HAL
archives-ouvertes.fr

Fertility, Longevity and International Capital Flows

Zsofia Barany, Nicolas Coeurdacier, Stéphane Guibaud

▶ **To cite this version:**

Zsofia Barany, Nicolas Coeurdacier, Stéphane Guibaud. Fertility, Longevity and International Capital Flows. 2015. <hal-01164462>

HAL Id: hal-01164462

https://hal-sciencespo.archives-ouvertes.fr/hal-01164462

Submitted on 16 Jun 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Fertility, Longevity, and Capital Flows

Zsófia Bárány
SciencesPo Paris

Nicolas Coeurdacier
SciencesPo Paris and CEPR

Stéphane Guibaud
SciencesPo Paris

June 10, 2015*

Abstract

The neoclassical growth model predicts large capital flows towards fast-growing emerging countries. We show that incorporating fertility and longevity into a lifecycle model of savings changes the standard predictions when countries differ in their ability to borrow inter-temporally and across generations through social security. In this environment, global aging triggers capital flows from emerging to developed countries, and countries' current account positions respond to growth adjusted by current and expected demographic composition. Data on international capital flows are broadly supportive of the theory. The fact that fast-growing emerging countries are also aging faster, while having less developed credit markets and pension systems, explains why they are more likely to export capital. Our quantitative multi-country overlapping-generations model explains a significant fraction of the patterns of capital flows, across time and across developed and emerging countries.

*We thank Pierre-Olivier Gourinchas, Espen Henriksen, and conference participants at the Chicago Booth International Macro Finance Conference, ESSIM, and BI-SHoF Conference for helpful comments. Assia Elgouacem and Benjamin Freyd provided excellent research assistance. Nicolas Coeurdacier thanks the ANR (Chaire d'Excellence INTPORT), the ERC (Starting Grant INFINHET) and Banque de France for financial support. Stéphane Guibaud gratefully acknowledges financial support from Banque de France. Contact details: zsofia.barany@sciencespo.fr; nicolas.coeurdacier@sciencespo.fr; stephane.guibaud@sciencespo.fr.

1 Introduction

The world is aging and is expected to age further in the years to come. Over the last decades, fertility and mortality have been falling worldwide, with emerging countries converging towards the demographic patterns of the developed world (Figure 1.1, panels (a) and (b); see also Lee (2003)). While the world was essentially bi-modal in the early sixties — with developed countries already characterized by low fertility and high life expectancy, versus emerging countries characterized by higher fertility and lower life expectancy — demographic patterns have become more homogenous across the globe (Figure 1.2).

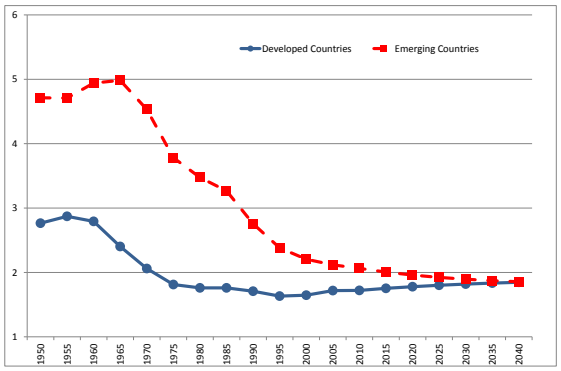
Despite overall convergence, panels (c) and (d) of Figure 1.1 reveal substantial heterogeneity in the timing and pace of demographic evolutions since the 1950s — especially among the group of emerging/developing countries. While South-East Asia has converged very fast towards the developed world both in terms of fertility and longevity, the rest of Asia, Latin America, the Middle East, and North Africa have converged at a slower pace; and Sub-Saharan Africa still has fairly high fertility and mortality rates.

The goal of this paper is to investigate how these broad demographic trends, featuring both common and country-specific components, can help explain patterns of international capital flows over time and across regions. Central to our analysis is the interaction between demographic evolutions and cross-country heterogeneity in access to credit (i.e., ability to transfer resources over time) and social security (i.e., ability to transfer resources across generations).

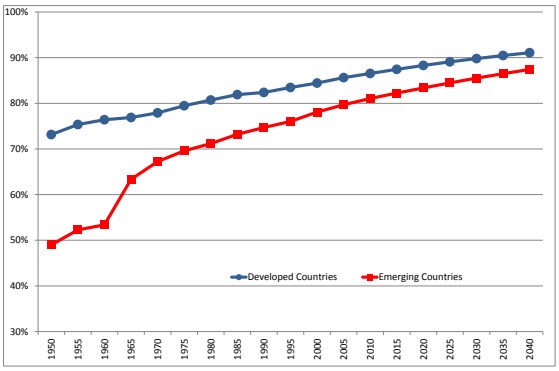
While the global trend in aging should affect world savings and investment, international capital flows should be left unaffected according to standard theory.[1] Capital flows would only arise if some countries were aging faster than others. Over time, as demographic patterns converge across countries, one would thus expect to see less capital flows between countries. In the data instead, the turn of the century was marked by large net capital flows — with vast amounts of capital flowing uphill from emerging to developed countries in the years 2000s (Bernanke (2005), Obstfeld and Rogoff (2009)).

The first contribution of our paper is to show that *global aging* can be a powerful driver

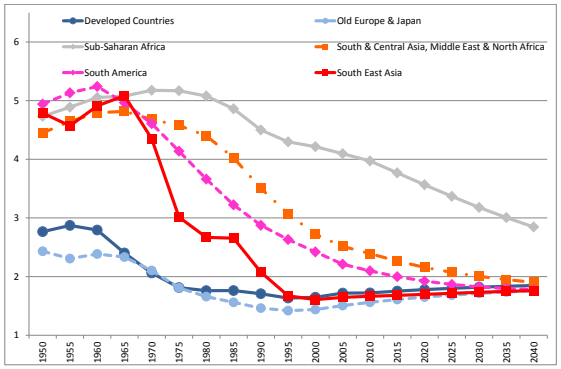
[1] In a standard OLG setting for instance, if all individuals across the globe are expected to live longer, savings should increase significantly on impact before falling as the share of retirees increases. However, all countries cannot simultaneously run a current account surplus, followed by deficits later on. In equilibrium, the world interest rate would adjust, leaving capital flows unchanged.



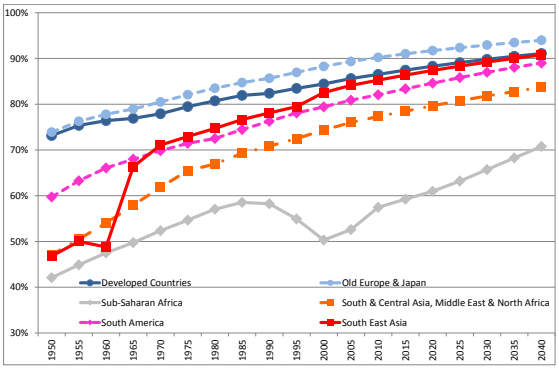
(a) Fertility: Developed vs Emerging countries



(b) Longevity: Developed vs Emerging countries



(c) Fertility across Regions



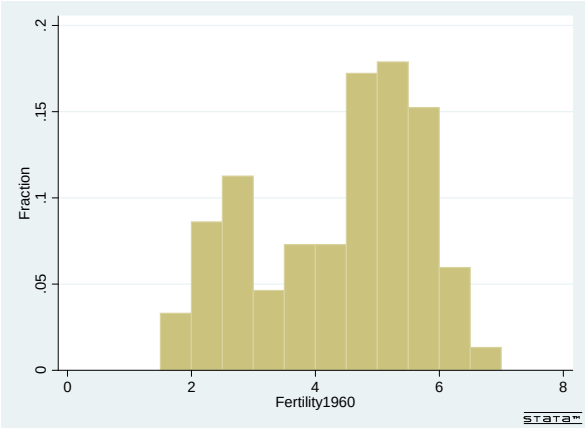
(d) Longevity across Regions

Figure 1.1: Global Aging: Fertility and Longevity across the World, 1950-2040

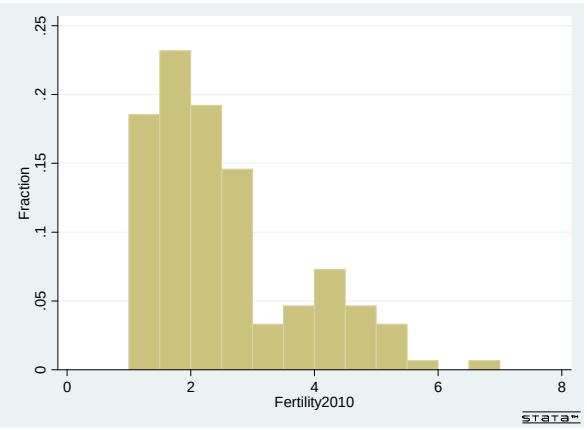
Notes: Panels (a) and (c) show fertility rate adjusted for infant mortality. Panel (b) and (d) show the probability of surviving until age 65 conditional on being 25. See Appendix A for data sources and list of countries.

of capital flows if the response of savings and investment to the *common* demographic trend differs across countries. Cross-country differences in the ability to borrow over the lifecycle (i.e., heterogenous levels of financial development across countries) as well as differences in the ability to borrow across generations (i.e., heterogeneity in retirement systems) can generate large differences in the savings responses to world aging, thus leading to diverging net foreign asset positions across countries. In particular, global aging can be a source of uphill capital flows — whereby emerging countries lend to richer countries, which typically have more developed credit markets and and a wider social security coverage.

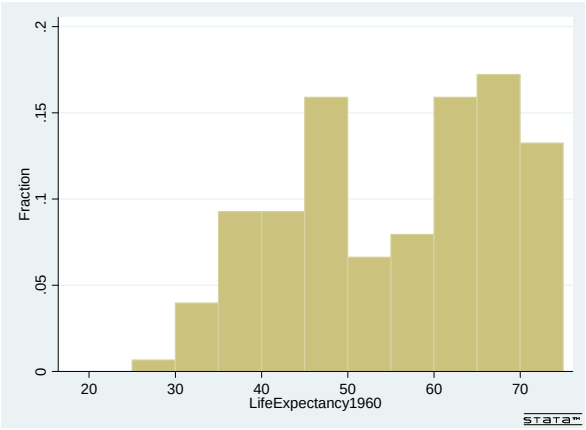
A second contribution of the paper is to investigate *differences in aging patterns* across



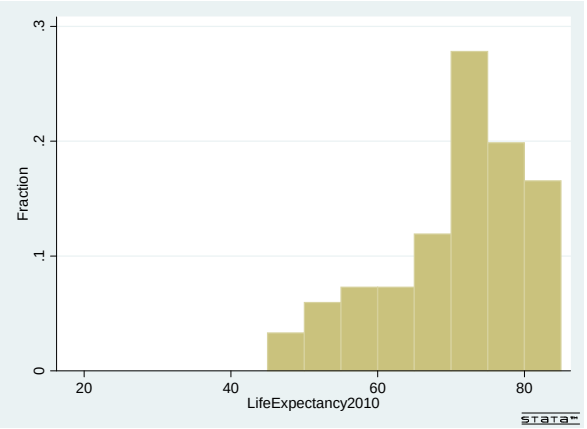
(a) Fertility Rates in 1960



(b) Fertility Rates in 2010



(c) Life Expectancy in 1960



(d) Life Expectancy in 2010

Figure 1.2: Fertility Rates and Life Expectancy: Cross-Country Distributions

Notes: The histograms in panels (a) and (b) represent the distribution across countries of fertility rates adjusted for infant mortality. The histograms in panels (c) and (d) represent the distribution across countries of life expectancy at birth. See Appendix A for data sources and list of countries.

countries as a potential driver of capital flows — with a particular focus on developing countries. While all countries are aging on average, they do so at a different speed and at different points in time. Whereas some developing countries are aging faster than the world average, others started their demographic transition later and at a much slower speed — Sub-Saharan Africa, in particular. In our model, emerging countries with faster aging prospects are more likely to export capital. Figure 1.3 suggests that this is also true in the data.[2] Interestingly, if the fast-aging developing countries have on average higher pro-

[2]This regularity confirms the findings of recent empirical work done for developed countries (see Domeij and Flodén (2006) and Ferrero (2010) among others), while extending the analysis to a large set of developing countries, for which demographic evolutions are arguably more dispersed.

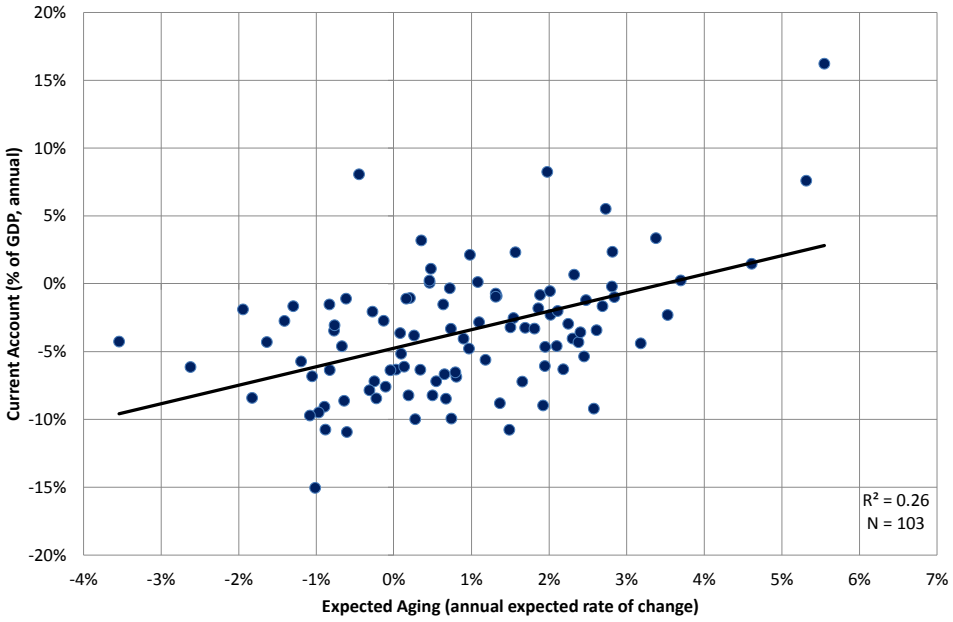


Figure 1.3: Country-Specific Aging and Capital Flows in Emerging Countries

Notes: For each country, expected aging is defined as the expected change in the old-dependency ratio between 2010 and 2035 (annualized). Current account (as a percentage of GDP) is the average of annual current account over GDP for the period 1990-2010. Sample of emerging countries excluding oil producers. See Appendix A for data sources and list of countries.

ductivity growth than the slow-aging ones, demographics could help explain why capital is not flowing towards the fast growing emerging countries — a phenomenon known as the 'allocation puzzle' (Gourinchas and Jeanne (2012)). Our model also has some distinct implications for developed countries: since Continental Europe and Japan have been aging faster than the US and other Anglo-Saxon economies, our theory predicts that the latter are more likely to be capital recipients in the recent period, especially as their household credit markets are the most developed.

We first articulate our theory of aging and capital flows in a stylized multi-country overlapping generations model. In a given country, three generations coexist: young agents borrow against future income; the middle-aged work, contribute to finance social security, and save for retirement; the elderly consume out of their savings and retirement benefits. The ability to borrow when young and the extent of contributions to social security (and the associated benefits) differ across countries and are assumed to be higher in developed countries. Global and country-specific demographic trends are captured by exogenous fertility and probability of surviving into old age.

This simple framework provides the minimum set of ingredients to explore the impact

of fertility and longevity on capital flows when countries differ in their ability to transfer resources over time and across generations. In this framework, we derive conditions under which global aging triggers uphill capital flows, from emerging to developed countries. This happens if the fall in the interest rate in response to world aging is large enough—i.e., for a low enough elasticity of intertemporal substitution and a low enough level of social security globally. Uphill capital flows are reinforced if the financing of social security in developed countries adjusts (to the pressure exerted by population aging) mostly through higher contribution rates, rather than through lower retirement benefits.

The uphill capital flows triggered by global aging are driven by a divergence in savings across countries. With a large fall in the interest rate and a small adjustment of retirement benefits, aggregate savings in the developed world tend to fall due to higher borrowing by the young and less savings by the middle aged — the latter being caused by a combination of higher contributions to social security and a higher present value of their future retirement benefits (wealth effect). By contrast, savings in emerging countries tend to increase in response to global aging, due to higher longevity combined with low social security and a different response to a drop in the interest rate. A lower ability to borrow against future income and future generations dampens the wealth and the substitution effect of the interest rate on savings while strengthening the income effect.

We also explore in the model how capital flows are affected by country-specific aging prospects — focusing our attention on emerging countries. Countries which experience their demographic transition at the time they integrate to the world capital market start to export capital very quickly (possibly as soon as they open up), even more so if their credit markets and social security system are underdeveloped. Countries which start their demographic transition at a later stage are more likely to experience capital inflows at opening.

While our stylized model of aging and capital flows is useful to deliver the main theoretical insights of the paper analytically (in closed form), our objective is to provide a quantitative framework that can be confronted with data on capital flows across countries and over time. In the extended version of the model, the crucial ingredients are the same but agents can live for a larger number of periods, with a given probability of dying at the end of each period. We consider several regions, and for each we calibrate mortality, fertility and productivity growth rates to the data, and to the extent of data availability,

we also calibrate the contribution rates to social security and the development of household credit markets. Based on our calibration, we simulate the world economy over the period 1960-2040, starting from complete autarky until 1980 when world capital markets integrate.[3]

Our quantitative model is broadly able to reproduce qualitatively and quantitatively the patterns of capital flows observed across countries, both emerging and developed, over the last three decades. Initially, in the eighties, the size of capital flows is relatively modest despite massive differences in demographic composition across countries, and capital tends to flow from Old Continental Europe and Japan to the younger regions. As emerging countries integrate to world capital markets, most of them initially import capital — but much less so than predicted by a standard neoclassical model. Due to its fast growth, East Asia attracts capital flows despite its fast aging prospects, while the rest of the developing countries also attract capital as they are expected to stay young relatively longer.

Later on in the late nineties, as some emerging countries (most notably East Asia) are aging faster than the rest of the world, they turn into creditors, slowly replacing Old Europe as an important world lender — a trend further reinforced by global aging. Instead, countries with a delayed demographic transition (Sub-Saharan Africa and to a lesser extent South and Central Asia) become the main debtors among the developing countries, despite their slower growth path. In the developed world, global aging reinforces the position of Anglo-Saxon countries as large debtors, while it brings Old Europe closer to balance — Japan remaining an important creditor as it is aging more and growing less.

Our paper relates to existing work on capital flows and demographics (Backus, Cooley and Henriksen (2014), Brooks (2003), Choukhmane (2012), Domeij and Floden (2006), Ferrero (2010), Obstfeld and Rogoff (1996)) which focus mostly on developed countries. Our paper also differs from theirs by investigating how global trends can generate capital flows when countries are heterogeneous in their level of development (financial markets and welfare systems). Recent studies by Attanasio, Kitao, Violante (2007), Krueger and Ludwig (2007), Borsch-Supan, Ludwig, Winter (2006) investigate retirement systems, potentially different across countries, in a global economy but these mostly focus on implications for

[3] In future iterations of the paper, we will also explore scenarios where different groups of countries integrate to the world capital market at different points of time.

the financing of social security and for welfare across countries and generations, rather than on capital flows. Finally, from a theoretical perspective, our work also relates to a large literature which investigates the mechanisms driving global imbalances and uphill capital flows.[4] We see our contribution as complementary to theirs as none of these papers focuses on demographics and global aging as an important driver of capital flows from developing countries to developed countries. With respect to our empirical contribution, our investigation of demographic changes as a source of capital flows in the model and in the data is related to a literature on the medium-run determinants of current account deficits.[5] Contrary to these papers, equipped with our theoretical predictions, we show that expected changes in aging are the main factor behind cross-country differences in capital flows — instead of current demographic compositions.

The paper is structured as follows. Section 2 develops a stylized model of aging and capital where we derive our main theoretical results and convey the main intuitions behind our findings. Section 3 presents the quantitative model calibrated to the data and confronts our quantitative predictions on capital flows across countries and over time to their empirical counterpart. Section 4 concludes.

2 Theory

The world consists of multiple countries, populated by overlapping generations of agents who live at most for three periods: youth (y), middle age (m), and retirement (o). Agents only work when they are middle-aged. Preferences are identical across countries, and all countries use the same technology to produce one homogeneous good — which is used for consumption and investment, and is traded freely and costlessly. Labor is immobile across countries, and firms are subject to changes in country-specific productivity and labor force. To begin with, we describe our setup by considering only one country in isolation (we therefore omit country indices at this stage to simplify notations).

[4]See among others Aguiar and Amador (2011), Bacchetta and Benhima (2013), Benhima (2013), Caballero, Farhi and Gourinchas (2008), Carroll and Jeanne (2009), Coeurdacier, Guibaud and Jin (2014), Gourinchas and Jeanne (2011), Mendoza, Quadrini and Rios-Rull (2009), Sandri (2010), Song, Storesletten and Zilibotti (2011). See the survey in Gourinchas and Rey (2013) for further references.

[5]See Alfaro et al. (2014), Chinn and Prasad (2003), Higgins (1998), Lane and Milesi-Ferretti (2002), Taylor and Williamson (1994) and a meta-analysis by Cazorzi et al. (2012) for further references. See Bloom et al. (2003) for the determinants of aggregate savings across countries and Leff (1969) for a seminal contribution.

Demographics. All agents reach middle age with probability one, but a middle-aged agent in period t survives to old age with probability p_t. Individuals of the same generation are grouped into households, each comprising a continuum of agents. Young and middle-aged households are of measure one, whereas the mass of an elderly household in period $t+1$ is given by p_t. Let $L_{y,t}$ (respectively $L_{m,t}$) denote the number of young (respectively middle-aged) households in period t. Each young household in period $t-1$ has n_t children, such that $L_{y,t} = n_t L_{y,t-1}$. It follows that $L_{m,t+1} = n_t L_{m,t}$, i.e., the growth rate of the labor force between t and $t+1$ is given by $(n_t - 1)$.

Production. There is a unique final good used for both consumption and investment. Production uses labour supplied inelastically by middle-aged agents:

$$Y_t = K_t^{\alpha} \left(A_t L_{m,t}\right)^{1-\alpha},$$

where K_t denotes the capital stock accumulated at the end of period $t-1$, and A_t is labour-augmenting productivity, which grows at an exogenous rate $\gamma_{A,t+1}$ between period t and $t+1$. Labour and capital markets are competitive. Assuming full capital depreciation over a generation, the wage rate w_t in period t and the gross rate of return R_t between periods $t-1$ and t are given by:

$$w_t = (1-\alpha) A_t k_t^{\alpha} \qquad \text{and} \qquad R_t = \alpha k_t^{\alpha-1}, \tag{1}$$

respectively, where $k_t \equiv K_t/(A_t L_{m,t})$ denotes the capital-effective-labour ratio.

