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Abstract

We study trade between a buyer and a seller who have existing inventories of assets similar to those being traded. We analyze how these inventories affect trade, information dissemination, and prices. We show that when traders' initial leverages are moderate, inventories increase price and trade volume (a market "run-up"), but when leverages are high, trade is impossible (a market "freeze"). Our analysis predicts a pattern of trade in which prices and volumes first increase, and then markets break down. Moreover, the presence of competing buyers may amplify the increased-price effect. We discuss implications for regulatory intervention in illiquid markets.

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1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than other market participants. One consequence, which has been much discussed, is that he will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade, information dissemination, and price formation. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset being traded.

Our main result is that the effect of inventories on trade depends on the buyer's and seller's initial leverage. When leverage is moderate, inventories increase price and trade volume (a market "run-up"); but when leverage is high, trade may become impossible (a market "freeze"), and information dissemination ceases. In a dynamic extension, our model predicts a pattern of trade in which prices and trade volume first increase, and then markets break down.

The intuition is as follows. In our setting, the motive for trade is that the buyer values the asset more than the seller does. Since the seller has an information advantage, a sale reveals information about the value of the traded asset. This information may be used to reassess the value of inventoried assets and the amount of leverage that the buyer and seller can take. To ensure neither agent violates his capital constraint (i.e., that the market value of each agent's assets is greater than the value of his liabilities), the buyer may need to offer a higher price, which increases the market's posterior about the value of the asset, and hence of inventories. The higher price also increases the probability that the seller will accept the offer. Hence, both prices and expected trade volume may increase. However, when the capital constraint is too tight, the buyer can no longer increase the price without

[1] See, for example, Ashcraft and Schuermann (2008); and Downing, Jaffee and Wallace (2009).

losing money. At this point, trade collapses and market participants learn nothing about asset values.

It is useful to compare the market freeze result above to that obtained in a standard lemons problem with gains from trade that are common knowledge, but without inventories and capital constraints. In this standard problem, trade does not break down completely as follows. If the buyer offers a price close to the minimum possible seller valuation, the information rent he surrenders to the seller is small, and in particular, it is smaller than the direct gains from trade. Consequently, there is a strictly positive probability that trade occurs. In contrast, when the buyer or seller is capital constrained and concerned about the effect of information revelation on the value of his inventories, there is an additional cost associated with the seller accepting a low offer, namely that it reveals that the asset's value is very low, which negatively impacts the buyer's and seller's capital constraints. Because of this additional cost, trade may break down *completely*, in the sense that there is zero probability of trade occurring.[2]

The notion that market participants adjust their trading behavior with an eye to influencing the dissemination of information is most natural when markets are "thin." Accordingly, in our baseline model we assume that there is a single potential buyer, who hence enjoys a monopoly position. Interestingly, our results continue to hold even when markets become "thicker," in the sense that the number of potential buyers increases. Some of the results are actually strengthened. In particular, a buyer may be forced to raise his bid not only because he is leveraged, but also because a *competing* buyer is leveraged; and he may even be forced to acquire assets at a *loss-making* price, just to make sure that a competing buyer does not acquire them at a lower price. The key insight here is that a purchase by one buyer may lead to the release of information that causes a violation of the capital constraint of a competing buyer, and this may force the competing buyer to increase the price. In each of the cases above, competition and capital constraints combine to push the price strictly higher than would be the case under either competition alone, or a single buyer with a capital constraint. However, when all buyers are highly leveraged, concerns about inventory valuation again lead to a market freeze and prevent the dissemination of information about

[2] An alternative explanation for a market freeze in situations in which there are gains from trade involves Knightian uncertainty; see, e.g., Easley and O'Hara (2010).

asset quality, just as in the single-buyer case.

Our baseline results, which are obtained in a static setting with a single round of trade, are suggestive of a dynamic process in which buyers increase leverage and prices until the market eventually breaks down. In a dynamic extension of our basic framework, we model this process explicitly and show that a market freeze may be preceded by a run-up in prices. This result is interesting because the run-up in prices occurs even though (by assumption) the underlying fundamentals remain unchanged. In this sense, the run-up shares features of a "bubble." In our model, this result reflects the fact that increasing inventories force the buyer to increase his bid. In particular, when the buyer adds assets to his balance sheet, he reduces the market value of his existing assets and increases his leverage. This forces him to bid a higher price in the next trade, or else not bid at all.

We use our model to discuss implications for regulatory intervention in illiquid markets. On the buyer's side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example); one implication is that by purchasing assets, the government may impose a cost on potential buyers who choose not to trade. On the seller's side, our analysis suggests potential limitations to the standard prescription that sellers should retain a stake in the assets they sell. We also relate the model's predictions to the freeze in the market for mortgage-backed securities during the recent financial crisis. Finally, we obtain some new testable implications regarding the relationship between broker-dealers' inventories and prices.

As a technical contribution, we show that out of all possible trading mechanisms, the one that maximizes the payoff of a monopolist buyer with inventories and a capital constraint is the simple mechanism in which the buyer makes a take-it-or-leave-it offer to buy up to q units of the asset at a price per unit p (the buyer selects both p and q). This result generalizes a classic result of Samuelson (1984), who analyzes essentially the same setting in the absence of capital constraints and inventories. Moreover, this result implies there is no loss in focusing on linear price schedules.

Related literature. Our paper relates to the literature on trade under asymmetric information, in which the seller is better informed and the gains from trade are common knowledge (e.g., Samuelson, 1984). As noted earlier, absent inventories and capital constraints, the market may *partially* break down in the sense that there is a positive probability that effi-

cient trade does not take place; however, the market does not break down *completely*, as in our paper.

When there are multiple buyers, the combination of capital constraints and inventories generates a situation in which the fact that the seller trades with one buyer has externalities for other buyers. In contrast to existing auction-theoretic papers dealing with externalities (e.g., Jehiel, Moldovanu, and Stacchetti, 1996), the externality depends on the price paid, rather than simply whether another buyer obtains the asset.

Two recent papers obtain periods of no trade in a dynamic lemons problem.[3] To do so, they add the assumption that some noisy information about the asset quality is revealed (exogenously). In Kremer and Skrzypacz (2007), information is revealed at some future point in time T and there exists $t < T$, such that trade ceases on the time interval $[t, T]$. In Daley and Green (forthcoming), information is revealed gradually. Instead, we obtain a no trade result by adding inventories and capital constraints to a standard lemons problem. We also show that concerns about the value of inventories can *increase* the probability of trade, as potential buyers are induced to increase the price. In our setting, trade is always efficient, and so increasing the price increases welfare. In this sense, our paper differs from papers in which price manipulation creates distortions that are suboptimal from a social point of view.[4]

Our paper also relates to the literature that explores the link between leverage and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices, while Diamond and Rajan (2011) show that the prospect of fire sales may lead to a market freeze. In their model, banks do not sell their assets because the gains from selling are captured by the bank's creditors rather than by the bank's equity holders. Other papers explore feedback effects between asset prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also; see, e.g., Kiyotaki and Moore (1997). In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Therefore, there is no need for fire sales or "cash in the market" pricing, as in Allen and Gale (1994). The only motive for trade in our paper is that the

[3] Also related, in a recent paper, Glode, Green, and Lowery (forthcoming) endogenize the extent of adverse selection in a *static* standard lemons problem, showing that firms may overinvest in financial expertise. The outcome of this is that if uncertainty increases, the probability of efficient trade is reduced.

[4] Examples include Allen and Gale (1992); Brunnermeier and Pedersen (2005); Goldstein and Guembel (2008).

buyer values the asset more than the seller does, and both agents know this.

In a contemporaneous paper, Milbradt (2012) shows that a trader who is subject to a mark-to-market capital constraint (i.e., one based on the last trade price rather than on the actual expected value of the asset) may suspend trade so that losses are not revealed. While the general idea relates to ours, there are big differences, including the following. First, in Milbradt (2012) the price is exogenous, whereas in our setting, the price is endogenous. This allows us to obtain predictions regarding the relationship between prices, inventories, and leverage. Second, in Milbradt (2012), trade suspension is bad news, whereas in our setting, a market freeze is neither good news nor bad news. The key difference is that in Milbradt (2012), the agent who acts strategically to prevent information dissemination is the informed agent, whereas in our setting, the agent who tries to prevent information dissemination (the buyer) is uninformed. Third, our results do not depend on a specific accounting or regulatory regime. Instead, the market value of existing assets is derived from Bayes' rule. Our main results continue to hold, however, even if we assume marking to market. Hence, our model is consistent with the view that marking to market is to blame for many of the phenomena discussed earlier, but it also predicts that one would see qualitatively the same phenomena even without marking to market.

The idea that inventory holdings affect price-setting behavior relates to market microstructure papers that study the effect of market-maker inventories on prices; see, e.g., Amihud and Mendelson (1980); Ho and Stoll (1981, 1983). These papers assume symmetric information and are therefore silent with respect to our main results. Moreover, these papers predict that as inventories increase, prices fall—a prediction that seems inconsistent with the empirical findings in Manaster and Mann (1996). In contrast, by interpreting buyers in our model as market-makers, our model predicts that higher inventories may lead to higher prices. The reason for the opposite predictions is as follows. In classic inventory models, the dealer wants to reduce the price when he has more inventories because he is either risk averse and concerned about future price movements, or else he is not allowed to carry too much inventory. Instead, in our setting, inventories serve as collateral and higher prices increase borrowing capacity. Our model also provides a new testable hypothesis, namely that a price offered by one dealer may increase when *other* dealers hold more inventories.

Finally, our paper relates to the literature on equity issuance, in which the issuing firm

cares about the market valuation of its remaining equity.[5] However, we do not focus on signaling. Instead, we show how leverage affects the bidding strategies of uninformed buyers.

Paper outline. Section 2 describes the model. Section 3 analyzes the simplest case, in which there is one buyer with inventories, and relates the results to the freeze in the market for mortgage-backed securities during the recent financial crisis. Section 4 analyzes a two-period dynamic extension, while Section 5 analyzes the effects of competition between multiple buyers. Section 6 discusses several extensions. Section 7 discusses policy and empirical implications, and Section 8 concludes. The appendices contain proofs and omitted details.

2 The Model

In the basic model, there is a risk-neutral buyer and a risk-neutral seller. The value of an asset is v to the seller and $v + \Delta$ to the buyer, where $\Delta > 0$ denotes the gains from trade. It is common knowledge that v is drawn from a uniform distribution on $[0, 1]$. The seller knows v. Everyone else is uncertain about the value of v. Consequently, trade affects posterior beliefs about v, and hence the market value of each unit of asset. Since $\Delta > 0$, trade is always efficient.

In one interpretation, the seller is a loan originator. The gains from trade may reflect the fact that the buyer has a lower cost than the seller of retaining risky assets on his balance sheet; for example, the buyer may face lower borrowing costs or less stringent regulation. Alternatively, the buyer might be a broker-dealer who helps with the matching process between the seller and other investors, who have higher valuations for the asset.

The seller owns x units of the asset for sale. The buyer has an inventory of M units of the asset, which he acquired earlier. The buyer also has cash and a short-term debt liability. The liability net of cash holdings is L, and so the buyer can roll over his liabilities only if the value of his noncash assets exceeds L. Assume, for simplicity, that the buyer holds only the asset traded. In addition, the purchase of additional units is financed out of existing cash holdings and/or new short-term borrowing.[6]

[5] See, for example, Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).

[6] The nature of the result remains if instead of holding units of the asset traded, the buyer holds units of an asset with correlated payoffs.

Specifically, suppose the buyer purchases q additional units at a price per unit p, and let h denote the "market value" of the asset, defined as the expected value of $v + \Delta$, conditional on the trading outcome, using Bayes' rule. Then the buyer can roll over his debt if

$$h(M + q) \geq L + pq, \tag{1}$$

where $M + q$ is the buyer's total inventory of assets net of trade, and $L + pq$ is the buyer's total liabilities, net of trade. We refer to equation (1) as the buyer's "capital constraint."

In Section 6.2, we discuss the generalization in which h is replaced with αh, reflecting constraints on the buyer's ability to pledge all his future cash flows.[7] In Section 6.3, we discuss the case in which the capital constraint is based on marking to market.

If the buyer violates his capital constraint, he defaults and incurs a cost, which represents lost growth opportunities due to bankruptcy or closure by a regulator. We focus on the case in which the capital constraint is satisfied before trading begins (i.e, when $q = 0$ and $h = \frac{1}{2} + \Delta$, so assets are evaluated at the prior). This assumption allows us to focus on the question of how the buyer changes his behavior to avoid violating the capital constraint, rather than on the much-studied fire sales that follow when the constraints are violated. We also assume that the cost of violating the constraint is sufficiently high so that the buyer's first priority is to satisfy his constraint.[8] Hence, the buyer's objective is to maximize the expected profits from buying the asset subject to not violating his capital constraint.[9]

Finally, we assume that the quantity of the asset available for trade is small relative to the buyer's existing asset holdings, specifically:

Assumption 1 $x < M$

Assumption 1 implies that the buyer's capital constraint is tightened when the seller accepts his offer (see discussion preceding Lemma 1 below). It also implies that increasing the bid loosens the buyer's capital constraint (see discussion preceding equation (3)).

We model trade by using a variant of the seminal Glosten and Milgrom (1985) model of price-setting in markets with asymmetric information, in which the uninformed party (the

[7] In Appendix D, we also analyze the case in which h is the expected value of $v + \gamma\Delta$, for some $\gamma \in [0, 1]$.

[8] For example, for the results in Section 3, it is enough to assume that the cost of violating the constraint is at least $x(1 + \Delta)$.

