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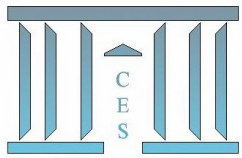

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When Trade Leads to Inefficient Public Good Provision: a Tax competition model

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Abstract

This paper analyses the tax competition mechanisms in a context of commodity trade. We show that the trade market equilibrium may restore the efficiency of the public good provision when agents from different countries have symmetric preferences. Asymmetry in preferences implies over or underprovision in public goods depending on the degree of asymmetry between countries. In both cases, the price adjustment leaves the capital stock unchanged so that the stock of capital is not affected by the taxes. Finally, we show that the centralized choice does not systematically restore the efficiency of the public good provision.

Keywords: Tax competition, Nash equilibrium, Interregional Trade.
JEL classification: H21, H41, E62, F12.

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1 Introduction

Our paper revisits the tax competition literature by introducing commodity trade in a standard tax competition model. While in an integrated world, both international trade and capital mobility are important issues, only little attention has been devoted to the analysis of tax competition in a context of trade. This is the purpose of this paper which aims to analyse the consequences of trade balance on tax competition mechanisms when countries are either symmetric or asymmetric.

The tax competition literature highlights the impact of capital mobility and the ajustments on the capital market that imply low taxes and the underprovision of the public goods (cf Zodow & Mieszkowki (1986), Wildasin (1989), Wilson (1986)). A huge literature based on these seminal articles has extended these results in a context of labor mobility (Bucovetsky & Wilson (1991), Wilson (1995)) or asymmetric countries (Bucovestky (1991)). The number of competing regions on the equilibrium tax has been also analysed in Hoyt (1991)[1].

Few papers have dealt with the introduction of trade in a tax competition model. Most of these papers have limited their analysis to a lump sum tax so that there is no distortionary effect of taxes on consumption (cf Turnovsky (1988), Chari and Kehoe (1990)). Other papers considered a production and/or consumption tax that avoids any capital tax competition effects (Devereux (1991), Devereux and Mansoorian (1992)). Another strand of the literature has dealt with the effect of tax exporting in models of trade on the public good provision. While it is widely held that tax exporting, by shifting the burden of taxes on the non residents, stimulates the provision of local public goods (see for instance Oates (1972)), Wildasin (1987a) and (1987b) mitigates these results by introductiong a labor tax that leads the exported tax to raise the same incremental revenue as the non exported tax. Wildasin (1993) goes further by introducing a capital tax rate in the model to analyze the consequences of fiscal competition with interindustry trade. However the analysis is limited to the case of two regions considered as rather small compared to the world economy so that the return of capital is taken as fixed and does not serve as a channel of policy transmission.

Exceptions are Wilson (1987) and Becker & Runkel (2012). In both models, the introduction of trade in a capital tax competition model crucially modifies the results of the standard tax competition literature. In line with these models, we show that trade

[1]See also Wildasin & Wilson (2004) for a survey on capital tax competition literature.

may reinforce an inefficient distribution of public goods in a context of trade equilibrium. In his paper, Wilson shows that trade creates, in addition to an inefficient distribution of public goods across regions, an inefficient pattern of trade. He develops a model with a large number of regions that may produce two types of goods. The after tax return is fixed since regions are assumed to be sufficiently small so that they have a negligible impact on the capital return. We definitely depart from Wilson by considering a two country model with endogeneous capital allocation and by analyzing the impact of the trade balance equilibrium on the tax competition game. In doing so, the constant level of capital in each region results from the market equilibrium and each price is determined by the trade balance and depends on the tax rate levels.

In their paper, Becker and Runkel consider the impact of transport cost in a model of trade with tax competition. Since the traded goods are perfect substitutes, there is no trade between symmetric regions at the equilibrium and even a small transport cost restores the efficiency of the public good provision. This result is due to the fact that transport cost in the product sector makes the capital sticky. In our model, we obtain a similar result in a quite different framework. We show that even without transport cost, efficiency of public good provision may be restored when trade balance is required under a case of symmetric countries. Our paper also highlights the impact of the preferences of the traded goods and the public goods on the level of public good provision. A strong taste for the public good implies an underprovision of the public good but a high asymmetry in preferences among countries for the traded good involves an overprovision of the public good through an upward distortion of the taxes due to the prices adjustment. Finally, the centralized equilibrium gives rise to additional results: while the decentralized and centralized choices perfectly match for symmetric countries, the constraint on the trade balance avoids the centralized choice to restore the efficiency of the public good provision when countries are asymmetric. These results are obtained in a two-country model where the price adjustment allowing for a trade equilibrium becomes the key element of our tax competition framework, especially when countries are asymmetric.

To our knowledge, very few papers deal with fiscal coordination in case of asymmetric countries. Among them, Cardarelli and al. (2002) analyze the sustainability of a tax harmonization in repeated games. They show that a small country may benefit from deviating from the harmonized equilibrium if asymmetry between countries is large. Our analysis of the centralized equilibrium differs also from the paper by Peralta and van

Ypersele (2006) by two different aspects. First, Peralta and van Ypersele do not consider trade and second, they consider peculiar types of fiscal coordination (a minimum capital tax level and a tax range). The purpose of their analysis is to determine the acceptability of these reforms.

This paper is organized as follows. The second section outlines the tax competition model with trade and derives the results for symmetric countries. In section 3 we derive the Nash equilibria according to the degree of asymmetry among countries. In section 4 we characterize the centralized equilibrium and compare the results with the decentralized equilibrium. The final section summarizes our conclusions.

2 The model

Consider an economy composed of two countries A and B. Each country is specialized in the production of a distinct good: jurisdiction A produces good a whereas jurisdiction B produces good b. For analytical simplicity, we assume that both jurisdictions are identical in size, and there is a single consumer in each jurisdiction who wants to consume both goods. In order to maximize each representative consumer's welfare, both jurisdictions are incited to trade with each other.

In each jurisdiction, each firm uses capital to produce its output, this capital being perfectly mobile between the two jurisdictions, and some locationally fixed factor, such as land which is held entirely by the representative consumer in each jurisdiction. Each firm provides its local private good by using the same production technology with decreasing returns to scale, that is to say, an increasing, twice continuously-differentiable and strictly concave function denoted by $f\left(k^i\right)$ which depends exclusively on capital demand k^i since fixed factors as explicit argument are suppressed from the production function. Capital being perfectly mobile between countries, the net of tax returns of capital equal between countries:

$$\rho = \rho^i = p_i f'(k^i) - t^i = r^i - t^i \tag{1}$$

where p_i stands for the price of good i and t^i for the tax on capital in region i. The demand for capital in country i can be rewritten as $k^i = \widehat{k^i}\left[\frac{t^i+\rho}{p_i}\right]$.

Each government provides a public good denoted by g^i, $i = A, B$ which is financed

by tax on capital. The government i's budget constraint writes:

$$p_i g^i = t^i k^i \tag{2}$$

Let c_a^A and c_b^A be the quantities of good a and good b consumed by country A's representative consumer, and let c_a^B and c_b^B be the quantities consumed by country B's representative consumer.

Goods a and b market equilibria write:

$$f\left(k^A\right) = c_a^A + c_a^B + g^A \tag{3}$$

and

$$f\left(k^B\right) = c_b^B + c_b^A + g^B \tag{4}$$

We assume that consumers of countries A and B have a Cobb-Douglas utility function U^A and U^B of the form[2]:

$$U^i = \left(\frac{c^i}{\eta}\right)^{\eta} \left(\frac{g^i}{1-\eta}\right)^{1-\eta} \text{ with } i = A, B$$

where c_i is the private consumption in country i, i.e. a bundle of goods produced in each country such that:

$$c^A = \left(c_a^A\right)^{\alpha} \left(c_b^A\right)^{1-\alpha};\ c^B = \left(c_b^B\right)^{\beta} \left(c_a^B\right)^{1-\beta}$$

with $\alpha \geqslant \beta$ which means that country A's household value equally or more the domestic good relative to the imported good than the country B's household.

