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The Expectation Hypothesis of the Term Structure of Very Short-Term Rates: Statistical Tests and Economic Value

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Abstract

This paper re-examines the validity of the Expectation Hypothesis (EH) of the term structure of US repo rates ranging in maturity from overnight to three months. We extend the work of Longstaff (2000) in two directions: (i) we implement statistical tests designed to increase test power in this context; (ii) we assess the economic value of departures from the EH based on criteria of profitability and economic significance. The EH is rejected throughout the term structure examined on the basis of the statistical tests. The results of our economic analysis are less uniform and more favorable to the EH, suggesting that the statistical rejections of the EH are not economically significant for rates with maturities of one month and longer.

Keywords: Expectation Hypothesis; Term Structure of Interest Rates; Vector Autoregression; Economic Value.

JEL Classification: G10; E43.

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1 Introduction

Ever since Fisher (1896) postulated the Expectations Hypothesis (EH) of the term structure of interest rates, this simple and intuitively appealing theory has attracted the attention of financial economists. Many authors have argued that interest rates at different maturities move together because they are linked by the EH and a number of studies have addressed the empirical validity of the EH. However, these studies, using a variety of tests and data, generally reject the EH (e.g. Roll, 1970; Fama, 1984; Fama and Bliss, 1987; Frankel and Froot, 1987; Stambaugh, 1988; Froot, 1989; Campbell and Shiller, 1991; Bekaert, Hodrick and Marshall, 1997; Longstaff, 2000; Bekaert and Hodrick, 2001; Clarida, Sarno, Taylor and Valente, 2006; Sarno, Thornton and Valente, 2007).

An important exception, in the sense that empirical evidence favorable to the EH is provided, is the study by Longstaff (2000), who presents the first tests of the EH at the extreme short end of the term structure, using repurchase (repo) rates with maturities measured in days or weeks. There are two reasons why Longstaff's study is particularly important. First, if the EH cannot explain the term structure at this extreme short end, it seems unlikely that it can be of value at longer maturities. Second, the use of repo rates is particularly appropriate for investigating the EH because repo rates represent the actual cost of holding riskless securities. Hence, repo rates provide potentially better measures of the short-term riskless term structure than other interest rates commonly used by the relevant literature. Longstaff (2000) finds that the EH is supported by the data.

This paper revisits the EH using an updated data set of repo rates from the same source as Longstaff (2000). Our motivation is twofold. First, the literature on testing the EH has made much progress in recent years by developing increasingly sophisticated testing procedures that are particularly useful in this context. Given the statistical problems afflicting conventional tests of the EH, in this paper we employ a test that was originally proposed by Campbell and Shiller (1987) and made operational by Bekaert and Hodrick (2001).[1] Bekaert and Hodrick (2001) develop a procedure for testing the parameter restrictions that the EH imposes on a vector autoregression (VAR) of the short- and long-term interest rates. The procedure is flexible and its size and power properties have been thoroughly investigated by Bekaert and Hodrick (2001) and Sarno, Thornton and Valente (2007). We apply this test to US repo rates ranging in maturity from overnight to three months over the sample period 1991-2005.

[1] It is well known that the tests that are commonly used to investigate the EH tend to generate paradoxical results (Campbell and Shiller, 1991). For example, Bekaert, Hodrick and Marshall (1997) and Thornton (2005, 2006) show how conventional tests may have generated misleading results in applied work because of a variety of finite sample biases, size distortions and power problems.

Second, we move beyond testing the validity of the EH from a purely statistical perspective and provide evidence on whether deviations from the EH reflected in the VAR tests are economically significant. Our economic analysis is important because statistical rejections of a hypothesis cannot directly be taken as economic rejections (e.g. Leitch and Tanner, 1991), and because statistical VAR tests of the EH do not allow for transactions costs which are essential if one wished to exploit departures from the EH in real world financial markets. Both of these reasons suggest that an assessment of the economic value of the EH is warranted to complement to statistical tests. Specifically, we calculate a measure of how much an investor is willing to pay for switching from a strategy that assumes that the EH holds (the restricted VAR) to a dynamic strategy which conditions on departures from the EH (the unrestricted VAR). We employ mean-variance analysis as a standard measure of portfolio performance and apply quadratic utility, which allows us to quantify how risk aversion affects economic value. With this procedure, we can also calculate directly a variety of common performance measures, including standard performance fees and Sharpe ratios.

To anticipate our results, we find that the EH is statistically rejected for all pairs of repo rates examined throughout the maturity spectrum from overnight to three months. Our results likely differ from Longstaff's (2000) because the VAR test is more powerful and because our sample period is somewhat longer than his. The results of our economic analysis are less uniform. We find that the statistical rejections of the EH can also be important economically at the shortest end of the repo term structure, but not for maturities of one month or longer. Hence, despite the statistical rejections of the EH, at maturities of one month or longer the EH provides a fairly reasonable first approximation to the repo rate term structure.

The outline of the paper is as follows. Section 2 briefly describes the data and preliminary statistics on repo rates. Section 3 presents the essential ingredients of the testing procedure proposed by Bekaert and Hodrick (2001). In Section 4, we outline the framework for measuring the economic value of departures from the EH in a mean-variance setting and describe the performance measures used to assess the economic significance of EH violations. Sections 5 and 6 report the results on the validity of the EH from VAR tests and economic value measures, respectively. The conclusions are presented in Section 7.

2 Data and Descriptive Statistics

The data set comprises daily observations of the closing overnight (r_t), one-week (r_t^{1w}), two-week (r_t^{2w}), three-week (r_t^{3w}), one-month (r_t^{1m}), two-month (r_t^{2m}), and three-month (r_t^{3m}) general collateral

government repo rates, from May 21, 1991 to December 9, 2005.[2] The data are obtained from Bloomberg and the source of the data is Garban, a large Treasury securities broker. Repo rates are quoted on a 360-day basis and the rate quotations in Bloomberg are given in increments of basis points (*bps*). Only days for which a complete set of rates for all maturities are available are included in the sample.[3] The total number of daily observations available is 3,625 and is essentially an update of the data set used by Longstaff (2000).[4]

Table 1 reports the descriptive statistics for the levels and the first differences in the short-term repo rates (expressed in percent). In general, the data display similar properties to those described by Longstaff (2000) for a shorter sample. The mean of the repo rates displays a small smile effect across the term structure. In particular, the mean overnight rate of 3.9600, is slightly higher than the mean one-week rate of 3.9492, which turns out to be lowest mean across the different maturities. The mean three-month rate is 3.9924, which is slightly more than 3 *bps* higher than the mean overnight rate. Table 1 also reports the mean repo rates for the different maturities by day of the week and shows a number of calendar regularities in the data. The mean repo tends to increase from Monday to Tuesday and to decrease afterwards, while the mean on Monday is always higher than the mean on Friday. For example, the mean overnight rate on Monday is 3.9718, which is about 5 *bps* higher than the mean overnight rate on Friday, equal to 3.9260. A similar pattern is observed for all other rates. We also report the standard deviations of daily changes in the rates. The overnight rate is much more volatile than the rates at other maturities. The standard deviation of daily changes in the overnight rate is about 18 *bps*, while the standard deviations for the other rates range from 5 to 6 *bps* per day. It is also worth noting that the standard deviations vary across days. Finally, the autocorrelation coefficients indicate a high level of persistence for all rates.

3 The Expectation Hypothesis

3.1 The VAR Framework

The EH states that the long-term interest rate is determined by the market's expectation for the short-term interest rate over the holding period of the long-term asset plus a constant risk premium:

$$r_t^n = k^{-1} \sum_{i=0}^{k-1} E_t r_{t+mi}^m + \pi^{n,m} \tag{1}$$

[2] The period September 11, 2001 through September 30, 2001 is not available and hence not included in the sample.
[3] This resulted in 42 days being dropped from the sample.
[4] Professor Longstaff kindly checked the consistency of our data set with the data used in Longstaff (2000).

where r_t^n is the n-period (long-term) rate; r_t^m is the m-period (short-term) rate; $\pi^{n,m}$ is the constant risk premium that may vary with the maturity of the rates; E_t denotes the mathematical expectation operator given information at time t; and $k = n/m$ is an integer.[5]

While much of the relevant literature relies on tests of the EH that are derived by parameterizing an algebraic manipulation of equation (1), Bekaert and Hodrick's (2001) reconsider the EH in a vector autoregressive (VAR) specification and test the restrictions implied by the EH on the VAR. Because a general VAR encompasses a wider range of potential data generating processes (DGPs) than equation (1), it is reasonable to conjecture, and is generally found, that tests based on the VAR specification are more powerful than tests derived from equation (1) (Bekaert and Hodrick, 2001; Sarno, Thornton and Valente, 2007).

The VAR restrictions implied by the EH are highly nonlinear, and Bekaert and Hodrick (2001) propose and study the properties of various tests for these restrictions, including the distance metric (DM) test, the Wald test and the Lagrange multiplier (LM) test.[6] To fit a VAR to interest rates data that satisfy the restrictions imposed by the EH, we use the constrained Generalized Method of Moments (GMM), which imposes orthogonality conditions defined by the EH.

Let $y_t \equiv (r_t^m,\ r_t^n)'$ be a vector of data available at time t. Then, a p-th order VAR for the demeaned y_t takes the form:

$$y_{t+1} = \sum_{i=1}^{p} \theta_i y_{t-i+1} + \eta_{t+1}, \tag{2}$$

where θ_i is a two-dimensional square matrix of coefficients; and η_{t+1} is the vector of innovations orthogonal to the information set available at time t. Let θ be formed by collecting the parameters in θ_i (for $i = 1, \ldots, p$) and denote the vector $z_t \equiv (y_t, x_{t-1}')'$. We can define the vector-valued function of the data and the parameters as $g(z_t, \theta) \equiv \eta_t \otimes x_{t-1}$, where x_{t-1} is a vector of instruments available at time $t-1$ and formed from stacking lagged values of y_t, and possibly a constant term. Then, GMM estimation imposes a standard set of orthogonality conditions of the form $E\left[g(z_t, \theta)\right] \equiv 0$.

[5] Shiller, Campbell, and Schoenholtz (1983) show that equation (1) is exact in some special cases and that it can be derived as a linear approximation to a number of nonlinear expectation theories of the term structure. Also, note that if the risk premium is zero, the resulting form of the EH is often termed the 'pure' EH.

[6] The LM test appears to perform best in Monte Carlo simulations. Bekaert and Hodrick (2001) show that, for a five percent nominal test, the empirical size of the LM test varies between 0.7 percent and 7.9 percent. In half of the cases considered, the empirical size is within one percentage point of the nominal size. Moreover, the LM test has reasonable power. The DM test displays less accurate size and power properties than the LM test, whereas the Wald test shows the worst properties among the three test statistics considered. In this paper, we rely on the LM and DM tests.

Using the corresponding sample moment condition for a sample of size T:

$$g_T(\theta) \equiv \frac{1}{T}\sum_{t=1}^{T} g(z_t, \theta), \tag{3}$$

the unconstrained parameters θ are estimated by minimizing the GMM criterion function:

$$J_T(\theta) \equiv g_T(\theta)' W g_T(\theta). \tag{4}$$

The optimal weighting matrix, W is defined as the inverse of $\Omega \equiv \sum_{k=-\infty}^{k=\infty} E\left[g(z_t,\theta)g(z_{t-k},\theta)'\right]$ (Hansen, 1982). Let Ω_T be a consistent estimator of Ω and define $G_T \equiv \nabla g_T(\theta)$. When the weighting matrix is chosen optimally as Ω_T^{-1}, the parameter estimates are asymptotically distributed as $\sqrt{T}\left(\widehat{\theta} - \theta_0\right) \longrightarrow N\left(0, \left(G_T'\Omega_T G_T\right)^{-1}\right)$, where θ_0 denotes the true parameter value.