[9] From the law of iterated expectations, the value of inventoried assets equals its prior and hence does not enter the objective function.

market-maker in their model, here the buyer) posts a "bid" price at which he is prepared to buy. We depart from Glosten and Milgrom by first analyzing the case in which the buyer is a monopolist and then modeling the effects of competition between multiple strategic buyers; in both cases, we assume that the seller is not subject to any capital constraints. In Section 6.1, we show that the basic result extends to the case in which the seller is capital constrained and must retain some fraction of his assets on his balance sheet.

3 A Monopolist Buyer

Formally, when the buyer is a monopolist the variant of the Glosten-Milgrom setting that we use to study price formation is as follows. The monopolist buyer makes a take-it-or-leave-it offer to buy up to $q \in [0, x]$ units of the asset at a price per unit p. The seller responds by choosing a quantity $q' \in [0, q]$ to sell, so as to maximize his profits $q'(p - v)$.[10] In other words, the buyer offers a linear price schedule.

As a preliminary, we note that in the monopolist case there is no loss of generality to assuming that the buyer offers a linear price schedule: he cannot increase his profits by instead offering a nonlinear schedule. Moreover, a take-it-or-leave-it offer of a linear price schedule is the best bargaining mechanism for the buyer out of *all* bargaining mechanisms in which the market observes the final outcome of the mechanism and only the final outcome. This is a generalization of the classic result of Samuelson (1984) to a setting with inventories and capital constraints; the proof is in Appendix C.[11]

Proposition 1 *A monopolist buyer cannot gain by designing a more complicated bargaining mechanism (e.g., offering a nonlinear price schedule).*

We start with the benchmark case $M = 0$, in which the buyer has no inventories. Then we analyze the main case with inventories, $M > 0$.

Note that whenever the seller chooses to sell, it is optimal for him to choose $q' = q$. In this case, we say that the seller accepts the buyer's offer; otherwise (if $q' = 0$), we say that the seller rejects the offer. To ensure that the seller's acceptance decision is nontrivial, we

[10] When $v = p$, and hence the seller is indifferent, we assume he picks $q' = q$.

[11] Our proof does not rely on a uniform distribution for v; the exact details are in Appendix C. Also note that as is standard in the mechanism design literature, we assume that the buyer can commit to using the mechanism he has chosen.

assume that the gains from trade are not too high, $\Delta < \frac{1}{2}$, so that the buyer always offers to pay $p < 1$.[12]

3.1 Buyer Does Not Have Inventories

In the benchmark case, the buyer offers a price-quantity pair (p, q) to maximize his expected profits subject to $q \leq x$. The seller accepts the offer if and only if $v \leq p$, which happens with probability p, since v is uniform on $[0, 1]$. Conditional on the seller accepting the offer, the expected value of the asset to the buyer is $\frac{1}{2}p + \Delta$. Since the buyer pays p, his expected profit per unit bought is $\Delta - \frac{1}{2}p$. Taking into account the probability of trade and the quantity traded, the buyer's expected profit is $\pi(p, q) \equiv pq(\Delta - \frac{1}{2}p)$. The buyer's profit-maximizing bid is to buy everything, $q = x$, for a price $p = \Delta$.

Proposition 2 *In the benchmark case of no inventories, $M = 0$, the buyer offers to buy x units at a price per unit Δ. The seller accepts this offer if and only if $v \leq \Delta$.*

For use below, observe that any $p \in [0, 2\Delta]$ gives the buyer nonnegative profits. Consequently, the buyer has room to increase his bid beyond the benchmark price Δ while still maintaining positive profits. These positive profits stem from the fact that the buyer makes the offer and so has some bargaining power.

3.2 Buyer Cares About the Value of His Inventory

As before, the seller accepts the buyer's offer if and only if $v \leq p$. Accepted offers reduce the market value of the asset and hence of existing inventories. However, the purchase of new units of asset may generate a profit. On net, these two forces tighten the capital constraint, since, by Assumption 1, inventories are large relative to new trades.

Lemma 1 *The acceptance of an offer tightens the capital constraint.*

In contrast, the rejection of an offer relaxes the capital constraint since it increases the market value of the asset. Hence, it is enough to ensure that the capital constraint is satisfied after an offer is accepted. In this case, the market value of the asset drops to

[12] Offering $p \geq 1$ is suboptimal, since the seller always accepts the offer and the buyer obtains an asset with an expected value of $\frac{1}{2} + \Delta < 1$.

$h = E[v|v \leq p] + \Delta = \frac{1}{2}p + \Delta$, and the capital constraint becomes

$$(\frac{1}{2}p + \Delta)(M + q) - pq \geq L. \tag{2}$$

Hence, the buyer's problem reduces to maximizing the expected profit from his trade, $\pi(p, q)$, subject to his capital constraint (2). In cases of indifference, we assume that the buyer makes the bid associated with the highest quantity q, thereby maximizing social welfare.

Define $\delta \equiv \frac{L}{(\frac{1}{2}+\Delta)M}$, a measure of the buyer's initial leverage (i.e., the ratio of his net liabilities to the initial market valuation of his assets). Since $q \leq x < M$ (Assumption 1), the buyer's capital constraint (2) can be rewritten as

$$p \geq \frac{\delta + 2\Delta(\delta - 1 - \frac{q}{M})}{1 - \frac{q}{M}} \equiv p(q), \tag{3}$$

where $p(q)$ is the minimum price that the buyer must offer to keep his capital constraint satisfied if the seller accepts the offer. In other words, increasing p loosens the capital constraint. Increasing the price increases the market value of existing inventories, which helps loosen the capital constraint, but it also increases the amount the buyer pays for the additional units he purchases, which tightens the capital constraint. When the amount of inventories is large relative to the amount for sale (Assumption 1), the first effect dominates.

Buying more assets also loosens the capital constraint. This follows from the assumption that the buyer can borrow against the full value of his assets. Formally, since the buyer makes nonnegative profits, we know $p \leq 2\Delta$, and so the left-hand side of equation (2) is increasing in q. Consequently, if the buyer finds it worthwhile to bid at all, he bids for the entire quantity available, $q = x$. Bidding for a lower quantity not only lowers the buyer's profits, but it also makes it harder to satisfy his capital constraint. In Section 6.2, we show that if the buyer can borrow only against a fraction of the market value of his assets, it may no longer be the case that increasing q loosens the capital constraint. In this case, we may obtain an interior solution in which the buyer offers to buy less than the full amount.

The buyer's problem reduces to choosing p to maximize his expected profits $\pi(p, x)$, such that $p \geq p(x)$. Since the buyer loses money from bids $p > 2\Delta$, trade is impossible if $p(x) > 2\Delta$, which reduces to $\delta > \frac{4\Delta}{1+2\Delta}$. If instead $\delta \leq \frac{4\Delta}{1+2\Delta}$, the buyer bids as close to his benchmark bid of Δ as possible; that is, $p = \max\{\Delta, p(x)\}$

Proposition 3 *When the buyer cares about the value of his inventories, trade can happen if and only if leverage is low, i.e.,* $\delta \leq \frac{4\Delta}{1+2\Delta}$. *In this case, the buyer offers to buy* x *units at a price per unit* $\max\{\Delta, p(x)\}$.

When the buyer's initial leverage is low, the price and the probability of trade are the same as in the benchmark case because the buyer has enough slack to satisfy his capital constraint even though purchasing assets tightens it. When leverage increases, so that the buyer has less slack, the buyer must increase his bid to ensure that his capital constraint is satisfied if the seller accepts the offer. Since a higher bid increases the probability that the seller will accept the offer, the probability of trade increases. Finally, if leverage is too high, the market breaks down because any bid that is high enough to satisfy the buyer's capital constraint provides him with negative expected profits.

We focus on an extreme case in which the buyer has all of the bargaining power, but the nature of the result remains even if the buyer has only some of the bargaining power. In particular, the result that, for some parameter values, the price increases in leverage depends on the fact that in the benchmark case the buyer can capture some of the surplus, and so when his leverage increases, he can increase the price, while still maintaining positive profits.

An immediate corollary to Proposition 3 concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

Corollary 1 *If initial leverage is high,* $\delta > \frac{4\Delta}{1+2\Delta}$, *market participants learn nothing about the value* v *of the asset.*

Finally, note that in the analysis above we implicitly assumed that market participants observe the terms of the buyer's offer only if the offer is accepted and trade actually occurs. The equilibrium outcomes are exactly the same under the alternative assumption that market participants observe the offer terms even when the offer is rejected. To see this, note that rejection of *any* offer increases the market's expectation about the asset value. Consequently, if the market observes no trade, the capital constraint is at least weakly slackened. Hence, the important capital constraint is the one following an accepted offer, which is observed.

3.3 Discussion

Our model implies that socially efficient trade can completely break down ("freeze") if the seller has an information advantage and if the buyer is both highly leveraged and holds significant inventories of similar assets. In Section 6.1, we show that trade can also break down when the seller is highly leveraged and must retain some assets on his balance sheet.

These implications are consistent with the freeze in the markets for mortgage-backed securities during the recent financial crisis. Adrian and Shin (2010) document a sharp increase in dealers' leverage, while many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*[13] as saying that "Other [companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio." Also related is the view expressed in Lewis's book (2010) that dealers who sold credit default swaps on subprime mortgage bonds did not make a market in these securities so that information is not revealed and their positions do not lose money. Moreover, and consistent with our results in Section 4 below, Lewis suggests that prior to the crisis, prices increased in a way not supported by fundamentals.[14]

4 Run-ups and breakdowns

The static model is suggestive of a dynamic process in which the buyer increases leverage and prices until the market breaks down eventually. To model this explicitly, we extend our single-period model to a two-period model in which the monopolist buyer trades sequentially with two potential sellers. Each seller sells a different asset, and the values of the two assets are assumed to be independent. Hence, one cannot infer anything about the value of one asset by observing trade in the other asset. This allows us to focus only on the effect of leverage, which changes endogenously: The outcome of trade with the first seller affects the

[13] "Nonperformance Space: Risky Assets Find Market" (*American Banker*, August 19, 2009).

[14] For example, on page 184, Lewis writes that "Burry [an investor who bought credit default swaps on subprime mortgage bonds] sent his list of credit default swaps to Goldman and Bank of America and Morgan Stanley with the idea that they would show it to possible buyers, so he might get some idea of the market price. That, after all, was the dealer's stated function: middleman. Market-makers. That is not the function they served, however. 'It seemed the dealers were just sitting on my lists and bidding extremely opportunistically themselves,' said Burry. The data from the mortgage servicers was worse every month...and yet the price of insuring those loans, they said, was falling." On page 185, he adds that "The firms always claimed that they had no position themselves...but their behavior told him otherwise."

buyer's leverage before he trades with the second seller. One of our results is that a market freeze may be preceded by a run-up in prices and increased trade volume.

Specifically, seller i ($i = 1, 2$) sells asset i and can trade only in period i. The value (per unit) of asset i is v_i to the seller and $v_i + \Delta$ to the buyer, where v_1, v_2 are independent random variables drawn from a uniform distribution on [0,1]. Each seller can sell at most x units. Before trading begins, the buyer has inventories of M units of asset 1 and M units of asset 2. In the first period, the buyer makes an offer (p_1, q_1) to the first seller, who can either accept or reject the offer, and in the second period, the buyer makes an offer (p_2, q_2) to the second seller, who can also either accept or reject it. For simplicity, we assume that the discount rate equals zero.[15]

Assume that $q_i \in \{0\} \cup \left[\underline{q}, x\right]$; that is, if the buyer offers to buy something, he must buy at least $\underline{q} > 0$ units. This assumption has no substantive effect on the results in the previous sections and is made to avoid an open set problem, as explained below. The parameter $\underline{q}$ can be made arbitrarily small.

Since the parameters in each period are the same, it is suboptimal to delay offers; if it is suboptimal to make an offer in the first period, it is also suboptimal to make an offer in the second period. Thus, a bidding strategy can be summarized by $(p_1, q_1; p_a, q_a; p_r, q_r)$, where (p_1, q_1) denotes the offer to the first seller, and $(p_a, q_q), (p_r, q_r)$ denote the offer to the second seller given that the first seller accepted or rejected the offer, respectively. The first seller accepts the buyer's offer with probability p_1. Hence, the buyer's expected profits are

$$\pi(p_1, q_1) + p_1\pi(p_a, q_a) + (1 - p_1)\pi(p_r, q_r). \tag{4}$$

As in the previous section, accepted offers tighten the capital constraint, while rejected offers relax the constraint. The potentially binding constraints are as follows:

The capital constraint must be satisfied if the first seller accepts the buyer's offer. For the case in which the buyer does not make a second offer (i.e., $p_a = q_a = 0$), we obtain

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2} + \Delta)M \geq L + p_1q_1, \tag{5}$$

and for the case in which the buyer makes a second offer (i.e., $p_a, q_a > 0$) and this second

[15] As before, the capital constraint is initially satisfied and the cost of violating it is sufficiently high to outweigh any profit gains obtained from doing so. Also as before, $x < M$ and $\Delta \in (0, 1/2)$.

offer is accepted, we obtain

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2}p_a + \Delta)(M + q_a) \geq L + p_1 q_1 + p_a q_a. \tag{6}$$

In both cases, the market value of the first asset is $\frac{1}{2}p_1 + \Delta$. In the first case, the market value of the second asset remains at its prior $(\frac{1}{2} + \Delta)$, while in the second case the market value of the second asset drops to $\frac{1}{2}p_a + \Delta$.

The capital constraint must also be satisfied if the first seller rejects the offer. In this case, the capital constraint is loosened after the first period, and the potentially binding constraint is when the buyer makes a second offer and this offer is accepted. Hence, the capital constraint is

$$(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)(M) + (\frac{1}{2}p_r + \Delta)(M + q_r) \geq L + p_r q_r. \tag{7}$$

The problem reduces to finding a bidding strategy that maximizes the buyer's expected profits such that if $p_a, q_a = 0$, constraints (5) and (7) are satisfied, and if $p_a, q_a > 0$, constraints (6) and (7) are satisfied.

