA theory

This paper has shown that in the standard two-country New Keynesian DSGE model, countries benefit from forming a monetary union when prices are relatively sticky but do not when prices are relatively flexible. This finding is clearly at odds with Friedman (1953)'s case for flexible exchange rates and with the traditional OCA theory.[30] First, note that in the model employed in this paper the need for macroeconomic adjustment is triggered by an asymmetric *temporary* change in *productivity*, whereas the traditional OCA analysis usually assumes a *permanent* shift in *demand* from the products of one country to the products of the other.

Second, and more importantly, the role of expectations differs quite substantially between the New Keynesian and the Old-Keynesian framework. In the Old-Keynesian framework, in which the key predictions of the traditional OCA theory were developed, expectations of economic agents were treated as exogenous. In contrast, in New Keynesian models private sector expectations are treated as endogenous. As shown, it is precisely the expectations channel that renders inflation rates more stable under the

[30] De Grauwe (2012) summarizes the key insights from the traditional OCA theory.

monetary union regime. This channel is naturally missing in models without such an expectational feedback mechanism.[31]

6 Conclusion

The main finding of this paper is that in the basic two-country New Keynesian DSGE model, in which monetary policy is conducted via Taylor-type interest rate rules, forming a monetary union is welfare-improving when prices are relatively sticky. In this case, the costs of higher output-gap and terms-of-trade-gap instability are outweighed by the benefit of higher inflation stability. Two endogenous effects are responsible for this. First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier. Second, fixing the exchange rate entails the inherent benefit that it stabilizes inflation expectations by inducing inertia into the economy.

The paper has also shown that whether forming a monetary union is beneficial or not depends heavily on the way monetary policy is conducted. When monetary policy responds to inflation aggressively or when it implements a high degree of interest rate smoothing, maintaining a flexible exchange rate is superior. In contrast, monetary policy mistakes (such as a reaction to output) are more harmful under a flexible exchange rate. In this sense, a monetary union provides a hedging mechanism against monetary policy mistakes.

These findings suggest that the conventional view of the costs and benefits of forming a monetary union in terms of macroeconomics stabilization may need to be revised, at least to the extent that real-world monetary policy is not optimal.

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[31] See King (1993) for a critical assessment of the Old-Keynesian, IS-LM model with respect to their treatment of expectations.

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A Variances

To obtain analytical expressions for the variance of the terms of trade gap and of the inflation differential under each monetary regime, I first derive the recursive laws of motion (RLOM). Then I set up the corresponding vector autoregressive (VAR) model of the system of equations. Finally, since the matrix algebra is very extensive, I use MATLAB Symbolic Math Toolbox to obtain the expressions of interest from the variance-covariance matrix.

The derivations in this Appendix are only valid if the degree of price stickiness and the persistence of productivity shocks are identical across countries ($\alpha^H = \alpha^F$ and $\rho_H = \rho_F$) and if monetary policy does not engage in interest rate smoothing ($\phi_R = 0$). As shown in section 3, the variables consumption and world inflation need not be considered as they behave identically across monetary regimes and independently of the variables terms of trade, nominal exchange rate and inflation differential.

A.1 Monetary union regime

The number of equations can be reduced by subtracting the New Keynesian Phillips curve of country H (B.42) from the one of country F (B.43). As a result, the consumption gap vanishes due to $k_C^H = k_C^F$. The second equation is given by the terms-of-trade identity (B.44). The resulting system of equations is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \tag{A.1}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \tag{A.2}$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \tag{A.3}$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \tag{A.4}$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1+\eta} \nu_t^R, \tag{A.5}$$

where $\nu_t^R = \nu_t^F - \nu_t^H$. Equation (A.5) is obtained by inserting the country-specific shock processes (B.38) into the equation of the flexible-price terms of trade (B.33).

To obtain the unknown coefficients as functions of the deep parameters of the model, I use the method of undetermined coefficients. First, inserting equations (A.3) through (A.5) into equations (A.1) and (A.2) and rearranging yields

$$\begin{aligned} b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t =& [-k_T b_1 + \beta b_2 b_1] \hat{T}_{t-1} \\ &+ [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H] \tilde{T}_t \end{aligned} \tag{A.6}$$

$$b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t = [1 + b_2] \hat{T}_{t-1} + c_2 \tilde{T}_t. \tag{A.7}$$

Setting $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$ and $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$ respectively gives the following four conditions for the four unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \tag{A.8}$$
$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \tag{A.9}$$
$$b_1 = 1 + b_2 \tag{A.10}$$
$$c_1 = c_2. \tag{A.11}$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - (1 + k_T + \beta) b_1 + 1 \tag{A.12}$$

and therefore two solutions for b_1. Only one solution fulfills the requirement for a stable equilibrium, i.e., $|b_1| < 1$. Using b_1 immediately yields the other coefficients. Thus, the coefficients of the RLOM take the following form:

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \tag{A.13}$$
$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \tag{A.14}$$
$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}. \tag{A.15}$$

The corresponding VAR model can be written as follows[32]:

$$\begin{pmatrix} \pi_t^R \\ \hat{T}_t \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & b_2 & \rho_H c \\ 0 & b_1 & \rho_H c \\ 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_{t-1}^R \\ \hat{T}_{t-1} \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c\frac{\eta}{1+\eta} \\ c\frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix Σ can be obtained in terms of the *vec* operator as follows:[33]

$$vec(\Sigma) = (I - A \otimes A)^{-1} vec(B), \tag{A.16}$$

where I denotes the identity matrix.

Since matrix A is of dimension 3×3, matrix $A \otimes A$ is of dimension 9×9. Although matrix $A \otimes A$ is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox. Further simplification of the

[32] The order of variables was chosen as to render matrix A and therefore matrix $A \otimes A$ triangular. This facilitates the calculation of the determinant considerably, since, in that case, the determinant is simply given by the product of the diagonal elements.

[33] See, e.g., Hamilton (1994).

resulting expressions finally yields

$$\operatorname{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[\frac{(1+\rho_H b_1)c^2}{(1-b_1^2)(1-\rho_H b_1)} - \frac{2c}{1-\rho_H b_1} + 1\right] \operatorname{var} \tilde{T}_t \tag{A.17}$$

$$\operatorname{var}_{MU} \pi_t^R = \frac{2c^2}{(1+b_1)(1-\rho_H b_1)(1+\rho_H)} \operatorname{var} \tilde{T}_t \tag{A.18}$$

$$\operatorname{var} \tilde{T}_t = \frac{1}{1-\rho_H^2}\left(\frac{\eta}{1+\eta}\right)^2 \left[\operatorname{var} \nu_t^H + \operatorname{var} \nu_t^F - 2\operatorname{cov}(\nu_t^H, \nu_t^F)\right]. \tag{A.19}$$

A.2 Flexible exchange rate regime

The derivation of the variances under the FX regime follows the exact same steps as under the MU regime. The number of equations can be reduced by subtracting the New Keynesian Phillips curves from each other. Furthermore, the expected change in the nominal exchange rate can be expressed as a function of the inflation differential and the terms of trade by inserting the interest rate rules (C.8) and (C.9) as well as the equations for aggregate demand (C.2) and (C.3) into the uncovered interest parity condition (C.7). The resulting system of equations is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \tag{A.20}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta\hat{S}_t \tag{A.21}$$

$$E_t \Delta\hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \tag{A.22}$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \tag{A.23}$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \tag{A.24}$$

$$\Delta\hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t \tag{A.25}$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1+\eta}\nu_t^R. \tag{A.26}$$

Inserting equations (A.23) through (A.26) into equations (A.20) through (A.21) and rearranging yields

$$\begin{aligned} b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t =& [-k_T b_1 + \beta b_2 b_1]\hat{T}_{t-1} \\ &+ [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H]\tilde{T}_t \end{aligned} \tag{A.27}$$

$$b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t = [1 + b_2 + b_3]\hat{T}_{t-1} + [c_2 + c_3]\tilde{T}_t \tag{A.28}$$

$$\begin{aligned} b_1 b_3 \hat{T}_{t-1} + [b_3 c_1 + c_3 \rho_H]\tilde{T}_t =& [-\phi_\pi b_2 + \phi_Y b_1]\hat{T}_{t-1} \\ &+ [-\phi_\pi c_2 + \phi_Y c_1]\tilde{T}_t. \end{aligned} \tag{A.29}$$

Setting $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$ and $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$ respectively gives the following six conditions for the six unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \tag{A.30}$$
$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \tag{A.31}$$
$$b_1 = 1 + b_2 + b_3 \tag{A.32}$$
$$c_1 = c_2 + c_3 \tag{A.33}$$
$$b_1 b_3 = -\phi_\pi b_2 + \phi_Y b_1 \tag{A.34}$$
$$b_3 c_1 + c_3 \rho_H = -\phi_\pi c_2 + \phi_Y c_1. \tag{A.35}$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - [1 + k_T + (1 + \phi_Y)\beta] b_1 + (1 + \phi_\pi k_T + \phi_Y). \tag{A.36}$$

In this case, there are either two real or two imaginary solutions for b_1, depending on the realizations of the deep parameters. However, neither solution fulfills the requirement for a stable equilibrium. Yet, $b_1 = 0$ is another solution to the above system of equations, and it implies a stable equilibrium, since $|b_1| < 1$. Given $b_1 = 0$, the coefficients of the RLOM take the following form:

$$b_1 = 0 \tag{A.37}$$
$$b_2 = 0 \tag{A.38}$$
$$b_3 = -1 \tag{A.39}$$
$$c_1 = \frac{(\phi_\pi - \rho_H) k_T}{(\phi_\pi - \rho_H) k_T + (1 - \rho_H + \phi_Y)(1 - \beta \rho_H)} \tag{A.40}$$
$$c_2 = \frac{(1 - \rho_H + \phi_Y) k_T}{(\phi_\pi - \rho_H) k_T + (1 - \rho_H + \phi_Y)(1 - \beta \rho_H)} \tag{A.41}$$
$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y) k_T}{(\phi_\pi - \rho_H) k_T + (1 - \rho_H + \phi_Y)(1 - \beta \rho_H)}. \tag{A.42}$$

The corresponding VAR model can be written as follows[34]:

$$\begin{pmatrix} \Delta \hat{S}_t \\ \hat{T}_t \\ \pi_t^R \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & 0 & \rho_H c_3 \\ 0 & 0 & 0 & \rho_H c_1 \\ 0 & 0 & 0 & \rho_H c_2 \\ 0 & 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \Delta \hat{S}_{t-1} \\ \hat{T}_{t-1} \\ \pi_{t-1}^R \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c_3 \frac{\eta}{1+\eta} \\ c_1 \frac{\eta}{1+\eta} \\ c_2 \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix Σ can be obtained in terms of the *vec* operator as follows:[35]

$$vec(\Sigma) = (I - A \otimes A)^{-1} vec(B), \tag{A.43}$$

[34] The order of variables was chosen as to render matrix A and therefore matrix $A \otimes A$ triangular. This facilitates the calculation of the determinant considerably, since, in that case, the determinant is simply given by the product of the diagonal elements.