3.2 Restrictions implied by the EH

The EH imposes a set of nonlinear restrictions on the parameters of the VAR in equation (2). These constraints can be seen by writing the VAR in first-order companion form as

$$\begin{pmatrix} r_t^m \\ r_t^n \\ r_{t-1}^m \\ r_{t-1}^n \\ \vdots \\ r_{t-p}^m \\ r_{t-p}^n \end{pmatrix} = \begin{pmatrix} \theta_1 & \theta_2 & \cdots & & \theta_p \\ I_2 & 0 & & & 0 \\ 0 & I_2 & & & \vdots \\ \vdots & & \ddots & & \\ 0 & & & I_2 & 0 \end{pmatrix} \begin{pmatrix} r_{t-1}^m \\ r_{t-1}^n \\ r_{t-2}^m \\ r_{t-2}^n \\ \vdots \\ r_{t-p-1}^m \\ r_{t-p-1}^n \end{pmatrix} + \begin{pmatrix} \eta_t \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{5}$$

or simply $x_t = \Gamma x_{t-1} + \upsilon_t$. Note that $E_t\left(x_{t+k}\right) = \Gamma^k x_t$, so that $E_t\left(r_{t+k}^m\right) = e_1'\Gamma^k x_t$ for $e_1 = (1, 0, \ldots, 0)'$. Note also that $r_t^n = e_2' x_t$ for $e_2 = (0, 1, 0, \ldots, 0)'$. Consequently, for any two interest rates such that $k = n/m$ is an integer, the EH implies that:

$$r_t^n = \frac{1}{k}\sum_{i=0}^{k-1} E_t\left(r_{t+mi}^m\right), \tag{6}$$

so that the EH can be expressed equivalently as:

$$e_2' x_t = \frac{1}{k}\sum_{i=0}^{k-1} e_1'\Gamma^{mi} x_t. \tag{7}$$

The constraints that satisfy the EH are given by:

$$a(\theta) \equiv e_2' - \frac{1}{k}\sum_{i=0}^{k-1} e_1'\Gamma^{mi} = 0. \tag{8}$$

Let these constraints be summarized in the null hypothesis that the EH holds, $H_0 : a(\theta_0) \equiv 0$. To estimate the parameters in θ under the restrictions in equation (8), define the Lagrangian from the usual GMM objective function and the vector of parameter constraints:

$$L(\theta, \gamma) = -\frac{1}{2} g_T(\theta)' \Omega_T^{-1} g_T(\theta) - a_T(\theta)' \gamma, \tag{9}$$

where γ is a vector of Lagrange multipliers (LMs), and $a_T(\theta)$ is the sample counterpart of $a(\theta)$. While direct maximization of the Lagrangian is difficult as the constraints are highly nonlinear, Bekaert and Hodrick (2001) develop a recursive algorithm which extends the estimator proposed by Newey and McFadden (1994). To obtain the estimates subjected to the constraints, we proceed as follows. Starting from a consistent estimator $\widetilde{\theta}$, which is usually the unconstrained estimator, the algorithm iterates on the following two equations derived from the Taylor expansions of the first-order conditions in (9):

$$\overline{\theta} \approx \widetilde{\theta} - B_T^{-1/2} M_T B_T^{-1/2} G_T' \Omega_T^{-1} g_T(\widetilde{\theta}) - B_T^{-1} A_T' \left(A_T B_T^{-1} A_T' \right)^{-1} a_T(\widetilde{\theta}) \tag{10}$$

$$\overline{\gamma} \approx - \left(A_T B_T^{-1} A_T' \right)^{-1} A_T B_T^{-1} G_T' \Omega_T^{-1} g_T(\widetilde{\theta}) + \left(A_T B_T^{-1} A_T' \right)^{-1} a_T(\widetilde{\theta}) \tag{11}$$

where $\overline{\theta}$ is the constrained estimate of θ, $M_T = I - B_T^{-1/2} A_T' \left(A_T B_T^{-1} A_T' \right)^{-1} A_T B_T^{-1/2}$ is an idempotent matrix; $B_T \equiv G_T' \Omega_T^{-1} G_T$; and $A_T \equiv \nabla a_T(\theta)$. In brief, we use the unconstrained VAR parameters, $\widetilde{\theta}$, as starting value and then iterate in equations (10) and (11) until the resulting constrained estimate satisfies the constraints, $a_T(\overline{\theta}) = 0$.

If the constraints have a significant impact on parameter estimation, then the value of the Lagrange multipliers should be significantly different from zero. To test the null hypothesis that the multipliers are jointly zero, i.e. the EH of the term structure of interest rates is valid, we consider the DM statistic:

$$T g_T(\overline{\theta})' \Omega_T^{-1} g_T(\overline{\theta}) \longrightarrow \chi^2(l) \tag{12}$$

and the LM statistic:

$$T \bar{\gamma} \left(A_T B_T^{-1} A_T' \right) \bar{\gamma} \longrightarrow \chi^2(l) \tag{13}$$

where l is the number of restrictions, which depends on the VAR order, imposed by the EH.

3.3 Unconstrained VAR

Bekaert, Hodrick and Marshall (1997) note that the estimated VAR parameters are biased in small samples and, thus, do not provide a consistent unconstrained estimate of θ that may be used as the

starting point of the iterative GMM procedure described in Section 3.2, equations (10) and (11). To examine the properties of the small-sample distribution of the unconstrained VAR parameters, we assume that the stochastic process driving the term structure follows two DGPs similar to those used by Bekaert and Hodrick (2001). Let $q_t = \left(r_t, S_t^{1w}, S_t^{2w}, S_t^{3w}, S_t^{1m}, S_t^{2m}, S_t^{3m}\right)'$, where r_t is the overnight repo rate and S_t^{1w}, S_t^{2w}, S_t^{3w}, S_t^{1m}, S_t^{2m}, S_t^{3m} denote the spreads between the relevant repo rate (from one-week to three-month maturities) and the overnight repo rate. Then, assume that a p-th order VAR characterizes the DGP for q_t:

$$q_t = \Upsilon_o + \sum_{j=1}^{p} \Upsilon_j q_{t-1} + \varepsilon_t. \tag{14}$$

Instead of the second-order process used by Bekaert, Hodrick and Marshall (1997), Longstaff (2000) suggests a fifth-order VAR for daily repo rates to take into account the day-of-the-week regularities in the short-term repo rates, documented in Table 1.

Specifically, the bias correction proceeds as follows. We use the estimated unconstrained VAR parameters described by equation (14) to simulate $100,000$ artificial data using an i.i.d. bootstrap of the residuals ε_t. Then, we reestimate the VAR parameters for each replication and compute the bias as the difference between the known parameters of the DGP and the average parameters from the bootstrap distributions. In each replication we simulate $3,625$ observations plus $1,000$ discarding values to avoid any dependence on the starting values. We bias correct the estimates by adding the biases to the original unconstrained estimates and generate a bias-corrected vector of time series for q_t of $71,000$ observations, with the initial $1,000$ observations discarded. Finally, given the bias-corrected vector q_t, we can generate the vector of interest, $\left(r_t, r_t^{1w}, r_t^{2w}, r_t^{3w}, r_t^{1m}, r_t^{2m}, r_t^{3m}\right)'$, by simply adding the overnight rate, r_t to each spread series. Given these new series of the seven repo rates, for each possible pairwise comparison of short-term and long-term repo rates, we use a bivariate fifth-order VAR to estimate the unconstrained parameter $\widetilde{\theta}$; and the recursive GMM algorithm described in Section 3.2 to estimate the constrained parameter $\overline{\theta}$.

In order to accommodate heteroskedasticity and its potential effects on small-sample distributions, we also model the overnight repo rate and the term spreads with conditionally heteroskedastic innovations. In particular, we model the innovation vector in equation (14), ε_t, as a factor structure with the innovations of the overnight rate and the three-month term spread as the factors (Bekaert, Hodrick and Marshall, 1997; Longstaff, 2000; Bekaert and Hodrick, 2001):

$$\varepsilon_t = F v_t \tag{15}$$

with

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ f_{21} & 1 & 0 & 0 & 0 & 0 & f_{27} \\ f_{31} & 0 & 1 & 0 & 0 & 0 & f_{37} \\ f_{41} & 0 & 0 & 1 & 0 & 0 & f_{47} \\ f_{51} & 0 & 0 & 0 & 1 & 0 & f_{57} \\ f_{61} & 0 & 0 & 0 & 0 & 1 & f_{67} \\ f_{71} & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

where v_t is a vector of idiosyncratic innovations, and F is the factor loading matrix governing the factor structure. Hence, $E_{t-1}\left[v_t' v_t\right] = H_t$, where H_t is a diagonal matrix. The conditional covariance matrix of the innovations, ε_t, is denoted as $\Sigma_t = F H_t F'$. Each diagonal element in H_t is assumed to follow a GARCH(1,1) process, augmented with the square root of the overnight rate – as in Gray (1996), Bekaert and Hodrick (2000), Longstaff (2000), and Ang and Bekaert (2002) – in order to accommodate shifts in the short-rate volatility:

$$h_{it} = \gamma_i \sqrt{r_{t-1}} + \beta_i h_{it-1} + \alpha_i v_{it-1}^2, \qquad i = 1, 2, \ldots 7. \tag{17}$$

The subscript i enumerates the components of q_t. The model in equations (14)-(17) requires a three-step estimation procedure. First, we obtain estimates of the unconstrained VAR parameters described by equation (14). Second, we correct the VAR parameters for small-sample bias using the above bootstrap procedure, estimate the unconditional covariance matrix for the innovations based on the least square point estimates, and generate a path of $4,625$ realizations for q_t by bootstrapping the least square residuals ε_t, discarding the first $1,000$ observations. This procedure is repeated $100,000$ times. The difference between the ordinary least squares (OLS) estimates and the average of the OLS parameters from the bootstrap experiment are then added to the original OLS estimates to obtain the bias-adjusted values.

Third, using these bias-adjusted VAR parameters, we compute the residual vector ε_t and estimate the parameters defined by the factor GARCH process via quasi-maximum likelihood.[7] We use the bias-corrected VAR parameters and factor GARCH parameters to generate a series of $71,000$ observations, with the initial $1,000$ observations discarded, for $\left(r_t, r_t^{1w}, r_t^{2w}, r_t^{3w}, r_t^{1m}, r_t^{2m}, r_t^{3m}\right)'$ by using an i.i.d. bootstrap of the standardized residual v_t. Each pair of these series for which $k = n/m$ is an integer is then subjected to the iterative GMM procedure described in Section 3.2,

[7] Hence, we assume normal innovations to construct the likelihood function although the true distribution may not be normal. White (1982) and Bollerslev and Woolridge (1992) show that the resulting estimator is consistent and asymptotically normal.

where we use a bivariate fifth-order VAR to estimate the unconstrained parameter $\widetilde{\theta}$; and the recursive GMM algorithm described in Section 3.2 to estimate the constrained parameter $\overline{\theta}$.

4 Measuring the Economic Value of Deviations from the EH

Econometric tests of the EH largely dominate the relevant empirical literature. An alternative way of evaluating the validity of the EH involves measuring whether departures from the EH provide information that is economically valuable, regardless of whether or not they are statistically significant on the basis of econometric tests. This section discusses the framework we use in order to evaluate the impact of allowing for deviations from the EH on the performance of dynamic allocation strategies in the repo market. We employ mean-variance analysis as a standard measure of portfolio performance assuming quadratic utility. Ultimately, we aim at measuring how much an investor is willing to pay for switching from a strategy that assumes that the EH holds (restricted VAR) to a dynamic strategy which conditions on departures from the EH (unrestricted VAR). From this setting we can calculate directly a variety of common performance measures, in the form of performance fees (Φ), the Modigliani and Modigliani (1997) (MM) measure, the Sharpe ratio (SR), and the Sortino ratio (SO).