It turns out that whenever the buyer's first-period offer p_1 is rejected, his capital constraint becomes sufficiently slack that he can make his unconstrained optimal bid of (Δ, x) in the second period. The intuition is that since constraint (6) implies constraint (5) (by Lemma 1), the first-period offer p_1 must satisfy constraint (5); this can happen only if p_1 is high, or the capital constraint is very slack to begin with. In either case, the buyer's capital constraint has a lot of slack if the first offer is rejected; therefore, the buyer can set his second offer equal to the benchmark offer (Δ, x).

Lemma 2 *If $p_1 > 0$, then $(p_r, q_r) = (\Delta, x)$.*

It remains to characterize (p_1, q_1) and (p_a, q_a). It follows from Proposition 3 that in the second period the buyer offers to buy either everything or nothing and makes nonnegative profits; hence, we can assume, without loss of generality, that $q_a = x$, with the interpretation that $p_a = 0$ corresponds to not making an offer.

In contrast, it is sometimes in the buyer's interest to make a loss-making offer of a very high price in the first period. The advantage of doing so is that if this high offer is rejected, the market valuation of the buyer's inventory rises to a commensurately high level, relaxing

the buyer's capital constraint. This allows the buyer to make a highly profitable offer in the second period. The buyer would like the loss-making first-period offer to be for the *smallest* quantity that still gives rise to the increase in market valuation—this quantity is $\underline{q}$ in our notation.

Our main result in this section is:

Proposition 4 *(i) When leverage is low, the buyer makes the benchmark bid* (Δ, x) *in both periods.*

(ii) When leverage is intermediate, the buyer offers to pay strictly more than the benchmark in the first period, and if the first offer is accepted, the buyer offers to pay even more in the second period (i.e., $p_a > p_1 > \Delta$*); in both periods the buyer bids for the maximum amount* x*.*

(iii) When leverage is high, the buyer withdraws from the market in the second period (i.e., $p_a = 0$*) if his first offer is accepted. The buyer's initial bid* (p_1) *is increasing in leverage. In particular, when initial leverage is sufficiently high, the buyer initially bids more than the benchmark; that is, the market freeze is preceded by high prices. The quantity the buyer bids for in the first period is decreasing in leverage.*

(iv) When leverage is very high, trade completely breaks downs.

Proposition 4 captures a few aspects of a dynamic behavior. If initial leverage is relatively moderate, the buyer has enough slack in his capital constraint to make two rounds of offers. But unless leverage is very low, the buyer still needs to consider his capital constraint, and this leads him to bid more than the benchmark price in both periods. If his first bid is accepted, his capital constraint is tightened, forcing him to bid even more in the second period. In other words, the price at which trade occurs rises with successful trades.

If instead initial leverage is high, the buyer has insufficient slack to have two bids accepted. Thus, there must be a period in which trade does not occur. In particular, if the buyer's first period offer is accepted, his capital constraint becomes too tight to make a bid in the second period and so the market freezes. The proposition also sheds light on the price path leading up to this market freeze. When initial leverage is sufficiently high, the capital constraint is binding, forcing the buyer to make a high bid in the first period. Thus, the market freeze may be preceded by a run-up in prices.

When leverage is high, and part (iii) of Proposition 4 applies, the quantity the buyer offers to buy in the first period is decreasing in leverage. Within this range, if leverage is *relatively* low, the buyer offers to buy the full amount to maximize his expected profits in the first period. If instead leverage is relatively high, the buyer offers to buy the lowest amount possible, since as we explained earlier, if the offer is accepted, he loses money and the only purpose of the offer is to relax his capital constraint if the offer is rejected.

5 Competition Among Buyers

Up till now we have focused on the case of a single buyer. As we have shown, concerns about preserving the market value of existing asset inventories affect the pattern of trade. When the buyer is very leveraged and so his capital constraint has little slack, such concerns lead to a trade breakdown and prevent the dissemination of information about asset quality. However, if the buyer is only moderately leveraged, these same concerns drive up both prices and trade volumes.

A natural question is how these results are affected by the presence of multiple competing buyers. One might conjecture that when multiple buyers are present, it is hard for any individual buyer to prevent the dissemination of information about asset values. In this section, we show that this conjecture is only partially correct. When all competing buyers are very leveraged, concerns about the market value of inventories again lead to a trade breakdown and prevent the dissemination of information about asset quality. However, under some circumstances in which one buyer is more leveraged than another, competition does indeed force trade and price dissemination to occur, even though the most leveraged buyer expects to lose money if his offer is accepted. In this sense, competition actually strengthens our previous finding that inventories may drive up prices: now, inventories of one buyer drive up the price offered by a second buyer.

Put slightly differently, the combination of capital constraints and inventories generates a situation where one buyer's bid has externalities for other buyers. Moreover, and in contrast to existing auction-theoretic papers dealing with externalities,[16] the externality depends on the price paid, rather than simply whether another buyer obtains the asset.

[16] See, for example, Jehiel, Moldovanu, and Stacchetti (1996).

To simplify exposition, we focus on the case of two buyers. Buyer i has an inventory of M_i units of the asset and a debt liability L_i. The gain from trade with buyer i is Δ_i. The seller has x units for sale. Everything is common knowledge, except for the true value of the asset (v), which is private information to the seller. As before, the capital constraint for each buyer is initially satisfied, the cost for violating it is large, $\Delta_i \in (0, \frac{1}{2})$, and the amount for sale is small relative to the buyer's existing inventories, $x < M_i$.

Both buyers make offers simultaneously. Buyer i offers a price and a quantity (p_i, q_i), meaning that he is willing to buy up to q_i units at a price per unit p_i. The seller selects quantities $q_i' \leq q_i$ to sell to each buyer, so as to maximize his profits $\sum_{i=1}^{2}(p_i - v)q_i'$, subject to $q_1' + q_2' \leq x$. Hence, if $v > \max\{p_1, p_2\}$, the seller rejects both offers and trade does not take place. Otherwise, the seller accepts the offer with the highest price; and if he has remaining assets to sell, he also accepts the lowest price offer if $v \leq \min\{p_1, p_2\}$.[17] The equilibrium outcomes discussed in this section (in particular, in Lemma 4 and in Propositions 5 and 6), remain even if a buyer can deviate by offering a nonlinear price schedule rather than a linear schedule; the proof, which extends that of Proposition 1, is in Appendix E.[18]

Parallel to the monopolist case, and in particular (3), define $p_i(q)$ as the minimum price that satisfies buyer i's capital constraint given that he offers to buy quantity q, and buyer $-i$ makes no offer. By continuity, define $p_i(0) = \lim_{q \to 0} p_i(q)$. Just as in the monopolist buyer case, $p_i(\cdot)$ is increasing in buyer i's leverage. Recall that a monopolist buyer makes an offer if and only if his leverage is low enough that $p_i(x) \leq 2\Delta_i$.

As in Section 4, to avoid technical issues we assume that the minimum quantity a buyer can offer to buy is $\underline{q}$, i.e., $q_i \in \{0\} \cap [\underline{q}, x]$. We also assume that the price space is finite, and the values $\{p_i(x), p_i(x) - \varepsilon, p_i(x) + \varepsilon, 2\Delta_i, 2\Delta_i - \varepsilon, 2\Delta_i + \varepsilon\}_{i \in \{1,2\}}$ lie within this space. The "tick" size ε is assumed to be close to zero, and for clarity, we exclude it from the statements of the results.

Because of the externalities generated by each buyer's bid on other buyers, there are typically Nash equilibria in which buyer i makes a bid that will violate buyer $-i$'s capital constraint and forces buyer $-i$ to make a higher bid himself. However, not all equilibria of

[17] If prices coincide, $p_1 = p_2$, the seller splits his trade between the buyers in proportion to the quantities q_1 and q_2; i.e., the seller's trade with buyer 2 is a fraction $\frac{q_2}{q_1}$ of his trade with buyer 1.

[18] Attar, Mariotti, and Salanié (2011) analyze a similar setting, but without inventories and capital constraints.

this type are robust, in the sense that there is no good reason for buyer i to make such a bid in the first place. Accordingly, we focus on equilibria that are robust in the sense of not entailing dominated strategies. Specifically, we characterize equilibria that survive the following iterated process of elimination of weakly dominated strategies. In the first stage we eliminate all strategies that are weakly dominated. In the second stage, we consider the game remaining after the first stage and eliminate strategies that are weakly dominated in this new game. And so on. Lemma 3 characterizes offers that survive the first elimination round.

Lemma 3 (First elimination round) *(A) If $p_i(x) < 2\Delta_i$, an offer (p_i, q_i) survives the first round of elimination of weakly dominated strategies if and only if $\max\{\Delta_i, p_i(x)\} \leq p_i < 2\Delta_i$ and $q_i = x$.*

(B) If $p_i(x) = 2\Delta_i$, the unique offer to survive the first round of elimination of weakly dominated strategies is $(p_i, q_i) = (2\Delta_i, x)$.

(C) If $p_i(x) > 2\Delta_i$, the offers $p_i = 0$ and $(p_i, q_i) = (p_i(x), x)$ survive the first round of elimination of weakly dominated strategies. In contrast, any offer (p_i, q_i) with $p_i \geq p_i(0)$ and $p_i \neq p_i(x)$ is eliminated.

Part (A) says that when a buyer has a profitable trade, he always tries to exploit it by making an offer that yields positive profits and does not violate his capital constraint. This behavior is similar to the single-buyer case previously analyzed. The result in part (C) that the loss-making offer $(p_i(x), x)$ is undominated reflects the fact that, with competition, a buyer may wish to make a "preemptive" bid to ensure that his capital constraint is not violated should the other buyer make an offer at a low price.

5.1 Equilibrium When Inventories Do Not Matter

We start with the benchmark case in which neither buyer is subject to a capital constraint. For ease of exposition, we assume in the main text that the two buyers have relatively similar valuations and state all results under this assumption. The appendix contains general statements of all results, which do not rely on this assumption.

Specifically, we assume that $\max\{\Delta_1, \Delta_2\} \leq \min\{2\Delta_1, 2\Delta_2\}$. In this case the buyer with the highest valuation acquires the asset at a price determined by the buyer with the

second-highest valuation, and the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$; this is a standard outcome for settings with public buyer valuations.[19] This result generalizes easily to the case in which both buyers have low leverage, so that their capital constraints are not binding in equilibrium.

Lemma 4 *If both buyers have low leverage, i.e.,* $\max\{p_1(x), p_2(x)\} \leq \min\{2\Delta_1, 2\Delta_2\}$, *then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is that whenever the seller agrees to sell, he sells everything to the buyer with the higher valuation for a price* $\min\{2\Delta_1, 2\Delta_2\}$.

5.2 Equilibrium When Inventories Matter

We start with the case in which at least one of the two buyers has a low-enough leverage such that he would acquire the asset if he were the only buyer. Without loss of generality, let this buyer be buyer 1; formally, $p_1(x) \leq 2\Delta_1$.

From Lemma 3, we know that buyer 1 bids for the full amount x and that the bid-price p_1 per unit is below $2\Delta_1$. The key observation is that if $p_1 < p_2(0)$, and the seller accepts buyer 1's offer, the expected value of v drops to $\frac{p_1}{2}$, and this causes buyer 2's capital constraint to be violated. Consequently, when buyer 2 is highly leveraged, so $p_2(0)$ is high, buyer 2 may bid more aggressively to ensure that the seller does not accept a lower bid from buyer 1. Our main result is:

Proposition 5 *Assume* $p_1(x) \leq 2\Delta_1$. *Then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is as follows:*

(A) If buyer 2 *has both low leverage and low valuation relative to buyer 1, i.e.,* $\max\{\Delta_2, p_2(0)\} \leq \max\{\Delta_1, p_1(x)\}$, *then whenever the seller agrees to sell, he sells everything for price* $\max\{p_1(x), \min\{2\Delta_1, 2\Delta_2\}\}$. *In particular, if* $p_1(x) > 2\Delta_2$, *buyer* 1 *acquires the asset at price* $p_1(x)$, *which increases in his leverage.*

(B) Otherwise, whenever the seller agrees to sell, he sells everything for price $\max\{p_2(x), \min\{2\Delta_1, 2\Delta_2\}\}$. *In particular, if* $p_2(x) \in (2\Delta_2, 2\Delta_1)$, *the seller sells everything to buyer* 1 *at price* $p_2(x)$ *and if* $p_2(x) \geq \max\{2\Delta_1, 2\Delta_2\}$, *the seller sells everything to buyer* 2 *who makes negative profits.*

[19] See, for example, Ho and Stoll (1983).

Recall that in the benchmark case without capital constraints, the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$. In part (A), capital constraints interact with competition in a straightforward way: when buyer 1's capital constraint is relatively tight, it forces him to increase his offer to $p_1(x)$.

In part (B), in contrast, the interaction between capital constraints and competition is less straightforward and can lead to a form of "spillover" of capital constraints. That is, if buyer 2's leverage is relatively high so that $p_2(x) \in (2\Delta_2, 2\Delta_1)$, buyer 2's capital constraint leads him to compete more aggressively with buyer 1, and consequently buyer 1 ends up paying an amount $p_2(x)$ that is determined by buyer 2's capital constraint. If $p_2(x) \geq \max\{2\Delta_1, 2\Delta_2\}$, buyer 1 can no longer compete; therefore, whenever trade occurs, buyer 2 acquires everything at a price $p_2(x)$. In the latter case, buyer 2 makes negative profits, even though he would not bid at all if he were the only buyer. Buyer 2 is forced to make this bid, since otherwise the seller trades with buyer 1 and buyer 2's capital constraint is violated.