The marginal rate of substitution between the national private good and the public good in country i, hereafter denoted by MRS^i is given by

$$\begin{aligned} MRS^A &= \frac{\partial U^A/\partial g^A}{\partial U^A/\partial c_a^A} = \frac{1}{\alpha}\frac{1-\eta}{\eta}\frac{c_a^A}{g^A} \\ MRS^B &= \frac{\partial U^B/\partial g^B}{\partial U^B/\partial c_b^B} = \frac{1}{\beta}\frac{1-\eta}{\eta}\frac{c_b^B}{g^B} \end{aligned}$$

[2]The general case without specifying the utility functions may be studied but with complex and demanding conditions on the primitives of the model.

so that

$$\frac{\partial MRS^i}{\partial g^i} < 0,\ \frac{\partial MRS^i}{\partial c_i^i} > 0 \text{ and } MRS^i_{g^i=0} > 0 \text{ for all } c_i^i > 0^3$$

Country A consumer's budget constraint writes:

$$p_a c_a^A + p_b c_b^A = p_a f(k^A) - \left(\rho^A + t^A\right) k^A + \rho\theta_A 2\overline{k} \tag{5}$$

and for the representative consumer in country B it becomes:

$$p_a c_a^B + p_b c_b^B = p_b f(k^B) - \left(\rho^B + t^B\right) k^B + \rho\theta_B 2\overline{k} \tag{6}$$

$2\overline{k}$ being the total amount of capital in the economy. The parameter θ_i stands for the proportion of the capital owned by the agent of country i and $\theta_A + \theta_B = 1$. Following Zodrow and Mieszkowski (1986) we consider $\theta_A = \theta_B = \frac{1}{2}$ so that $\theta_A 2\overline{k} = \theta_B 2\overline{k} = \overline{k}$.

Finally, each country being specialized in the production of a specific good that is consumed in both countries, the trade balance equilibrium between both countries requires:

$$p_b c_b^A = p_a c_a^B \tag{7}$$

Inserting (2), (3) and (7) in (5) for country A, and symmetrically for country B, gives the following relation:[4]

$$k^i = \overline{k}\ \forall i \tag{8}$$

With a trade balance equilibrium, the level of the capital demand remains unchanged when the production function faces decreasing returns to scale. A change in the capital tax rates impacts both the relative price $\left(\frac{p_a}{p_b}\right)$ and the net return of capital (ρ) so that the level of capital demand is not affected by capital tax changes since the effects of the relative price and the net return of capital offset. This result is in line with Becker and

[3]These conditions are all assumed in Bucovetsky (1991).

[4]If instead to have two different markets, which is a consequence of the fact that each country is specialized in the production of a distinct good, we have a world or a common product market, that is goods a and b are essentially the same, then market equilibria write:

$$f\left(k^A\right) + f\left(k^B\right) = c_a^A + c_a^B + g^A + c_b^B + c_b^A + g^B.$$

In this case the capital demand may be arbitrary (depending on the taxes).

Runkel (2012)[5]. This is a key difference with the standard tax competition models in which the capital tax base is affected by a tax rate modification.

The arbitrage condition allowing for the capital market equilibrium yields:

$$p_a - p_b = \frac{t^A - t^B}{f'} \tag{9}$$

so that the difference in prices directly depends on the difference in taxes, the marginal production being fixed due to the adjustments of the prices.

The country i's representative consumer chooses his level of consumption of both goods so as to maximize his welfare function subject to his budget constraint. The maximization program in both jurisdictions gives the following relationships:

$$\frac{\alpha}{1-\alpha}\frac{c_b^A}{c_a^A} = \frac{p_a}{p_b} = \frac{1-\beta}{\beta}\frac{c_b^B}{c_a^B} \tag{10}$$

and consumptions in goods a and b are given by:

$$c_b^A = \frac{(1-\alpha)}{p_b}\left[p_a f - t^A\overline{k}\right]; \; c_a^A = \frac{\alpha}{p_a}\left[p_a f - t^A\overline{k}\right]$$

$$c_a^B = \frac{(1-\beta)}{p_a}\left[p_b f - t^B\overline{k}\right]; \; c_b^B = \frac{\beta}{p_b}\left[p_b f - t^B\overline{k}\right]$$

Contrary to Wilson (1986) and Becker & Runkel (2012), relative prices are not sufficient to specify the equilibrium of the economy so that one of the price cannot be set as a numeraire. Indeed, our equilibrium requires not only the equilibrium on the capital and product markets but also on the external market through the balance of trade. Both the difference in prices and the relative prices matter in the analysis.

Let us briefly develop the symmetric case, which is commonly studied in most of the tax competition models.

Proposition 1. *With symmetric countries,*

i) when $\mathbf{C}_0$ *holds, the symmetric Nash equilibrium tax rate is given by* $t^{A*} = t^{B*} = t = (1-\eta)\frac{f'}{\varepsilon_k}$ *and tax competition with trade induces an optimal provision of public goods in both countries.*

[5]In Wilson (1987), the fixed stock of capital is given by the assumptions of the model and does not result from the market equilibrium.

ii) when $\mathbf{C}_0$ *does not hold, the symmetric Nash equilibrium tax rate is given by* $t^{A*} = t^{B*} = t = f'$ *and tax competition with trade induces an under provision of public goods in both countries.*

with $\mathbf{C}_0 : \varepsilon_k \geq 1 - \eta$ *and* $\varepsilon_k = \frac{f'\overline{k}}{f}$ *stands for the production function elasticity of capital.*

Proof. See Appendix 1 □

When the production elasticity dominates the preference for the public good, the condition ensuring the balance of trade equilibrium restores the efficiency of the capital tax rates at the symmetric equilibrium. Both countries being perfectly symmetric, the balance of trade equilibrium requires $t^A = t^B$ so that $p_a = p_b$ and $\frac{p_a}{p_b} = 1$. The provision of the public good is optimal ($MRS^i = 1$) because the strategic effects implied by the standard tax competition mechanism is canceled by the prices adjustment. The symmetry in preferences for the national good does not distort the external market in favor of one of the countries[6]. This result is consistent with Becker and Runkel (2012) while the mechanisms allowing for the optimality of the public good provision is different. In the Becker and Rundel's paper, the price adjustment works through the existence of transport costs.

Proposition 1 also highlights that the optimality of the public good provision is no longer valid when the preference for the public good dominates the production elasticity. The high taste for the public good relative to the production elasticity implies an underprovision of the public good at the equilibrium since more public goods would be desired by the agents but the arbitrage condition limits its maximum level. Note that the level of the production elasticity ε_k is fixed since the level of capital remains unchanged. In addition, $1 > \varepsilon_k > 0$ due to decreasing returns to scale.

3 Asymmetric Countries

In this section, we state $\alpha > \beta$ so that countries differ in their preferences for the national good relative to the imported good. In other words, country A's agent values more the

[6]We follow the standard literature by defining the inefficient provision of public goods an allocation characterized by the inequality between the marginal rate of substitution (MRS) and the marginal rate of transformation (MRT) (see Atkinson and Stern (1974)). Here $MRT = 1$.

national good than the country B's agent. This creates an asymmetry in the trade market that may induce a kind of leadership in the fiscal decision in favor of country A.