Social Security. Middle-aged workers pay social security contributions proportional to their wage income, at rate τ_t. An agent who reaches retirement in period $t+1$ receives social benefits $\sigma_{t+1} w_t$, where σ_{t+1} denotes the replacement rate in period $t+1$. The social security system ('Pay-as-you-go') runs a balanced budget in every period, which requires that the contribution and replacement rates satisfy:

$$p_t \sigma_{t+1} w_t = n_t \tau_{t+1} w_{t+1}. \tag{2}$$

A higher old-dependency ratio ($L_{o,t+1}/L_{m,t+1} = p_t/n_t$) or smaller wage growth (w_{t+1}/w_t) exert pressure on the financing of the retirement system, which needs to adjust through a

combination of higher contribution rates (τ_{t+1}) or smaller replacement rates (σ_{t+1}).

Household Preferences and Budget Constraints. Let $c_{\chi,t}$ denote the consumption in period t of an agent of generation $\chi \in \{y, m, o\}$. A young household in period $t-1$ maximizes the following lifetime utility with a discount factor $\beta \in (0, 1)$,

$$U_{t-1} = u(c_{y,t-1}) + \beta u(c_{m,t}) + \beta^2 p_t u(c_{o,t+1}),$$

which is the expected lifetime utility of any individual in the household, or is the total lifetime utility of all members. We assume that the per period utility is given by $u(c) = \frac{c^{1-1/\omega}}{1-1/\omega}$, where $\omega \leq 1$ denotes the elasticity of intertemporal substitution.[6] The maximization is subject to the following budget constraints:

$$c_{y,t-1} + a_{y,t-1} = 0, \tag{3}$$

$$c_{m,t} + a_{m,t} = (1 - \tau_t)\, w_t + R_t a_{y,t-1}, \tag{4}$$

$$c_{o,t+1} = \frac{R_{t+1} a_{m,t}}{p_t} + \sigma_{t+1} w_t. \tag{5}$$

Since the young do not work, the household needs to borrow to finance consumption, $a_{y,t-1} < 0$ denotes the value of their end-of-period net asset holdings. Gross labor income earned in middle age is used for debt repayment, contribution to social security, consumption, and asset accumulation, where $a_{m,t}$ denotes the amount of assets accumulated for retirement. At the end of the middle-age period, the assets held by the mass of agents who do not survive are transferred to the mass of those who do within the household, so that an individual who reaches retirement earns gross income from savings $R_{t+1} a_{m,t}/p_t$. Elderly agents consume all available resources, including social security benefits, leaving no bequests.

Credit Constraints. Young households are subject to a credit constraint, whereby they cannot borrow more than a given fraction of their discounted future gross labor income. A

[6]The assumption that $\omega \leq 1$ is standard and in line with the empirical evidence. Since the seminal paper of Hall (1988), estimates of the elasticity of intertemporal substitution are typically below 0.5 (see Attanasio and Weber (1993), Ogaki and Reinhart (1998), Vissing-Jørgensen (2002), and Yogo (2004) among others). The macro and asset pricing literature discussed in Guvenen (2006) typically assumes values between 0.5 and 1.

young household in period $t-1$ faces the constraint:

$$a_{y,t-1} \geq -\theta_{t-1}\frac{w_t}{R_t},$$

where θ_{t-1} measures the level of development of credit markets in period $t-1$. We assume that the constraint is binding in every period, which implies that[7]

$$c_{y,t-1} = -a_{y,t-1} = \theta_{t-1}\frac{w_t}{R_t}. \tag{6}$$

Saving Decisions. The first order condition with respect to middle-age consumption yields the following Euler equation:

$$(c_{o,t+1})^{1/\omega} = \beta R_{t+1}(c_{m,t})^{1/\omega}. \tag{7}$$

Using the above equation along with (4), (5), and (6), accumulated wealth at middle age can be expressed as:

$$a_{m,t} = \frac{p_t(1-\tau_t-\theta_{t-1})}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}w_t - \frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}\frac{p_t\sigma_{t+1}w_t}{R_{t+1}}. \tag{8}$$

The first term in (8) involves the fraction of wages net of taxes and debt repayment that is saved for the next period.[8] This fraction is increasing in the probability of survival p_t, and as long as $\omega < 1$, decreasing in the rate of interest R_{t+1}. The second term in (8) captures the impact on middle-aged savings of the retirement benefits received in old age. The reduction in middle-aged savings coming from this term can be interpreted as intergenerational borrowing: effectively, middle-aged agents borrow against their future pension benefits, i.e., against the social security contributions levied on the middle-aged workers of next period. Through a wealth effect, this amount of borrowing is decreasing in R_{t+1}. Using (1) and (2), we can rewrite (8) as:

$$a_{m,t} = \frac{p_t(1-\tau_t-\theta_{t-1})}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}(1-\alpha)A_tk_t^{\alpha} - \frac{1-\alpha}{\alpha}\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}n_t\tau_{t+1}A_{t+1}k_{t+1}. \tag{9}$$

[7] See Appendix B for conditions under which this assumption holds.

[8] Note that net disposable income in middle age is only positive if $\theta_{t-1} < 1-\tau_t$. We assume that this holds in all periods, otherwise there would be no savers in the economy.

2.1 Autarky Equilibrium

We first characterize equilibrium under financial autarky, focusing on the determinants of the interest rate. The insights derived here will be useful to understand the determinants of the world interest rate and capital flows in Section 2.2. Under autarky, market clearing in period t requires that total assets accumulated at the end of period t equals the total capital stock at the beginning of period $t+1$:

$$L_{y,t}a_{y,t} + L_{m,t}a_{m,t} = K_{t+1}.$$

Substituting the expression for $a_{y,t}$ and $a_{m,t}$ from (6) and (9) and re-arranging yields:

$$n_t(1+\gamma_{A,t+1})\left[1+\frac{1-\alpha}{\alpha}\left(\theta_t+\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}\right)\right]k_{t+1} = \frac{p_t(1-\tau_t-\theta_{t-1})}{p_t+\beta^{-\omega}R_{t+1}^{1-\omega}}(1-\alpha)k_t^{\alpha}. \quad (10)$$

The left-hand side corresponds to the supply of assets, which is increasing in the growth rate of productivity, $\gamma_{A,t+1}$, and of the labor force, n_t, between periods t and $t+1$, as well as in the ability to borrow over the lifecycle, θ_t, and across generations, τ_{t+1}. The latter effect, as discussed earlier, comes from a reduction in middle-aged savings in anticipation of social security benefits received in period $t+1$. The right-hand side corresponds to the demand for assets by the middle aged that would prevail absent future social security benefits. It is increasing in the survival probability, p_t, decreasing in the current tax rate, τ_t, paid by middle-aged savers, decreasing in their ability to borrow when young, θ_{t-1}, and decreasing in the interest rate, R_{t+1} as long as $\omega < 1$. Equations (1) and (10) summarize the dynamics of the economy.

Autarky Steady State. Suppose productivity grows at a constant rate γ_A, the tightness of credit constraints remains constant, $\theta_t = \theta$, and demographic and social security variables are constant, $n_t = n$, $p_t = p$, $\tau_t = \tau$, implying a constant replacement rate, $\sigma = \frac{n}{p}(1+\gamma_A)\tau$. The following proposition defines the equilibrium steady-state interest rate.

Proposition 1. *An economy where γ_A, n, p, θ and τ are all constant converges to its unique stable steady-state. The steady-state gross interest rate R is implicitly given by:*

$$R = \frac{n(1+\gamma_A)}{p(1-\tau-\theta)}\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]. \quad (11)$$

For $\omega = 1$, this simplifies to

$$R = \frac{n(1+\gamma_A)}{\beta p(1-\tau-\theta)}\left[(1+\beta p)\left(\frac{\alpha}{1-\alpha}+\theta\right)+\tau\right]. \tag{12}$$

The autarky rate is increasing in the level of social security contributions, τ, and increasing in the looseness of the borrowing constraint, θ.

Proof. See Appendix B.

Proposition 1 shows that countries with better social security or looser borrowing constraints have higher autarky rates. Indeed, more developed social security (resp., better access to credit) increases borrowing against future generations (resp., future labor income), while reducing the savings of the the middle-aged by reducing their net disposable income.

Based on Equation (11), we now analyze the impact of demographics on the steady-state interest rate. The steady-state comparative statics results presented here are useful to understand the impact of aging on the interest rate in the transition. In the absence of social security ($\tau = \sigma = 0$), population aging (i.e., a fall in n or a rise in p) unambiguously leads to a lower autarky rate, as stated in the following corollary.

Corollary 1. *In a country without social security, the steady-state autarky rate, R_0, satisfies:*

$$R_0 = \frac{n(1+\gamma_A)}{p(1-\theta)}\left(\frac{\alpha}{1-\alpha}+\theta\right)\left(p+\beta^{-\omega}R_0^{1-\omega}\right).$$

The autarky rate, R_0, is increasing in fertility, n, and decreasing in longevity, p. The elasticities of the autarky interest rate with respect to fertility and longevity, denoted by $\eta_{0,x} = \frac{\partial R_0/R_0}{\partial x/x}$ for $x = \{n, p\}$, can be expressed as:

$$\eta_{0,n} = \frac{1}{1-(1-\omega)\phi_0} \quad \textit{and} \quad \eta_{0,p} = -\frac{\phi_0}{1-(1-\omega)\phi_0},$$

where $\phi_0 = \frac{(\beta R_0)^{1-\omega}}{\beta p+(\beta R_0)^{1-\omega}} \in [0,1]$. The magnitude of these elasticities is falling in ω, i.e. $\frac{\partial|\eta_{0,x}|}{\partial\omega} < 0$ for $x = \{n, p\}$.

Proof. See Appendix B.

The effect of demographic variables on autarky rates is intuitive. Higher fertility, n, leads to higher autarky rates by increasing the marginal productivity of capital and the share

of borrowers in the economy. Rising longevity, p, increases the incentives to save of the middle-aged, thus lowering the autarky rate. Moreover, a lower elasticity of intertemporal substitution, ω, reduces the sensitivity of net savings to the interest rate, implying that the equilibrium requires a larger adjustment of the interest rate. This in turn implies that the lower is ω, the stronger is the response of interest rates to demographic variables.[9]

In the presence of social security, the impact of demographics on the interest rate is slightly more involved — due to the indirect effect of demographics going through adjustments of the pension system. Indeed, a lower fertility, n, or a higher longevity, p, require higher contribution and/or lower replacement rates for the social security budget to remain balanced. To characterize the dependence of the steady-state interest rate on demographic variables, we assume that the elasticity of the contribution rate to population aging is constant and independent of the source of aging, i.e., we proceed under the assumption that[10]

$$-\frac{\partial\tau/\tau}{\partial n/n} = \frac{\partial\tau/\tau}{\partial p/p} \equiv \varepsilon, \qquad \varepsilon \in [0,1].$$

For instance, if the social security system adjusts to population aging entirely through an increase in the contribution rate, with no adjustment in the replacement rate, then ε is equal to 1. The following corollary summarizes the response of the interest rate to aging in an economy with social security.

Corollary 2. *In a country with social security the steady-state autarky rate, R, defined by* (11)*, is increasing in fertility, n, and decreasing in longevity, p, as long as the contribution rate is not too high, $\tau < \tau_{\max}$, where $\tau_{\max}$ is defined in Appendix B. The elasticities of the autarky rate with respect to aging (i.e., fall in n or increase in p) can be decomposed into two terms of opposite sign as follows:*

$$-\eta_n = -\frac{\partial R/R}{\partial n/n} = -\frac{1}{1-(1-\omega)\phi_\tau} + \tau\varepsilon\frac{\phi_\tau/\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)+1/(1-\theta-\tau)}{1-(1-\omega)\phi_\tau}, \tag{13}$$

$$\eta_p = \frac{\partial R/R}{\partial p/p} = -\frac{\phi_\tau}{1-(1-\omega)\phi_\tau} + \tau\varepsilon\frac{\phi_\tau/\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)+1/(1-\theta-\tau)}{1-(1-\omega)\phi_\tau}, \tag{14}$$

where $\phi_\tau \in [0,1]$ is defined in Appendix B and satisfies $\lim_{\tau\to 0}\phi_\tau = \phi_0$.

[9] In particular, by evaluating the elasticities at $\omega = 1$, one can see that for $\omega < 1$, $\eta_{0,n} > 1$ and $|\eta_{0,p}| > \frac{1}{1+\beta p}$.

[10] This is an assumption on the *steady-state* level of the contribution rate $\tau = \tau(n, p, \gamma_A)$. By assuming that $\varepsilon \in [0,1]$, we are deriving comparative statics under the natural assumption that the steady-state contribution rate τ (resp. the steady-state replacement rate σ) does not fall (resp. increase) when the steady-state old dependency ratio p/n increases.

Proof. See Appendix B.

Corollary 2 shows that the rate of adjustment of the interest rate to demographic changes depends on the extent of social security. For τ not too high, the autarky rate falls as the population ages, which corroborates the insight from Corollary 1. However in the presence of social security, when $\varepsilon > 0$, the response of the interest rate is limited due to the increase in contribution rate (second term in (13) and (14)). Indeed, an increase in the old dependency ratio triggers a rise in contribution rates, which limits the increase of net savings and dampens the fall in the autarky rate.[11]

Figure 2.1 summarizes the values of the elasticity of the interest rate to aging as a function of the level of social security contributions, τ, and of the elasticity of intertemporal substitution, ω, for $\varepsilon = 0.8$. Essentially, a higher contribution rate or a higher elasticity of intertemporal substitution lowers the sensitivity of the autarky rate to aging.

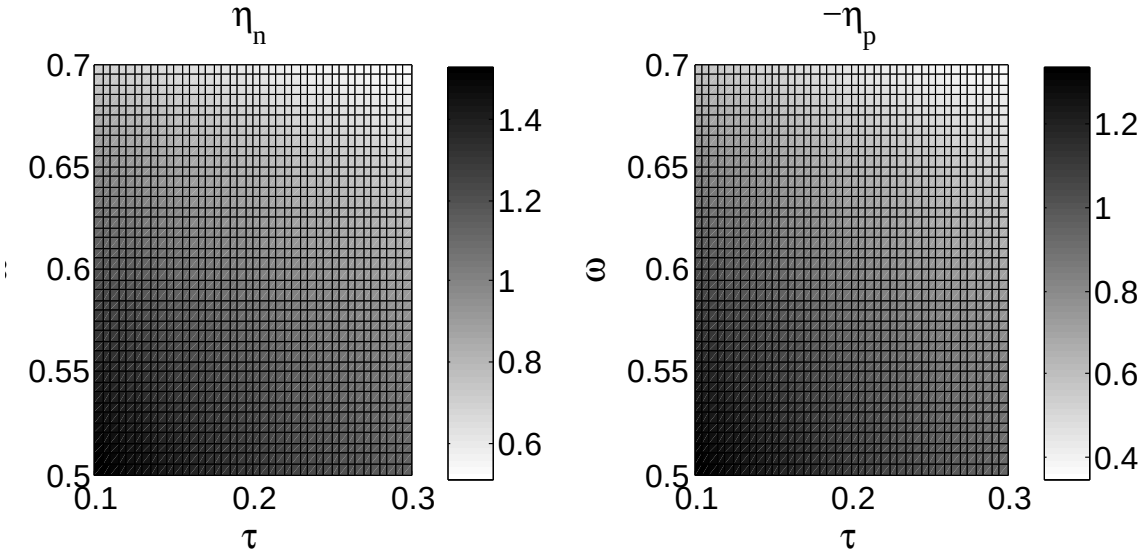


Figure 2.1: Magnitude of the Interest Rate Response to Aging

Notes: The left panel shows the value of the elasticity of the autarky interest rate with respect to fertility, n, while the right panel shows the negative of the elasticity with respect to longevity, p, for different values of social security contribution rates, τ, and of inter-temporal elasticity of substitutions, ω. Parameter values are $\beta = 0.975$ (annual basis), $\alpha = 0.3$, $\theta = 0.15$, and $\varepsilon = 0.8$, $\gamma_A = 1.5\%$ (annual basis), $p = 0.8$, $n = 1.1$. A period is 25 years.

[11]Note that if taxes did not adjust in response to changes in demographic composition (i.e., $\varepsilon = 0$), population aging would affect the autarky rate very similarly as in the absence of social security. The behaviour is almost identical for low levels of taxes, as $\phi_\tau \approx \phi_0$ in the neighbourhood of $\tau = 0$.

2.2 Integrated Equilibrium

We now consider the world economy, which consists of N countries indexed by i, each characterized by an OLG structure as previously described. Let $\gamma^i_{A,t}$, n^i_t, p^i_t, τ^i_t, and θ^i_t denote country-specific exogenous variables for $i \in \{1, ..., N\}$.

Financial integration in period t implies that capital flows freely across borders until country interest rates are equal to the world interest rate: $R^i_{t+1} = R_{t+1}$ and $k^i_{t+1} = k_{t+1}$ for all $i \in \{1, ..., N\}$. The capital market clearing condition is given by:

$$\sum_i \left(L^i_{y,t} a^i_{y,t} + L^i_{m,t} a^i_{m,t}\right) = \sum_i K^i_{t+1} = k_{t+1} \sum_i A^i_{t+1} L^i_{m,t+1}$$

Using optimal individual asset holdings, described by (6) and (9), the above yields the equivalent of equation (10) at the world level:

$$\begin{aligned} &k_{t+1} \sum_i n^i_t (1 + \gamma^i_{A,t+1}) A^i_t L^i_{m,t} \left[1 + \frac{1-\alpha}{\alpha}\left(\theta^i_t + \frac{\beta^{-\omega} R^{1-\omega}_{t+1}}{p^i_t + \beta^{-\omega} R^{1-\omega}_{t+1}} \tau^i_{t+1}\right)\right] \\ &= k^\alpha_t \sum_i A^i_t L^i_{m,t} \frac{p^i_t (1 - \tau^i_t - \theta^i_{t-1})}{p^i_t + \beta^{-\omega} R^{1-\omega}_{t+1}} (1 - \alpha). \end{aligned} \tag{15}$$

The top line is the supply of assets at the world level, which has to equal the total demand by middle-aged savers, shown in the bottom line.

Integrated Steady State. Assume that the effective labour supply in all countries grows at the same constant rate, $n^i(1 + \gamma^i_A) = 1 + \gamma$ for all $i \in \{1, ..., N\}$. Moreover assume that all other demographic and social security variables are constant in each country: $p^i_t = p^i$, $\tau^i_t = \tau^i$, $\sigma^i_t = \sigma^i$, and $\theta^i_t = \theta^i$ for all t and all $i \in \{1, ..., N\}$.

Proposition 2. *Under the above assumptions there exists a unique integrated steady state, where the world interest rate is implicitly defined by:*

$$\sum_i \lambda^i n^i (1 + \gamma^i_A) \left[\frac{\alpha}{1-\alpha} + \left(\theta^i + \frac{\beta^{-\omega} R^{1-\omega}}{p^i + \beta^{-\omega} R^{1-\omega}} \tau^i\right)\right] = R \sum_i \lambda^i \frac{p^i (1 - \tau^i - \theta^i)}{p^i + \beta^{-\omega} R^{1-\omega}} \tag{16}$$

where λ^i denotes the relative size of country i in terms of effective labour force, $\lambda^i \equiv \frac{A^i_t L^i_{m,t}}{\sum_i A^i_t L^i_{m,t}}$. The

net foreign asset position of country i normalized by its GDP is:

$$\frac{NFA_t^i}{Y_t^i} = \frac{p^i(1-\alpha)(1-\tau^i-\theta^i)}{p^i+\beta^{-\omega}R^{1-\omega}} - \frac{n^i(1+\gamma_A^i)}{R}\left[\alpha+(1-\alpha)\left(\theta^i+\frac{\beta^{-\omega}R^{1-\omega}}{p^i+\beta^{-\omega}R^{1-\omega}}\tau^i\right)\right], \quad (17)$$

where R is implicitly given by (16).

2.2.1 Global Aging and Capital Flows

Consider an integrated steady state where countries are identical in terms of demographics and productivity growth, with $p^i = p$, $n^i = n$, $1+\gamma_A^i = 1+\gamma_A$, and only differ in the extent of social security, τ^i, and the ease of borrowing against future income, θ_i. Equation (16) can be rewritten as:

$$\overline{R} = \frac{n(1+\gamma_A)}{p(1-\overline{\tau}-\overline{\theta})}\left[p\left(\frac{\alpha}{1-\alpha}+\overline{\theta}\right)+\beta^{-\omega}\overline{R}^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\overline{\theta}+\overline{\tau}\right)\right], \quad (18)$$

where $\overline{\theta} = \sum_i \lambda^i\theta^i$ and $\overline{\tau} = \sum_i \lambda^i\tau^i$ correspond to the world average level of credit constraints and of contribution rates, respectively. Comparing Equations (18) and (11), it is immediate that the world interest rate coincides with the interest rate that would prevail under autarky in a country characterized by $\overline{\theta}$ and $\overline{\tau}$. Importantly, in light of Corollaries 1 and 2, this implies that the world interest rate tends to fall with global aging. Indeed, if we denote by $\overline{\eta}_n \equiv \frac{\partial\overline{R}/\overline{R}}{\partial n/n}$ (resp. $\overline{\eta}_p \equiv \frac{\partial\overline{R}/\overline{R}}{\partial p/p}$) the elasticity of the world interest rate with respect to the common fertility rate n (resp. survival probability p), we know that $\overline{\eta}_n > 0$ and $\overline{\eta}_p < 0$ as long as the world average contribution rate $\overline{\tau}$ is not too high.

Turning to countries' net foreign asset positions, we first observe that in the integrated steady state with symmetric demographics and productivity growth, a country characterized by (θ^i, τ^i) tends to export capital if $\theta^i < \overline{\theta}$ and/or if $\tau^i < \overline{\tau}$. This result can be viewed as a direct consequence of Proposition 1. Indeed, the autarky interest rate of a country characterized by (θ^i, τ^i) tends to be lower than the world interest rate, $\overline{R}$, if $\theta^i < \overline{\theta}$ and/or if $\tau^i < \overline{\tau}$.

Corollary 3. *If countries only differ in their level of social security contributions and in the ease of borrowing against future income, then the net foreign asset position of a country characterized by*

(θ^i, τ^i) *in the steady state is given by:*

$$\begin{aligned}\frac{NFA_t^i}{Y_t^i} =&(1-\alpha)(\overline{\theta}-\theta^i)\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right]\\ &+\frac{p(1-\alpha)(\overline{\tau}-\tau^i)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right],\end{aligned} \tag{19}$$

where $\overline{R}$ *is given by* (18)*. This implies that a country with tighter credit constraints than the world average level,* $\theta^i < \overline{\theta}$*, and/or with lower social security than the world average level,* $\tau^i < \overline{\tau}$*, tends to export capital.*

Proof. See Appendix B.

Next, we analyze the impact of global aging on capital flows, driven by cross-country differences in credit markets and social security. To do so, we characterize the impact of demographic variables, n and p, on the dispersion in net foreign asset positions at the integrated steady state. We then complement the steady-state comparative statics with numerical results showing the impact of global aging in the transition.

Global Aging and Heterogeneity in Credit Constraints. We start by considering two countries, denoted by H and L, differing only in the tightness of the borrowing constraint on young households, with $\theta^L < \overline{\theta} < \theta^H$. Corollary 3 implies that in steady state, country H is a debtor, while country L is a creditor. Using (19), the steady-state difference in net foreign asset positions relative to GDP across the two countries is given by:

$$\frac{NFA_t^L}{Y_t^L}-\frac{NFA_t^H}{Y_t^H}=(1-\alpha)(\theta^H-\theta^L)\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right]. \tag{20}$$

The following proposition summarizes the impact of global aging on capital flows between countries H and L.