It is worth contrasting this last result, in which competition induces buyer 2 to bid when he would otherwise have exited the market, with the existing literature on nonexclusive contracting. In this literature,[20] latent offers deter entry. In contrast, in our setting, latent offers induce entry: buyer 2 enters precisely because of buyer 1's latent offer.

Finally, consider the case in which both buyers are so leveraged, that, if bidding individually, trade collapses in the sense that no one makes an offer. Clearly, no trade is an equilibrium that survives iterated elimination of weakly dominated strategies, since given that one buyer is unwilling to make an offer, the unique best response for the other buyer is also not to make an offer. Moreover, no trade is the only outcome to survive iterated elimination of weakly dominated strategies when $p_1(x) \neq p_2(x)$.[21]

Proposition 6 *If both buyers are highly leveraged (i.e., $p_i(x) > 2\Delta_i$ for $i \in \{1, 2\}$), then a no-trade equilibrium survives iterated elimination of weakly dominated strategies. When $p_1(x) \neq p_2(x)$, this is the unique equilibrium that survives iterated elimination.*

[20] See, for example, Bisin and Guaitoli (2004), who study a moral hazard environment; and Attar, Mariotti, and Salanié (2011), and Ales and Maziero (2011), who study an adverse selection environment.

[21] In the nongeneric case $p_1(x) = p_2(x) > \max\{p_1(0), p_2(0)\}$, we cannot rule out other equilibria in which one buyer makes a "latent" offer, knowing that it will not be accepted in equilibrium, and the second buyer makes a "loss making" offer to rule out a situation in which the seller trades with the first buyer and the capital constraint of the second buyer is violated.

Proposition 6 shows that when both buyers have tight capital constraints, the conclusions of the single-buyer case continue to hold: trade collapses, and price dissemination stops. Indeed, the condition $p_i(x) > 2\Delta_i$ in Proposition 6 is equivalent to the condition for no trade ($\delta_i > \frac{4\Delta_i}{1+2\Delta_i}$) in Proposition 3.

6 Extensions

6.1 Seller Cares About the Value of His Inventory

In the analysis so far, we assume that the buyer is capital constrained, but the seller is not. A similar intuition applies when the seller is capital constrained. In particular, a seller close to his capital constraint may not accept an offer p that exceeds the true value v of the asset, because acceptance reduces the market value of his assets from the ex ante value $\frac{1}{2}$ to $\frac{p}{2}$. In words, the seller's trading decision is distorted by the seller's desire to prevent the dissemination of bad news.

More formally, suppose the seller has a stock of M_s assets and is able to sell a maximum $x \leq M_s$. For example, regulatory requirements may force the seller to retain $M_s - x$ assets on his balance sheet; or $M_s - x$ of the assets may be much more valuable to the seller than the buyer, so that the gains from trade Δ exist only on x of the assets. For ease of exposition, we analyze the case in which the seller is constrained and the buyer is not. We denote the seller's liabilities by L_s and define $\delta_s \equiv \frac{L_s}{\frac{1}{2}M_s}$, which is a measure of the seller's initial leverage. Similar to before, we assume the seller's capital constraint is satisfied initially and that the cost of violating it is very high.

The potentially binding constraint is when the seller accepts the buyer's offer. In this case, the market learns that $E(v) = \frac{1}{2}p$, and the seller's capital constraint becomes

$$\frac{1}{2}p(M_s - q) + pq \geq L_s, \tag{8}$$

where $M_s - q$ is the seller's total inventory net of trade and pq is the sale proceeds. Note that the seller's capital constraint is based on the value of asset to the seller and so does not include the term Δ. Equation (8) can be rewritten as

$$p \geq \frac{\delta_s}{1 + \frac{q}{M_s}}. \tag{9}$$

The seller accepts the buyer's offer if and only if $v \leq p$ and the offer satisfies the capital constraint. Hence, the buyer's problem reduces to choosing an offer (p, q) such that equation (9) is satisfied. As in the buyer's case, it is optimal for the buyer to offer either $q = 0$ or $q = x$, which leads us to Proposition 7.

Proposition 7 *When only the seller cares about the value of his inventory, trade can occur if and only if leverage is sufficiently low, i.e.,* $\delta_s \leq 2\Delta(1 + \frac{x}{M_s})$. *In this case, the buyer offers to buy* x *units at a price per unit* $\max\{\Delta, \frac{\delta_s}{1+\frac{x}{M_s}}\}$.

The relationship between leverage, prices and probability of trade is similar to the relationship we obtained earlier for the case in which the buyer was capital constrained. The seller's case also provides several predictions regarding $M_s - x$, the assets that the seller must retain. Trade can happen only if $M_s - x$ is sufficiently low. However, once trade happens, a further reduction in $M_s - x$ reduces the probability of trade. Intuitively, when retained assets $M_s - x$ decrease, the market value of these retained assets has a less important role. This means that the market is less likely to break down, but it also means that if trade happens, the buyer does not need to increase the price as much to satisfy the seller's capital constraint, and so the probability of trade is reduced.

Finally, the analysis above treats $M_s - x$ of the assets as absolutely unavailable for sale. If instead the buyer could purchase them even though there are no immediate gains from trade, we conjecture that under some circumstances he might be willing to do so, in order to relax the seller's capital constraint and obtain a better price on the x assets he really wants to buy.

6.2 Limited Borrowing Capacity

What if the buyer and seller can borrow against only a fraction of the market value of their assets, as opposed to the full value as we have thus far assumed? The analysis of the one-period case is easily extended to capture this. Replace h in the capital constraint with αh, where α is a constant, such that $\alpha \in (0, 1]$. The parameter α represents "haircuts" set by a regulator or by potential lenders to account for the asset's risk. Alternatively, $\alpha < 1$ represents limitations on the ability of potential lenders to seize borrowers' assets. Our basic results are qualitatively unchanged by the possibility of haircuts; in addition, we obtain a

couple of new results (Appendix D contains the details).

First, the comparative static in α sheds light on the effects of government interventions. Increases in α increase the region in which trade can happen. Thus, a regulator might be able to defrost the market by reducing capital requirements or providing loan guarantees, as both increase α. However, once α is large enough so that trade can occur, but not too large so that the benchmark solution is not achieved, a further increase in α reduces the bid price and the probability of trade. Intuitively, a higher α increases the borrowing capacity of existing assets and therefore has a similar effect to that of reducing initial leverage.

The case $\alpha < 1$ also provides some new results in the buyer's case. First, the quantity offered is continuous in leverage and as leverage increases, the quantity gradually drops from the full amount x to zero. Second, expected volume, $q \Pr(v \leq p)$, is continuous in leverage. It first rises and then drops gradually to zero. The initial increase in expected volume occurs because at moderate levels of leverage, the buyer increases the price but keeps the quantity unchanged, at $q = x$. As leverage increases further, the buyer continues to increase the price, but he also reduces the quantity until it reaches zero.

6.3 Marking to Market

In our main analysis above, we assumed that the market value of assets is derived using all available information; that is, using Bayes' rule. However, as we show below, one obtains qualitatively similar results if instead assets are "marked to market"; that is, valued at the most recent transaction price. Hence, our model is consistent with the interpretation that marking to market is to blame for many of the phenomena we discussed earlier, but it also predicts that one would see qualitatively the same phenomena even without marking to market.

Denote by p_0 the price of the last offer accepted. Under marking to market with a borrowing capacity $\hat{\alpha} \in (0, 1]$, the initial borrowing capacity of each unit of the asset is $\hat{\alpha} p_0$, and if an offer (p, q) is accepted, the borrowing capacity changes to $\hat{\alpha} p$. A rejected offer has no effect on borrowing capacity. Hence, the relevant capital constraint for the buyer is $\hat{\alpha} p(M + q) \geq L + pq$. As before, the buyer's objective is to maximize the profits from trading subject to satisfying the capital constraint.

For ease of exposition, consider the case $\hat{\alpha} = 1$, so the capital constraint reduces to

$pM \geq L$.[22] Denoting the buyer's initial leverage by $\delta' \equiv \frac{L}{p_0 M}$, the capital constraint becomes $p \geq \delta' p_0$. If leverage is low or intermediate, i.e., $2\Delta \geq \delta' p_0$, the buyer offers to buy x units at a price per unit $\max\{\Delta, \delta' p_0\}$. Otherwise, the buyer does not make any offer. The results for competing buyers are also similar to those in our previous analysis.[23]

For the two-period case, the relevant capital constraints, which are the analogues of equations (5), (6), and (7), are $p_1 M + p_0 M \geq L$, $p_1 M + p_a M \geq L$, and $p_0 M + p_r M \geq L$. Our previous analysis of the dynamic case reveals that the buyer sometimes makes a high loss-making bid early on (i.e., date 1) in order to raise market beliefs about the value of the first asset and relax his capital constraint. In the case of marking to market, the level of last price p_0 has a big effect on whether such a strategy is worthwhile. Specifically, offers p above 2Δ are loss-making, with the expected loss equal to $q \min\{p, 1\}\left(p - \frac{1}{2}\min\{p, 1\} - \Delta\right)$, which is independent of the last price p_0; while the extent to which the capital constraint is relaxed depends on $p - p_0$. So ceteris paribus, a given relaxation of the capital constraint is more costly when the markets are falling (so the last price p_0 was high) than when markets are rising (so that the last price p_0 is low).

Our benchmark analysis also establishes that when the buyer makes a profitable offer in the first period and his offer is accepted, he either increases the offer price in the second period, or else stops bidding. This result also continues to hold.[24]

7 Policy and Empirical Implications

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning. It also has empirical implications regarding the relationship between dealers' inventories and prices.

7.1 Defrosting Frozen Markets

Consider the case in which only the buyer cares about inventory values and in which trade has completely broken down; that is, $\delta > \frac{4\Delta}{1+2\Delta}$(Proposition 3). The discussion can easily be

[22] The case $\hat{\alpha} < 1$ corresponds to $\alpha = 2\hat{\alpha}$ and $\gamma = 0$ in our general analysis in Appendix D. Our main results are qualitatively unchanged by this generalization.

[23] Denote buyer i's initial leverage by $\delta_i' \equiv \frac{L_i}{p_0 M_i}$ and replace $p_i(x)$ and $p_i(0)$ with $\delta_i' p_0$.

[24] To see that define $H = \frac{L}{M}$ and $\sigma = p_0$ in the proof of Proposition 4. (Note that since the capital constraint is satisfied initially ($2p_0 M \geq L$), it follows that $p_0 > 2\Delta$ whenever the benchmark solution is not achieved; hence, the interval $(4\Delta, 2\Delta + \sigma)$, which is discussed in the proof, exists.)

extended to the case of more than one buyer, as in Proposition 6.

One option open to a government is to offer to buy the seller's assets. A central question is whether such government purchases can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect. First, if the government faces the same lemons problem that the potential buyer does, a subsidy-free purchase scheme is possible *only if* the asset is worth more to the government than to the seller; formally, if the government's valuation of the asset is $v+\Delta_g$, we must have $\Delta_g > 0$. Second, even if $\Delta_g > 0$, a subsidy-free purchase scheme imposes a cost on the original buyer, assuming he is a more efficient holder of the asset than the government ($\Delta_g < \Delta$). Recall that this buyer does not purchase the asset himself because doing so violates his capital constraint. However, if the government buys the asset at unsubsidized terms, this also leads to a violation of the buyer's capital constraint.[25] A similar issue arises if the government subsidizes a second private buyer to purchase the asset.

Another option is to remove assets from the buyer's balance sheet; that is, to replace assets with cash. If the buyer can borrow against the full value of his assets, as assumed in our analysis so far, then again, purchasing assets from the buyer can relax his capital constraint only if the purchase involves a taxpayer subsidy. If instead the buyer has limited borrowing capacity, as in Section 6.2, purchasing assets from the buyer might relax his capital constraint even if the purchase does not involve a taxpayer subsidy.[26]

7.2 Should Regulation Mandate Some Retention of the Asset by the Seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to retain some stake in the assets they sell.[27] Our analysis identifies a potential cost to this proposal,

[25] Formally, since the market has broken down, we know that $p(q) > 2\Delta$, for every $q \in (0, x]$; and since $\Delta_g < \Delta$, it follows that any unsubsidized offer $p \leq 2\Delta_g$ satisfies $p < 2\Delta$. Therefore, $p < p(q)$ for every $q \in (0, x]$, and any unsubsidized offer violates the buyer's capital constraint.

[26] In particular, if the government plans to spend Z_g dollars, it can buy $Z_g/(\frac{1}{2}+\Delta_g)$ units without a taxpayer subsidy. Since each of these units allows the buyer to borrow $a(\frac{1}{2}+\Delta)$ dollars, the government purchase loosens the buyer's capital constraint only if the sale proceeds exceed the assets' borrowing capacity; that is, only if $Z_g > \alpha(\frac{1}{2}+\Delta)\frac{Z_g}{\frac{1}{2}+\Delta_g}$. This condition reduces to $\Delta_g > \alpha\Delta - \frac{1}{2}(1-\alpha)$ and can be satisfied even if $\Delta_g < \Delta$.

[27] See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

namely, that under some circumstances it leads to a market breakdown. To see this, interpret the parameter x in our model as stemming from a regulation mandating that the seller retain a fraction $\frac{M_s - x}{M_s}$ of the asset he is selling. From Proposition 7, whenever x is sufficiently low, trade is impossible because the seller cares too much about the market's perception of the value of the assets he is forced to retain. Moreover, notice that this case arises more easily when the seller is highly leveraged (measured by δ_s).

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of asset sellers; for example, to discourage loan originators from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this regulation, namely, that it can lead to the breakdown of socially efficient trade.

7.3 Dealers' Inventories and Prices

A further implication of our analysis is that prices are increasing in inventory levels. Intuitively, larger inventories make buyers more concerned about the market value of these inventories, leading them to make higher offers.

