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Robust viable management of a harvested ecosystem model

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Abstract

The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the ecosystem approach by 2010. In this perspective, we propose a theoretical management framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties. More specifically, we define the robust viability kernel as the set of initial species biomasses such that at least one harvesting strategy guarantees minimal production and preservation levels for all times, whatever the uncertainties. We apply our approach to the anchovy-hake couple in the Peruvian upwelling ecosystem. We find that accounting for uncertainty significantly reduces the robust viability kernel compared to the deterministic one (without uncertainties). We observe that, when we increase the set of uncertainties, the robust viability kernel very slightly decreases, expressing a moderate sensibility with respect to refining the set of uncertainties. We comment on the management implications of comparing robust viability kernels (with uncertainties) and the deterministic one (without uncertainties).

Key words: optimization; viability; uncertainty; robustness; sustainability; ecosystem management; fisheries; Peruvian upwelling.

1 Introduction

There is a growing demand for moving from single species management schemes to an ecosystemic approach of fisheries management [5]. The World Summit on Sustainable Development (Johannes-

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burg, 2002) encouraged the application of an ecosystem approach by 2010.

Furthermore, uncertainty inherent to fisheries is recognized to play an important role in the failure of management regimes. Fisheries modeling requires estimations of stock status and total withdrawal from stock; such information remains imprecise and error prone. Uncertainty can also concern the structure and dynamics of ecosystems, which are poorly known. At last, uncertain climatic hazards or technical progress are likely to affect fisheries productivity. Some claim that fishing decreases the resilience of fish populations, rendering them more vulnerable to environmental change [7], and that not accounting for uncertainty can lead to excessive harvest of a resource [6].

We propose a theoretical management framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties.

We set forward the robust viability theory [3] as a relevant approach to address dynamical control problems under constraints with uncertainty. This approach aims at describing all the evolutions of a dynamical system under uncertainty that satisfy, at each instant, given objectives, whatever the uncertainties. Starting from a so-called *robust viable state*, there exists a control strategy guaranteeing constraints, here production and preservation objectives, for all dates of a time span, and for all uncertainties. The set of robust viable states is called the robust viability kernel. Notice that, in the robust viability theory, no trade-offs are allowed between pursued objectives or time periods: all constraints must be satisfied for all times, whatever the uncertainties.

We apply this theory to a discrete-time two-species dynamical model, where states are biomasses and where two harvesting efforts act as controls. Uncertainty takes the form of additive disturbances affecting each species dynamics, and are assumed to take their values in a known given set. We consider two nested sets in order to appraise the sensibility of our results to uncertainties. Constraints are imposed for each species: a minimum safe biomass level, usually identified by biologists, and a minimum required harvesting level assumed to ensure economic needs. Thus, starting from a robust viable biomass couple, it is possible to drive the system on a sustainable path along which catches and biomasses stand above production and biological minimums, despite of uncertainties.

Reducing uncertainties to zero amounts to formulate the problem within the deterministic viability framework [1]. Comparison of these deterministic viable states with the sets of robust viable states shades light on the potential interest of adopting a precautionary approach in harvested

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ecosystem management.

The paper is organized as follow. Section 2 introduces a generic class of harvested nonlinear ecosystem models, the sustainability constraints, and presents the robust viability kernel. The deterministic viability kernel is also displayed for comparison purpose. In section 3, we proceed to an application of the robust and deterministic viability analysis to the Peruvian hake-anchovy upwelling ecosystem between 1971 and 1981. After exposing our assumptions on uncertainty sets, robust viability kernels are computed numerically and compared to the deterministic viability kernel, whose expression can be obtained analytically. Section 4 concludes.

2 The Robust Viability Approach

In what follows, we present a class of generic harvested *nonlinear* ecosystem models with uncertainty. Next, we introduce the concept of robust viable state, that is, a state starting from which conservation and production constraints can be guaranteed over a given time span, despite of uncertainty. We compare it with the set of deterministic viable states — states guaranteeing conservation and production constraints in absence of uncertainties — for which we are able to provide an analytical expression.

2.1 A generic ecosystem model with uncertainty and the associated sustainability constraints

For simplicity, we consider a dynamical model with two species, each targeted by a specific fleet, but this approach can be easily extended to extended to more than two species in interaction. Each species is described by its biomass: the two–dimensional state vector (y, z) represents the biomasses of both species. The two–dimensional control vector (v_y, v_z) comprises the harvesting effort for each species, respectively, each lying in $[0, 1]$. The two additive terms, ε_y and ε_z, correspond to uncertainties affecting each species biomass, respectively (the fact that uncertainties are additive is not crucial for the theory). The discrete-time control dynamical system we consider is given by

$$\left\{ \begin{array}{lcl} y(t+1) & = & y(t)R_y\big(y(t),z(t)\big)\big(1-v_y(t)\big)+\varepsilon_y(t)\ , \\ z(t+1) & = & z(t)R_z\big(y(t),z(t)\big)\big(1-v_z(t)\big)+\varepsilon_z(t)\ , \end{array} \right. \tag{1}$$

where t stands for time (typically, periods are years), and ranges from the initial time t_0 to the time horizon T. The two functions $R_y : \mathbb{R}^2 \to \mathbb{R}$ and $R_z : \mathbb{R}^2 \to \mathbb{R}$ represent biological growth factors.

