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Abstract

In a sticky-price model where firms finance their production inputs, there is both a lower and an upper bound on the central bank's inflation response necessary to rule out the possibility of self-fulfilling inflation expectations. This paper shows that real wage rigidities decrease this upper bound, but coefficients in the range of those on the Taylor rule place the economy well within the determinacy region. However, when there is time-variation in the share of firms who finance their inputs (i.e. Markov-Switching) then inflation targeting interest rate rules are often found to result in indeterminacy, even if the central bank also targets output. In this case, adding money growth as an intermediate target in the Taylor rule can alleviate this indeterminacy and anchor inflation expectations. Whether the money growth target should be a constant feature of the central bank's policy rule or Markov-Switch depends on the weight the central bank places on output stability relative to inflation stability and the size of money demand shocks.

Keywords: Cost Channel, Taylor Principle, Determinacy, Regime Switching, Money
JEL codes: E3, E4, E5, C62

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1 Introduction

The advice to central banks that a well designed interest rate reaction function mechanically adjusts the policy rate more than one for one to deviations of inflation from target (c.f. Taylor (1993)) is one of the most robust policy prescriptions in monetary theory. However, the Taylor principle as described above is not without its caveats. Importantly, Bruckner and Schabert (2003), Surico (2008) and Christiano, Trabandt, and Walentin (2010) show that an upper bound may need to be placed on the central bank's reaction to inflation in order to ensure expectations remain well anchored.[1] This constraint on the central bank arises when there is a timing mismatch between when the firm produces its product and when it gets paid for the product. Firms in this situation will typically have to finance inputs with short-term loans called working capital. This environment introduces a cost channel which works counter to the typical transmission mechanism of monetary policy in sticky-price models.

The typical transmission mechanism of monetary policy in New-Keynesian models suggests that central banks can eliminate self-fulfilling inflation episodes by increasing the nominal interest rate more than the increase in inflation. Such an aggressive interest rate hike shifts consumption to the future and therefore decreases current demand, marginal costs, and prices. Whenever the cost-channel is present, however, the increase in nominal interest rates may actually confirm the expected inflation. This could happen, for example, if the central bank tries to eliminate the inflation scare by aggressively raising nominal interest rates. The aggressive interest rate rise increases firms' financing costs and leads to higher marginal costs and, through the Phillips curve, higher inflation.

It is not clear in practice whether the cost-channel poses a significant hurdle to inflation targeting central banks. For example, targeting current inflation instead of expected future inflation significantly enlarges the determinacy regions of interest rate rules in the presence of the cost-channel (Bruckner and Schabert, 2003). In addition, Surico (2008) shows that a "flexible" inflation target (in the sense that stabilizing prices is not the only concern of the central bank) is less likely to induce self-fulfilling equilibria from an overly-hawkish central bank. This type of central bank reaction function is consistent with a Taylor rule which includes a reaction to output as well as inflation. Therefore, the presence of the cost channel may only be a theoretical curiosity and not a serious challenge to inflation targeting central banks.

The practical appeal of the cost channel is its ability to explain the empirical regularity that an exogenous monetary policy tightening results in a lower price level only several periods after the policy disturbance (See for example Barth and Ramey (2001)). VAR evidence suggests that prices may increase in the short-run in response to a monetary policy shock before falling. This "price-puzzle" was originally identified by Sims (1992) as a robust feature across several economies, including the U.S., Germany, and France. In an effort to reconcile the puzzling, or at least inertial, response of prices following a monetary policy shock in the data with sticky-price equilibrium models, Christiano, Eichenbaum, and Evans (2005) and Henzel, Hulsewig, Mayer, and Wollmershauser (2009) use a minimum distance estimator which reveals that, on average, there is a cost-channel present in the U.S. and the Euro area which helps to explain the gradual fall in prices following a monetary contraction.[2]

These findings are conditional on underlying real-rigidities in addition to the cost-channel and sticky prices. Intuitively, to generate the gradual response of prices in the data to a monetary policy shock, it is necessary to have other frictions, in addition to the cost-channel, which prevent the demand

[1] Other apparent failures of the Taylor principle involve the interaction of the inflation response coefficient and trend-inflation (Coibion and Gorodnichenko, 2011) and the timing of the stock of money which enters the utility function (Carlstrom and Fuerst, 2001).

[2] These findings corroborate the single-equation estimates of Phillips Curves (Ravenna and Walsh, 2006; Chowdhury, Hoffmann, and Schabert, 2006) and multi-equation decompositions of inflation (Tillmann, 2008) which find that short-term interest rate plays a significant role in shaping inflation in the U.S. and Euro area.

channel from affecting marginal cost in the short-run. Of these frictions, sluggish wage-adjustment is key since it prevents non-financial input cost from changing for firms who rely on labor for production. Therefore, the cost channel, when paired with wage rigidities, is able to generate empirically plausible responses to monetary policy shocks. However, the aforementioned studies of equilibrium determinacy in the presence of the cost channel have typically focused on the case of no wage frictions.

In this paper, I characterize the determinacy regions of interest rate rules when real wages only gradually adjust to the marginal rate of substitution between consumption and labor. I show that for interest rate rules which satisfy the Taylor principle, the determinacy regions shrink when the degree of wage rigidity increases. Although the resulting determinacy regions are smaller, they still imply that interest rate rules with coefficients in line with the Taylor rule place the economy well within the determinacy region.

In addition to relaxing the assumption that real wages are flexible, I also allow the share of firms that need to finance their wage bill to vary over time. After all, the cost-channel represents a financial imperfection which is likely to vary over time with financial market conditions. Whereas, Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010) assume a constant fraction of firms finance their inputs, I model the share of firms who are subject to the cost-channel as a 2-state Markov-Switching process. In this Markov-Switching DSGE (MS-DSGE) model, inflation targeting regimes are almost always indeterminate regardless of whether they include a reaction to output. Therefore, the Taylor rule is indeterminate for a wide range of parameters governing the Markov-Switching process.