More formally, this result is a corollary of our Proposition 3, which characterizes trade in the basic (single buyer) version of our model. To see this, observe that the derivative of $p(x)$ with respect to inventories M has the same sign as $4\Delta - \delta(1 + 2\Delta)$, which by Proposition 3 is positive whenever trade occurs. Note that here we are characterizing only the direct effect of inventories, in the sense that we are holding leverage δ constant. If changes in inventory levels also affect leverage, there is an additional indirect effect on prices; this indirect effect reinforces the direct effect if higher inventories are associated with greater leverage (as seems likely).

Moreover, the positive relationship between inventories and prices continues to hold when there are multiple competing dealers. Indeed, by part (B) of Proposition 5, the price offered by one buyer may increase when a *competing* buyer has larger inventories.

As noted in the introduction, this comparative static result has implications for the relationship between market-maker inventories and prices that is different from that produced by existing models. In particular, existing models generally predict a negative relationship

between market-maker inventories and prices, while our analysis predicts a positive relationship, consistent with the empirical findings of Manaster and Mann (1996).

8 Summary

We analyze how existing stocks of assets — inventories — affect trade, information dissemination, and price formation. When market participants are close to their maximal leverage, concerns about the revelation of bad news prevents socially beneficial trade and information dissemination. However, when market participants are further from their maximal leverage, inventories lead to overbidding (in the sense that the buyer pays more than he would like), which stimulates socially beneficial trade. Because trade increases buyer inventories and often increases buyer leverage, these predictions imply that prices and trade volumes may first increase before collapsing, as we show formally in the dynamic extension of our basic model. Our results continue to hold when buyers compete with one another, and some of the results are even strengthened. We use our model to comment on several prominent policy questions, and we also derive several new empirical predictions. As a technical contribution, we generalize a classic result of Samuelson (1984) to cover buyers with capital constraints and inventories.

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Appendix A: Proofs (Monopolist Buyer)

We use the notation $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$ to denote the buyer's expected profits per unit.

Proof of Lemma 1. Suppose the buyer offers (p, q). The acceptance of this offer has two effects: First, the value of existing assets falls from $(\frac{1}{2}+\Delta)M$ to $(\frac{1}{2}p+\Delta)M$, with a net effect $\frac{1}{2}(1-p)M > \frac{1}{2}(1-p)x$. Second, the buyer adds q units, each with a borrowing capacity of $\frac{1}{2}p + \Delta$, but he also pays p per unit. If profits $\frac{1}{2}p + \Delta - p$ are negative, this second effect is also negative, and the proof is complete. Otherwise, the added borrowing capacity from this is $(\Delta - \frac{1}{2}p)q$, which is at most $\frac{1}{2}(1-p)x$, since $q \leq x$ and $\Delta < \frac{1}{2}$. Combining the two effects, it follows that the overall effect is negative and the capital constraint is tightened. Q.E.D.

Proof of Lemma 2. Since choosing $(p_r, q_r) = (\Delta, x)$ maximizes second-period profits, it is enough to show that the capital constraint is not violated after choosing this offer; that is, we need to show that $(p_r, q_r) = (\Delta, x)$ satisfies equation (7): $(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{3}{2}\Delta)(M + x) \geq L + \Delta x$. Since $q_1 \leq x$, it is enough to show that $(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{3}{2}\Delta)(M + q_1) \geq L + \Delta q_1$, which can be rewritten as $(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2} + \Delta)M + \frac{1}{2}\Delta(M - q_1) + \frac{1}{2}p_1 q_1 \geq L + p_1 q_1$. The last equation follows since the offer (p_1, q_1) satisfies equation (5), and since $M > x \geq q_1$. Q.E.D.

Lemma A-1 *The first-period offer satisfies $q_1 \in \{0, \underline{q}, x\}$. If $q_1 = \underline{q}$, expected profits in the first period are negative; that is, $p_1 > 2\Delta$. If $q_1 = x$, expected profits in the first period are nonnegative; that is, $p_1 \leq 2\Delta$.*

Proof of Lemma A-1. The quantity q_1 enters the capital constraint with a coefficient $(\frac{1}{2}p_1 + \Delta) - p_1$, which is the expected value of the asset acquired minus the price paid. This expression has the same sign as the per-unit profit $\pi(p_1)$. Consequently, if the buyer finds it worthwhile to bid at all, then if $\pi(p_1) \geq 0$, he bids for the maximum amount x, since doing so relaxes the capital constraint and increases profits; while if $\pi(p_1) < 0$, he bids for the minimum amount $\underline{q}$. Q.E.D.

Proof of Proposition 4. As we show in the text, $q_a = x$, $p_a \leq 2\Delta$, and $(p_r, q_r) = (\Delta, x)$. Hence, the buyer's problem reduces to choosing p_1, q_1, p_a that maximize expected profits (4)

subject to the capital constraint: namely, equation (5) if $p_a = 0$, and equation (6) if $p_a > 0$. For use below, define $\sigma \equiv \frac{\frac{1}{2}M - \Delta x}{\frac{1}{2}(M-x)}$ and $H \equiv \frac{L - 2\Delta(M+x)}{\frac{1}{2}(M-x)}$. Observe that $\sigma > 1 > 2\Delta$, and H is a monotone transformation of the buyer's initial leverage, $\delta = \frac{L}{(\frac{1}{2}+\Delta)2M}$. In addition, when $q_1 = x$, equation (5) reduces to $p_1 \geq H - \sigma$.

We first establish that if $H \leq 2\Delta + \sigma$ (i.e., leverage is not very high), $q_1 = x$. Define $\overline{p} = \max\{\Delta, H - \sigma\}$. Since offering $(p_1, q_1) = (\overline{p}, x)$ and $p_a = 0$ satisfies the capital constraint and provides nonnegative profits (since $H - \sigma \leq 2\Delta$), trade can always happen. By Lemma A-1, it is enough to show that it is suboptimal to choose $q_1 = \underline{q}$. The proof is by contradiction. Suppose the optimal bidding strategy is $(p_1, q_1; p_a, q_a; p_r, q_r)$ with $q_1 = \underline{q}$, so from Lemma A-1, $p_1 > 2\Delta$ and the buyer loses money in the first period. We obtain a contradiction as follows: If $p_a = 0$, the buyer is better off choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (\overline{p}, x; p_a, q_a; p_r, q_r)$. The alternate strategy satisfies the capital constraint $(\widetilde{p}_1 \geq H - \sigma)$ and increases the buyer's expected profits from $q_1\pi(p_1) + (1 - p_1)x\pi(\Delta)$ to $x\pi(\overline{p}) + (1 - \overline{p})x\pi(\Delta)$. In particular, the alternate strategy provides nonnegative profits in the first period, and increases the probability that the buyer can make the benchmark bid in the second period. If instead $p_a > 0$, the buyer is better off choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (p_a, q_a; p_1, q_1; p_r, q_r)$. Since the original strategy satisfies the capital constraint, the alternate strategy also satisfies it. The alternate strategy also increases the buyer's expected profits from $q_1\pi(p_1) + p_1q_a\pi(p_a) + (1-p_1)x\pi(\Delta)$ to $q_a\pi(p_a) + p_aq_1\pi(p_1) + (1-p_a)x\pi(\Delta)$. In particular, the alternate strategy reduces the probability of obtaining negative profits and increases the probability that the buyer can make the benchmark bid in the second period.

Hence, whenever $H \leq 2\Delta + \sigma$, $q_1 = x$ and by Lemma A-1, $p_1 \leq 2\Delta$. For strategies in which $p_a > 0$, the optimal (p_1, p_a) maximizes expected profits per unit $V(p_1, p_a) \equiv \pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(\Delta)$, subject to the capital constraint (6), which reduces to $p_1 + p_a \geq H$. For strategies in which $p_a = 0$, the optimal p_1 maximizes $V(p_1, 0)$, subject to $p_1 \geq H - \sigma$. The benchmark solution $p_1 = p_a = \Delta$ is achieved if and only if $H \leq 2\Delta$. In contrast, when $H \in (4\Delta, 2\Delta + \sigma]$, it is optimal to choose $p_a = 0$, since otherwise either $p_a > 2\Delta$, which is suboptimal, or $p_1 > 2\Delta$, which violates Lemma A-1 (recall $q_1 = x$). By continuity, there exist $H_1, H_2 \in (2\Delta, 4\Delta)$, such that whenever $H \in (2\Delta, H_1)$ (i.e., intermediate leverage), it is optimal to choose $p_a > 0$ and whenever $H \in (H_2, 2\Delta + \sigma)$ (i.e., higher leverage), it is optimal to choose $p_a = 0$. In the first case, the capital constraint (6) is binding because if it is not,

either $p_1 > \Delta$ and $\frac{\partial V}{\partial p_1} = \pi'(p_1) + \pi(p_a) - \pi(\Delta) < 0$, which contradicts the optimality of p_1; or else $p_a > \Delta$, which is clearly suboptimal. Hence, p_1 maximizes $V(p, H-p)$, which is cubic in p with a negative coefficient on the cubic term. To show that $p_a > p_1 > \Delta$, we need to show that $p_1 \in (\Delta, \frac{H}{2})$. This result follows if $\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=\Delta} > 0 > \frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=H/2}$. Evaluating, $\frac{d}{dp_1}V(p_1, H-p_1) = \pi'(p_1) + \pi(H-p_1) - p_1\pi'(H-p_1) - \pi(\Delta)$. Since π is a quadratic with its maximum at Δ, for any p, $\pi(p) = \pi(\Delta) + \frac{1}{2}(p-\Delta)\pi'(p)$. Given this, $\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=\Delta} = \pi(H-\Delta) - \Delta\pi'(H-\Delta) - \pi(\Delta) = \left(\frac{1}{2}(H-\Delta-\Delta) - \Delta\right)\pi'(H-\Delta)$, which is positive. Similarly, $\frac{d}{dp_1}V(p_1, H-p_1)\Big|_{p_1=H/2} = (1-\frac{H}{2})\pi'(\frac{H}{2}) + \pi(\frac{H}{2}) - \pi(\Delta) = (1-\frac{H}{2}+\frac{1}{2}(\frac{H}{2}-\Delta))\pi'(\frac{H}{2}) = (1-H/4-\Delta/2)\pi'(H/2)$, which is negative. In the second case $H \in (4\Delta, 2\Delta+\sigma]$, the objective function is quadratic and the optimal solution is $\widetilde{p}_1 = \max\{\Delta - \frac{1}{2}\Delta^2, H-\sigma\}$, which is increasing in H (and hence, in leverage). In particular, when $H > \Delta + \sigma$, we obtain $\widetilde{p}_1 > \Delta$, and so the buyer bids more than the benchmark.

Finally, consider the case $H > 2\Delta + \sigma$. If there is trade, we must have $p_1 > 2\Delta$; that is, the buyer bids more than the benchmark and loses money. To see that, suppose to the contrary that $p_1 \leq 2\Delta$; then $q_1 = x$ (by Lemma A-1), and (p_1, q_1) violates the capital constraint (5), which is a contradiction. Hence, by Lemma A-1, $q_1 \in \{0, \underline{q}\}$. Next, we show that $p_a = 0$. To see that, recall that accepted offers tighten the capital constraint (Lemma 1). Therefore, an offer with $p_a > 0$ must satisfy equation (6) when $q_1 = 0$ and when the first asset is evaluated at its prior; that is, $p_a \geq H - \sigma$. But then $p_a > 2\Delta$, which is suboptimal. Hence, $p_a = 0$, and whenever there is trade, the buyer's expected utility is $\underline{q}\pi(p_1) + (1-p_1)x\pi(\Delta)$. This expression is strictly decreasing in p_1 when $p_1 > 2\Delta$. Thus, if there is trade, the optimal p_1 satisfies the capital constraint (5) with equality. It then follows that $p_1 = \underline{H} \equiv \frac{L-Z-2\Delta(M+\underline{q})}{\frac{1}{2}(M-\underline{q})} - \frac{\frac{1}{2}M-\Delta\underline{q}}{\frac{1}{2}(M-\underline{q})}$. Observe that $\underline{H}$ is a monotone transformation of the buyer's initial leverage.

Combining the results above, we obtain that whenever $p_a = 0$, the initial bid is increasing in leverage and the quantity offered in the first period is decreasing in leverage (from x to $\underline{q}$). To establish that trade breaks down if leverage is too high, observe that if $\underline{H} = 2\Delta$, the buyer's expected utility is strictly positive, whereas if $\underline{H} = 1$, the buyer's expected utility is strictly negative. Q.E.D

Appendix B: Proofs (Competing Buyers)

Lemma A-2 *For every $i \in \{1,2\}$, one of the following is true: (i) $p_i(\cdot) \equiv 2\Delta_i$; (ii) for all $q \in [0,x)$, $p_i(x) > p_i(q) > 2\Delta_i$; or (iii) for all $q \in [0,x)$, $p_i(x) < p_i(q) < 2\Delta_i$.*

Proof of Lemma A-2. The capital constraint (2) can be written as $(\frac{1}{2}p + \Delta_i)M_i + q(\Delta_i - \frac{1}{2}p) \geq L_i$. By Assumption 1, the left-hand side is increasing in p, and by definition, $p_i(q)$ satisfies the capital constraint with equality given quantity q. In particular, $\left(\frac{1}{2}p_i(0) + \Delta_i\right) M_i = L_i$. Hence, (i) if $p_i(0) = 2\Delta_i$, the capital constraint is satisfied with equality for any q when $p_i = 2\Delta_i$, and so $p_i(\cdot) \equiv 2\Delta_i$; (ii) if $p_i(0) > 2\Delta_i$, the capital constraint is violated when $q = x$ and $p = p_i(0)$, and so $p_i(x) > p_i(0)$; (iii) if $p_i(0) < 2\Delta_i$, the capital constraint is slack when $q = x$ and $p = p_i(0)$, and so $p_i(x) < p_i(0)$. The result then follows from monotonicity of $p_i(\cdot)$ in q. Q.E.D.

