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Efficiency Enhancing Taxation in Two-sided Markets

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Efficiency Enhancing Taxation in Two-sided Markets*

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Abstract

This paper examines the efficient provision of goods in two-sided markets and characterizes optimal specific and ad-valorem taxes. We show that (i) a monopoly may have too high output compared to the social optimum; (ii) output may be reduced by imposing negative value-added taxes (subsidy) or positive specific taxes.

JEL Codes: D4; D43; H21;H22; L13

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1 Introduction

Two-sided platform firms cater to two distinct groups of customers that are connected through quantity spillovers, and the firms maximize profit by facilitating value-creating interactions between these groups.[1] Two-sided platforms operate in many economically significant industries, such as the media sector, the financial sector (payment card systems), real-estate brokerage, and the computing industry (computer operating systems, software, game consoles etc.). The pricing strategies of a platform firm must account for interactions between the demands of different customer groups and the externalities that arise in these relationships. For instance, in the credit card industry there are positive quantity spillovers between merchants and cardholders. Merchants who accept a credit card value an increase in the number of households joining the credit card system, and vice versa.

We analyze the efficient provision and taxation of goods in two-sided markets. We find that under both perfect competition and monopoly goods may be over- or underprovided depending on the size and sign of the customer group spillover effects. In particular, if there are positive intergroup spillovers, both goods will be underprovided in the competitive equilibrium. In contrast, a monopoly platform may actually overprovide both goods. Our analysis shows that if both sides of the market can be taxed, ad valorem and unit taxes can be used to achieve the social optimum. However, there is an inherent asymmetry between ad valorem and unit taxes under monopoly. In particular, a firm always reduces output of a good which is subject to an increased unit tax. This is the only way the firm can reduce its tax burden. In contrast, the monopoly can raise output of a good subject to a higher ad-valorem tax and still reduce its tax burden by lowering the end-user price. Depending on the interrelationship between the two markets, we show that this may be a profitable strategy for the firm. If so, it could be optimal for the government to increase the ad-valorem tax rate if the firm produces above the socially optimal quantity. This result differs markedly from findings in one-sided markets (e.g. Guesnerie and Laffont, 1978).

Our analysis is related to recent papers in Industrial Organization which study two-sidedness (e.g. Anderson and Coate, 2005, and Rochet and Tirole, 2003, 2007). This literature, however, does not study taxation. The public finance literature has a long tradition for investigating efficiency enhancing

[1] Evans (2003) provides examples and classifications of two-sided markets.

taxation (see e.g. Anderson et al., 2001, and Auerbach and Hines, 2002), but to our knowledge the issue of two-sidedness has not received any attention. Our study seeks to remedy this oversight.

2 Model

We consider a two-sided market with two different groups of customers, where group $i = 1, 2$ buys x^i units of good i at price p^i. Each customer group has a quasi-linear utility function of the form

$$u^i = m^i + \phi^i\left(x^i, x^j\right) \qquad i \neq j \text{ and } i = 1, 2,$$

where $m^i \gtreqless 0$ is consumption of the outside good (the numeraire good).[2] For ease of exposition, we suppress the fact that $i \neq j$ and $i, j = 1, 2$ in the remainder of the text. The utility function $\phi^i\left(x^i, x^j\right)$ satisfies all the usual conditions for utility maximization, in particular that $\partial\phi^i/\partial x^i \equiv \phi^i_{x^i} > 0$ and $\phi^i_{x^i x^i} < 0$ (in what follows subscripts denote partial derivatives). If higher output of good j increases (decreases) the utility from good i we have $\phi^i_{x^j} > 0$ $\left(\phi^i_{x^j} < 0\right)$.