We realize that a portfolio consisting only of interest (repo) rates is unlikely to be a realistic portfolio managed by a US investor. The repurchase agreements involving US Treasury securities are mainly used by banks in order to manage the quantity of reserves on a short-term basis and, hence, play an important role in the Federal Reserve's implementation of monetary policy. Moreover, the repo market plays a fundamental role in dealers' hedging activities and repos are used by investment managers who sell short Treasury securities in order to hedge the interest rate risk in other securities. Our main objective is not to design a realistic (executable) asset allocation strategy, but rather to measure the economic significance of deviations from the EH. Our measures of economic value complement the LM tests for statistical significance of the EH by showing whether the constraints imposed on the VAR by the EH have economic value. On the one hand, departures from the EH may be statistically insignificant, and yet provide considerable value to an investor. On the other hand, the departures might be statistically significant, but be of almost no economic value to a repo market investor.[8] This economic evaluation is easier to carry out and assess by focusing exclusively on a VAR where the only assets being modelled are repo rates at various maturities, because the only source of risk in the resulting repo portfolio is interest rate risk.

[8] See Leitch and Tanner (1991) for an early treatment of the relationship between statistical significance and economic value.

4.1 The EH in a Dynamic Mean-Variance Framework

In mean-variance analysis, the maximum expected return strategy leads to a portfolio allocation on the efficient frontier. Specifically, consider the trading strategy of an investor who has a one-period horizon and constructs a dynamically rebalanced portfolio that maximizes the conditional expected return subject to achieving a target conditional volatility. Computing the time-varying weights of this portfolio requires one-step ahead predictions of the conditional mean and the conditional variance-covariance matrix.

Let $\mathbf{R}_{t+1}$ denote the $N \times 1$ vector of risky asset returns; $\boldsymbol{\mu}_{t+1|t} \equiv E_t\left[\mathbf{R}_{t+1}\right]$ is the conditional expectation of $\mathbf{R}_{t+1}$; and $\boldsymbol{\Sigma}_{t+1|t} = E_t\left[\left(\mathbf{R}_{t+1} - \boldsymbol{\mu}_{t+1|t}\right)\left(\mathbf{R}_{t+1} - \boldsymbol{\mu}_{t+1|t}\right)'\right]$ is the conditional variance-covariance matrix of $\mathbf{R}_{t+1}$. At each period t, the investor solves the following problem:

$$\begin{aligned} \max_{w_t} \ & \left\{\boldsymbol{\mu}_{p,t+1} = \mathbf{w}_t'\boldsymbol{\mu}_{t+1|t} + \left(1 - \mathbf{w}_t'\mathbf{1}\right) R_f\right\} \\ & \text{s.t.} \ \ \left(\sigma_p^*\right)^2 = \mathbf{w}_t'\boldsymbol{\Sigma}_{t+1|t}\mathbf{w}_t \end{aligned} \tag{18}$$

where $\mathbf{w}_t$ is the $N \times 1$ vector of portfolio weights on the risky assets, $\mu_{p,t+1}$ is the conditional expected return of the portfolio, σ_p^* is the target conditional volatility of the portfolio returns, and R_f is the return on the riskless asset. The solution to this optimization problem delivers the following risky asset weights:

$$\mathbf{w}_t = \frac{\sigma_p^*}{\sqrt{C_t}}\boldsymbol{\Sigma}_{t+1|t}^{-1}\left(\boldsymbol{\mu}_{t+1|t} - \mathbf{1}R_f\right) \tag{19}$$

where $C_t = \left(\boldsymbol{\mu}_{t+1|t} - \mathbf{1}R_f\right)'\boldsymbol{\Sigma}_{t+1|t}^{-1}\left(\boldsymbol{\mu}_{t+1|t} - \mathbf{1}R_f\right)$. The weight on the riskless asset is $1 - \mathbf{w}_t'1$.

By design, in this setting the optimal weights will vary across models only to the extent that predictions of the conditional moments will vary, which is precisely what the empirical models provide. In our setting, the restricted VAR represent the benchmark against which we compare the alternative model, that is the unrestricted VAR. Thus, we carry out the economic value analysis comparing the outcome from a dynamic trading strategy that is based on the unrestricted VAR – a strategy that exploits deviations from the EH (denoted as the DEH strategy) – to the benchmark strategy based on the restricted VAR obtaining under the EH constraints (denoted as the EH strategy). We carry out the calculations both for the case where the VARs assume a homoskedastic variance-covariance matrix and for the case of a GARCH covariance matrix. Given that we consider bivariate models of repo rates, where one of the rates is assumed to have a safe nominal return, $\boldsymbol{\Sigma}$ is a scalar and is simply a variance term.[9] In short, our objective is to determine whether there is economic value in

[9] The optimal weights of the dynamically rebalanced portfolio are computed using the unconditional variance, which

using the unrestricted VAR which relaxes the constraints imposed by the EH.

4.2 Quadratic Utility

We rank the performance of the competing repo rate models using the West, Edison, and Cho (1993) methodology, which is based on mean-variance analysis with quadratic utility. The investor's realized utility in period $t+1$ can be written as:

$$U\left(W_{t+1}\right) = W_{t+1} - \frac{\lambda}{2}W_{t+1}^2 = W_t\Pi_{p,t+1} - \frac{\lambda W_t^2}{2}\Pi_{p,t+1}^2 \tag{20}$$

where W_{t+1} is the investor's wealth at $t+1$, λ determines his risk preference, and

$$\Pi_{p,t+1} = 1 + R_{p,t+1} = 1 + \left(1 - \mathbf{w}_t'1\right)R_f + \mathbf{w}_t'R_{t+1} \tag{21}$$

is the period $t+1$ gross return on his portfolio.

We quantify the economic value of deviations from the EH (the unrestricted VAR) by setting the investor's degree of relative risk aversion $\delta_t = \lambda W_t/\left(1 - \lambda W_t\right)$ equal to a constant value δ. In this case, West, Edison, and Cho (1993) demonstrate that one can use the average realized utility, $\overline{U}\left(\cdot\right)$, to consistently estimate the expected utility generated by a given level of initial wealth. Specifically, the average utility for an investor who allocates \$1 in every time period is equal to:

$$\overline{U}\left(\cdot\right) = \sum_{t=0}^{T-1}\left\{\Pi_{p,t+1} - \frac{\delta}{2\left(1+\delta\right)}\Pi_{p,t+1}^2\right\}. \tag{22}$$

4.3 Performance Measures

At any point in time, one set of estimates of the conditional mean and variance is better than a second set if investment decisions based on the first set lead to higher average realized utility, $\overline{U}$. Alternatively, the optimal model requires less wealth to yield a given level of $\overline{U}$ than a suboptimal model. Following Fleming, Kirby, and Ostdiek (2001) we measure the economic value of the interest rate strategies by equating the average utilities for selected pairs of portfolios. Suppose, for example, that holding a portfolio constructed using the optimal weights based on the benchmark model (restricted VAR) yields the same average utility as holding the alternative portfolio implied by the

implies that the optimal weights will vary only to the extent that the forecasts of the conditional means will vary. In a different experiment, we use a conditional estimator based on the backward-looking estimator of Foster and Nelson (1996), but the economic value of EH remains largely unchanged when using this specification and, therefore, we do not report these results.

unrestricted VAR that exploits departures from the EH. The latter portfolio is subject to daily management expenses Φ, expressed as a fraction of wealth invested in the portfolio. Since the investor would be indifferent between these two strategies, we interpret Φ as the maximum performance fee the investor would be willing to pay to switch from the EH to the DEH strategy. In general, this utility-based criterion measures how much an investor with a mean-variance utility function is willing to pay for conditioning on the deviations from the EH, as modelled in the unrestricted VAR model. The performance fee depends on the investor's degree of risk aversion. To estimate the fee, we find the value of Φ that satisfies

$$\sum_{t=0}^{T-1}\left\{\left(\Pi_{p,t+1}^{a}-\Phi\right)-\frac{\delta}{2\left(1+\delta\right)}\left(\Pi_{p,t+1}^{a}-\Phi\right)^{2}\right\}=\sum_{t=0}^{T-1}\left\{\Pi_{p,t+1}^{b}-\frac{\delta}{2\left(1+\delta\right)}\left(\Pi_{p,t+1}^{b}\right)^{2}\right\} \tag{23}$$

where $\Pi_{p,t+1}^{a}$ is the gross portfolio return constructed using the predictions from the unrestricted VAR model, and $\Pi_{p,t+1}^{b}$ is the gross portfolio return implied by the benchmark, EH-consistent model. Under the EH, $\Phi=0$, while if the EH is violated $\Phi>0$.

We also calculate other performance measures, including the most commonly used measure in financial markets, namely the Sharpe ratio (SR). As it is well known, the *ex ante* SR for a strategy s is commonly defined as $SR_s=\left(E[R_s]-R_f\right)/\sigma_s$, where $E[R_s]$ is the expected return on the strategy, R_f is the risk-free interest rate, and σ_s is the standard deviation of the returns to the strategy. The Sharpe ratio is the expected excess return from speculation per unit of risk. Because we are interested in evaluating different strategies *ex post*, we compute the *ex post* (realized) Sharpe ratios using the sample means and standard deviations from the realized portfolio returns from the DEH and EH strategies, i.e. $SR_s=\left(R_s-R_f\right)/\sigma_s$.

A performance measure that is directly related to the SR is the MM measure developed by Modigliani and Modigliani (1997). The MM measure is defined as the abnormal return that the dynamic strategy would have earned if it had the same risk as the benchmark. It is defined as

$$MM=\frac{\sigma_b}{\sigma_a}(\mu_a-r_f)-(\mu_b-r_f) \tag{24}$$

where μ_b and σ_b are the average return and standard deviation, respectively, of the benchmark portfolio (EH); and μ_a and σ_a are the average return and standard deviation, respectively, of the alternative investment strategy (DEH). By using this performance measure, the dynamic DEH portfolio is leveraged downwards or upwards, so that it has the same volatility as the EH portfolio. MM may be seen as a risk-adjusted abnormal return and is also directly related to the SR as $MM=\sigma_b(SR_a-SR_b)$.

Finally, we calculate the Sortino ratio (SO). While the Sharpe ratio takes into account the volatility in the portfolio return, the SO differentiates volatility due to 'up' and 'down' movements. The up movements are considered desirable and not accounted in the volatility. The *ex ante* SO is calculated by dividing the expected excess return by the downside volatility or semi-standard deviations σ_d, i.e., standard deviations of the negative returns: $SO = \frac{E(r_p - r_f)}{\sigma_d}$. As for the SR, we calculate the *ex post* (realized) SO from an active trading strategy, $SO = \frac{(r_p - r_f)}{\sigma_d}$. A large Sortino ratio indicates a low risk of large losses occurring.

4.4 The Dynamic Strategies

In this mean-variance quadratic-utility framework, we design the following strategy. Consider a US investor with one period horizon who builds a dynamic portfolio by allocating his wealth between two interest rates, a long-term rate r_t^n and a short-term rate r_t^m. At time t the investor decides to allocate his wealth between a buy-and-hold k-period asset and a rolling 1-period asset.[10] However, while the return on the long-term asset is known at time t and, hence, delivers a risk-free return R_f, the return on the short-term asset turns out to be risky at time t since $k-1$ interest rates are not known at time t.[11] Hence, the conditional expected return for the investment period $(t, t+1)$ can be defined as

$$E_t[R_{t+1}] = k^{-1} \exp[\textstyle\sum_{i=0}^{k-1} \ln(1 + E_t r_{t+i}^m)] - 1. \tag{25}$$

In brief, at the beginning of each period the long-term rate r_t^n yields a riskless return while a sequence of the short-term rate r_t^m implies a risky investment decision. We consider two alternative trading strategies. The benchmark dynamic trading strategy assumes that EH holds exactly and, hence, the investor takes position using the forecasts based on the constrained VAR (the EH strategy). The alternative strategy, instead, assumes departure from the EH (the DEH strategy), and uses the forecasts based on the unconstrained VAR.