Proof of Lemma 3. *Part (A):* From Lemma A-2, $p_i(x) < p_i(q) < 2\Delta_i$, for any $q \in [0,x)$. Consider an offer (p_i, q_i). If $p_i \in [\max\{\Delta_i, p_i(x)\}, 2\Delta_i)$ and $q_i = x$, the offer (p_i, q_i) survives the first stage of elimination, since it is a unique best response for buyer i when buyer $-i$ bids $p_{-i} = p_i - \varepsilon$. If p_i is in this same interval, but $q_i < x$, the offer (p_i, q_i) is weakly dominated by the offer (p_i, x), since raising the quantity to x weakly increases both buyer i's profits, with a strict increase if $-i$'s offer is lower, and moreover weakly increases the probability that i's capital constraint is satisfied. Any offer with $p_i < \Delta_i$ is weakly dominated by raising the offer to Δ_i. Finally, if $p_i \geq 2\Delta_i$ or $p_i < p_i(x)$, the offer (p_i, q_i) is weakly dominated by the offer $(p_i(0), x)$, as follows. This offer produces positive profits whenever it is accepted (e.g., if $p_{-i} = 0$) and guarantees that buyer i's capital constraint is satisfied, regardless of $-i$'s offer. In contrast, offering $p_i \geq 2\Delta_i$ never leads to positive profits, and offering $p_i < p_i(x)$ violates i's capital constraint for at least some offers of buyer $-i$ (e.g., $p_{-i} = 0$).

Part (B): From Lemma A-2, $p_i(\cdot) \equiv 2\Delta_i$. Offering $(2\Delta_i, x)$ weakly dominates any other offer (p_i, q_i): Offering $(2\Delta_i, x)$ produces zero profits and guarantees that buyer i's capital constraint is satisfied, regardless of $-i$'s offer. In contrast, if $p_i > 2\Delta_i$, the offer (p_i, q_i) produces negative profits whenever it is accepted (e.g., if $p_{-i} = 0$), and if $p_i < 2\Delta_i$, the offer violates i's capital constraint for at least some offers of buyer $-i$ (e.g., $p_{-i} = 0$).

Part (C): From Lemma A-2, $p_i(x) > p_i(q) > 2\Delta_i$ for any $q \in [0,x)$. The offer $p_i = 0$ survives the first elimination round, since it is the unique best response for buyer i when

buyer $-i$ bids $p_{-i} = 0$. Specifically, if buyer i bids $p_i = 0$, he obtains zero profits and his capital constraint is satisfied; if he bids $p_i > 0$ and his offer is accepted, he either makes negative profits or his capital constraint is violated. The offer $(p_i, q_i) = (p_i(x), x)$ survives the first elimination round, since it is a unique best response when buyer $-i$ bids $p_{-i} \in (0, p_i(0) - \varepsilon)$. Specifically, the offer $(p_i, q_i) = (p_i(x), x)$ guarantees that buyer i's capital constraint is satisfied, while if he bids $p_i < p_i(x)$ or $q_i < x$, his capital constraint is violated with a positive probability, and if he bids $p_i > p_i(x)$, his expected profits are reduced.

Any offer (p_i, q_i) with $p_i > p_i(x)$ is weakly dominated by the alternate offer $(p_i(x), q_i)$. The alternate offer weakly increases buyer i's profits, with a strict increase whenever $-i$'s offer is lower, and leaves the probability that his capital constraint is satisfied unchanged.

Finally, any offer (p_i, q_i) such that $p_i \in [p_i(0), p_i(x))$ gives negative profits whenever it is accepted (e.g., if $p_{-i} = p_i(0)$) and is weakly dominated by $\tilde{p}_i = 0$ (which gives zero profits), as follows. If $p_{-i} \geq p_i(0)$, the offer $\tilde{p}_i = 0$ guarantees that his capital constraint is satisfied. If instead $p_{-i} < p_i(0)$, the offer $\tilde{p}_i = 0$ weakly reduces the probability that buyer i's capital constraint will be violated; under $\tilde{p}_i = 0$, the constraint is violated with probability p_{-i}, while under (p_i, q_i), it is violated with probability p_i if $q_i = x$ and a probability of at least p_{-i}, otherwise. Q.E.D.

Proof of Lemma 4 (The lemma holds as stated in the main text, subject to the assumption that $\max\{\Delta_1, \Delta_2\} \leq \min\{2\Delta_1, 2\Delta_2\}$. The case $\max\{\Delta_1, \Delta_2\} > \min\{2\Delta_1, 2\Delta_2\}$ is covered by the general version of Proposition 5, stated below.)

First, consider the case $\Delta_1 \neq \Delta_2$, and assume, without loss of generality, that $\Delta_1 > \Delta_2$. If $p_2(x) = 2\Delta_2$, buyer 2 bids $p_2 = 2\Delta_2$ (Lemma 3); therefore, buyer 1's best response is to bid $p_1 = 2\Delta_2 + \varepsilon$. If $p_2(x) < 2\Delta_2$, then from Lemma 3, $p_2 < 2\Delta_2$, and standard competition arguments imply that buyer 2 bids $p_2 = 2\Delta_2 - \varepsilon$ and buyer 1 bids $p_1 = 2\Delta_2$. Next, consider the case $\Delta_1 = \Delta_2 = \Delta$. If $\max\{p_1(x), p_2(x)\} = 2\Delta$, then by Lemma 3, the equilibrium price is 2Δ and is posted by the buyer with the highest $p_i(x)$. If $\max\{p_1(x), p_2(x)\} < 2\Delta$, Lemma A-2 and standard competition arguments imply that both buyers post the price $2\Delta - \varepsilon$ for the full amount. Q.E.D.

Lemma A-3 (Second elimination round) *Suppose* $p_1(x) \leq 2\Delta_1$ *and* $p_2(x) > 2\Delta_2$.

(i) If $p_2(0) \leq \max\{\Delta_1, p_1(x)\} \leq p_2(x)$*, no offer* (p_2, q_2) *with* $p_2 = p_2(x)$ *survives two rounds of elimination of weakly dominated strategies.*

(ii) If $p_2(0) > \max\{\Delta_1, p_1(x)\}$*, the only offer* (p_2, q_2) *with* $p_2 > \max\{2\Delta_2, \Delta_1, p_1(x)\}$ *to survive two rounds of elimination of weakly dominated strategies is* $(p_2, q_2) = (p_2(x), x)$*.*

Proof of Lemma A-3. From Lemma A-2, $p_2(x) > p_2(0) > 2\Delta_2$. From the first round of elimination (Lemma 3), buyer 1 bids $p_1 \in [\max\{\Delta_1, p_1(x)\}, 2\Delta_1]$ and $q_1 = x$, buyer 2 bids either $p_2 = p_2(x)$ or $p_2 < p_2(0)$, and the offers $p_2 = 0$ and $(p_2, q_2) = (p_2(x), x)$ survived.

Part (i): Since $p_1 \geq \max\{\Delta_1, p_1(x)\} \geq p_2(0)$, the offer $\tilde{p}_2 = 0$ guarantees that buyer 2 makes zero profits and his capital constraint is satisfied. In contrast, offering $p_2 = p_2(x)$ generates strictly negative profits whenever it is accepted (e.g., if $p_1 = \max\{\Delta_1, p_1(x)\}$).

Part (ii): The offer $(p_2, q_2) = (p_2(x), x)$ survives the second elimination round, since it is a unique best response for buyer 2 if buyer 1 offers $(p_1, q_1) = (\max\{\Delta_1, p_1(x)\}, x)$. Specifically, under $(p_2(x), x)$ buyer $2's$ capital constraint is satisfied, while under any other offer that survives the first round, the capital constraint is violated with a positive probability. Moreover, any offer (p_2, q_2) remaining after the first elimination round, such that $p_2 \in (\max\{2\Delta_2, \Delta_1, p_1(x)\}, p_2(x))$, or $p_2 = p_2(x)$ and $q_2 < x$, is weakly dominated by $\tilde{p}_2 = 0$, which weakly increases profits (strictly if $p_1 = 0$) and weakly decreases the probability of violating the capital constraint. Q.E.D.

Proof of Proposition 5 (As stated in the text, but to cover the case $\max\{\Delta_1, \Delta_2\} > \min\{2\Delta_1, 2\Delta_2\}$, $p_i(x)$ is replaced by $\max\{\Delta_i, p_i(x)\}$ for $i \in \{1, 2\}$.)

From the first elimination round (Lemma 3), we know that buyer 1 bids for the entire amount; therefore, whenever the seller accepts buyer 1's offer, the expected value of v drops to $\frac{1}{2}p_1$, regardless of buyer 2's offer. In addition, $\max\{\Delta_1, p_1(x)\} \leq p_1 \leq 2\Delta_1$.

Part (A), $\max\{\Delta_2, p_2(0)\} \leq \max\{\Delta_1, p_1(x)\}$: First, consider the case $\max\{\Delta_1, p_1(x)\} \leq 2\Delta_2$. Since $p_2(0) \leq 2\Delta_2$, Lemma A-2 implies that $p_2(x) \leq p_2(0) \leq 2\Delta_2$. Moreover, it is easy to show that $p_2(x) \leq 2\Delta_1$. So Lemma 4 applies, completing the proof of this case. Second, consider the case $\max\{\Delta_1, p_1(x)\} > 2\Delta_2$; we must show the sale price is $\max\{\Delta_1, p_1(x)\}$. If $p_2(x) \leq 2\Delta_2$, then from the first elimination round, buyer 2 bids at most $2\Delta_2$, and since $\max\{\Delta_1, p_1(x)\} > 2\Delta_2$, buyer 1 bids $(p_1, q_1) = (\max\{\Delta_1, p_1(x)\}, x)$, which is his unique best response. If instead $p_2(x) > 2\Delta_2$, then any offer (p_2, q_2) with

$p_2 \geq p_2(0)$ and $p_2 \neq p_2(x)$ is eliminated in the first stage, while the offer $\tilde{p}_2 = 0$ remains. If $p_2(x) < \max\{\Delta_1, p_1(x)\}$, then since $p_2 \leq p_2(x)$, the unique best response for buyer 1 is to offer $(p_1, q_1) = (\max\{\Delta_1, p_1(x)\}, x)$. If $p_2(x) \geq \max\{\Delta_1, p_1(x)\}$, a second round of elimination (Lemma A-3) implies that buyer 2 offers $p_2 < p_2(0)$; hence, the unique best response for buyer 1 is to offer $(p_1, q_1) = (\max\{\Delta_1, p_1(x)\}, x)$.

Part (B), $\max\{\Delta_2, p_2(0)\} > \max\{\Delta_1, p_1(x)\}$**:** Start with the case $p_2(x) \leq 2\Delta_2$. If $p_2(x) > 2\Delta_1$, then $p_2(0) > 2\Delta_1$ (Lemma A-2), and since buyer 1 bids $p_1 \leq 2\Delta_1$, buyer 2's unique best response is to offer $(p_2, q_2) = (\max\{\Delta_2, p_2(x)\}, x)$; the seller accepts whenever $v \leq p_2$. If instead $p_2(x) \leq 2\Delta_1$, then since $p_2(0) \leq 2\Delta_2$ (Lemma A-2), it follows that $\max\{\Delta_1, p_1(x)\} \leq 2\Delta_2$. Lemma 4 then applies, completing the proof.

The remainder of the proof deals with the case $p_2(x) > 2\Delta_2$; we must show the sale price is $p_2(x)$ $(\pm\ \varepsilon)$. From the first elimination round, buyer 2's offer must satisfy $p_2 \leq p_2(x)$. From Lemma A-2, $\max\{2\Delta_2, \Delta_1, p_1(x)\} < p_2(0) < p_2(x)$.

There is no equilibrium in which buyer 2 bids $p_2 \leq \max\{2\Delta_2, \Delta_1, p_1(x)\}$ and buyer 1 bids $p_1 > p_2$, as follows. In any such equilibrium, $q_1 = x$, and p_1 would be either $\max\{2\Delta_2, \Delta_1, p_1(x)\}$ or $\max\{2\Delta_2, \Delta_1, p_1(x)\} + \varepsilon$, and hence below $p_2(0)$. So buyer 2's capital constraint is violated whenever $v \leq p_1$; but then buyer 2 would deviate to offer $(p_2, q_2) = (p_2(x), x)$. Nor is there an equilibrium in which buyer 2 bids $p_2 \leq \max\{2\Delta_2, \Delta_1, p_1(x)\}$ and buyer 1 bids $p_1 \leq p_2$. To see this, note that $\max\{2\Delta_2, \Delta_1, p_1(x)\} < p_2(0)$, and so buyer 2's capital constraint would certainly be violated whenever $v \leq p_1$, and again buyer 2 would deviate to offer $(p_2, q_2) = (p_2(x), x)$. Consequently, the second elimination round (Lemma A-3) implies that $(p_2, q_2) = (p_2(x), x)$ in any candidate equilibrium. Hence, if $p_2(x) < 2\Delta_1$, the unique equilibrium is that buyer 2 bids $(p_2, q_2) = (p_2(x), x)$ and buyer 1 bids $(p_1, q_1) = (p_2(x) + \varepsilon, x)$. If instead, $p_2(x) \geq 2\Delta_1$, the unique equilibrium outcome is that buyer 2 bids $(p_2, q_2) = (p_2(x), x)$, and the equilibrium price is $p_2(x)$. Q.E.D.