We assume that both groups are price takers with budget constraints $\omega^i = m^i + p^i x^i$. Maximization of the utility function subject to the budget constraint for each group yields the inverse demand function for good i

$$p^i\left(x^i, x^j\right) = \phi^i_{x^i}\left(x^i, x^j\right). \tag{1}$$

The profit of a monopoly platform (or of a representative platform in a perfectly competitive market) equals

$$\pi = \sum_{i=1}^{2} \frac{\left(p^i - \tau^i\right) x^i}{\left(1 + t^i\right)} - C\left(x^i, x^j\right), \tag{2}$$

where $\tau^i \gtreqless 0$ and $t^i > -1$ are the per unit and ad valorem tax, respectively, on good i. The cost function $C\left(x^i, x^j\right) \geq 0$ satisfies $C_{x^i} > 0$ and $C_{x^i x^i} \geq 0$.

[2] We allow for $m^i < 0$ so that we do not have to consider corner solutions (see Mas-Colell, Whinston and Green, 1995).

2.1 The social optimum

A social planner chooses output to maximize the sum of consumer surplus minus the costs of providing the two goods. This amounts to solving

$$\left\{x^1, x^2\right\} = \arg\max \left\{ \sum_{i=1}^{2} \phi^i \left(x^i, x^j\right) - C\left(x^1, x^2\right) \right\}.$$

Given that the second-order conditions for interior solutions are satisfied, the socially optimal quantities (marked by an asterix) are implicitly given by

$$p^{i*} = C_{x^i}\left(x^{i*}, x^{j*}\right) - \phi^j_{x^i}\left(x^{i*}, x^{j*}\right). \tag{3}$$

By noting that

$$\phi^j_{x^i}\left(x^i, x^j\right) = \int_0^{x^j} \phi^j_{x^i x^j}\left(x^i, \tilde{x}^j\right) d\tilde{x}^j = \int_0^{x^j} p^j_{x^i}\left(x^i, \tilde{x}^j\right) d\tilde{x}^j, \tag{4}$$

we can rewrite equation (3) as

$$p^{i*} = C_{x^i}\left(x^{i*}, x^{j*}\right) - \int_0^{x^{j*}} p^j_{x^i}\left(x^{i*}, \tilde{x}^j\right) d\tilde{x}^j. \tag{5}$$

The term $\int_0^{x^{j*}} p^j_{x^i}\left(x^{i*}, \tilde{x}^j\right) d\tilde{x}^j$ in (5) measures the externality that the marginal unit of good i imposes on the total utility of customer group j. If this term is positive, the price of good i should be set below its marginal cost (and vice versa for the opposite constellation).

In what follows we discuss efficiency enhancing taxation in the competitive equilibrium as well as under monopoly. In either case it is worth pointing out that efficiency can be preserved for any distribution of the tax revenue among consumers. This is due to the quasi-linearity of the preferences.

2.2 Competitive equilibrium

A competitive firm maximizes (2) with respect to x^1 and x^2, taking the end-user prices as given. In absence of taxes ($t^i = \tau^i = 0$) this yields the first-order condition for good i

$$p^i\left(x^i, x^j\right) = C_{x^i}\left(x^i, x^j\right). \tag{6}$$

Equation (6) shows that under perfect competition, each platform firm fails to account for the spillover in demand between the two goods. Depending on the sign of these spillovers, there will consequently be underprovision or overprovision of x^1 and x^2 relative to the first-best allocation. Market failure in provision can be corrected for by levying appropriate pigovian taxes, and it suffices to use either ad valorem or unit taxes. By comparing the two conditions in (5) to (6), we see that the competitive equilibrium yields first-best output if the per-unit pigovian tax scheme is characterized by

$$\tau^{i*} = -\int_0^{x^{j*}} p^j_{x^i}\left(x^{i*}, \tilde{x}^j\right) d\tilde{x}^j, \qquad t^1 = t^2 = 0. \tag{7}$$

Alternatively, the government can use solely pigovian ad valorem taxes to achieve first-best outputs:

$$t^{i*} = -\frac{\int_0^{x^{j*}} p^j_{x^i}\left(x^{i*}, \tilde{x}^j\right) d\tilde{x}^j}{C_{x^i}}, \qquad \tau^1 = \tau^2 = 0. \tag{8}$$

In both cases the sign of the optimal pigovian tax of good i is equal to the sign of the externality it imposes on the buyers of good j. If there are positive spillovers between the groups $\left(p^i_{x^j} > 0,\ i = 1, 2\right)$, both goods will be underprovided in the competitive equilibrium. A first-best solution can then be obtained by appropriately subsidizing the production of both goods.