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The property that the growth factor $R_y(y,z)$ of species y depends on the other species biomass z (and vice versa) captures ecosystemic features of species interactions.

The catches are given by $v_y y R_y\big(y,z\big)$ and $v_z z R_z\big(y,z\big)$ (measured in biomass). This model is generic in that no explicit or analytic assumptions are made on how the growth factors R_y and R_z indeed depend upon both biomasses (y,z).

An uncertainty *scenario* is defined by a sequence of uncertainty couples of length $T-t_0$,

$$\big(\varepsilon_y(\cdot),\varepsilon_z(\cdot)\big) = \big((\varepsilon_y(t_0),\varepsilon_z(t_0)),\ldots,(\varepsilon_y(T-1),\varepsilon_z(T-1))\big) \in \Omega = \prod_{t=t_0}^{T-1} \mathbb{S}(t)\ , \tag{2}$$

where uncertainties $(\varepsilon_y(t),\varepsilon_z(t))$ are assumed to take there values in a two-dimensional set:

$$(\varepsilon_y(t),\varepsilon_z(t)) \in \mathbb{S}(t) \subset \mathbb{R}^2. \tag{3}$$

We now propose to define sustainability as the ability to respect preservation and production minimal levels for all times, building upon the original approach of [2]. For this purpose we consider:

- on the one hand, *minimal biomass levels* $y^\flat \geq 0$, $z^\flat \geq 0$, one for each species,
- on the other hand, *minimal catch levels* $Y^\flat \geq 0$, $Z^\flat \geq 0$, one for each species.

2.2 The robust viability kernel

A control strategy γ is defined as a sequence of mappings from biomasses towards efforts as follows:

$$\gamma = \{\gamma_t\}_{t=t_0,\ldots,T-1}, \quad \text{with} \quad \gamma_t : \mathbb{R}^2 \to [0,1]^2\ . \tag{4}$$

A control strategy γ as in (4) and the dynamic model (1) produce state paths by the closed-loop dynamics

$$\left\{ \begin{array}{rcl} y(t+1) &=& y(t) R_y\big(y(t),z(t)\big)\big(1-\gamma_t(y(t),z(t))\big) + \varepsilon_y(t)\ , \\ z(t+1) &=& z(t) R_z\big(y(t),z(t)\big)\big(1-\gamma_t(y(t),z(t)) + \varepsilon_z(t)\ , \end{array} \right. \tag{5}$$

and control paths by

$$(v_y(t),v_z(t)) = \gamma_t(y(t),z(t))\ , \quad t=t_0,\ldots,T-1\ . \tag{6}$$

Notice that, as in (6), controls $(v_y(t),v_z(t))$ are determined by constantly adapting to the state $(y(t),z(t))$ of the system, itself affected by uncertainties $(\varepsilon_y(t-1),\varepsilon_z(t-1))$.

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The *rrobust viability kernel* $\mathbb{V}^R(t_0)$ [3] is the set of initial states $(y(t_0), z(t_0))$ — called *robust viable states* — for which there exists a control strategy γ as in (4) producing state paths $\{(y(t), z(t))\}_{t=t_0,\ldots,T}$ as in (5), and control paths $\{(v_y(t), v_z(t))\}_{t=t_0,\ldots,T-1}$ as in (6), such that, for all uncertainty scenarios $(\varepsilon_y(\cdot), \varepsilon_z(\cdot)) \in \Omega$ in (2), the following goals are satisfied:

- preservation (minimal biomass levels)

$$y(t) \geq y^\flat \ , \quad z(t) \geq z^\flat \ , \quad \forall t = t_0, \ldots, T \ , \tag{7}$$

- production requirements (minimal catch levels)

$$v_y(t)y(t)R_y\big(y(t), z(t)\big) \geq Y^\flat \ , \quad v_z(t)z(t)R_z\big(y(t), z(t)\big) \geq Z^\flat \ , \quad \forall t = t_0, \ldots, T-1 \ . \tag{8}$$

Characterizing robust viable states makes it possible to test whether or not minimal biomass and catch levels can be guaranteed for all time, despite of uncertainty. By *guaranteed* we mean that biomasses and catches never fall below the minimal thresholds as in the inequalities (7) and (8).

The robust viability kernel can be computed numerically by means of a dynamic programming equation associated with dynamics (1), state constraints (7) and control constraints (8) [3]. Numerical simulations have been performed with the scientific software Scicoslab.