[35] See, e.g., Hamilton (1994).

where I denotes the identity matrix.

Since matrix A is of dimension 4×4, matrix $A \otimes A$ is of dimension 16×16. Although matrix $A \otimes A$ is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox. Further simplification of the resulting expressions finally yields

$$\operatorname{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \operatorname{var} \tilde{T}_t \tag{A.44}$$

$$\operatorname{var}_{FX} \pi_t^R = c_2^2 \operatorname{var} \tilde{T}_t \tag{A.45}$$

$$\operatorname{var} \tilde{T}_t = \frac{1}{1-\rho_H^2} \left(\frac{\eta}{1+\eta} \right)^2 \left[\operatorname{var} \nu_t^H + \operatorname{var} \nu_t^F - 2 \operatorname{cov}(\nu_t^H, \nu_t^F) \right]. \tag{A.46}$$

B Monetary union regime

This appendix contains the full derivation of the model under the monetary union regime. The world, which consists of two countries labeled H and F, is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ lives in country H, the population on the segment $[n, 1]$ lives in country F. Thus, n measures the population size as a fraction of world population. An agent is both consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

B.1 Consumer problem

Agent j in country $i = H, F$ derives positive utility from consumption C^j and negative utility from producing the differentiated good y^j. The present discounted value of lifetime utility U^j is thus given by[36]

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j) - V(y_t^j, z_t^i) \right], \tag{B.1}$$

where E_t denotes the expectations operator and β the discount factor.

V is an increasing, convex function of agent j's supply of his product y_t^j and a decreasing convex function of productivity z_t^i, which is common to all agents in country i. One can think of V as the combination of the agent's disutility of working and the production function. If the disutility of working is given by $g(N_t^j)$, where N_t^j is the number of hours worked, and the production function is given by $y_t^j = f(N_t^j, z_t^i)$, then $V = g(f^{-1}(y_t^j, z_t^i))$.

U is an increasing, concave function of consumption C_t^j. The agent consumes both a bundle of differentiated goods from country H and from country F with a preference structure of the Cobb-Douglas type, so that

$$C_t^j = \left(\frac{C_{H,t}^j}{n} \right)^n \left(\frac{C_{F,t}^j}{1-n} \right)^{1-n}, \tag{B.2}$$

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$C_{H,t}^j = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \tag{B.3}$$

$$C_{F,t}^j = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

[36] In Benigno (2004), the agent derives utility also from holding money. However, money in the utility function is not necessary if monetary policy is conducted via the interest rate.

These preferences imply (1) that the elasticity of substitution between differentiated goods c_t^j from one country is σ, which is assumed to be greater than one and equal across countries, (2) that the elasticity of substitution between the bundle of goods from the two countries $C_{H,t}$ and $C_{F,t}$ is one and equal across countries, and (3) that the share of a bundle of goods from one country in the overall consumption expenditures of an agent coincides with the country's share in world population, i.e., there is no home bias in consumption.

Accordingly, the aggregate price index in country i is given by

$$P_t^i = (P_{H,t}^i)^n (P_{F,t}^i)^{1-n}, \tag{B.4}$$

where the price indices for the bundles of differentiated goods in each country are defined by

$$\begin{aligned} P_{H,t}^i &= \left[\frac{1}{n}\int_0^n p_t^i(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}} \\ P_{F,t}^i &= \left[\frac{1}{1-n}\int_n^1 p_t^i(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{B.5}$$

In their role as producers, agents charge only one price for their good irrespective of whether the good remains in the country or is exported (no price discrimination). Furthermore, exporting does not entail transportation costs. These assumptions imply that a single good has the same price in both countries, i.e., $p_t^H(h) = p_t^F(h)$ and $p_t^H(f) = p_t^F(f)$. Given identical consumption preferences across countries, this immediately leads purchasing power parity to hold, so $P_t^H = P_t^F = P_t$. Consequently, the superscript i can be dropped from all the price indices.[37]

Agent j takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption C_t^j.[38] Second, given C_t^j he optimally allocates expenditures between the bundles of differentiated goods from the two countries $C_{H,t}^j$ and $C_{F,t}^j$ by minimizing total expenditure $P_t C_t^j$ with respect to (B.2). As a result, demand for these bundles is given by

$$C_{H,t}^j = n\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t^j, \qquad C_{F,t}^j = (1-n)\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t^j. \tag{B.6}$$

Third, given $C_{H,t}$ and $C_{F,t}$ the agent optimally allocates expenditures between the differentiated goods by minimizing $P_{H,t}C_{H,t}$ and $P_{F,t}C_{F,t}$ with respect to equations (B.3). This yields

$$c_t^j(h) = \frac{1}{n}\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma} C_{H,t}^j, \qquad c_t^j(f) = \frac{1}{1-n}\left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma} C_{F,t}^j. \tag{B.7}$$

[37] Note that P_t can be interpreted as a consumer price index, $P_{H,t}$ and $P_{F,t}$ as producer price indices.

[38] As shown below, C_t^j is determined by the usual Euler consumption equation (B.14).

Combining (B.6) and (B.7) yields

$$c_t^j(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma} \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t^j, \quad c_t^j(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma} \left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t^j. \tag{B.8}$$

The terms of trade are defined from the perspective of country F, i.e., the ratio of the price of the bundle of goods produced in country F to the price of the bundle of goods imported from country H:

$$T_t = \frac{P_{F,t}}{P_{H,t}}. \tag{B.9}$$

Equations (B.8) can then be expressed in terms of the terms of trade as

$$c_t^j(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma} T_t^{1-n} C_t^j, \quad c_t^j(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma} T_t^{-n} C_t^j, \tag{B.10}$$

where the terms of trade were inserted by rearranging the aggregate price equation (B.4) and by using the definition of the terms of trade (B.9).

Aggregating over all agents living in both countries, global demand for the differentiated goods h and f can be written as

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma} T_t^{1-n} C_t^W, \quad y_t(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma} T_t^{-n} C_t^W, \tag{B.11}$$

where world consumption is given by

$$C_t^W = \int_0^1 C_t^j dj. \tag{B.12}$$

There are two types of assets agents can trade in. Within each country, agents can insure against all possible states of nature by holding a portfolio of contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by $B_t^{i,j}$ and whose vector of prices is denoted by q_t^i. Across countries, agents can trade in a non-contingent, one-period bond whose nominal value (denominated in the currency of the union) is denoted by B_t^j and whose nominal interest rate is denoted by R_t. Thus, asset markets are incomplete across countries, but complete within countries. The intertemporal budget constraint of agent j in country i is then given by

$$C_t^j + q_t^i B_t^{i,j} + \frac{B_t^j}{P_t(1+R_t)} = B_{t-1}^{i,j} + \frac{B_{t-1}^j}{P_t} + (1-\tau^i)\frac{p_t(j)y_t(j)}{P_t}, \tag{B.13}$$

where the left-hand side represents the agent's expenditures and the right-hand side his income. The latter stems also from sales revenues $p_t(j)y_t(j)$ net of a proportional, country-specific tax τ^i.[39]

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period, so $B_0^{i,j} = B_0^j = 0$ for all i and j. Together with the facts that

[39] The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.

agents have identical preferences and that asset markets are complete within countries, this assumption implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country H and country F.

The representative agent in country i maximizes his lifetime utility (B.1) with respect to the budget constraint (B.13). By combining the resulting first order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$U_C(C_t^i) = (1 + R_t)\beta E_t \left\{ U_C(C_{t+1}^i) \frac{P_t}{P_{t+1}} \right\}. \tag{B.14}$$

One important implication of the Cobb-Douglas type consumption preferences given by (B.2) together with the initial condition $B_0^H = B_0^F = 0$ is that risk sharing is perfect across countries as well despite incomplete asset markets at the international level, in the sense that[40]

$$C_t^H = C_t^F = C_t. \tag{B.15}$$

To gain intuition, first note that, similar to (B.3), aggregate demand for the bundles of goods in the two countries can be expressed as

$$\begin{aligned} Y_t^H &= \left[\left(\frac{1}{n} \right) \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\ Y_t^F &= \left[\left(\frac{1}{1-n} \right) \int_n^1 y_t(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \tag{B.16}$$

Then, applying (B.16) to (B.11) and using (B.15) yields

$$Y_t^H = T_t^{1-n} C_t, \qquad Y_t^F = T_t^{-n} C_t. \tag{B.17}$$

Making use of the definition of the terms of trade (B.9) and the aggregate price equation (B.4), this can be rearranged to

$$P_{H,t} Y_t^H = P_t C_t, \qquad P_{F,t} Y_t^F = P_t C_t. \tag{B.18}$$

Finally, the ratio of the two equations is given by

$$\frac{P_{F,t}}{P_{H,t}} \frac{Y_t^F}{Y_t^H} = T_t \frac{Y_t^F}{Y_t^H} = 1. \tag{B.19}$$

Nominal output equals nominal consumption in both countries at all times, as can be seen from (B.18). Thus, current accounts are always balanced. The reason is that any variation in the terms of trade is accompanied by an exact proportional variation in relative output across countries, as shown by (B.19). Agents shift consumption from the good that has become relatively expensive to the good that has become relatively cheap (expenditure switching effect) in such a way that a one percent increase in the relative price (terms of trade) leads to a one percent decrease in relative quantities. This is ultimately due to the fact that the elasticity of substitution between the bundles of goods from the two countries is one (Cobb-Douglas preferences).