Other nominal targets are not subject to the same pitfalls that plague inflation targeting rules. A small coefficient on money growth in the central bank's reaction function can eliminate the multiple equilibria that appear under inflation targeting in this MS-DSGE model. In other words, once the central bank establishes an intermediate target, aggressive inflation targeting can be implemented. In the baseline calibration of the model, raising nominal rates 10 basis points for every 100 basis point increase in nominal money growth is sufficient to restore determinacy of the standard Taylor rule.

This finding is perhaps not surprising considering the existing literature on the global equilibrium determinacy of Taylor rules. In particular, Christiano and Rostagno (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002) find that the global determinacy of the Taylor rule can be restored once the central bank commits to switching to a money-growth targeting regime. The results in this paper are similar in spirit. However, this is the first study which shows that a switch to a money growth target can restore local determinacy of an otherwise indeterminate Taylor rule.[3]

In light of these new results, I estimate the euro area reaction function and find a small, but statistically significant coefficient on M3 money growth. The estimated rule is evidence of the ECB's "Two-Pillar" approach to price stability which uses monetary analysis as a cross-check to achieve its inflation target. As for the U.S., I calibrate an interest rate rule which features an occasional switch to a money growth targeting regime, as identified by Sims and Zha (2006). The small coefficient on money-growth in the ECB's reaction function and the Federal Reserve's past money-growth targeting regime are sufficient to restore determinacy of otherwise indeterminate Taylor rules for most calibrations of the MS-DSGE model. Additionally, I demonstrate that a switching policy rule results in greater inflation volatility, but less output volatility. Therefore, the difference between the single-mandate given to the ECB and the dual mandate given to the Federal Reserve helps to explain why the ECB has been reluctant to deemphasize monetary aggregates. If the ECB were to switch to a U.S. style policy rule they could achieve less volatility in the output gap but at the cost of greater volatility in inflation.

[3]Another important distinction is that I (implicitly) assume throughout that fiscal policy is Ricardian. Or, in the sense of Leeper (1991), fiscal policy is passive.

2 The Model

This section describes a log-linearized sticky price model in which firms have to finance their inputs prior to production. In the appendix, the model is derived in detail, however, here I present the relavent model equations to answer the questions of interest in this paper regarding local equilibrium determinacy. The non-policy block of the model, in large part, follows from Ravenna and Walsh (2006) and Blanchard and Gali (2007).

$$x_t = \mathbb{E}_t x_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) + \varepsilon_t^x \tag{1}$$
$$w_t = \rho w_{t-1} + (1-\rho)(1+\varphi) x_t + \varepsilon_t^w \tag{2}$$
$$m_t = x_t - \eta r_t + \varepsilon_t^m \tag{3}$$
$$\pi_t = \kappa(w_t + \alpha(s_t) r_t) + \beta \mathbb{E}_t \pi_{t+1} + \varepsilon_t^\pi \tag{4}$$

In the above equations, x_t is the output gap, π_t denotes the quarterly inflation rate, w_t is the real wage rate, r_t is the nominal interest rate on one-period bonds, and m_t denotes real money balances. All variables are expressed as percent deviations from their steady state values. The exogenous processes ε_t^x, ε_t^w, ε_t^π and ε_t^m are linear combinations of structural preference, technology, and money-demand shocks. Since the focus of the paper is on local determinacy, the exact way the structural shocks enter these exogenous processes is left in the appendix.

Equation (1) is the hosuehold's Euler equation which relates the expected rate of output growth to the real return on a 1-period bond. Equation (2) shows the evolution of the real wage rate, where following Blanchard and Gali (2007) and Hall (2005), I assume the real wage only adjusts gradually to the marginal rate of substitution. The speed of adjustment is determined by the magnitude of $0 \leq \rho \leq 1$. When $\rho = 0$, the model collapses to the flexible wage model previously studied in the cost-channel determinacy literature. The parameter $\varphi > 0$ is the labor supply elasticity (or inverse Frisch elasticity).

Equation (3) is the household's money demand equation in which $\eta > 0$ is the interest semi-elasticity. Equation (4) summarizes the pricing decision that firms face. The parameter $0 < \beta < 1$ is the household's discount factor. The parameter $\kappa = (1-\omega)(1-\beta\omega)/\omega$ is the slope of the Phillips curve, in which $1/(1-\omega)$ is the average duration of prices. The strength of the (potentially time-varying) cost channel is governed by the term $0 \leq \alpha(s_t) \leq 1$. The micro-foundations for motivating the parameter α stem from two possible modeling strategies. Rabanal (2007) interprets α as the share of firms in the aggregate who must finance their wage bill, while Christiano et al. (2010) interpret α as the share of each firm's wage bill which is financed each period. In either interpretation, this parameter determines the supply-side effects of monetary policy.

One deviation from the standard models in Woodford (2003) and Gali (2008) is m_t is an aggregate of interest bearing and non-interest-bearing assets. The monetary aggregate is modeled following Belongia and Ireland (2014). This adjustment is made to allow for a more direct interpretation of the use of monetary aggregates by central banks. For example, the ECB monitors the growth rate of M3 (according to their press conference transcripts) while Sims and Zha (2006) present evidence that, in the U.S., the pre-Greenspan regimes focused on M2 growth. Although the micro-foundations of monetary aggregation are more clearly spelled out with this specification, after log-linearizing, the money demand equation is isomorphic to those found in Woodford (2003) and Gali (2008). The model is closed with a specification of monetary policy which I assume can be described by an interest rate feedback rule of the type specified by Taylor (1993); however, I also allow for a possible time-varying

reaction to the nominal growth rate of money:

$$r_t = \phi_\pi \pi_t + \phi_x x_t + \phi_\mu(s_t)\mu_t + \varepsilon_t^{mp}, \tag{5}$$

$$\mu_t = m_t - m_{t-1} + \pi_t, \tag{6}$$

where ε_t^{mp} is an i.i.d. monetary policy shock.

3 Baseline model

Consider first a model most similar to that analyzed by Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010) in which wages are flexible ($\rho = 0$), and a constant fraction of firms must finance their inputs prior to production $\alpha(s_t) = \alpha$. In this section, I show that if the central bank is too aggressive in adjusting its policy rate to movements in inflation, unwanted volatility may emerge due to a multiplicity of equilibria. This is the point raised by Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010). Therefore, the following proposition is not novel, but creates a baseline to compare the determinacy regions of inflation targeting rules with wage rigidities and a time-varying cost channel.