Combining Equations (5), (6), (7) and (10) we obtain:

$$p_b c_b^A = (1-\alpha)\left[p_a f - t^A \overline{k}\right] = (1-\beta)\left[p_b f - t^B \overline{k}\right] = p_a c_a^B \tag{11}$$

Condition (11) together with condition (1) given the level of capital (8) allow us to characterize the prices p_a and p_b as functions of the taxes:

$$p_a = \frac{(1-\beta)\left(t^A - t^B\right)(1-\varepsilon_k)}{f'(\alpha-\beta)} + \frac{\varepsilon_k t^A}{f'} \tag{12}$$

$$p_b = \frac{(1-\alpha)\left(t^A - t^B\right)(1-\varepsilon_k)}{f'(\alpha-\beta)} - \frac{\varepsilon_k t^B}{f'} \tag{13}$$

which implies

$$\frac{dp_a}{dt^A} = \frac{dp_b}{dt^A} + \frac{1}{f'} \text{ and } \frac{dp_b}{dt^B} = \frac{dp_a}{dt^B} + \frac{1}{f'} \tag{14}$$

Expressions (12) and (13) show that a large asymmetry between countries impact the equilibrium prices. Moreover, for given tax rates, a large asymmetry impacts more p_a than p_b since it stimulates the demand for good a at the expense of good b. Since production of both goods is fixed to maintain the capital market equilibrium, prices adjust so that the trade balance stays in equilibrium.

From the expressions above, we directly derive the following lemma:

Lemma 1. *Positive prices requires* $t^A \geqslant t^B$

Proof. $p_a > 0 \iff \left(t^A - t^B\right) > -\frac{\varepsilon_k}{(1-\varepsilon_k)}\frac{(\alpha-\beta)}{(1-\beta)}t^A$
$p_b > 0 \iff \left(t^A - t^B\right) > \frac{\varepsilon_k}{(1-\varepsilon_k)}\frac{(\alpha-\beta)}{(1-\alpha)}t^B \geqslant 0$ for any $t^B \geqslant 0$ □

In country A, households have stronger preferences for the national good which implies a trade advantage for country A and the possibility to set a higher tax rate without supporting capital outflows or trade imbalance. As a result, the difference in prices is always positive (Equation (9)). An increase in the country A's capital tax implies an increase in the difference in prices whereas an increase in the country B's capital tax implies a decrease in the difference in prices. The difference in prices adjusts so as to maintain an equal stock of capital in each country. Note that the difference

in prices does not depend on the parameters that characterize the asymmetry between countries (α and β).

Let us derive the impact of the capital taxes on the prices. With $\alpha > \beta$, we obtain:

$$\begin{aligned}
\frac{dp_a}{dt^A} &= \frac{f}{\Omega}\left((\alpha-\beta)+(1-\alpha)(1-\varepsilon_k)\right) > 0;\ \frac{dp_a}{dt^B} = \frac{(1-\beta)}{\Omega} f(\varepsilon_k - 1) < 0 \\
\frac{dp_b}{dt^A} &= \frac{(1-\alpha)}{\Omega} f(1-\varepsilon_k) > 0; \qquad (15) \\
\frac{dp_b}{dt^B} &= \frac{f}{\Omega}\left((\alpha-\beta)-(1-\beta)(1-\varepsilon_k)\right) \begin{matrix} > \\ = \\ < \end{matrix} 0 \qquad (16) \\
\frac{d\left(\frac{p_a}{p_b}\right)}{dt^A} &= \frac{t^B\overline{k}}{ff'}\frac{1}{p_b^2} > 0; \qquad \frac{d\left(\frac{p_a}{p_b}\right)}{dt^B} = -\frac{t^A\overline{k}}{ff'}\frac{1}{p_b^2} < 0
\end{aligned}$$

with $\Omega = ff'(\alpha-\beta) > 0$

The increase in tax in one country has clear-cut effects on the price of the foreign good. A rise in the tax rate of country A undoubtedly increases the price of good b by decreasing the demand from country A in good b and increasing the difference in prices to maintain the capital market equilibrium. In order to maintain the balance trade in equilibrium, p_b increases. The impact of t^B on p_a is negative because t^B tends to decrease the difference in prices. The impact of t^A on p_a implies an additional argument based on the asymmetry between countries: the negative direct impact on c_b^A is relatively smaller compared to the positive impact on c_b^A derived from the difference in prices. In order to adjust the balance of trade, p_a has to increase. Finally, contrary to the difference in prices, the relative price reaction to a change in taxes depends on the asymmetry between countries through the level of p_b. The bigger the asymmetry, the higher the relative price reaction to a change in taxes.

The impact of t^B on p_b is not clear-cut. Let us derive the following lemma:

Lemma 2. *When the asymmetry between countries is strong enough,* $\frac{dp_b}{dt^B} > 0$.

Proof. Directly from Equation (16) we deduce:

$$\frac{dp_b}{dt^B} > 0 \Longleftrightarrow (\alpha-\beta) > (1-\beta)(1-\varepsilon_f)$$

□

Due to the asymmetry, the negative impact on c_a^B following a rise in t^B can be either higher or smaller than the negative impact on c_b^A resulting from the decrease in the difference in prices. This depends on the size of the asymmetry relative to the production elasticity. When the asymmetry between countries is high, the impact on c_b^A is rather limited so that the impact on c_a^B is relatively high and p_b has to increase to reestablish the balance of trade equilibrium. The opposite result applies when the asymmetry between countries is rather small.

According to our model, capital supply $\overline{k}$ is fixed, whenever the solution of (1) and (8) requires that capital earns a non-negative net return, $\rho \geq 0$. When $\rho < 0$, we assume that capital owners do not supply any capital:

Claim 1. *When $\rho < 0$, the net return of capital is negative so that the capitalists keep their capital outside the economy. Utilities of countries A and B vanish.*

Before determining the optimal level of taxes we have to specify the set of strategies. According to the previous claim, the following lemma defines the sets of strategies:

Lemma 3. *The profile of strategies $\mathbf{t} = \left(t^A, t^B\right)$ is defined on $\left[t^B \frac{(1-\beta)}{(1-\alpha)}, T\right] \times \left[0, \frac{(1-\alpha)}{(1-\beta)} t^A\right]$*

Proof. See Appendix 2. □

The sets of strategies are deduced from the different constraints of the model i.e. positive consumptions and non negative returns to capital. The asymmetry in preferences for the traded good allows country A to set a higher tax. With no particular restrictions, $T \to \infty$. If restrictions on taxes or price levels are given, T can be bounded. Even if $T \to \infty$ appears to be a particular unrealistic and peculiar case, let us insist on the fact that only the difference in taxes and therefore the difference in prices and relative prices matter.

Claim 2. *When $\rho = 0$ the amount of capital in each country is equal to $\overline{k}$.*

Proof. When $t^A > \frac{(1-\beta)}{(1-\alpha)} t^B$ that implies $\rho > 0$, we have $k_i = \overline{k}$. Then $\lim k_i = \overline{k}$ when $t^A \longrightarrow \left[\frac{(1-\alpha)}{(1-\beta)} t^B\right]^+$ so that $k_i = \overline{k}$ for $\rho = 0$. □

The analysis of the marginal rate of substitution between the public and the private goods is particularly important to analyze the distortive effects of preference asymmetry on the public good provision.

Lemma 4. *Asymmetric countries with $\alpha > \beta$ imply $MRS^B \gtrless MRS^A$ with $\frac{\partial MRS^A}{\partial t^A} > 0$, $\frac{\partial MRS^B}{\partial t^A} > 0$, $\frac{\partial MRS^A}{\partial t^B} < 0$ and $\frac{\partial MRS^B}{\partial t^B} < 0$*

Proof.

$$MRS^A = \frac{\partial U^A/\partial g^A}{\partial U^A/\partial c_a^A} = \frac{1-\eta}{\eta}\frac{\left[p_a f - t^A\overline{k}\right]}{t^A\overline{k}} = \frac{1-\eta}{\eta}\frac{(1-\beta)\left(t^A - t^B\right)\left(f - f'\overline{k}\right)}{t^A\overline{k}f'(\alpha-\beta)}$$

and

$$MRS^B = \frac{\partial U^B/\partial g^B}{\partial U^B/\partial c_b^B} = \frac{1-\eta}{\eta}\frac{\left[p_b f - t^B\overline{k}\right]}{t^B\overline{k}} = \frac{1-\eta}{\eta}\frac{(1-\alpha)\left(t^A - t^B\right)\left(f - f'\overline{k}\right)}{t^B\overline{k}f'(\alpha-\beta)}$$

$$\begin{array}{rcl} MRS^A & > & MRS^B \\ & \Longleftrightarrow & \\ \frac{1-\eta}{\eta}\frac{(1-\beta)\left(t^A - t^B\right)\left(f - f'\overline{k}\right)}{t^A\overline{k}f'(\alpha-\beta)} & > & \frac{1-\eta}{\eta}\frac{(1-\alpha)\left(t^A - t^B\right)\left(f - f'\overline{k}\right)}{t^B\overline{k}f'(\alpha-\beta)} \\ t^B & > & t^A\frac{(1-\alpha)}{(1-\beta)} \end{array}$$

which is a condition that cannot be satisfied according to the constraint of the model (see proof of lemma 3).