Specifically, each period the investor takes two steps. First, he uses the selected VAR model to generate the predictions of the one-day ahead conditional mean and volatility of the repo rate returns. Second, conditional on the predictions of the model, he dynamically rebalances his portfolio by computing the new optimal weights for the maximum return strategy. This setup is designed to inform us whether using one particular conditional mean (and volatility) specification affects

[10] The position on the short-term asset implies a rolling investment decision ($k-1$ times) over the period $(t, t+1)$, $(t+1, t+2)$, ..., $(t+k-1, t+k)$. For instance, if we invest 1 dollar at time t, after k periods, the final wealth will be $\prod_{i=0}^{k-1}(1 + r_{t+i}^m)$, where r_{t+i}^m is the interest rate for the period $(t+i, t+i+1)$ since it is known at the beginning of the holding period.

[11] For simplicity, we drop out the subscript t from the riskless return R_f.

the performance of a short-horizon allocation strategy in an economically meaningful way. The predictions are all in sample predictions, since our focus is not to provide forecasting models of the repo term structure but to evaluate the measured departures from the EH as determined by the unrestricted VAR model.[12] We compute the performance fee Φ, the Sharpe ratio SR, the MM measure, and the Sortino ratio SO for each scenario as annualized values in basis points.[13]

4.5 Transaction Costs

With daily rebalancing, transaction costs play an important role and should be considered in evaluating the relative performance of different strategies. We consider transactions costs as follows. At time t, the investor decides his allocation strategy for the period $(t, t+1)$ by setting the weight $w_{i,t}$ for the i-th asset.[14] At time $t+1$, before rebalancing the portfolio for the next period, the actual weight for the i-th asset has changed to $\widehat{w}_{i,t+1} = w_{i,t}\frac{1+R_{i,t+1}}{1+R_{p,t+1}}$. Then, conditioning on information at time $t+1$, the investor's optimization problem will define the new weight $w_{i,t+1}$ for the i-th asset. Hence, the difference between the end-of-period and the beginning-of-period portfolio allocation, given by $(w_{i,t+1} - \widehat{w}_{i,t+1})$, will determine the level of rebalancing at time $t+1$ in the i-th asset. Following Balduzzi and Lynch (1999), we assume transaction costs to comprise two components:

$$\tau_{t+1} = \tau \sum_{i=1}^{N+1} |w_{i,t+1} - \widehat{w}_{i,t+1}| + f \times \mathbf{I}\left(w_{i,t+1} \neq \widehat{w}_{i,t+1}\right) \tag{26}$$

where τ and f represent a proportional transaction cost to the value traded and a fixed cost of rebalancing the portfolio, respectively; $\mathbf{I}(\cdot)$ is the indicator function that equals unity if $w_{i,t+1} \neq \widehat{w}_{i,t+1}$ and zero otherwise. Accounting for the effect of transaction costs on the performance of our competitive dynamic strategies implies redefining the gross return in equation (21) as $\widetilde{\Pi}_{p,t+1} = 1 + (R_{p,t+1} - \tau_{t+1})$.

In our empirical settings, we use the average bid-ask spread across repo rates as the proportional transaction cost and assume that the roundtrip commissions and fees for large institutions are about 10 dollars per transaction. We end up with $\tau = 0.00065$ *bps*, and $f = 0.00001$ *bps* for an investor with wealth $W = \$1,000,000$. Whenever the portfolio is rebalanced daily, the investor must pay transaction costs to roll in the subsequent contract and liquidate the position at the end of the sample. The transaction costs are introduced by subtracting the appropriate level, τ_t, each day from the daily portfolio returns.

[12] Moreover, recent work by Inoue and Kilian (2004) suggests that in-sample predictability tests may be more robust than out-of-sample analysis.

[13] We annualize our economic measures by accounting for an average of 252 trading days.

[14] Recall that we consider N risky assets plus 1 riskless asset, so that the portfolio return is $R_{p,t+1} = \sum_{i=1}^{N+1} w_{i,t}R_{i,t+1}$.

5 Empirical Results I: The VAR Test of the EH

5.1 The VAR with Homoskedastic Innovations

We begin our statistical evaluation of the EH by performing estimation of the parameters of our 10 pairwise combinations of repo rates – these are all possible repo rates combinations of the integer $k = n/m$. Tables 2 and 3 report the bias corrected coefficients for the unconstrained VARs and the constrained VARs that satisfy the EH, respectively, when the DGP used to bias correct the parameters assumes homoskedastic innovations. Comparing the coefficients in Tables 2 and 3, we note that there are sharp differences in the constrained and unconstrained estimated dynamics. In particular, for each pairwise comparison, we find that the standard errors are quite large in the constrained VAR. Also, the absolute size of the constrained coefficients is much larger than the corresponding unconstrained ones, and, perhaps more importantly, the constrained coefficients measuring the response of the short-term rate to the long-term rate sometimes have a different *sign* from the corresponding unconstrained estimates. This is *prima facie* evidence that the EH restrictions may be generally inconsistent with the data, although this evidence does not constitute a formal statistical test.

5.2 The VAR with Time-Varying Volatility

Table 4 reports the estimation results for the VAR-GARCH model. Panel A reports the factor loadings, which are found to be statistically significant at standard significance levels. In Panel B, we also notice that the conditional variance turns out to be persistent for the overnight repo and moderately persistent for the spreads. Hence, departing from the assumption of homoskedasticity is likely to yield more accurate estimates of the VAR parameters and, consequently, more precise tests of the EH.

Tables 5 and 6 report the bias corrected coefficients for the unconstrained VARs and the constrained VARs that satisfy the EH, respectively, when the DGP used to bias correct the parameters assumes GARCH innovations. These results are quantitatively different from but qualitatively identical to the results for the VAR with homoskedastic innovations given in Tables 2-3. Specifically, the standard errors of parameters estimates in the constrained VAR are large, the absolute size of the constrained coefficients is larger than the corresponding unconstrained ones, and the constrained coefficients measuring the response of the short-term rate to the long-term rate have sometimes a different sign from the corresponding estimates in the unconstrained VAR.

5.3 LM and DM Tests of the EH

The LM and DM tests results are presented in Table 7, where we report the p-values for the null hypothesis that the EH holds for all possible repo rates combinations of the integer $k = n/m$. The figures in Table 7 suggest that the EH is rejected for each rate pair with p-values that are well below standard significance levels. Table 7 also reports the p-values from the J-test for the validity of the choice of the optimal weighting matrix in the GMM estimation. The p-values are comfortably larger than conventional significance levels, validating our GMM estimation and, hence, the LM and DM tests.

These results differ from Longstaff (2000), who does not reject the EH using conventional tests, because the VAR test is more powerful – and, hence, more likely to detect fine departures from the null hypothesis in finite sample – and because our sample is larger than Longstaff's (2000). However, given the very high power of the VAR testing procedure, a legitimate concern is whether the rejection of the EH may be due to small departures from the null hypothesis (due, for example, to tiny data imperfections or very small departures from the EH) which are not economically significant, but which appear as statistically significant given our very large sample size and powerful test statistics.[15] Moreover, the VAR tests are not designed to incorporate the fact that if one wanted to trade on departures from the EH – rather than assuming that the EH holds in a simple buy-and-hold allocation strategy – transactions costs are incurred that create a wedge between returns from an active strategy exploiting departures from the EH and a simple buy-and-hold strategy. In order to address these issues and to shed light on the economic significance of the statistical rejections of the EH recorded in this section, we proceed to an economic evaluation of the EH departures.

6 Empirical Results II: The Economic Value of EH Departures

Given the parameter estimates reported in Tables 2-6, we assume that a US investor dynamically updates his portfolio weights daily on the basis of the input from reestimating the VAR model period by period with the latest available data. The key question is whether the dynamic strategy that allows for departures from the EH generates economic gains relative to a benchmark dynamic strategy that assumes that the EH holds. We assess the economic value embedded in departures from the EH by analyzing the performance of the dynamically rebalanced portfolio constructed using

[15] Leamer (1978, Chapter 4) points out that classical hypothesis testing will lead to rejection of any null hypothesis with a sufficiently large sample: 'Classical hypothesis testing at a fixed level of significance increasingly distorts the interpretation of the data against a null hypothesis as the sample size grows. The significance level should consequently be a decreasing function of sample size' (p. 114).

pairwise combinations of repo rates.[16]

We compute the performance fee Φ, the MM measure, the Sharpe ratio SR, and the Sortino ratio SO for each scenario. We compare the performance fees for the combinations corresponding to the following cases: (i) two target annualized portfolio volatilities ($\sigma_p^* = \{0.1\%, 0.3\%\}$)[17]; (ii) two pairs of dynamic strategies, DEH strategy versus EH strategy; (iii) degrees of relative risk aversion $\delta = 1, 2, \ldots, 10$; (iv) for each pairs of repo maturities where the long maturity is an exact multiple of the short maturity; and (v) two different DGPs for the parameter estimates, with homoskedastic and heteroskedastic innovations (GARCH). We report the calculations of Φ, MM, SR and SO as annualized values in basis points.

6.1 Performance Fees and the MM Measure

Panel A of Table 8 displays the in-sample performance fees and the MM measure for the alternative strategy (DEH) against the benchmark strategy (EH) when the bootstrap experiment for bias correction assumes homoskedastic innovations and allowing for transaction costs as described in Section 4.5. We report results only for risk aversion $\delta = 5$ as no additional insight is obtained by changing the degree of risk aversion.

The results in Table 8 suggest that the performance fees for switching from a model that assumes the EH holds to a model that exploits departures from the EH is generally fairly modest and can be substantial only at the shorter end of the maturity spectrum with an aggressive strategy (target volatility $\sigma_p^* = 0.3\%$), but not at the longer end. For example, if we set the target volatility to be $\sigma_p^* = 0.1\%$, the annual management fee a risk-averse investor would be willing to pay to switch from the EH strategy to the alternative strategy conditioning on the EH departures is 41 *bps* when the overnight repo rate is the short-term rate and the 1-month repo rates is the long-term rate. Only if we calibrate the target volatility to be $\sigma_p^* = 0.3\%$, the corresponding annual management fee a risk-averse investor would be willing to pay reaches 122 *bps*. For longer maturities, however, the economic gains are modest, even miniscule. Interestingly, the risk-adjusted abnormal returns, as measured by MM, are of similar magnitude as the performance fees. Panel B of Table 8 reports the same performance criteria when the bootstrap experiment assumes GARCH innovations and allowing for transaction costs. The results are largely comparable to those in Panel A, but there are quantitative differences, confirming that modelling the time-varying covariance structure changes in conditional moments significantly.

[16] For weekends and holidays we consider the rate on the previous business day for which a rate was reported.

[17] Note that the annualized volatility of the overnight repo rate is about 0.1%. Hence a strategy targeting a volatility of 0.3% is aggressive in this context.

6.2 Sharpe Ratios and Sortino Ratios

The results from computing Sharpe ratios and Sortino ratios, reported in Table 9, are consistent with the results in the previous sub-section. Panel A of Table 9 presents the SR and SO for both the DEH strategy and the EH strategy when the bootstrap experiment assumes homoskedastic innovations and allowing for transaction costs. These results suggest that conditioning on departures from the EH offers, respectively, a return to risk ratio and a reward to downside volatility ratio higher than the corresponding ratios obtained assuming that the EH holds. For instance, when combining the overnight and 1-month repo rates and setting $\sigma_p^* = 0.1\%$, the annualized realized Sharpe ratio is positive and sizable (0.73) for the DEH strategy and negative for the EH strategy. As was the case with the performance fees and MM measures, the differences in Sharpe ratios tend to become modest at the longer end of the maturity spectrum. The results from calculating the Sortino Ratio confirm that departures from the EH can take into account downside risk more accurately than a dynamic portfolio based on the EH-consistent model. In particular, when selecting the combination of overnight vs. 1-month repo rates and setting $\sigma_p^* = 0.1\%$, the Sortino ratio for the DEH strategy is 1.56, and negative for the EH strategy.[18] These findings remain qualitatively unchanged when the bootstrap experiment assumes GARCH innovations, as reported in Panel B of the same table.