3 Monopoly

Under imperfect competition the firms' price setting behavior on the two sides of the market may at least partially internalize demand spillovers. In order to abstract from strategic interactions, we only consider a monopoly platform. Let us again start out by considering the equilibrium if none of the goods are taxed. Differentiating (2) w.r.t. x^1 and x^2 and using (1), the platform's first-order conditions are now:

$$p^i\left(x^i, x^j\right) + x^i p^i_{x^i}\left(x^i, x^j\right) = C_{x^i}\left(x^i, x^j\right) - x^j p^j_{x^i}\left(x^i, x^j\right). \tag{9}$$

Compared to the first-order conditions in the competitive equilibrium there are two additional terms under monopoly. The first of these appears

on the left-hand side of equation (9) and is well known; by selling one extra unit of good i, the platform will have to reduce the price of good i by $\left|p^i_{x^i}(x^i,x^j)\right|$ units. This generates a revenue loss equal to $\left|x^i p^i_{x^i}(x^i,x^j)\right|$. Taken in isolation, this term implies underprovision of commodity i due to monopoly pricing.

The last term on the right-hand side of equation (9) is also new compared to the competitive equilibrium. It captures the fact that a marginally higher output of good i increases the willingness to pay for good j by $p^j_{x^i}(x^i,x^j)$ units, generating extra income equal to $x^j p^j_{x^i}(x^i,x^j)$. This reduces the alternative cost of producing good i if $p^j_{x^i}>0$, and acts like a reduction in the marginal costs of producing the good. Thereby the monopolist internalizes the intergroup externalities, but not perfectly so from a social point of view. To clarify this, we use equations (5) and (9) to define the difference in the spillovers internalized by the monopolist and the social planner as

$$\Omega^j \equiv x^j p^j_{x^i} - \int_0^{x^j} p^j_{x^i} d\tilde{x}^j. \tag{10}$$

The sign of Ω^j is in general ambiguous, showing that the monopolist may put higher weight on intergroup externalities than what a social planner would do.[3] The reason is that the monopolist accounts for how a larger output of good i affects the willingness to pay for the marginal unit of good j, while a social planner also cares for the change in the valuation for inframarginal units. Suppose, for instance, that the externalities are positive ($p^j_{x^i}>0$) and that $p^j_{x^i}$ is increasing in x^j. In this case the value of the externality is smaller on average than it is for the marginal unit, $\int_0^{x^j} p^j_{x^i} d\tilde{x}^j / x^j < p^j_{x^i}$, implying that $\Omega^j > 0$.[4]

The fact that Ω^j might be positive opens up the interesting possibility that the monopolist has too high output compared to the social optimum

[3]It is straightforward to give examples where Ω^i may be positive or negative.

[4]Note that this outcome resembles the result that a monopolist may oversupply quality compared to what a social planner would do for any given output. To see the analogy we may interpret a higher output of good i as having the effect of increasing the perceived quality of good j if $p^j_{x^i}>0$. Thus, we may perceive the monopolist as choosing a too high quality level of good j (i.e., producing too much of good i) if the marginal willingness to pay for the "quality improvement" is higher than the increased average valuation (e.g. Tirole, 1988, pp. 100-104, Spence, 1975).

even if there are positive intergroup externalities. To see this, suppose that x^i is exogenously given. Then there is a one-to-one relationship between p^j and x^j. Assuming that $x^i = x^{i*}$, it follows from equations (5) and (9) that $x^j > x^{j*}$ if $p^j < p^{j*}$, or

$$\Omega^i + x^j p^j_{x^j} > 0. \tag{11}$$

We shall say that the monopoly has an overprovision incentive of good j if inequality (11) holds, while it has an underprovision incentive if the inequality is reversed. In what follows we examine how the government can use tax policy to correct for such over- and underprovision incentives. We start out by examining a case where the government only taxes one side of the market.