[40] For a proof, see Benigno (2003), Appendix A.

As a result, relative nominal output and therefore relative income between the two countries are constant at all times. Thus, there are no gains from asset trade across countries, and the internationally traded bond becomes redundant ($B_t^H = B_t^F = 0 \ \forall \ t$).[41]

B.2 Producer problem

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change his price in a given period with a fixed probability, as in Calvo (1983). The probability of being able to change the price may differ across countries and is given by $1 - \alpha^i$.

Agent j in country i maximizes expected, discounted profits by choosing the price $\tilde{p}_t(j)$ taking into account that demand for his good depends on the chosen price and that the price may remain unchanged for some periods. Formally, the agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \left[\lambda_{t+k}(1-\tau^i)\tilde{p}_t(j)\tilde{y}_{t,t+k}(j) - V(\tilde{y}_{t,t+k}(j), z^i_{t+k}) \right] \tag{B.20}$$

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} T_{t+k}^{1-n} C_{t+k} \tag{B.21}$$

if the agent lives in country H or

$$\tilde{y}_{t,t+k}(f) = \left(\frac{\tilde{p}_t(f)}{P_{F,t+k}} \right)^{-\sigma} T_{t+k}^{-n} C_{t+k}, \tag{B.22}$$

if the agent lives in country F, where $\tilde{y}_{t,t+k}(j)$ denotes total demand of good j at time $t+k$ if the price $\tilde{p}_t(j)$ prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes $(1-\tau^i)\tilde{p}_t(j)\tilde{y}_{t,t+k}(j)$ are converted into utility units using the marginal utility of nominal revenues $\lambda_{t+k} = \frac{U_C(C_{t+k})}{P_{t+k}}$, which is the same for all agents in both countries due to perfect risk sharing within and across countries and due to purchasing power parity. The cost of production expressed in utility units is given by the function V.

The first order condition yields the optimal price

$$\tilde{p}_t(j) = \frac{\sigma}{(\sigma-1)(1-\tau^i)} \frac{E_t \sum_{k=o}^{\infty} (\alpha^i \beta)^k V_y(\tilde{y}_{t,t+k}(j), z^i_{t+k}) \tilde{y}_{t,t+k}(j)}{E_t \sum_{k=o}^{\infty} (\alpha^i \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(j)}, \tag{B.23}$$

[41] The result that, under Cobb-Douglas preferences, the terms of trade provide perfect insurance against output variations was already shown by Cole and Obstfeld (1991). Note that the result does not hinge upon the specification in which the expenditure share in the Cobb-Douglas function coincides with the population size n (a feature also common to the Obstfeld and Rogoff (2001) model). If the expenditure share does not coincide with the population size, as in Corsetti and Pesenti (2001), relative consumption across countries as well as relative income across countries are still constant over time. However, they are not equal to one, as in (B.15) and (B.19) respectively. Consumption and nominal output, then, differ across countries.

where V_y denotes the derivative of function V with respect to output $\tilde{y}(j)$. All agents that live in the same country and are able to reset their price in a certain period will set the same price, since they share identical preferences (function V) and face the same demand curves, which depend only on aggregate variables such as P_H, P_F, T, and C, and the common elasticity of substitution σ. Hence, in a given period, a fraction $1-\alpha^i$ of agents will set the same optimal price, while for a fraction α^i of agents the price from the previous period remains effective:

$$P_{H,t} = [\alpha^H P_{H,t-1}^{1-\sigma} + (1-\alpha^H)\tilde{p}_t(h)^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{B.24}$$
$$P_{F,t} = [\alpha^F P_{F,t-1}^{1-\sigma} + (1-\alpha^F)\tilde{p}_t(f)^{1-\sigma}]^{\frac{1}{1-\sigma}}.$$

When prices are flexible, the optimal price equation (B.23) for country H simplifies to

$$T_t^{n-1} = \frac{\sigma}{(\sigma-1)(1-\tau^H)} \frac{V_y(y_t^H, z_t^H)}{U_C(C_t)}, \tag{B.25}$$

and for country F to

$$T_t^n = \frac{\sigma}{(\sigma-1)(1-\tau^F)} \frac{V_y(y_t^F, z_t^F)}{U_C(C_t)}. \tag{B.26}$$

Note that the closed-economy counterpart is given by

$$1 = \frac{\sigma}{(\sigma-1)(1-\tau)} \frac{V_y(y_t, z_t)}{U_C(C_t)}. \tag{B.27}$$

Moreover, variations in the marginal disutility of production of one country relative to the other country are reflected in variations in the terms of trade. Dividing (B.26) by (B.25) yields

$$T_t = \frac{1-\tau^H}{1-\tau^F} \frac{V_y(y_t^F, z_t^F)}{V_y(y_t^H, z_t^H)}. \tag{B.28}$$

B.3 Terms of trade

It is necessary to express the terms of trade equation (B.9) in changes, since the model will only contain price changes (i.e., inflation) rather than price levels. Thus

$$\frac{T_t}{T_{t-1}} = \frac{P_{F,t}}{P_{F,t-1}} \frac{P_{H,t-1}}{P_{H,t}}. \tag{B.29}$$

B.4 Log-linearization

Since the model is not solvable in non-linear form, the equations need to be log-linearized. Given a variable X_t, the following definitions will be used:

$$X_t^W = nX_t^H + (1-n)X_t^F \tag{B.30}$$
$$X_t^R = X_t^F - X_t^H \tag{B.31}$$

Furthermore, deviations of the logarithm of a variable X_t from its steady state are denoted by $\tilde{X}_t$ under flexible prices and by $\hat{X}_t$ under sticky prices.

B.4.1 Flexible-price equilibrium

Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only. Accordingly, consumption, world output, and the terms of trade evolve as follows:[42]

$$\tilde{C}_t = \frac{\eta}{\rho + \eta} \overline{Y}_t^W \tag{B.32}$$

$$\tilde{T}_t = -\frac{\eta}{1 + \eta} \overline{Y}_t^R \tag{B.33}$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho + \eta} \overline{Y}_t^W . \tag{B.34}$$

The first equation is derived by log-linearizing (B.25) and (B.26) and taking the weighted average with weight n. The second equation is derived by subtracting the log-linear approximation of (B.25) from the log-linear approximation of (B.26). The third equation is derived by inserting the first two equations into the weighted average of the log-linear approximations of equations (B.17).

The following definitions were used:

$$\rho = -\frac{U_{CC}\overline{C}}{U_C} \tag{B.35}$$

denotes the inverse of the elasticity of intertemporal substitution in consumption,

$$\eta = \frac{V_{yy}\overline{C}}{V_y} \tag{B.36}$$

denotes the inverse of the elasticity of producing the differentiated good, and finally

$$\overline{Y}_t^i = -\frac{V_{yz}}{V_{yy}\overline{C}} \hat{z}_t^i \tag{B.37}$$

reparameterizes the productivity shock in country i.

The productivity shock in country i follows an AR(1) process of the form

$$\overline{Y}_t^i = \rho_i \overline{Y}_{t-1}^i + \nu_t^i, \tag{B.38}$$

where ν_t^i is a white noise process with var $\nu_t^i = 1$.

[42] In contrast to Benigno (2004), I abstract from fiscal policy shocks.

B.4.2 Sticky-price equilibrium

Under sticky prices, the system of equations is given by

$$E_t\hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}(\hat{R}_t - E_t\pi^W_{t+1}) \tag{B.39}$$

$$\hat{Y}^H_t = (1-n)\hat{T}_t + \hat{C}_t \tag{B.40}$$

$$\hat{Y}^F_t = -n\hat{T}_t + \hat{C}_t \tag{B.41}$$

$$\pi^H_t = (1-n)k^H_T(\hat{T}_t - \tilde{T}_t) + k^H_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^H_{t+1} \tag{B.42}$$

$$\pi^F_t = -nk^F_T(\hat{T}_t - \tilde{T}_t) + k^F_C(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi^F_{t+1} \tag{B.43}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi^F_t - \pi^H_t \tag{B.44}$$

$$\hat{R}_t = \phi_R\hat{R}_{t-1} + (1-\phi_R)(\phi_\pi\pi^W_t + \phi_Y\hat{Y}^W_t). \tag{B.45}$$

Equation (B.39) is the log-linear approximation of the Euler consumption equation (B.14), where $C^i_t = C_t$ and $\pi_t = \ln(P_t/P_{t-1})$. Recall that, due to perfect risk sharing, consumption is the same across countries, which implies that there is only one Euler equation. Equations (B.40) and (B.41) are log-linear approximations of the equations for aggregate demand (B.17).