6.3 Summing Up

At first glance, the finding that the economic value of departures from the EH may be larger at shorter maturities seems paradoxical; however, it might be due to the fact that the larger the difference between the maturity of the short rate and the maturity of the long rate, the longer the sequence of forecasts of the short rate needed to cover the maturity of the long rate. If one assumes that the accuracy of the forecasts of the short-term rate declines with the horizon over which it is being forecast, then at some forecast horizon (some relative maturity of long- and short-term rates) the benefit of conditioning on deviations from the EH and, consequently, the economic value of conditioning on EH departures in the unrestricted VAR become small. Although our results do not suggest a monotonic decline in economic value across maturities, they do suggest that forecasting repo rates beyond one month is much harder – simply conditioning on the basis of departures from

[18] The Sharpe ratios and the Sortino ratios are virtually unchanged across different volatility targets, and we only report results for a volatility target of 0.1%. Furthermore, note that the annualized Sharpe ratios and Sortino ratios reported in Table 9 are adjusted for the serial correlation in the daily portfolio returns generated by the dynamic strategies. Specifically, following Lo (2002), we multiply the daily Sharpe ratios and the Sortino ratios by the adjustment factor $\frac{252}{\sqrt{252+2\sum_{k=1}^{251}(252-k)\rho_k}}$, where ρ_k is the autocorrelation coefficient of portfolio returns at lag k.

the EH – than forecasting repo rates for horizons shorter than one month.[19]

The results from economic value calculations are more favorable to the EH than the VAR tests reported in the previous section. This difference confirms that statistical rejections of a hypothesis cannot directly be taken as economic rejections (e.g. Leitch and Tanner, 1991). Although the VAR tests reject the EH throughout the term structure examined, the economic value of the EH violations recorded here is probably not appealing to investors and is confined to the shortest end of the term structure with maturities of one month or less. Activities in the repo market at maturities of days or weeks are largely driven by liquidity considerations and by the attempts of banks to manage the quantity of reserves and to hedge interest rate risk on a short-term basis, rather than to speculate in search of excess returns. Hence, it seems unlikely, also on the basis of our conversations with traders in Treasury services departments at Wall Street, that investors would be actively exploiting EH departures on a very short-term basis. Our interpretation of these results is that the EH violations, while statistically significant, are unlikely to be economically very important.

7 Conclusions

The EH plays important roles in economics and finance and, not surprisingly, has been widely tested using a variety of tests and data. Much empirical literature has struggled to find evidence supporting the validity of the EH across a variety of data sets and countries, and employing increasingly sophisticated testing procedures. This paper re-examines an important exception in this literature: the result that the EH appears to fit the behavior of US repo rates at the shortest end of the term structure, measured at daily frequency from overnight to the 3-month maturity (Longstaff, 2000). We innovate in this context on two grounds. First, we extend this research by testing the restrictions implied by the EH on a VAR of the long- and short-term repo rate using the test proposed by Bekaert and Hodrick (2001). These results are not encouraging for the EH, which is statistically rejected across the term structure considered.

Second, we move beyond merely statistical tests and provide complementary evidence on the validity of the EH using some economic value calculations. We assess the economic value of exploiting departures from the EH – i.e. using empirical models which condition on information contained in

[19] At first glance, this statement may seem in contrast with related research that finds forecasting power to be stronger for long-maturity rates than for short-maturity rates in time series models (e.g. Diebold and Li, 2006). However, the models considered in this paper are not designed to be forecasting models of the term structure of interest rates. Instead we are interested in evaluating empirically the EH, using a model that is EH-consistent and a model that conditions only on departures from the EH. Therefore, our results say nothing about the degree of forecastability in interest rates at different maturities with conditioning information. All our statements only relate to the ability of EH deviations to forecast repo rates up to the three-month maturity.

EH deviations – relative to the economic value of using a model that assumes the EH holds. This is also done in a framework that allows for time-varying volatility. The empirical results indicate that the economic value of departures from the EH can at times be sizable at the short-end of the repo yield curves, i.e., using overnight and weekly rates. For repo rates of one month and longer, however, differences in performance measures are very small.

Overall, the evidence in this paper confirms the power of the testing procedure proposed by Bekaert and Hodrick (2001) as well as the usefulness of the economic value framework designed by Fleming, Kirby, and Ostdiek (2001), which we amended to suit our context in this paper. Taking together both the results from statistical tests and economic value calculations, departures from the EH are statistically significant but we cannot find robust evidence that they are economically significant. Our conclusion is that, even though the EH is rejected statistically for repo rates of all maturities, it may still provide a reasonable approximation to the term structure of repo rates, especially at the longer end of the repo term structure.

Table 1
Descriptive Statistics for Daily Repo Rates

	Panel A: Percent Values							*Panel B: Percent Daily Changes*						
	Over-night	One-week	Two-week	Three-week	One-month	Two-month	Three-month	Over-night	One-week	Two-week	Three-week	One-month	Two-month	Three-month
$Mean$	3.9600	3.9492	3.9521	3.9544	3.9623	3.9752	3.9924	−0.0004	−0.0005	−0.0005	−0.0005	−0.0005	−0.0005	−0.0004
$Mean_{Mon}$	3.9718	3.9433	3.9420	3.9428	3.9483	3.9599	3.9764	−0.0360	−0.0106	−0.0057	−0.0051	−0.0016	−0.0004	0.0000
$Mean_{Tue}$	3.9728	3.9628	3.9657	3.9672	3.9757	3.9890	4.0051	−0.0040	−0.0064	−0.0037	−0.0029	−0.0036	−0.0035	−0.0030
$Mean_{Wed}$	3.9616	3.9496	3.9544	3.9571	3.9650	3.9784	3.9952	0.0036	−0.0022	−0.0049	−0.0039	−0.0037	−0.0022	0.0002
$Mean_{Thu}$	3.9683	3.9492	3.9526	3.9563	3.9642	3.9780	3.9969	−0.0330	−0.0028	−0.0006	−0.0010	0.0004	−0.0016	−0.0027
$Mean_{Fri}$	3.9260	3.9403	3.9445	3.9474	3.9565	3.9692	3.9866	0.0643	0.0191	0.0123	0.0103	0.0060	0.0055	0.0033
$Std\ Dev$	1.6998	1.6944	1.6973	1.6990	1.7003	1.7007	1.7013	0.1738	0.0648	0.0524	0.0517	0.0488	0.0507	0.0567
$Std\ Dev_{Mon}$	1.7039	1.7008	1.7017	1.7032	1.7023	1.7019	1.7010	0.1533	0.0672	0.0584	0.0621	0.0550	0.0517	0.0656
$Std\ Dev_{Tue}$	1.6951	1.6927	1.6959	1.6978	1.6999	1.6997	1.7009	0.1818	0.0640	0.0486	0.0498	0.0516	0.0540	0.0609
$Std\ Dev_{Wed}$	1.7115	1.7015	1.7054	1.7069	1.7072	1.7075	1.7074	0.2081	0.0621	0.0484	0.0448	0.0466	0.0484	0.0492
$Std\ Dev_{Thu}$	1.6975	1.6884	1.6927	1.6948	1.6978	1.6993	1.7009	0.1383	0.0447	0.0447	0.0463	0.0471	0.0518	0.0570
$Std\ Dev_{Fri}$	1.6953	1.6935	1.6956	1.6969	1.6991	1.6996	1.7006	0.1580	0.0779	0.0590	0.0535	0.0426	0.0470	0.0500
Min	0.8400	0.8900	0.8800	0.8700	0.8600	0.8300	0.8300	−1.5500	−0.8200	−0.8300	−0.8400	−0.8600	−0.8100	−0.8600
Max	6.7500	6.7000	6.5000	6.4900	6.4700	6.5000	6.5800	3.4000	1.1000	0.4100	0.6300	0.2900	0.3700	0.6200
ρ_1	0.9948	0.9993	0.9995	0.9995	0.9996	0.9996	0.9994	−0.3226	−0.0308	−0.1077	−0.1885	−0.1806	−0.2354	−0.2882
ρ_2	0.9929	0.9986	0.9991	0.9992	0.9993	0.9993	0.9992	−0.0921	−0.0150	0.0420	0.0399	0.0209	−0.0158	0.0467
ρ_3	0.9920	0.9979	0.9987	0.9989	0.9990	0.9991	0.9989	−0.0287	−0.0650	−0.0112	−0.0200	−0.0449	0.0123	−0.0345
ρ_4	0.9914	0.9973	0.9983	0.9986	0.9988	0.9989	0.9987	−0.0041	−0.1112	0.0101	0.0388	0.0491	0.0500	0.0494
ρ_5	0.9909	0.9969	0.9979	0.9983	0.9985	0.9986	0.9984	−0.0350	−0.0270	−0.0022	−0.0097	0.0276	−0.0225	0.0019

The table summarizes the descriptive statistics for repo rates and daily changes in repo rates. The data set consists of 3,625 daily observations of the indicated term government general collateral repo rates from May 21, 1991 to December 9, 2005. The period September 10, 2001 to September 30, 2001 is not included. The daily change in the repo rate for the indicated weekday is measured from the indicated day to the next business day. ρ_i denotes the i-th order serial correlation coefficient. All variables are measured in percentage points per annum.

Table 2

Unconstrained VAR Dynamics with Bias-Corrected Coefficients

Panel A: overnight (r_t^m) vs. one–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.2662	0.7015	−0.0455	0.0347	−0.0146	0.0423	−0.0093	−0.0219	−0.0087	0.0548
	(0.0040)	(0.0086)	(0.0041)	(0.0115)	(0.0041)	(0.0115)	(0.0041)	(0.0115)	(0.0040)	(0.0087)
r_t^n	0.0462	0.9267	−0.0238	0.0305	−0.0034	−0.0438	−0.0128	−0.0427	0.0008	0.1219
	(0.0018)	(0.0040)	(0.0019)	(0.0053)	(0.0019)	(0.0053)	(0.0019)	(0.0053)	(0.0018)	(0.0040)

Panel B: overnight (r_t^m) vs. two–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3258	0.4357	−0.0210	0.2842	−0.0094	−0.0262	−0.0048	−0.0774	−0.0223	0.1135
	(0.0039)	(0.0104)	(0.0041)	(0.0137)	(0.0041)	(0.0138)	(0.0041)	(0.0137)	(0.0039)	(0.0106)
r_t^n	0.0241	0.8714	−0.0194	0.1566	−0.0176	−0.0164	−0.0071	0.0026	0.0095	−0.0042
	(0.0014)	(0.0039)	(0.0015)	(0.0051)	(0.0015)	(0.0051)	(0.0015)	(0.0051)	(0.0014)	(0.0040)

Panel C: overnight (r_t^m) vs. three–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3544	0.3889	0.0036	0.2738	0.0122	−0.0763	0.0060	−0.0514	−0.0023	0.0884
	(0.0039)	(0.0108)	(0.0041)	(0.0137)	(0.0041)	(0.0139)	(0.0041)	(0.0138)	(0.0038)	(0.0111)
r_t^n	0.0126	0.7984	−0.0162	0.1985	−0.0190	0.0083	−0.0096	0.0466	−0.0024	−0.0177
	(0.0014)	(0.0039)	(0.0015)	(0.0049)	(0.0015)	(0.0050)	(0.0015)	(0.0049)	(0.0014)	(0.0040)

Panel D: overnight (r_t^m) vs. one-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4106	0.3061	0.0346	0.1944	0.0362	−0.1804	0.0221	0.1004	0.0123	0.0611
	(0.0038)	(0.0116)	(0.0041)	(0.0149)	(0.0041)	(0.0150)	(0.0041)	(0.0149)	(0.0038)	(0.0119)
r_t^n	0.0179	0.8146	−0.0164	0.1630	−0.0118	−0.0162	−0.0085	0.0848	−0.0038	−0.0240
	(0.0013)	(0.0038)	(0.0014)	(0.0049)	(0.0014)	(0.0049)	(0.0013)	(0.0049)	(0.0012)	(0.0039)