3.1 Only one good taxed

In a two-sided market a government will in general need two tax instruments to ensure that the monopolist provides the socially optimal outputs. However, the government can still use tax policy to improve resource allocation even if it taxes only one side of the market with either ad valorem or unit taxes.

3.1.1 Ad valorem taxes

Suppose that only good 1 is taxed. With an ad-valorem tax (and no unit taxes) we have

$$\pi = \frac{x^1 p^1}{1+t^1} + x^2 p^2 - C(x^1, x^2).$$

Maximization of profit yields the following first-order conditions

$$\pi_{x^1} = \frac{p^1 + x^1 p^1_{x^1}}{1+t^1} + x^2 p^2_{x^1} - C_{x^1} = 0 \tag{12}$$

$$\pi_{x^2} = \frac{x^1 p^1_{x^2}}{1+t^1} + p^2 + x^2 p^2_{x^2} - C_{x^2} = 0. \tag{13}$$

and the second-order conditions require

$$H = \pi_{x^1x^1}\pi_{x^2x^2} - \pi^2_{x^1x^2} > 0, \qquad \pi_{x^1x^1} < 0 \text{ and } \pi_{x^2x^2} < 0.$$

Taking the total differential of the first-order conditions yields:

$$\frac{dx^1}{dt^1} = \frac{-\pi_{x^2x^2}\left(x^2 p^2_{x^1} - C_{x^1}\right)}{H\left(1+t^1\right)} + \frac{1}{\left(1+t^1\right)^2}\frac{-\pi_{x^1x^2}x^1 p^1_{x^2}}{H} \tag{14}$$

$$\frac{dx^2}{dt^1} = \frac{\pi_{x^1x^2}\left(x^2 p^2_{x^1} - C_{x^1}\right)}{H\left(1+t^1\right)} + \frac{1}{\left(1+t^1\right)^2}\frac{\pi_{x^1x^1}x^1 p^1_{x^2}}{H} \tag{15}$$

In order to have a two sided market, there must be positive externalities from at least one side of the market to the other.[5] The implication of this is that $p^i_{x^j} > 0$ for at least one good. The interesting feature of equations (14) and (15) is that a higher value-added tax on good 1 may increase output of both goods. To illustrate this possibility, assume that the following holds:

Example: *Suppose that; (a)* $p^2_{x^1} > 0$ and $p^1_{x^2} = 0$*; (b)* $\left(x^2 p^2_{x_1} - C_{x^1}\right) > 0$*, and; (c)* $\pi_{x^1x^2} > 0$.

With assumption (a) in the Example the willingness to pay for good 2 is increasing in the sales of good 1 ($p^2_{x^1} > 0$), while consumers of good 1 are indifferent about the output level of good 2 ($p^1_{x^2} = 0$). The latter implies that the last term on the right-hand side of (14) and (15) vanishes. We then have $dx^1/dt^1 > 0$ and $dx^2/dt^1 > 0$ if assumptions (b) and (c) hold.[6]

We thus see that a higher value-added tax might lead to higher sales, contrary to what is the standard result in one-sided markets. The reason is that a higher tax on good 1 makes it profitable for the firm to shift revenue from the more heavily taxed to the untaxed side of the market (good 2). To see why this induces the platform to increase production in the Example, note that a marginally higher output of good 1 increases the willingness to pay for good 2 by $p^2_{x^1}$ units. This gives rise to an extra income equal to $x^2 p^2_{x^1}$. With our assumption that $\left(x^2 p^2_{x_1} - C_{x^1}\right) > 0$, the income gain is higher than the marginal cost of producing good 1. This explains why $dx^1/dt^1 > 0$. Since we further have presupposed that the marginal profitability of selling good 2

[5] See Evans (2003a,b) for an informal definition of two-sidedness and Rochet and Tirole (2003) for a formal definition.