Proposition 1. *If the central bank follows the policy rule $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$ then there exists a unique REE if and only if*

$$1 < \phi_\pi < \begin{cases} \infty, & for\ \alpha \leq \frac{(1+\varphi)}{2} \\ \frac{2(1+\beta)+\kappa(1+\varphi)}{\kappa(2\alpha-(1+\varphi))}, & for\ \alpha > \frac{(1+\varphi)}{2}. \end{cases}$$

To get a better sense of how the cost channel alters the determinacy regions of inflation targeting rules, consider the supply and demand side effects of monetary policy separately. Higher values of φ imply larger demand side effects, where as higher values of α imply larger supply-side effects. The demand side effects work through the household's Euler equation and labor supply curve. On the other hand, the supply-side effects work directly through the Phillips curve.

The demand side effects of monetary policy work through the Euler equation and the labor supply curve. An increase in real interest rates causes consumption today to decrease and, therefore, hours worked to fall since agents work to consume today. The extent to which the decrease in labor hours supplied puts downward pressure on wages is calibrated by φ. When φ is large, a given change in labor supply results in a larger movement in the real wage. For example, setting $\varphi = 1$ implies there is no upper bound on the inflation response since $0 \leq \alpha \leq 1$. At the other extreme, if $\varphi = 0$, then labor supply has no effect on the real wage. This acts to dampen the demand-side effects of monetary policy and in turn limits the aggressiveness with which the central bank can target inflation.

The supply-side effects of monetary policy work directly through the Phillips curve via the cost-channel parameter α. When α is large, increases in the nominal interest rate put more upward pressure on inflation. This happens because when a greater number of firms must borrow funds to finance their inputs, the increase in the nominal rate implies higher borrowing costs which translate into more inflationary pressure. In this sense, raising interest rates when inflation is high is akin to "throwing gasoline on a fire" to quote U.S. Congressman Wright Patman's description of the cost channel.

Combining both demand and supply effects, Proposition 1 shows that inflation targeting rules are subject to an upper bound on the inflation coefficient. The likelihood that monetary policy enters the region of the parameter space in which the inflation coefficient is bounded above depends on the strength of these demand and supply-side effects. Once in this region of the parameter space, it is up to monetary policy to temper its aggressiveness.

I now show that inflation targeting rules are particularly susceptible to indeterminacy with the cost channel. Rules which target the growth rate of the monetary aggregate have stable determinacy regions independent of the number of firms who need to finance their wage bill. Although money growth oriented rules have fallen from favor amongst most central bankers, they have a rich history in monetary theory.[4] Arguably, all policy rules trace their roots to Friedman's constant money growth rule which called for the central bank to fix the growth rate of a broad monetary aggregate (such as M2) at k-percent.

Friedman argued that stabilizing the growth rate of money shielded the economy from potential mistakes by central bankers. One such mistake would be to passively allow the money supply to collapse, a chief explanation for the severity of the Great Depression according to Milton Friedman and Anna Schwartz (1971). In a similar spirit, when the cost channel is active, a money growth targeting rule prevents the central bank from becoming too aggressive. In particular, if inflation rises above target, a central bank which attempts to stabilize the growth rate of nominal money will react by increasing interest rates (since nominal money growth implicitly includes inflation). However, the increase in nominal rates decreases the demand for money. This liquidity effect tempers the central bank's contraction, preventing the possibility of sunspot equilibria emerging regardless of the strength of the cost channel.

Proposition 2. *If the central bank follows the policy rule* $r_t = \phi_\mu \mu_t + \varepsilon_t^{mp}$ *then there exists a unique REE if and only if* $\phi_\mu > 1$.

While policy rules like those specified in Proposition (2) are not typically used to describe central bank policy, some reaction to money, either in previous regimes as in the U.S. experience during the pre-Greenspan Fed (Sims and Zha, 2006) or as part of their inflation targeting strategy as in the case of the European Central Bank, is common across central banks. Therefore, the rest of this article will explore how money-growth targeting can be used to anchor inflation expectations in the presence of the cost channel when typical Taylor rules may be prone to producing multiple equilibria.