Panel E: overnight (r_t^m) vs. two-months (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4539	0.2259	0.0589	0.1228	0.0596	−0.1476	0.0455	0.1073	0.0451	0.0262
	(0.0038)	(0.0116)	(0.0042)	(0.0143)	(0.0042)	(0.0144)	(0.0042)	(0.0144)	(0.0038)	(0.0117)
r_t^n	0.0293	0.7349	−0.0302	0.1841	−0.0095	0.0742	−0.0095	0.0686	−0.0124	−0.0301
	(0.0012)	(0.0038)	(0.0014)	(0.0047)	(0.0014)	(0.0047)	(0.0014)	(0.0047)	(0.0012)	(0.0039)

(continued)

Table 2 *(continued)*

Panel F: overnight (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4780	0.2409	0.0786	0.0645	0.0743	−0.1137	0.0592	−0.0243	0.0706	0.0699
	(0.0038)	(0.0106)	(0.0042)	(0.0129)	(0.0042)	(0.0131)	(0.0042)	(0.0129)	(0.0037)	(0.0108)
r_t^n	0.0226	0.6935	−0.0184	0.2493	−0.0104	0.0231	−0.0068	0.0756	−0.0160	−0.0131
	(0.0014)	(0.0038)	(0.0015)	(0.0046)	(0.0015)	(0.0047)	(0.0015)	(0.0046)	(0.0013)	(0.0039)

Panel G: one-week (r_t^m) vs. two-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6103	0.3270	−0.0389	0.0793	−0.1263	0.1010	−0.1026	0.0647	0.0132	0.0706
	(0.0048)	(0.0055)	(0.0056)	(0.0067)	(0.0056)	(0.0067)	(0.0056)	(0.0067)	(0.0047)	(0.0059)
r_t^n	0.0377	0.8525	−0.0320	0.1683	−0.0916	0.0476	−0.0220	0.0249	0.0311	−0.0171
	(0.0043)	(0.0049)	(0.0050)	(0.0059)	(0.0050)	(0.0060)	(0.0050)	(0.0060)	(0.0042)	(0.0052)

Panel H: one-week (r_t^m) vs. three-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7264	0.1871	−0.0187	0.0822	−0.1119	0.1230	−0.0749	0.0284	0.0732	−0.0164
	(0.0046)	(0.0054)	(0.0056)	(0.0066)	(0.0056)	(0.0067)	(0.0056)	(0.0066)	(0.0045)	(0.0058)
r_t^n	0.0201	0.7837	−0.0437	0.2176	−0.0392	0.0331	−0.0629	0.0876	0.0167	−0.0138
	(0.0039)	(0.0046)	(0.0048)	(0.0056)	(0.0048)	(0.0057)	(0.0048)	(0.0056)	(0.0038)	(0.0049)

Panel I: one-month (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6411	0.1791	0.1533	−0.0186	−0.0528	0.0345	0.0745	−0.0047	−0.0159	0.0090
	(0.0054)	(0.0052)	(0.0061)	(0.0058)	(0.0062)	(0.0058)	(0.0061)	(0.0058)	(0.0053)	(0.0054)
r_t^n	0.1690	0.6119	0.0047	0.1651	−0.1572	0.1768	−0.0625	0.1200	−0.0500	0.0219
	(0.0057)	(0.0054)	(0.0064)	(0.0060)	(0.0064)	(0.0061)	(0.0064)	(0.0061)	(0.0055)	(0.0056)

Panel J: one-month (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6952	0.1253	0.1603	−0.0171	−0.0267	−0.0066	0.0688	0.0136	−0.0051	−0.0080
	(0.0047)	(0.0041)	(0.0055)	(0.0047)	(0.0056)	(0.0048)	(0.0055)	(0.0047)	(0.0045)	(0.0041)
r_t^n	0.1163	0.6336	0.0127	0.2296	−0.0956	0.0671	0.0210	0.0745	−0.0977	0.0382
	(0.0053)	(0.0047)	(0.0063)	(0.0053)	(0.0064)	(0.0054)	(0.0063)	(0.0053)	(0.0052)	(0.0048)

The table presents the unconstrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes homoskedastic innovations. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-tem repo rates such that $k = n/m$ is an integer. Standard errors are reported in paranthesis.

Table 3

Constrained VAR Dynamics with Bias-Corrected Coefficients

Panel A: overnight (r_t^m) vs. one–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.2286	0.8225	−0.5215	0.6936	−0.2054	−0.1770	−0.2616	−0.0859	0.1196	0.3863
	(0.1751)	(0.3164)	(0.1746)	(0.4348)	(0.1817)	(0.4436)	(0.1711)	(0.4316)	(0.1773)	(0.3267)
r_t^n	−0.1347	1.2090	−0.0385	0.0089	−0.0320	−0.0419	−0.0141	0.0079	0.0084	0.0272
	(0.0894)	(0.1305)	(0.0926)	(0.1884)	(0.0987)	(0.1886)	(0.0959)	(0.1871)	(0.0842)	(0.1456)

Panel B: overnight (r_t^m) vs. two–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7535	0.9498	−0.5332	2.1630	−0.0431	−2.7820	0.2823	−0.5580	0.3245	0.4431
	(0.2055)	(0.5970)	(0.2291)	(0.8370)	(0.2345)	(0.8645)	(0.2224)	(0.9049)	(0.2017)	(0.6167)
r_t^n	−0.0633	1.2590	−0.0745	0.0563	−0.0104	−0.2213	0.0280	−0.0241	0.0214	0.0293
	(0.0546)	(0.1538)	(0.0576)	(0.2237)	(0.0587)	(0.2347)	(0.0587)	(0.2277)	(0.0566)	(0.1723)

Panel C: overnight (r_t^m) vs. three–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.5064	2.1260	0.4321	−0.5844	0.1015	−0.7700	0.2523	−1.2270	0.1721	−0.0069
	(0.2210)	(0.6373)	(0.2362)	(0.9172)	(0.2336)	(0.8275)	(0.2351)	(0.8132)	(0.2133)	(0.6715)
r_t^n	−0.0898	0.9670	−0.0293	0.0775	−0.0160	0.0661	−0.0145	0.0446	−0.0062	0.0002
	(0.0543)	(0.1397)	(0.0566)	(0.1853)	(0.0592)	(0.1886)	(0.0570)	(0.1921)	(0.0497)	(0.1507)

Panel D: overnight (r_t^m) vs. one-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.9455	1.2760	−0.3140	1.1240	0.5022	−2.8020	0.0288	0.8825	0.1478	−0.7888
	(0.1866)	(0.5981)	(0.2179)	(0.7717)	(0.2246)	(0.8165)	(0.2080)	(0.8894)	(0.1735)	(0.6722)
r_t^n	−0.0707	0.9640	−0.0082	0.0393	−0.0211	0.0867	−0.0052	−0.0062	−0.0049	0.0260
	(0.0444)	(0.1205)	(0.0480)	(0.1619)	(0.0476)	(0.1657)	(0.0480)	(0.1691)	(0.0402)	(0.1396)

Panel E: overnight (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7475	0.9542	0.1531	0.3933	0.4843	−0.3941	−0.1387	−0.4929	−0.0651	−0.6399
	(0.1841)	(0.5065)	(0.2052)	(0.5847)	(0.1991)	(0.5977)	(0.2048)	(0.6083)	(0.1945)	(0.4921)
r_t^n	−0.0483	0.9502	−0.0123	0.0237	−0.0087	0.0387	0.0056	0.0305	0.0019	0.0183
	(0.0452)	(0.1608)	(0.0468)	(0.1838)	(0.0494)	(0.1776)	(0.0469)	(0.1737)	(0.0447)	(0.1550)

(continued)

Table 3 *(continued)*

Panel F: overnight (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7324	0.7945	0.1271	0.2549	0.4525	0.2989	−0.1255	−0.9856	−0.0764	−0.4717
	(0.1788)	(0.5197)	(0.1994)	(0.5724)	(0.1871)	(0.6151)	(0.2020)	(0.5764)	(0.1812)	(0.4872)
r_t^n	−0.0310	0.9698	−0.0070	0.0133	−0.0050	0.0193	0.0037	0.0265	0.0014	0.0088
	(0.0512)	(0.1415)	(0.0538)	(0.1703)	(0.0558)	(0.1651)	(0.0544)	(0.1673)	(0.0489)	(0.1429)

Panel G: one-week (r_t^m) vs. two-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6383	0.7791	−0.7206	0.8372	0.9606	−1.0300	−1.2080	0.4947	0.6610	−0.4139
	(0.2936)	(0.3515)	(0.3955)	(0.4998)	(0.3996)	(0.5078)	(0.3849)	(0.4794)	(0.2983)	(0.4074)
r_t^n	−0.2584	1.4060	0.0007	0.1942	0.1562	−0.3139	−0.4770	0.1818	0.2953	−0.1849
	(0.1946)	(0.2209)	(0.2292)	(0.2792)	(0.2279)	(0.2946)	(0.2244)	(0.2747)	(0.1950)	(0.2586)

Panel H: one-week (r_t^m) vs. three-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6635	0.3950	−0.2646	0.3161	0.3033	−0.2055	−0.6291	0.5706	0.1752	−0.3269
	(0.2598)	(0.2815)	(0.3411)	(0.3520)	(0.3318)	(0.3629)	(0.3085)	(0.3373)	(0.2474)	(0.3081)
r_t^n	−0.0064	1.0090	−0.1196	0.1536	0.0726	−0.0245	−0.2311	0.2025	0.0699	−0.1272
	(0.2071)	(0.2002)	(0.2615)	(0.2228)	(0.2481)	(0.2341)	(0.2513)	(0.2137)	(0.1665)	(0.1855)

Panel I: one-month (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7047	0.1758	0.1700	−0.0967	−0.5617	0.3737	0.4135	−0.1273	−0.2036	0.1514
	(0.2284)	(0.1910)	(0.2531)	(0.2134)	(0.2592)	(0.2085)	(0.2603)	(0.2124)	(0.2246)	(0.2008)
r_t^n	−0.1482	1.0880	0.0859	−0.0489	−0.2815	0.1872	0.2070	−0.0638	−0.1019	0.0758
	(0.2451)	(0.1911)	(0.2794)	(0.2258)	(0.2925)	(0.2083)	(0.2933)	(0.2144)	(0.2214)	(0.2041)

Panel J: one-month (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6712	0.2068	0.2236	−0.1122	−0.4692	0.2558	0.3072	−0.0090	−0.0269	−0.0472
	(0.1905)	(0.1495)	(0.2260)	(0.1677)	(0.2270)	(0.1647)	(0.2292)	(0.1733)	(0.1762)	(0.1594)
r_t^n	−0.1101	1.0690	0.0747	−0.0375	−0.1565	0.0853	0.1024	−0.0030	−0.0090	−0.0157
	(0.2277)	(0.1742)	(0.2659)	(0.1925)	(0.2753)	(0.1930)	(0.2715)	(0.2000)	(0.2165)	(0.1809)

The table presents the constrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes homoskedatsic innovations. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-tem repo rates such that $k = n/m$ is an integer. Standard errors are reported in paranthesis.