2.3 The deterministic viability kernel

The deterministic version of the framework exposed in § 2.2 corresponds to the case where the uncertainties $(\varepsilon_y(t), \varepsilon_z(t)) = (0, 0)$, for all $t = t_0, \ldots, T-1$, that is, the uncertainty sets in (3) are reduced to the singleton $\mathbb{S}(t) = \{(0, 0)\}$. Then, the robust viability kernel coincides with the so-called *viability kernel* [1], defined in § A.

The following Proposition 1 gives an analytical expression of the deterministic viability kernel under some conditions on the guaranteed levels in (7) and (8). The proof, adapted from [4] is given in § A.

Proposition 1 *Consider $T \geq t_0 + 2$. If the minimal biomass thresholds $y^\flat$, $z^\flat$ and catch thresholds $Y^\flat$, $Z^\flat$ are such that*

$$y^\flat R_y\big(y^\flat, z^\flat\big) - y^\flat \geq Y^\flat \ \ \textit{and} \ \ z^\flat R_z\big(y^\flat, z^\flat\big) - z^\flat \geq Z^\flat \ , \tag{9}$$

the deterministic viability kernel is given by

$$\mathbb{V}(t_0) = \left\{ \ (y,z) \in \mathbb{R}^2_+ \mid y \geq y^\flat, z \geq z^\flat, yR_y\big(y,z\big) - y^\flat \geq Y^\flat, zR_z\big(y,z\big) - z^\flat \geq Z^\flat \ \right\} . \tag{10}$$

Conditions (9) mean that, at the point $(y^\flat, z^\flat)$ of minimum biomass thresholds, the surplus $y^\flat R_y\big(y^\flat, z^\flat\big) - y^\flat \geq Y^\flat$ and $z^\flat R_z\big(y^\flat, z^\flat\big) - z^\flat \geq Z^\flat$ are at least equal to the minimum catch thresholds $Y^\flat$ and $Z^\flat$, respectively.

3 Application to the anchovy-hake couple in the Peruvian upwelling ecosystem (1971–1981)

We now apply the above robust viability analysis to the Peruvian hake-anchovy fisheries between 1971 and 1981. For this, we extend the model in [4] to the uncertain case. We compute the robust viability kernel numerically, testing different assumptions on the uncertainty sets $\mathbb{S}(t)$ in (3), to appraise the sensitivity of robust viable states to uncertainty scenarios.

3.1 Lotka-Volterra dynamical model with uncertainties

The Peruvian anchovy-hake system is modeled as prey-predator system, where the anchovy growth rate is decreasing in the hake population. We describe this interaction by the following discrete-time Lotka-Volterra system

$$\left\{ \begin{array}{rcl} y(t+1) & = & y(t) \overbrace{\big(R - \dfrac{R}{\kappa} y(t) - \alpha z(t)\big)}^{R_y\left(y(t),z(t)\right)} \big(1 - v_y(t)\big) + \varepsilon_y(t) \\ z(t+1) & = & z(t) \underbrace{\big(L + \beta y(t)\big)}_{R_z\left(y(t),z(t)\right)} \big(1 - v_z(t)\big) + \varepsilon_z(t), \end{array} \right. \tag{11}$$

where $R > 1$, $0 < L < 1$, $\alpha > 0$, $\beta > 0$ and $\kappa = \frac{R}{R-1}K$, with $K > 0$ the carrying capacity for the prey. The variable y stands for anchovy biomass and z for hake biomass. The purpose of this compact model is not to provide biological "knowledge" on the Peruvian upwelling ecosystem, but rather to capture the essential features of the system in what concerns decision making.

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The 5 parameters of the model (11) have been estimated in [4], based on 11 yearly observations of the Peruvian anchovy-hake biomasses and catches over the time period 1971–1981. Their values are given in Table 1.

Parameters	**Estimates**
R	2.25 $year^{-1}$
L	0.945 $year^{-1}$
κ	67113 10^3 $tons$
K	37285 10^3 $tons$
α	1.220 10^6 $tons^{-1}$
β	4.845 10^{-8} $tons^{-1}$

Table 1: Parameter estimates of the model (11)

3.2 Choice of uncertainty sets

We are now more specific about the uncertainty sets $\mathbb{S}(t)$ in (3), in which the uncertainties $\varepsilon_y(t)$ and $\varepsilon_z(t)$ in (11) take their values.

Figure 1 represents the observed biomasses of Peruvian anchovy and hake over the years 1971–1981 and the simulated biomasses with the deterministic version of the Lotka-Volterra model (that is, with $\varepsilon_y(t) = 0$ and $\varepsilon_z(t) = 0$ in the dynamical system (1)).