The signs of $\frac{\partial MRS^A}{\partial t^A}$, $\frac{\partial MRS^B}{\partial t^A}$ and $\frac{\partial MRS^B}{\partial t^B}$ are obvious from the expressions above. Finally, $\frac{\partial MRS^A}{\partial t^A} = \frac{1-\eta}{\eta}\frac{(1-\beta)\left(f-f'\overline{k}\right)}{\overline{k}f'(\alpha-\beta)}\frac{t^B}{(t^A)^2} > 0$ □

Before determining the properties of the Nash equilibrium, let us define the equilibrium:

Definition 1. *A profile $\mathbf{t}^* = \left(t^{A*}, t^{B*}\right)$ is a Nash equilibrium of the game $\Gamma\left(2, \left[t^B\frac{(1-\beta)}{(1-\alpha)}, T\right] \times \left[0, \frac{(1-\alpha)}{(1-\beta)}t^A\right]\right)$, if none of the unilateral deviation is profitable $\forall i = A, B$.*

Let us assume that a Nash equilibrium exists and let us denote this equilibrium by $\left(t^{A*}, t^{B*}\right)$[7]. The following propositions will determine the nature of the equilibrium.

Proposition 2. *An equilibrium with interior solutions for both countries does not exist.*

[7]The question of the existence of a Nash equilibrium is very difficult and complex. In this paper we specifically focus on the impact of trade on the tax competition results.

Proof. See Appendix 3 □

An interior equilibrium for country A requires a marginal rate of substitution higher than 1 because an increase in the country A's capital tax implies a decrease in the public good g^A due to a high elasticity of p_a $\left(\frac{\partial p_a}{\partial t^A}\frac{t^A}{p_a}\right)$. The opposite mechanism works for country B: a tax rate increase in country B implies a rise in the public good g^B due to a negative or low elasticity of price p_b. Hence, an interior solution in country B requires a marginal rate of substitution lower than 1. According to Lemma 4, both conditions cannot be fulfilled at the same time. As a result, one of the tax rate will be constrained by the boundary of its set of strategies. The asymmetry between countries works as an additional constraint that avoids one of the country to reach an optimal interior solution.

Proposition 3. *Assume that condition* $\mathbf{C}_1$ *holds (large asymmetry between countries). The asymmetric Nash equilibrium is characterized by* $t^{A*} = T$ *and* t^{B*} *solution of* $\frac{\partial V^B}{\partial t^B} = 0$ *with* $t^{A*} = T$.

where $\mathbf{C}_1 : (\alpha - \beta) > \left(\frac{1}{\varepsilon} - 1\right)(1 - \beta)\frac{(1-\eta)}{\eta}$

Proof. see Appendix 4 □

Because of the high price elasticity of good a, the provision of the public good in country A decreases with an increase in t^A.[8] This works for the good a consumption on the good market. When the asymmetry between countries is large, the marginal rate of substitution $\left(MRS^A\right)$ is lower than one and the increase in the private goods, thanks to the price adjustment, always dominates the decrease in the public good in the utility function. Then, the welfare of country A's agent increases whatever the level of t^A. The country A tax rate is set to its maximum. In country B, the government limits the level of t^B relative to t^A since a too high level of t^B would imply a higher level of public good at the expense of the private one.

Corollary 1. *Under condition* $\mathbf{C}_1$ *(large asymmetry between countries) tax competition with trade induces an over provision of public good in both countries.*

Proof. According to Lemma 2, we know that $MRS^B > MRS^A$.

In Appendix 3, it is shown that an interior solution for country B implies $1 > MRS^B$.

Both relations induce $1 > MRS^B > MRS^A$. □

[8] $\frac{dg_A}{dt_A} = \frac{\overline{k}}{p_a}\left(1 - \frac{\partial p_a}{\partial t^A}\frac{t^A}{p_a}\right)$ with $\frac{\partial p_a}{\partial t^A}\frac{t^A}{p_a}$ high.

Asymmetry between countries introduces distortive mechanisms in the provision of public goods but crucially different from the ones observed in the standard tax competition models. Here, distortions are not driven by the capital market but by the external market tensions. With no trade, tax competition implies a low level of tax rates and an underprovision of public goods because of the adjustment on the capital market in order to verify the arbitrage condition. In our model, since country A's tax rate is fixed to its maximum, taxes are distorted upwards which implies an overprovision of public goods in both countries. As discussed above, a large asymmetry between countries requires strong adjustments of prices that are harmful for the economic efficiency.

Proposition 4. *Assume that conditions* $\mathbf{C}_2$ *and* $\mathbf{C}_3$ *hold (small asymmetry between countries), the asymmetric Nash equilibrium is characterized by* $\left(t^{A*}, \frac{1-\alpha}{1-\beta}t^{A*}\right)$ *with* $t^{A*} \neq 0$.
where $\mathbf{C}_2 : \ln(\alpha-\beta) < \frac{\eta}{(1-\eta)}\left[(1-\alpha)\ln(1-\varepsilon_f) - \alpha\ln\frac{(1-\beta)}{(1-\beta)-(1-\alpha)\varepsilon_f}\right] + \ln\left((1-\beta)-(1-\alpha)\varepsilon_f\right)$
and $\mathbf{C}_3 : (\alpha-\beta) < \left(\frac{1}{\varepsilon_k}-1\right)\frac{(1-\eta)}{\eta} - 1$

Proof. See Appendix 5 □

Both conditions $\mathbf{C}_2$ and $\mathbf{C}_3$ specify that the asymmetry between countries is small. A small asymmetry limits the extent of the relative price adjustment to a change in tax rates. This vanishes the unlimited positive effect of the country A tax rate on the agent's utility since a too high level of t^A would imply a negative effect on welfare because a small asymmetry can no longer compensate the negative effect of the public good. Conversely, country B benefits from this small asymmetry, and now, is able to fix a tax rate on the upper bound of its set of strategies without suffering from strong price adjustments. Note that this equilibrium leads to a zero return of capital but a constant level of capital, as specified in Claim 2.

Corollary 2. *Under conditions* $\mathbf{C}_2$ *and* $\mathbf{C}_3$ *(small asymmetry between countries) tax competition with trade induces an underprovision of the public good in both countries.*

Proof. When $t^{B*} = \frac{1-\alpha}{1-\beta}\, t^{A*}$, $MRS^A = MRS^B = \left(\frac{1}{\varepsilon_f}-1\right)\frac{1-\eta}{\eta}$. Condition $\mathbf{C}_3$ immediatly implies $MRS^B = MRS^A > 1$ □

A rather small asymmetry between countries leads to an underprovision of public goods in both countries. Either in a symmetric or an asymmetric case, when the taste

for the public good is high relative to the private good, it leads to an underprovision of the public good since the maximum tax rate is not high enough to ensure an optimal provision of the public good.