Proposition 3. *A fall in fertility leads to a larger dispersion of net foreign assets over GDP if* $\overline{\eta}_n \geq 1$*. A rise in longevity,* p*, unambiguously leads to a larger dispersion of net foreign assets over GDP. The stronger is the interest rate response to aging (i.e., the larger* $\overline{\eta}_n$ *and* $|\overline{\eta}_p|$*), the larger is the increase in dispersion.*

Proof. See Appendix B.

Following a fall in global fertility, n, the dispersion of net foreign assets over GDP is governed by two conflicting forces: a *direct effect* and a world *interest rate channel*. As fertility falls globally, the world interest rate, $\overline{R}$, falls, leading to more borrowing by young households and, for $\omega < 1$, more savings by middle-aged households. The former effect is stronger in the country with looser borrowing constraints, the H country, while the latter is stronger in the country with stricter borrowing constraints, the L country. As a result, the interest rate channel unambiguously implies that global aging induces greater dispersion of net foreign asset positions between the H and the L countries, with a stronger divergence when $\omega < 1$. The direct effect, which works through a change in *demographic composition*, goes in the opposite direction: lower fertility implies a smaller share of young in the population, and thus lower weight on their borrowing. Since the borrowing of the young is higher in the H country (where credit constraints are less tight), the change in demographic composition increases savings more in that country. A sufficient condition (although not necessary) for the interest rate channel to dominate is that the elasticity of the world interest rate to fertility, $\overline{\eta}_n$, is above unity. In this configuration, lower fertility implies more dispersion in the steady-state net foreign asset position (as a fraction of GDP) between the high- and low-θ country — i.e., a fall in fertility leads to increased capital flows from the L to the H country.[12]

When it is driven by a rise in longevity, p, global aging *unambiguously* leads to an increase in capital flows from the L to the H country. In this case, the direct effect and the interest rate channel both work in the same direction. The interest rate channel operates in a similar way, and is stronger when the interest rate response measured by $\left|\overline{\eta}_p\right|$ is larger. The direct effect of the increase in the probability of survival to old age, p, is to increase the propensity to save of the middle-aged, which increases national savings more in the L country (where a smaller fraction of middle-age labor income is spent on debt repayment), leading to further divergence in net foreign assets across countries.

Thus, if the elasticity of intertemporal substitution is low enough, global aging triggers an increase in capital flows from a low-θ to a high-θ country. This is illustrated in Figure 2.2, where, for realistic parameters values, we plot the transition dynamics of net foreign

[12]Note that if $\omega = 1$ and the average level of taxes $\overline{\tau}$ is not too large, then $\overline{\eta}_n \approx 1$. In this knife-edge case, the dispersion in net foreign assets is left unchanged when fertility changes, as the interest rate and composition channels perfectly offset each other.

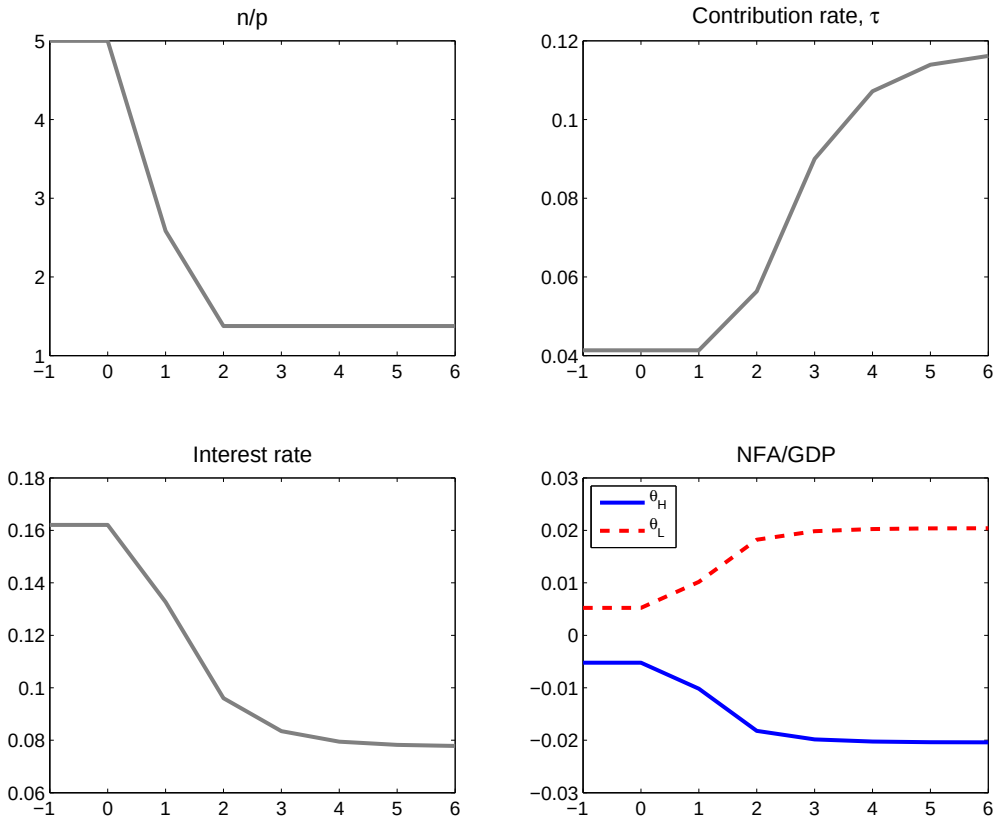


Figure 2.2: Global Aging and NFA with Heterogenous Credit Constraints

Notes: Parameter values are $\omega = 1/2$, $\beta = 0.975$ (annual basis), $\alpha = 0.3$, $\theta_H = 0.15$, $\theta_L = 0.015$, and $\varepsilon = 0.8$. Initial integrated steady state is for $\gamma_A = 1.5\%$ (annual basis), $p = 0.4$, $n = 2$, and $\sigma = 0.3$, with countries of equal size. Final steady state is for $\gamma_A = 1.5\%$, $p = 0.8$ and $n = 1.1$. A period is 25 years.

asset positions for a high-θ and a low-θ country, when between $t = 0$ and $t = 2$, global fertility falls and longevity increases.

Global Aging and Heterogeneity in Social Security. Consider now two countries H and L differing from the average only in their level of social security contributions τ^i, which are such that $\tau^L < \overline{\tau} < \tau^H$. Corollary 3 implies that in steady state, country L is a creditor, while country H is a debtor. Using (19), the steady-state difference in net foreign asset positions relative to GDP across the two countries can be expressed as:

$$\frac{NFA_t^L}{Y_t^L} - \frac{NFA_t^H}{Y_t^H} = \frac{p(1-\alpha)(\tau^H - \tau^L)}{p + \beta^{-\omega}\overline{R}^{1-\omega}} \left[1 + \frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right]. \tag{21}$$

The impact of global aging on capital flows between country H and L is summarized in the following proposition, under the additional assumption that the elasticity of the contribution rate to aging is identical across countries, i.e., $\varepsilon^H = \varepsilon^L = \varepsilon$.

Proposition 4. *A fall in fertility leads to a larger dispersion of net foreign assets over GDP if* $\overline{\eta}_n \geq \frac{1-\varepsilon}{\omega}$. *A rise in longevity leads to a larger dispersion of net foreign assets over GDP if* $|\overline{\eta}_p| \geq \frac{1-\varepsilon}{\omega}$.

Proof. See Appendix B.

A fall in world fertility leads to increased capital flows from a low-τ to a high-τ country if $\overline{\eta}_n$ is large enough. This is the outcome of three distinct forces: the *direct effect*, the *interest rate channel*, and the *contribution rate channel*. The direct effect, working through a change in demographic composition, and the interest rate channel, are similar as before — the latter boosting, the former limiting capital flows from the low-τ to the high-τ country. However, due to the cross-country heterogeneity in social security, a new channel operates, through the adjustment of contribution rates: lower fertility requires an increase in contribution rates, which reduces savings more in a high-τ country, and therefore reinforces the capital flows from the low-τ to the high-τ country.

An increase in world longevity also leads to increased capital flows from a low-τ to a high-τ country, provided $|\overline{\eta}_p|$ is large enough. As before, the *interest rate channel* induces a greater increase in net national savings in the low-τ country. The *contribution rate channel* plays in the same direction: higher longevity increases contribution rates more in the high-τ country, implying further divergence of national savings across countries, and an even larger dispersion of net foreign assets. Finally, the *direct effect* of higher longevity increases savings due to two forces. First, as before, individuals save more, as their probability of survival to old age is higher; this effect is larger in the low-τ country. Second, higher longevity reduces the present value of future social security benefits, and this effect is stronger in the high-τ country, partially offsetting the previous forces.

It is worth noting that the value of ε, the elasticity of the contribution rate to demographic variables, affects the impact of global aging on the dispersion of net foreign assets. The effect is ambiguous in general: a higher value of ε limits the response of the world interest rate to world aging (i.e., $\overline{\eta}_n$ and $\left|\overline{\eta}_p\right|$ are lower), which dampens the interest rate channel; but it also makes the contribution rate channel stronger, as the contribution rates react more to aging. For realistic parameter values, the latter effect typically dominates, and a higher tax elasticity amplifies the impact of global aging on capital flows.

In sum, if the the response of the world interest rate to global aging is large enough,

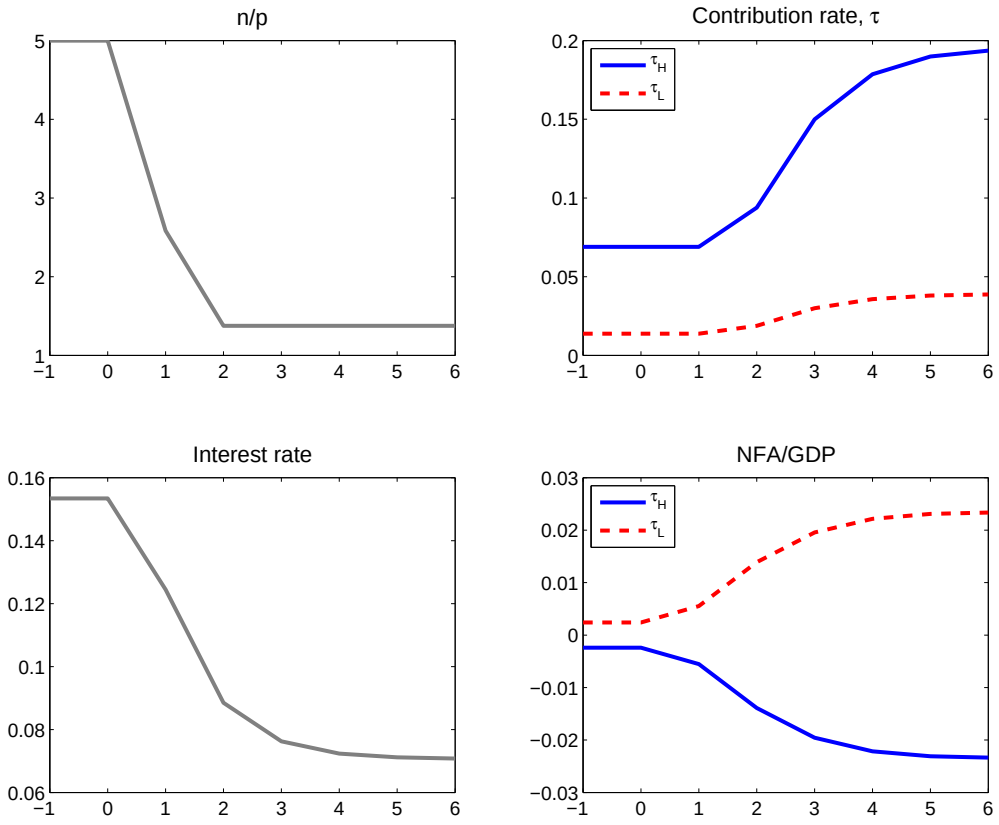


Figure 2.3: Global Aging and NFA with Heterogenous Social Security

Notes: Parameter values are $\omega = 1/2$, $\beta = 0.975$ (annual basis), $\alpha = 0.3$, $\theta = 0.05$, and $\varepsilon = 0.8$. Initial integrated steady state is for $\gamma_A = 1.5\%$ (annual basis), $p = 0.4$, $n = 2$, and initial replacement rates $\sigma_H = 0.5$ and $\sigma_L = 0.1$, with countries of equal size. Final steady state is for $\gamma_A = 1.5\%$, $p = 0.8$ and $n = 1.1$. A period is 25 years.

global aging triggers an increase in capital flows from low to high social security countries. Figure 2.3 illustrates this effect in the transition. The figure depicts, for realistic parameters values, the evolution of current account balances between a low-τ and a high-τ country when between $t = 0$ and $t = 2$, fertility falls and longevity increases globally.

2.2.2 Country-Specific Aging and Capital Flows

We now turn our attention to the effects of *country-specific* demographic patterns on capital flows — i.e., we analyze how a country's net foreign asset position is affected by its own demographic characteristics. In particular, our goal here is to characterize the net foreign asset position (and the dynamics thereof) of a country whose demographic evolutions deviate from the global trend. We consider a *small* country, such that changes in its demographic characteristics do not affect the world interest rate. Furthermore, we assume

that the world interest rate is at its steady state, $\overline{R}$, reflecting (see Equation (18)) the average ease of borrowing and the average level of social security worldwide, $(\overline{\theta}, \overline{\tau})$, as well as demographic and productivity growth parameters (p, n, γ_A) common across the rest of the world.[13] In this case, the net foreign asset position of the small open country (indexed by i) in period t is given by:

$$\begin{aligned}\frac{NFA_t^i}{Y_t^i} =& \frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}\overline{R}^{1-\omega}}(1-\alpha) \\ & - \frac{n_t^i(1+\gamma_{A,t+1}^i)}{\overline{R}}\left[\alpha+(1-\alpha)\left(\theta_t^i+\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p_t^i+\beta^{-\omega}\overline{R}^{1-\omega}}\tau_{t+1}^i\right)\right], \end{aligned} \tag{22}$$

Differentiating with respect to fertility n_t^i and longevity p_t^i yields:

$$\frac{\partial}{\partial n_t^i}\frac{NFA_t^i}{Y_t^i} = -\frac{(1+\gamma_{A,t+1}^i)}{\overline{R}}\left[\alpha+(1-\alpha)\left(\theta_t^i+\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p_t+\beta^{-\omega}\overline{R}^{1-\omega}}\tau_{t+1}^i(1-\varepsilon_{t+1}^i)\right)\right] < 0, \tag{23}$$

and

$$\begin{aligned}\frac{\partial}{\partial p_t^i}\frac{NFA_t^i}{Y_t^i} =& \frac{(1-\alpha)\beta^{-\omega}\overline{R}^{1-\omega}}{p_t^i+\beta^{-\omega}\overline{R}^{1-\omega}}\times \\ & \left[\frac{1-\tau_t^i-\theta_{t-1}^i}{p_t^i+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n_t^i(1+\gamma_{A,t+1}^i)}{\overline{R}}\tau_{t+1}^i\left(\frac{(1-\varepsilon_{t+1}^i)p_t^i-\varepsilon_{t+1}^i\beta^{-\omega}\overline{R}^{1-\omega}}{p_t^i\left(p_t^i+\beta^{-\omega}\overline{R}^{1-\omega}\right)}\right)\right], \end{aligned} \tag{24}$$

where $\varepsilon_{t+1}^i \equiv \frac{\partial\tau_{t+1}^i/\tau_{t+1}^i}{\partial p_t^i/p_t^i} = -\frac{\partial\tau_{t+1}^i/\tau_{t+1}^i}{\partial n_t^i/n_t^i}$ captures the adjustment of the contribution rate τ_{t+1}^i to demographic changes affecting the old dependency ratio in period $t+1$. Both a fall in fertility and an increase in longevity operate via a direct effect and an indirect effect, working through the contribution rate. The direct effect in either case leads to an increase in the country's net foreign asset position, while the contribution rate channel goes in the opposite direction. In the case of a drop in fertility, under the reasonable assumption that $\varepsilon_{t+1}^i \in [0,1]$, one can see from (23) that the positive direct effect dominates the negative effect due to the adjustment of the contribution rate, and thus the net foreign asset position of the country (expressed as a fraction of GDP) unambiguously increases. The effect of a rise in longevity, however, is ambiguous in general. From (24), one can see that the smaller is

[13] In the long run, fertility and productivity growth in country i are such that $\lim_{t\to\infty} n_t^i(1+\gamma_{A,t+1}^i) \leq n(1+\gamma_A)$. This guarantees that the country does not become too large and affect the world interest rate.

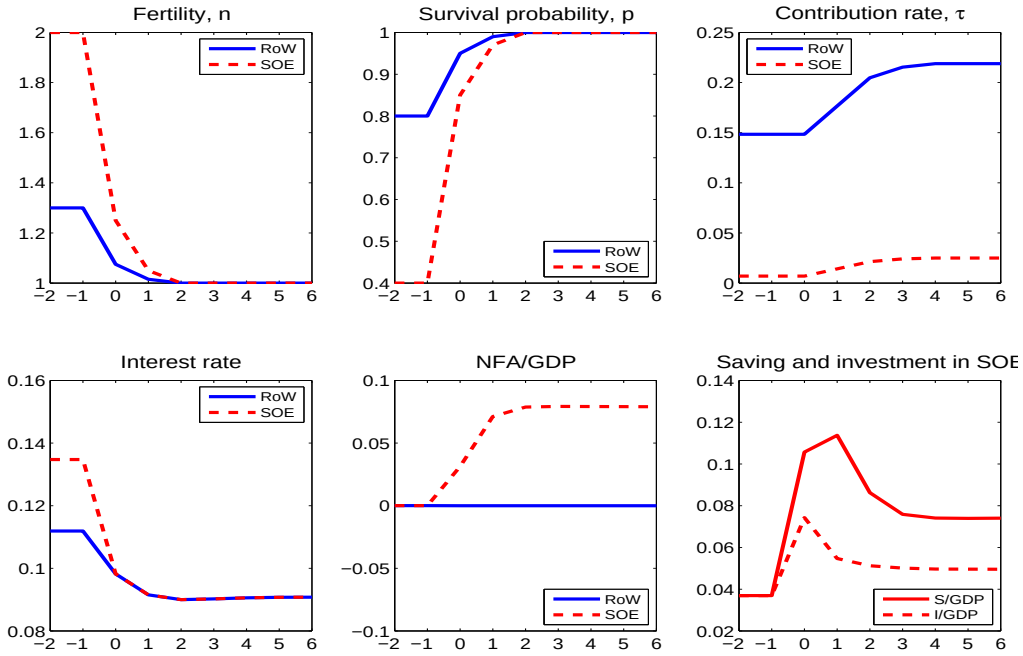
the contribution rate, τ^i_{t+1}, and/or its elasticity, ε^i_{t+1}, the more likely it is that the direct effect dominates the effect of the contribution rate adjustment.[14] In other words, the net foreign asset position should improve less when the rise in longevity occurs in countries with more developed social security, and/or which are more reluctant to decrease their replacement rates. As far as emerging countries are concerned, given that their social security systems are typically less developed (notably in terms of coverage), our analysis suggests that they are likely to experience an improvement of their net foreign asset positions and to export capital when their fertility (resp., longevity) drops (resp., rises) faster than in the rest of the world.

Finally, countries where productivity grows at a faster pace than in the rest of the world tend to have lower net foreign asset positions and tend to import capital (the impact of a temporary shock on productivity growth $\gamma^i_{A,t+1}$ can be seen from Equation (22)). However, if the countries which experience faster productivity growth are also the ones aging faster, they may well be (or soon turn into) capital exporters.

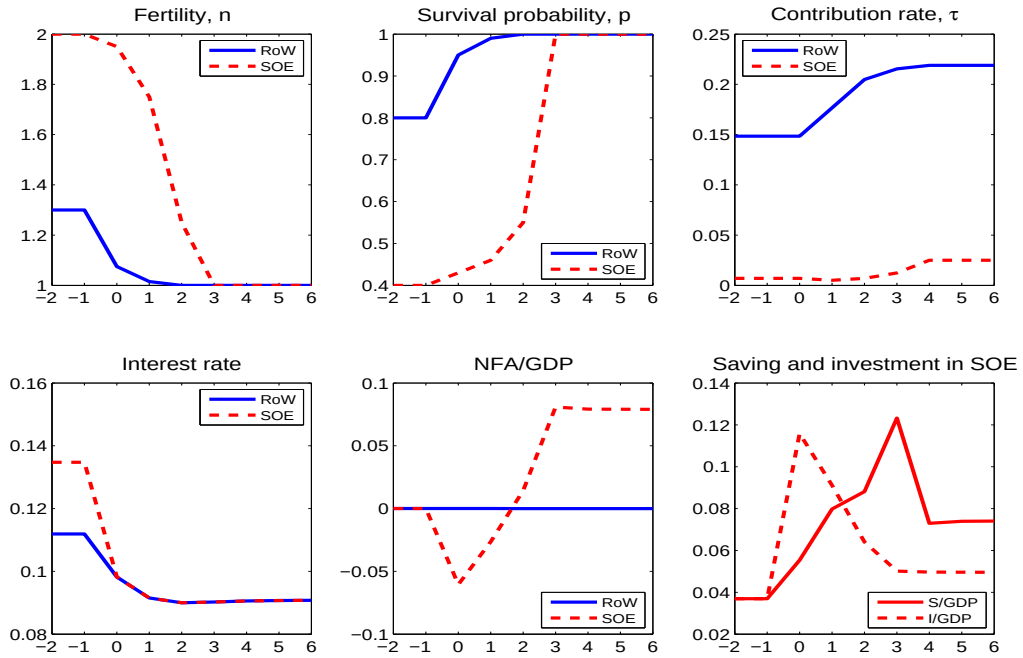
Illustrative Example: Demographic Convergence in a Small Open Economy. Having established that a typical small emerging economy, when it ages faster than the global economy, tends to become a capital exporter, we now explore in more details how the speed of demographic convergence towards developed countries affects its net foreign asset position and current account dynamics.

Figure 2.4 illustrates the response of capital flows to aging in a small emerging economy. The emerging country (SOE) is characterized by credit constraints which are constant over time and tighter than in the rest of the world, so that $\theta^{SOE} < \overline{\theta}$. Social security contribution rates are also significantly lower, so that $\tau^{SOE}_t < \overline{\tau}$ for all t. We abstract from productivity growth differentials and assume that productivity grows at the same constant rate γ_A in the SOE as in the rest of the world. The small emerging economy starts from an initial autarky steady state and integrates to the world economy at date $t = 0$. While the world economy and the SOE are both aging, the emerging country is initially characterized by a younger population, $p_t < \overline{p}_t$ and $n_t > \overline{n}_t$. Over time, the demographic patterns of the SOE converge to those of the rest of the world. Panel (a) of Figure 2.4 corresponds to a case of fast demographic convergence, while panel (b) corresponds to a case where convergence is

[14] In the limit as τ^i_{t+1} or ε^i_{t+1} goes to 0, the direct effect clearly dominates.