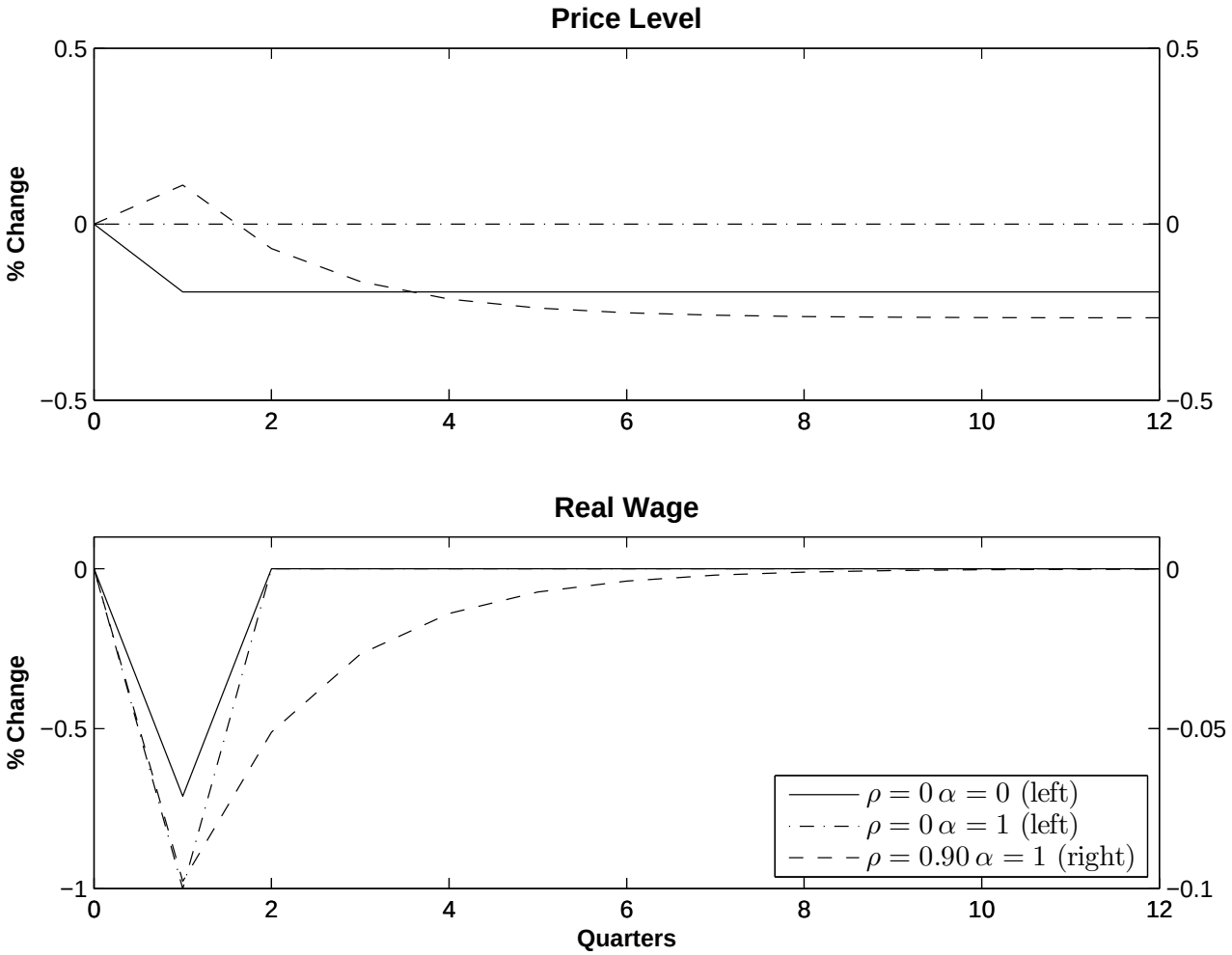
4 Adding Real Wage Rigidities

Wage rigidities have proven to be an important source of propagating business cycles and are considered a stock feature of DSGE models (Blanchard and Gali (2007); Smets and Wouters (2007); Christiano et al. (2005)). What is especially interesting about wage rigidities in the presence of a cost channel is the ability to generate the so called "price puzzle." The price puzzle is said to be present when a monetary contraction leads to an initial increase in the price level/inflation. Sims (1992) first noted the puzzle as a prevalent feature of monetary vector autoregressions across multiple countries. Christiano et al. (2005) go on to show that the slow response of prices to a monetary contraction can be captured by DSGE models with both a cost channel and wage rigidities, in a larger model with capital and other real frictions. Intuitively, from the Phillips Curve in equation (4), if wages adjust slowly to a monetary contraction then inflation will increase when interest rates rise. To better understand this interaction, it is useful to analyze the model's reaction to a monetary contraction under various assumption on the degree of wage flexibility and the cost channel.

The impulse response functions to a monetary contraction verify this small-scale model with the cost channel and real wage rigidities is capable of generating the price puzzle. Notice, the cost channel alone is not capable of generating an initial rise in prices for the baseline calibration. In fact, the response of prices to a monetary policy shock is nil with flexible wages as the demand-side effects of

[4]See for example Kahn and Benolkin (2007) and the quote within from Lawrence Meyer, former Federal Reserve Governor, "money plays no role in today's consensus macro model ... and virtually no role in the conduct of monetary policy, at least in the United States."

Figure 1: Impulse response functions to an exogenous monetary contraction for different degrees of wage rigidity and the cost channel.



Note: The responses above are graphed for parameters set to the baseline U.S. calibration described in Table (4).

higher interest rates, which pushes wages down, are canceled out by higher borrowing costs. With both partial adjustment of real wages and the cost channel prices initially raise before falling.[5] These impulse responses suggest that the cost channel, when coupled with wage rigidities, can produce impulse responses that are more similar to those in the data. The initial rise in prices and muted response of the real wage are all features of empirical impulse responses, as shown in Christiano et al. (2005).

Figure (1) provides evidence that the strength of the demand-side channel of monetary policy is weakened by the sluggish response of wages. This increased relative strength of the cost channel suggests the determinacy regions for interest rate rules will be further reduced by the addition of real wage rigidities. The following proposition more formally states this conclusion.

[5]Prices ultimately fall more in the wage rigidity/cost-channel model than in the model with flexible wages and the cost channel. This is because the endogenous portion of policy calls for keeping rates high after the policy shock due to the elevated inflation rate that initially follows the policy contraction in the wage-rigidity/cost-channel model.

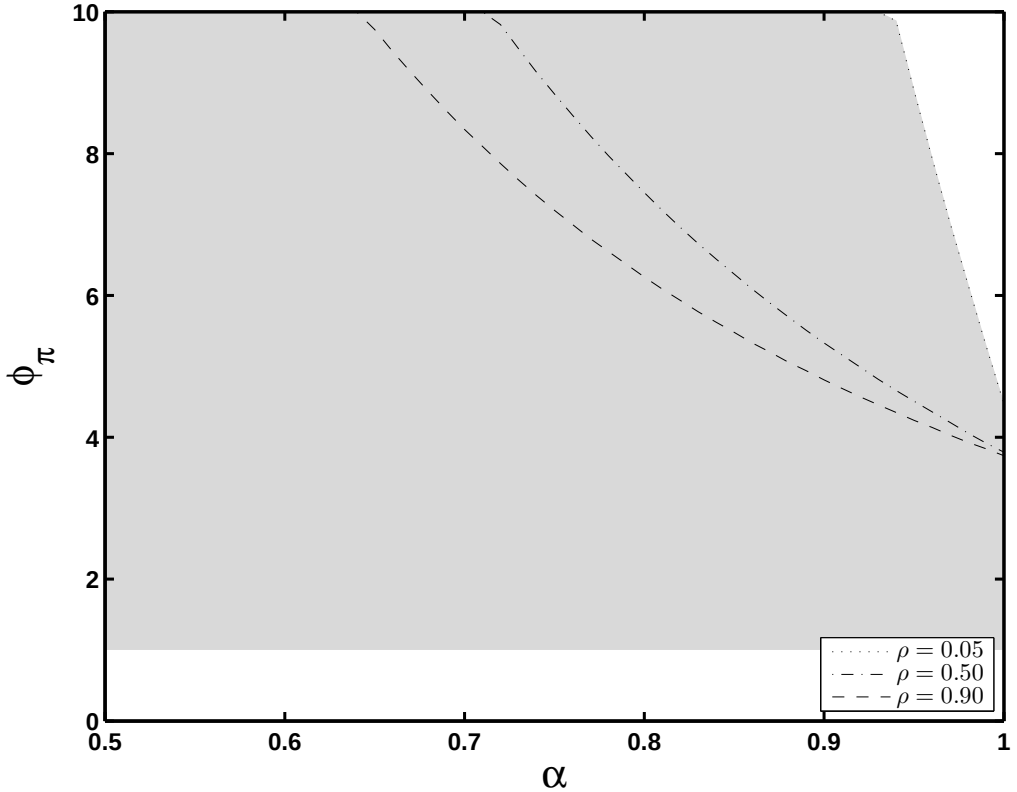
Proposition 3. *Suppose the central bank follows the policy rule* $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$ *with* $\phi_\pi > 1$. *Let* α_{min}^{ρ} *denote the lower bound on the strength of the cost channel necessary to induce the upper bound* $\phi_{\pi,max}^{\rho}$ *on the inflation response for a given value of* ρ. *Then, if* $0 < \rho < 1$*:*