[6] The condition (b) $\left(x^2 p^2_{x^1} - C_{x^1}\right) > 0$ is unambiguously satisfied for $C_{x^1} = 0$. This is particularly likely for electronic communications services. Consider for instance an internet newspaper which sells its product to x^1 readers and has an advertising volume equal to x^2. A larger number of readers increases the advertisers willingness to pay for inserting an ad (generating an extra platform income equal to $x^2 p^2_{x^1}$). Since the marginal cost of increasing the number of readers is approximately equal to zero, we have $\left(x^2 p^2_{x^1} - C_{x^1}\right) \approx x^2 p^2_{x^1} > 0$.

is increasing in output of good 1 ($\partial \pi_{x^2}/\partial x^1 = \pi_{x^1x^2} > 0$), it follows directly that $dx^2/dt^1 > 0$.

Note that the results which follow from the example are strengthened if we relax the assumption $p^1_{x^2} = 0$ to $p^1_{x^2} \leq 0$, and that they survive at least in the neighborhood of $p^1_{x^2} = 0$ if there are positive intergroup externalities across both customer groups. Hence, assumption (a) is not crucial. Neither is assumption (c), in that we still have $dx^1/dt^1 > 0$ even if $\pi_{x^1x^2} < 0$. However, since $\pi_{x^1x^2} < 0$ implies that the marginal profitability of good 2 is decreasing in x^1, it follows that $dx^2/dt^1 < 0$. In this case we therefore get the interesting result that output of the good which is subject to a higher tax increases while output of the untaxed good falls.[7]

We shall now see what implications equations (14) and (15) have for the optimal tax policy. To this end, define welfare as the sum of consumer surplus (S^i), producer surplus (π), and tax revenue (T);

$$W = \underbrace{\phi^1\left(x^1, x^2\right) - x^1 p^1}_{S^1} + \underbrace{\phi^2\left(x^1, x^2\right) - x^2 p^2}_{S^2} + \pi + T, \tag{16}$$

where T is defined as

$$T = \frac{t^1}{1+t^1} x^1 p^1.$$

Differentiating consumer surplus, profit and tax revenue with respect to t^1 and using the envelope theorem we obtain[8]

$$\begin{aligned} \frac{dS^i}{dt^1} &= -\Omega^i \frac{dx^j}{dt^1} - x^i p^i_{x^i} \frac{dx^i}{dt^1}; \quad \frac{d\pi}{dt^1} = -\frac{x^1 p^1}{\left(1+t^1\right)^2} \text{ and} \\ \frac{dT}{dt^1} &= \frac{x^1 p^1}{\left(1+t^1\right)^2} + \frac{t^1}{1+t^1}\left[\left(p^1 + x^1 p^1_{x^1}\right)\frac{dx^1}{dt^1} + x^1 p^1_{x^2}\frac{dx^2}{dt^1}\right]. \end{aligned} \tag{17}$$

With only one tax instrument it is generally not possible to achieve socially optimal outputs. However, by solving $dW/dt^1 = 0$ and assuming that the second-order conditions hold, we find that a second-best equilibrium can

[7] The price of good 2 will then rise, both because own quantity falls and because output of good 1 increases. See Kind, Koethenbuerger and Schjelderup (2006) for a detailed discussion of the sign of $\pi_{x^1x^2}$ and of how the strength and size of the externalities affect output responses to a tax increase.

[8] If the platform's first-order conditions yield an interior solution we have $\pi = \pi(x^1(t^1), x^2(t^1), t^1)$, so that $d\pi/dt^1 = \partial\pi/\partial t^1$.

be obtained by setting the tax rate equal to

$$\frac{\hat{t}^1}{1+\hat{t}^1} = \frac{\left(\hat{\Omega}^1 + \hat{x}^2\hat{p}^2_{x^2}\right)\frac{d\hat{x}^2}{dt^1} + \left(\hat{\Omega}^2 + \hat{x}^1\hat{p}^1_{x^1}\right)\frac{d\hat{x}^1}{dt^1}}{\left(\hat{p}^1 + \hat{x}^1\hat{p}^1_{x^1}\right)\frac{d\hat{x}^1}{dt^1} + \hat{x}^1\hat{p}^1_{x^2}\frac{d\hat{x}^2}{dt^1}}, \tag{18}$$

where the hat indicates that we consider equilibrium values (and derivatives evaluated at the equilibrium values).