4 Centralized equilibrium

In this section, we compare the decentralized with the centralized equilibrium. The centralized equilibrium aims to feature the results that would arise in the case of a centralized European government that would fix the level of the capital tax on behalf of each country. Before determining the properties of such an equilibrium, let us first discuss the symmetric case and the impossibility of tax harmonization when countries are asymmetric.
First, for symmetric countries prices are equal (and so the taxes) so that there is no difference between the centralized and the decentralized equilibrium. Second, for asymmetric countries tax harmonization would imply that the centralized government chooses the level of the uniform tax $\left(t^A = t^B = t\right)$ that maximizes the sum of the welfare. The arbitrage condition (9) implies that the prices should be equal if a uniform tax is implemented. As a result, the trade balance cannot be in equilibrium when the preferences for the national good are asymmetric among countries (Equation (11)). From now, we concentrate on the case of unequal national taxes determined at the centralized level.

In the centralized equilibrium, the social planner aims to maximize the welfare of both representative consumers with respect to their strategic variables t^i. The program of the social planner is the following:

$$\max_{t^A, t^B} V^C \text{ where } V^C = U^A + U^B$$

Comparing the centralized results to the decentralized results leads to the following proposition

Proposition 5. *If V^C is concave, the asymmetric centralized equilibrium implies lower or equal tax rates compared to the Nash equilibrium when either $\mathbf{C}_1$ or $\mathbf{C}_2$ and $\mathbf{C}_3$ hold (large or small asymmetry between countries).*

Proof. See Appendix 6. □

This result is definitely different from the standard tax competition results that highlight the too low level of the tax rates at the Nash equilibrium compared to the centralized choice. An analysis of the impact of the tax rates on the marginal rate of substitution implies the following lemma:

Lemma 5. *A decrease (resp. increase) in* t^A *and* t^B

- *implies an increase (resp. decrease) in the marginal rates of substitution in both countries* (MRS^A *and* MRS^B) *if and only if the tax response elasticity* ($\frac{t^B}{t^A}\frac{dt^A}{dt^B}$) *is lower (resp. higher) than* 1
- *does not modify the marginal rates of substitution in both countries if and only if the tax response elasticity is equal to* 1.

Proof. See Appendix 7. □

Following a decrease in t^B, the increase or decrease in the marginal rates of substitution depends on the *tax response elasticity* $\left(\frac{dt^A}{dt^B}\frac{t^B}{t^A}\right)$[9]. The tax reponse elasticity characterizes the response of country A's tax rate to a change of country B's tax rate. A small tax response elasticity implies a weak reaction of country A's tax rate to a change in country B's tax rate. Since MRS^A responds positively to a decrease in country B's tax rate, this effect dominates the effect on MRS^A of the country A's tax rate response. As a result, the MRS increases in country A. For country B the same mechanism applies.

Proposition 6. *Compared to the Nash equilibrium;*

- *If* $\frac{dt^A}{dt^B}\frac{t^B}{t^A} > 1$, *the centralized equilibrium worsens the overprovision of public goods in Country* A *and* B *when* $\mathbf{C}_1$ *holds.*
- *If* $\frac{dt^A}{dt^B}\frac{t^B}{t^A} < 1$, *the centralized equilibrium worsens the underprovision of public goods in Country* A *and* B *when* $\mathbf{C}_2$ *and* $\mathbf{C}_3$ *hold.*
- *If* $\frac{dt^A}{dt^B}\frac{t^B}{t^A} = 1$, *the centralized equilibrium does not modify the provision of public goods and inefficiencies of public goods provision remain unchanged.*

[9]The term "tax response elasticity" has been introduced by Hindriks and Nishimura (2014) by contrast with the tax base elasticity.

Proof. Using Proposition 6, we know that $t_N^{*A} > t_C^{*A}$ and $t_N^{*B} > t_C^{*B}$ so that from the Nash to the centralized equilibrium we have $\frac{dt^A}{dt^B} > 0$. Given that $\frac{dMRS^B}{dt^B} < 0$ and $\frac{dMRS^A}{dt^A} > 0$, we can deduce that

$$dMRS^B \geqslant 0 \Longleftrightarrow \frac{dt^A}{dt^B}\frac{t^B}{t^A} \leqslant 1 \quad \text{and} \quad dMRS^A \geqslant 0 \Longleftrightarrow \frac{dt^A}{dt^B}\frac{t^B}{t^A} \leqslant 1 \tag{17}$$

Finally, using Corollary 1 and 2, we obtain the Proposition 6 results. □

While a centralized equilibrium is supposed to limit inefficiencies by taking into account the externalities of the taxes, this result is no longer systematically valid in the particular case of asymmetric countries that we developed. This is due to the constraint on the external equilibrium which avoids both a tax harmonization to exist when countries are asymmetric, and a centralized equilibrium to reach the optimum. The introduction of such a constraint enables to avoid any tax competition by maintaining the capital level unchanged in each country; however at the expense of price adjustment that implies tax levels leading to an inefficient provision of public goods even in the centralized equilibrium. The tax response elasticity together with the degree of asymmetry is particularly crucial in determining the degree of inefficiency of the public good provision. Two cases are particularly interesting to comment. When the tax response elasticity is high and the asymmetry is small, the centralized equilibrium tends to exacerbate the overprovision of the public goods. Indeed, a high tax response elasticity implies a decrease in the marginal rate of substitution following a decrease in the tax rates because the country A's tax rate response is high (in absolute value) relative to the country B's tax rate response. Conversely, when the tax response elasticity is lower that 1 and the asymmetry is large, the centralized equilibrium worsens the underpovision. Finally a tax response elasticity equal to one does not modify the inefficiencies of public good provision arising from the Nash equilibrium with a centralized choice. Note that we are not able to give clear-cut results when both the response elasticity and th asymmetry are high or low. Indeed, the centralization may either diminish the inefficiency of the public good provision or imply an overprovision of public good while the Nash equilibrium involved an underprovision (and vice versa).

5 Conclusion

In this paper, we developed a capital tax competition model with trade. We show that the trade balance equilibrium crucially modifies the tax competition mechanisms by maintaining the level of capital unchanged between countries. This may lead to an optimal level of public good provision in a symmetric countries framework. When agents have asymmetric preferences among countries, the Nash equilibrium is conditional to the degree of asymmetry. It may imply either an overprovision or an underprovision of public goods. The link between the trade market and the capital market creates pressure on prices that are not investigated in standard tax competition models. In concordance with several papers that mitigate the benefit of fiscal cooperation among asymmetric countries, we show that a centralized choice may worsen the inefficient provision of public goods. In light with these papers, another conclusion arising from our work is that inefficiencies of public good provision may be mitigated by a centralized choice, but at the expense of trade imbalances. If we try to apply our results for European countries, our paper shows that a centralized choice will not mitigate the inefficiency of public goods provision for asymmetric countries (for example Germany and Ireland) if the tax response elasticity is rather small or equal to one; a scenario which is the more consistent with the European case. The mitigation of the inefficiency may apply under trade imbalances.

6 Appendix

6.1 Appendix 1

Let us assume that $\alpha = \beta$ so that countries are perfectly symmetric.

The constraint on the balance of trade together with the arbitrage condition imply

$$p_a - p_b = \left(t^A - t^B\right) \frac{\overline{k}}{f} = \left(t^A - t^B\right) \frac{1}{f'}$$

so that for any $f'\overline{k} \neq f$ we have $p_a = p_b$, $t^A = t^B = t$ and $g^A = g^B = g$. We deduce

the expressions of the private and public consumptions as

$$\begin{aligned} g^A &= g^B = \frac{t\overline{k}}{p} \\ c_a^A &= c_b^B = \alpha\left(f - \frac{t\overline{k}}{p}\right) \\ c_b^A &= c_a^B = (1-\alpha)\left(f - \frac{t\overline{k}}{p}\right) \end{aligned}$$

Normalizing the common price to the unity ($p = 1$), we deduce that the indirect utility functions denoted by $V^A \equiv V^A(t)$ and $V^B \equiv V^B(t)$ with $t \in [0, f']$ according to the arbitrage condition.