Proof of Proposition 6. (The proposition as stated in the main text continues to hold even without the assumption $\max\{\Delta_1, \Delta_2\} \leq \min\{2\Delta_1, 2\Delta_2\}$.)

From Lemma A-2, $p_i(x) > p_i(0) > 2\Delta_i$ for $i \in \{1, 2\}$. From the first elimination round (Lemma 3), either $p_i = p_i(x)$, or $p_i < p_i(0)$, for $i \in \{1, 2\}$. In addition, we know that for $i \in \{1, 2\}$, the offers $p_i = 0$ and $(p_i, q_i) = (p_i(x), x)$ survive the first round. In fact, the

offers $p_1 = 0$ and $p_2 = 0$ cannot be eliminated in any round, since each one is a unique best response against the other. Hence, the no trade equilibrium survives the elimination process. The remainder of the proof establishes that, provided $p_1(x) \neq p_2(x)$, this is the unique equilibrium to survive the elimination process.

Start with the case $p_1(0) > p_2(x)$. The offer $(p_1, q_1) = (p_1(x), x)$ survives the second elimination round, since it is a unique best response if buyer 2 bids $(p_2, q_2) = (p_2(x), x)$. However, any offer (p_1, q_1) with $p_1 \in (0, p_1(0))$ is eliminated, since it violates buyer 1's capital constraint with a positive probability, while the offer $(p_1(x), x)$ never violates the constraint.[28] Hence, buyer 1 bids either $p_1 = 0$ or $(p_1, q_1) = (p_1(x), x)$. In the third elimination round, the offer $p_2 = 0$ weakly dominates any other remaining offer for buyer 2, since $p_2 = 0$ gives buyer 2 zero profits and guarantees that his capital constraint is satisfied, while any other remaining offer gives zero profits if $p_1 = p_1(x)$ and leads to negative profits and/or violates buyer 2's capital constraint, if $p_1 = 0$. Hence, the unique equilibrium that survives the third elimination round is that both buyers bid nothing. The case $p_2(0) > p_1(x)$ is similar.

The rest of the proof deals with the case in which both $p_1(0) \leq p_2(x)$ and $p_2(0) \leq p_1(x)$; and without loss, assume $p_1(x) < p_2(x)$.

Note first that if there is no offer $(\bar{p}_i, \bar{q}_i)$ with $\bar{p}_i \in (0, p_{-i}(0))$ that survives the first elimination round, then (from the second elimination round) the other buyer $-i$ must offer $p_{-i} = 0$, and the unique equilibrium is no trade, and the proof is complete. The rest of the proof deals with the case in which for both buyers an offer $(\bar{p}_i, \bar{q}_i)$ with $\bar{p}_i \in (0, p_{-i}(0))$ survives the first round. Consequently, for each buyer i the offer $(p_i(x), x)$ survives the second round, since it is a unique best response when the buyer $-i$ bids $(\bar{p}_{-i}, \bar{q}_{-i})$.

Next, we show that no offer (p_2, q_2) with $q_2 = x$ and $p_1(0) \leq p_2 < p_2(0)$ survives the second elimination round. In particular, any such offer is weakly dominated by $\tilde{p}_2 = 0$, as follows. If $(p_1, q_1) = (p_1(x), x)$, buyer 2 is indifferent between $\tilde{p}_2 = 0$ and (p_2, q_2), since in both cases he obtains zero profits and his capital constraint is satisfied. If $p_1 = 0$, or if $p_1 = p_1(x)$ and $q_1 < x$, buyer 2 strictly prefers $\tilde{p}_2 = 0$. It remains to show that buyer 2 weakly prefers $\tilde{p}_2 = 0$ to (p_2, q_2), given all other potential offers from buyer 1 that survive the first

[28] If $p_2 = 0$ or ($p_2 < p_1$ and $q_1 = x$), (p_1, q_1) violates with probability p_1. If $0 < p_2 < p_1$ and $q_1 < x$, it violates with probability of at least p_2. If $p_2 \geq p_1$, we know from the first round that $p_2 \leq p_2(x) < p_1(0)$, and so the offer (p_1, q_1) violates the constraint with probability of at least p_1.

round; any such offer would have $p_1 \in (0, p_1(0))$. Given such offer, if buyer 2 offers $\tilde{p}_2 = 0$, his capital constraint is violated with probability p_1, but if he chooses $p_2 \in [p_1(0), p_2(0))$ and $q_2 = x$, his capital constraint is violated with a higher probability, namely p_2.

In the third elimination round, any offer (p_1, q_1) such that $p_1 \in (0, p_1(0))$ is eliminated, since it is weakly dominated by the offer $(p_1(x), x)$, as follows. If $(p_2, q_2) = (p_2(x), x)$, both offers provide the same utility. If instead, $p_2 < p_2(0)$, either $p_2 \in [p_1(0), p_2(0))$ and (from the argument immediately above) $q_2 < x$; or $p_2 < p_1(0)$. In either case, buyer 1's capital constraint is violated with a positive probability under the offer (p_1, q_1) but is never violated under the offer $(p_1(x), x)$. Hence, buyer 1 bids either $p_1 = 0$ or $p_1 = p_1(x)$. If $p_1 = p_1(x)$ is eliminated in the third round, we are done and the unique equilibrium is no trade; otherwise, since $p_2(0) \leq p_1(x)$, the only offer for buyer 2 that survives the fourth round of elimination is $p_2 = 0$, and the unique equilibrium is no trade. Q.E.D.

Appendix C: Optimality of Linear Price Schedules

Proof of Proposition 1. We prove the proposition for the more general case in which the asset value v is not restricted to be uniformly distributed. Instead, we assume only that the conditional expectation $E(v|v \leq p)$ is differentiable with respect to p, with the derivative bounded away from zero. We write $h(p) \equiv E(v|v \leq p) + \Delta$ and replace Assumption 1 with the more general assumption $h'(p) \geq \frac{x}{M+x}$; note that when v is uniformly distributed, $h'(p) = \frac{1}{2}$, and so this reduces to $M \geq x$. We also assume that the asset is divisible into N individual units and write $x_N \equiv \frac{x}{N}$. N can be arbitrarily large.

We first show that the revelation principle holds in our setting. Consider a nondirect mechanism. The outcome is the number of units transferred from the seller to the buyer and the total monetary payment from the buyer to the seller. From this (and taking as given the mechanism and the buyer's and seller's strategies), the market determines the market value of each unit h, using Bayes' rule. To show that we can achieve the same outcome using a direct revelation mechanism, we can use standard arguments plus the observation that since the market observes the final outcome and only the final outcome, inferences regarding the asset value (v) are the same in both mechanisms.

Any stochastic mechanism can be replaced by a deterministic mechanism. The latter

can leave expected payments unchanged and make it easier to satisfy the buyer's capital constraint. Any deterministic (direct) mechanism specifies a transfer of $q(v)$ units of the asset in exchange for a monetary payment of $P(v)$ and is equivalent to giving the seller the choice of quantity-price pairs in the menu $\{(q(v), P(v))\}$. Since the asset is divisible into N units, and since for a given quantity the seller will always choose the quantity-price pair that gives him the highest payment, without loss we can restrict attention to menus of the type $\{(q, P_q) : q = 1, \dots N\}$. In turn, such menus are equivalent to offering a price schedule $p(\cdot) = (p_1, p_2, ..., p_N)$, where $p_i x_N$ denotes the price paid for the i'th unit bought; that is, if the buyer purchases n units (i.e., a quantity nx_N of the asset), he pays $x_N \sum_{i=1}^{n} p_i$.

Next, we establish that for any price schedule $p(\cdot)$, there exists an alternate price schedule $\tilde{p}(\cdot)$ that satisfies $\tilde{p}_1 \geq \tilde{p}_2 \geq ... \geq \tilde{p}_N$ and leaves all payoffs and capital constraints unchanged. The basic idea is that if there exists k such that $p_k < p_{k+1}$, the seller would never sell just k units, and hence, we can replace p_k and p_{k+1} with their average. Formally, suppose $p_k < p_{k+1}$. Let $\underline{k}$ be the smallest integer such that $p_i = p_k$ for every integer $i \in [\underline{k}, k]$, and let $\overline{k}$ be the largest integer such that $p_i = p_{k+1}$ for every integer $i \in [k+1, \overline{k}]$. Define $\overline{p} = \frac{1}{\overline{k}-\underline{k}+1} \sum_{i=\underline{k}}^{\overline{k}} p_i$, and define a new price schedule by $\tilde{p}_i = \overline{p}$ for $i \in [\underline{k}, \overline{k}]$, and $\tilde{p}_i = p_i$ otherwise. Recall that we assume that if the seller is indifferent between selling and not selling, he chooses to sell. Note that if the seller chooses to sell only j units of asset even though the buyer is willing to buy more than j units, we must have $p_{j+1} < v \leq \frac{1}{j-j'+1} \sum_{i=j'}^{j} p_i$ for every $j' \leq j$. It then follows that under both schedules, the seller never sells any quantity $q \in \left[\underline{k}x_N, (\overline{k}-1)x_N\right]$. In addition, by construction, if he sells any other quantity, he gets the same payoff under both schedules. Consequently, the seller's response to and payoff from the two schedules are exactly the same. Hence, the buyer's profits are also the same, and since the seller's behavior is the same, the information revealed in equilibrium is the same. Therefore, the capital constraints are the same. Iteration of the argument above completes the proof of the claim: the iteration process ends in a finite number of steps, since under the new price schedule, the number of prices p_i that are different from their follower p_{i+1} is strictly lower than that in the original schedule.

Hence, we can assume, without loss of generality, that an optimal price schedule satisfies $p_1 \geq p_2 \geq ... \geq p_N$. Under such a schedule, the seller sells the i'th unit if and only if $v \leq p_i$.

The capital constraint must be satisfied given any amount that the seller might choose

to sell. In particular, denote by n the largest $i \in \{1, \ldots, N\}$ such that $p_i > 0$. Then the capital constraint associated with the sale of n units must be satisfied; that is, $(M + nx_N)h(p_n) - x_N(p_1 + ... + p_n) \geq L$. Since p_n is the lowest price in the set $\{p_1, ..., p_n\}$, the capital constraint would also be satisfied under a linear schedule in which the buyer offers to buy up to n units at a price per unit p_n, i.e., $(M + nx_N)h(p_n) - nx_N p_n \geq L$. For such a linear price schedule, this is the only relevant constraint, since whenever the seller chooses to sell anything, he sells as much as he can. Note that the left-hand side of the capital constraint is increasing as a function of p_n: either $h'(p_n) \geq 1$, in which case this is immediate, or $h'(p_n) < 1$, in which case the derivative of the left-hand side with respect to p_n is $Mh'(p_n) + nx_N(h'(p_n) - 1) \geq Mh'(p_n) + x(h'(p_n) - 1) \geq 0$, where the second inequality follows from $h'(p) \geq \frac{x}{M+x}$. Hence, the capital constraint would be satisfied under any linear schedule in which the buyer offers to buy up to n units at a price per unit of $p \geq p_n$.

To complete the proof, we must show that the buyer can always weakly raise his profits by deviating from his original nonlinear schedule and instead offering a linear schedule with a price $p \geq p_n$. Writing $p_{N+1} = 0$, his profits under the original schedule are $x_N \sum_{i=1}^{N} \Pr(v \in (p_{i+1}, p_i]) \sum_{k=1}^{i} (\Delta + E[v|v \in (p_{i+1}, p_i]] - p_k)$. Changing the order of summation, this reduces to $x_N \sum_{k=1}^{n} \Pr(v \leq p_k)(\Delta + E[v|v \leq p_k] - p_k)$, which is necessarily weakly less than the profits $x_N \sum_{k=1}^{n} \max_{\tilde{p} \in \{p_1, ..., p_n\}} \Pr(v \leq \tilde{p})(\Delta + E[v|v \leq \tilde{p}] - \tilde{p})$, which the buyer can attain under a linear schedule in which he offers to buy up to n units of the asset for a fixed price. Finally, the proof extends easily to the case in which the buyer has a limited borrowing capacity as in Section 6.2. Q.E.D.

Appendix D: Limited Borrowing Capacity.

Part 1: Buyer is capital constrained

In this appendix, we characterize the optimal bidding strategy of a monopolist buyer when his capital constraint (2) is replaced with $\alpha(\frac{1}{2}p + \gamma\Delta)(M + q) - pq \geq L$, where $\alpha \in (0, 2)$ and $\gamma \in [0, 1]$. The case $\alpha > 1$ does not reflect borrowing constraints but is useful when we discuss marking to market in Section 6.3. We replace Assumption 1 with $x < \frac{\alpha}{2-\alpha}M$ and define leverage as $\delta = \frac{L}{(\frac{1}{2}+\gamma\Delta)M}$, which generalizes the definition in the text. We also generalize the definition of $p(q)$ in equation (3), so that $p(q) \equiv \frac{2\theta - cq}{1-\beta q}$, where $\theta \equiv \frac{L}{\alpha M} - \gamma\Delta$,

$\beta \equiv \frac{2-\alpha}{\alpha M}$, and $c \equiv \frac{2\gamma\Delta}{M}$.

Proposition A-1 *(i) If $\frac{\alpha\gamma}{2-\alpha} < 1$, trade can happen if and only if leverage is low; that is, $\delta < \frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$. If leverage is very low, the buyer offers to buy the entire quantity x for a price Δ. As leverage increases, the buyer increases the price; and as leverage increases further, the buyer also reduces the quantity. Both price and quantity are continuous in leverage. As leverage approaches $\frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$, the price approaches 2Δ and the quantity approaches zero. Expected volume is continuous in leverage; it first increases and then drops to zero.*

(ii) If $\frac{\alpha\gamma}{2-\alpha} \geq 1$, trade can happen if and only if leverage is sufficiently low, so that $p(x) \leq 2\Delta$. If trade happens, the buyer offers to buy the entire quantity x for a price per unit $\max\{\Delta, p(x)\}$, which is weakly increasing in initial leverage δ.