The differences between these curves over the time span give us indications on the ranges in which the uncertainties $\varepsilon_y(t)$ and $\varepsilon_z(t)$ in (11) take there values. The time period 1971–1981 is denoted by $t = t_0, \ldots, T$, with $t_0 = 0$, and $T = 10$. Let $(\overline{y}(t), \overline{z}(t))_{t=t_0,\ldots,T}$ and $(\overline{v}_y(t), \overline{v}_z(t))_{t=t_0,\ldots,T-1}$ denote the observed biomass and effort trajectories. We set $\overline{\varepsilon}_y(t)$ and $\overline{\varepsilon}_z(t)$ defined by

$$\left\{\begin{array}{rcl} \overline{y}(t+1) & = & \overline{y}(t)\big(R - \frac{R}{\kappa}\overline{y}(t) - \alpha\overline{z}(t)\big)\big(1 - \overline{v}_y(t)\big) + \overline{\varepsilon}_y(t) \\ \overline{z}(t+1) & = & \overline{z}(t)\big(L + \beta\overline{y}(t)\big)\big(1 - \overline{v}_z(t)\big) + \overline{\varepsilon}_z(t), \end{array}\right.$$

so that (11) is satisfied.

Figure 2 displays the points $\{(\overline{\varepsilon}_y(t), \overline{\varepsilon}_z(t)) | t = t_0, \ldots, T-1\}$, that is, the difference between simulated and observed biomasses (there are 10 points as 1971 observations are used as starting points for simulating biomasses). We denote $\overline{\varepsilon}_y^{min} = \min_{t=t_0,\ldots,T-1} \overline{\varepsilon}_y(t) = -1610.95$ ktons,

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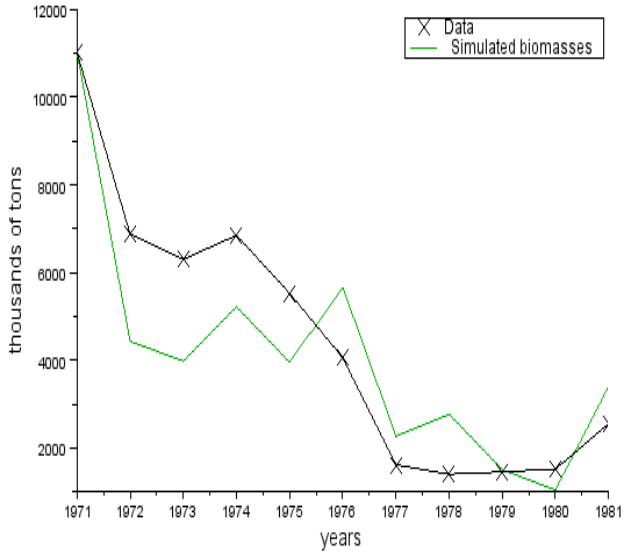
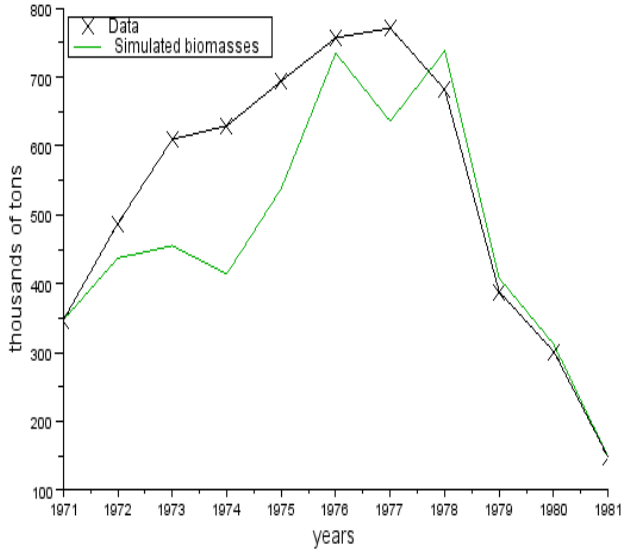


(a) Anchovy



(b) Hake

Figure 1: Observed and simulated biomasses over 1971–1981

$\overline{\varepsilon}_y^{max} = \max_{t=t_0,\ldots,T-1} \overline{\varepsilon}_y(t) = 2441.35$ ktons, $\overline{\varepsilon}_z^{min} = \min_{t=t_0,\ldots,T-1} \overline{\varepsilon}_z(t) = -55.64$ ktons and $\overline{\varepsilon}_z^{max} = \max_{t=t_0,\ldots,T-1} \overline{\varepsilon}_z(t) = 215.24$ ktons.

We will compute robust viable kernels corresponding to the following two cases of the uncertainty sets $\mathbb{S}(t)$ in (3).