They can be rewritten as

$$V^i = U^i\left[f - g - c_i^j, c_i^j\ ,\ g\right] \text{ with } i = A, B,\ j = A, B \text{ and } i \neq j$$

The symmetric Nash equilibrium is obtained by maximizing the indirect utility function of both countries:

$$\frac{\partial V^i}{\partial t} = -\frac{\partial U^i}{\partial c_i^i}\frac{dg}{dt} + \frac{\partial U^i}{\partial g}\frac{dg}{dt} = \frac{\partial U^i}{\partial c_i^i}\frac{dg}{dt}\left(MRS^i - 1\right) = 0$$

so that

$$MRS^i = 1$$

Since

$$MRS^i = \frac{\partial U^i/\partial g^i}{\partial U^i/\partial c_i^i} = \frac{1-\eta}{\eta}\frac{f-g}{g}$$

$$MRS^i = 1 => t^* = (1-\eta)\frac{f}{\overline{k}}$$

Let us check that $t = (1-\eta)\frac{f}{\overline{k}}$ is an interior solution.

$$(1-\eta)\frac{f}{\overline{k}} \leqslant f' \Longleftrightarrow \varepsilon + \eta \geqslant 1$$

When $\varepsilon + \eta < 1$, $MRS^i > 1\ \forall\ t \in [0, f']$ and the Nash solution is on the boundary so that $t^* = f'$.

To complete the proof we have to check the concavity of the indirect utility functions:

At the equilibrium we have $\frac{\partial(V)^2}{\partial t^2} = \frac{\partial U^i}{\partial c_i^i}\frac{dg}{dt}\frac{\partial MSR^i}{\partial t} = -\frac{\partial U^i}{\partial c_i^i}\left(\frac{dg}{dt}\frac{1}{g}\right)^2\frac{1-\eta}{\eta} < 0$. This condition ensures that there exists only one maximum.

6.2 Appendix 2

A non negative return of capital ($\rho \geqslant 0$) implies

$$p_a f' \geqslant t^A \text{ and } p_b f' \geqslant t^B$$

and replacing p_a and p_b gives the same constraint:

$$t^A \geqslant \frac{(1-\beta)}{(1-\alpha)} t^B$$

Consumptions in goods a and b are given by:

$$c_b^A = \frac{(1-\alpha)}{p_b}\left[p_a f - t^A\overline{k}\right] ; c_a^A = \frac{\alpha}{p_a}\left[p_a f - t^A\overline{k}\right]$$

$$c_a^B = \frac{(1-\beta)}{p_a}\left[p_b f - t^B\overline{k}\right] ; c_b^B = \frac{\beta}{p_b}\left[p_b f - t^B\overline{k}\right]$$

According to the budget constraint of the household, positive consumptions in both countries require

$$p_a f \geqslant t^A\overline{k} \text{ and } p_b f \geqslant t^B\overline{k}$$

Replacing p_a and p_b by their expressions (12) and (13) leads to the same constraint for both countries:

$$\left(t^B - t^A\right)\left(f - f'\overline{k}\right) \leqslant 0 \Longleftrightarrow t^B \leqslant t^A \tag{18}$$

Compiling these conditions gives:

$$t^A \geqslant \frac{(1-\beta)}{(1-\alpha)} t^B > t^B \geqslant 0$$

and

$$0 \leqslant t^B \leqslant \frac{(1-\alpha)}{(1-\beta)} t^A < t^A$$

6.3 Appendix 3

In the case of a non-cooperative game, each government i, $i = A, B$ aims to maximize the welfare of its representative consumer with respect to its strategic variable t^i, taking the tax rate of the other government as given.

$$\frac{\partial V^A}{\partial t^A} = \frac{\partial U^A}{\partial c_a^A}\left[-\frac{dg_A}{dt_A} - \frac{1}{p_a}c_b^A\left(\frac{dp_b}{dt^A} - \frac{p_b}{p_a}\frac{dp_a}{dt^A}\right)\right] + \frac{\partial U^A}{\partial g^A}\frac{dg_A}{dt_A} \tag{19}$$

$$\frac{\partial V^B}{\partial t^B} = \frac{\partial U^B}{\partial c_b^B}\left[-\frac{dg_B}{dt_B} - \frac{1}{p_b}c_a^B\left(\frac{dp_a}{dt^A} - \frac{p_a}{p_b}\frac{dp_b}{dt^A}\right)\right] + \frac{\partial U^B}{\partial g^B}\frac{dg_B}{dt_B} \tag{20}$$

For country A, (19) rewrites

$$\frac{\partial V^A}{\partial t^A} = \frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g_A}{\partial t_A}\left(MRS^A - 1\right) - \frac{1}{p_a}c_b^A\left(\frac{\partial p_b}{\partial t^A} - \frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right)\right]$$

with

$$\left(\frac{\partial p_b}{\partial t^A} - \frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right) = \frac{\left(\beta - \alpha\right)t^B\overline{k}}{\Omega p_a} < 0$$

and

$$\frac{\partial g_A}{\partial t_A} = \frac{\left(1-\beta\right)t^B\overline{k}\left(f'\overline{k} - f\right)}{\Omega\left(p_a\right)^2} < 0$$

An interior solution with positive consumptions requires

$$MRS^A = 1 + \frac{c_b^A\left(\alpha - \beta\right)}{\left(1-\beta\right)\left(f - f'\overline{k}\right)} > 1$$

For country B we obtain

$$\frac{\partial V^B}{\partial t^B} = \frac{\partial U^B}{\partial c_b^B}\left[\frac{\partial g^B}{\partial t^B}\left(MRS^B - 1\right) - \frac{1}{p_b}c_a^B\left(\frac{\partial p_a}{\partial t^B} - \frac{p_a}{p_b}\frac{\partial p_b}{\partial t^B}\right)\right]$$

with

$$\left(\frac{\partial p_a}{\partial t^B} - \frac{p_a}{p_b}\frac{\partial p_b}{\partial t^B}\right) = \frac{\left(\beta - \alpha\right)t^A\overline{k}}{\Omega p_b} < 0$$

and

$$\frac{\partial g_B}{\partial t_B} = \frac{\left(1-\alpha\right)t^A\overline{k}\left(f - f'\overline{k}\right)}{\Omega\left(p_b\right)^2} > 0$$

An interior solution with positive consumptions requires

$$MRS^B = 1 + \frac{c_a^B (\alpha - \beta)}{(1-\alpha)(f'k - f)} < 1$$

Then an interior solution in both countries requires

$$MRS^A > MRS^B$$

which is inconsistent with Lemma 4.

6.4 Appendix 4:

For country A, (19) rewrites

$$\frac{\partial V^A}{\partial t^A} = \frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g_A}{\partial t_A}\left(MRS^A - 1\right) - \frac{1}{p_a} c_b^A \left(\frac{\partial p_b}{\partial t^A} - \frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right)\right]$$

with

$$\left(\frac{\partial p_b}{\partial t^A} - \frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right) = \frac{(\alpha - \beta) t^B \overline{k}}{\Delta p_a} < 0$$

Then if $MRS^A < 1$, we have $\frac{\partial V^A}{\partial t^A} > 0 \ \forall t^A$ for $t^B \neq 0$

$$\begin{aligned} MRS^A &= \frac{(1-\eta)}{\eta}\frac{\left(t^B - t^A\right)}{t^A \overline{k}}\frac{(1-\beta)\left(f - f'\overline{k}\right)}{f'(\beta - \alpha)} \\ &= \frac{\left(t^A - t^B\right)}{t^A}\frac{(1-\beta)}{(\beta - \alpha)}\left(\frac{1}{\varepsilon} - 1\right)\frac{(1-\eta)}{\eta} \end{aligned}$$

since $\frac{\left(t^A - t^B\right)}{t^A} < 1$, a sufficient condition that ensures $MRS^A < 1 \ \forall t^A$ is $\frac{(1-\beta)}{(\alpha-\beta)}\left(\frac{1}{\varepsilon} - 1\right)\frac{(1-\eta)}{\eta} < 1$. Let us denote this condition by $\mathbf{C}_1$ and rewrite it as:

$$\mathbf{C}_1 : \alpha - \beta > \left(\frac{1}{\varepsilon} - 1\right)\frac{(1-\eta)}{\eta}(1-\beta)$$