(a) Early Demographic Transition in SOE



(b) Delayed Demographic Transition in SOE

Figure 2.4: Impact of Country-Specific Aging in a Small Open Emerging Economy

Notes: The small open economy (SOE, dashed line) integrates to the rest of the world (RoW, solid line) at time 0. The exogenous evolution of fertility and longevity for the SOE and the RoW is represented in the first two subplots in the top row of each panel. Parameter values are $\omega = 1/2$, $\beta = 0.975$, $\gamma_A = 1.5\%$ (both on an annual basis), $\alpha = 0.3$. The world is characterized by $\overline{\theta} = 0.15$, initial replacement rate $\overline{\sigma}_{-1} = 0.35$, and final replacement rate $\overline{\sigma}_6 = 0.3174$. The small open emerging economy is characterized by $\theta = 0.015$, $\sigma_{-1} = 0.05$, and $\sigma_6 = 0.0362$. A period is 25 years.

slower. To understand the dynamics of the SOE's net foreign assets under the two scenarios, it is informative to consider the bottom-right corner of each panel, which depicts the savings-to-GDP (solid line) and the investment-to-GDP (dashed line) in the small emerging economy. Independently of the pace of demographic convergence, the SOE's saving and investment rates increase at the time it integrates to the world economy. However, the speed of convergence makes a difference for the *relative magnitude* of the increase in savings and investment. When demographic convergence is faster, savings increase faster than investment, leading the SOE to export capital initially. On the contrary, with a slower demographic convergence, the small emerging country initially borrows from the rest of the world: investment increases more as fertility is expected to stay higher for longer, while savings rise slowly — as both the composition effect of a fall in fertility (i.e., less young borrowers relative to middle-aged savers), and the direct effect of longevity on the middle-aged's propensity to save, kick-in at a slower pace.

3 Quantitative Analysis

We now turn to the quantitative part of the analysis, where we assess the ability of our theory to account for the evolution of savings as well as patterns of capital flows across the world over the last decades. Our quantitative framework enriches the simple model of Section 2 along several dimensions. We start with a brief description of the setup before presenting our calibration procedure and discussing the quantitative performance of the model.

3.1 Multi-Period OLG Model

Countries are populated with agents whose economic life runs for at most $\bar{J}+1$ periods. Age is indexed by $j = 0, ..., \bar{J}$. We denote by $p^i_{j,t}$ the conditional probability in country i that an agent of age j in period t survives into the next period (by construction, $p^i_{\bar{J},t} = 0$ for all i and t). Let $L^i_{j,t}$ denote the number of agents who reach age j in period t. Taking mortality into account, this number is related to the size of the youngest cohort in period

$t-j$ by[15]

$$L^i_{j,t} = \left(\prod_{\ell=0}^{j-1} p^i_{\ell,t-j+\ell}\right) L^i_{0,t-j}, \qquad 0 \leq j \leq \bar{J}. \tag{25}$$

The size of the youngest cohort itself evolves as $L^i_{0,t+1} = (1+\gamma^i_{L,t+1})L^i_{0,t}$.

Preferences. Let $c^i_{j,t}$ denote the consumption of an agent of age j in period t in country i. The lifetime expected utility of this agent is

$$U^i_t = \sum_{j=0}^{\bar{J}} \left(\prod_{\ell=0}^{j-1} p^i_{\ell,t+\ell}\right) \beta^j u(c^i_{j,t+j}), \tag{26}$$

with standard isoelastic preferences $u(c) = (c^{1-\frac{1}{\omega}}-1)/(1-\frac{1}{\omega})$.

Production. Agents work until period $\underline{J}$ of their life, after which they retire. Gross output in country i is

$$Y^i_t = (K^i_t)^\alpha \left[A^i_t \sum_{j=0}^{\underline{J}} e^i_{j,t} L^i_{j,t}\right]^{1-\alpha} = A^i_t \hat{L}^i_t (k^i_t)^\alpha, \tag{27}$$

where $\hat{L}^i_t \equiv \sum_{j=0}^{\underline{J}} e^i_{j,t} L^i_{j,t}$ denotes the total efficiency-weighted population, and $k^i_t = K^i_t/(A^i_t \hat{L}^i_t)$ denotes the capital-effective-labor ratio. The set of efficiency weights $\{e^i_{j,t}\}_{j=0}^{\underline{J}}$ captures the shape of the age-income profile in period t and country i. Indeed, the labor income received by an agent of age j in country i in period t is

$$w^i_{j,t} = e^i_{j,t} w^i_t, \tag{28}$$

where $w^i_t \equiv (1-\alpha) A^i_t (k^i_t)^\alpha$. Finally, the gross rate of return between $t-1$ and t is

$$R^i_t = 1 - \delta + \alpha (k^i_t)^{\alpha-1}, \tag{29}$$

where δ denotes the depreciation rate of capital over one period.

Social Security. Social security contributions are levied on labor income, at a flat rate τ^i_t.

[15] We adopt the convention $\prod_{\ell=0}^{-1} p^i_{\ell,t+\ell} = 1$.

The pension transfer received by a retiree of age $j = \underline{J}+1, \ldots, \bar{J}$ in period t is

$$\pi^i_{j,t} = \sigma^i_t w^i_{\underline{J},t-j+\underline{J}}, \tag{30}$$

where σ^i_t denotes the replacement rate in that period. For the social security system to be balanced in every period, the paths of contribution and replacement rates must be such that

$$\tau^i_t \sum_{j=0}^{\underline{J}} L^i_{j,t} w^i_{j,t} = \sum_{j=\underline{J}+1}^{\bar{J}} L^i_{j,t} \pi^i_{j,t} = \sigma^i_t \sum_{j=\underline{J}+1}^{\bar{J}} L^i_{j,t} w^i_{\underline{J},t-j+\underline{J}}. \tag{31}$$

Unintentional Bequests. Let $a^i_{j,t}$ denote the end-of-period net asset holdings of an agent of age j in period t. The total net wealth of agents who die at the end of period t in country i is

$$Q^i_t \equiv \sum_{j=0}^{\bar{J}-1} (1 - p^i_{j,t}) L^i_{j,t} a^i_{j,t}. \tag{32}$$

Unintentional bequests are redistributed as lump-sum transfers to all agents of age $j \in \mathcal{B}$ who survive into the next period.[16] Hence the amount of wealth transfer received by a surviving agent of age $j < \bar{J}$ at the end of period t is

$$q^i_{j,t} = \begin{cases} 0 & j \notin \mathcal{B} \\ \frac{Q^i_t}{\sum_{j \in \mathcal{B}} p^i_{j,t} L^i_{j,t}} =: q^i_t, & j \in \mathcal{B}. \end{cases} \tag{33}$$

Household Constraints. Consider the consumption-saving problem of an agent born in period t and country i. This agent faces a (potential) sequence of gross rates of return $\{R^i_{t+j+1}\}_{j=0}^{\bar{J}-1}$, labor income $\{w^i_{j,t+j}\}_{j=0}^{\underline{J}}$, pension transfers $\{\pi^i_{j,t+j}\}_{j=\underline{J}+1}^{\bar{J}}$, and bequest transfers $\{q^i_{j,t+j}\}_{j=0}^{\bar{J}-1}$. Flow budget constraints are

$$c^i_{j,t+j} + a^i_{j,t+j} = R^i_{t+j}(a^i_{j-1,t+j-1} + q^i_{j-1,t+j-1}) + (1 - \tau^i_t) w^i_{j,t}, \qquad 0 \le j \le \underline{J}, \tag{34}$$

$$c^i_{j,t+j} + a^i_{j,t+j} = R^i_{t+j}(a^i_{j-1,t+j-1} + q^i_{j-1,t+j-1}) + \pi^i_{j,t}, \qquad \underline{J}+1 \le j \le \bar{J}, \tag{35}$$

[16] Note that when an agent dies without having fully repaid his debt, the loan is effectively repaid by the "heirs".

with $a^i_{-1,t-1} = q^i_{-1,t-1} = 0$. Define the expected discounted present value of current and future labor income for an agent who reaches age j in period t

$$H^i_{j,t} \equiv w^i_{j,t} + \sum_{\tau=1}^{\underline{J}-j} \frac{\left(\prod_{s=0}^{\tau-1} p^i_{j+s,t+s}\right) w^i_{j+\tau,t+\tau}}{\prod_{s=1}^{\tau} R^i_{t+s}}, \qquad 0 \leq j \leq \underline{J}-1, \tag{36}$$

$$H^i_{\underline{J},t} \equiv w^i_{\underline{J},t} \tag{37}$$

$$H^i_{j,t} \equiv 0, \qquad j \geq \underline{J}+1. \tag{38}$$

The credit constraint faced at age $j \leq \bar{J}-1$ by an agent born in period t in country i is

$$a^i_{j,t+j} \geq -\theta^i \frac{p^i_{j,t+j} H^i_{j+1,t+j+1}}{R^i_{t+j+1}}. \tag{39}$$

Equilibrium. We solve for the autarkic and integrated steady states of the model, as well as its transitory dynamics for a given evolution of productivity, demographics, and social security contribution rates. In autarky, the model equilibrium is given by a path for the capital-effective-labor ratio k^i_t, bequest transfers $\left\{q^i_{j,t}\right\}_{j=0}^{\bar{J}-1}$, and replacement rates σ^i_t such that: (i) all agents maximize their intertemporal utility (Eq. 26) with respect to their consumption decisions, subject to the sequence of budget constraints (Eqs. 34-35) and credit constraints (Eq. 39); (ii) the consistency condition (Eq. 33) between the bequest transfers and the amount of unintentional bequests left is satisfied; (iii) the social security budget is balanced (Eq. 31); and (iv) the market for capital clears in every period. Under financial integration, a similar definition of an equilibrium holds, with the market for capital clearing globally. When solving for equilibrium, the presence of bequests and social security transfers adds layers of complexity as the paths of capital, bequests and replacement rates have to be determined together in a dynamic fixed point problem. A detailed description of the numerical solution method is provided in Appendix C.

3.2 Calibration

In our experiments, the world consists of multiple groups of countries (regions), as defined in Appendix A.1. For illustrative purposes, we start our simulations with two groups of countries, Developed and Emerging, and extend progressively the number of regions. Each

period lasts for 10 years and agents live for (at most) 7 periods — which map into age brackets 20-30, 30-40, ... , 70-80, and above 80. Agents work during four periods and are retired in the last three periods of their life. All regions start from their initial autarky steady state in 1950 and stay in autarky until 1980, date at which capital can freely move across regions. We now turn to the calibration of common and region-specific parameters. The values of calibrated parameters are summarized in Table 1.

Preferences and Technology. We set the discount factor β to 0.985 annually and the elasticity of intertemporal substitution ω to $1/2$ — in the mid-range of empirical estimates. The capital share α is set to 0.3 and the depreciation rate of capital to 7.5% (on annual basis).

Demographics. In the initial autarky steady state of region i, the survival probabilities, p_j^i, are set to match the demographic composition by age groups of the region in 1950, given an initial steady-state population growth γ_L^i calibrated over the period 1920-1950. Note that the obtained survival probabilities are fairly close to the ones estimated by the United Nations (World Population Prospects) in 1950. For years 1960, 1970, ..., 2010, the growth rate of the size of the youngest age group $\gamma_{L,t}^i$ and the survival probabilities $p_{j,t}^i \in [0,1]$ are chosen to match the growth rate of the population and its composition by age group.[17] For the years post-2010, we proceed similarly but we match population forecasts by the United Nations for those years, based on their medium scenario. After 2100, survival probabilities in a region are held constant to their 2100 value and demographic growth $\gamma_{L,t}^i$ is set to zero in all countries.

Age-income Profiles and Distribution of Unintentional Bequests. The relative efficiency parameters, e_j^i, are set to match the wage-income profile (net of income taxes) of the U.S., based on income data by age groups from the Consumption Expenditure Survey (CEX) for the year 2008 (normalizing to 1 the efficiency parameter of the 40-50 age group). Note that the age-income profile is very stable over time in the U.S. for the period of availability of the CEX (1986-2008) and results are not sensitive to the choice of the year 2008. However, it is likely that age-income profiles differ across countries (see Coeurdacier, Guibaud, and Jin (2015) for a U.S.-China comparison) but due to the lack of individual income data for

[17] With $p_{j,t}^i \in [0,1]$ one cannot match the demographic data perfectly due to potential migration but our predicted demographic compositions are very close to the true ones and our survival probabilities are also very close to the ones estimated by the United Nations.

Table 1: Calibration Summary

Age-income profiles, $e_{j,t}$							
		20-30	*30-40*	*40-50*	*50-60*	*60-70*	*70+*
All regions	All periods	0.33	0.75	1	0.75	0.05	0
Conditional survival probabilities, $p_{j,t}$, *percent*							
		20-30	*30-40*	*40-50*	*50-60*	*60-70*	*70-80*
Developed	1960	99.4	98.0	94.2	86.2	68.2	43.6
	2010	100	100	100	90.5	82.0	58.8
Emerging	1960	93.7	91.6	86.1	74.7	53.5	28.9
	2010	97.6	96.7	94.4	87.4	71.6	44.6
South-East Asia	1960	94.7	92.6	86.4	74.2	51.2	22.8
	2010	98.9	98.2	96.3	89.9	74.8	45.3
Rest of Emerging	1960	93.1	90.6	84.9	73.6	54.1	30.7
	2010	97.5	96.0	94.1	84.5	66.3	39.5
U.S.	1960	99.4	98.0	94.2	86.2	68.2	43.6
	2010	100	100	100	90.5	82.0	58.8
Europe	1960	100	98.8	94.6	87.1	68.8	41.1
	2010	100	100	100	94.9	87.3	65.8
Demographic growth, $\gamma_{L,t}$, *percent per year*							
		D	E	SEA	RoE	U.S.	Eu
	1950-60	0.2	1.8	1.9	1.6	0.2	0.5
	2000-10	0.3	1.6	0.9	2.1	0.3	-0.8
Productivity growth, $\gamma_{A,t}$, *percent per year*							
		D	E	SEA	RoE	US	Eu
	1950-60	2.5	2.5	2.5	1.5	2.5	4.0
	1960-70	2.5	2.5	3.0	1.5	2.5	3.5
	1970-80	1.5	2.5	3.0	2.0	1.5	1.8
	1980-90	1.5	2.5	3.0	2.3	1.5	1.3
	1990-00	2.0	2.5	3.0	2.5	2.0	1.0
	2000-10	2.0	2.5	3.0	2.5	2.0	1.0
	2010-20	1.5	2.0	2.3	2.5	1.5	1.0
Credit constraint, θ, *percent*							
		D	E	SEA	RoE	U.S.	Eu
	All periods	15.0	1.8	2.5	1.4	19.1	13.5
Social security contribution rate, τ_t, *percent*							
		D	E	SEA	RoE	U.S.	Eu
	1950	10.22	1.27	1.71	1.20	8.18	12.72
	2010	13.19	1.74	2.22	1.67	9.63	23.21
Other parameters							
Discount factor, β, *per year*		0.985		E.i.s. coefficient, ω			0.5
Depreciation rate, δ, *per year*		0.075		Capital share, α			0.3

a large sample of countries, we assume that all countries share common relative efficiency parameters.

For now, we assume that unintentional bequests are distributed evenly across individuals in their working age, reflecting bequests towards children (and potentially grandchildren). Results are barely sensitive to different redistribution schemes of bequests.

Heterogeneity in Credit Constraints [Preliminary]. We use household debt over GDP data to calibrate the (time-invariant) region-specific credit constraint parameters. As data on household debt across countries are scarce and missing for many emerging countries in our sample, we compute forecasts of household debt over GDP for each country using variables correlated with household debt and observed for a large cross-section of countries in 2011 (such as the fraction of the population above the age of 15 with a mortgage, and the same fraction with a credit card), where the forecasting equation is estimated on a sample of countries for which household debt over GDP is observed in 2010.[18] Using observed and predicted household debt over GDP, credit constraints parameters across regions are set such that for a given region i, the ratio θ_i/θ_D matches the ratio of household debt between the region i and the group of developed countries (D) in 2010.[19] The remaining free parameter θ_D is set to match aggregate savings in the group of developed countries (D) in 2010.[20] Note that estimated differences in the level of financial development across countries are fairly large, with a θ about 5 to 10 times smaller in emerging countries than in developed countries (see Figure A.1 in Appendix A).

Heterogeneity in Social Security [Preliminary]. Data on contribution and replacement rates of pay-as-you-go systems are available for OECD countries over the period 1990-2010, and for a large sample of emerging countries over the period 2002-2010. A major issue is the extent of coverage of social security programs. In emerging countries, most workers do not contribute and do not qualify for any benefits. In our framework, this would translate into lower contribution and replacement rates. To adjust for coverage, we

[18]Household debt over GDP in 2010 is observed for 36 countries. Both predicting variables are significant and the R^2 of the forecasting equation is 0.88. Forecasting equations with alternative predictors correlated with the development of credit markets give very similar measures of household debt. The dispersion of household debt over GDP as predicted by our baseline regression is shown in Figure A.1 (Appendix A.3).

[19]The household debt over GDP of a given region is the population-weighted average of the different countries in the region.

[20]The value obtained is close to the one estimated for the U.S. in Coeurdacier, Guibaud, and Jin (2015).

use data from the World Social Security Report 2010/11 (ILO) for one large cross-section of countries. As shown in Figure A.2 in Appendix A, large disparities in coverage exist across countries; in low income countries less than 20% of the population is covered by social security schemes compared to about 90% in the most developed countries. In the most recent period, the *effective* replacement rate is thus an order of magnitude smaller in emerging countries, and most likely even more so in earlier periods. At this stage, we target an effective replacement rate of about 10% in emerging countries — corresponding to a replacement rate of 50% and a coverage of 20%. For developed countries, we target the effective replacement rate in 2010. In the time-series dimension, starting in 1950, we set a path of taxes such that the replacement rates in developed countries exhibit a small downward trend — assuming that by 2040, 70% of the adjustment in the financing of social security due to aging is done through higher contribution rates. For emerging countries, the path of taxes is set so as to maintain replacement rates at their 2010 level over the entire period. [The calibration is illustrative at this stage but note that our results are mostly sensitive to cross-sectional differences.]

Initial Size and Productivity Growth. In 1950, the initial relative size of all regions is set to match their relative share in world GDP. Over the period 1950-2010, given demographics and other structural parameters, world productivity growth is set to match the evolution of world GDP (at constant 1990 USD), and the productivity growth rate in a region (relative to the world) is set to match the share of the region in world GDP. Post 2030, all regions share a common productivity growth rate of 1.5% annually, towards which productivity growth in each region converges in two decades.

3.3 Results

Developed vs. Emerging. We start with a two-region version of our quantitative model, the world being divided between developed and emerging countries (see Appendix A.1 for a list of countries in each region). Emerging countries are characterized by significantly tighter credit constraints and lower replacement rates, as well as a much younger population initially and faster productivity growth post-1970.

Figure 3.1 shows the evolution of the variables of interest in our baseline two-region

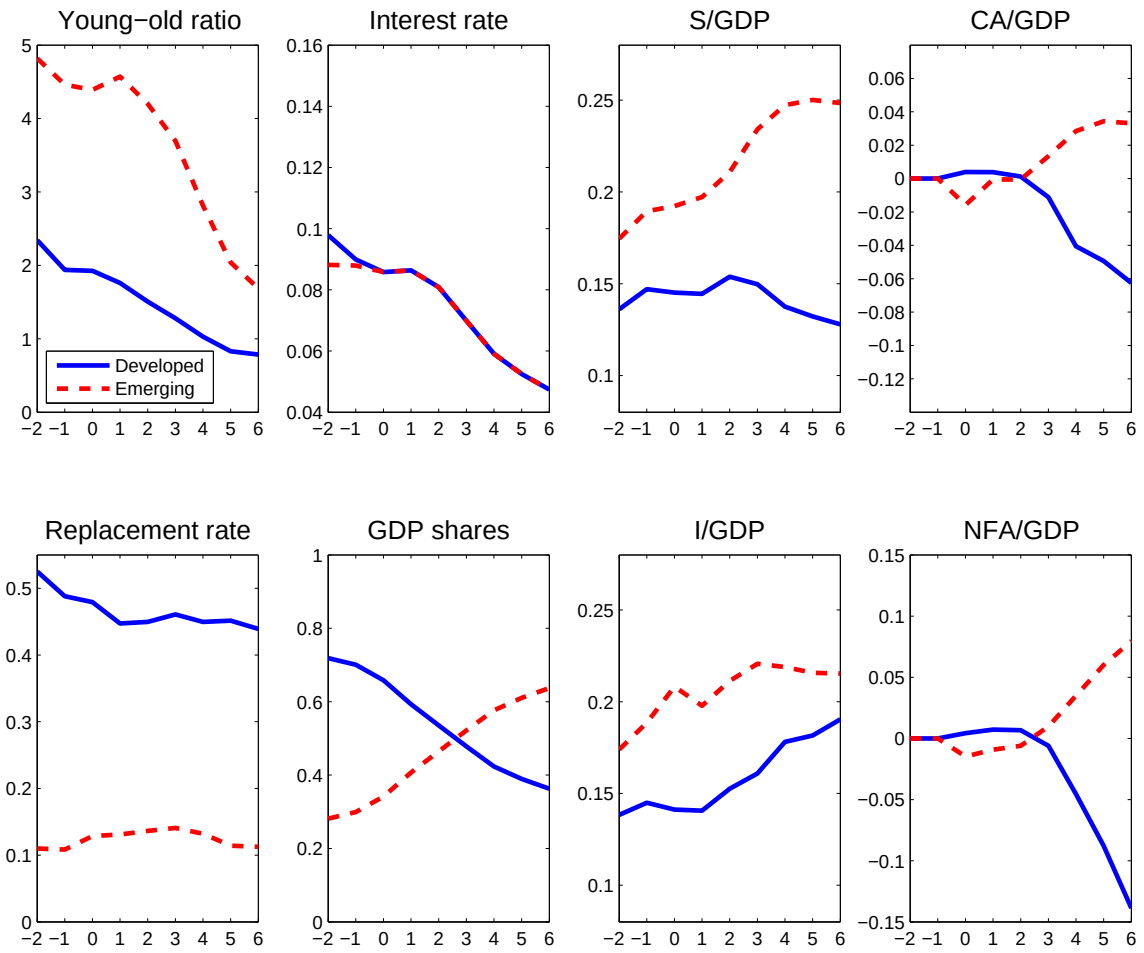


Figure 3.1: Capital Flows Between a Developed and an Emerging Region

Notes: The young-old ratio corresponds to the ratio of the size of the group of agents of age $j = 1, 2$ (i.e., 20-40) over the size of the group of agents of age $j = 5, 6, 7$ (i.e., 60-90). The values of calibrated parameters are given in Table 1.

experiment. Emerging countries initially run a small current account deficit in the 1980s before slowly starting to run a surplus as they converge to the developed countries in terms of demographics. This initial current account deficit is driven by faster growth and a younger population initially. In a simulation where productivity growth is equal across regions, emerging countries accumulate net foreign assets as soon as they open up (see Figure D.1 in Appendix D). Aging in both regions triggers a fall in the world interest rate, amplified by the increasing size of emerging countries which have tighter credit constraints and lower replacement rates. Falling interest rates trigger a savings divergence between emerging and developed countries. The contribution rates have to increase in developed countries to maintain the financing of the pension system, which exerts further downward

pressure on their savings. The increase in savings in emerging countries eventually outpace their higher investment rates, driven by faster growth, turning these countries into world creditors. They remain creditors as long as the initial differences in credit markets and social security coverage persist.