Using the assumptions in the Example we can now sign the optimal tax rate in (18) as follows:[9]

$$\frac{\hat{t}^1}{1+\hat{t}^1} = \frac{\overbrace{\hat{x}^2\hat{p}^2_{x^2}}^{-}\overbrace{\frac{d\hat{x}^2}{dt^1}}^{+} + \left(\hat{\Omega}^2 + \hat{x}^1\hat{p}^1_{x^1}\right)\overbrace{\frac{d\hat{x}^1}{dt^1}}^{+}}{\underbrace{\left(\hat{p}^1 + \hat{x}^1\hat{p}^1_{x^1}\right)}_{-}\underbrace{\frac{dx^1}{dt^1}}_{+}}. \tag{19}$$

Suppose that $\left(\hat{\Omega}^2 + \hat{x}^1\hat{p}^1_{x^1}\right) < 0$. Other things equal the monopoly then has incentives to underprovide good 1. In order to correct for this, we see from equation (19) that the government must set a positive tax ($\hat{t}^1 > 0$). Thereby it induces the monopoly to increase production. A positive tax is warranted also if $\left(\hat{\Omega}^2 + \hat{x}^1\hat{p}^1_{x^1}\right) > 0$ but not high enough to make the numerator in (19) positive. The reason why $\hat{t}^1 > 0$ in this case is that the monopoly always underprovides good 2 in absence of taxes (since a higher output of this good does not generate higher income on the other side of the market when $p^1_{x^2} = 0$). Thus, the fact that $dx^2/dt^1 > 0$ makes it optimal to set a positive tax rate unless the overprovision incentive for good 1 is sufficiently high.

3.1.2 Unit taxes

In this section we assume that there are no ad valorem taxes and that the unit tax only falls on good 1. Profits can then be written as

$$\pi = x^1\left(p^1 - \tau^1\right) + x^2p^2 - C(x^1, x^2)$$

[9] From equation (12) we see that assumption (b) in the example implies that $\left(p^1 + x^1p^1_{x^1}\right) < 0$.

Maximizing profit yields the first order conditions

$$\begin{aligned} \pi_{x^1} &= p^1 - \tau^1 + x^1 p^1_{x^1} + x^2 p^2_{x^1} - C_{x^1} = 0 \quad (20) \\ \pi_{x^2} &= x^1 p^1_{x^2} + p^2 + x^2 p^2_{x^2} - C_{x^2} = 0. \quad (21) \end{aligned}$$

The second order conditions are the same as under ad valorem taxation, and totally differentiating (20) and (21) we find

$$\frac{dx^1}{d\tau^1} = \frac{\pi_{x^2x^2}}{H} < 0 \qquad \frac{dx^2}{d\tau^1} = -\frac{\pi_{x^1x^2}}{H}. \quad (22)$$

As in a one-sided market, we see that a higher specific tax on a good leads to lower output of that good. This simply reflects the fact that an increase in τ^1 is like an increase in the marginal costs of producing good 1. If the marginal profit of selling good 2 is increasing in output of good 1 ($\pi_{x^1x^2} > 0$), the reduced output of good 1 further makes it optimal to reduce output of good 2 ($dx^2/d\tau^1 < 0$). Otherwise the firm's tax burden will increase. We thus have an asymmetry between unit taxes and ad-valorem taxes; as shown above it might be optimal for the firm to increase output of a good which is subject to higher ad-valorem taxes.

Welfare is still given by (16), but tax revenue is now equal to $T = \tau x^1$. We also have a similar expression for the changes in consumer surplus as under ad valorem taxation, that is:

$$\frac{dS^1}{d\tau^1} = -\Omega^1 \frac{dx^2}{d\tau^1} - x^1 p^1_{x^1} \frac{dx^1}{d\tau^1} \text{ and } \frac{dS^2}{d\tau^1} = -\Omega^2 \frac{dx^1}{d\tau^1} - x^2 p^2_{x^2} \frac{dx^2}{d\tau^1}. \quad (23)$$

For the changes in profits and tax revenue we have:

$$\frac{d\pi}{d\tau^1} = -x^1, \qquad \frac{dT}{d\tau^1} = x^1 + \tau^1 \frac{dx^1}{d\tau^1}.$$