The best reply for country B is neither $t^B = 0$ which implies $V^B = 0$, nor $t^B = t^A \frac{(1-\alpha)}{(1-\beta)}$ which implies $t^{A*} = t^{B*}\frac{1-\beta}{1-\alpha}$ which contradicts $\mathbf{C}_1$. Then the best response t^{B*} is interior. Let us rewrite $\frac{\partial V^B}{\partial t^B}$

$$\begin{aligned}\frac{\partial V^B}{\partial t^B} &= \frac{\partial U^B}{\partial c_b^B}\left(\frac{1}{p_b}\right)^2 \frac{(1-\alpha)\,t^A\left(f'\overline{k}-f\right)}{\Delta\eta t^B f'\,(\beta-\alpha)\,p_a\Omega}\cdot \\ &\quad \left[\left((1-\eta)\,(1-\alpha)\left(t^B-t^A\right)\left(f-f'\overline{k}\right)-\eta t^B\overline{k}f'\,(\beta-\alpha)\right)p_a\Omega\right. \\ &\quad \left.-\eta t^B\overline{k}\,(1-\beta)\left(t^B-t^A\right)ff'\,(\alpha-\beta)^2\right]\end{aligned}$$

Let denote by $Z\left(t^B\right)$

$$\begin{aligned}Z\left(t^B\right) &= \left[\left((1-\eta)\,(1-\alpha)\left(t^B-t^A\right)\left(f-f'\overline{k}\right)-\eta t^B\overline{k}f'\,(\beta-\alpha)\right)p_a\Omega\right. \\ &\quad \left.-(1-\beta)\,t^B\left(t^B-t^A\right)f\eta\overline{k}f'\,(\alpha-\beta)^2\right]\end{aligned}$$

$Z\left(t^B\right)$ is a second degree polynomial of the form

$$Z\left(t^B\right)=z_1{t^B}^2+z_2t^B+z_3$$

so that $Z\left(t^B\right)=0$ admits only two roots.

The existence of two maxima is impossible since a minimum would necessarily exist between the maxima and more than two roots would exist. The best response t^{B*} is then unique and $t^{B*}\in\left]0,\frac{(1-\alpha)}{(1-\beta)}t^A\right[$.

6.5 Appendix 5:

Let us consider that $\mathbf{C}_1$ does not hold. From Proposition 1 we know that an equilibrium with two interior solutions does not exist. Then at least one strategy of the equilibrium must be on the boundary.

1. $t^B=0$ implies $V^B=0$ so that $t^B=0$ is dominated by any strategies which insure $V^B\neq 0$.

2. Let analyze the case $t^A=T$

   Without particular restrictions, t^A is defined on $\left[t^B\frac{(1-\beta)}{(1-\alpha)},\infty\right]$. Let us compare the values of V^A for $t^A=t^B\frac{(1-\beta)}{(1-\alpha)}$ and $t^A\mapsto\infty$.

- When $t^A = t^B \frac{(1-\beta)}{(1-\alpha)}$ we have

$$p_a = \frac{t^A}{f'} \text{ and } p_b = \frac{t^B}{f'}$$

$$g^A = f'\overline{k}$$

$$c_a^A = \alpha\left[f - f'\overline{k}\right] \text{ and } c_a^A = (1-\beta)\left[f - f'\overline{k}\right]$$

Then the utility writes

$$V^A\left(t^B\frac{(1-\beta)}{(1-\alpha)}\right) = \left(\frac{\alpha^\alpha(1-\beta)^{1-\alpha}\left(f - f'\overline{k}\right)}{\eta}\right)^\eta \left(\frac{f'\overline{k}}{1-\eta}\right)^{1-\eta}$$

- When $t^A \mapsto \infty$ we have

$$\lim_{t^A\mapsto\infty}\frac{p_a}{p_b} = \frac{(1-\beta)f - (1-\alpha)f'\overline{k}}{(1-\beta)f - (1-\beta)f'\overline{k}} > 1$$

$$\lim_{t^A\mapsto\infty} g^A = \frac{ff'(\alpha-\beta)\overline{k}}{(1-\beta)f - (1-\alpha)f'\overline{k}}$$

$$\lim_{t^A\mapsto\infty} c_a^A = \alpha(1-\beta)f\frac{f - f'\overline{k}}{(1-\beta)f - (1-\alpha)f'\overline{k}} \text{ and } \lim_{t^A\mapsto\infty} c_a^A = (1-\alpha)f$$

Then the utility writes

$$\lim_{t^A\mapsto\infty} V^A = \left(\frac{f\left(\alpha(1-\beta)\frac{f-f'\overline{k}}{(1-\beta)f-(1-\alpha)f'\overline{k}}\right)^\alpha(1-\beta)^{1-\alpha}}{\eta}\right)^\eta \left(\frac{f'f(\alpha-\beta)\overline{k}}{(1-\eta)(1-\beta)f-(1-\alpha)f'\overline{k}}\right)^{1-\eta}$$

And we have

$$V^A\left(t^B\frac{(1-\beta)}{(1-\alpha)}\right) > \lim_{t^A\mapsto\infty} V^A$$

$$\iff \left(\left(\frac{(1-\beta)}{(1-\beta)-(1-\alpha)\varepsilon_f}\right)^\alpha\left(\frac{1}{(1-\varepsilon_f)}\right)^{1-\alpha}\right)^\eta\left(\frac{(\alpha-\beta)}{(1-\beta)-(1-\alpha)\varepsilon_f}\right)^{1-\eta} < 1$$

This last condition can be rewritten as:

$$\mathbf{C}_2 : \eta\left[\alpha \ln \frac{(1-\beta)}{(1-\beta)-(1-\alpha)\,\varepsilon_f} - (1-\alpha)\ln(1-\varepsilon_f)\right] + (1-\eta)\ln\frac{(\alpha-\beta)}{(1-\beta)-(1-\alpha)\,\varepsilon_f} < 0$$

Then under Condition $\mathbf{C}_2$, $V^A\left(t^B\frac{(1-\beta)}{(1-\alpha)}\right) > \lim_{t^A\mapsto\infty} V^A$ and since $V^A\left(t^B\frac{(1-\beta)}{(1-\alpha)}\right)$ is of constant value whatever the value of t^B, then there exists a finite $\overline{T}$ such that for any $t^A > \overline{T}$, $\lim_{t^A\mapsto\infty} V^A$ is strictly dominated by $V^A\left(t^B\frac{(1-\beta)}{(1-\alpha)}\right)$.

3. Let us assume $t^A = \frac{1-\beta}{1-\alpha}\, t^B$. At this point we know that we have

$\frac{\partial V^A}{\partial t^A}\left(t^{A*}, t^{B*}\right) < 0$ which implies

$$\begin{aligned}
\frac{\partial V^A}{\partial t^A}\left(t^{A*}, t^{B*}\right) &= \frac{\partial U^A}{\partial c_a^A}\left(\frac{1}{p_a}\right)^2 \frac{t^{B2}\left(f - f'\overline{k}\right)}{\Delta\eta f' f' p_b}(1-\beta)\cdot \\
&\quad \left[\left((1-\eta)\left(f - f'\overline{k}\right)\right) + \eta\overline{k}f'\left(\beta-\alpha-1\right)\right] < 0 \\
&\iff \left((1-\eta)\left(f - f'\overline{k}\right)\right) + \eta\overline{k}f'\left(\beta-\alpha-1\right) > 0 \\
&\iff \frac{1}{\varepsilon_f} - 1 > \frac{\eta}{(1-\eta)}\left(1+\alpha-\beta\right) : \mathbf{C}_3
\end{aligned}$$

According to Appendix 3, an interior solution for B implies $MRS^B < 1$.
For $t^A = \frac{1-\beta}{1-\alpha}\, t^B$, we obtain $MRS^B = \left(\frac{1}{\varepsilon_f} - 1\right)\frac{1-\eta}{\eta} > 1$ under $\mathbf{C}_3$ which eliminates any interior solution for t^B.