Figure 3.2 shows the importance of the asymmetry between regions in the ability to borrow — intertemporally and across generations — for the dynamics of savings and capital flows. With homogenous credit constraints and replacement rates, set to the level in developed countries, the model predicts low savings and very large current account deficits in emerging countries for over forty years after integration.

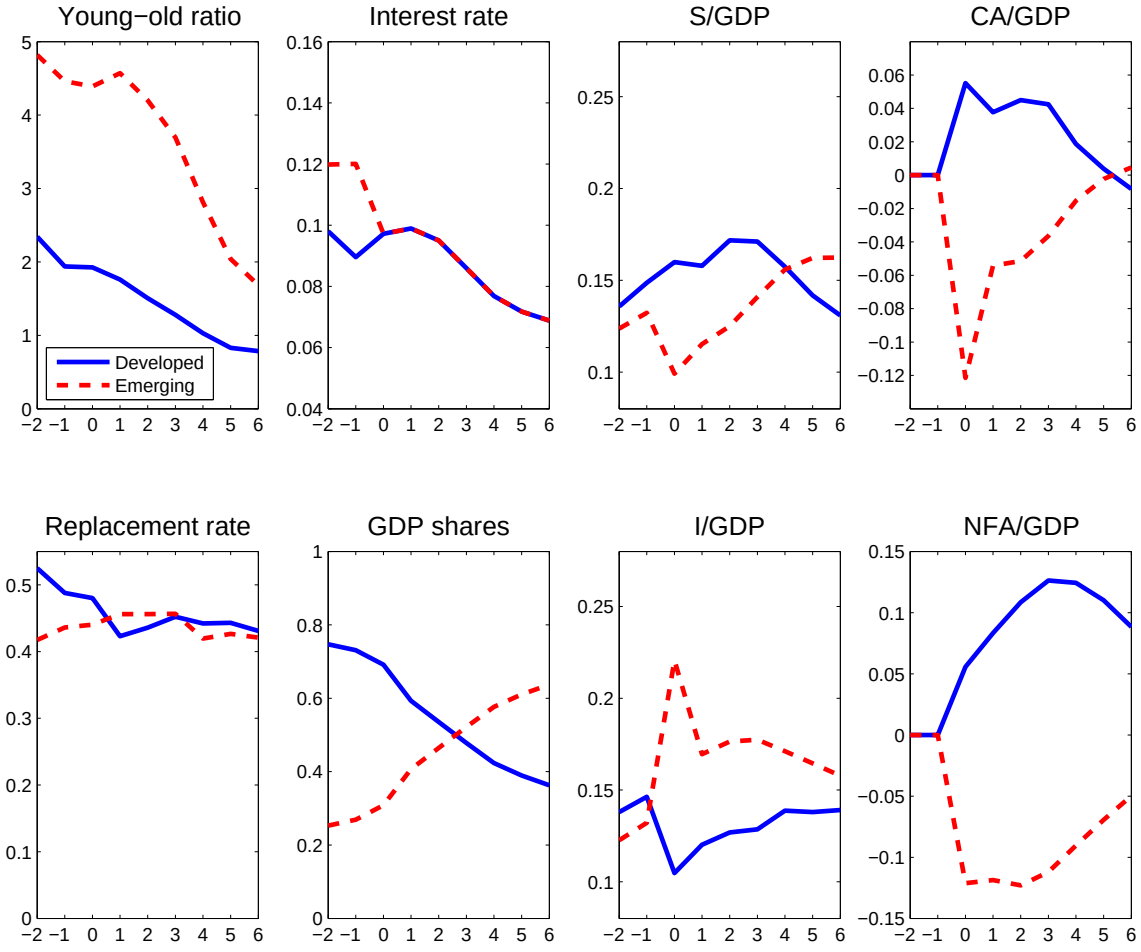


Figure 3.2: Capital Flows Between a Developed and an Emerging Region, Identical τ and θ

Notes: The young-old ratio corresponds to the ratio of the size of the group of agents of age $j = 1, 2$ (i.e., 20-40) over the size of the group of agents of age $j = 5, 6, 7$ (i.e., 60-90). The values of calibrated parameters are given in Table 1.

Three Regions. We now turn to our second main experiment, which takes into account

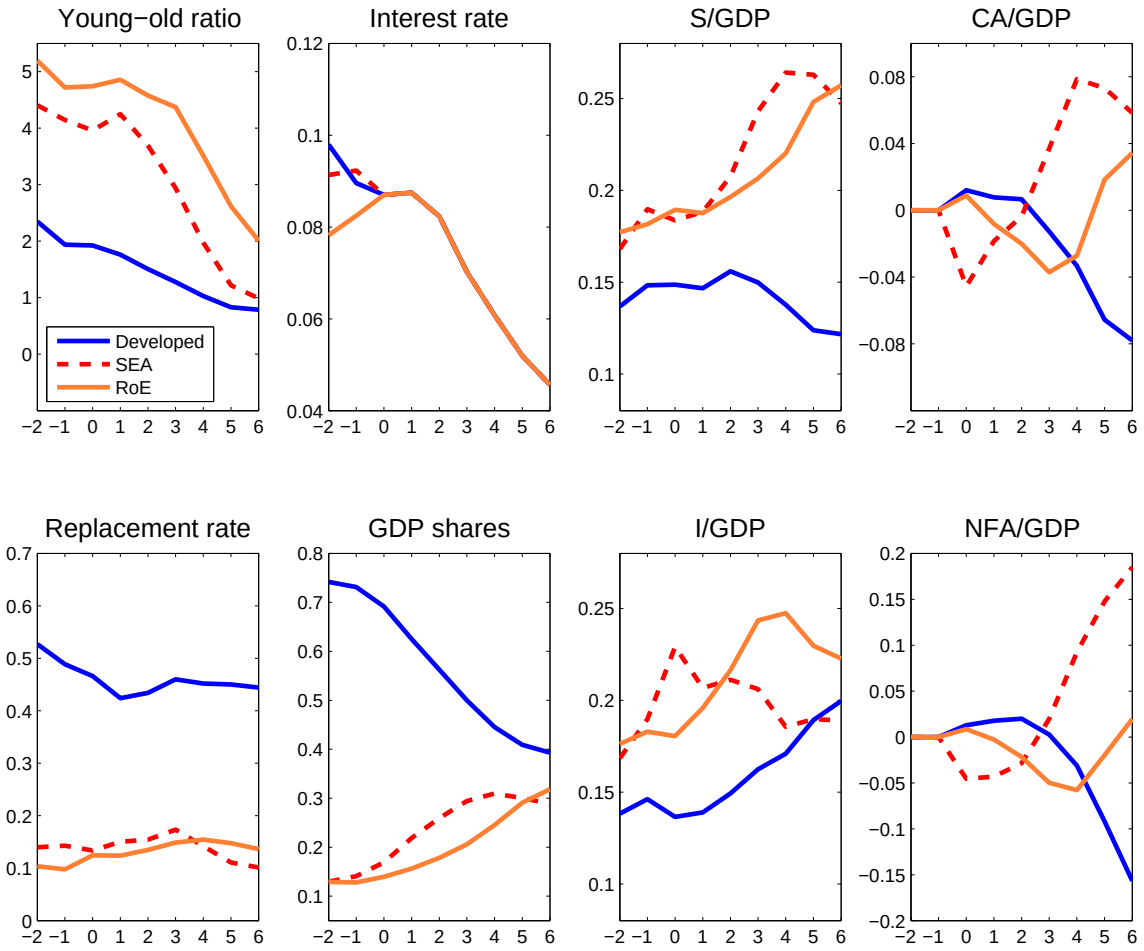


Figure 3.3: Capital Flows Between a Developed and Two Emerging Regions

Notes: The young-old ratio corresponds to the ratio of the size of the group of agents of age $j = 1, 2$ (i.e., 20-40) over the size of the group of agents of age $j = 5, 6, 7$ (i.e., 60-90). The values of calibrated parameters are given in Table 1.

some heterogeneity among emerging countries. We split emerging countries into two regions: South-East Asia (SEA) and a large region including South-Central Asia, the Middle East and Northern Africa, which we refer to as rest of emerging (RoE).[21] The former is characterized by faster productivity growth (starting from 1970) and faster aging, while the latter experienced slower growth and a delayed demographic transition. Both regions have similar credit constraints and replacement rates.

Faster growth in South-East Asia together with the small initial differences in demographics triggers larger capital flows towards this region initially, but the pattern quickly

[21]The region including South-Central Asia, the Middle East and Northern Africa had a slightly slower demographic transition than Latin America but significantly faster than the rest of Africa (see Figure 1.1, panels (c) and (d)). A region made of all emerging countries but South-East Asia exhibits a very similar demographic evolution.

reverses (Figure 3.3). Due to a delayed demographic transition and despite slower productivity growth, the RoE region turns into the major world borrower in the 1990s. Faster aging in South-East Asia turns the region into a creditor in the most recent period. At the date of integration, developed countries are financing faster growth in both emerging regions, whether due to demographics or productivity, before slowly turning into debtors.

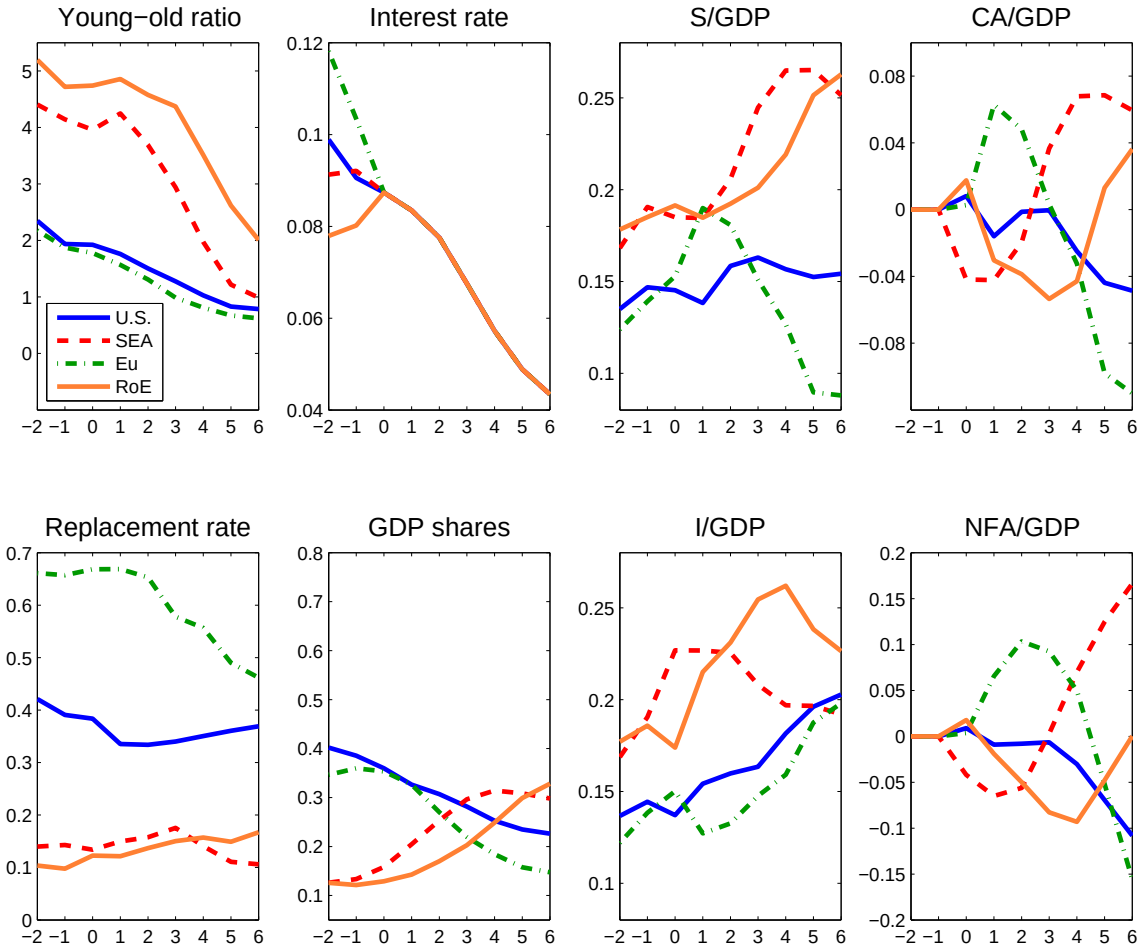


Figure 3.4: Capital Flows Between Two Developed and Two Emerging Regions

Notes: The young-old ratio corresponds to the ratio of the size of the group of agents of age $j = 1, 2$ (i.e., 20-40) over the size of the group of agents of age $j = 5, 6, 7$ (i.e., 60-90). The values of calibrated parameters are given in Table 1.

Four Regions. Our last experiment takes into account some heterogeneity among developed countries as well. We consider a region comprising the U.S. and other Anglo-saxon countries (which we refer to as U.S.), with more developed credit markets, and another region comprising continental Western Europe and Japan (which we refer to as Europe (Eu)), with more generous social security systems. This latter region is also aging more than the

U.S. and their productivity stagnates after 1990.

As a consequence, due to depressed investment, Europe is the major world creditor until the most recent period, where it is replaced by South-East Asia (see Figure 3.4). Interestingly, Europe is expected to become a debtor in the near future as the financing of social security and the larger share of elderly in the population starts to lower their savings. On the other hand, the U.S., being slightly younger and growing faster than the rest of the developed countries, imports capital over the entire period. For the two emerging regions, results are very similar to those in the previous experiment.

Multiple Countries and the Cross Section of Capital Flows. [TO BE DONE]

3.4 Sensitivity analysis and extensions

[TO BE DONE]

4 Conclusion

[TO BE WRITTEN]

References

[1] Abel, A., 2001. Will Bequests Attenuate The Predicted Meltdown In Stock Prices When Baby Boomers Retire?. The Review of Economics and Statistics, vol. 83(4), 589-595.

[2] Aguiar, M. and Amador, M., 2011. Growth in the Shadow of Expropriation. The Quarterly Journal of Economics, Oxford University Press, vol. 126(2), 651-697.

[3] Angeletos, G.M. and Panouzi, V., 2011. Financial Integration, Entrepreneurial Risk and Global Imbalances. Journal of Economic Theory 146:3, 863-896.

[4] Attanasio, O., Kitao, S., and Violante, G., 2007. Global demographic trends and social security reform. Journal of Monetary Economics, 54 (1), 144-198.

[5] Attanasio, O., and Weber, G., 1993. Consumption Growth, the Interest Rate and Aggrega- tion. Review of Economic Studies 60, 631-649.

[6] Bacchetta P. and Benhima, K., 2011. The Demand for Liquid Assets, Corporate Saving, and Global Imbalances, Journal of the European Economic Association, forthcoming.

[7] Backus, D., Cooley, T., and Henriksen, E., 2014. Demography and Low-Frequency Capital Flows. Journal of International Economics, 92(1), 94102.

[8] Benhima, K., 2013. A Reappraisal of the Allocation Puzzle through the Portfolio Approach. Journal of International Economics, 89(2), 331346.

[9] Benigno, G. and L. Fornaro, 2012. Reserve Accumulation, Growth and Financial Crisis. CEP Discussion Paper No 1161.

[10] Bernanke, B., 2005. The Global Saving Glut and the U.S. Current Account Deficit. Sandridge Lecture, Virginia Association of Economists, Richmond, V irginia.

[11] Bloom, D., Canning, D., Mansfield, R., and Moore, M., 2003. Demographic change, social security systems, and savings. Journal of Monetary Economics,54(1), 92-114.

[12] Bolt, J. and van Zanden, J.L., 2014. The Maddison Project: collaborative research on historical national accounts. The Economic History Review, 67(3), 627-651.

[13] Borsch-Supan, A., Ludwig, A., and Winter, J., 2005. Aging, Pension Reform, and Capital Flows: A Multi-Country Simulation Model. Economica, 73(292), 625-658.

[14] Broner, F. and Ventura, J., 2013. Rethinking the Effects of Financial Liberalization. Manuscript, CREI.

[15] Brooks, R., 2003. Population Aging and Global Capital Flows in a Parallel Universe. IMF Staff Papers, 50 (2), 200-221.

[16] Buera, F. and Shin, Y., 2011. Productivity Growth and Capital Flows: The Dynamics of Reforms. Manuscript, UCLA.

[17] Caballero, R., Farhi E., and Gourinchas P.-O., 2008. An Equilibrium Model of 'Global Imbalances' and Low Interest Rates, American Economic Review, 98(1), 358-93.

[18] Campbell, J., 2003. Consumption-Based Asset Pricing. in: G.M. Constantinides, M. Harris, R. M. Stulz (ed.), Handbook of the Economics of Finance, Elsevier. 1 (13), 803-887.

[19] Carroll, Christopher and Olivier Jeanne. 2009. A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds. Manuscript, Johns Hopkins University.

[20] Ca'Zorzi, M., Chudik, A. and Dieppe, A., 2012. Thousands of models, one story: current account imbalances in the global economy. ECB WP 1441.

[21] Coeurdacier, N., S. Guibaud, and K. Jin. 2015. Growing like China. Credit Constraints and Growth in a Global Economy, American Economic Review, Forthcoming.

[22] Chinn, M.D. and Prasad, E.S., 2003. Medium-term Determinants Of Current Accounts In Industrial And Developing Countries: An Empirical Exploration. Journal of International Economics, 59(1), 47-76.

[23] Choukhmane, T., 2012. Demography, Domestic Savings and the Current Account: a Demographic Perspective on Global Imbalances. Mimeo.

[24] Curtis, Chadwick, Steven Lugauer, and Nelson Mark. 2011. Demographic Patterns and Household Saving in China. mimeo.

[25] De Nardi, M., 2004. Wealth Inequality and Intergenerational Links. Review of Economic Studies, 71 (3): 743-768.

[26] De Nardi, M., E. French and J.B. Jones, 2010. Why Do the Elderly Save? The Role of Medical Expenses. Journal of Political Economy, vol. 118(1), pp 39-75.

[27] Domeij, D. and Flodén, M., 2006. Population Aging And International Capital Flows. International Economic Review, vol. 47(3), 1013-1032.

[28] Ferrero, A., 2010. A Structural Decomposition of the U.S. Trade Balance: Productivity, Demographics and Fiscal Policy. Journal of Monetary Economics, 57 (4), 478-49

[29] Gale, W.G and J. K. Scholz, 1994. Intergenerational Transfers and the Accumulation of Wealth. Journal of Economic Perspectives 8(4): 145-60.

[30] Ge, S., Yang, D., and J. Zhang., 2012. Population Policies, Demographic Structural Changes, and the Chinese Household Saving Puzzle. IZA Discussion paper 7026.

[31] Gollin, D., 2002. Getting Income Shares Right. Journal of Political Economy, 110(2), 458-474.

[32] Gourinchas, P-O., and O. Jeanne, 2012, Capital Flows to Developing Countries: The Allocation Puzzle, Review of Economic Studies, forthcoming.

[33] Gourinchas, P-O., and H. Rey, 2013. External Adjustment, Global Imbalances, Valuation Effects. Prepared for the Handbook Chapter in International Economics, vol IV, Gopinath, Helpman and Rogoff eds.

[34] Guo, S. and F. Perri, 2021. The Allocation Puzzle is Not as Bad as You Think. University of Minnesota, Manuscript.

[35] Guvenen, F., 2006. Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective. Journal of Monetary Economics, 53, 1451-1472.

[36] Hall, R., 1988. Intertemporal Substitution in Consumption. Journal of Political Economy, 96, 339-357.

[37] Hall, R.E and Jones C., 1999. Why Do Some Countries Produce So Much More Output Per Worker Than Others?. The Quarterly Journal of Economics, Vol. 114 (1), 83-116.

[38] Hendricks, L., 2001. Bequests and Retirement Wealth in the United States. Mimeo Arizona State University.

[39] Higgins, M., 1998. Demography, national savings and international capital flows. International Economic Review, 39(2), 343-69.

[40] IMF, 2006, Household Credit Growth in Emerging Market Countries, Global Financial Stability Report Market Developments and Issues, Chapter 2, September.

[41] Jappelli, T. and Pagano, M., 1994. Saving, Growth, and Liquidity Constraints. The Quarterly Journal of Economics, vol. 109(1), 83-109.

[42] Karabarbounis, L. and Neiman, B., 2012. Declining Labor Shares and the Global Rise of Corporate Saving. Mimeo Chicago Booth.

[43] Krueger D., and Ludwig, A., 2007. On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. Journal of Monetary Economics, vol. 54(1), 49-87.

[44] Laitner, J. and D. Silverman, 2005. Estimating Life-Cycle Parameters from Consumption Behavior at Retirement. NBER Working Papers 11163.

[45] Lane, P.R. and Milesi-Ferretti, G.M., 2002. Long-Term Capital Movements. NBER Chapters, NBER Macroeconomics Annual 2001, vol. 16, 73-136.

[46] Lane, P.R. and Milesi-Ferretti, G.M., 2007. The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004. Journal of International Economics 73, 223-250.

[47] Lee, R., 2003. The Demographic Transition: Three Centuries of Fundamental Change. Journal of Economic Perspectives, 17(4), 167-190.

[48] Leff, N.H., 1969. Dependency Rates and Savings Rates. American Economic Review, 59(5), 886-96.

[49] Mendoza E., Quadrini V. and Ríos-Rull J.-V., 2009. Financial Integration, Financial Development and Global Imbalances. Journal of Political Economy, vol. 117(3), 371-416.

[50] Meng, X., and Yeo, C., 2005. aging and health-care expenditure in urban China. Mimeo Australian National University.

[51] Obstfeld, M., and Rogoff, K., 1996. Foundations of International Macroeconomics, MIT Press, Cambridge.

[52] Obstfeld, M., and Rogoff, K., 2009. Global Imbalances and the Financial Crisis: Products of Common Causes. Asia Economic Policy Conference, Santa Barbara, CA, Federal Reserve Bank of San Francisco.

[53] Ogaki M. and C.M. Reinhart, 1998. Measuring Intertemporal Substitution: The Role of Durable Goods. The Journal of Political Economy, vol. 106(5), 1078-1098.

[54] Sandri, D., 2010. Growth and Capital Flows with Risky Entrepreneurship, Working Paper, International Monetary Fund.

[55] Song, M., K. Storesletten, and F. Zilibotti. 2011. Growing like China. American Economic Review, vol. 101(1), 196-233.

[56] Song, M., and Yang D.T., 2010. Life Cycle Earnings and Saving in a Fast-Growing Economy, mimeo, Chinese University of Hong Kong.

[57] Taylor, A.M, and J. Williamson, 1994. Capital Flows to the New World as an Intergenerational Transfer. Journal of Political Economy, 102(2), 348-71.

[58] Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. Journal of Political Economy, vol. 110(4), 825-853.

[59] Warnock, V., and F. Warnock, 2008. Markets and Housing Finance. Journal of Housing Economics, 17, 239-251.

[60] World Social Security Report 2010–11, International Labour Organization (Social Security Department).

[61] Yang, D.T., 2012. Aggregate Savings and External Imbalances in China. Journal of Economic Perspectives, vol. 26(4), 125-146.