- The set $\mathbb{S}_L$ is made of the 10 empirical uncertainty couples (see diamonds in Figure 3) and

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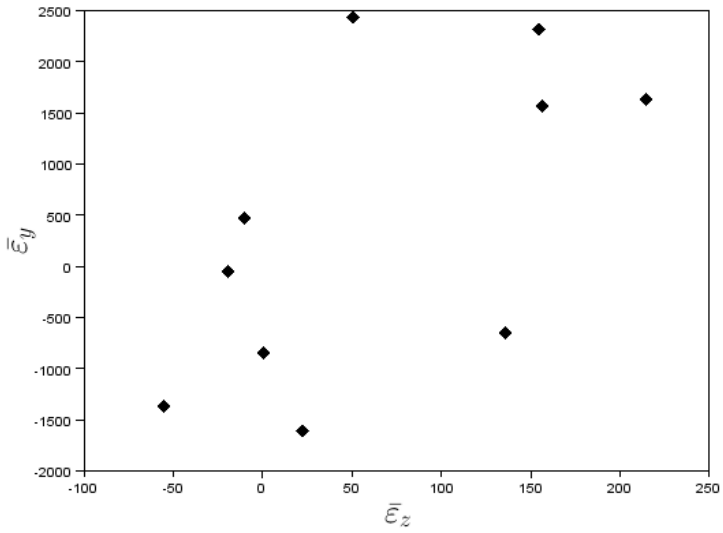

Figure 2: Empirical uncertainties

the uncertainty couple $(\varepsilon_y, \varepsilon_z) = (0, 0)$ (corresponding to the deterministic case):

$$\mathbb{S}_L = \{(\bar{\varepsilon}_y(t), \bar{\varepsilon}_z(t)) | t = t_0, \ldots, T-1\} \cup \{(0,0)\} \; . \tag{12}$$

- The set $\mathbb{S}_H$ is made of 400 uncertainty couples delineated by a 20×20 grid over the surface $[\bar{\varepsilon}_y^{min}, \bar{\varepsilon}_y^{max}] \times [\bar{\varepsilon}_z^{min}, \bar{\varepsilon}_z^{max}]$, including all the uncertainty couples of $\mathbb{S}_L$ (see the grid in Figure 3).

Since $\{(0,0)\} \subset \mathbb{S}_L \subset \mathbb{S}_H$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}_H^R(t_0) \subset \mathbb{V}_L^R(t_0) \subset \mathbb{V}(t_0) \; . \tag{13}$$

Indeed, the larger the uncertainty sets $\mathbb{S}(t)$ in (3), the smaller the robust viability kernel $\mathbb{V}^R(t_0)$, because the constraints are much harder to satisfy, since they have to be respected for all uncertainties.

3.3 Robust and deterministic viability kernels

We consider the minimal biomasses $y^\flat = 7,000,000$ tons and $z^\flat = 200,000$ tons, and minimal catches $Y^\flat = 2,000,000$ tons and $Z^\flat = 5,000$ tons (IMARPE, 2000, 2004). The condition (9) in Proposition 1 is satisfied for the above minimal threshold values and for the Lotka-Volterra model coefficient estimates in Table 1.

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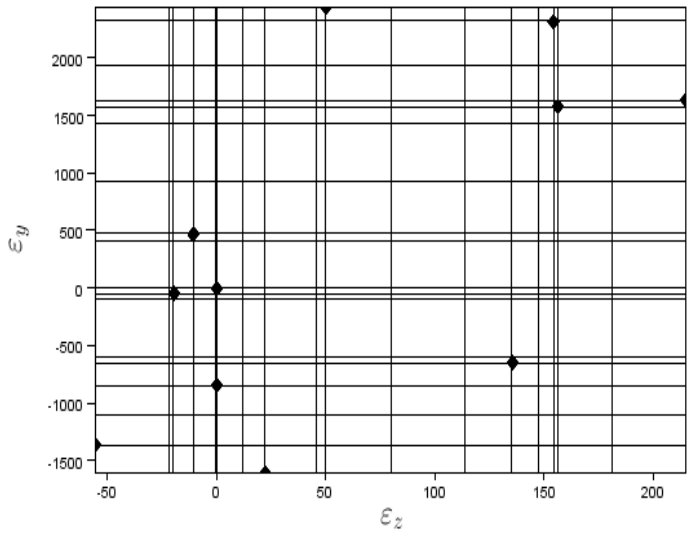

Figure 3: Uncertainty sets $\mathbb{S}_L$ (dimamonds) and $\mathbb{S}_H$ (grid)

Figure 4 displays the two robust viability kernels associated with dynamics (11), constraints (7) and (8), and with the uncertainty sets $\mathbb{S}_L$ and $\mathbb{S}_H$, respectively. As expected, the set of robust viable states is decreasing in the number of scenarios as stated in (13). However, notice that whether we consider the set of uncertainty scenarios derived from the rough grid $\mathbb{S}_L$ or from the finer grid $\mathbb{S}_H$ does not substantially change the robust viability kernel. This fact traduces a rather moderate sensibility of robust viable states to uncertainty scenarios, at least as long as uncertainty values are contained in the rectangle $[\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_y^{max}] \times [\overline{\varepsilon}_z^{min}, \overline{\varepsilon}_z^{max}]$ and comprise the empirical estimates in $\mathbb{S}_L$.

