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Heterogeneous Firms and Cost Sharing in China's Marketplaces

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Abstract

This study extends Melitz's model with heterogeneous firms by introducing shared fixed costs in a marketplace. It aims to explain heterogeneous firms' choice between traditional marketplaces and modern distribution channels on the basis of their productivities. The results reveal that the co-existence of a traditional marketplace and modern distribution channels improves social welfare. In addition, a deregulation policy for firm entry outside a marketplace and accumulation of human capital are factors that contribute to improve the social welfare.

Keywords: Heterogeneous firms; Marketplace; Cost sharing; Multiplicity
JEL classification: D04, R12

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1 Introduction

This paper investigates the impact of cost sharing in a marketplace on heterogeneous firms' sales strategy. Sharing costs means that firms in a marketplace do not need to build independent sales channels, can easily collect information on competitors and consumers, and share various services in the marketplace. As a result, less productive firms often prefer to locate in marketplaces, whereas more productive ones tend to locate outside of it.

To analyze the effects of cost sharing on a firms' sales choice, we develop a Melitzstyle model in which firms share fixed costs only in the marketplace. To sell its products to consumers, each firm is required to locate in a marketplace or establish its own store outside the marketplace (modern distribution channels). While locating in a marketplace gives firms the advantage of share fixed costs, doing so also disadvantages them through higher transaction costs (North, 1991). A modern distribution channel, on the other hand, allows for lower transaction costs but higher fixed costs from independently establishing a sales channel.

Under these settings, we find that introducing cost sharing provides a qualitative change of conditions for firms' sales strategies. Importantly, the size of population or human capital influences the number of varieties, which further affects the size of a marketplace. Through shared fixed costs in the marketplace, firms tend to benefit much more than competitors utilizing modern distribution channels. More precisely, less productive firms can not survive without the marketplace but become profitable and survive if a marketplace is available; this in turn increases the number of firms operating in the marketplace. As a result, even consumers are better off because of the wider varieties produced in the economy. Without sharing fixed costs in a marketplace, firms equally benefit from the increasing size of population or human capital. In this case, there is no reallocation of resources among firms in the marketplace and their competitors utilizing modern distribution channels.

It is noteworthy that lower fixed costs for modern distribution channels due to the deregulation of establishing such channels keeps the ratio of the indifferent productivity to

threshold productivity unchanged. However, cost sharing increases threshold productivity during the deregulation process, which in turn increases both the number of firms in the marketplace and the total number of varieties, even though the fixed costs of firms only outside the marketplace decreases. Thus, social welfare improves during the deregulation process.

An existing paper that introduces cost sharing in a model for heterogeneous firms is Krautheim (2012), which accounts for fixed costs of exporting which decreases with the number of exporters. To determine the number of exporters, Krautheim (2012) assumes that the total number of firms in an industry is fixed; under these conditions, the entry and exit of firms into an industry is not affected, although cost sharing may impact the degree of externalities. However, the assumption of Krautheim (2012) appears to be unrealistic since, in reality, industries frequently experience the entry and exit of firms. The present paper also constructs a model with heterogeneous firms and fixed cost sharing; however, we consider an endogenous number of firms in an industry by introducing sharing fixed costs, which thus qualitatively changes results in Melitz (2003). In doing so, we clarify the gap between the case with and without shared fixed costs in a marketplace.

The remainder of this paper is organized as follows. Section 2 presents the background of our model. Section 3 develops a Melitz-style model to characterize the co-existence of a marketplace and modern distribution channels as a benchmark model. Section 4 introduces costs sharing of the marketplace in the model. Section 5 clarifies the importance of cost sharing and its implications for market equilibrium and social welfare. Section 6 concludes.

2 Background

Developing countries consist of several distribution channels, of which traditional marketplaces are the most popular worldwide. Marketplaces have played an important role in China's domestic trade circulation. During 1978–2003, the total number of marketplaces in China increased from 33,302 to 81,017. In a mere decade, from 1990 to 2000, the transaction volume of consumer goods in marketplaces accounted for 26.162.1% of

Chinese total retail sales of social consumer goods.[1] However, from 2006 to 2014, the share of markets with a turnover of above 100 million yuan declined from 37% to 19%.[2] Nevertheless, marketplaces remain key in China's domestic distribution for the following reasons. First, in addition to the above100millionyuan markets, there are more than 50,000 marketplaces whose transaction volume is below 100 million yuan, which have not been accounted for in the data. Second, in recent years, e-commerce platforms have proliferated in China. In 2015, the total online retail sales amounted to 2.79 trillion yuan with a growth rate of 49.7%.[3] From our definition of distribution channels shared by a large number of smallscale firms with higher transaction costs and lower fixed costs, e-commerce platforms are essentially similar to marketplaces.

On the other hand, as an economy develops, firms tend to own their distribution channels, hereinafter "modern distribution channels." An increasing number of firms with high productivity levels in the manufacturing sector have begun establishing their own sales networks. They organize a wider scale of sales agents to sell products; for example, there are 300 apparel companies in the Rui'an wear cluster in Wenzhou, China (Ding 2012, Chapter 10). By 2005, these companies opened nearly 10,000 stores in China's domestic market.

Company L is representative of Rui'an's casual wear company. In 2005, its production output reached 6 million pieces, amounting to 80 million yuan in sales. Company L's products are mainly sold to midincome consumers of domestic mid and smallsized cities and countylevel cities. It established more than 400 chain stores to cover the broad geographical scope of Shanghai (5060 stores), Zhejiang Province (110 stores), Jiangsu Province (just under 100 stores), and three provinces in Northeast China (100 stores). Contrary to a firm in the marketplace, a company with its own sale network must bear

[1]Data are taken from National Statistics Trading, Goods and Materials Statistics Secretary (NST-GMSS), ed. 1991-2001, Zhongguo Shichang Tongji Nianjian [Market Statistical Yearbook of China], Beijing: China Statistics Press.

[2]Data are taken from China Statistical Yearbooks, National Bureau of Statistics of China, for the calculation. We were able to retrieve data for only the so-called "above 100 million yuan markets."

[3]Source: http://www.stats.gov.cn/tjsj/zxfb/201601/t20160119_1306083.html (accessed on March 1, 2016).

higher fixed costs but lower transaction costs as imitation is more difficult.

Most booth keepers in marketplaces tend to be lessproductive SMEs. A good example is the narrow fabric industry in Yiwu China Commodity City (Yiwu market). According to Fah (2008), there are three types of firms in this industry: workshops (with an average of less than 19 machines), factories (20100 machines), and companies (more than 100 machines). The number of machines represents the size of fixed costs. An ISO certificate can be regarded an indicator of each firm's productivity. Fah (2008) showed that of those surveyed, 90% companies held ISO certificates, whereas only 33% factories and no workshops were certified. In other words, a firm whose productivity is low incurs lower fixed costs.

Marketplaces allow firms to have low fixed costs to sell their products, which stimulates the development of small-scale firms in the following manner.

First, marketplaces provide a sales channel shared by small-scale firms. A firm can meet numerous buyers every day and the larger the number of booths in a marketplace, the greater the number of buyers. To access these buyers, a firm must pay a booth rent and taxes as fixed costs and thus, saves various advertising and promotion costs. Consequently, the necessary costs for each firm to search for a new buyer is considerably low. In Yiwu's narrow fabric industry, the average share of sales is 57% for workshops, 56% for factories, and 32% for companies (Fah 2008). In sum, the lower the productivity, the smaller the firm size and higher the sales share in Yiwu Market.