Table 4
Factor GARCH Model

Panel A: Factor Loadings

Maturity	f_{i1}	f_{i2}	f_{i3}	f_{i4}	f_{i5}	f_{i6}	f_{i7}
r_t	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_t^{1w}	−0.8436 (0.00693)	1.0000	0.0000	0.0000	0.0000	0.0000	0.2447 (0.01446)
S_t^{2w}	−0.8933 (0.00638)	0.0000	1.0000	0.0000	0.0000	0.0000	0.3058 (0.01298)
S_t^{3w}	−0.9116 (0.00616)	0.0000	0.0000	1.0000	0.0000	0.0000	0.3786 (0.01231)
S_t^{1m}	−0.9463 (0.00615)	0.0000	0.0000	0.0000	1.0000	0.0000	0.4794 (0.01172)
S_t^{2m}	−0.9559 (0.00635)	0.0000	0.0000	0.0000	0.0000	1.0000	0.6792 (0.00924)
S_t^{3m}	−0.9653 (0.00718)	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Panel B: GARCH (1,1)

Parameter	h_{1t}	h_{2t}	h_{3t}	h_{4t}	h_{5t}	h_{6t}	h_{7t}
γ_i	0.0001 (0.00001)	0.0006 (0.00004)	0.0003 (0.00003)	0.0004 (0.00004)	0.0005 (0.00012)	0.0001 (0.00003)	0.0008 (0.00006)
β_i	0.9014 (0.00888)	0.1995 (0.03783)	0.5305 (0.04272)	0.2212 (0.05774)	0.1121 (0.0614)	0.4940 (0.06973)	0.1427 (0.04294)
α_i	0.0886 (0.00888)	0.0795 (0.00773)	0.0240 (0.00340)	0.0300 (0.00411)	0.0144 (0.00298)	0.0091 (0.00159)	0.0650 (0.00833)

The table reports the volatility parameters estimated by quasi-maximum likelihood from the VAR-GARCH model. The time series modeled in the VAR are the overnight repo rate r_t, the spread S_t^{1w} between the one-week rate and the overnight rate, the spread S_t^{2w} between the two-week rate and the overnight rate, the spread S_t^{3w} between the three-week rate and the overnight rate, the spread S_t^{1m} between the one-mont rate and the overnight rate, the spread S_t^{2m} between the two-month rate and the overnight rate, and the spread S_t^{3m} between the three-month rate and the overnight rate. The residuals from the bias-corrected fifth-order VAR are assumed to follow a factor structure with the overnight repo rate and the spreads as factors, where F is the factor loadings matrix governing the factor structure given in equation (16). The idiosyncratic innovations are assumed to follow an Augmented GARCH (1,1) process $h_{it} = \gamma_i\sqrt{r_{t-1}} + \beta_i h_{it-1} + \alpha_i v_{it-1}^2$, where $i = 1, 2, 3, 4, 5, 6, 7$. Asymptotic standard errors are given below the parameter estimates.

Table 5

Unconstrained VAR Dynamics with Bias-Corrected Coefficients and GARCH Innovations

Panel A: overnight (r_t^m) vs. one–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.2754	0.6997	−0.0456	0.0491	−0.0179	0.0445	−0.0285	−0.0368	−0.0182	0.0763
	(0.0044)	(0.0128)	(0.0045)	(0.0168)	(0.0045)	(0.0168)	(0.0045)	(0.0168)	(0.0044)	(0.0119)
r_t^n	0.0301	0.9498	−0.0332	0.0416	−0.0110	−0.0308	−0.0206	−0.0348	−0.0108	0.1192
	(0.0015)	(0.0044)	(0.0015)	(0.0058)	(0.0016)	(0.0058)	(0.0016)	(0.0058)	(0.0015)	(0.0041)

Panel B: overnight (r_t^m) vs. two–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3455	0.4422	−0.0186	0.2476	−0.0036	−0.0154	−0.0182	−0.0881	−0.0252	0.1302
	(0.0041)	(0.0144)	(0.0043)	(0.0188)	(0.0043)	(0.0189)	(0.0043)	(0.0189)	(0.0041)	(0.0139)
r_t^n	0.0247	0.8858	−0.0197	0.1562	−0.0196	−0.0212	−0.0139	0.0081	0.0031	−0.0040
	(0.0012)	(0.0041)	(0.0012)	(0.0054)	(0.0012)	(0.0054)	(0.0012)	(0.0054)	(0.0012)	(0.0040)

Panel C: overnight (r_t^m) vs. three–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3719	0.3720	0.0038	0.2369	0.0160	−0.0575	−0.0092	−0.0512	−0.0119	0.1248
	(0.0040)	(0.0143)	(0.0042)	(0.0181)	(0.0042)	(0.0182)	(0.0042)	(0.0181)	(0.0040)	(0.0140)
r_t^n	0.0218	0.8077	−0.0140	0.1948	−0.0187	0.0039	−0.0143	0.0504	−0.0033	−0.0290
	(0.0011)	(0.0040)	(0.0012)	(0.0051)	(0.0012)	(0.0051)	(0.0012)	(0.0051)	(0.0011)	(0.0039)

Panel D: overnight (r_t^m) vs. one-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4114	0.2689	0.0329	0.1777	0.0383	−0.1292	0.0009	0.0825	0.0053	0.1068
	(0.0039)	(0.0142)	(0.0042)	(0.0182)	(0.0042)	(0.0184)	(0.0042)	(0.0183)	(0.0039)	(0.0142)
r_t^n	0.0240	0.8122	−0.0103	0.1622	−0.0118	−0.0133	−0.0083	0.0787	−0.0041	−0.0297
	(0.0010)	(0.0039)	(0.0011)	(0.0049)	(0.0011)	(0.0050)	(0.0011)	(0.0049)	(0.0011)	(0.0038)

Panel E: overnight (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4470	0.1867	0.0565	0.1016	0.0606	−0.1241	0.0228	0.0967	0.0344	0.1136
	(0.0038)	(0.0150)	(0.0042)	(0.0188)	(0.0042)	(0.0190)	(0.0042)	(0.0188)	(0.0039)	(0.0149)
r_t^n	0.0336	0.7628	−0.0259	0.1750	−0.0097	0.0664	−0.0074	0.0621	−0.0126	−0.0449
	(0.0010)	(0.0038)	(0.0011)	(0.0048)	(0.0011)	(0.0048)	(0.0011)	(0.0048)	(0.0010)	(0.0038)

(continued)

Table 5 *(continued)*

Panel F: overnight (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.4692	0.1991	0.0739	0.0364	0.0744	−0.0822	0.0385	−0.0270	0.0624	0.1512
	(0.0038)	(0.0136)	(0.0042)	(0.0167)	(0.0042)	(0.0170)	(0.0042)	(0.0168)	(0.0038)	(0.0137)
r_t^n	0.0268	0.7143	−0.0158	0.2416	−0.0133	0.0174	−0.0022	0.0712	−0.0184	−0.0224
	(0.0011)	(0.0038)	(0.0012)	(0.0047)	(0.0012)	(0.0047)	(0.0012)	(0.0047)	(0.0011)	(0.0038)

Panel G: one-week (r_t^m) vs. two-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6555	0.3337	−0.0672	0.0616	−0.1200	0.0931	−0.1038	0.0540	0.0168	0.0743
	(0.0054)	(0.0061)	(0.0064)	(0.0074)	(0.0064)	(0.0074)	(0.0063)	(0.0074)	(0.0053)	(0.0064)
r_t^n	0.0500	0.8743	−0.0608	0.1650	−0.1000	0.0472	−0.0226	0.0309	0.0272	−0.0118
	(0.0048)	(0.0054)	(0.0057)	(0.0066)	(0.0057)	(0.0066)	(0.0057)	(0.0066)	(0.0048)	(0.0057)

Panel H: one-week (r_t^m) vs. three-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7871	0.1764	−0.0575	0.0738	−0.1066	0.1144	−0.0790	0.0231	0.0840	−0.0175
	(0.0048)	(0.0055)	(0.0060)	(0.0067)	(0.0060)	(0.0068)	(0.0060)	(0.0067)	(0.0047)	(0.0058)
r_t^n	0.0672	0.7766	−0.0786	0.2205	−0.0517	0.0352	−0.0513	0.0830	0.0124	−0.0142
	(0.0042)	(0.0048)	(0.0052)	(0.0059)	(0.0052)	(0.0059)	(0.0052)	(0.0059)	(0.0041)	(0.0051)

Panel I: one-month (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6449	0.2023	0.1449	−0.0272	−0.0467	0.0257	0.0751	−0.0102	−0.0098	0.0004
	(0.0054)	(0.0055)	(0.0061)	(0.0062)	(0.0061)	(0.0062)	(0.0061)	(0.0062)	(0.0053)	(0.0056)
r_t^n	0.1784	0.6350	−0.0067	0.1523	−0.1542	0.1733	−0.0631	0.1177	−0.0470	0.0139
	(0.0053)	(0.0054)	(0.0059)	(0.0060)	(0.0059)	(0.0060)	(0.0059)	(0.0060)	(0.0052)	(0.0055)

Panel J: one-month (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7057	0.1360	0.1508	−0.0221	−0.0258	−0.0107	0.0687	0.0120	−0.0046	−0.0107
	(0.0046)	(0.0043)	(0.0055)	(0.0049)	(0.0055)	(0.0050)	(0.0055)	(0.0049)	(0.0045)	(0.0044)
r_t^n	0.1337	0.6479	−0.0023	0.2213	−0.1025	0.0650	0.0203	0.0745	−0.0923	0.0338
	(0.0049)	(0.0046)	(0.0059)	(0.0053)	(0.0059)	(0.0054)	(0.0059)	(0.0053)	(0.0048)	(0.0047)

The table presents the unconstrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes GARCH innovations. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-tem repo rates such that $k = n/m$ is an integer. Standard errors are reported in paranthesis.

Table 6

Constrained VAR Dynamics with Bias-Corrected Coefficients and GARCH Innovations

Panel A: overnight (r_t^m) vs. one–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3251	1.7610	−0.4489	1.2170	−0.6072	−0.4146	0.4183	−2.3480	0.2903	0.8060
	(0.2008)	(0.7283)	(0.2135)	(0.9923)	(0.2034)	(0.9415)	(0.1991)	(0.9386)	(0.1912)	(0.5594)
r_t^n	−0.0542	1.1950	−0.1091	0.2705	−0.0905	−0.1230	0.0480	−0.2598	0.0325	0.0903
	(0.0407)	(0.1287)	(0.0421)	(0.1788)	(0.0430)	(0.1801)	(0.0425)	(0.1865)	(0.0427)	(0.1129)

Panel B: overnight (r_t^m) vs. two–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.3725	4.2410	−0.5280	−0.1566	−0.1095	−3.3550	0.9648	−3.5840	0.2606	2.8940
	(0.2266)	(1.1830)	(0.2552)	(1.4590)	(0.2489)	(1.4720)	(0.2654)	(1.5570)	(0.2092)	(1.0480)
r_t^n	−0.0026	0.8769	−0.0323	0.3286	−0.0610	0.0505	−0.0046	−0.1853	0.0024	0.0273
	(0.0300)	(0.1282)	(0.0340)	(0.1724)	(0.0340)	(0.1791)	(0.0323)	(0.1742)	(0.0321)	(0.1151)

Panel C: overnight (r_t^m) vs. three–week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	1.0480	1.5620	−0.1600	1.5160	−0.7218	−1.3830	0.6094	−2.6810	0.5887	0.6252
	(0.2449)	(1.1850)	(0.2659)	(1.4880)	(0.2751)	(1.5010)	(0.2651)	(1.5000)	(0.2298)	(1.0660)
r_t^n	−0.0036	0.9236	−0.0326	0.1892	−0.0622	0.0377	−0.0116	−0.0574	0.0080	0.0085
	(0.0280)	(0.1101)	(0.0301)	(0.1367)	(0.0307)	(0.1476)	(0.0299)	(0.1424)	(0.0298)	(0.1042)

Panel D: overnight (r_t^m) vs. one-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.9087	2.6450	−0.1655	0.3231	−0.5085	−1.2960	0.5701	−2.7540	0.5065	0.7747
	(0.1952)	(1.1030)	(0.2189)	(1.3950)	(0.2154)	(1.4340)	(0.2073)	(1.5210)	(0.1961)	(1.0260)
r_t^n	−0.0137	0.8960	−0.0093	0.1264	−0.0336	0.0706	−0.0198	−0.0099	−0.0029	−0.0043
	(0.0281)	(0.1044)	(0.0329)	(0.1266)	(0.0317)	(0.1290)	(0.0325)	(0.1352)	(0.0300)	(0.1052)