Replacing the growth rates R_y and R_z in (10) by their expressions (11) yields the expression of the deterministic viability kernel:

$$\mathbb{V}(t_0) = \left\{ \ (y,z) \mid y \geq y^\flat, z^\flat \leq z \leq \tfrac{1}{\alpha}[R - \tfrac{R}{\kappa}y - \tfrac{y^\flat + Y^\flat}{y}] \ \right\}. \tag{14}$$

This set $\mathbb{V}(t_0)$ is delineated by the outer dashed curve in Figure 4. We observe an important gap between the deterministic kernel and the robust ones. A share of the states identified as viable by the deterministic approach are in fact excluded when introducing uncertainty. Indeed, there exists no effort strategy capable of guaranteeing preservation and production minima for biomass couples standing outside the robust kernels, given the chosen scenarios sets and time horizon. Furthermore, we cannot tell whether the effort strategies advocated by the deterministic approach for an initial biomass couple belonging to the robust kernels may guarantee sustainability objectives over time

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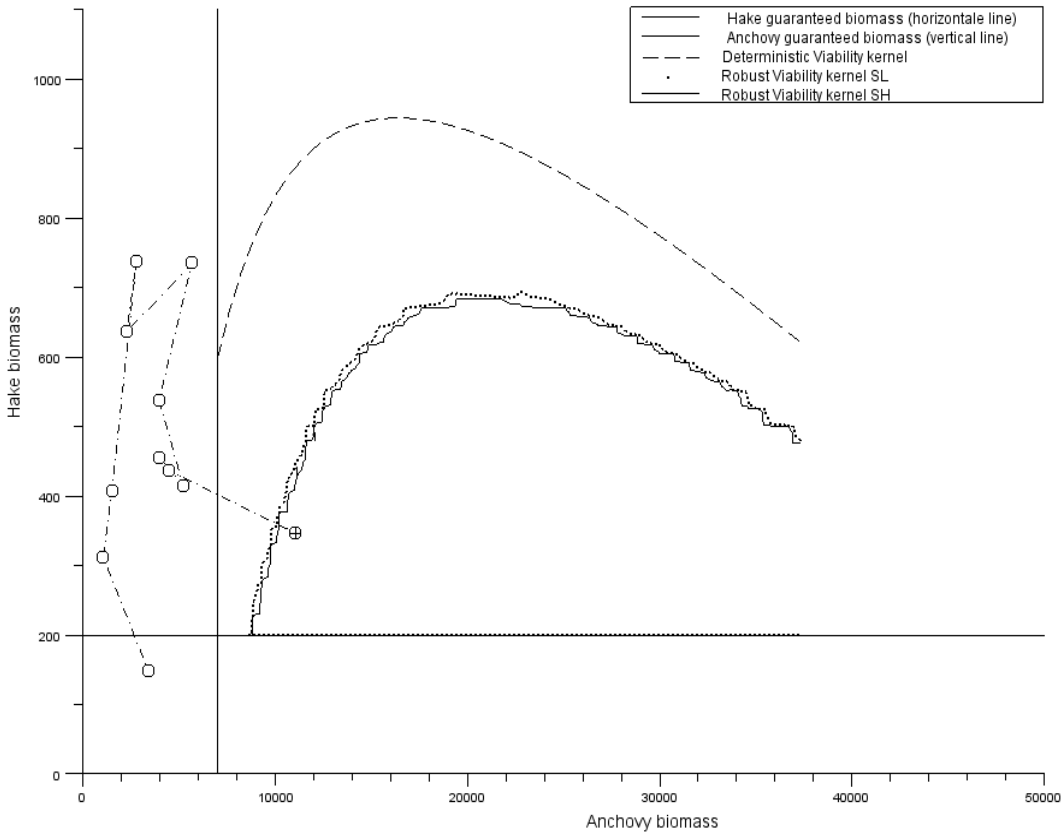

Figure 4: Comparing robust and deterministic viable states

in presence of uncertainty.

The horizontal and vertical lines represent the minimal biomass safety levels and the circles indicate the biomass observations of the anchovy-hake couple over 1971–1981. Only one circle, marked by a cross, stands within the robust sets of states, corresponding to the initial biomass couple observed in 1971. Starting from that date, there theoretically existed a control strategy providing, for the next 10 years, at least the sustainable yields $Y^\flat$ and $Z^\flat$, and guaranteeing biomasses over the preservation thresholds $y^\flat$, $z^\flat$, whatever the scenario stemming from $\mathbb{S}_H$. In reality, the catches of year 1971 were very high, and the biomass trajectories were well below the biological minimal levels for 14 years.

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4 Conclusion

This work is a theoretical and practical contribution to ecosystem sustainable management under uncertainties. The robust viable kernel is an interesting mean to display the impact of uncertainty on the possibility of a sustainable management. By exposing the ecosystem dynamics to uncertainties, fewer initial states are likely to allow for effort strategies guaranteeing all sustainability constraints at all times, compared to the deterministic framework.