Second, marketplaces help small-scale firms collect information on competitors and consumers. A marketplace with a large number of sellers and buyers offer more opportunities to access information. An example is the Huaqiang North Market in Shenzhen, which comprises 20,000 booths and 600,000 daily visitors (of these, 10,000 are professional buyers). According to a questionnaire survey of 56 local cellphone companies[4], 45 companies consider Huaqiang North Market important or comparatively important to acquire consumer demand information and 42 believe so for competitor information.

[4]Data are collected by Ke Ding and Shiro Hioki, the members of the research project the upgrading of Chinas industrial agglomeration: an interdisciplinary approach of spatial economics and area-study funded by a grant from the Japan Society for the Promotion of Science (JSPS).

However, given the easy accessibility to competitors information, the infringement of intellectual property or imitation is more common in a marketplace. According to the above mentioned questionnaire, 31 companies consider the Huaqiang North Market to have intensified imitation activities among firms. As a result, firms pay more money to continuously differentiate their products, which however are horizontally differentiated. The products of a firm with lower fixed costs tend to be imitated more easily and thus, the transaction costs for these lessproductive firms are higher. By contrast, firms with high productivity generally construct their own sales networks and formulate their own brand strategies. In this case, although the fixed costs are higher, the transaction costs become much cheaper and the total profit margins are greater.

Finally, marketplaces provide various services to small-scale firms: the larger the number of firms in a marketplace, the greater the economies of scale at the level of a marketplace in providing services. For example, the Yiwu market established an international logistics center that includes various facilities such as container yards, warehouses, delivery centers, unloading zones, shipment zones and parking areas.[5] These logistic facilities are shared by the large numbers of smallscale firms in Yiwu.

3 Benchmark model

The economy comprises a continuum of firms and under Dixit-Stiglitz (1977) monopolistic competition, each firm uses a unique production factor, that is, labor, to produce a horizontally differentiated manufactured good with increasing returns to scale technology. We denote the population of a country as E and each individual inelastically supplies one unit of labor. Without loss of generality, we take labor as numéraire. Thus, the wage rate $w = 1$ holds.

The utility function of a representative consumer is given by:

$$U \equiv \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where Ω is the set of available varieties and $\sigma > 1$ is the elasticity of substitution between any two varieties. Following Melitz (2003), horizontally differentiated varieties are

[5]Source: http://baike.baidu.com (accessed on February 18, 2016).

produced by firms bearing a fixed entry cost F_e (measured in units of labor). The intertemporal discounting of capital is ignored, but firms die according to a Poisson process with the hazard rate δ. After paying F_e, each firm draws an efficiency coefficient a from the distribution function $G(a)$ and density function $g(a)$ over interval $(0, a_0]$. Without loss of generality, we assume that $a_0 = 1$ holds.

Upon observing this draw, a firm may decide to produce and sell its products through a modern distribution channel or marketplace, or exit immediately. We suppose that firms in the marketplace enjoy low fixed costs f (measured in units of labor), whereas those in modern distribution channels bear higher fixed costs F (measured in units of labor), that is, $F > f$. Firms in the marketplace bear higher transaction costs, $t > 1$, while those with modern distribution channels enjoy lower iceberg-form transaction costs T, with $1 < T < t$. Specifically, $T < t$ indicates that the transaction cost in the marketplace is larger than that in the modern distribution channel due to imitation in the marketplace. In other words, the marketplace poses negative externalities. Further, the following assumption holds:

$$F/f > \Phi/\phi > 1. \tag{1}$$

The profit maximization of firm a entering the marketplace or utilizing modern distribution channel yields:

$$p(a) = \frac{\sigma}{\sigma - 1} at, \qquad P(a) = \frac{\sigma}{\sigma - 1} aT.$$

, respectively. Substituting the above pricing strategies into their profit functions and setting $\pi(a) = \Pi(a)$ yields the *indifferent productivity* $\underline{a}$, who is indifferent to entering the marketplace and utilizing modern distribution channel.

$$\underline{a} = \frac{(\sigma - 1)\mathcal{P}}{\sigma} \left[\frac{(\Phi - \phi)E}{(F - f)\sigma} \right]^{\frac{1}{\sigma - 1}} \tag{2}$$

where $\phi \equiv t^{1-\sigma}$, $\Phi \equiv T^{1-\sigma}$ and $\mathcal{P}$ is the consumer price index (CPI). Since $\Phi > \phi$ and $F > f$, we obtain $\Pi(a) > \pi(a)$ iff $a < \underline{a}$. Thus, firm $a \in (0, \underline{a})$ prefers the modern distribution channel to the marketplace.

The zero cutoff profit condition to enter the marketplace $\pi(a) = 0$ yields the *threshold productivity of marketplace* $\bar{a}_1$, which is the lowest productivity level of active firms in the

marketplace:

$$\bar{a}_1 = \frac{(\sigma-1)\mathcal{P}}{\sigma}\left(\frac{\phi E}{\sigma f}\right)^{\frac{1}{\sigma-1}}.$$

Correspondingly, the zero cutoff profit condition to utilize a modern distribution channel $\Pi(a) = 0$ yields the *threshold productivity of modern distribution channel* $\bar{a}_2$:

$$\bar{a}_2 = \frac{(\sigma-1)\mathcal{P}}{\sigma}\left(\frac{\Phi E}{\sigma F}\right)^{\frac{1}{\sigma-1}}.$$

From assumption (1), *threshold productivity* $\bar{a}$ is determined by[6]

$$\bar{a} = max\{\bar{a}_1, \bar{a}_2\} = \frac{(\sigma-1)\mathcal{P}}{\sigma}\left(\frac{\phi E}{\sigma f}\right)^{\frac{1}{\sigma-1}} > \underline{a}. \tag{3}$$

Therefore, firm $a \in (\underline{a}, \bar{a})$ chooses to produce and sell in the marketplace, and firm $a \in (0, \underline{a})$ chooses a modern distribution channel.

An equilibrium is characterized by mass $\mathcal{N}$ of firms and distribution $\mu(a)$ of productivity levels over a subset of $(0, 1)$. Since any entering firm drawing productivity level $a > \bar{a}$ will never produce and exit immediately, $\mu(a)$ is the conditional distribution of $g(a)$ on $(0, \bar{a}]$:

$$\mu(a) = \begin{cases} \frac{g(a)}{G(\bar{a})} & \text{if} \quad 0 < a < \bar{a}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Therefore, CPI is given by

$$\mathcal{P} \equiv \left[\int_0^{\underline{a}} \left(\frac{\sigma}{\sigma-1}aT\right)^{1-\sigma} N\mu(a)da + \int_{\underline{a}}^{\bar{a}} \left(\frac{\sigma}{\sigma-1}at\right)^{1-\sigma} n\mu(a)da\right]^{\frac{1}{1-\sigma}}, \tag{5}$$

where N and n are the mass of available varieties sold in the modern distribution channel and marketplace, respectively. Thus, we have

$$N \equiv \mathcal{N}\int_0^{\underline{a}} \mu(a)da = \mathcal{N}\frac{G(\underline{a})}{G(\bar{a})}, \qquad n \equiv \mathcal{N}\int_{\underline{a}}^{\bar{a}} \mu(a)da = \mathcal{N}\frac{G(\bar{a}) - G(\underline{a})}{G(\bar{a})}.$$

Combining (2) and (3), we obtain:

$$\underline{a}/\bar{a} = \left[\frac{(\Phi-\phi)f}{\phi(F-f)}\right]^{\frac{1}{\sigma-1}} \equiv \Lambda \in (0, 1) \tag{6}$$

[6]Otherwise, all active firms will choose to set up a modern distribution channel, that is, $\bar{a} = \frac{(\sigma-1)\mathcal{P}}{\sigma}\left(\frac{\Phi E}{\sigma F}\right)^{\frac{1}{\sigma-1}} < \underline{a}$.