i. $\alpha_{min}^{\rho} < \alpha_{min}^{0}$, *the minimum strength of the cost channel necessary to induce an upper bound on the inflation response is decreasing in* ρ.

ii. $\phi_{\pi,max}^{\rho} < \phi_{\pi,max}^{0}$, *the upper bound on the inflation response necessary for a unique REE to exist is decreasing in* ρ.

Real wage rigidities modify the determinacy conditions under the cost channel in two ways. First, it decreases the threshold strength of the cost channel needed to induce an upper bound on the policy response to inflation. Second, partial wage adjustment lowers this upper bound on the inflation coefficient once the cost channel is sufficiently strong. These two effects shrink the determinacy regions of inflation targeting rules that satisfy the Taylor principle relative to the case with flexible wages. Figure (2) quantifies this change in the determinacy regions. When wages are perfectly flexible the upper bound on the inflation coefficient is larger than 15 and therefore for values of $1 < \phi_\pi < 10$ the entire region is determinate. However, introducing even a small bit of inertia into real wages lowers the upper bound to less than 5. Further increasing the wage rigidity doesn't lower the upper bound for $\alpha = 1$ much, but it does reduce the upper bound for values of $\alpha < 1$.

Figure 2: Determinacy regions with a cost channel and various degrees of real wage rigidity when the central bank follows the inflation targeting rule: $r_t = \phi_\pi \pi_t$.



Note: The gray area to the southwest of the respective curves are determinate while the white areas are indeterminate. Parameters are set to the baseline U.S. calibration (see Table (4)).

Even in the presence of wage rigidities, inflation targeting rules still have reasonably large determinacy regions. For parameters typically used to calibrate central bank reaction functions such as the Taylor rule, the coefficient of $\phi_\pi = 1.5$ is well within the determinacy region. Adding an output gap reaction further increases the upper-bound on inflation targeting rules. This suggests the cost channel, even in the presence of real-wage rigidities, is unlikely to impose a binding constraint on the central bank's ability to achieve its inflation target via a Taylor rule. However, the next section shows that once a time-varying cost-channel is introduced, in addition to real-wage rigidities, Taylor rules are surprisingly susceptible to multiple equilibrium. In such an environment adding an intermediate money-growth target proves to be a useful tool to the central bank.[6]

5 A Time Varying Cost Channel

The cost-channel represents a financial imperfection which is likely to vary over time with financial market conditions. For example, Tillmann (2009) argues the cost-channel varies in the U.S. according to the cyclical nature of financial frictions. In a similar vein, Sim, Schoenle, Zakrajsek, and Gilchrist (2013) present evidence that firms who were liquidity constrained were much more likely to raise prices than firms with strong cash-flows during the 2008-2009 financial crisis. One way to capture this time-variation in firms pricing decisions is to allow the share of firms who are subject to the cost-channel to vary over time. In this section, I model the share of firms who are subject to the cost-channel as a 2-state Markov-Switching process. From the above discussion, a cost-channel regime has the interpretation of a period of financial distress in which some firms are liquidity constrained to the point they must borrow to pay their wage bill.

While there are positive implications of allowing the cost channel to vary over time, it is not clear how this will impact the determinacy regions of Taylor rules. In principle, introducing a time-varying cost channel could either enlarge or shrink the determinacy regions of Taylor rules in this sticky wage model. For example, in the seminal work of Davig and Leeper (2007), allowing the policy-rule coefficients to switch between active and passive can enlarge determinacy regions because the passive regime is known to be temporary and therefore inflation expectations remain anchored. Similarly here, if the cost channel is only temporarily binding, then even an aggressive inflation targeting central bank may be able to keep inflation expectations anchored during an inflation scare because firms expect, with a certain probability, that their future financing costs will decrease.

Of course, Davig and Leeper (2007) also show the cross-regime interactions can work in the other direction and shrink determinacy regions. If one regime is too passive, the passive regime can spillover and result in indeterminacy despite the presence of an active regime. In the work of Foerster (2013) for example, having both regimes active is neither necessary nor sufficient for equilibrium determinacy. In this cost-channel model the analogue could occur if policy is overly aggressive. If policy makers raise interest rates in response to an inflation scare in the no cost-channel regime, firms recognize that even though they have low financing costs today, there is a possibility that they will have to finance their inputs in the future. This expected future borrowing cost leads them to post higher prices now, making the potential inflation scare self-fulfilling.