Our results show the robust viable kernels attached to two different sets of uncertainty scenarios. Comparing the robust viable kernels, it appears that accounting for a larger set of uncertainties hardly modifies the robust viability kernel. This traduces a rather moderate sensibility of robust viable states to the set of uncertainties.

The viability stochastic theory is an alternative approach to address dynamical control problems under uncertainty and constraints. This approach allows for constraints violations with a low probability. Thus, it can be used to affect to each biomass couples taken as potential initial states, the maximum probability with which constraints may be satisfied at all times. This issue is under current investigation.

A The deterministic viability kernel

The *deterministic viability kernel* $\mathbb{V}(t_0)$ is the set of viable states defined as follows. A couple (y_0, z_0) of initial biomasses is said to be a *viable state* if there exist a trajectory of harvesting efforts (controls) $\big(v_y(t), v_z(t)\big) \in [0,1],\ t = t_0, \ldots, T-1$, such that the state path $\{\big(y(t), z(t)\big)\}_{t=t_0,\ldots,T}$, and control path $\{\big(v_y(t), v_z(t)\big)\}_{t=t_0,\ldots,T-1}$, solution of[1]

$$\left\{\begin{array}{rcl} y(t+1) & = & y(t)\overbrace{\big(R - \frac{R}{\kappa}y(t) - \alpha z(t)\big)}^{R_y\left(y(t),z(t)\right)}\big(1 - v_y(t)\big) \\ z(t+1) & = & z(t)\underbrace{\big(L + \beta y(t)\big)}_{R_z\left(y(t),z(t)\right)}\big(1 - v_z(t)\big) \end{array}\right. \tag{15}$$

starting from $\big(y(t_0), z(t_0)\big) = (y_0, z_0)$ satisfy the following goals:

[1] Equation (15) is (11) with the uncertainty couple $(\varepsilon_y, \varepsilon_z) = (0,0)$ (corresponding to the deterministic case).

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- preservation (minimal biomass levels)

$$y(t) \geq y^\flat \ , \quad z(t) \geq z^\flat \ , \quad \forall t = t_0, \ldots, T \ , \tag{16}$$

- and production requirements (minimal catch levels)

$$v_y(t)y(t)R_y\big(y(t),z(t)\big) \geq Y^\flat \ , \quad v_z(t)z(t)R_z\big(y(t),z(t)\big) \geq Z^\flat \ , \quad \forall t = t_0, \ldots, T-1 \ . \tag{17}$$

We now turn to the proof of Proposition 1.

Proof. Consider $y^\flat \geq 0$, $z^\flat \geq 0$, $Y^\flat \geq 0$, $Z^\flat \geq 0$. We set

$$\mathbb{V}_0 = \Big\{ (y,z) \in \mathbb{R}^2_+ \,\Big|\, y \geq y^\flat, z \geq z^\flat \Big\}$$

and we define a sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ inductively by

$$\begin{aligned} \mathbb{V}_{k+1} = \{\ (y,z) \in \mathbb{V}_k \mid \exists (v_y, v_z) \in [0,1] \ \text{ such that } \ & yv_yR_y(y,z) \geq Y^\flat, zv_zR_z(y,z) \geq Z^\flat, \\ \text{and } \ & y' = yR_y(y,z)(1-v_y), \ z' = zR_z(y,z)(1-v_z), \\ & \text{are such that } \ (y',z') \in \mathbb{V}_k\} \ . \end{aligned}$$

For $k = 0$, we obtain

$$\begin{aligned} \mathbb{V}_1 &= \left\{ (y,z) \,\middle|\, \begin{array}{l} y \geq y^\flat, z \geq z^\flat \ \text{ and, for some } \ (v_y, v_z) \in [0,1], \\ v_yyR_y(y,z) \geq Y^\flat, v_zzR_z(y,z) \geq Z^\flat, \\ yR_y(y,z)(1-v_y) \geq y^\flat, zR_z(y,z)(1-v_z) \geq z^\flat \end{array} \right\} \\ &= \left\{ (y,z) \,\middle|\, \begin{array}{l} y \geq y^\flat, z \geq z^\flat \ \text{ for which there exist } (v_y, v_z) \text{ such that} \\ \frac{Y^\flat}{yR_y(y,z)} \leq v_y \leq \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)} \quad \text{and} \quad 0 \leq v_y \leq 1, \\ \frac{Z^\flat}{zR_z(y,z)} \leq v_z \leq \frac{zR_z(y,z)-z^\flat}{zR_z(y,z)} \quad \text{and} \quad 0 \leq v_z \leq 1 \end{array} \right\} \\ &= \left\{ (y,z) \,\middle|\, \begin{array}{l} y \geq y^\flat, z \geq z^\flat, \\ \sup\{0, \frac{Y^\flat}{yR_y(y,z)}\} \leq \inf\{1, 1 - \frac{y^\flat}{yR_y(y,z)}\} \\ \sup\{0, \frac{Z^\flat}{zR_z(y,z)}\} \leq \inf\{1, 1 - \frac{z^\flat}{zR_z(y,z)}\} \end{array} \right\} \\ &= \left\{ (y,z) \,\middle|\, y \geq y^\flat, z \geq z^\flat, \frac{Y^\flat}{yR_y(y,z)} \leq \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)}, \frac{Z^\flat}{zR_z(y,z)} \leq \frac{zR_z(y,z)-z^\flat}{zR_z(y,z)} \right\} \\ &= \left\{ (y,z) \,\middle|\, y \geq y^\flat, z \geq z^\flat, Y^\flat \leq yR_y(y,z) - y^\flat, Z^\flat \leq zR_z(y,z) - z^\flat \right\} \ . \end{aligned}$$