We assume $\Phi f < \phi F$ holds and thus, $\Lambda \in (0,1)$. Assuming $G(a) = a^{\rho}, a \in (0,1)$ and $\rho > \sigma$, Eq. (5) can be written as

$$\mathcal{P}^{1-\sigma} = \frac{\rho\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{1+\rho-\sigma}\left[\Phi\Lambda^{1+2\rho-\sigma} + \phi(1-\Lambda^{\rho})\left(1-\Lambda^{1+\rho-\sigma}\right)\right]\bar{a}^{1-\sigma}\mathcal{N}. \tag{7}$$

Substituting Eq. (3) into (7), we obtain the equilibrium mass of available varieties:

$$\mathcal{N}^{*} = \frac{(1+\rho-\sigma)E}{\rho\sigma f}\frac{\phi}{\Phi\Lambda^{1+2\rho-\sigma} + \phi(1-\Lambda^{\rho})\left(1-\Lambda^{1+\rho-\sigma}\right)}. \tag{8}$$

In this paper, we focus on the interior solution that both the marketplace and modern distribution channel are active, that is, $\Lambda \in (0,1)$.

The free entry condition is given by

$$\begin{aligned} F_e =& \frac{G(\bar{a})}{\delta}\left\{\int_0^{\underline{a}}\left[\frac{E}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{aT}{\mathcal{P}}\right)^{1-\sigma} - F\right]\mu(a)da + \int_{\underline{a}}^{\bar{a}}\left[\frac{E}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{at}{\mathcal{P}}\right)^{1-\sigma} - f\right]\mu(a)da\right\} \\ =& \frac{\rho\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}E}{\delta\sigma(1+\rho-\sigma)\mathcal{P}^{1-\sigma}}\left[\Phi\underline{a}^{1+\rho-\sigma} + \phi(\bar{a}^{1+\rho-\sigma} - \underline{a}^{1+\rho-\sigma})\right] - \frac{\bar{a}^{\rho}}{\delta}\left\{F\left(\frac{\underline{a}}{\bar{a}}\right)^{\rho} + f\left[1-\left(\frac{\underline{a}}{\bar{a}}\right)^{\rho}\right]\right\}. \end{aligned}$$

Substituting Eqs. (2) and (3) into the free entry condition yields

$$F_e = \frac{\sigma-1}{\delta(1+\rho-\sigma)}\left[\Lambda^{\rho}F + (1-\Lambda^{\rho})f\right]\bar{a}^{\rho}$$

Thus, indifferent productivity in equilibrium $\underline{a}^{*}$ and threshold productivity in equilibrium $\bar{a}^{*}$ are, respectively, given by

$$\bar{a}^{*} = \left[\frac{\delta(1+\rho-\sigma)F_e}{(\sigma-1)\widetilde{F}}\right]^{\frac{1}{\rho}} \tag{9}$$

$$\underline{a}^{*} = \left[\frac{(\Phi-\phi)f}{\phi(F-f)}\right]^{\frac{1}{\sigma-1}}\bar{a}^{*} = \Lambda\bar{a}^{*} \tag{10}$$

where $\widetilde{F} \equiv \Lambda^{\rho}F + (1-\Lambda^{\rho})f$. Thus, we have the following proposition.

Proposition 1 *If $1 < \frac{\Phi}{\phi} < \frac{F}{f}$ holds, the coexistence of a marketplace and modern distribution channel occurs. Furthermore, there will be a sorting of productivity between both, that is, $0 < \underline{a}^{*} < \bar{a}^{*} < 1$.*

4 Endogenous positive externalities

We now assume that there exist positive externalities in the marketplace through shared fixed costs. In the marketplace, firms share advertising costs, gather consumer and competitor information, and enjoy public services supported by the marketplace. However, firms in the marketplace face a high probability of being imitated by competitors, which results in a loss of revenue in the form of iceberg transaction costs. Thus, a firm entering the marketplace benefits from low fixed costs at the expense of high variable costs. We assume that each firm in the marketplace is required to pay fixed cost as follows:

$$f_x = \begin{cases} f/n_x, & \text{if} \quad n_x \geq 1 \\ f, & \text{if} \quad n_x \in [0,1). \end{cases}$$

where n_x is the mass of available varieties sold in the marketplace.

Setting $\pi(a) = \Pi(a)$ and using the above fixed costs yield *indifferent productivity* $\underline{a}_x$:

$$\underline{a}_x = \frac{(\sigma-1)\mathcal{P}_x}{\sigma}\left[\frac{(\Phi-\phi)E}{(F-f/n_x)\sigma}\right]^{\frac{1}{\sigma-1}}. \tag{11}$$

Furthermore, the zero cutoff profit condition $\pi(a) = 0$ yields *threshold productivity* $\bar{a}_x$:

$$\bar{a}_x = \frac{(\sigma-1)\mathcal{P}_x}{\sigma}\left(\frac{\phi E}{\sigma f/n_x}\right)^{\frac{1}{\sigma-1}}. \tag{12}$$

We focus on the case in which both the marketplace and modern distribution channels exist, that is, $\Pi(\underline{a}_x) = \pi(\underline{a}_x)$ and $\pi(\bar{a}_x) = 0$, with $\underline{a}_x < \bar{a}_x \in (0,1)$. Thus, a firm who draws productivity level $a > \bar{a}_x$ will never produce and exit immediately and $\mu(a)$ is the conditional distribution of $g(a)$ on $(0, \bar{a}_x]$:

$$\mu(a) = \begin{cases} \frac{g(a)}{G(\bar{a}_x)} & \text{if} \quad 0 < a < \bar{a}_x, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Therefore, CPI is given by:

$$\mathcal{P}_x \equiv \left[\int_0^{\underline{a}_x}\left(\frac{\sigma}{\sigma-1}aT\right)^{1-\sigma} N_x\mu(a)da + \int_{\underline{a}_x}^{\bar{a}_x}\left(\frac{\sigma}{\sigma-1}at\right)^{1-\sigma} n_x\mu(a)da\right]^{\frac{1}{1-\sigma}}, \tag{14}$$

where N_x is the mass of available varieties sold in the modern distribution channel. Thus, we have:

$$N_x \equiv \mathcal{N}_x\int_0^{\underline{a}_x}\mu(a)da = \mathcal{N}_x\frac{G(\underline{a}_x)}{G(\bar{a}_x)}, \qquad n_x \equiv \mathcal{N}_x\int_{\underline{a}_x}^{\bar{a}_x}\mu(a)da = \mathcal{N}_x\frac{G(\bar{a}_x)-G(\underline{a}_x)}{G(\bar{a}_x)}.$$