[62] Yang, D.T., Zhang, J. and S. Zhou, 2011. Why are saving rates so high in China?. NBER Chapters, in: Capitalizing China, pp 249-278.

[63] Yogo, M., 2004. Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak. Review of Economics and Statistics , 86(3), 797–810.

A Data

A.1 List of Countries

Developed countries.

Western offshoots and Anglo-saxon countries. Australia, Canada, Iceland, Ireland, New Zealand, United Kingdom, United States.

Old Europe and Japan. Austria, Belgium, Cyprus, Denmark, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland.

Central and Eastern Europe. Albania, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Macedonia, Moldova, Poland, Romania, Russian Federation, Serbia, Slovak Republic, Slovenia, Ukraine.

Emerging countries.

South East Asia. Cambodia, China, Hong Kong, Indonesia, Korea Rep., Lao PDR, Macao SAR, China, Malaysia, Philippines, Singapore, Thailand, Vietnam.

South and Central Asia. Afghanistan, Armenia, Azerbaijan, Bangladesh, Bhutan, Georgia, India, Kazakhstan, Kyrgyz Republic, Maldives, Mongolia, Nepal, Pakistan, Sri Lanka, Tajikistan.

Middle-East and North Africa. Algeria, Bahrain, Egypt Arab Rep., Iran Islamic Rep., Israel, Jordan, Kuwait, Lebanon, Morocco, Oman, Saudi Arabia, Syrian Arab Republic, Tunisia, Turkey, Yemen Rep.

Central and Western Africa. Benin, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Congo Rep., Cote d'Ivoire, Equatorial Guinea, Ethiopia, Gambia, Ghana, Guinea, Kenya, Liberia, Madagascar, Mali, Mauritius, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, Sudan, Tanzania, Togo, Uganda.

Southern Africa. Angola, Botswana, Lesotho, Mozambique, Namibia, South Africa, Swaziland, Zambia, Zimbabwe.

Pacific Islands. Fiji, Kiribati, Papua New Guinea, Vanuatu.

A.2 Data sources

Demographics. United Nations World Population Prospects. Maddison Project for population data prior to 1950 (Bolt and van Zanden (2014)).

GDP Data. Maddison Project and Penn World Tables.

Data on Savings, Investment, and Current Account. World Development Indicators (World Bank), Penn World Tables and OECD.

Data on Net Foreign Asset Positions. External Wealth of Nations Dataset, 1970-2011 (Lane and Milesi Feretti (2007)).

Data on Social Security. OECD and Social Security Administration (Social Security Programs Throughout the World in collaboration with the International Social Security Association). Coverage data are from the International Labour Organization (Social Security Department), latest available year.

Household Debt Data. Global Financial Inclusion Database (World Bank), OECD, GFDD (World Bank).

A.3 Household Debt and Social Security across the World

Fig A.1 below shows the level of household debt over GDP across regions in 2010. For each region, the household debt over GDP is the population-weighted average of countries household debt over GDP (with 2010 population-weights). As household debt is not observable for most developing countries, we use a forecasted value when missing. Our predicted household debt over GDP is a linear combination of two explanatory variables — percentage of individuals above 15 with a mortgage and percentage of individuals above 15 with a credit card in 2011 (from the Global Financial Inclusion Database, World Bank). The coefficients in the forecasting equation are estimated using a linear regression on OECD countries for which household debt over GDP in 2010 is observed (both coefficients significant at 5% with an R^2 of 88%). Fig A.2 shows the level of official replacement rates of paygo systems as well as the coverage of the system across regions in 2010. Two measures of coverage from ILO are used (latest available year between 2002-2008): the percentage of active contributors in percentage of the working age population and the percentage of pension recipients in old age population (60+ or 65+ according to retirement age). For each

region, the replacement rate and the measures of coverage are the population-weighted average across countries (with 2010 population-weights).

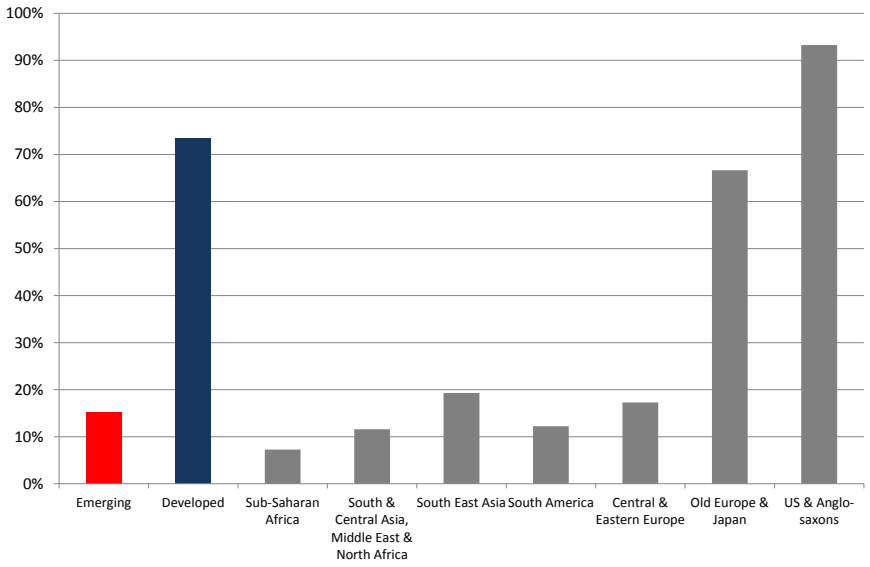


Figure A.1: Household Debt to GDP across the World.

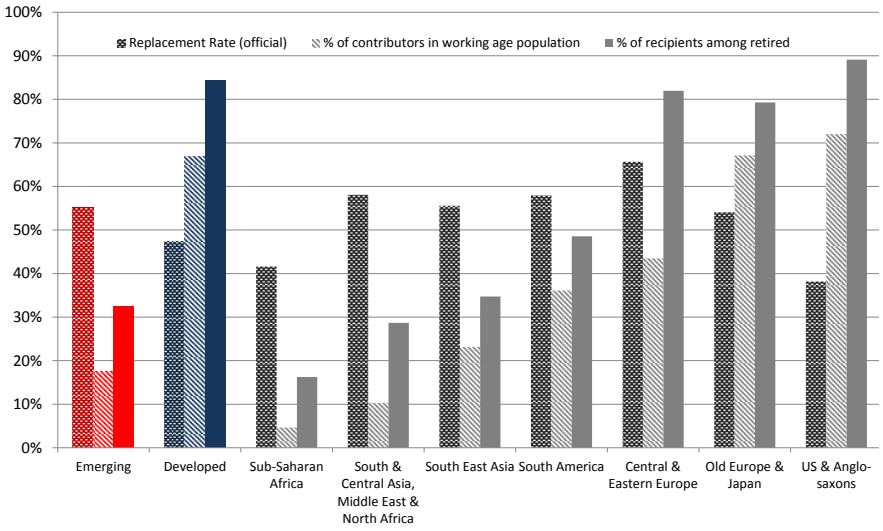


Figure A.2: Social Security across the World.

B Proofs

Condition for a Binding Credit Constraint in the Three-Period Model of Section 2.
The borrowing constraint is binding if the young without any borrowing constraints would prefer to consume more than what the borrowing constraint allows them to consume, i.e. $\theta_{t-1} w_t / R_t < c^*_{y,t-1}$. Solving the household's problem, this condition can be expressed as:

$$\theta_{t-1} < \frac{1 - \tau_t + \frac{p_t \sigma_{t+1}}{R_{t+1}}}{1 + \beta^\omega R_t^{\omega-1} + p_t \beta^{2\omega} (R_t R_{t+1})^{\omega-1}}.$$

For $\omega = 1$, this simplifies to: $\theta_{t-1} < \frac{1}{1+\beta+\beta^2 p_t} \left(1 - \tau_t + \frac{p_t \sigma_{t+1}}{R_{t+1}}\right)$.
Note that the condition does not only entail parameters of the model, but also R_t and R_{t+1}, which are endogenous, and hence depend on the values of $\theta_{t-1}, \tau_t, \sigma_{t+1}, p_t$ and β. We assume that θ_{t-1} and τ_t are jointly such that this condition is satisfied. For a given θ_{t-1}, we will later denote by τ_y the maximum value of τ which satisfies this condition in the steady state.
Note also that if the borrowing constraint is binding, then the net income of the middle aged (after taxes and repayment of debt accumulated when young) is $w_t(1 - \theta_{t-1} - \tau_t)$. If $\theta_{t-1} > 1 - \tau_t$, then the middle aged cannot accumulate any savings, and the model would not have a solution in the autarky case. Therefore we also assume that $\theta_{t-1} < 1 - \tau_t$.

Proof of Proposition 1. Re-arranging (10) and using the fact that all parameters are constant, we get:

$$\begin{aligned}
R_{t+1} &= \left(\frac{k_{t+1}}{k_t}\right)^\alpha \frac{n(1+\gamma_A)}{p(1-\tau-\theta)} \left(p + \beta^{-\omega} R_{t+1}^{1-\omega}\right) \left(\frac{\alpha}{1-\alpha} + \theta + \frac{\beta^{-\omega} R_{t+1}^{1-\omega}}{p + \beta^{-\omega} R_{t+1}^{1-\omega}} \tau\right) \\
&= \left(\frac{k_{t+1}}{k_t}\right)^\alpha \frac{n(1+\gamma_A)}{p(1-\tau-\theta)} \left[\left(p + \beta^{-\omega} R_{t+1}^{1-\omega}\right) \left(\frac{\alpha}{1-\alpha} + \theta\right) + \left(p + \beta^{-\omega} R_{t+1}^{1-\omega}\right) \frac{\beta^{-\omega} R_{t+1}^{1-\omega}}{p + \beta^{-\omega} R_{t+1}^{1-\omega}} \tau\right] \\
&= \left(\frac{k_{t+1}}{k_t}\right)^\alpha \frac{n(1+\gamma_A)}{p(1-\tau-\theta)} \left[p \left(\frac{\alpha}{1-\alpha} + \theta\right) + \beta^{-\omega} R_{t+1}^{1-\omega} \left(\frac{\alpha}{1-\alpha} + \theta + \tau\right)\right].
\end{aligned}$$

Re-arranging further yields

$$\left(\frac{k_t}{k_{t+1}}\right)^\alpha = \frac{n(1+\gamma_A)}{p(1-\tau-\theta)} \left[\frac{p\left(\frac{\alpha}{1-\alpha} + \theta\right)}{R_{t+1}} + \frac{\beta^{-\omega}\left(\frac{\alpha}{1-\alpha} + \theta + \tau\right)}{R_{t+1}^\omega}\right]. \tag{B-1}$$

Existence and uniqueness. If a steady state k and hence R exist, they have to satisfy the above equation for $k_t = k_{t+1}$ and $R_{t+1} = R$:

$$1 = \frac{n(1+\gamma_A)}{p(1-\tau-\theta)}\left[\frac{p\left(\frac{\alpha}{1-\alpha}+\theta\right)}{R} + \frac{\beta^{-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{R^{\omega}}\right] \tag{B-2}$$

The above clearly has a unique solution in terms of R as long $\tau < 1-\theta$. The left hand side is constant, while the right hand side is decreasing in R. In the limit as $R \to 0$, the right hand side converges to ∞, while as $R \to \infty$, the right hand side converges to 0.

Convergence. We now have to show that the economy converges to the above defined k and R from any initial k_t. Without loss of generality consider $k_t < k$. We need to show that a k_{t+1} and hence R_{t+1}, which satisfies (B-1), is necessarily such that $k_t < k_{t+1} < k$. We proceed by showing that neither $k_{t+1} < k_t$, nor $k_{t+1} > k$ cannot satisfy (B-1).

Consider first a $k_{t+1} < k_t < k$. In this case $R_{t+1} > R$, and hence the right hand side of (B-1) is smaller than 1. However, the left hand side of (B-1) is larger than 1.

Now consider a $k_{t+1} > k > k_t$. In this case $R_{t+1} < R$, and hence the right hand side of (B-1) is larger than 1. However, the left hand side is now smaller than 1.

Comparative statics. Multiplying (B-2) by R we get

$$R = \frac{n(1+\gamma_A)}{p(1-\tau-\theta)}\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right) + \beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right],$$

which is exactly (11) as stated in the Proposition. Taking derivatives of (11) with respect to τ:

$$\begin{aligned}
\frac{\partial R}{\partial \tau} =& \frac{n(1+\gamma_A)\beta^{-\omega}R^{1-\omega}}{p(1-\theta-\tau)} + \frac{n(1+\gamma_A)\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]}{(p(1-\theta-\tau))^2}p \\
&+ \frac{n(1+\gamma_A)(1-\omega)\beta^{-\omega}R^{-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p(1-\theta-\tau)}\frac{\partial R}{\partial \tau} \\
=& \frac{n(1+\gamma_A)\beta^{-\omega}R^{1-\omega}}{p(1-\theta-\tau)} + \frac{R}{1-\theta-\tau} \\
&+ \frac{(1-\omega)\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}\frac{\partial R}{\partial \tau} \\
\frac{\partial R}{\partial \tau} =& \frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}}\left[\frac{n(1+\gamma_A)\beta^{-\omega}R^{1-\omega}}{p(1-\theta-\tau)} + \frac{R}{1-\theta-\tau}\right].
\end{aligned}$$

Since $(1-\omega)\frac{\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)} < 1$, $\frac{\partial R}{\partial \tau} \geq 0$.
Similarly taking the derivative with respect to θ and re-arranging we get:

$$\frac{\partial R}{\partial \theta} = \frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}}\left[\frac{n(1+\gamma_A)\beta^{-\omega}R^{1-\omega}}{p(1-\tau-\theta)}+\frac{R}{(1-\tau-\theta)}\right]. \quad \text{(B-3)}$$

implying that $\frac{\partial R}{\partial \theta} \geq 0$. ∎

Proof of Corollary 1. Plug $\tau = 0$ into (11) to get

$$R_0 = \frac{n(1+\gamma_A)}{p(1-\theta)}\left(\frac{\alpha}{1-\alpha}+\theta\right)\left(p+\beta^{-\omega}R_0^{1-\omega}\right),$$

which is the equation pinning down R_0 given in the Corollary. Now differentiate with respect to n:

$$\begin{aligned}
\frac{\partial R_0}{\partial n} =& \frac{R_0}{n}+\frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)(1-\omega)\beta^{-\omega}R_0^{-\omega}}{p\,(1-\theta)}\frac{\partial R_0}{\partial n} = \frac{R_0}{n}+(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{\partial R_0}{\partial n},\\
\frac{\partial R_0}{\partial n} =& \frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}}\frac{R_0}{n}.
\end{aligned}$$

Let $\phi_0 \equiv \frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}} \in [0,1]$. The elasticity is then

$$\eta_{0,n} \equiv \frac{\partial R_0/R_0}{\partial n/n} = \frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}} = \frac{1}{1-(1-\omega)\phi_0} > 0.$$

Note that with $\omega = 1$, $\eta_{0,n} = 1$.

Now let's do the same by differentiating with respect to p:

$$\begin{aligned}
\frac{\partial R_0}{\partial p} =& \frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)}{p(1-\theta)} - \frac{n(1+\gamma_A)\left(p+\beta^{-\omega}R_0^{1-\omega}\right)\left(\frac{\alpha}{1-\alpha}+\theta\right)(1-\theta)}{p^2(1-\theta)^2} \\
&+ \frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)(1-\omega)\beta^{-\omega}R_0^{-\omega}}{p\,(1-\theta)}\frac{\partial R_0}{\partial p} \\
=& \frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)}{p(1-\theta)}\left(1-\frac{\left(p+\beta^{-\omega}R_0^{1-\omega}\right)}{p}\right) + (1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{\partial R_0}{\partial p} \\
=& -\frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)}{p(1-\theta)}\frac{\beta^{-\omega}R_0^{1-\omega}}{p} + (1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{\partial R_0}{\partial p} \\
=& -\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{R_0}{p} + (1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{\partial R_0}{\partial p} = -\phi_0\frac{R_0}{p} + (1-\omega)\phi_0\frac{\partial R_0}{\partial p} \\
\frac{\partial R_0}{\partial p} =& -\frac{\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}}{1-(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}}\frac{R_0}{p} = -\frac{\phi_0}{1-(1-\omega)\phi_0}\frac{R_0}{p} < 0.
\end{aligned}$$

The elasticity is then:

$$\eta_{0,p} \equiv \frac{\partial R_0/R_0}{\partial p/p} = -\frac{\phi_0}{1-(1-\omega)\phi_0} < 0$$

Again note that with $\omega = 1$, $\eta_{0,p} = -\frac{1}{1+\beta p}$.

To characterize how these elasticities behave for different values of ω, first differentiate R_0 wrt ω:

$$\begin{aligned}
\frac{\partial R_0}{\partial \omega} &= \frac{n(1+\gamma_A)\left(\frac{\alpha}{1-\alpha}+\theta\right)}{p(1-\theta)}\left[-\beta^{-\omega}R_0^{1-\omega}\ln\beta - \beta^{-\omega}R_0^{1-\omega}\ln R_0 + (1-\omega)\beta^{-\omega}R_0^{-\omega}\frac{\partial R_0}{\partial \omega}\right] \\
&= -R_0\underbrace{\ln(\beta R_0)}_{>0 \text{ iff } \beta R_0 > 1}\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}} + (1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\frac{\partial R_0}{\partial \omega} \\
\frac{\partial R_0}{\partial \omega} &= -\frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}}\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}R_0\ln(\beta R_0) < 0.
\end{aligned}$$

The above holds as long as $\beta R_0 > 1$.

Now differentiate $\eta_{0,n}$ wrt ω:

$$\frac{\partial \eta_{0,n}}{\partial \omega} = \frac{\partial \left(1-(1-\omega)\frac{\beta^{-\omega}R_0^{1-\omega}}{p+\beta^{-\omega}R_0^{1-\omega}}\right)^{-1}}{\partial \omega} = -\left(1-(1-\omega)\left(1-\frac{p}{p+\beta^{-\omega}R_0^{1-\omega}}\right)\right)^{-2} \cdot \underbrace{\left[\left(1-\frac{p}{p+\beta^{-\omega}R_0^{1-\omega}}\right) - \frac{(1-\omega)p}{\left(p+\beta^{-\omega}R_0^{1-\omega}\right)^2}\left[-\beta^{-\omega}R_0^{1-\omega}\underbrace{\ln(\beta R_0)}_{>0 \text{ iff } \beta R_0>1} + (1-\omega)\beta^{-\omega}R_0^{-\omega}\underbrace{\frac{\partial R_0}{\partial \omega}}_{<0}\right]\right]}_{>0} < 0.$$

Since for $\omega = 1$ the elasticity $\eta_{0,n} = 1$, and $\frac{\partial \eta_{0,n}}{\partial \omega} < 0$, for $\omega < 1$ the elasticity $\eta_{0,n} > 1$.

Now differentiate $\eta_{0,p}$ wrt ω:

$$\frac{\partial \eta_{0,p}}{\partial \omega} = -\frac{\partial \left[\eta_{0,n}\left(1-\frac{p}{p+\beta^{-\omega}R_0^{1-\omega}}\right)\right]}{\partial \omega} = -\left(1-\frac{p}{p+\beta^{-\omega}R_0^{1-\omega}}\right)\underbrace{\frac{\partial \eta_{0,n}}{\partial \omega}}_{<0} - \eta_{0,n}\frac{p}{\left(p+\beta^{-\omega}R_0^{1-\omega}\right)^2}\underbrace{\left[-\beta^{-\omega}R_0^{1-\omega}\underbrace{\ln(\beta R_0)}_{>0 \text{ iff } \beta R_0>1} + (1-\omega)\beta^{-\omega}R_0^{-\omega}\underbrace{\frac{\partial R_0}{\partial \omega}}_{<0}\right]}_{<0} > 0.$$

Since for $\omega = 1$ the elasticity $\eta_{0,p} = -\frac{1}{1+\beta p}$, and $\frac{\partial \eta_{0,p}}{\partial \omega} > 0$, for $\omega < 1$ the elasticity $\eta_{0,p} < -\frac{1}{1+\beta p}$, i. e. the elasticity in absolute terms is larger.

To summarize, for $x = \{n, p\}$ and $\tau = 0$, we get that:

$$\frac{\partial \left|\eta_{0,x}\right|}{\partial \omega} > 0.$$

∎

Proof of Corollary 2. In general, (11) for $\tau > 0$ is:

$$R_\tau = \frac{n(1+\gamma_A)}{p(1-\tau-\theta)}\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right) + \beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right].$$

Differentiating with respect to n:

$$\begin{aligned}
\frac{\partial R_\tau}{\partial n} =& \frac{R_\tau}{n} + \frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{1-\omega}}{p(1-\theta-\tau)}\frac{\partial \tau}{\partial n} + \frac{n(1+\gamma_A)\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]}{p^2(1-\theta-\tau)^2}p\frac{\partial \tau}{\partial n} \\
&+ (1-\omega)\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}\frac{\partial R_\tau}{\partial n} \\
=& \frac{R_\tau}{n} - \frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{1-\omega}}{p(1-\theta-\tau)}\varepsilon\frac{\tau}{n} - \frac{R_\tau}{1-\theta-\tau}\varepsilon\frac{\tau}{n} \\
&+ (1-\omega)\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}\frac{\partial R_\tau}{\partial n} \\
=& \frac{R_\tau}{n}\left(1 - \frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right]\right) \\
&+ (1-\omega)\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}\frac{\partial R_\tau}{\partial n} \\
\frac{\partial R_\tau}{\partial n} =& \frac{1}{1-(1-\omega)\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}}\frac{R_\tau}{n}\left(1 - \frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right]\right).
\end{aligned}$$

Let $\phi_\tau \equiv \frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)} \in [0,1]$. Using this, the elasticity can be expressed as:

$$\eta_{\tau,n} \equiv \frac{\partial R_\tau/R_\tau}{\partial n/n} = \frac{1}{1-(1-\omega)\phi_\tau}\left(1 - \frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right]\right),$$

which is positive if and only if

$$\frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right] < 1.$$

Now let's do the same analysis for p. First differentiate R_τ wrt p:

$$\begin{aligned}
\frac{\partial R_\tau}{\partial p} =& \frac{n(1+\gamma_A)\left[\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\frac{\partial\tau}{\partial p}+(1-\omega)\beta^{-\omega}R_\tau^{-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\frac{\partial R_\tau}{\partial p}\right]}{p(1-\theta-\tau)} \\
& - \frac{n(1+\gamma_A)\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]}{(p(1-\theta-\tau))^2}\left[(1-\theta-\tau)-p\frac{\partial\tau}{\partial p}\right] \\
=& -\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}\frac{R_\tau}{p}+\left[\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p(1-\theta-\tau)}+\frac{1}{(1-\theta-\tau)}\right]\frac{R_\tau}{p}\varepsilon\tau \\
& + (1-\omega)\frac{\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]}\frac{\partial R_\tau}{\partial p} \\
\frac{\partial R_\tau}{\partial p} =& \frac{1}{1-(1-\omega)\phi_\tau}\frac{R_\tau}{p}\left(-\phi_\tau+\frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right]\right)
\end{aligned}$$

the elasticity can be expressed as:

$$\eta_{\tau,p} \equiv \frac{\partial R_\tau/R_\tau}{\partial p/p} = \frac{1}{1-(1-\omega)\phi_\tau}\left(-\phi_\tau+\frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right]\right),$$

which is negative if and only if

$$\frac{\varepsilon\tau}{1-\theta-\tau}\left[1+\frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p}\right] < \phi_\tau. \tag{B-4}$$

Note that as $\phi_\tau \leq 1$, this implies that as long as $\eta_{\tau,p} < 0$ it necessarily has to hold that $\eta_{\tau,n} > 0$.