In the numerical analysis that follows, I find this latter spillover to dominate the former. In fact, indeterminacy can arise in the MS-DSGE model for any interest rate response that satisfies the Taylor principle. Targeting output in addition to inflation does not alleviate the indeterminacy problem,

[6] Although there are no analytic results available, numerically I find there is no upper bound on the money growth response regardless of the degree of wage rigidity. The money demand equation provides some intuition. Since money demand depends on output, not wages, and the nominal interest rate, real wage rigidities don't alter the fact that a monetary contraction leads to lower real balances. This liquidity effect is driven by lower output and higher interest rates, two features which are not eliminated by the gradual wage adjustment process.

implying that Taylor rules are indeterminate for a wide range of parameter values. This indeterminacy can be alleviated by targeting nominal money growth in addition to inflation and output. While this money-growth augmented Taylor rule stabilizes inflation expectations, it also opens the door for money demand shocks to influence output and inflation. I explore whether an occasional switch to a money-growth target is preferred to a constant parameter policy rule. I find the answer generally depends on the central banks preferences for inflation stability relative to output stability. However, when money-demand shocks are very large a switching policy rule generates both lower inflation and output variances.

5.1 Solution Procedure, Equilibrium Refinement, and Calibration

I find solutions to the linearly approximated MS-DSGE model using the perturbation procedure developed by Foerster, Rubio-Ramirez, Waggoner, and Zha (2013). This model includes endogenous state variables, which precludes the use of Sims (2002) solution procedure, for example, to find the minimum state variable (MSV) solutions. Meanwhile, the solution method of Farmer, Waggoner, and Zha (2011) can analyze determinacy in such a regime-switching model in principle, but doesn't address how to perturb the non-linear model and relies on numerical analysis to find all possible equilibria. The method of Foerster et al. (2013) sidesteps both of these issues.

First, the Foerster et al. (2013) method develops the *Partition Principle* which partitions the parameters of the non-linear model between those that affect the steady state and those that don't. This is particularly useful because it allows for the analysis of local determinacy in the constant parameter model and the switching parameter model from the same non-linear DSGE model. Second, their method uses Groebner Bases to find all possible equilibria. By not relying on numerical methods to search for equilibria, the use of Groebner Bases provides a tractable solution procedure since it can find all equilibria. The primary drawback of the Foerster et al. (2013) procedure is the computational time. To analyze determinacy regions I perform a grid search over about 400 evenly spaced points. Even after parallelizing the computations in Mathematica, analyzing determinacy regions on a Beowulf Cluster often times takes up to 24 hours for one grid.

Once the set of all possible solutions to the Markov-Switching rational expectations problem are found, this large set of equilibria is further refined by selecting those which satisfy a stability concept. The possible solution concepts are mean-squared stability (MSS) which requires the model solution to admit finite first and second moments, but allows for potentially unbounded realized paths, and both regimes stable (BRS) which requires the model's values not to wonder off in a simulation if the model permanently stayed in any given regime. Although there is no consensus on the appropriate refinement strategy, the MSS solution refinement has been the most widely proposed concept (Foerster et al., 2013; Farmer, Waggoner, and Zha, 2009). Therefore, to align the results in this paper with those in the previous literature, I use the MSS concept here as well.

Introducing Markov Switching to the DSGE model implies the switching process needs to be calibrated as well. In particular, the parameter $\alpha(s_t)$ can take on two values: $\alpha(s_t = 1)$ and $\alpha(s_t = 2)$. These different values of α are determined by an exogenous 2-state Markov process with transition matrix,

$$\mathcal{P} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

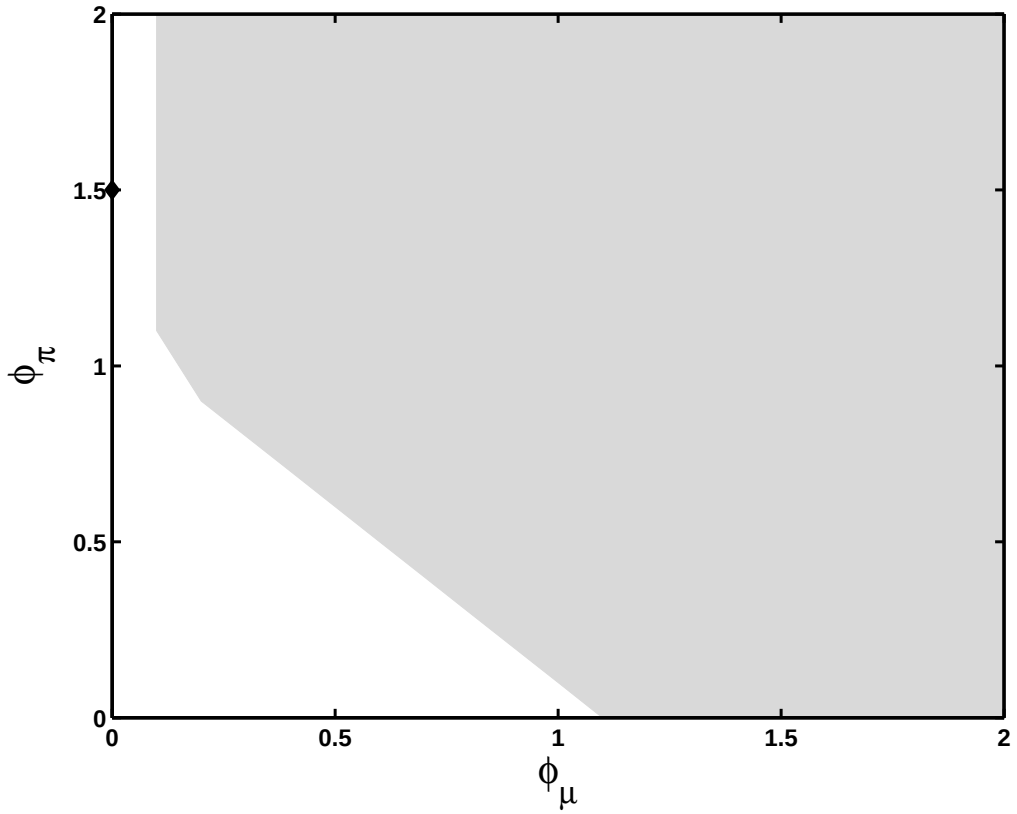
where $0 < p < 1$ and $0 < q < 1$. For the baseline calibration, I set $p = 0.9$ and $q = 0.05$ so that conditional on not being in the cost channel regime, there is only a 10% chance the cost channel becomes active. Similarly, once in the cost channel regime, with a 95% chance, on average, the cost channel will not be active in the following period. Therefore, the average duration of a liquidity constraint is a little more than 3 months. Once in the cost channel regime, all firms are subject to the

borrowing constraint so that $\alpha(s_t = 1) = 0$ and $\alpha(s_t = 2) = 1$. Since there is little empirical guidance on calibrating the Markov-Switching cost-channel model, in what follows I will vary these parameters to highlight how the results depend on p, q, and $\alpha(s_t = 2)$.