Equations (B.42) and (B.43) represent the New Keynesian Phillips curves for country H and country F respectively, where $\pi^H_t = \ln(P_{H,t}/P_{H,t-1})$ and $\pi^F_t = \ln(P_{F,t}/P_{F,t-1})$. They are derived by combining the log-linear approximation of the optimal price (B.23) with the log-linear approximation of (B.24) for each country separately. The parameters in front of the terms of trade gap $(\hat{T}_t - \tilde{T}_t)$ and the consumption gap $(\hat{C}_t - \tilde{C}_t)$ are defined as follows (for $i = H, F$):

$$k^i_C = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i}\frac{\rho+\eta}{1+\sigma\eta} \tag{B.46}$$

$$k^i_T = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i}\frac{1+\eta}{1+\sigma\eta} \tag{B.47}$$

Equation (B.44) is the log-linear approximation of the terms of trade equation (B.29). Finally, equation (B.45) represents the Taylor-type interest rate rule, according to which the common monetary policy reacts to union-wide inflation and to union-wide output (measured as the weighted average of country-specific inflation and output respectively) with coefficients ϕ_π and ϕ_Y and engages in interest rate smoothing with coefficient ϕ_R.

C Flexible exchange rate regime

The main difference to the MU regime, of course, is that both countries possess their own currency and independent monetary policy. Notwithstanding, the model structure is to a large extent identical. The behavior of output, consumption, and the terms of trade under flexible prices is given by equations (B.32) through (B.34).

Under sticky prices, the system of equations is given by

$$E_t\hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}\left(n(\hat{R}_t^H - E_t\pi_{t+1}^H) + (1-n)(\hat{R}_t^F - E_t\pi_{t+1}^F)\right) \tag{C.1}$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \tag{C.2}$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \tag{C.3}$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi_{t+1}^H \tag{C.4}$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t\pi_{t+1}^F \tag{C.5}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta\hat{S}_t \tag{C.6}$$

$$E_t\Delta\hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F \tag{C.7}$$

$$\hat{R}_t^H = \phi_R\hat{R}_{t-1}^H + (1-\phi_R)(\phi_\pi\pi_t^H + \phi_Y\hat{Y}_t^H) \tag{C.8}$$

$$\hat{R}_t^F = \phi_R\hat{R}_{t-1}^F + (1-\phi_R)(\phi_\pi\pi_t^F + \phi_Y\hat{Y}_t^F). \tag{C.9}$$

Given the same assumptions as in the MU regime on the set of assets agents can trade in, on the agents' preferences, and on the law of one price, the result of perfect risk sharing carries over to the FX regime.[43] Therefore, consumption is described by one Euler equation (C.1). In contrast to the MU regime, the Euler equation contains two interest rates, since monetary policy is country-specific.

The equations for aggregate demand (C.2) and (C.3) as well as the New Keynesian Phillips curves (C.4) and (C.5) are the same as under the MU regime. Agents are assumed to set their price in the currency of their country (producer currency pricing). The assumption of no price discrimination and no transportation costs implies that the law of one price holds, which in turn implies that exchange rate pass-through is complete. The law of one price together with identical consumption preferences implies that purchasing power parity holds as well.

The terms of trade are now given by

$$T_t = \frac{S_t P_{F,t}}{P_{H,t}}, \tag{C.10}$$

where $P_{H,t}$ denotes the price of the bundle of differentiated goods produced in country H denominated in country H's currency, $P_{F,t}$ denotes the price of the bundle of differentiated goods produced in country F denominated in country F's currency, and S_t is the nominal exchange rate defined as the price of country F's currency in terms of country H's currency. First-differencing and log-linearizing the definition of the terms of trade yields (C.6).

[43] See Obstfeld and Rogoff (2001). By adopting the assumption from the MU regime that asset markets are incomplete across countries I deviate from Benigno and Benigno (2008), who assume asset markets across countries to be complete. With identical Cobb-Douglas preferences, however, risk sharing is perfect regardless of whether asset markets are complete or not.

Equation (C.7) represents the uncovered interest parity condition, which can be obtained by subtracting the log-linearized Euler equation of country F from the one of country H, using the fact that purchasing power parity holds. Thus, the expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, equations (C.8) and (C.9) represent the Taylor-type interest rate rules, according to which monetary policy reacts to country-specific inflation and output with coefficients ϕ_π and ϕ_Y and engages in interest rate smoothing with coefficient ϕ_R.

D Welfare loss function

This appendix contains the full derivation of the world welfare loss function. The world welfare loss function is the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion in the spirit of Woodford (2003).[44]

The average utility among agents in country H is given by

$$w_t^H = U(C_t) - \frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh, \tag{D.1}$$

and average utility among agents in country F is given by

$$w_t^F = U(C_t) - \frac{1}{1-n}\int_{1-n}^1 V(y_t(f), z_t^F)df. \tag{D.2}$$

The discounted value of the weighted average of the two flows is then given by

$$\tilde{W}_t = E_t \sum_{k=0}^{\infty} \beta^k (n w_{t+k}^H + (1-n) w_{t+k}^F). \tag{D.3}$$

Each term of the utility function is treated separately.

D.1 The term $U(C_t)$

Taking a second-order linear expansion of $U(C_t)$ around the steady state value $\overline{C}$ yields

$$U(C_t) = U(\overline{C}) + U_C(C_t - \overline{C}) + \frac{1}{2}U_{CC}(C_t - \overline{C})^2 + O(\|\xi\|^3), \tag{D.4}$$

where the term $O(\|\xi\|^3)$ groups all the terms that are of third or higher order in the deviations of the various variables from their steady state.

Furthermore, a second-order Taylor expansion to C_t yields

$$\frac{C_t - \overline{C}}{\overline{C}} = \hat{C}_t + \frac{1}{2}\hat{C}_t^2 + O(\|\xi\|^3) \Leftrightarrow C_t - \overline{C} = \overline{C}\hat{C}_t + \frac{1}{2}\overline{C}\hat{C}_t^2 + O(\|\xi\|^3), \tag{D.5}$$

where $\hat{C}_t = \ln(C_t) - \ln(\overline{C})$.

[44] The derivation follows Benigno (2003), Appendix D. Here, I do not abstract from exogenous government expenditures. The loss function without government expenditure shocks is identical.

Inserting (D.5) into (D.4) yields

$$\begin{aligned}
U(C_t) &= \underbrace{U(\overline{C})}_{=t.i.p} + U_C(\overline{C}\hat{C}_t + \frac{1}{2}\overline{C}\hat{C}_t^2) + \frac{1}{2}U_{CC}(\overline{C}\hat{C}_t + \frac{1}{2}\overline{C}\hat{C}_t^2)^2 + O(\|\xi\|^3) \\
&= U_C\overline{C}\hat{C}_t + \frac{1}{2}U_C\overline{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}(\overline{C}^2\hat{C}_t^2 + \underbrace{\overline{C}^2\hat{C}_t^3 + \frac{1}{4}\overline{C}^2\hat{C}_t^4}_{=O(\|\xi\|^3)}) + t.i.p. + O(\|\xi\|^3) \\
&= U_C\overline{C}\hat{C}_t + \frac{1}{2}U_C\overline{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}\overline{C}^2\hat{C}_t^2 + t.i.p. + O(\|\xi\|^3) \\
&= U_C\overline{C}\left[\hat{C}_t + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}\underbrace{\frac{U_{CC}}{U_C}}_{=-\rho}\overline{C}\,\hat{C}_t^2\right] + t.i.p. + O(\|\xi\|^3) \\
&= U_C\overline{C}\left[\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2\right] + t.i.p. + O(\|\xi\|^3), \qquad \text{(D.6)}
\end{aligned}$$

where the term $t.i.p.$ collects all the terms that are independent of monetary policy and independent of whether the two countries form a monetary union or not.

D.2 The term $\frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh$

A second-order Taylor expansion of the second term in (D.1) around a steady state, where $y_t(h) = \overline{Y}^H$ for all h and t, and where $z_t^H = 0$ for all t yields

$$\begin{aligned}
V(y_t(h), z_t^H) = V(\overline{Y}^H, 0) + V_y\left(y_t(h) - \overline{Y}^H\right) + V_z z_t^H + \frac{1}{2}V_{yy}\left(y_t(h) - \overline{Y}^H\right)^2 \\
+ V_{yz}\left(y_t(h) - \overline{Y}^H\right)z_t^H + \frac{1}{2}V_{zz}z_t^{H2} + O(\|\xi\|^3). \qquad \text{(D.7)}
\end{aligned}$$

Global demand for a differentiated good produced in country H (including demand from government expenditures G^H) can be expressed by

$$\begin{aligned}
y(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma}\left[T^{1-n}C^W + G^H\right] \\
&= \underbrace{\left(\frac{p(h)}{P_H}\right)^{-\sigma}T^{1-n}C^W}_{=y^d(h)} + \underbrace{\left(\frac{p(h)}{P_H}\right)^{-\sigma}G^H}_{=y^g(h)} \\
&= y^d(h) + y^g(h). \qquad \text{(D.8)}
\end{aligned}$$

A second-order Taylor expansion to $y_t^d(h)$ yields

$$y_t^d(h) - \overline{Y}^H = \overline{Y}^H\hat{y}_t^d(h) + \frac{1}{2}\overline{Y}^H\hat{y}_t^d(h)^2 + O(\|\xi\|^3), \qquad \text{(D.9)}$$

where $\hat{y}_t^d(h) = \ln(y_t^d(h)) - \ln(\overline{Y}^H)$.