Using this information in the expressions for change in welfare we find that $dW/d\tau^1 = 0$ implies

$$\hat{\tau}^1 = \left[\left(\hat{\Omega}^1 + \hat{x}^2 \hat{p}^2_{x^2} \right) \frac{d\hat{x}^2}{d\tau^1} + \left(\hat{\Omega}^2 + \hat{x}^1 \hat{p}^1_{x^1} \right) \frac{d\hat{x}^1}{d\tau^1} \right] \left[\frac{d\hat{x}^1}{d\tau^1} \right]^{-1}. \quad (24)$$

With the assumptions in the Example we have $dx^1/d\tau^1 < 0$ and $dx^2/d\tau^1 < 0$. If the monopoly has underprovision incentives for both goods ($\hat{\Omega}^i + \hat{x}^j \hat{p}^j_{x^j} < 0$), we thus see that the government should subsidize production in order to

increase output, while it is optimal to tax production if $\hat{\Omega}^i + \hat{x}^j \hat{p}^j_{x^j} > 0$. These results are in accordance with insight from traditional tax analysis. Only if there exists an overprovision incentive for one good and an underprovision incentive for the other is the sign of the specific tax ambiguous.

3.2 Taxation of both goods

The government can achieve first-best outputs under monopoly by appropriately choosing unit or ad valorem taxes. Setting $t^1 = t^2 = 0$, it follows from equation (2) that the platform's first-order conditions in the presence of only unit taxes are equal to

$$p^i\left(x^i, x^j\right) = \tau^i - x^i p^i_{x^i}\left(x^i, x^j\right) - x^j p^j_{x^i}\left(x^i, x^j\right) + C_{x^i}\left(x^i, x^j\right). \tag{25}$$

Equating the monopolist's equilibrium prices with those of a social planner (c.f. equation (5)), it follows that the optimal unit taxes are given by[10]

$$\tau^{i*} = \Omega^{j*} + x^{i*} p^{i*}_{x^i}. \tag{26}$$

The intuition behind equation (26) is straightforward; the specific tax on good i should be positive (negative) if the monopoly otherwise oversupplies (undersupplies) the good.

The government can also reproduce the first best solution by use of ad valorem taxes. Setting $\tau^1 = \tau^2 = 0$, we find that the firm's first-order conditions equal

$$\pi_{x^1} = 0 => \frac{p^1 + x^1 p^1_{x^1}}{1+t^1} + \frac{x^2 p^2_{x^1}}{1+t^2} - C_{x^1} = 0 \tag{27}$$

$$\pi_{x^2} = 0 => \frac{x^1 p^1_{x^2}}{1+t^1} + \frac{p^2 + x^2 p^2_{x^2}}{1+t^2} - C_{x^2} = 0. \tag{28}$$

Using the same procedure as under unit taxes, it can be shown that the optimal ad valorem taxes that reproduce the first-best output levels are

$$t^{i*} = \frac{\left(p^{j*} + x^{j*} p^{j*}_{x^j}\right)\left(\Omega^{j*} + x^{i*} p^{i*}_{x^i}\right) - x^{j*} p^{j*}_{x^i}\left(\Omega^{i*} + x^{j*} p^{j*}_{x^j}\right)}{\left(p^{j*} + x^{j*} p^{j*}_{x^j}\right) C^*_{x^i} - x^{j*} p^{j*}_{x^i} C^*_{x^j}}. \tag{29}$$

[10] Recall, variables marked by an asterix denote first-best allocations.