5.2 Taylor Rules and Indeterminacy with a MS Cost Channel

The primary finding when introducing an occasionally active (Markov-Switching) cost channel is that Taylor rules can induce indeterminacy under a wide range of parameter values. Shifting the baseline parameters in ways which increase the demand-side effects of monetary policy do not alleviate the indeterminacy of the Taylor rule. In particular, lowering the degree of wage rigidity, increasing the nominal rigidities (which flattens the Phillips curve), and increasing the value of the inverse Frisch elasticity of the labor supply still leads to determinacy regions in which the Taylor rule is indeterminate. Additionally, shifting the baseline parameters in ways which diminish the supply-side effects of monetary policy don't lead to determinate Taylor rules either. In particular, assuming there is incomplete pass-through from policy rates to borrowing costs as in Chowdhury et al. (2006), $0 < \alpha < 1$, then the Taylor rule is still indeterminate.

Figure 3: Determinacy regions with a Markov-Switching cost channel when the central bank follows the inflation targeting rule: $r_t = \phi_\pi \pi_t + 0.125 x_t + \phi_\mu \mu_t$



Note: The gray area is determinate while the white area is indeterminate. The black diamond denotes the baseline Taylor rule with $\phi_\pi = 1.5$ and $\phi_\mu = 0$. Other parameters are set to the baseline U.S. calibration described in Table (4). The same region emerges when $\rho = 0.5$, $\eta = 0.05$, $\omega = 0.9$, or $\varphi = 2.0$.

In addition to showing that Taylor rules are susceptible to indeterminacy when the cost channel varies over time, Figure (3) also highlights one way this indeterminacy can be resolved. Adding a small

reaction to nominal money growth restores determinacy of the Taylor rule. The intuition from the constant parameter model seems to carry over to the Markov Switching model. A money growth response inhibits the severity of the upper bound on the inflation response since money demand moves opposite of nominal rates. This liquidity effect doesn't depend on the cost channel and therefore variation in the cost channel doesn't have the extreme spillovers that make the Taylor rule indeterminate.

This raises the question as to why an output response doesn't alleviate the indeterminacy of the Taylor rule in the markov-switching model. The equilibrium effect of reacting to output in the presence of the cost channel is highlighted by Surico (2008). He shows that a reaction to output can cause the output gap to fall less than the rise in nominal rates in response to a monetary contraction in the presence of the cost channel. Larger reactions to output typically require larger reactions to inflation to counteract this effect and result in a unique rational expectations equilibrium. In the presence of real-wage rigidities, this effect is working to increase the lower-bound on the inflation response at the same time the wage-rigidity is working to decrease the upper-bound on the inflation response. In this MS-DSGE model, these competing channels and cross-regime spillovers effectively eliminate the determinacy regions of Taylor rules. Although the graph is not interesting, the Taylor rule in (ϕ_x, ϕ_π) space is always indeterminate under the baseline calibration.

Since nominal money demand is simply a function of the price level, nominal interest, output and exogenous money demand shocks, the determinacy regions of money growth rules should be recoverable via a nominal GDP growth targeting rule with interest rate smoothing. While it would be interesting to analyze the relative importance of nominal GDP growth targeting versus interest-rate smoothing, computational limitations in the solution procedure limit the size of model for which determinacy regions can be analyzed.[7] In particular, introducing lagged interest rates or output increases the number of state variables and makes determinacy analysis infeasible in the current framework.[8] Additionally, since central banks have not practiced nominal GDP growth targeting, considering policy rules which feature responses to inflation, output, and nominal money-growth seems like a reasonable starting point for understanding the implications of a time-varying cost channel.

6 Revisiting ECB and Federal Reserve Policy

Taylor rules have typically served as the linchpin linking policy in DSGE models to descriptions of actual central bank actions. In light of the results in the previous section, it is therefore worthwhile to evaluate how central banks can achieve determinacy by extending the typical Taylor (1993) rule to include a reaction to money growth. How central banks use monetary aggregates varies considerably. Some central banks, such as the Federal Reserve, no longer use monetary aggregates when setting interest rates but have in previous regimes. Meanwhile, the European Central Bank has institutionalized money as part of their strategy to achieve price stability.

In this section, I find that whether policy follows a Taylor rule which occasionally switches to include a money growth target or always targets money growth, the probability of determinacy is greatly increased relative to the baseline Taylor rule. Additionally, I demonstrate that a switching policy rule results in greater inflation volatility, but less output volatility. Therefore, the difference between the single-mandate given to the ECB and the dual mandate given to the Federal Reserve helps to explain why the ECB has been reluctant to deemphasize money.

[7] Although it is far from definitive, the results below vary η and find similar determinacy regions. Therefore, this would suggest it is the nominal GDP growth targeting that is driving the economy into the determinacy region more so than the interest rate smoothing induced by reacting to nominal money growth.

[8] The Groebner Bases procedure is can be sensitive to the model parameterization. Even for some parameterizations of the baseline model (for example setting $\omega < 0.6$), the Groebner-Bases procedure is unable to make progress in finding the number of solutions after more than 48 hours.

6.1 A Switching Money Growth Target and Determinacy

Although the role of money has been downgraded by the Federal Reserve since the Greenspan Fed, Sims and Zha (2006) provide evidence that money growth has served an important role in describing past Federal Reserve policies. In fact, when Sims and Zha (2006) allow for changes in the policy regimes the pre-Greenspan era emerges as a time when the Federal Reserve focused on money more so than interest rates. An appealing approach to understand how these past money targeting regimes might anchor expectations would be to allow for the cost channel and the policy regime to switch independently. Unfortunately, initial attempts in solving for the determinacy regions of a four-regime model proved to be too computationally burdensome. Therefore, I instead assume the money-growth response regime switches precisely with the cost channel.