A second-order Taylor expansion to $y_t^g(h)$ yields

$$y_t^g(h) = \overline{Y}^H \hat{y}_t^g(h) + \frac{1}{2}\overline{Y}^H \hat{y}_t^g(h)^2 + O(\|\xi\|^3). \tag{D.10}$$

Combining (D.8), (D.9), and (D.10) gives

$$\begin{aligned} y_t(h) - \overline{Y}^H &= y_t^d(h) + y_t^g(h) - \overline{Y}^H \\ &= \overline{Y}^H \hat{y}_t^d(h) + \frac{1}{2}\overline{Y}^H \hat{y}_t^d(h)^2 + \overline{Y}^H \hat{y}_t^g(h) + \frac{1}{2}\overline{Y}^H \hat{y}_t^g(h)^2 \\ &= \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2}\hat{y}_t^g(h)^2 \right). \end{aligned} \tag{D.11}$$

Inserting into (D.7) and simplifying yields

$$
\begin{aligned}
V(y_t(h), z_t^H) = &\underbrace{V(\overline{Y}^H, 0)}_{=t.i.p.} + V_y \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \underbrace{\hat{y}_t^g(h) + \frac{1}{2}\hat{y}_t^g(h)^2}_{=t.i.p.} \right) \\
&+ \underbrace{V_z z_t^H}_{=t.i.p.} + \frac{1}{2} V_{yy} \overline{Y}^{H2} \underbrace{\left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2}\hat{y}_t^g(h)^2 \right)^2}_{=\hat{y}_t(h)^2 + t.i.p. + O(\|\xi\|^3)} \\
&+ V_{yz} \overline{Y}^H \underbrace{\left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2}\hat{y}_t^g(h)^2 \right) z_t^H}_{=\hat{y}_t^d(h) z_t^H + t.i.p. + O(\|\xi\|^3)} + \underbrace{\frac{1}{2} V_{zz} z_t^{H2}}_{=t.i.p.} \\
&+ O(\|\xi\|^3) \\
&= V_y \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 \right) + \frac{1}{2} V_{yy} \overline{Y}^{H2} \hat{y}_t(h)^2 + V_{yz} \overline{Y}^H \hat{y}_t^d(h) z_t^H \\
&+ t.i.p. + O(\|\xi\|^3) \\
&= V_y \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{1}{2} \underbrace{\frac{V_{yy}}{V_y} \overline{Y}^H}_{\equiv \eta} \hat{y}_t(h)^2 + \frac{V_{yz}}{V_y} \hat{y}_t^d(h) z_t^H \right) \\
&+ t.i.p. + O(\|\xi\|^3) \\
&= V_y \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2 + \underbrace{\frac{V_{yy}}{V_y} \overline{Y}^H}_{=\eta} \underbrace{\frac{V_{yz} z_t^H}{V_{yy} \overline{Y}^H}}_{\equiv -\overline{Y}_t^H} \hat{y}_t^d(h) \right) \\
&+ t.i.p. + O(\|\xi\|^3) \\
&= V_y \overline{Y}^H \left(\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2 - \eta \hat{y}_t^d(h) \overline{Y}_t^H \right) \\
&+ t.i.p. + O(\|\xi\|^3)
\end{aligned} \tag{D.12}
$$

Next, a relationship between V_y and U_C will be derived. In the steady state, equations (B.25) and (B.26) can be expressed as

$$(1 - \tau^H) U_C(\overline{C}) = \frac{\sigma}{\sigma - 1} \overline{T}^{1-n} V_y \left(\overline{T}^{1-n} \overline{C}, 0 \right) \tag{D.13}$$

$$(1 - \tau^F) U_C(\overline{C}) = \frac{\sigma}{\sigma - 1} \overline{T}^{-n} V_y \left(\overline{T}^{-n} \overline{C}, 0 \right), \tag{D.14}$$

which can be rearranged to

$$(1 - \Phi^H) U_C(\overline{C}) = \overline{T}^{1-n} V_y \left(\overline{T}^{1-n} \overline{C}, 0 \right) \tag{D.15}$$

$$(1-\Phi^F)U_C(\overline{C}) = \overline{T}^{-n}V_y\left(\overline{T}^{-n}\overline{C}, 0\right) \tag{D.16}$$

with

$$(1-\Phi^H) = (1-\tau^H)\frac{\sigma-1}{\sigma} \tag{D.17}$$

$$(1-\Phi^F) = (1-\tau^F)\frac{\sigma-1}{\sigma}. \tag{D.18}$$

The analysis must be restricted to the case in which distortions from the efficient steady state are small, i.e., the deviations of Φ^H and Φ^F are at least of order $O(\|\xi\|)$. Furthermore, for reasons of tractability, it is assumed that $\Phi^H = \Phi^F$. If $\tau^H = \tau^F$, it follows that $\overline{T} = 1$ and $\overline{Y}^H = \overline{Y}^F = \overline{C}$. Then, equation (D.15) yields

$$\begin{aligned}(1-\Phi^H)U_C(\overline{C}) &= \overline{T}^{1-n}V_y\left(\overline{T}^{1-n}\overline{C}, 0\right)\\ &= V_y\left(\overline{Y}^H, 0\right)\\ &= V_y.\end{aligned} \tag{D.19}$$

Plugging into (D.12) yields

$$\begin{aligned}V(y_t(h), z_t^H) = (1-\Phi^H)U_C(\overline{C})\overline{Y}^H\Bigg(&\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2\\ &- \eta\hat{y}_t^d(h)\overline{Y}_t^H\Bigg) + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.20}$$

With $\overline{Y}^H = \overline{C}$ and $U_C(\overline{C}) = U_C$ together with the small distortion assumption, i.e., the product of Φ^H with second-order terms can be neglected, the last equation can be written as

$$\begin{aligned}V(y_t(h), z_t^H) = U_C\overline{C}\Bigg((1-\Phi^H)&\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2\\ &- \eta\hat{y}_t^d(h)\overline{Y}_t^H\Bigg) + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.21}$$

Integrating across agents belonging to country H yields

$$\begin{aligned}\frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh = U_C\overline{C}\Bigg(&(1-\Phi^H)\frac{1}{n}\int_0^n \hat{y}_t^d(h)dh + \frac{1}{2}\frac{1}{n}\int_0^n \hat{y}_t^d(h)^2dh\\ &+ \frac{\eta}{2}\frac{1}{n}\int_0^n \hat{y}_t(h)^2dh - \eta\frac{1}{n}\int_0^n \hat{y}_t^d(h)dh\overline{Y}_t^H\Bigg)\\ &+ t.i.p. + O(\|\xi\|^3)\\ = U_C\overline{C}\Bigg(&(1-\Phi^H)E_h\hat{y}_t^d(h) + \frac{1}{2}E_h\hat{y}_t^d(h)^2 + \frac{\eta}{2}E_h\hat{y}_t(h)^2\\ &- \eta E_h\hat{y}_t^d(h)\overline{Y}_t^H\Bigg) + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.22}$$

Recall the basic relationship

$$\text{var}(X) = E(X^2) - (E(X))^2 \Leftrightarrow E(X^2) = \text{var}(X) + (E(X))^2. \tag{D.23}$$

Thus,

$$\begin{aligned}\frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh = U_C\overline{C}\Bigg((1-\Phi^H)E_h\hat{y}_t^d(h) + \frac{1}{2}\left(\text{var}_h\,\hat{y}_t^d(h) + [E_h\hat{y}_t^d(h)]^2\right) \\ + \frac{\eta}{2}\left(\text{var}_h\,\hat{y}_t(h) + [E_h\hat{y}_t(h)]^2\right) - \eta E_h\hat{y}_t^d(h)\overline{Y}_t^H\Bigg) \\ + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.24}$$

D.3 Expanding Y_t^H

Recall the aggregator

$$Y_t^H = \left\{\frac{1}{n}\int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}}dh\right\}^{\frac{\sigma}{\sigma-1}}. \tag{D.25}$$

I conduct a second-order Taylor series expansion of both sides of the equation. Note that the more general case of (D.5) is given by

$$\frac{C_t^a - \overline{C}^a}{\overline{C}^a} = a\hat{C}_t + \frac{1}{2}a^2\hat{C}_t^2 + O(\|\xi\|^3). \tag{D.26}$$

Thus, approximating $y_t(h)^{\frac{\sigma-1}{\sigma}}$ up to second-order yields

$$y_t(h)^{\frac{\sigma-1}{\sigma}} = \overline{Y}^{H\frac{\sigma-1}{\sigma}}\left[1 + \frac{\sigma-1}{\sigma}\hat{y}_t(h) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)^2\hat{y}_t(h)^2\right] + O(\|\xi\|^3). \tag{D.27}$$

Inserting into (D.25) yields

$$\begin{aligned}Y_t^{H\frac{\sigma-1}{\sigma}} &= \frac{1}{n}\int_0^n \overline{Y}^{H\frac{\sigma-1}{\sigma}}\left[1 + \frac{\sigma-1}{\sigma}\hat{y}_t(h) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)^2\hat{y}_t(h)^2\right]dh + O(\|\xi\|^3) \\ &= \overline{Y}^{H\frac{\sigma-1}{\sigma}}\left\{\frac{1}{n}\int_0^n 1dh + \frac{\sigma-1}{\sigma}\frac{1}{n}\int_0^n \hat{y}_t(h)dh + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)^2\frac{1}{n}\int_0^n \hat{y}_t(h)^2dh\right\} \\ &+ O(\|\xi\|^3) \\ &= \overline{Y}^{H\frac{\sigma-1}{\sigma}}\left\{1 + \frac{\sigma-1}{\sigma}E_h\hat{y}_t(h) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)^2 E_h\hat{y}_t(h)^2\right\} + O(\|\xi\|^3).\end{aligned} \tag{D.28}$$

A second-order Taylor expansion to $Y_t^{H\frac{\sigma-1}{\sigma}}$ yields

$$Y_t^{H\frac{\sigma-1}{\sigma}} = \overline{Y}^{H\frac{\sigma-1}{\sigma}}\left\{1 + \frac{\sigma-1}{\sigma}\hat{Y}_t^H + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)^2\hat{Y}_t^{H^2}\right\} + O(\|\xi\|^3). \tag{D.29}$$

Equating the previous two equations yields

$$\hat{Y}_t^H + \frac{1}{2}\frac{\sigma-1}{\sigma}\hat{Y}_t^{H^2} = E_h\hat{y}_t(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}E_h\hat{y}_t(h)^2 + O(\|\xi\|^3). \tag{D.30}$$