Let us check that we also have $\frac{\partial V^B}{\partial t^B}\left(t^{A*}, t^{B*}\right) > 0$

$$\begin{aligned}
\frac{\partial V^B}{\partial t^B}\left(t^{A*}, t^{B*}\right) &= \frac{\partial U^B}{\partial c_b^B}\frac{(1-\alpha)\left(f'\overline{k} - f\right)}{\Delta\eta}\left(\frac{1-\beta}{1-\alpha}\right)\cdot \\
&\quad \left[\left((1-\eta)\left(f - f'\overline{k}\right) - \eta\overline{k}f'\right) - \eta\overline{k}f'\left(\beta-\alpha\right)\right] > 0 \\
&\iff (1-\eta)\left(f - f'\overline{k}\right) - \eta\overline{k}f'\left(1+\beta-\alpha\right) > 0 \\
\frac{1}{\varepsilon_k} - 1 &> \frac{\eta}{(1-\eta)}\left(1+\beta-\alpha\right)
\end{aligned}$$

which is always true under $\mathbf{C}_3$.
As a result, under $\mathbf{C}_3$, when a Nash equilibrium exists, it is given by $\left(t^{A*}, \frac{1-\alpha}{1-\beta}t^{A*}\right)$.

6.6 Appendix 6

- The first derivative of the indirect utility functions V^C are:

$$\begin{aligned}\frac{\partial V^C}{\partial t^A} &= \frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g_A}{\partial t_A}\left(MRS^A-1\right)-\frac{1}{p_a}c_b^A\left(\frac{\partial p_b}{\partial t^A}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right)\right]+\\ &\quad \frac{\partial U^B}{\partial c_b^B}\left[\frac{\partial g^B}{\partial t^A}\left(MRS^B-1\right)-\frac{1}{p_b}c_a^B\left(\frac{\partial p_a}{\partial t^A}-\frac{p_a}{p_b}\frac{\partial p_b}{\partial t^A}\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial V^C}{\partial t^B} &= \frac{\partial U^B}{\partial c_b^B}\left[\frac{\partial g^B}{\partial t^B}\left(MRS^B-1\right)-\frac{1}{p_b}c_a^B\left(\frac{\partial p_a}{\partial t^B}-\frac{p_a}{p_b}\frac{\partial p_b}{\partial t^B}\right)\right]+\\ &\quad \frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g_A}{\partial t_B}\left(MRS^A-1\right)-\frac{1}{p_a}c_b^A\left(\frac{\partial p_b}{\partial t^B}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^B}\right)\right]\end{aligned}$$

- If $\mathbf{C}_1$ holds, $\frac{\partial V^C}{\partial t^B}$ evaluated at the Nash equilibrium, gives

$$\left.\frac{\partial V^C}{\partial t^B}\right|_{Nash}=\frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g^A}{\partial t^B}\left(MRS^A-1\right)-\frac{1}{p_a}c_b^A\left(\frac{\partial p_b}{\partial t^B}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^B}\right)\right] \tag{21}$$

with $\frac{\partial p_b}{\partial t^B}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^B}>0$; $MRS^A<1$; $\frac{dg_A}{dt^B}=-\frac{t^A\overline{k}}{(p_a)^2}\frac{dp_a}{dt^B}>0$; so that $\frac{\partial V^S}{\partial t^B}\left(t_N^{*B}\right)<0$. If a centralized equilibrium exists and V^C is concave, then $\frac{\partial V^S}{\partial t^B}\left(t_N^{*B}\right)<0$ implies that $t_N^{*B}>t_C^{*B}$.

Since the Nash equilibrium implies the highest tax rate for country A, the tax rate at the centralized equilibrium cannot be higher ($t_N^{*A}\geq t_C^{*A}$).

- If $\mathbf{C}_2$ and $\mathbf{C}_3$ hold, $\frac{\partial V^C}{\partial t^B}$ evaluated at the Nash equilibrium, gives

$$\begin{aligned}\left.\frac{\partial V^C}{\partial t^A}\right|_{Nash} &= \underbrace{\frac{\partial U^A}{\partial c_a^A}\left[\frac{\partial g_A}{\partial t_A}\left(MRS^A-1\right)-\frac{1}{p_a}c_b^A\left(\frac{\partial p_b}{\partial t^A}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right)\right]}_{<0}+\\ &\quad \frac{\partial U^B}{\partial c_b^B}\left[\frac{\partial g^B}{\partial t^A}\left(MRS^B-1\right)-\frac{1}{p_b}c_a^B\left(\frac{\partial p_a}{\partial t^A}-\frac{p_a}{p_b}\frac{\partial p_b}{\partial t^A}\right)\right]<0\end{aligned}$$

since $\frac{\partial g^B}{\partial t^A}=\frac{1-\alpha}{\alpha-\beta}\frac{f'\overline{k}}{f}\frac{1}{t^A}\left(f'\overline{k}-f\right)<0$, $MRS^B>1$ and $\left(\frac{\partial p_a}{\partial t^A}-\frac{p_a}{p_b}\frac{\partial p_b}{\partial t^A}\right)=-\frac{p_a}{p_b}\left(\frac{\partial p_b}{\partial t^A}-\frac{p_b}{p_a}\frac{\partial p_a}{\partial t^A}\right)>0$.

Under the assumption that V^C is concave, we can deduce that $t_N^{*A} > t_S^{*A}$. Finally, the collective utility V^C is defined on $\left[t^B \frac{(1-\beta)}{(1-\alpha)}, T\right] \times \left[0, \frac{(1-\alpha)}{(1-\beta)} t^A\right]$. Under $\mathbf{C}_2$ and $\mathbf{C}_3$, we have $t_N^{*B} = \frac{1-\alpha}{1-\beta} t_N^{*A} > \frac{1-\alpha}{1-\beta} t_C^{*A}$ where $\frac{1-\alpha}{1-\beta} t_C^{*A}$ is the upper level that t^B could take in response to t_C^A. This implies that $t_N^{*B} > t_C^{*B}$.

6.7 Appendix 7

According to Lemma 1 and its proof we can write

$$\frac{\partial MRS^A}{\partial t^A} = \frac{1-\eta}{\eta} \frac{(1-\beta)\left(f - f'\overline{k}\right)}{\overline{k} f' (\alpha - \beta)} \frac{t^B}{(t^A)^2} = -\frac{\partial MRS^A}{\partial t^B} \frac{t^B}{t^A}$$

so that the total derivative of MRS^A is given by

$$dMRS^A = \frac{\partial MRS^A}{\partial t^A} dt^A + \frac{\partial MRS^A}{\partial t^B} dt^B = \frac{\partial MRS^A}{\partial t^A} \frac{t^A}{t^B} dt^B \left(\frac{t^B}{t^A} \frac{dt^A}{dt^B} - 1\right)$$

with $\frac{\partial MRS^A}{\partial t^A} > 0$.

For country B

$$\frac{\partial MRS^B}{\partial t^B} = \frac{1-\eta}{\eta} \frac{(1-\alpha)\left(f - f'\overline{k}\right)}{\overline{k} f' (\alpha - \beta)} \frac{t^A}{(t^B)^2} = -\frac{\partial MRS^B}{\partial t^B} \frac{t^B}{t^A}$$

$$dMRS^B = \frac{\partial MRS^B}{\partial t^B} dt^B + \frac{\partial MRS^B}{\partial t^A} dt^A = \frac{\partial MRS^B}{\partial t^B} dt^B \left(1 - \frac{t^B}{t^A} \frac{dt^A}{dt^B}\right)$$

with $\frac{\partial MRS^B}{\partial t^B} < 0$.

For $\frac{t^B}{t^A} \frac{dt^A}{dt^B} = 1$, $dMRS^B = 0$ and $dMRS^A = 0$.

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