Combining Eqs. (11) and (12), we obtain:

$$\underline{a}_x/\bar{a}_x = \left[\frac{(\Phi-\phi)f/n_x}{\phi(F-f/n_x)}\right]^{\frac{1}{\sigma-1}} \equiv \Lambda_x, \tag{15}$$

If $n_x > \big(\Phi f\big)/\big(\phi F\big)$ holds, we have $\Lambda_x \in (0,1)$. Therefore, CPI (14) can be rewritten as

$$\mathcal{P}_x^{1-\sigma} = \frac{\rho(\frac{\sigma}{\sigma-1})^{1-\sigma}}{1+\rho-\sigma}\big[\Phi\Lambda_x^{1+2\rho-\sigma} + \phi(1-\Lambda_x^{\rho})(1-\Lambda_x^{1+\rho-\sigma})\big]\bar{a}_x^{1-\sigma}\mathcal{N}_x. \tag{16}$$

Substituting Eq. (12) into (16), the mass of available varieties is determined by:

$$\mathcal{N}_x = \frac{(1+\rho-\sigma)E}{\rho\sigma f}\frac{\phi n_x}{\Phi\Lambda_x^{1+2\rho-\sigma} + \phi(1-\Lambda_x^{\rho})(1-\Lambda_x^{1+\rho-\sigma})}, \tag{17}$$

Substituting $n_x = \frac{G(\bar{a}_x)-G(\underline{a}_x)}{G(\bar{a}_x)}\mathcal{N}_x = (1-\Lambda_x^{\rho})\mathcal{N}_x$ into Eq. (17), we obtain

$$\mathcal{H}(\Lambda_x) \equiv \Phi\frac{\Lambda_x^{1+\rho-\sigma}}{\Lambda_x^{-\rho}-1} + \phi\big(1-\Lambda_x^{1+\rho-\sigma}\big) - \phi\frac{(1+\rho-\sigma)E}{\rho\sigma f} = 0. \tag{18}$$

As shown in Appendix A[7], Eq. (18) has at most two roots: $\Lambda_{x,L}^* \in [0,1]$ and $\Lambda_{x,H}^* \in [0,1]$, with $\Lambda_{x,L}^* < \Lambda_{x,H}^*$.[8] We refer to $\Lambda_{x,L}^*$ and $\Lambda_{x,H}^*$ as the L-equilibrium and H-equilibrium, respectively. Note that Eq. (18) shows the relationship between Φ/ϕ and E/f, even though Proposition 1 focuses on the relationship between Φ/ϕ and F/f.

Substituting Λ_x^* into Eq. (15), the equilibrium mass of varieties sold in the marketplace is given by

$$n_x^* = \frac{f}{F} + \frac{(\Phi-\phi)f}{(\Lambda_x^*)^{\sigma-1}\phi F}. \tag{19}$$

Since $n_x > 1$ and $\partial\Lambda_x/\partial n_x < 0$, we find that $\Lambda_x < \Lambda$ holds. In other words, *the share of firms in the marketplace in the total number of firms increases because of the shared fixed costs.*

Taking the derivative of n_x^* with respect to Λ_x^*, we have:

$$\frac{\partial n_x^*}{\partial \Lambda_x^*} < 0. \tag{20}$$

[7]We derive the following results: (1) if E is small enough such that $\mathcal{H}(\Lambda_x^0) \geq 0$, the condition for (iv) holds (2) if E is large enough such that $0 \geq \lim_{\Lambda_x \to 0}\mathcal{H}(\Lambda_x)$, the condition for (iii) holds, and (3) otherwise, the condition for (i) or (ii) holds.

[8]Both equilibria $\Lambda_{x,L}^*$ and $\Lambda_{x,H}^*$ are stable because the following conditions hold: $\partial\Pi(\underline{a}_x^*)/\partial\underline{a}_x^* < 0$, and $\partial\Pi(\bar{a}_x^*)/\partial\bar{a}_x^* = 0$.

To explain the existence of the two roots of Λ_x, following Melitz (2003), we denote the reverse index of weighted average productivity $\widetilde{a}_x$ as follows:

$$\mathcal{P}_x^{1-\sigma} = \mathcal{N}_x \Big[\frac{\sigma}{\sigma-1}\widetilde{a}_x\Big]^{1-\sigma}. \tag{21}$$

Combining Eqs. (16) and (21), we obtain

$$\frac{\widetilde{a}_x^{1-\sigma}}{\bar{a}_x^{1-\sigma}} = \Gamma^{\mathrm{CPI}}(\Lambda_x) \equiv \frac{\rho}{1+\rho-\sigma}\big[\Phi\Lambda_x^{1+2\rho-\sigma} + \phi(1-\Lambda_x^{\rho})(1-\Lambda_x^{1+\rho-\sigma})\big]. \tag{22}$$

Eq. (22) indicates the relationship between the two ratios of productivity $\widetilde{a}_x^{1-\sigma}/\bar{a}_x^{1-\sigma}$ and $\Lambda_x = \underline{a}_x/\bar{a}_x$ based on the weighted average productivity (WAP), such that the definition of price index holds. Thus, we call it the *WAP curve*. Furthermore, Eq. (22) takes the same form when there is no positive externality of sharing fixed costs. Specifically, Λ_x^{ρ} and $1-\Lambda_x^{\rho}$ indicate the share of firms that chooses the modern distribution channel in the total number of firms and the share of firms located in the marketplace in the total number of firms, respectively. Note that $\Lambda_x^{1+\rho-\sigma}$ and $1-\Lambda_x^{1+\rho-\sigma}$ represent the indexes of the average productivity of firms choosing a modern distribution channel and that of firms located in the marketplace, respectively. Thus, when Λ_x increases, the share of firms choosing the modern distribution channel Λ_x^{ρ} and the index of the average productivity of these firms, $\Lambda_x^{1+\rho-\sigma}$, increase. Therefore, the first term in the bracket on the RHS of Eq. (22) increases, whereas the second term on the RHS decreases because $1-\Lambda_x^{\rho}$ and $1-\Lambda_x^{1+\rho-\sigma}$ decrease when Λ_x increases. It can be readily verified that Γ^{CPI} in Eq. (22) is a convex function of Λ_x.[9] Therefore, when Λ_x gradually increases, the first term is dominated by the second term in Eq. (22) and then the former dominates the latter.