This expression raised to the power of two gives

$$\hat{Y}_t^{H^2} = (E_h\hat{y}_t(h))^2 + O(\|\xi\|^3). \tag{D.31}$$

Inserting back into (D.30) and simplifying yields

$$\begin{aligned}
\hat{Y}_t^H + \frac{1}{2}\frac{\sigma-1}{\sigma}(E_h\hat{y}_t(h))^2 &= E_h\hat{y}_t(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}E_h\hat{y}_t(h)^2 + O(\|\xi\|^3) \\
\hat{Y}_t^H &= E_h\hat{y}_t(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}\left[E_h\hat{y}_t(h)^2 - (E_h\hat{y}_t(h))^2\right] + O(\|\xi\|^3) \\
\hat{Y}_t^H &= E_h\hat{y}_t(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h\hat{y}_t(h) + O(\|\xi\|^3).
\end{aligned} \tag{D.32}$$

Analogously,

$$\hat{Y}_t^{H,d} = E_h\hat{y}_t^d(h) + \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h\hat{y}_t^d(h) + O(\|\xi\|^3). \tag{D.33}$$

Using the previous two equations to substitute out $E_h\hat{y}_t(h)$ and $E_h\hat{y}_t^d(h)$ in (D.24) gives

$$
\begin{aligned}
&\frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh \\
&= U_C\overline{C}\Bigg((1-\Phi^H)\left[\hat{Y}_t^{H,d} - \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h)\right] \\
&+ \frac{1}{2}\left(\operatorname{var}_h \hat{y}_t^d(h) + \left[\hat{Y}_t^{H,d} - \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h)\right]^2\right) \\
&+ \frac{\eta}{2}\left(\operatorname{var}_h \hat{y}_t(h) + \left[\hat{Y}_t^{H} - \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t(h)\right]^2\right) \\
&- \eta\left[\hat{Y}_t^{H,d} - \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h)\right]\overline{Y}_t^H\Bigg) + t.i.p. + O(\|\xi\|^3) \\
&= U_C\overline{C}\Bigg((1-\Phi^H)\hat{Y}_t^{H,d} - \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h) + \underbrace{\Phi^H\frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h)}_{=O(\|\xi\|^3)} \\
&+ \frac{1}{2}\left(\operatorname{var}_h \hat{y}_t^d(h) + \hat{Y}_t^{H,d^2} - \underbrace{\hat{Y}_t^{H,d}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h) + \frac{1}{4}\left(\frac{\sigma-1}{\sigma}\right)^2(\operatorname{var}_h \hat{y}_t^d(h))^2}_{=O(\|\xi\|^3)}\right) \\
&+ \frac{\eta}{2}\left(\operatorname{var}_h \hat{y}_t(h) + \hat{Y}_t^{H^2} - \underbrace{\hat{Y}_t^{H}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t(h) + \frac{1}{4}\left(\frac{\sigma-1}{\sigma}\right)^2(\operatorname{var}_h \hat{y}_t(h))^2}_{=O(\|\xi\|^3)}\right) \\
&- \eta\hat{Y}_t^{H,d}\overline{Y}_t^H + \underbrace{\frac{\eta}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h)\overline{Y}_t^H}_{=O(\|\xi\|^3)}\Bigg) + t.i.p. + O(\|\xi\|^3) \\
&= U_C\overline{C}\Bigg((1-\Phi^H)\hat{Y}_t^{H,d} + \frac{1}{2}\hat{Y}_t^{H,d^2} + \frac{\eta}{2}\hat{Y}_t^{H^2} - \eta\hat{Y}_t^{H,d}\overline{Y}_t^H \\
&- \frac{1}{2}\frac{\sigma-1}{\sigma}\operatorname{var}_h \hat{y}_t^d(h) + \frac{1}{2}\operatorname{var}_h \hat{y}_t^d(h) + \frac{\eta}{2}\operatorname{var}_h \hat{y}_t(h)\Bigg) + t.i.p. + O(\|\xi\|^3). \qquad \text{(D.34)}
\end{aligned}
$$

Note that since $\hat{y}_t(h) = \hat{y}_t^d(h)$, $\operatorname{var}_h \hat{y}_t(h) = \operatorname{var}_h \hat{y}_t^d(h)$. Therefore, the previous expression can be simplified to

$$
\begin{aligned}
\frac{1}{n}\int_0^n V(y_t(h), z_t^H)dh = U_C\overline{C}\Bigg((1-\Phi^H)\hat{Y}_t^{H,d} + \frac{1}{2}\hat{Y}_t^{H,d^2} + \frac{\eta}{2}\hat{Y}_t^{H^2} - \eta\hat{Y}_t^{H,d}\overline{Y}_t^H \\
+ \frac{1}{2}(\sigma^{-1}+\eta)\operatorname{var}_h \hat{y}_t(h)\Bigg) + t.i.p. + O(\|\xi\|^3). \qquad \text{(D.35)}
\end{aligned}
$$

D.4 Combining the results

Inserting (D.35) and (D.6) into (D.1) yields

$$w_t^H = U_C\overline{C}\Bigg(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - (1-\Phi^H)\hat{Y}_t^{H,d} - \frac{1}{2}\hat{Y}_t^{H,d^2} - \frac{\eta}{2}\hat{Y}_t^{H^2} + \eta\hat{Y}_t^{H,d}\overline{Y}_t^H - \frac{1}{2}(\sigma^{-1}+\eta)\operatorname{var}_h \hat{y}_t(h)\Bigg) + t.i.p. + O(\|\xi\|^3). \tag{D.36}$$

Average utility among agents living in country F is derived completely analogously. Thus,

$$w_t^F = U_C\overline{C}\Bigg(\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - (1-\Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2}\hat{Y}_t^{F,d^2} - \frac{\eta}{2}\hat{Y}_t^{F^2} + \eta\hat{Y}_t^{F,d}\overline{Y}_t^F - \frac{1}{2}(\sigma^{-1}+\eta)\operatorname{var}_f \hat{y}_t(f)\Bigg) + t.i.p. + O(\|\xi\|^3). \tag{D.37}$$

World welfare consists of the linear combination of country H's and country F's welfare with weight n and $1-n$:

$$\begin{aligned} w_t &= nw_t^H + (1-n)w_t^F \\ &= U_C\overline{C}\Bigg(\underbrace{n\hat{C}_t + (1-n)\hat{C}_t}_{=\hat{C}_t} + \frac{1}{2}(1-\rho)(\underbrace{n\hat{C}_t^2 + (1-n)\hat{C}_t^2}_{=\hat{C}_t^2}) \\ &\quad - n(1-\Phi^H)\hat{Y}_t^{H,d} - (1-n)(1-\Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2}\left(n\hat{Y}_t^{H,d^2} + (1-n)\hat{Y}_t^{F,d^2}\right) \\ &\quad - \frac{\eta}{2}\left(n\hat{Y}_t^{H^2} + (1-n)\hat{Y}_t^{F^2}\right) + \eta\left(n\hat{Y}_t^{H,d}\overline{Y}_t^H + (1-n)\hat{Y}_t^{F,d}\overline{Y}_t^H\right) \\ &\quad - \frac{1}{2}(\sigma^{-1}+\eta)[n\operatorname{var}_h \hat{y}_t(h) + (1-n)\operatorname{var}_f \hat{y}_t(f)]\Bigg) + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.38}$$

Inserting the expressions

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t + g_t^H \tag{D.39}$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^F \tag{D.40}$$

$$\hat{Y}_t^{H,d} = (1-n)\hat{T}_t + \hat{C}_t \tag{D.41}$$

$$\hat{Y}_t^{F,d} = -n\hat{T}_t + \hat{C}_t \tag{D.42}$$

and simplifying yields

$$
\begin{aligned}
w_t = U_C\overline{C}\Bigg(&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\
&- n(1-\Phi^H)[(1-n)\hat{T}_t + \hat{C}_t] - (1-n)(1-\Phi^F)[-n\hat{T}_t + \hat{C}_t] \\
&- \frac{1}{2}\left(n[(1-n)\hat{T}_t + \hat{C}_t]^2 + (1-n)[-n\hat{T}_t + \hat{C}_t]^2\right) \\
&- \frac{\eta}{2}\left(n[(1-n)\hat{T}_t + \hat{C}_t + g_t^H]^2 + (1-n)[-n\hat{T}_t + \hat{C}_t + g_t^F]^2\right) \\
&+ \eta\left(n[(1-n)\hat{T}_t + \hat{C}_t]\overline{Y}_t^H + (1-n)[-n\hat{T}_t + \hat{C}_t]\overline{Y}_t^F\right) \\
&- \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Bigg) \\
&+ t.i.p. + O(\|\xi\|^3) \qquad \text{(D.43)} \\
= U_C\overline{C}\Bigg(&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\
&- (1-\Phi^H)n(1-n)\hat{T}_t - n\hat{C}_t + n\Phi^H\hat{C}_t + (1-\Phi^F)n(1-n)\hat{T}_t - (1-n)\hat{C}_t + (1-n)\Phi^F\hat{C}_t \\
&- \frac{1}{2}\left(n(1-n)^2\hat{T}_t^2 + 2n(1-n)\hat{T}_t\hat{C}_t + n\hat{C}_t^2 + (1-n)n^2\hat{T}_t^2 - 2n(1-n)\hat{T}_t\hat{C}_t + (1-n)\hat{C}_t^2\right) \\
&- \frac{\eta}{2}\Bigg(n(1-n)^2\hat{T}_t^2 + n\hat{C}_t^2 + \underbrace{ng_t^{H2}}_{=t.i.p} + 2n(1-n)\hat{T}_t\hat{C}_t + 2n(1-n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^H \\
&+ (1-n)n^2\hat{T}_t^2 + (1-n)\hat{C}_t^2 + \underbrace{(1-n)g_t^{F2}}_{=t.i.p.} - 2n(1-n)\hat{T}_t\hat{C}_t - 2n(1-n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^F\Bigg) \\
&+ \eta\left(n(1-n)\hat{T}_t\overline{Y}_t^H + n\hat{C}_t\overline{Y}_t^H - (1-n)n\hat{T}_t\overline{Y}_t^F + (1-n)\hat{C}_t\overline{Y}_t^F\right) \\
&- \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Bigg) \\
&+ t.i.p. + O(\|\xi\|^3). \qquad \text{(D.44)}
\end{aligned}
$$