We can rewrite (B-4) as:

$$\frac{\varepsilon\tau}{\frac{\alpha}{1-\alpha}+\theta+\tau} < \frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p+n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}.$$

If we can show that there exists a $\tau_{\max}$ such that for any $\tau < \tau_{\max}$:

$$\frac{\tau}{\frac{\alpha}{1-\alpha}+\theta+\tau} < \frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}{p+n(1+\gamma_A)\beta^{-\omega}R_\tau^{-\omega}}, \tag{B-5}$$

then for any $\tau < \tau_{\max}$ it also has to hold that $\eta_{\tau,p} < 0$ and $\eta_{tau,n} > 0$, as we assumed that $\varepsilon \in [0,1]$.

To show the existence of such a $\tau_{\max}$, note that the left hand side of (B-5) is increasing in τ,

at $\tau = 0$ its value is 0.

To evaluate the dependence of the right hand side of (B-5) on τ first take the derivative of R_τ with respect to τ:

$$\begin{aligned}
\frac{\partial R_\tau}{\partial \tau} =& \frac{n(1+\gamma_A)\beta^{-\omega}R_\tau^{1-\omega}}{p(1-\theta-\tau)} + \frac{n(1+\gamma_A)\left[p\left(\frac{\alpha}{1-\alpha}+\theta\right)+\beta^{-\omega}R_\tau^{1-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)\right]}{(p(1-\theta-\tau))^2}p \\
&+ \frac{n(1+\gamma_A)(1-\omega)\beta^{-\omega}R_\tau^{-\omega}\left(\frac{\alpha}{1-\alpha}+\theta+\tau\right)}{p(1-\theta-\tau)}\frac{\partial R_\tau}{\partial \tau} \\
=& \left(\phi_\tau + \frac{1}{1-\theta-\tau}\right)R_\tau + (1-\omega)\phi_\tau\frac{\partial R_\tau}{\partial \tau} \\
\frac{\partial R_\tau}{\partial \tau} =& \frac{1}{1-(1-\omega)\phi_\tau}\left(\phi_\tau + \frac{1}{1-\theta-\tau}\right)R_\tau > 0,
\end{aligned}$$

implying that R_τ is increasing in τ. Since the right hand side of (B-5) is decreasing in R_τ, it is immediate that it is also decreasing in τ. At $\tau = 0$ the value of the right hand side of (B-5) is positive, and hence larger than the value on the left hand side. This together with the facts that the left hand side is monotone increasing, while the right hand side is monotone decreasing in τ, implies that there exists a $\widetilde{\tau} > 0$ that equalizes the left and the right hand side of (B-5). Now it is easy to see that for any $\tau < \tau_{\max}$, where $\tau_{\max}$ is defined as $\tau_{\max} \equiv \min\{1-\theta, \widetilde{\tau}, \tau_y\}$, the inequality (B-5) holds. ∎

Proof of Proposition 2. INSERT PROOF OF EXISTENCE OF STEADY STATE.

The wealth (asset) holdings in country i in period t are given by:

$$\begin{aligned}
W_t^i &= L_{y,t}^i a_{y,t}^i + L_{m,t}^i a_{m,t}^i = L_{m,t}^i\left(n_t^i a_{y,t}^i + a_{m,t}^i\right) \\
&= L_{m,t}^i\left[(1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}A_t^i k_t^\alpha - \frac{1-\alpha}{\alpha}\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right)n_t^i A_{t+1}^i k_{t+1}\right] \\
&= A_t^i L_{m,t}^i\left[(1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}k_t^\alpha - \frac{1-\alpha}{\alpha}\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right)n_t^i(1+\gamma_{A,t+1}^i)k_{t+1}\right],
\end{aligned}$$

where we used the optimal saving decisions of households (6) and (9), as well as the market clearing wage and interest rates (1).

The net foreign assets of country i is the difference between their wealth at the end of

period t (assets) and their capital stock used in period $t+1$:

$$\begin{aligned}
\frac{NFA_t^i}{Y_t^i} &= \frac{W_t^i - K_{t+1}^i}{\left(A_t^i L_{m,t}^i\right)^{1-\alpha}\left(K_t^i\right)^{\alpha}} = \frac{W_t^i - A_{t+1}^i L_{m,t+1}^i k_{t+1}}{A_t^i L_{m,t}^i k_t^{\alpha}} \\
&= \frac{A_t^i L_{m,t}^i \left[(1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}k_t^{\alpha} - \frac{1-\alpha}{\alpha}\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right) n_t^i(1+\gamma_{A,t+1}^i)k_{t+1}\right]}{A_t^i L_{m,t}^i k_t^{\alpha}} \\
&- \frac{A_t^i L_{m,t}^i n_t^i(1+\gamma_{A,t+1}^i)k_{t+1}}{A_t^i L_{m,t}^i k_t^{\alpha}} \\
&= \frac{(1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}k_t^{\alpha} - \left[\frac{1-\alpha}{\alpha}\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right) + 1\right] n_t^i(1+\gamma_{A,t+1}^i)k_{t+1}}{k_t^{\alpha}} \\
&= (1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}} - \left[\frac{1-\alpha}{\alpha}\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right) + 1\right] n_t^i(1+\gamma_{A,t+1}^i)\frac{k_{t+1}}{k_t^{\alpha}}
\end{aligned}$$

The world capital accumulation equation, (15), implies:

$$\frac{k_{t+1}}{k_t^{\alpha}} = \frac{\sum_i A_t^i L_{m,t}^i \frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}(1-\alpha)}{\sum_i n_t^i(1+\gamma_{A,t+1}^i)A_t^i L_{m,t}^i\left[1+\frac{1-\alpha}{\alpha}\left(\theta_t^i + \frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i\right)\right]} \equiv \Psi_{t+1}.$$

Using this we can substitute for $k_{t+1}/(k_t^{\alpha})$ in the equation for NFA_t^i/Y_t^i to get the following expression:

$$\frac{NFA_t^i}{Y^i} = (1-\alpha)\frac{p_t^i(1-\tau_t^i-\theta_{t-1}^i)}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}} - \left[(1-\alpha)\left(\frac{\beta^{-\omega}R_{t+1}^{1-\omega}}{p_t^i+\beta^{-\omega}R_{t+1}^{1-\omega}}\tau_{t+1}^i + \theta_t^i\right) + \alpha\right] n_t^i(1+\gamma_{A,t+1}^i)\frac{\Psi_{t+1}}{\alpha}$$

Using that in the steady state all parameters are constants, and that $k_t = k_{t+1}$, and hence $\Psi_{t+1}/\alpha = R^{-1}$, we get the expression in the Proposition:

$$\frac{NFA_t^i}{Y_t^i} = (1-\alpha)\frac{p^i(1-\tau^i-\theta^i)}{p^i+\beta^{-\omega}R^{1-\omega}} - \left[(1-\alpha)\left(\frac{\beta^{-\omega}R^{1-\omega}}{p^i+\beta^{-\omega}R^{1-\omega}}\tau^i + \theta^i\right) + \alpha\right]\frac{n^i(1+\gamma_A^i)}{R}. \quad \text{(B-6)}$$

∎

Proof of Corollary 3. In a world with integrated capital markets, where countries share common, constant values for p, γ_A, n, and only differ in their values for θ^i, τ^i, the steady state interest rate, $\overline{R}$, is such that the global asset supply equals the global asset demand. This is equivalent to saying that the net foreign asset position of a hypothetical country characterized by the values $(\overline{\theta}, \overline{\tau})$ in the steady state is zero.

The net foreign asset position of a country with parameters θ^i and τ^i using (B-6) can be expressed as:

$$\frac{NFA_t^i}{Y_t^i} = (1-\alpha)\frac{p(1-\tau^i-\theta^i)}{p+\beta^{-\omega}\overline{R}^{1-\omega}} - \left[(1-\alpha)\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\tau^i+\theta^i\right)+\alpha\right]\frac{n(1+\gamma_A)}{\overline{R}}.$$

From the above we can subtract the same expression for the hypothetical average country, for which the value of the above is zero to get the expression given in the corollary:

$$\begin{aligned}
\frac{NFA_t^i}{Y_t^i} &= \frac{NFA_t^i}{Y_t^i} - \frac{\overline{NFA}_t}{\overline{Y}_t} \\
&= (1-\alpha)\frac{p(1-\tau^i-\theta^i)}{p+\beta^{-\omega}\overline{R}^{1-\omega}} - \left[(1-\alpha)\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\tau^i+\theta^i\right)+\alpha\right]\frac{n(1+\gamma_A)}{\overline{R}} \\
&- (1-\alpha)\frac{p(1-\overline{\tau}-\overline{\theta})}{p+\beta^{-\omega}\overline{R}^{1-\omega}} - \left[(1-\alpha)\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\tau+\theta\right)+\alpha\right]\frac{n(1+\gamma_A)}{\overline{R}} \\
&= (1-\alpha)\frac{p(\overline{\tau}-\tau^i)+p(\overline{\theta}-\theta^i)}{p+\beta^{-\omega}\overline{R}^{1-\omega}} + (1-\alpha)\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}(\overline{\tau}-\tau^i)+(\overline{\theta}-\theta^i)\right)\frac{n(1+\gamma_A)}{\overline{R}} \\
&= (1-\alpha)(\overline{\theta}-\theta^i)\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right] + \frac{p(1-\alpha)(\overline{\tau}-\tau^i)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right].
\end{aligned}$$

∎

Proof of Proposition 3. The difference in net foreign assets as stated in (20) is:

$$\Delta(\theta)\frac{NFA_t}{Y_t} \equiv \frac{NFA_t^L}{Y_t^L} - \frac{NFA_t^H}{Y_t^H} = (1-\alpha)(\theta^H-\theta^L)\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right].$$

Taking its derivative with respect to n we get:

$$\begin{aligned}
\frac{\partial \Delta(\theta)\frac{NFA_t}{Y_t}}{\partial n} &= (1-\alpha)(\theta^H-\theta^L)\left(\underbrace{\frac{\partial \overline{R}}{\partial n}\frac{\partial}{\partial \overline{R}}\left(\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right)}_{\text{interest rate channel}}+\underbrace{\frac{(1+\gamma_A)}{\overline{R}}}_{\text{direct effect}}\right)\\
&= (1-\alpha)(\theta^H-\theta^L)\left(-\overline{\eta}_n\left(\frac{p\,(1-\omega)\,\beta^{-\omega}\overline{R}^{1-\omega}}{n\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)^2}+\frac{(1+\gamma_A)}{\overline{R}}\right)+\frac{(1+\gamma_A)}{\overline{R}}\right)\\
&= (1-\alpha)(\theta^H-\theta^L)\left(-\overline{\eta}_n\frac{p\,(1-\omega)\,\beta^{-\omega}\overline{R}^{1-\omega}}{n\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)^2}+\frac{(1+\gamma_A)}{\overline{R}}\left(1-\overline{\eta}_n\right)\right).
\end{aligned}$$

From applying Corollary 2 to the world interest rate, $\overline{R}$, we know that $\overline{\eta}_n > 0$, hence the first term is negative. As long as $\overline{\eta}_n > 1$ also holds, the second term is also negative, which is exactly the condition stated in the proposition.

Similarly take the derivative of the above with respect to p to get:

$$\begin{aligned}
\frac{\partial \Delta(\theta)\frac{NFA_t}{Y_t}}{\partial p} &= (1-\alpha)(\theta^H-\theta^L)\left(\underbrace{\frac{\partial \overline{R}}{\partial p}\frac{\partial}{\partial \overline{R}}\left(\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}+\frac{n(1+\gamma_A)}{\overline{R}}\right)}_{\text{interest rate channel}}+\underbrace{\frac{\partial}{\partial p}\left(\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\right)}_{\text{direct effect}}\right)\\
&= (1-\alpha)(\theta^H-\theta^L)\left(-\overline{\eta}_p\left(\frac{(1-\omega)\beta^{-\omega}\overline{R}^{1-\omega}}{\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)^2}+\frac{n(1+\gamma_A)}{p\overline{R}}\right)+\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)^2}\right)\\
&= (1-\alpha)(\theta^H-\theta^L)\left(\left(1+(1-\omega)|\overline{\eta}_p|\right)\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)^2}+|\overline{\eta}_p|\frac{n(1+\gamma_A)}{p\overline{R}}\right)>0,
\end{aligned}$$

where we used that $-\overline{\eta}_p = |\overline{\eta}_p|$, as we know that $\overline{\eta}_p < 0$ from applying Corollary 2 to the world interest rate, $\overline{R}$. As $\omega \leq 1$, both the first and the second term in the above sum are positive.

To summarize, the difference in the net foreign asset position of a high-θ and a low-θ country increase as n falls if $\overline{\eta}_n > 1$, and as p increases. ∎

Proof of Proposition 4. The difference in net foreign assets as stated in (21) is:

$$\Delta(\tau)\frac{NFA_t}{Y_t} \equiv \frac{NFA_t^L}{Y_t^L} - \frac{NFA_t^H}{Y_t^H} = \frac{p(1-\alpha)(\tau^H-\tau^L)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right].$$

Taking its derivative with respect to n we get:

$$\begin{aligned}
\frac{\partial\Delta(\tau)\frac{NFA_t}{Y_t}}{\partial n} &= p(1-\alpha)(\tau^H-\tau^L)\left(\underbrace{\frac{1+\gamma_A}{\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)p\beta^{\omega}\overline{R}^{\omega}}}_{\text{direct effect}} + \underbrace{\frac{\partial\overline{R}}{\partial n}\frac{\partial}{\partial\overline{R}}\left(\frac{\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right]}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\right)}_{\text{interest rate channel}}\right)\\
&\quad + \underbrace{\frac{\partial(\tau^H-\tau^L)}{\partial n}\frac{p(1-\alpha)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right]}_{\text{contribution rate channel}}\\
&= \frac{p(1-\alpha)(\tau^H-\tau^L)}{n\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)}\left[\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right.\\
&\quad \left.-\overline{\eta}_n\left(\frac{(1-\omega)\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)+\omega\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)-\varepsilon\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)\right]\\
&= \frac{p(1-\alpha)(\tau^H-\tau^L)}{n\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)}\left[\underbrace{\Theta}_{\text{direct effect}}-\underbrace{\overline{\eta}_n\left((1-\omega)\widetilde{\phi}_0\left(1+\Theta\right)+\omega\Theta\right)}_{\text{interest rate ch.}}-\underbrace{\varepsilon\left(1+\Theta\right)}_{\text{contribution rate ch.}}\right]\\
&= \frac{p(1-\alpha)(\tau^H-\tau^L)}{n\left(p+\beta^{-\omega}\overline{R}^{1-\omega}\right)}\cdot\\
&\quad\left[-\varepsilon+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}(1-\varepsilon-\overline{\eta}_n\omega)-\overline{\eta}_n\frac{(1-\omega)\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right]\right]
\end{aligned}$$

Since $\overline{\eta}_n > 0$, a sufficient condition for the above to be negative is $1-\varepsilon-\overline{\eta}_n\omega < 0$, which is exactly the statement in the proposition. If this holds, then as n falls, the net foreign asset positions of a high- and a low-τ country diverge.

The same derivation for p:

$$
\begin{aligned}
\frac{\partial \Delta(\tau)\frac{NFA_t}{Y_t}}{\partial p} &= \underbrace{(1-\alpha)(\tau^H-\tau^L)\frac{\partial}{\partial p}\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)\right]}_{\text{direct effect}} \\
&+ \underbrace{(1-\alpha)(\tau^H-\tau^L)\frac{\partial \overline{R}}{\partial p}\frac{\partial}{\partial \overline{R}}\left[\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)\right]}_{\text{interest rate channel}} \\
&+ \underbrace{(1-\alpha)\left[1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right]\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\frac{\partial(\tau^H-\tau^L)}{\partial p}}_{\text{contribution rate channel}} \\
&= \frac{(1-\alpha)(\tau^H-\tau^L)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left(1-\frac{n(1+\gamma_A)}{\overline{R}}\right)\right. \\
&\left. - \overline{\eta}_p\left(\frac{(1-\omega)\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)+\omega\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)+\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)\varepsilon\right] \\
&= \frac{(1-\alpha)(\tau^H-\tau^L)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[\underbrace{\widetilde{\phi}_0\left[1-\frac{n(1+\gamma_A)}{\overline{R}}\right]}_{\text{direct effect}}+\underbrace{|\overline{\eta}_p|\left((1-\omega)\widetilde{\phi}_0\left(1+\Theta\right)+\omega\Theta\right)}_{\text{interest rate ch.}}+\underbrace{\varepsilon\left(1+\Theta\right)}_{\text{contribution r ch.}}\right] \\
&= \frac{(1-\alpha)(\tau^H-\tau^L)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\cdot\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[(1-\omega)|\overline{\eta_p}|\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)+\left(1-\frac{n(1+\gamma_A)}{\overline{R}}\right)\right]\right. \\
&\left. + \omega\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}|\overline{\eta_p}|+\left(1+\frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\right)\varepsilon\right) \\
&= \frac{(1-\alpha)(\tau^H-\tau^L)}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\cdot\left(\frac{\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\left[(1-\omega)|\overline{\eta_p}|+1\right]+\varepsilon\right. \\
&\left. + \frac{n(1+\gamma_A)}{p\beta^{\omega}\overline{R}^{\omega}}\left(\frac{\omega p+\beta^{-\omega}\overline{R}^{1-\omega}}{p+\beta^{-\omega}\overline{R}^{1-\omega}}|\overline{\eta_p}|+\varepsilon-\frac{p}{p+\beta^{-\omega}\overline{R}^{1-\omega}}\right)\right)
\end{aligned}
$$

As long as the last term in brackets is positive, as longevity increases the net foreign asset positions between a high- and a low-τ country diverge. A sufficient condition for the last term to be positive is that $|\overline{\eta}_p| \geq \frac{1-\varepsilon}{\omega}$. ∎

C Technical Appendix for Quantitative Section

This appendix provides further details on the solution method for the quantitative model of Section 3.

C.1 Individual Optimization

The agent's problem consists in choosing a path of consumption and asset holdings to maximize expected lifetime utility (26) subject to flow budget constraints (34)-(35) and credit constraints (39).

C.2 Autarky Steady State

Consider a steady state in country i where $e^i_{j,t} = e^i_j$, conditional survival probabilities remain constant over time ($p^i_{j,t} = p^i_j$) and $L^i_{0,t+1} = (1+\gamma^i_L)L^i_{0,t}$, so that the total population grows at constant rate γ^i_L, productivity grows at constant rate γ^i_A, and $\sigma^i_t = \sigma^i$ in every period. The steady-state contribution rate τ^i must satisfy

$$\tau^i \sum_{j=0}^{\underline{J}} L^i_{j,t} e^i_j w^i_t = \sigma^i e^i_{\underline{J}} \sum_{j=\underline{J}+1}^{\bar{J}} L^i_{j,t} w^i_{t+\underline{J}-j},$$

which implies

$$\begin{aligned} \tau^i &= \sigma^i e^i_{\underline{J}} \sum_{j=\underline{J}+1}^{\bar{J}} \left(\frac{L^i_{j,t}}{\sum_{\ell=0}^{\underline{J}} e^i_\ell L^i_{\ell,t}} \right) \frac{w^i_{t+\underline{J}-j}}{w^i_t} \\ &= \sigma^i e^i_{\underline{J}} \sum_{j=\underline{J}+1}^{\bar{J}} \left(\frac{(1+\gamma^i_L)^{-j} \prod_{m=0}^{j-1} p^i_m}{\sum_{\ell=0}^{\underline{J}} e^i_\ell (1+\gamma^i_L)^{-\ell} \prod_{m=0}^{\ell-1} p^i_m} \right) (1+\gamma^i_A)^{\underline{J}-j}. \end{aligned} \tag{C-1}$$

Let k^i denote the autarky steady state level of the capital-efficient-labor ratio in country i,

$$\frac{K^i_t}{A^i_t \hat{L}^i_t} = k^i.$$

The market clearing condition at the end of period t is

$$K^i_{t+1} = A^i_{t+1}\hat{L}^i_{t+1}k^i = \sum_{j=0}^{\bar{J}-1} L^i_{j,t}a^i_{j,t}. \tag{C-2}$$

In order to determine the steady state equilibrium, we need to characterize net asset holdings $a^i_{j,t}$ for a given value of k^i. To do so, we observe that in steady state, $a^i_{j,t}$ grows over time at rate γ^i_A, while Q^i_t grows over time at rate $\gamma^i_Y = (1+\gamma^i_A)(1+\gamma^i_L)-1$. Thus, there exists $\{\tilde{a}^i_j\}_{j=0}^{\bar{J}-1}$ and $\tilde{Q}^i$ such that

$$a^i_{j,t} = \tilde{a}^i_j w^i_{t-j}, \qquad \text{for all } j,t \tag{C-3}$$

$$Q^i_t = \tilde{Q}^i L_{0,t} w^i_t, \qquad \text{for all } t. \tag{C-4}$$

As a preliminary step, we analyze the joint determination of the values of $\{\tilde{a}^i_j\}_{j=0}^{\bar{J}-1}$ and $\tilde{Q}^i$ for a candidate steady state interest rate R^i.