In order to see that it may be optimal to set a positive tax on good 1 if it is otherwise overproduced and vice versa, we continue to use the assumptions specified in the Example. From (29) we then find

$$\begin{aligned} t^{1*} &= \frac{\overbrace{\left(p^{2*}+x^{2*}p^{2*}_{x^2}\right)}^{+}\left(\Omega^{2*}+x^{1*}p^{1*}_{x^1}\right)+\overbrace{(x^{2*})^2p^{2*}_{x^1}\left(-p^{2*}_{x^2}\right)}^{+}}{\underbrace{\left(p^{2*}+x^{2*}p^{2*}_{x^2}\right)C^*_{x^1}}_{+}+\underbrace{\left(-x^{2*}p^{2*}_{x^1}C^*_{x^2}\right)}_{-}} \\ t^{2*} &= \frac{x^{2*}p^{2*}_{x^2}}{C^*_{x^2}}<0. \end{aligned} \tag{30}$$

Given the assumption $p^1_{x^2}=0$ the monopoly does not have any externalities to internalize when it decides the output level of good 2. Totally differentiating first-order conditions (27) and (28) we therefore find that $\frac{dx^2}{dt^2}=-\frac{p^2+x^2p^2_{x^2}}{(1+t^2)^2}\left(\frac{-\pi_{x^2x^2}}{H}\right)<0$, similar to what we would have in a one-sided market. Since the good in absence of taxes is undersupplied compared to social optimum, we consequently get the standard result that $t^{2*}<0$.

Whether also t^{1*} is negative depends on the sizes of $\left(\Omega^{2*}+x^{1*}p^{1*}_{x^1}\right)$ and $C^*_{x^1}$.[11] To see why, suppose in accordance with the assumptions in the Example that the marginal costs of producing good I are "low". We then know that $dx^1/dt^1>0$.[12] If the firm otherwise overproduces good 1 ($\Omega^{2*}+x^{1*}p^{1*}_{x^1}>0$), the government would like the firm to produce less of it. This can be achieved by setting a negative tax rate ($t^{1*}<0$).[13] If the underprovision incentive for good 1 is sufficiently strong ($\Omega^{2*}+x^{1*}p^{1*}_{x^1}<<0$), on the other hand, it is optimal for the government to set $t^1>0$. This positive tax induces the monopoly to produce more of good 1. Note that these results are qualitatively equivalent to those we arrived at when only Good 1 was taxed.

Finally, it should be noted that the crucial assumption in our working example is that $(x^2p^2_{x^1}-C_{x^1})>0$; the assumptions that $\pi_{x^1x^2}>0$ and

[11] Note that with $t^2<0$ and $x^2p^2_{x^1}-C_{x^1}>0$ we have $\left(p^1+x^1p^1_{x^1}\right)<0$. This follows from the first-order condition (27).

[12] With two ad valorem taxes the quantity response is equivalent to equation (14) except that $p^2_{x^1}$ has to be replaced by $\frac{1}{1+t^2}p^2_{x^1}$. In this case assumption (b) reads $\frac{1}{1+t^2}x^2p^2_{x^1}-C_{x^1}$.

[13] When $\left(\Omega^2+x^1p^1_{x_1}\right)>0$ and C_{x^1} is "small" the numerator in (30) is positive and the denominator negative.

$p^1_{x^2} = 0$ are made only to verify as simply as possible that output responses to a higher tax and the sign of the optimal taxes may be the opposite of those we typically find in one-sided markets. If in contrast $(x^2 p^2_{x^1} - C_{x^1}) < 0$, tax responses are more likely to be qualitatively similar in one-sided and two-sided markets. This makes intuitive sense, since the two-sidedness is then not very pronounced.

4 Conclusion

In this paper we have shown that in absense of taxes, a monopoly platform in a two-sided market may have too high outputs compared to the social optimum. We have also identified situations where higher value-added taxes increase output, such that a negative VAT rate (subsidy) may be warranted if the monopoly otherwise produces too much. These results are in sharp contrast to those we typically find in one-sided markets, and they are more probable the smaller the marginal production costs and the larger the marginal value of the externalility of a taxed good. Through a formal analysis we have further demonstrated that the signs of the optimal VAT rates may be the opposite to those of the optimal specific taxes. In particular, while a higher VAT rate in some cases increases output, the monopoly will always produce less of a good that faces a higher specific tax. The reason is that there is a one-to-one relationship between tax payments and quantity under specific taxes, while there is no direct link between output and the burden of taxation under ad valorem taxation. In fact, subsequent to a higher ad valorem tax the firm can in principle both reduce tax payments and increase the quantity by lowering the price.

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