Further simplification gives

$$
\begin{aligned}
w_t = U_C \overline{C} \Bigg(& \hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\
& - \underbrace{\Big((1-\Phi^H) - (1-\Phi^F)\Big)}_{=0 \text{ if } \Phi^H = \Phi^F} n(1-n)\hat{T}_t \underbrace{-n\hat{C}_t - (1-n)\hat{C}_t}_{=-\hat{C}_t} + \hat{C}_t \left[n\Phi^H + (1-n)\Phi^F \right] \\
& - \frac{1}{2} \left(\hat{C}_t^2 + \underbrace{\left[n(1-n)^2 + (1-n)n^2 \right]}_{=n(1-n)} \hat{T}_t^2 \right) \\
& - \frac{\eta}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2n(1-n)\hat{T}_t \underbrace{\left[g_t^H - g_t^F \right]}_{=-g_t^R} + 2\hat{C}_t \underbrace{\left[n g_t^H + (1-n) g_t^F \right]}_{=g_t^W} \right) \\
& + \eta \left(\hat{C}_t \underbrace{\left[n\overline{Y}_t^H + (1-n)\overline{Y}_t^F \right]}_{=\overline{Y}_t^W} + n(1-n)\hat{T}_t \underbrace{\left[\overline{Y}_t^H - \overline{Y}_t^F \right]}_{=-\overline{Y}_t^R} \right) \\
& - \frac{1}{2}(\sigma^{-1} + \eta)[n \operatorname{var}_h \hat{y}_t(h) + (1-n) \operatorname{var}_f \hat{y}_t(f)] \Bigg) \\
& + t.i.p. + O(\|\xi\|^3).
\end{aligned} \tag{D.45}
$$

This yields[45]

$$
\begin{aligned}
w_t = U_C \overline{C} \Bigg(& \hat{C}_t \left[n\Phi^H + (1-n)\Phi^F \right] + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\
& + \eta \left(\hat{C}_t \overline{Y}_t^W - n(1-n)\hat{T}_t \overline{Y}_t^R \right) \\
& - \frac{1}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 \right) \\
& - \frac{\eta}{2} \left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2\hat{C}_t g_t^W - 2n(1-n)\hat{T}_t g_t^R \right) \\
& - \frac{1}{2}(\sigma^{-1} + \eta)[n \operatorname{var}_h \hat{y}_t(h) + (1-n) \operatorname{var}_f \hat{y}_t(f)] \Bigg) \\
& + t.i.p. + O(\|\xi\|^3).
\end{aligned} \tag{D.46}
$$

[45] This equation corresponds to equation (E.21) in Benigno (2003), Appendix D, except for a typo: There must be a minus sign in front of $n(1-n)\hat{T}_t \overline{Y}_t^R$.

Factoring out a minus sign yields

$$\begin{aligned} w_t = -U_C\overline{C}\Bigg(& -\hat{C}_t\left[n\Phi^H + (1-n)\Phi^F\right] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \\ & - \eta\left(\hat{C}_t\overline{Y}_t^W - n(1-n)\hat{T}_t\overline{Y}_t^R\right) \\ & + \frac{1}{2}\left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2\right) \\ & + \frac{\eta}{2}\left(\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2\hat{C}_t g_t^W - 2n(1-n)\hat{T}_t g_t^R\right) \\ & + \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.47}$$

Expanding gives

$$\begin{aligned} w_t = -U_C\overline{C}\Bigg(& -\hat{C}_t\left[n\Phi^H + (1-n)\Phi^F\right] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \\ & - \eta\hat{C}_t\overline{Y}_t^W + \eta n(1-n)\hat{T}_t\overline{Y}_t^R \\ & + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}n(1-n)\hat{T}_t^2 \\ & + \frac{\eta}{2}\hat{C}_t^2 + \frac{\eta}{2}n(1-n)\hat{T}_t^2 + \eta\hat{C}_t g_t^W - \eta n(1-n)\hat{T}_t g_t^R \\ & + \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.48}$$

Rearranging yields

$$
\begin{aligned}
w_t = &-U_C\overline{C}\Bigg(-\hat{C}_t\left[n\Phi^H+(1-n)\Phi^F\right]\\
&+\frac{1}{2}(\rho+\eta)\hat{C}_t^2-\underbrace{\eta\left[\overline{Y}_t^W-g_t^W\right]}_{=(\rho+\eta)\tilde{C}_t}\hat{C}_t\\
&+\frac{1}{2}(1+\eta)n(1-n)\hat{T}_t^2-n(1-n)\underbrace{\eta\left[g_t^R-\overline{Y}_t^R\right]}_{=(1+\eta)\tilde{T}_t}\hat{T}_t\\
&+\frac{1}{2}(\sigma^{-1}+\eta)[n\operatorname{var}_h\hat{y}_t(h)+(1-n)\operatorname{var}_f\hat{y}_t(f)]\Bigg)\\
&+t.i.p.+O(\|\xi\|^3) \qquad \text{(D.49)}\\
= &-U_C\overline{C}\Bigg(-\hat{C}_t\left[n\Phi^H+(1-n)\Phi^F\right]\\
&+\frac{1}{2}(\rho+\eta)\left[\hat{C}_t^2-2\tilde{C}_t\hat{C}_t\right]\\
&+\frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t^2-2\tilde{T}_t\hat{T}_t\right]\\
&+\frac{1}{2}(\sigma^{-1}+\eta)[n\operatorname{var}_h\hat{y}_t(h)+(1-n)\operatorname{var}_f\hat{y}_t(f)]\Bigg)\\
&+t.i.p.+O(\|\xi\|^3) \qquad \text{(D.50)}\\
= &-U_C\overline{C}\Bigg(-\hat{C}_t\left[n\Phi^H+(1-n)\Phi^F\right]\\
&+\frac{1}{2}(\rho+\eta)\underbrace{\left[\hat{C}_t^2-2\tilde{C}_t\hat{C}_t+\tilde{C}_t^2\right]}_{=\left[\hat{C}_t-\tilde{C}_t\right]^2}-\underbrace{\frac{1}{2}(\rho+\eta)\tilde{C}_t^{W^2}}_{=t.i.p}\\
&+\frac{1}{2}(1+\eta)n(1-n)\underbrace{\left[\hat{T}_t^2-2\tilde{T}_t\hat{T}_t+\tilde{T}_t^2\right]}_{=\left[\hat{T}_t-\tilde{T}_t\right]^2}-\underbrace{\frac{1}{2}(1+\eta)n(1-n)\tilde{T}_t^2}_{=t.i.p.}\\
&+\frac{1}{2}(\sigma^{-1}+\eta)[n\operatorname{var}_h\hat{y}_t(h)+(1-n)\operatorname{var}_f\hat{y}_t(f)]\Bigg)\\
&+t.i.p.+O(\|\xi\|^3) \qquad \text{(D.51)}\\
= &-U_C\overline{C}\Bigg(-\hat{C}_t\left[n\Phi^H+(1-n)\Phi^F\right]\\
&+\frac{1}{2}(\rho+\eta)\left[\hat{C}_t-\tilde{C}_t\right]^2+\frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t-\tilde{T}_t\right]^2\\
&+\frac{1}{2}(\sigma^{-1}+\eta)[n\operatorname{var}_h\hat{y}_t(h)+(1-n)\operatorname{var}_f\hat{y}_t(f)]\Bigg)\\
&+t.i.p.+O(\|\xi\|^3). \qquad \text{(D.52)}
\end{aligned}
$$

The difference between steady-state consumption under the presence of the monopolistic distortion $\overline{C}$ and the efficient level of consumption C^* (situation without the distortion) is given by

$$\bar{c} = -\ln\left(\frac{\overline{C}}{C^*}\right) = \frac{n\Phi^H + (1-n)\Phi^F}{\rho+\eta}. \tag{D.53}$$

Inserting yields

$$\begin{aligned}
w_t = &-U_C\overline{C}\Big(-\hat{C}_t(\rho+\eta)\bar{c} \\
&+ \frac{1}{2}(\rho+\eta)\left[\hat{C}_t - \tilde{C}_t\right]^2 + \frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t - \tilde{T}_t\right]^2 \\
&+ \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Big) \\
&+ t.i.p. + O(\|\xi\|^3) \qquad \text{(D.54)} \\
= &-U_C\overline{C}\Big(\frac{1}{2}(\rho+\eta)\left(\left[\hat{C}_t - \tilde{C}_t\right]^2 - 2\hat{C}_t\bar{c}\right) + \frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t - \tilde{T}_t\right]^2 \\
&+ \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Big) \\
&+ t.i.p. + O(\|\xi\|^3) \qquad \text{(D.55)} \\
= &-U_C\overline{C}\Big(\frac{1}{2}(\rho+\eta)\left(\underbrace{\left[\hat{C}_t - \tilde{C}_t\right]}_{\equiv c_t} - \bar{c}\right)^2 + \frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t - \tilde{T}_t\right]^2 \\
&+ \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Big) \\
&+ t.i.p. + O(\|\xi\|^3). \qquad \text{(D.56)}
\end{aligned}$$

Thus,

$$\begin{aligned}
w_t = &-U_C\overline{C}\Big(\frac{1}{2}(\rho+\eta)\left[c_t - \bar{c}\right]^2 + \frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_t - \tilde{T}_t\right]^2 \\
&+ \frac{1}{2}(\sigma^{-1}+\eta)[n\,\mathrm{var}_h\,\hat{y}_t(h) + (1-n)\,\mathrm{var}_f\,\hat{y}_t(f)]\Big) \\
&+ t.i.p. + O(\|\xi\|^3). \qquad \text{(D.57)}
\end{aligned}$$