Substituting Eq. (21) and $n_x/\mathcal{N}_x = 1-\Lambda_x^{\rho}$ into (12), we have:

$$\frac{\widetilde{a}_x^{1-\sigma}}{\bar{a}_x^{1-\sigma}} = \Gamma^{\mathrm{ZCP}}(\Lambda_x) \equiv \frac{\phi E}{\sigma f}\big(1-\Lambda_x^{\rho}\big). \tag{23}$$

Eq. (23) represents the relationship between the two ratios of productivity $\widetilde{a}_x^{1-\sigma}/\bar{a}_x^{1-\sigma}$ and Λ_x such that the zero cutoff profit condition holds. Correspondingly, we call it the *ZCP curve*. In particular, the first term on the RHS of Eq. (23) represents the size of

[9] Let $\Lambda^{\rho} = \lambda$ and $\Gamma^{\mathrm{CPI}}(\Lambda_x) \equiv F(\lambda)$ in Eq. (22); accordingly, we have $\lim_{\lambda\to 0}\frac{\partial F}{\partial \lambda} = -\phi$, and $\lim_{\lambda\to 1}\frac{\partial F}{\partial \lambda} = \frac{1-\sigma+2\rho}{\rho}\Phi$. We also obtain $\partial F^2/\partial^2\lambda = \frac{1-\sigma+\rho}{\rho}\lambda^{\frac{1-\sigma}{\rho}}[(\Phi+\phi)\frac{1-\sigma+\rho}{\rho} + \frac{\phi(\sigma-1)}{\lambda\rho}] > 0$. Thus, $\Gamma^{\mathrm{CPI}}(\Lambda_x)$ is a convex function of Λ_x and has a U-shaped curve.

the economy, whereas the second term on the RHS denotes the share of firms located in the marketplace, that is, the magnitude of positive externalities in the marketplace from sharing fixed cost. Therefore, the ratio between the weighted average productivity and least productivity $\widetilde{a}_x^{1-\sigma}/\bar{a}_x^{1-\sigma}$ decreases when Λ_x increases.

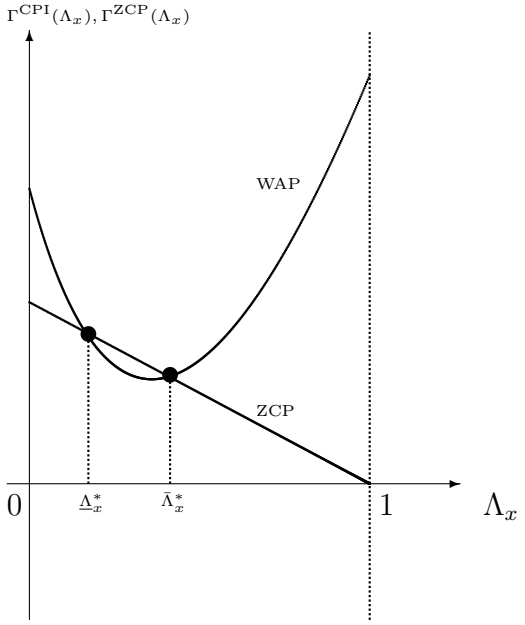

Figure 1: Multiple equilibria

In sum, Eq. (18) is a combination of the definition of the price index, zero cutoff profit condition, and the number of firms in the marketplace by the total number of firms that determines Λ_x. In Eq. (18), Λ_x can be interpreted in three ways: Λ_x is the ratio of threshold to indifferent productivity, Λ_x^ρ is the share of firms outside the marketplace in the total number of firms, and $\Lambda_x^{1+\rho-\sigma}$ are the indexes of the average productivity of firms choosing a modern distribution channel. The combination of the zero cutoff profit condition and indifference condition, Eq. (15), determines the number of firms in the marketplace. Using two determined variables, Λ_x and n_x, the mass of available varieties, $\mathcal{N}$, is determined by the definition of the number of firms in the marketplace in the total number of firms. In what follows, using the determined value of Λ_x, the combination of free entry condition, zero cutoff profit condition, and indifferent condition, we determine threshold productivity. Then, we use the definition of Λ_x to determine indifferent productivity.

The free entry condition is given by

$$F_e = \frac{G(\bar{a}_x)}{\delta}\left\{\int_0^{\underline{a}_x}\left[\frac{E}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{aT}{\mathcal{P}_x}\right)^{1-\sigma}-F\right]\mu(a)da+\int_{\underline{a}_x}^{\bar{a}_x}\left[\frac{E}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{at}{\mathcal{P}_x}\right)^{1-\sigma}-\frac{f}{n_x}\right]\mu(a)da\right\}$$
$$=\frac{\rho\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}E}{\delta\sigma(1+\rho-\sigma)\mathcal{P}_x^{1-\sigma}}\left[\Phi\underline{a}_x^{1+\rho-\sigma}+\phi(\bar{a}_x^{1+\rho-\sigma}-\underline{a}_x^{1+\rho-\sigma})\right]-\frac{\bar{a}_x^{\rho}}{\delta}\left\{F\left(\frac{\underline{a}_x}{\bar{a}_x}\right)^{\rho}+\frac{f}{n_x}\left[1-\left(\frac{\underline{a}_x}{\bar{a}_x}\right)^{\rho}\right]\right\}.$$

Substituting Eqs. (11) and (12) into the free entry condition yields

$$F_e = \frac{\sigma-1}{\delta(1+\rho-\sigma)}\widetilde{F}_x\bar{a}_x^{\rho}, \tag{24}$$

where $\widetilde{F}_x = \Lambda_x^{\rho}F + (1-\Lambda_x^{\rho})f/n_x$. Therefore, the indifferent productivity in equilibrium $\underline{a}_x^*$ and threshold productivity in equilibrium $\bar{a}_x^*$ are

$$\bar{a}_x^* = \left[\frac{\delta(1+\rho-\sigma)F_e}{(\sigma-1)\widetilde{F}_x^*}\right]^{\frac{1}{\rho}} = \left[\frac{\delta(1+\rho-\sigma)F_e}{(\sigma-1)F}\right]^{\frac{1}{\rho}}\left[\frac{\phi+(\Phi-\phi)(\Lambda_x^*)^{1-\sigma}}{\phi+(\Phi-\phi)(\Lambda_x^*)^{\rho-\sigma+1}}\right]^{\frac{1}{\rho}}, \tag{25}$$

$$\underline{a}_x^* = \Lambda_x^*\bar{a}_x^* = \left[\frac{\delta(1+\rho-\sigma)F_e}{(\sigma-1)F}\right]^{\frac{1}{\rho}}\left[\frac{\Phi-\phi+\phi(\Lambda_x^*)^{\sigma-1}}{\Phi-\phi+\phi(\Lambda_x^*)^{\sigma-1-\rho}}\right]^{\frac{1}{\rho}}. \tag{26}$$

Because $n_x > 1$ and $\Lambda_x < \Lambda$, it is readily verified that $\widetilde{F}_x < \widetilde{F}$ holds from (9) and (25), which results in $\bar{a}_x^* > \bar{a}^*$. In other words, *sharing fixed costs allows less productive firms to survive in the marketplace.*

From Eqs. (25) and (26), we further have

$$\frac{\partial\bar{a}_x^*}{\partial\Lambda_x^*} < 0 \qquad \text{and} \qquad \frac{\partial\underline{a}_x^*}{\partial\Lambda_x^*} > 0.^{10} \tag{27}$$

Thus, we obtain $\bar{a}_{x,L}^* > \bar{a}_{x,H}^*$ and $\underline{a}_{x,L}^* < \underline{a}_{x,H}^*$ since $\Lambda_{x,L}^* < \Lambda_{x,H}^*$.