To start with, we show how to solve for $\{\tilde{a}^i_j\}$ taking $\tilde{Q}^i$ as given. (33) implies that in steady state, the level of bequests q^i_t received at the end of period t must satisfy

$$q^i_t \sum_{j\in\mathcal{B}} p^i_j L^i_{j,t} = Q^i_t.$$

Therefore using (C-4), we can write for all $j \in \mathcal{B}$

$$\begin{aligned} q^i_{j,t} = q^i_t &= \tilde{Q}^i \frac{L_{0,t}}{\sum_{j\in\mathcal{B}} p^i_j L^i_{j,t}} w^i_t \\ &= \tilde{Q}^i \left(\sum_{m\in\mathcal{B}} \frac{p^i_m \prod_{\ell=0}^{m-1} p^i_\ell}{(1+\gamma^i_L)^m} \right)^{-1} w^i_t \\ &= \tilde{Q}^i \left(\sum_{m\in\mathcal{B}} \frac{p^i_m \prod_{\ell=0}^{m-1} p^i_\ell}{(1+\gamma^i_L)^m} \right)^{-1} (1+\gamma^i_A)^j w^i_{t-j} \quad =: \tilde{q}^i_j w^i_{t-j}. \end{aligned} \tag{C-5}$$

Moreover the labor income received in period t by a worker of age j is

$$w^i_{j,t} = e^i_j w^i_t = e^i_j (1+\gamma^i_A)^j w^i_{t-j} =: \tilde{w}^i_j w^i_{t-j},$$

and the amount of pensions received in period t by a retiree of age j is

$$\pi^i_{j,t} = \sigma^i w^i_{\underline{J},t+\underline{J}-j} = \sigma^i e^i_{\underline{J}} w^i_{t+\underline{J}-j} = \sigma^i e^i_{\underline{J}} (1+\gamma^i_A)^{\underline{J}} w^i_{t-j} =: \tilde{\pi}^i_j w^i_{t-j}, \quad \underline{J} < j \leq \bar{J}.$$

Now, consider the individual optimization problem of an agent born in period $t-j$. This agent faces a sequence of labor income $w^i_{j,t} = \tilde{w}^i_j w^i_{t-j}$, constant tax rate τ^i, pension transfers $\pi^i_{j,t} = \tilde{\pi}^i_j w^i_{t-j}$ and wealth transfers $q^i_{j,t} = \tilde{q}^i_j w^i_{t-j}$. Since in our environment the solution to the individual optimization problem is homogenous of degree one in the level of wages and transfers, the optimal path of asset holdings $\{a^i_{j,t}\}$ for this agent satisfies $a^i_{j,t} = \tilde{a}^i_j w^i_{t-j}$, where $\{\tilde{a}^i_j\}$ denotes the optimal path of asset holdings in the *normalized* optimization problem of an individual who faces a sequence of wages $\{\tilde{w}^i_j\}$, tax rate τ^i, and transfers $\{\tilde{\pi}^i_j\}$ and $\{\tilde{q}^i_j\}$. Note that since $\tilde{q}^i_j = \tilde{q}^i_j(\tilde{Q}^i)$ as per (C-5), the normalized amount of unintentional bequests $\tilde{Q}^i$ affects wealth accumulation, that is $\{\tilde{a}^i_j = \tilde{a}^i_j(\tilde{Q}^i)\}$.

Conversely individual wealth accumulation, encapsulated in $\{\tilde{a}^i_j\}$, affects the normalized amount of unintentional bequests $\tilde{Q}^i$. Indeed, using the definition of Q^i_t in (32), we have in steady state

$$\begin{aligned}
Q^i_t &= \sum_{j=0}^{\bar{J}-1} (1-p^i_j) L^i_{j,t} a^i_{j,t} \\
\Rightarrow \quad \tilde{Q}^i L^i_{0,t} w^i_t &= \sum_{j=0}^{\bar{J}-1} (1-p^i_j) L^i_{j,t} \tilde{a}^i_j w^i_{t-j} \\
\Rightarrow \quad \tilde{Q}^i &= \sum_{j=0}^{\bar{J}-1} (1-p^i_j) \frac{L^i_{j,t}}{L^i_{0,t}} \frac{w^i_{t-j}}{w^i_t} \tilde{a}^i_j \\
\Rightarrow \quad \tilde{Q}^i &= \sum_{j=0}^{\bar{J}-1} \frac{(1-p^i_j)\prod_{\ell=0}^{j-1} p^i_\ell}{[(1+\gamma^i_L)(1+\gamma^i_A)]^j} \tilde{a}^i_j. \qquad \text{(C-6)}
\end{aligned}$$

The value of $\tilde{Q}^i$ in the autarky steady state is obtained by solving (C-6) with $\tilde{a}^i_j = \tilde{a}^i_j(\tilde{Q}^i)$, which immediately pins down normalized asset holdings $\{\tilde{a}^i_j\}$.

Finally, using the fact that $a^i_{j,t} = \tilde{a}^i_j w^i_{t+1}/(1+\gamma^i_A)^{j+1}$ and $L^i_{j,t} = \left(\prod_{\ell=0}^{j-1} p^i_\ell\right) L^i_{0,t+1}/(1+$

$\gamma_L^i)^{j+1}$, the market clearing condition (C-2) can be rewritten as

$$\left\{\sum_{j=0}^{\bar{J}} \frac{e_j^i \prod_{\ell=0}^{j-1} p_\ell^i}{(1+\gamma_L^i)^j}\right\} (k^i)^{1-\alpha} = (1-\alpha) \sum_{j=0}^{\bar{J}-1} \frac{\prod_{\ell=0}^{j-1} p_\ell^i}{[(1+\gamma_L^i)(1+\gamma_A^i)]^{j+1}} \tilde{a}_j^i(k^i), \tag{C-7}$$

where the notation $\tilde{a}_j^i(k^i)$ captures the fact that (normalized) net asset positions depend on the steady-state rate of return, as previously described. Equation (C-7) implicitly defines the level of the capital-effective-labor ratio in the autarky steady state.

C.3 Integrated Steady State

Consider an integrated steady state where $(1+\gamma_A^i)(1+\gamma_L^i) = 1+\gamma_Y$ in all countries, although countries may be heterogenous in other respects. Social security systems are balanced in all countries, so that τ^i and σ^i satisfy (C-1) for all i. Capital-effective-labor ratios are equalized across countries,

$$\frac{K_t^i}{A_t^i \hat{L}_t^i} = k.$$

The integrated steady state can be determined along the same logic as for the autarky steady state. First, taking the steady state value of the gross rate of return R as given, we consider the (normalized) optimization problem of an individual in country i with labor income sequence $\{\tilde{w}_j^i\}_{j=0}^{\underline{J}}$ with $\tilde{w}_j^i = e_j^i(1+\gamma_A^i)^j$ and constant tax rate τ^i, pension transfers $\tilde{\pi}_j^i = \sigma^i e_{\underline{J}}^i (1+\gamma_A^i)^{\underline{J}-1}$ for $\underline{J} < j \leq \bar{J}$, and wealth transfers $\{\tilde{q}_j^i\}_{j\in\mathcal{B}}$ satisfying (C-5) for some $\tilde{Q}^i$. Let $\{\tilde{a}_j^i(\tilde{Q}^i)\}_{j=0}^{\bar{J}}$ denote the optimal path of wealth. For each country i, define $\tilde{Q}^i(R)$ the unique value of $\tilde{Q}$ solution to (C-6), i.e.,

$$\tilde{Q} = \sum_{j=0}^{\bar{J}-1} \frac{(1-p_j^i)\prod_{\ell=0}^{j-1} p_\ell^i}{(1+\gamma_Y)^j} \tilde{a}_j^i(\tilde{Q}).$$

Stationarity and homogeneity imply that, if the world steady-state interest rate is R, the wealth at age j of an agent born in period t in country i is $a_{j,t+j}^i = \tilde{a}_j^i w_t^i$, where $\tilde{a}_j^i = \tilde{a}_j^i[\tilde{Q}^i(R)]$.

The market clearing condition at the end of period t is

$$\sum_i K^i_{t+1} \equiv k \sum_i A^i_{t+1} \hat{L}^i_{t+1} = \sum_i \sum_{j=0}^{\bar{J}-1} L^i_{j,t} a^i_{j,t}, \tag{C-8}$$

which is equivalent to

$$\left\{ \sum_i A^i_{t+1} L^i_{0,t+1} \sum_{j=0}^{J} \frac{e^i_j \prod_{\ell=0}^{j-1} p^i_\ell}{(1+\gamma^i_L)^j} \right\} k^{1-\alpha} = (1-\alpha) \sum_i A^i_{t+1} L^i_{0,t+1} \sum_{j=0}^{\bar{J}-1} \frac{\prod_{\ell=0}^{j-1} p^i_\ell}{(1+\gamma_Y)^{j+1}} \tilde{a}^i_j(k).$$

Let $\eta^i \equiv \sum_{j=0}^{J} \frac{e^i_j \left(\prod_{\ell=0}^{j-1} p^i_\ell\right)}{(1+\gamma^i_L)^j}$, and introduce country weights

$$\lambda^i \equiv \frac{\eta^i A^i_{t+1} L^i_{0,t+1}}{\sum_n \eta^n A^n_{t+1} L^n_{0,t+1}}.$$

The market clearing condition can be rewritten

$$\begin{aligned} k^{1-\alpha} &= (1-\alpha) \sum_i \frac{\lambda^i}{\eta_i} \sum_{j=0}^{\bar{J}-1} \frac{\prod_{\ell=0}^{j-1} p^i_\ell}{(1+\gamma_Y)^{j+1}} \tilde{a}^i_j(k) \\ &= (1-\alpha) \sum_{j=0}^{\bar{J}-1} \frac{1}{(1+\gamma_Y)^{j+1}} \sum_i \frac{\lambda^i}{\eta_i} \left(\prod_{\ell=0}^{j-1} p^i_\ell \right) \tilde{a}^i_j(k). \end{aligned}$$

Special case: when $\eta^i = \eta$ in every country, the market clearing condition simplifies to

$$k^{1-\alpha} = \frac{1-\alpha}{\eta} \sum_{j=0}^{\bar{J}-1} \frac{1}{(1+\gamma_Y)^{j+1}} \left[\sum_i \lambda^i \left(\prod_{\ell=0}^{j-1} p^i_\ell \right) \tilde{a}^i_j(k) \right],$$

where λ^i corresponds to the constant share of country i in world effective labor.

C.4 Dynamics

The law of motion for $\mathbf{k}_t \equiv (k^i_t)_{i=1}^N$ depends on whether countries are financially integrated or in financial autarky. If countries are closed financially in period t, the market clearing

condition in country i is

$$A^i_{t+1}\hat{L}^i_{t+1}k^i_{t+1} = \sum_{j=0}^{\bar{J}-1} L^i_{j,t}a^i_{j,t}. \tag{C-9}$$

The generations who matter in period t are those born in periods $t-\bar{J}+1$ to t. Thus market clearing in period t pins down k^i_{t+1} given

- lagged values $\mathbf{K}^i_{L,t+1} \equiv \{k^i_\tau\}^t_{\tau=t-\bar{J}+1}$ and future values $\mathbf{K}^i_{F,t+1} \equiv \{k^i_\tau\}^{t+\bar{J}}_{\tau=t+2}$,
- past, current, and future productivity $\{A^i_\tau\}^{t+\underline{J}}_{\tau=t-\bar{J}+1}$,
- past, current, and future age-income profile, i.e., $\{e^i_{j,\tau+j}\}^{\underline{J}}_{j=0}$ for $\tau = t-\bar{J}+1, ..., t$,
- tax rates τ^i_τ for $\tau = t-\bar{J}+1, ..., t+\underline{J}$,
- replacement rates $\mathbf{\Sigma}^i_{t+1} \equiv \{\sigma^i_\tau\}^{t+\bar{J}}_{\tau=t-\bar{J}+\underline{J}+2}$,
- demographic composition in period t, driven by $\{L^i_{0,\tau}\}^t_{\tau=t-\bar{J}+1}$ and $\{p^i_{j,\tau+j}\}^{t-1-\tau}_{j=0}$ for $\tau = t-\bar{J}+1, ..., t-1$,
- mortality tables in period t, i.e., $\{p^i_{j,\tau+j}\}^{\bar{J}-1}_{j=t-\tau}$ for $\tau = t-\bar{J}+1, ..., t$,
- past, current,[22] and future amounts of unintentional bequests $\mathcal{Q}^i_{t+1} \equiv \{Q^i_\tau\}^{t+\bar{J}-1}_{\tau=t-\bar{J}+1}$.

If instead countries are financially integrated in period t, then rates of return are equalized across countries

$$R^i_{t+1} = R_{t+1}, \quad \text{for all } i,$$

and so are their capital-effective-labor ratios

$$k^i_{t+1} \equiv K^i_{t+1}/(A^i_{t+1}\hat{L}^i_{t+1}) = k_{t+1}, \quad \text{for all } i.$$

The market clearing condition in period t is

$$\sum_i K^i_{t+1} = k_{t+1}\sum_i A^i_{t+1}\hat{L}^i_{t+1} = \sum_i \sum_{j=0}^{\bar{J}-1} L^i_{j,t}a^i_{j,t}. \tag{C-10}$$

[22] Alternatively, the amount of unintentional bequests left in period t, Q^i_t, could be determined along with k^i_{t+1}, given $\{Q^i_\tau\}_{\tau\in\{t-\bar{J}+1,...,t-1\}\bigcup\{t+1,...,t+\bar{J}-1\}}$, by imposing that the pair (k^i_{t+1}, Q^i_t) jointly solves (32) and (C-9).

Thus market clearing in period t pins down k_{t+1} given

- lagged and future values, $\mathbf{K}_{L,t+1} \equiv (\mathbf{K}^i_{L,t+1})_{i=1}^N$ and $\mathbf{K}_{F,t+1} \equiv (\mathbf{K}^i_{F,t+1})_{i=1}^N$,
- productivity $\{A^i_\tau\}_{\tau=t-\bar{J}+1}^{t+\underline{J}}$ for $i = 1, ..., N$,
- age-income profile, i.e., $\{e^i_{j,\tau+j}\}_{j=0}^{\underline{J}}$ for $\tau = t - \bar{J} + 1, ..., t$ and $i = 1, ..., N$,
- tax rates τ^i_τ for $\tau = t - \bar{J} + 1, ..., t + \underline{J}$ and $i = 1, ..., N$,
- replacement rates $\mathbf{\Sigma}_{t+1} \equiv (\mathbf{\Sigma}^i_{t+1})_{i=1}^N$,
- demographic composition in period t, driven by $\{L^i_{0,\tau}\}_{\tau=t-\bar{J}+1}^t$ and $\{p^i_{j,\tau+j}\}_{j=0}^{t-1-\tau}$ for $\tau = t - \bar{J} + 1, ..., t - 1$, and $i = 1, ..., N$,
- mortality tables in period t, i.e., $\{p^i_{j,\tau+j}\}_{j=t-\tau}^{\bar{J}-1}$ for $\tau = t - \bar{J} + 1, ..., t$, and $i = 1, ..., N$,
- total amounts of unintentional bequests $\mathcal{Q}_{t+1} \equiv (\mathcal{Q}^i_{t+1})_{i=1}^N$.

C.5 Shooting Algorithm

Consider an experiment where countries start in financial autarky and integrate in period X (say, $X = 0$). Hence for $t \geq X + 1$, $k^i_t = k_t$, for all i. Around the integration period, we feed the model with "shocks" to productivity, demography, social security, and age-income profiles. We allow for shocks over the window $[X - \mathfrak{T}, X + \mathfrak{T}]$. All shocks are perfectly anticipated. The sequence of contribution rates $\{\tau^i_t\}$ is taken as given. Adjustments to the replacement rates $\{\sigma^i_t\}$ are obtained as part of the equilibrium construction.

In order to determine how the global economy responds to financial integration and other contemporaneous shocks, we use the following algorithm.

1. We assume each country starts at its autarkic steady state, and that the economy does not react to future shocks before period $X - T$, for $T > \mathfrak{T}$ large. That is, $k^i_t = k^{i*}$ for $t \leq X - T - 1$. The initial steady state for country i is determined by the country-specific parameters θ^i, $\{p^i_j\}_{j=0}^{\bar{J}-1}$, along with (γ^i_L, γ^i_A), $\{e^i_j\}_{j=0}^{\underline{J}}$, and (τ_i, σ_i) as described in Section C.2. From the initial steady state, we obtain unintentional bequests Q^i_t for

$t \leq X - T - 1$. In particular,

$$Q^i_{X-T-1} = (1-\alpha) A^i_{X-T-1} (k^{i*})^\alpha L^i_{0,X-T-1} \tilde{Q}^i[k^{i*}].$$

2. We assume the global economy has converged to its integrated steady state k^* in period $X+T+1$. That is, $k_t = k^*$ for $t \geq X+T+1$. The final steady state is determined by country-specific parameters, along with final relative weights $\{\lambda^i\}_{i=1}^N$, as described in Section C.3. Unintentional bequests in period $X+T+1$ are

$$Q^i_{X+T+1} = (1-\alpha) A^i_{X+T+1} (k^*)^\alpha L^i_{0,X+T+1} \tilde{Q}^i[k^*].$$

3. The transition path of the economy is obtained iteratively as follows.

   (a) We start with a guess for the paths of capital-effective-labor ratios $\{\mathbf{k}_t^{(0)}\}_{t=X-T}^{X+T}$, replacement rates $\{\boldsymbol{\sigma}_t^{(0)}\}_{t=X-T}^{X+T}$, and unintentional bequests $\{\mathbf{Q}_t^{(0)}\}_{t=X-T}^{X+T}$. We set $k_t^{i(0)} = k^{i*}$ for $t \leq X$ and $k_t^{i(0)} = k^*$ for all i for $t \geq X+1$. We set initial values $\{Q_t^{i(0)}\}$ by interpolating between Q^i_{X-T-1} and Q^i_{X+T+1}. Given $\{k_t^{i(0)}\}$, the initial path $\{\sigma_t^{i(0)}\}$ follows from the balanced budget condition, Eq. (31).

   (b) For $n \geq 0$, given the paths $\{\mathbf{k}_t^{(n)}\}_{t=X-T}^{X+T}$, $\{\boldsymbol{\sigma}_t^{(n)}\}_{t=X-T}^{X+T}$, and $\{\mathbf{Q}_t^{(n)}\}_{t=X-T}^{X+T}$, the updated paths $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X+T}$, $\{\boldsymbol{\sigma}_t^{(n+1)}\}_{t=X-T}^{X+T}$, and $\{\mathbf{Q}_t^{(n+1)}\}_{t=X-T}^{X+T}$ are obtained as follows.

      i. First, we solve for the updated path $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X+T}$, given replacement rates $\{\boldsymbol{\sigma}_t^{(n)}\}_{t=X-T}^{X+T}$ and transfers $\{\mathbf{Q}_t^{(n)}\}_{t=X-T}^{X+T}$.

         – We obtain $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X}$ by iterating on the autarkic forward-backward difference equation (FBDE) in each country (see Section C.4). Specifically, for each country i, and for $t = X-T, \ldots, X$, we compute $k_t^{i(n+1)}$ as the solution to the autarkic FBDE, given $\mathbf{K}_{L,t}^{i(n)}$, $\mathbf{K}_{F,t}^{i(n)}$, $\boldsymbol{\Sigma}_t^{i(n)}$, and $\mathcal{Q}_t^{i(n)}$.

         – We determine the common path $\{k_t^{(n+1)}\}_{t=X+1}^{X+T}$ by iterating on the integrated FBDE. Specifically, for $t = X+1, \ldots, X+T$, we compute $k_t^{(n+1)}$ as the solution to the integrated FBDE given $\mathbf{K}_{L,t}^{(n)}$, $\mathbf{K}_{F,t}^{(n)}$, $\boldsymbol{\Sigma}_t^{(n)}$, and $\mathcal{Q}_t^{(n)}$.

ii. Given $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X+T}$, the updated sequence of replacement rates $\{\boldsymbol{\sigma}_t^{(n+1)}\}_{t=X-T}^{X+T}$ is obtained from the balanced budget condition, Eq. (31).

iii. Finally, we obtain the updated path of transfers $\{\mathbf{Q}_t^{(n+1)}\}_{t=X-T}^{X+T}$ as per Eq. (32), where the asset holdings of all generations in each period are computed given $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X+T}$, $\{\boldsymbol{\sigma}_t^{(n+1)}\}_{t=X-T}^{X+T}$, and $\{\mathbf{Q}_t^{(n)}\}_{t=X-T}^{X+T}$.

(c) We iterate on n until convergence, based on the distance between two consecutive paths $\{\mathbf{k}_t^{(n)}\}_{t=X-T}^{X+T}$ and $\{\mathbf{k}_t^{(n+1)}\}_{t=X-T}^{X+T}$.

4. We verify that T is large enough for the distances $|k_{X-T}^{i(\infty)} - k^{i*}|$ and $|k_{X+T}^{(\infty)} - k^*|$ to fall below some convergence threshold.

D Results from Additional Experiment

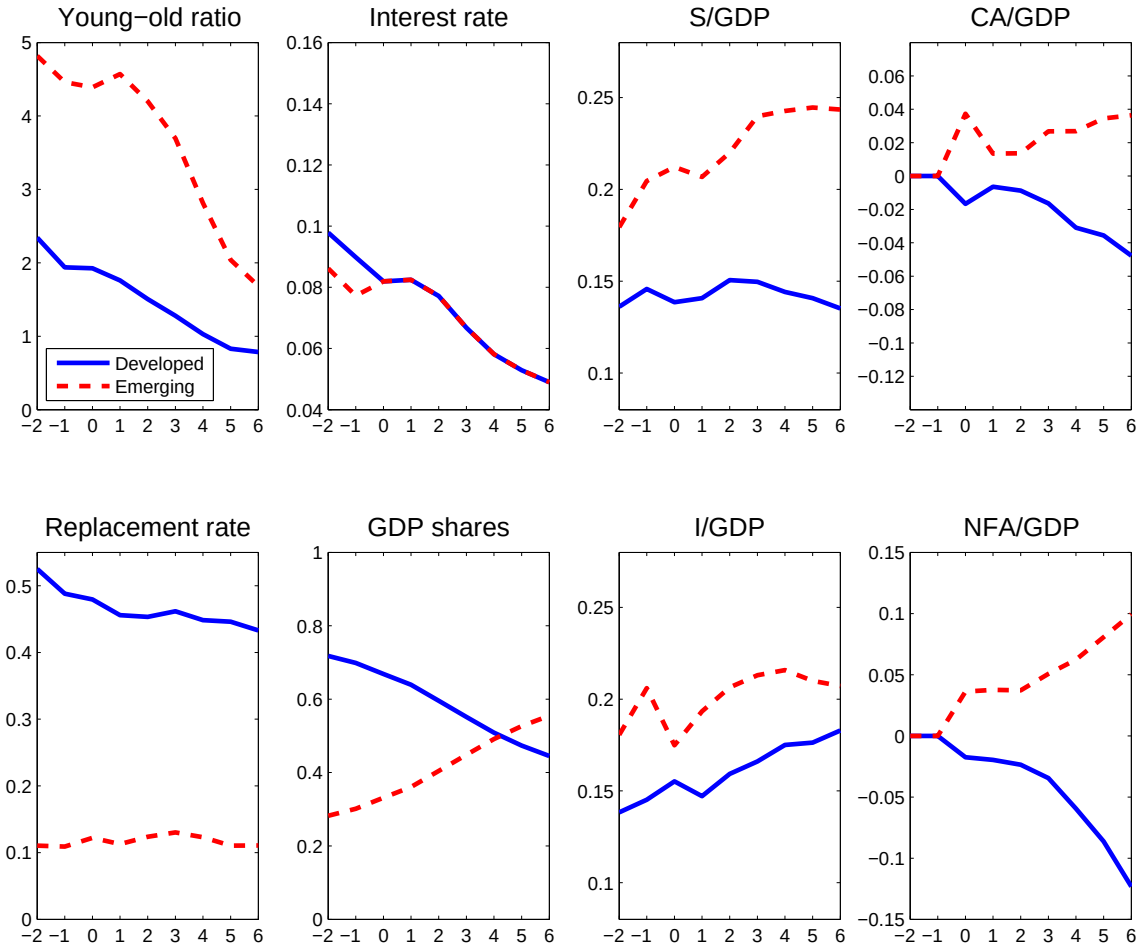


Figure D.1: Capital Flows Between a Developed and an Emerging Region, with Equal Productivity Growth

Notes: The young-old ratio corresponds to the ratio of the size of the group of agents of age $j = 1, 2$ (i.e., 20-40) over the size of the group of agents of age $j = 5, 6, 7$ (i.e., 60-90). Credit constraint and replacement rates for the group of emerging countries are set equal to their counterparts in developed countries. The values of other calibrated parameters are given in Table 1.