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Then, for $k=1$, we obtain

$$\begin{aligned}\mathbb{V}_2 &= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y,v_z) \in [0,1], \\ v_y y R_y(y,z) \geq Y^\flat,\ v_z z R_z(y,z) \geq Z^\flat \\ \text{and such that } (y',z') \in \mathbb{V}_k \\ \text{where } y' = yR_y(y,z)(1-v_y),\ z' = zR_z(y,z)(1-v_z) \end{array} \right\} \\ &= \left\{ (y,z) \;\middle|\; \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y,v_z) \in [0,1], \\ v_y y R_y(y,z) \geq Y^\flat,\ v_z z R_z(y,z) \geq Z^\flat,\ y' \geq y^\flat,\ z' \geq z^\flat, \\ Y^\flat \leq y' R_y(y',z') - y^\flat,\ Z^\flat \leq z' R_z(y',z') - z^\flat \\ \text{where } y' = yR_y(y,z)(1-v_y),\ z' = zR_z(y,z)(1-v_z) \end{array} \right\}. \end{aligned}$$

We now make use of the property (see [4]) that, when the decreasing sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ is stationary, its limit is the viability kernel $\mathbb{V}(t_0)$. Hence, it suffices to show that $\mathbb{V}_1 \subset \mathbb{V}_2$ to obtain that $\mathbb{V}(t_0) = \mathbb{V}_1$.

Let $(y,z) \in \mathbb{V}_1$. We have that

$$y \geq y^\flat, \quad z \geq z^\flat \text{ and } \quad yR_y(y,z) - y^\flat \geq Y^\flat, \quad zR_z(y,z) - z^\flat \geq Z^\flat\ .$$

Let us set $\hat{v}_y = \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)}$, which has the property that $y' = yR_y(y,z)(1-\hat{v}_y) = y^\flat$. We prove that $\hat{v}_y \in [0,1]$. Indeed, on the one hand, we have that $\hat{v}_y \leq 1$ since $1-\hat{v}_y = y^\flat / yR_y(y,z)$, where $y^\flat \geq 0$. On the other hand, since $yR_y(y,z) - y^\flat \geq Y^\flat \geq 0$, we deduce that $\hat{v}_y \geq 0$. The same holds true for $\hat{v}_z$ and $z' = zR_z(y,z)(1-\hat{v}_z) = z^\flat$. By (9), we deduce that

$$y'R_y(y',z') - y^\flat = y^\flat R_y(y^\flat,z^\flat) - y^\flat \geq Y^\flat \text{ and } z'R_z(y',z') - z^\flat = z^\flat R_z(y^\flat,z^\flat) - z^\flat \geq Z^\flat\ .$$

The inclusion $\mathbb{V}_1 \subset \mathbb{V}_2$ follows, hence $\mathbb{V}(t_0) = \mathbb{V}_1$, and (10) holds true. □

The *viable controls* attached to a given viable state $(y,z) \in \mathbb{V}(t_0)$ are the admissible controls such that the image by the dynamics is in $\mathbb{V}(t_0)$.

Corollary 2 *Suppose that the assumptions of Proposition 1 are satisfied. The set of viable controls is given by*

$$\mathbb{U}_{\mathbb{V}(t_0)}(y,z) = \left\{ (v_y,v_z) \in [0,1]^2 \;\middle|\; \begin{array}{l} \frac{yR_y(y,z)-y^\flat}{yR_y(y,z)} \geq v_y \geq \frac{Y^\flat}{yR_y(y,z)}, \quad \frac{zR_z(y,z)-z^\flat}{zR_z(y,z)} \geq v_z \geq \frac{Z^\flat}{zR_z(y,z)}, \\ y'R_y(y',z') - y^\flat \geq Y^\flat,\ z'R_z(y',z') - z^\flat \geq Z^\flat \end{array} \right\},$$

where $y' = yR_y(y,z)(1-v_y),\ z' = zR_z(y,z)(1-v_z)$.

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