To clarify the difference between L-equilibrium and H-equilibrium, we turn to the welfare analysis. Combining Eqs. (16), (17) and (19), the relative price indexes of two equilibria is given by

$$\left(\frac{\mathcal{P}_{x,H}^*}{\mathcal{P}_{x,L}^*}\right)^{1-\sigma} = \frac{\left[\phi\left(\Lambda_{x,H}^*\right)^{1-\sigma}+(\Phi-\phi)\right]}{\left[\phi\left(\Lambda_{x,L}^*\right)^{1-\sigma}+(\Phi-\phi)\right]}\left(\frac{\underline{a}_{x,L}^*}{\underline{a}_{x,H}^*}\right)^{\sigma-1} < 1. \tag{28}$$

Eq. (28) implies that individuals in L-equilibrium are happier than those in H-equilibrium; therefore, the economy chooses the former.

[10] It can be readily verified that $\widetilde{F}_x^* = (\Lambda_x^*)^{\rho}F + [1-(\Lambda_x^*)^{\rho}]f/n_x^* = F\left[\frac{\phi(\Lambda_x^*)^{\sigma-1}+(\Phi-\phi)(\Lambda_x^*)^{\rho}}{\phi(\Lambda_x^*)^{\sigma-1}+\Phi-\phi}\right]$ and $\frac{\partial\widetilde{F}_x^*}{\partial\Lambda_x^*} > 0$ hold.

Finally, combining Eqs. (7), (8), (16), and (17), the relative price indexes in equilibrium between the cases with and without shared fixed costs in the marketplace is given by

$$\begin{aligned}\left(\frac{\mathcal{P}_x^*}{\mathcal{P}^*}\right)^{1-\sigma} =& \frac{n_x^*(\bar{a}_x^*)^{1-\sigma}}{(\bar{a}^*)^{1-\sigma}}, \\ =& n_x^* \left\{ \frac{(\Lambda^*)^\rho F + [1-(\Lambda^*)^\rho] f}{(\Lambda_x^*)^\rho F + [1-(\Lambda_x^*)^\rho] f/n_x^*} \right\}^{\frac{1-\sigma}{\rho}}. \end{aligned} \tag{29}$$

Thus, we have

$$\left(\frac{\mathcal{P}_x^*}{\mathcal{P}^*}\right)^{1-\sigma} > 1 \Rightarrow \mathcal{P}_x^* < \mathcal{P}^*.^{11} \tag{30}$$

Therefore, when shared fixed costs exist in the marketplace, social welfare is higher. The results are not obvious in the case of heterogeneous firms. Since $n_x^* > 1$ and $(\bar{a}_x^*)^{1-\sigma} < (\bar{a}^*)^{1-\sigma}$, cost sharing allows for a higher number of firms in a marketplace and a lower threshold productivity. The former increases the price index, whereas the latter decreases it. The result shows that the impact of the former dominates that of the latter.

Proposition 2 *Because of the shared fixed costs in the marketplace, the threshold productivity level for firms to survive in the economy decreases and the number of firms in the marketplace increases. Furthermore, the impact of greater varieties dominates the impact of a lower threshold productivity. Thus, individuals are better off sharing fixed costs in the marketplace.*

5 Discussion and Extension

5.1 Human capital accumulation

First, we focus on the impact of increasing human capital E on the economy. From Eq. (18), we have

$$\frac{\partial \Lambda_x^*}{\partial E} = \frac{(1+\rho-\sigma)\phi[1-(\Lambda_x^*)^\rho]^2}{\rho\sigma f(\Lambda_x^*)^{\rho-\sigma}\Psi(\Lambda_x^*)} \tag{31}$$

[11] By conducting some simple calculations, we derive $\frac{\mathcal{P}_x^{1-\sigma}}{\mathcal{P}^{1-\sigma}} \gtrless 1 \Leftrightarrow n_x^{\frac{\rho}{\sigma-1}} \Lambda_x^\rho F + n_x^{\frac{\rho-\sigma+1}{\sigma-1}}(1-\Lambda_x^\rho) f \gtrless \Lambda^\rho F + (1-\Lambda^\rho) f$. Since $\Lambda_x < \Lambda$ and $n_x > 1$, the inequality $n_x^{\frac{\rho-\sigma+1}{\sigma-1}}(1-\Lambda_x^\rho) f > (1-\Lambda^\rho) f$ holds. On the other hand, from (6) and (15), the inequality $\left(\frac{\Lambda}{\Lambda_x}\right)^{\sigma-1} = \frac{F-f}{n_x F - f} < 1$ holds because $n_x > 1$. Therefore, we have $\frac{\mathcal{P}_x^{1-\sigma}}{\mathcal{P}^{1-\sigma}} > 1$.

where $\Psi(\Lambda_x) \equiv -(\phi+\Phi)(1+\rho-\sigma)\Lambda_x^{2\rho} + [\rho\Phi + (2\phi+\Phi)(1+\rho-\sigma)]\Lambda_x^{\rho} - \phi(1+\rho-\sigma)$. Since $\Psi(\Lambda_{x,L}^*) < 0$, the following inequality holds:

$$\left.\frac{\partial \Lambda_x^*}{\partial E}\right|_{\Lambda_x^*=\Lambda_{x,L}^*} < 0. \tag{32}$$

In other words, an increase in human capital/population leads to an increase in the share of the number of firms within the marketplace in the total number of firms under cost sharing. We obtain $\partial\Lambda^*/\partial E = 0$ from Eq. (6), which implies that the existence of a scale economy at the marketplace level affects the share of firms in the marketplace.

Furthermore, from Eqs. (25), (26) and (32), we have:

$$\left.\frac{\partial \bar{a}_x^*}{\partial E}\right|_{\Lambda_x^*=\Lambda_{x,L}^*} > 0, \qquad \left.\frac{\partial \underline{a}_x^*}{\partial E}\right|_{\Lambda_x^*=\Lambda_{x,L}^*} < 0. \tag{33}$$

Since $\partial\bar{a}^*/\partial E = 0$ and $\partial\underline{a}^*/\partial E = 0$ hold from Eqs. (9) and (10), the scale economy at the marketplace level allows less productive firms that could not survive without a marketplace to now make profits and some productive firms that could establish a modern distribution channel to now operate in a marketplace.

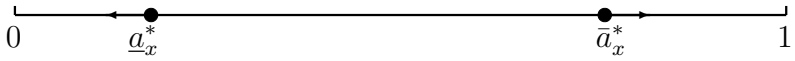

Figure 2: Impacts of increasing E in L-equilibrium

Likewise, from Eqs. (19), (17), and (32), we obtain

$$\left.\frac{\partial n_x^*}{\partial E}\right|_{\Lambda_x^*=\Lambda_{x,L}^*} > 0, \qquad \left.\frac{\partial}{\partial E}\left(\frac{n_x^*}{N_x^*}\right)\right|_{\Lambda_x^*=\Lambda_{x,L}^*} > 0. \tag{34}$$

We obtain $\partial(n^*/N^*)/\partial E = 0$ since $\partial\Lambda^*/\partial E = 0$. That is, owing to cost sharing, an increase in human capital accumulation leads to a larger number of firms in the marketplace in the ratio of varieties within the marketplace to those outside of it.