Next, $\mathrm{var}_h\,\hat{y}_t(h)$ can be linked to inflation π_t^H and $\mathrm{var}_f\,\hat{y}_t(f)$ to π_t^F. Note that

$$\mathrm{var}_h\,\hat{y}_t(h) = \mathrm{var}_h\,y_t(h) = \sigma^2\,\mathrm{var}_h\,\hat{p}_t(h) = \sigma^2\,\mathrm{var}_h\,p_t(h). \tag{D.58}$$

Then, the following relationship is derived in a completely analogous way as in Woodford (2003):

$$\sum_{t=0}^{\infty} \beta^t \operatorname{var}_h p_t(h) = \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^{H^2} + t.i.p. + O(\|\xi\|^3). \tag{D.59}$$

Finally, calculating the discounted value of all future utility flows yields

$$\begin{aligned}\tilde{W}_t &= E_t \sum_{k=0}^{\infty} \beta^k w_{t+k} \\ &= E_t \sum_{k=0}^{\infty} \beta^k (-U_C \overline{C}) \Bigg(\frac{1}{2}(\rho+\eta)\left[c_{t+k} - \bar{c}\right]^2 + \frac{1}{2}(1+\eta)n(1-n)\left[\hat{T}_{t+k} - \tilde{T}_{t+k}\right]^2 \\ &+ \frac{1}{2}(\sigma^{-1}+\eta)[n \operatorname{var}_h \hat{y}_{t+k}(h) + (1-n)\operatorname{var}_f \hat{y}_{t+k}(f)] \Bigg) \\ &+ t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.60}$$

Thus,

$$\begin{aligned}\tilde{W}_t = -\frac{1}{2} U_C \overline{C} E_t \sum_{k=0}^{\infty} \beta^k \Bigg(&(\rho+\eta)\left[c_{t+k} - \bar{c}\right]^2 + (1+\eta)n(1-n)\left[\hat{T}_{t+k} - \tilde{T}_{t+k}\right]^2 \\ &+ \sigma(1+\sigma\eta) n \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)} \pi_{t+k}^{H^2} \\ &+ \sigma(1+\sigma\eta)(1-n) \frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)} \pi_{t+k}^{F^2} \Bigg) \\ &+ t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.61}$$

This expression is equivalent to equation (26) in Benigno (2004) with $\bar{c} = 0$, i.e., the monopolistic distortion is perfectly neutralized by an appropriate subsidy, and with $c_t = y_t^W$.

Dividing both sides by $U_C \overline{C}$, letting $\beta \to 1$, and with $\bar{c} = 0$, the loss function can be written as

$$\begin{aligned} W_t = -\frac{1}{2} \Bigg(&(\rho+\eta)\operatorname{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\operatorname{var}(\hat{T}_t - \tilde{T}_t) \\ &+ \sigma(1+\sigma\eta) n \frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)} \operatorname{var} \pi_t^H \\ &+ \sigma(1+\sigma\eta)(1-n) \frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)} \operatorname{var} \pi_t^F \Bigg) \\ &+ t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.62}$$

This equation corresponds to equation (2.24) in the main text.

D.5 Special case: $\alpha^H = \alpha^F$

When prices are equally rigid in the two countries ($\alpha^H = \alpha^F$), the world welfare loss function can be simplified further in a useful way.

When $\alpha^H = \alpha^F = \alpha$, it immediately follows that

$$\begin{aligned}\tilde{W}_t = -\frac{1}{2}U_C\overline{C}E_t\sum_{k=0}^{\infty}\beta^k\Bigg((\rho+\eta)\left[c_{t+k}-\bar{c}\right]^2 + (1+\eta)n(1-n)\left[\hat{T}_{t+k}-\tilde{T}_{t+k}\right]^2 \\ + \sigma(1+\sigma\eta)\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\left[n\pi_{t+k}^{H^2}+(1-n)\pi_{t+k}^{F^2}\right]\Bigg) \\ + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.63}$$

The last term in square brackets can be modified in the following way:

$$\begin{aligned}&n\pi_t^{H^2}+(1-n)\pi_t^{F^2} \\ &= n\underbrace{\left(n\pi_t^{H^2}+(1-n)\pi_t^{H^2}\right)}_{=\pi_t^{H^2}} + (1-n)\underbrace{\left(n\pi_t^{F^2}+(1-n)\pi_t^{F^2}\right)}_{=\pi_t^{F^2}} \\ &= n^2\pi_t^{H^2}+n(1-n)\pi_t^{H^2}+n(1-n)\pi_t^{F^2}+(1-n)^2\pi_t^{F^2}.\end{aligned} \tag{D.64}$$

Adding $2n(1-n)\pi_t^H\pi_t^F - 2n(1-n)\pi_t^H\pi_t^F$ and simplifying yields

$$\begin{aligned}n\pi_t^{H^2}+(1-n)\pi_t^{F^2} &= n^2\pi_t^{H^2}+2n(1-n)\pi_t^H\pi_t^F+(1-n)^2\pi_t^{F^2} \\ &\quad + n(1-n)\pi_t^{F^2}-2n(1-n)\pi_t^H\pi_t^F+n(1-n)\pi_t^{H^2} \\ &= \underbrace{n^2\pi_t^{H^2}+2n(1-n)\pi_t^H\pi_t^F+(1-n)^2\pi_t^{F^2}}_{\left(n\pi_t^H+(1-n)\pi_t^F\right)^2} \\ &\quad + n(1-n)\underbrace{\left(\pi_t^{F^2}-2\pi_t^H\pi_t^F+\pi_t^{H^2}\right)}_{\left(\pi_t^F-\pi_t^H\right)^2} \\ &= \underbrace{\left(n\pi_t^H+(1-n)\pi_t^F\right)^2}_{\pi_t^{W^2}} + n(1-n)\underbrace{\left(\pi_t^F-\pi_t^H\right)^2}_{\pi_t^{R^2}} \\ &= \pi_t^{W^2}+n(1-n)\pi_t^{R^2}.\end{aligned} \tag{D.65}$$

The world welfare loss function is, then, given by

$$\begin{aligned}\tilde{W}_t = -\frac{1}{2}U_C\overline{C}E_t\sum_{k=0}^{\infty}\beta^k\Bigg((\rho+\eta)\left[c_{t+k}-\bar{c}\right]^2 + (1+\eta)n(1-n)\left[\hat{T}_{t+k}-\tilde{T}_{t+k}\right]^2 \\ + \sigma(1+\sigma\eta)\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\left[\pi_{t+k}^{W^2}+n(1-n)\pi_{t+k}^{R^2}\right]\Bigg) \\ + t.i.p. + O(\|\xi\|^3).\end{aligned} \tag{D.66}$$

Dividing both sides by $U_C\overline{C}$, letting $\beta \to 1$, and with $\bar{c} = 0$, the loss function can be written as

$$\begin{aligned} W_t = -\frac{1}{2}\Bigg(& (\rho+\eta)\,\mathrm{var}(\hat{C}_t - \tilde{C}_t) + (1+\eta)n(1-n)\,\mathrm{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\left[\mathrm{var}\,\pi_t^W + n(1-n)\,\mathrm{var}\,\pi_t^R\right]\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.67}$$

This equation corresponds to equation (2.25) in the main text.

D.6 Version containing country-specific output gaps

The welfare loss function can be expressed alternatively in terms of the country-specific output gaps instead of the consumption gap. This makes the analogy to the closed-economy counterpart, which is expressed in terms of the output gap as well, more obvious.

Inserting the gap version of the equations for aggregate demand

$$\hat{Y}_t^H - \tilde{Y}_t^H = (1-n)(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \tag{D.68}$$

$$\hat{Y}_t^F - \tilde{Y}_t^F = -n(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \tag{D.69}$$

into the weighted average of the squared output gaps yields

$$n(\hat{Y}_t^H - \tilde{Y}_t^H)^2 + (1-n)(\hat{Y}_t^F - \tilde{Y}_t^F)^2 = n(1-n)(\hat{T}_t - \tilde{T}_t)^2 + (\hat{C}_t - \tilde{C}_t)^2. \tag{D.70}$$

Solving this equation for $(\hat{C}_t - \tilde{C}_t)^2$ and inserting the resulting expression into equation (D.61) with $\bar{c} = 0$ yields

$$\begin{aligned} \tilde{W}_t = -\frac{1}{2}U_C\overline{C}E_t\sum_{k=0}^{\infty}\beta^k\Bigg(& (\rho+\eta)\left[n(\hat{Y}_{t+k}^H - \tilde{Y}_{t+k}^H)^2 + (1-n)(\hat{Y}_{t+k}^F - \tilde{Y}_{t+k}^F)^2\right] \\ & + (1-\rho)n(1-n)\left[\hat{T}_{t+k} - \tilde{T}_{t+k}\right]^2 \\ & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\pi_{t+k}^{H^2} \\ & + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\pi_{t+k}^{F^2}\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.71}$$

Expressed in variances, the welfare loss function is then given by

$$\begin{aligned} W_t = -\frac{1}{2}\Bigg(& (\rho+\eta)\left[n\,\mathrm{var}(\hat{Y}_t^H - \tilde{Y}_t^H) + (1-n)\,\mathrm{var}(\hat{Y}_t^F - \tilde{Y}_t^F)\right] \\ & + (1-\rho)n(1-n)\,\mathrm{var}(\hat{T}_t - \tilde{T}_t) \\ & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\,\mathrm{var}\,\pi_t^H \\ & + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\,\mathrm{var}\,\pi_t^F\Bigg) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \tag{D.72}$$

This welfare loss function closely resembles those in Benigno and Benigno (2006, eq. 21) as well as in Corsetti, Dedola, and Leduc (2011, eq. 40).