Proof of Proposition A-1.

For use below, note that $\theta = \delta\frac{1+2\gamma\Delta}{2\alpha} - \gamma\Delta$; that is, θ is a monotone increasing function of leverage. Since $1-\beta q > 0$ (from $x < \frac{\alpha}{2-\alpha}M$), the capital constraint can be written as $p \geq p(q)$. Observe that $p'(q) = \frac{2\beta\left(\theta - \frac{c}{2\beta}\right)}{(1-\beta q)^2} = \frac{2\beta\left(\theta - \frac{\alpha\gamma\Delta}{2-\alpha}\right)}{(1-\beta q)^2}$. Thus, $\min_{q\in[0,x]} p(q) = \begin{cases} p(0) & \text{if } \theta \geq \frac{\alpha\gamma\Delta}{2-\alpha} \\ p(x) & \text{if } \theta \leq \frac{\alpha\gamma\Delta}{2-\alpha}. \end{cases}$ Trade is possible if and only if there exists a quantity $q \in (0, x]$, such that $p(q) \leq 2\Delta$, so that the buyer makes nonnegative profits; that is, if either $p(x) \leq 2\Delta$ or $p(0) = 2\theta < 2\Delta$. Hence, trade is possible if and only if θ falls in some lower interval.

Case 1: $\frac{\alpha\gamma}{2-\alpha} < 1$. At $\theta = \Delta$, $p(x) > p(0) = 2\Delta$. Thus, trade is impossible for $\theta \geq \Delta$ but is possible for all $\theta < \Delta$, or equivalently, $\delta < \frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$. Next, we characterize the buyer's offer when $\theta < \Delta$. If $p(x) \leq \Delta$, which is equivalent to $\theta \leq \frac{1}{2}(\Delta - \beta\Delta x + cx)$, the capital constraint does not bind and the buyer makes the benchmark offer (x, Δ). If $\theta \leq \frac{\alpha\gamma}{2-\alpha}\Delta$, increasing q relaxes the capital constraint, and since $\theta < \Delta$ we know the buyer has a strictly profitable trade. Consequently, the buyer bids for the entire amount x available and chooses a price $\max\{\Delta, p(x)\}$. This price is weakly increasing in θ, and hence in initial leverage δ.

The remainder of the proof of this case deals with the open interval of θ values above $\max\left\{\frac{\alpha\gamma}{2-\alpha}\Delta, \frac{1}{2}(\Delta - \beta\Delta x + cx)\right\}$ but below Δ. Since $\theta < \Delta$, we know that the buyer has a strictly profitable trade. Moreover, any strictly profitable trade in which the capital constraint is slack is strictly dominated by one in which it binds: either $q < x$, in which case q can be increased, or $q = x$ and $p > p(x) > \Delta$, in which case p can be decreased. Consequently, the buyer's best offer is the solution to the more constrained maximization problem in which he

must keep the capital constraint binding; that is,

$$\max_{q\in[0,x]} q\pi\left(p\left(q\right)\right). \tag{A-1}$$

Observe that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)=\pi\left(p\left(q\right)\right)+qp'\left(q\right)\pi'\left(p\left(q\right)\right)$, and recall $p'\left(q\right)>0$ in the interval under consideration. Hence, (A-1) has a unique solution, as follows: if $q\pi\left(p\left(q\right)\right)>0$ and $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)\leq 0$ for some q, then $\pi'\left(p\left(q\right)\right)<0$, and so by the strict concavity of π and the strict convexity of p, it follows that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)<0$ for all higher q. Moreover, the maximizer of (A-1) must be such that $\pi'\left(p\left(q\right)\right)<0$ (if the maximizer is the corner $q=x$, this follows from $p\left(x\right)>\Delta$). Hence, in the interval from which the maximizer of (A-1) is drawn, $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)$ strictly decreases in θ. Hence, the buyer's choice of q weakly decreases as θ (and hence initial leverage δ) increases. Note also that if q is strictly decreasing at any θ, the same is true for all higher θ.

For the effect of θ (and hence leverage) on the price offered, note first that if the buyer offers to buy everything ($q=x$) for price $p\left(x\right)$, it follows immediately that the price increases. If instead the buyer offers to buy $q<x$, the price satisfies $p=p(q)$, and the optimal p solves $\max_{p\in[0,1]}q(p)\pi(p)$, where $q(p)=\frac{p-2\theta}{\beta p-c}$ is the inverse function of $p(q)$. Observe that $\frac{\partial}{\partial p}q(p)\pi(p)=q'(p)\pi(p)+q(p)\pi'(p)$; $q'(p)=\frac{2\theta\beta-c}{(\beta p-c)^2}>0$; and recall that the optimal p satisfies $\pi'(p)<0$. Hence, in the interval in which the (unique) optimal p is drawn, $\frac{\partial}{\partial p}q(p)\pi(p)$ strictly increases in θ and strictly decreases in p (the last part follows from the concavity of π and q). Hence, the optimal p increases in θ.

From the analysis above, the buyer's offer is continuous as a function of θ. Finally, as θ approaches Δ, only offers with q close to 0 can satisfy the capital constraint (with a price below 2Δ). It follows easily that as θ approaches Δ, the buyer's offer converges to $(q,p)=(0,2\Delta)$. The expected volume (pq) converges to 0.

To show that expected volume first increases in leverage, it is enough to show that there exists some interval to the right of $\frac{1}{2}\left(\Delta-\beta\Delta x+cx\right)$ such that when θ falls in this interval, the buyer offers to buy everything, $q=x$. If $\frac{\alpha\gamma}{2-\alpha}\Delta>\frac{1}{2}\left(\Delta-\beta\Delta x+cx\right)$, this is immediate from the analysis above. Otherwise, note that at $\theta=\frac{1}{2}(\Delta-\beta\Delta x+cx)$, we know $p(x)=\Delta$; thus, $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)\Big|_{q=x}=\pi\left(\Delta\right)+xp'\left(x\right)\pi'\left(\Delta\right)=\pi\left(\Delta\right)>0$. By continuity, it follows that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)\Big|_{q=x}>0$ over some interval to the right of $\frac{1}{2}(\Delta-\beta\Delta x+cx)$, implying that the buyer offers to buy $q=x$ in this interval.

From the analysis above, q must eventually be strictly decreasing in θ, (and if it is strictly decreasing at some θ, the same is true for all higher θ up to Δ, when trade becomes impossible). In this case, expected volume changes by $\frac{\partial(qp(q))}{\partial\theta} = \frac{\partial q}{\partial\theta}p(q)+qp'(q)\frac{\partial q}{\partial\theta} = \frac{\partial q}{\partial\theta}[p(q)+qp'(q)]$, which is strictly negative in the interval under consideration.

Case 2: $\frac{\alpha\gamma}{2-\alpha} \geq 1$ (contains Proposition 3 as a special case) In this case, at $\theta = \Delta$, $p(x) \leq p(0) = 2\theta$, and so $p(x) \leq 2\Delta$. Hence, trade is certainly possible up to $\theta = \Delta$. Hence, trade is possible for all θ weakly below the cutoff value of θ such that $p(x) = 2\Delta$. The characterization of the buyer's offer for θ below this cutoff is the same as for the first part of the case $\frac{\alpha\gamma}{2-\alpha} < 1$.

Changes in α: From the capital constraint $\alpha(\frac{1}{2}p+\gamma\Delta)(M+q)-pq \geq L$, it follows that if trade is possible when $\alpha = \alpha'$, it is also possible when $\alpha \geq \alpha'$. Also observe that β, $p(q)$, and $p'(q)$ strictly decrease in α. Hence, following similar steps as above, one can show that increasing α has a similar effect on the price and quantity as does reducing θ. Q.E.D.

Part 2: Seller is capital constrained

Next, we analyze the case in which the seller faces borrowing restrictions, and his capital constraint (8) is given by $\frac{1}{2}\alpha p(M_s - q) + pq \geq L_s$, where $a \in (0,2)$. Equation (9) becomes $p \geq \frac{\delta_s}{\alpha+(2-\alpha)\frac{q}{M_s}}$. Since increasing q relaxes the capital constraint, the claim that the buyer offers either $q = 0$ or $q = x$ continues to hold. Intuitively, since the expected value of the asset conditional on the seller accepting the offer is less than the sale price (for standard adverse selection reasons), replacing assets with cash relaxes the capital constraint. Trade can occur if and only if $\delta \leq 2\Delta[\alpha+(2-\alpha)\frac{x}{M_s}]$, and if trade occurs, the buyer offers to buy x units at a price per unit $\max\{\Delta, \frac{\delta_s}{\alpha+(2-\alpha)\frac{x}{M_s}}\}$.

Appendix E: Competition With Nonlinear Schedules

Proposition A-2 *The equilibrium outcomes discussed in Section 5 (in particular, Lemma 4 and Propositions 5 and 6) remain even if a buyer can deviate by offering a nonlinear price schedule rather than a linear schedule.*

Proof of Proposition A-2. Let (p_1^*, q_1^*) and (p_2^*, q_2^*) be a pair of equilibrium offers when buyers are restricted to use linear price schedules. Without loss, $q_1^* = q_2^* = N$, with the

interpretation that $p_i^* = 0$ is the same as not making an offer. We show that neither buyer has an incentive to deviate to a nonlinear schedule. We show that buyer 1 has no incentive to deviate; the proof that buyer 2 has no incentive to deviate is symmetric.

Step 1: First, we show that without loss we can restrict attention to deviations to a nonincreasing price schedule. The proof is along the lines of the proof of Proposition 1. Specifically, as before denote buyer 1's nonlinear price schedule by $p_1, p_2, \ldots, p_N$, where p_i is the price buyer 1 pays for the i'th unit acquired. We then follow the same iterative process as before and replace any subsequence $p_{\underline{k}} = \ldots = p_k < p_{k+1} = \ldots = p_{\bar{k}}$ with the average of these $\bar{k} - \underline{k} + 1$ prices. If this replacement leaves outcomes unaffected for all realizations of the seller valuation v, then we proceed to the next step, exactly as in the monopoly case.

The additional step necessitated by competition is the case in which the replacement affects outcomes for some realization v' of v. This case arises only if, under the original schedule, there is a positive probability that the seller sells $k' \in \{\underline{k}, \ldots, \bar{k} - 1\}$ units to buyer 1. Because $p_{\underline{k}} = \ldots = p_k < p_{k+1} = \ldots = p_{\bar{k}}$, this can happen only if buyer 2's offered price p_2^* is weakly above both p_{k+1} and v', and if buyer 2's maximum quantity is weakly above $N - \bar{k} + 1$. Moreover, buyer 2's price p_2^* must be weakly above all averages of the form $\frac{1}{l}\sum_{j=k'+1}^{k'+l} p_j$ $(l \leq N - k')$, since otherwise the seller could do better by selling more to buyer 1.

Consider the price schedule formed by replacing prices $p_{k'+1}, \ldots, p_N$ with zeros. In the case that $p_2^* > \frac{1}{l}\sum_{j=k'+1}^{k'+l} p_j$ for all l, this leaves outcomes unchanged, since the seller never bought more than k' units from buyer 1 anyway. If instead $p_2^* = \frac{1}{l}\sum_{j=k'+1}^{k'+l} p_j$ for some l, the new schedule gives buyer 1 the same profits: if instead the new schedule strictly lowered profits, this would imply that buyer 1 could have improved profits under the original schedule by slightly increasing his offered price. Moreover, because under the original schedule there was some probability of the seller accepting buyer 2's offer, buyer 1's capital constraint remains satisfied for all realizations of v under the new schedule.

Step 2: From step 1, buyer 1 can restrict himself to deviations that satisfy $p_1 \geq p_2 \geq \ldots \geq p_N$. The seller effectively sees a nonincreasing schedule that is a combination of the schedules offered by the two buyers. If $p_2^* = 0$ or $p_2^* < p_N$, the seller never sells to buyer 2 and the proof is as in Proposition 1. Below, we deal with the case in which $p_2^* \geq p_N$. Denote by n^* the largest $i \in \{1, 2, \ldots, N\}$, such that $p_i \geq p_2^*$.

For use below denote $f(N) = x_N \sum_{k=1}^{N} \Pr(v \leq p_k)(\Delta + E[v|v \leq p_k] - p_k)$, which is the monopolist buyer's expected profits from Appendix C. Since $q_2^* = N$, the seller sells at most n^* units to buyer 1. In particular, if $v \leq p_2^*$, the seller sells all N units, and the market valuation of the asset is $h(p_2^*)$. Since there is a positive probability that the seller sells n^* units to buyer 1, the following capital constraint for buyer 1 must hold: $(M + n^* x_N)h(p_2^*) - x_N(p_1 + ... + p_{n^*}) \geq L$. The rest of the proof then follows as in Proposition 1, but we need to take into account potential ties. In particular, if there are no ties, the buyer's expected profits under the original schedule are at most $f(n^*)$, and if there are ties (e.g., $p_{n^*} = p_2^*$), profits are less than $f(n^*)$. In contrast, if the buyer offers to buy up to n^* units at a price per unit $p \in \{p_1, ..., p_{n^*}\}$, he obtains $x_N n^* \sum_{k=1}^{n} \Pr(v \leq p)(\Delta + E[v|v \leq p] - p)$, unless $p = p_2^*$. In the special case $p = p_2^*$, the seller might sell less than n^* to buyer 1. However, if buyer 1 makes positive profits when he purchases n^* units at a price per unit p_2^*, he also makes positive profits when he purchases N units at a slightly higher price. By offering to buy up to N units at a price per unit slightly higher than p_2^*, buyer 1 can avoid ties and guarantee that the seller sells everything to him. Q.E.D.