Combining Eqs. (16), (17), and (19), we have:

$$(\mathcal{P}_x^*)^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{E}{\sigma F}\left[\phi(\Lambda_x^*)^{1-\sigma} + \Phi - \phi\right](\underline{a}_x^*)^{1-\sigma}. \tag{35}$$

Thus, the following inequality holds:

$$\left.\frac{\partial \mathcal{P}_x^*}{\partial E}\right|_{\Lambda_x^*=\Lambda_{x,L}^*} < 0.$$

Therefore, the welfare of equilibrium $(\Lambda^*_{x,L}, \underline{a}^*_{x,L}, \bar{a}^*_{x,L})$ increases when the human capital of the economy, E, gradually increases. Furthermore, substituting Eq. (8) into (7) and using $\partial \bar{a}^*/\partial E = 0$, we obtain $\partial \mathcal{P}^*/\partial E < 0$. That is, social welfare also increases in human capital even when there is no cost sharing.

Furthermore, combining Eqs. (7), (8), and (35), we obtain:

$$\left(\frac{\mathcal{P}^*_x}{\mathcal{P}^*}\right)^{1-\sigma} = \frac{f}{F}\frac{\left[\phi(\Lambda^*_x)^{1-\sigma} + \Phi - \phi\right](\underline{a}^*_x)^{1-\sigma}}{\phi(\bar{a}^*)^{1-\sigma}} \tag{36}$$

Using Eqs. (32) and (33) and $\partial \bar{a}^*/\partial E = 0$, we obtain

$$\frac{\partial(\mathcal{P}^*_x|_{\Lambda^*_x=\Lambda^*_{x,L}}/\mathcal{P}^*)}{\partial E} < 0.$$

Although Proposition 2 concludes that individuals are better off sharing fixed costs in the marketplace, we find that the ratio of relative price indices with and without shared fixed costs decrease when human capital in the economy increases. The intuition underlying the shared fixed costs in the marketplace helps less productive firms survive and thus, increases the total number of varieties in the economy.

Proposition 3 *As for increasing human capital with the same magnitude, social welfare increases far more when there exists cost sharing than without.*

5.2 Deregulation policy

We now turn to the impact of deregulating entry control on the establishment of modern distribution channels such as the reform and open door policy in China. In 1992, China's central government relaxed its regulation policy and permitted foreign retail companies to establish stores and branches in 10 cities in the Hainan Province. Since then, Walmart, Carrefour, Auchan, and other foreign companies have established stores across China. In this paper, the deregulation policy allows firms to utilize modern distribution channels more easily, which implies a decrease in F.

Eq. (18) determines the equilibrium value Λ^*_x, which is independent of F. Thus, we have

$$\frac{\partial \Lambda^*_x}{\partial F} = 0. \tag{37}$$

For firms' profits to be the same within and outside the marketplace, the value of F must be met as a condition. However, this condition is used to determine the zero cutoff profit condition, not the value of Λ. From Eq. (19), we find that the number of firms in modern distribution channels in equilibrium n_x^* is determined by Λ_x^* and F. Thus, the following inequality holds:

$$\frac{dn_x^*}{dF} = \frac{\partial n_x^*}{\partial F} + \frac{\partial n_x^*}{\partial \Lambda_x^*}\frac{\partial \Lambda_x^*}{\partial F} = \frac{\partial n_x^*}{\partial F} < 0. \tag{38}$$

This result is obtained from the combination of the zero cutoff profit condition and indifferent condition. Without cost sharing, Eq. (6) provides that $\partial \Lambda^*/\partial F < 0$, which implies that the share of firms using modern distribution channels increases by reducing fixed costs outside the marketplace under no cost sharing. This results from the choice of firms between the production condition within and outside the marketplace.

Combining Eqs. (25), (26), and (37), we have

$$\frac{d\bar{a}_x^*}{dF} = \frac{\partial \bar{a}_x^*}{\partial F} + \frac{\partial \bar{a}_x^*}{\partial \Lambda_x^*}\frac{\partial \Lambda_x^*}{\partial F} = \frac{\partial \bar{a}_x^*}{\partial F} < 0, \tag{39}$$

$$\frac{d\underline{a}_x^*}{dF} = \frac{\partial \underline{a}_x^*}{\partial F} + \frac{\partial \underline{a}_x^*}{\partial \Lambda_x^*}\frac{\partial \Lambda_x^*}{\partial F} = \frac{\partial \underline{a}_x^*}{\partial F} < 0. \tag{40}$$

Eqs. (39) and (40) imply that both indifferent productivity $\underline{a}_x^*$ and threshold productivity $\bar{a}_x^*$ increase when F decreases. The decrease of F implies reduced average fixed costs $\widetilde{F}_x$ in the economy under the constant share of firms in the marketplace, which induces further firms entries. Thus, deregulating the establishment of modern distribution channels allows less productive firms to survive in the economy (see Fig. 3). In the absence of cost sharing, since $\partial \widetilde{F}/\partial F = \Lambda^{\rho}[1-\rho/(\sigma-1)] < 0$, we obtain $\partial \bar{a}^*/\partial F > 0$. In other words, deregulating the establishment of modern distribution channels creates opposing impacts on threshold productivity between the two cases.

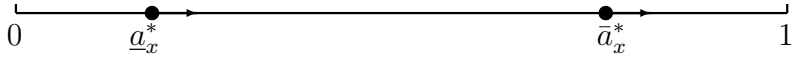

Figure 3: Impacts of decreasing F in L-equilibrium

From Eqs. (17) and (19), we obtain

$$\mathcal{N}_x^* = \frac{(1+\rho-\sigma)E}{\rho\sigma F}\frac{\Phi-\phi+\phi(\Lambda_x^*)^{\sigma-1}}{\Phi(\Lambda_x^*)^{2\rho}+\phi[1-(\Lambda_x^*)^{\rho}][(\Lambda_x^*)^{\sigma-1}-(\Lambda_x^*)^{\rho}]}. \tag{41}$$

From Eq. (41), we obtain

$$\begin{aligned}\frac{d\mathcal{N}_x^*}{dF} =& \frac{\partial \mathcal{N}_x^*}{\partial F} + \frac{\partial \mathcal{N}_x^*}{\partial \Lambda_x^*}\frac{\partial \Lambda_x^*}{\partial F} \\ =& \underbrace{\frac{\partial \mathcal{N}_x^*}{\partial F}}_{(-)} < 0\end{aligned} \tag{42}$$

Eqs. (19) and (41) imply that the following equality holds:

$$\frac{d}{dF}\Big(\frac{n_x^*}{\mathcal{N}_x}\Big) = 0. \tag{43}$$

Eq. (43) implies that the numbers of active firms in the marketplace and modern distribution channels increase, while the ratio of indifferent productivity to threshold productivity remains unchanged during the process of deregulating the establishment of modern distribution channels.