Panel E: overnight (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.9254	1.1460	0.1275	0.2946	−0.4429	0.9320	0.3702	−2.5520	0.2059	−0.0044
	(0.1606)	(0.9667)	(0.1857)	(1.1410)	(0.1850)	(1.1700)	(0.1725)	(1.2540)	(0.1631)	(0.8935)
r_t^n	−0.0339	0.9697	−0.0032	0.0165	−0.0012	0.0229	−0.0090	0.0409	−0.0033	0.0001
	(0.0300)	(0.1347)	(0.0355)	(0.1697)	(0.0365)	(0.1665)	(0.0358)	(0.1675)	(0.0308)	(0.1296)

(continued)

Table 6 *(continued)*

Panel F: overnight (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.8504	1.3090	0.1023	−0.2667	−0.3512	1.1180	0.3024	−2.2630	0.1693	0.0312
	(0.1403)	(0.9041)	(0.1688)	(1.1020)	(0.1670)	(1.0980)	(0.1566)	(1.1480)	(0.1445)	(0.7819)
r_t^n	−0.0186	0.9923	−0.0014	0.0093	−0.0007	0.0076	−0.0034	0.0162	−0.0012	−0.0002
	(0.0307)	(0.1233)	(0.0378)	(0.1327)	(0.0371)	(0.1366)	(0.0369)	(0.1329)	(0.0319)	(0.1137)

Panel G: one-week (r_t^m) vs. two-week (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.9011	0.8882	−0.1610	−0.2609	−0.3622	0.4408	−0.0055	−0.4077	0.5327	−0.5652
	(0.2267)	(0.2760)	(0.2827)	(0.3567)	(0.2752)	(0.3508)	(0.2945)	(0.3640)	(0.2282)	(0.3030)
r_t^n	0.0226	1.2010	−0.1065	−0.0195	−0.2151	0.2979	−0.0620	−0.1045	0.2194	−0.2328
	(0.1540)	(0.1775)	(0.1839)	(0.2253)	(0.1815)	(0.2253)	(0.1785)	(0.2116)	(0.1470)	(0.1995)

Panel H: one-week (r_t^m) vs. three-week ($r_t^n n$)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	1.4440	−0.4138	−0.8129	0.7940	−0.1750	0.4348	−0.4230	0.0078	0.5080	−0.3659
	(0.2134)	(0.2671)	(0.3113)	(0.3196)	(0.3124)	(0.3207)	(0.3037)	(0.3227)	(0.2319)	(0.2912)
r_t^n	−0.2060	1.2570	−0.2410	0.1966	0.0251	0.0161	−0.0307	−0.0559	0.1335	−0.0961
	(0.1840)	(0.1944)	(0.2428)	(0.2277)	(0.2354)	(0.2305)	(0.2398)	(0.2304)	(0.1784)	(0.1962)

Panel I: one-month (r_t^m) vs. two-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.6018	0.2211	−0.0800	0.1783	−0.2035	0.0155	0.2014	0.1004	−0.0995	0.0638
	(0.1682)	(0.1672)	(0.1766)	(0.1894)	(0.1694)	(0.1823)	(0.1722)	(0.1891)	(0.1535)	(0.1722)
r_t^n	−0.1992	1.1110	−0.0400	0.0892	−0.1018	0.0077	0.1007	0.0502	−0.0498	0.0319
	(0.1764)	(0.1637)	(0.1897)	(0.1801)	(0.1851)	(0.1704)	(0.1915)	(0.1840)	(0.1558)	(0.1647)

Panel J: one-month (r_t^m) vs. three-month (r_t^n)

	r_{t-1}^m	r_{t-1}^n	r_{t-2}^m	r_{t-2}^n	r_{t-3}^m	r_{t-3}^n	r_{t-4}^m	r_{t-4}^n	r_{t-5}^m	r_{t-5}^n
r_t^m	0.7011	0.2237	0.0444	0.0875	−0.2645	0.0844	0.1491	−0.0607	−0.0083	0.0431
	(0.1649)	(0.1458)	(0.1933)	(0.1680)	(0.1896)	(0.1686)	(0.1885)	(0.1718)	(0.1522)	(0.1574)
r_t^n	−0.0997	1.0750	0.0148	0.0292	−0.0882	0.0281	0.0497	−0.0202	−0.0028	0.0144
	(0.1701)	(0.1377)	(0.1994)	(0.1690)	(0.2064)	(0.1697)	(0.2123)	(0.1659)	(0.1646)	(0.1461)

The table presents the constrained VAR parameter estimates adjusted for small-sample bias. The data generating process (DGP) used for the bias-correction assumes GARCH innovations. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each panel reports different combinations of short-term and long-tem repo rates such that $k = n/m$ is an integer. Standard errors are reported in paranthesis.

Table 7

Test Statistics

Panel A: Bivariate VAR										
n/m	r_t/r_t^{1w}	r_t/r_t^{2w}	r_t/r_t^{3w}	r_t/r_t^{1m}	r_t/r_t^{2m}	r_t/r_t^{3m}	r_t^{1w}/r_t^{2w}	r_t^{1w}/r_t^{3w}	r_t^{1m}/r_t^{2m}	r_t^{1m}/r_t^{3m}
LM	0.0001	0	0	0	0	0.0055	0	0.0001	0.0037	0.0023
DM	0	0	0	0	0	0	0	0	0	0
$J-Test$	0.34	0.40	0.18	0.18	0.59	0.81	0.16	0.16	0.48	0.78
Panel B: Bivariate VAR-GARCH										
n/m	r_t/r_t^{1w}	r_t/r_t^{2w}	r_t/r_t^{3w}	r_t/r_t^{1m}	r_t/r_t^{2m}	r_t/r_t^{3m}	r_t^{1w}/r_t^{2w}	r_t^{1w}/r_t^{3w}	r_t^{1m}/r_t^{2m}	r_t^{1m}/r_t^{3m}
LM	0	0	0	0	0	0	0	0.0001	0.0001	0.0005
DM	0	0	0	0	0	0	0	0	0	0
$J-Test$	0.63	0.37	0.38	0.30	0.91	0.71	0.16	0.58	0.43	0.96

The table reports the p-values for the Lagrange Multiplier (LM) and distance metric (DM) statistics under the null hypothesis that the EH is validated by the data for each possible combination of short-term and long-tem repo rates such that $k = n/m$ is an integer. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. The p-values are calculated by bootstrap as described in the text. Panel A reports the results when the data generating process (DGP) used for bias-correction assumes homoskedastic innovations. Panel B reports the results when the DGP used for bias-correction assumes GARCH innovations. 0 denotes p-values below 10^{-5}. The $J-Test$ is the test for the validity of the choice of the optimal weighting matrix in the GMM estimation, and figures reported are p-values.

Table 8
Economic Value Results: Performance Fees and MM Measures

	Panel A: VAR				*Panel B: VAR-GARCH*			
	Target σ_p^*				*Target* σ_p^*			
	0.1%		0.3%		0.1%		0.3%	
r_t^m *vs* r_t^n	Φ	MM	Φ	MM	Φ	MM	Φ	MM
overnight vs $1-$*week*	37	38	111	116	21	23	64	69
overnight vs $2-$*week*	38	36	114	120	23	20	69	64
overnight vs $3-$*week*	41	36	123	117	45	37	135	120
overnight vs $1-$*month*	41	39	122	119	55	53	165	162
overnight vs $2-$*month*	20	19	59	56	30	28	89	85
overnight vs $3-$*month*	8	8	25	25	14	13	42	42
$1-$*week vs* $2-$*week*	48	58	145	180	28	36	84	113
$1-$*week vs* $3-$*week*	45	51	135	157	21	30	64	95
$1-$*month vs* $2-$*month*	11	11	34	32	7	6	20	17
$1-$*month vs* $3-$*month*	2	1	5	4	2	2	6	6

The table reports the in-sample performance fees (Φ) and MM measure (MM) for the alternative strategy (DEH) against the benchmark strategy (EH) for two target portfolio volatilities (0.1%, 0.3%), and daily rebalancing. Panel A reports the performance measures when the data generating process (DGP) used for the bias-correction assumes homoskedatic innovations, while Panel B gives the results when the DGP used for the bias-correction assumes GARCH innovations. Each panel reports the results for all combinations of short-term and long-term repo rates such that $k = n/m$ is an integer. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each strategy is consistent with an optimizing investor allocates capital in two assets delivering repo rates: the long-term repo rate, known with certainty at the time of trading, and a risky return generated by rolling the short-term asset for k periods. The EH strategy assumes that the Expectation Hypothesis holds exactly and, hence, uses the conditional moments implied by either the constrained VAR (Panel A) or the constrained VAR-GARCH (Panel B). The DEH strategy conditions on the departures from the Expectation Hypothesis and, hence, uses the conditional moments implied by either the unconstrained VAR (Panel A) or the unconstrained VAR-GARCH (Panel B). The performance fees denote the amount an investor with quadratic utility and a degree of relative risk aversion equal to 5 is willing to pay for switching from the benchmark strategy (EH) to the alternative strategy (DEH). The MM measure is the abnormal retun that the alternative strategy would have earned if it had the same risk as the benchmark strategy. The performance fees and the MM measure are expressed in annual basis points.

Table 9
Economic Value Results: Sharpe Ratios and Sortino Ratios

	Panel A: VAR				*Panel B: VAR-GARCH*			
	DEH		*EH*		*DEH*		*EH*	
r_t^m *vs* r_t^n	*SR*	*SO*	*SR*	*SO*	*SR*	*SO*	*SR*	*SO*
overnight vs $1-week$	0.23	0.47	−0.05	−0.11	0.54	1.08	0.24	0.46
overnight vs $2-week$	0.76	1.54	−0.37	−0.85	0.91	1.81	0.15	0.31
overnight vs $3-week$	0.81	1.68	−0.14	−0.30	0.99	2.01	−0.16	−0.35
overnight vs $1-month$	0.73	1.56	−0.20	−0.52	0.93	1.92	−0.23	−0.60
overnight vs $2-month$	0.70	1.59	−0.14	−0.40	0.89	1.96	0.02	0.05
overnight vs $3-month$	0.51	1.25	−0.01	−0.02	0.85	2.05	0.04	0.14
$1-week$ *vs* $2-week$	0.33	0.55	−0.12	−0.19	0.31	0.47	−0.13	−0.21
$1-week$ *vs* $3-week$	0.22	0.37	−0.05	−0.08	0.18	0.30	−0.06	−0.09
$1-month$ *vs* $2-month$	0.08	0.14	0.05	0.09	0.08	0.14	0.05	0.08
$1-month$ *vs* $3-month$	0.12	0.28	0.09	0.21	0.12	0.29	0.07	0.17

The table reports the in-sample Sharpe ratio (SR) and Sortino ratio (SO) for the alternative strategy (DEH) and the benchmark strategy (EH) for one target portfolio volatility (0.1%), and daily rebalancing. Panel A reports the performance measures when the data generating process (DGP) used for the bias-correction assumes homoskedatic innovations, while Panel B gives the results when the DGP used for the bias-correction assumes GARCH innovations. Each panel reports the result for a different combination of short-term and long-term repo rates such that $k = n/m$ is an integer. r_t^n is the n-period (long-term) rate and r_t^m is the m-period (short-term) rate. Each strategy is consistent with an optimizing investor allocates capital in two assets delivering repo rates: the long-term repo rate, known with certainty at the time of trading, and a risky return generated by rolling the short-term asset for k periods. The EH strategy assumes that the Expectation Hypothesis holds exactly and, hence, uses the conditional moments implied by either the constrained VAR (Panel A) or the constrained VAR-GARCH (Panel B). The DEH strategy conditions on the departures from the Expectation Hypothesis and, hence, uses the conditional moments implied by either the unconstrained VAR (Panel A) or the unconstrained VAR-GARCH (Panel B). The Sharpe ratio defines return per unit of risk. The Sortino ratio measures the reward to negative volatility trade-off. The Sharpe ratio and Sortino ratio are expressed on annualized basis.

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