Finally, from Eqs. (16), (19), and (41), we have

$$\mathcal{P}_x^* = \frac{\sigma}{\sigma-1}\Big(\frac{E}{\sigma}\Big)^{\frac{1}{1-\sigma}}\left[\frac{\delta(1+\rho-\sigma)F_e}{\sigma-1}\right]^{\frac{1}{\rho}}\frac{\big[\phi+(\Phi-\phi)(\Lambda_x^*)^{1-\sigma}\big]^{\frac{1}{\rho}+\frac{1}{1-\sigma}}}{\big[\phi+(\Phi-\phi)(\Lambda_x^*)^{1+\rho-\sigma}\big]^{\frac{1}{\rho}}}F^{\frac{1+\rho-\sigma}{\rho(\sigma-1)}}. \tag{44}$$

Thus, we have

$$\begin{aligned}\frac{d\mathcal{P}_x^*}{dF} =& \frac{\partial \mathcal{P}_x^*}{\partial F} + \frac{\partial \mathcal{P}_x^*}{\partial \Lambda_x^*}\frac{\partial \Lambda_x^*}{\partial F} \\ =& \underbrace{\frac{\partial \mathcal{P}_x^*}{\partial F}}_{(+)} > 0.\end{aligned} \tag{45}$$

Proposition 4 *If the government relaxes the regulation on the establishment of modern distribution channels (i.e., the fixed costs of firms outside the marketplace F decreases), less productive firms can survive in the economy and the number of available varieties increases. However, the numbers of active firms in the marketplace and in modern distribution channels increase, while the ratio of indifferent productivity to threshold productivity remains unchanged. Finally, the effect of expanding varieties dominates that of decreasing survival productivity, which results in the improvement of social welfare during deregulation process.*

Using Eqs. (16) and (17), we have:

$$\mathcal{P}_x^* = \frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} E}{\sigma f} \phi \bar{a}_x^{1-\sigma} n_x \tag{46}$$

In both cases, threshold productivity and the size of marketplace increase when E increases or F decreases. Thus, the two variables have opposing impacts on social welfare. This is because increasing the size of a marketplace has two effects: (i) less efficient firms can survive and sell products at higher prices and (ii) all firms within the marketplace benefit from the marketplacelevel scale economy because of greater varieties. The propositions show that the latter dominates the former.

Finally, in the benchmark case, substituting Eq. (8) into (7), we obtain $\partial \mathcal{P}^*/\partial F > 0$ since $\partial \bar{a}^*/\partial F > 0$. That is, in the case without shared fixed costs in the marketplace, the decrease of fixed costs outside the marketplace leads to the exit of less productive firms, which results in higher social welfare.

6 Conclusions

In this paper, we extended Melitz's (2003) model by introducing lower fixed costs and higher transaction costs in a marketplace. In doing so, we attempted to identify the conditions for the coexistence of heterogeneous firms within and outside the marketplace. To examine the impact of shared fixed cost among firms in a marketplace, we compared two cases with and without shared cost. In addition, we examined the impacts of population size and fixed costs of modern distribution channels.

We clarified that the decreasing fixed costs of modern distribution channels leads to a larger marketplace with cost sharing. This is because lower fixed costs in modern distribution channels positively affect firms' entry decisions. Thus, a large number of entry firms triggers scale economy in the marketplace, which results in an increase in the size of the marketplace.

Given the flexibility of our model, there are numerous research directions for future works. It is natural to extend this setting to a two-region model, which can further clarify the impact of market size during the integration process. Furthermore, an empirical study

that tests the relationship between country size and the size of a marketplace can identify externalities that foster the development of marketplaces.

A Number of roots

Proof: First, we have:

$$\lim_{\Lambda_x \to 0} \mathcal{H}(\Lambda_x) = \phi\Big[1 - \frac{(1+\rho-\sigma)E}{\rho\sigma f}\Big], \quad \lim_{\Lambda_x \to 1} \mathcal{H}(\Lambda_x) = +\infty.$$

Taking the derivative of $\mathcal{H}(\Lambda_x)$ with respect to Λ_x, we have:

$$\frac{\partial \mathcal{H}(\Lambda_x)}{\partial \Lambda_x} = \frac{\Lambda_x^{\rho-\sigma}}{(1-\Lambda_x^{\rho})^2}\Psi(\Lambda_x).$$

where $\Psi(\Lambda_x) \equiv -(\phi+\Phi)(1+\rho-\sigma)\Lambda_x^{2\rho} + [\rho\Phi + (2\phi+\Phi)(1+\rho-\sigma)]\Lambda_x^{\rho} - \phi(1+\rho-\sigma)$. Thus, we have:

$$\lim_{\Lambda_x \to 0} \mathcal{H}'(\Lambda_x) = 0, \quad \lim_{\Lambda_x \to 1} \mathcal{H}'(\Lambda_x) = +\infty.$$

Furthermore, we have:

$$\lim_{\Lambda_x \to 0} \Psi(\Lambda_x) = -\phi(1+\rho-\sigma) < 0, \quad \lim_{\Lambda_x \to 1} \Psi(\Lambda_x) = \rho\Phi > 0.$$

Thus, the equation $\Psi(\Lambda_x) = 0$ has only one root in the domain $\Lambda_x \in (0,1)$. Specifically, we have:

$$\Lambda_x^0 = \left[1 - \frac{2\rho\sqrt{\Phi}}{(\sigma-1)\sqrt{\Phi} + \sqrt{(1+2\rho-\sigma)^2\Phi + 4\rho(1+\rho-\sigma)\phi}}\right]^{\frac{1}{\rho}} \in (0,1), \quad \text{and} \quad \Psi(\Lambda_x^0) = 0.$$

Thus, we have:

$$\lim_{\Lambda_x \to \Lambda_x^0} \mathcal{H}'(\Lambda_x) = 0.$$

Therefore, the number of roots of Eq. (18) can be concluded as follows:

(i) there are two roots with the same value: $\Lambda_{x,L}^* = \Lambda_{x,H}^* = \Lambda_x^0$ if and only if $\mathcal{H}(\Lambda_x^0) = 0$.

(ii) there are two roots: $\Lambda_{x,L}^* \in (0, \Lambda_x^0)$ and $\Lambda_{x,H}^* \in (\Lambda_x^0, 1)$ if and only if $\mathcal{H}(\Lambda_x^0) \leq 0 \leq \lim_{\Lambda_x \to 0} \mathcal{H}(\Lambda_x)$.

(iii) there are two roots: $\Lambda_{x,L}^* = 0$ and $\Lambda_{x,H}^* \in (\Lambda_x^0, 1)$ if and only if $0 \geq \lim_{\Lambda_x \to 0} \mathcal{H}(\Lambda_x)$.

(iv) otherwise, Eq. (18) does not have any root. (See Fig. 4.) □

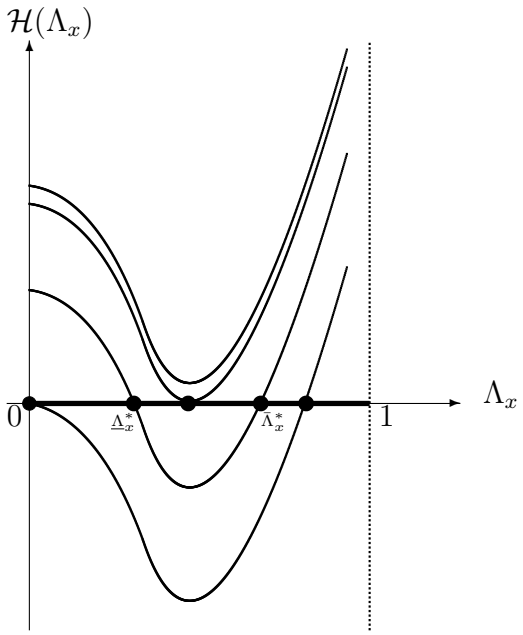

Figure 4: Multiple equilibria

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