Table 1: Determinacy of the Money Growth Switching Taylor Rule

Calibration		Minimum $\phi_\mu(s_t = 2)$
Baseline U.S.		0.1
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 2$	0.1
Small Interest Semi-Elasticity of Money Demand	$\eta = 0.05$	0.1
Flat Phillips Curve	$\omega = 0.9$	0.1
Fewer Cost Channel Firms	$\alpha = 0.5$	0.1
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.1
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	0.1
More Frequent Cost Channel Regime	$p = 0.5$	0.4
More Frequent Cost Channel Regime	$p = 0.25$	0.8
More Persistent Cost Channel Regime	$q = 0.25$	0.1
More Persistent Cost Channel Regime	$q = 0.5$	0

Notes: This table is constructed by fixing $\phi_\mu = 1.5$, $\phi_x = 0.125$, and searching a grid of values of $\phi_\mu(s_t = 2)$ starting from from 0 and increasing by 0.1 steps. The first value of $\phi_\mu(s_{t=2})$ which yields a unique MSS equilibrium is reported in the third column. All other parameters are fixed at the baseline U.S. calibration from Table (4).

There is some empirical evidence to appeal to for this specification. First, rolling-window estimates of the cost channel in the United States by Tillmann (2009) provide some evidence that the cost channel was a significant factor in shaping inflation in the pre-Greenspan era, and has been less of a factor since. Moreover, Barth and Ramey (2001) report evidence that the price puzzle emerges consistently in pre-1980 U.S. data and weakens thereafter. All of this, when combined with the evidence in Sims and Zha (2006), suggests the period in the U.S. when the Federal Reserve used monetary aggregates when setting monetary policy aligns with a time when the cost channel was a feature shaping inflation dynamics.

From a normative perspective, the simultaneous occurrence of the cost channel and the money-growth targeting policy regime is also appealing. It offers one rationalization why the Federal Reserve focused on money growth prior to Allen Greenspan's tenure as Chair of the FOMC despite the volatility associated with using money as a policy target due to money-demand shocks. To further analyze the implications of a Taylor rule with an occasional money-growth target, I assume the central bank follows the Markov-Switching policy rule in equation (5) where $\phi_\mu(s_t = 1) = 0$ in the no cost-channel regime.

Table (1) shows the minimum money-growth coefficient in the cost-channel regime necessary to make the Taylor rule determinate under various calibrations.

For all variations of the preference and production parameters the minimum response in the money growth switching Taylor rule necessary to ensure determinacy is 0.1, the first positive value of $\phi_\mu(s_t = 2)$ in the grid search. Changing the Markov-Switching parameters on the other hand changes the determinacy regions in important ways. If the cost channel regime occurs more frequently, then a larger response to money growth is required to make the Taylor rule determinate. However, if the cost channel regime becomes more persistent then no response to money growth is needed to make the Taylor rule determinate.

6.2 A Constant Money Growth Target and Determinacy

Unlike the Federal Reserve, the ECB continues to use information from monetary aggregates when setting monetary policy. In particular, the governing council uses a two-pillar approach to stabilizing prices. The first pillar rests on economic analysis of dynamics and shocks which may affect the short and medium-term developments in prices. The second pillar rests on monetary analysis with a focus on the medium to long-run prospects for price stability implied by the growth rate of a broad monetary aggregate (M3). The governing council takes into consideration information from both pillars, using one another to "cross-check" the risks to the ECB's objective of price-stability over the medium term.

Table 2: Econometric Estimation of the ECB's Reaction Function

$r_t = \phi_0 + \phi_\pi \pi_t + \phi_\mu \mu_t + \phi_y \Delta y_t + e_t$			
	$\hat{\phi}_\pi$	$\hat{\phi}_\mu$	$\hat{\phi}_y$
	Baseline		
GMM Estimates	1.547^{***}	0.134^{***}	–
Standard-Error	0.082	0.038	–
	With Output Growth		
GMM Estimates	1.529^{***}	0.140^{***}	0.050
Standard-Error	0.087	0.050	0.065
	With Alternative Instrument Set		
GMM Estimates	1.553^{***}	0.101^{*}	–
Standard-Error	0.073	0.052	–

[a] The instrument set includes a constant and 1 to 4 lags of: real unit labor cost, HP-filtered output gap, GDP deflator inflation, commodity price inflation, the term-spread. The instrument set follows closely from Ravenna and Walsh (2006).

[b] The alternative instrument set includes a constant and 1 to 4 lags of detrended output, GDP deflator inflation, the short-term interest rate, M3 growth, commodity price inflation, and the term-spread. The instrument set follows closely from Clarida, Gali, and Gertler (2000).

Although ECB press conferences always feature an overview of both the economic and monetary analysis (including M3 money growth), it is still not clear how much weight the ECB places on money

growth. Therefore, for the purposes of calibrating their policy rule, I estimate a simple reaction function for the ECB using the Area-Wide Model (AWM) dataset constructed by Fagan, Henry, and Mestre (2001). The estimation strategy uses two-step GMM to efficiently deal with the endogeneity inherent in central bank reaction functions. The data used covers the sample from 1981-Q1 to 2012-Q4. The short-term interest rate is used to measure r_t, the GDP deflater is used to measure π_t and the growth rate of M3 is used to measure μ_t. All standard errors are Newey-West HAC to correct for any residual auto-correlation in the error terms.[9]

The estimated rule is found to have a similar coefficient on inflation as the Taylor rule (Taylor, 1993), and importantly, a statistically significant reaction to M3 money growth. The coefficient estimates are rather robust to varying the instrument sets. One concern is that by specifying the rule without any inclusion of real economic activity, the coefficient estimate on M3 growth is biased as a result of an omitted variable. However, including real GDP growth in the reaction function yields an insignificant coefficient on real activity and has no significant effect on the other estimated parameters.[10]

Table 3: Determinacy of the Estimated Euro Area Policy Rule

Calibration		Minimum ϕ_μ
Baseline Eruo Area		0.1
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 2$	0.1
Small Interest Semi-Elasticity of Money Demand	$\eta = 0.05$	0.1
Flat Phillips Curve	$\omega = 0.9$	0.1
Fewer Cost Channel Firms	$\alpha = 0.5$	0.1
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.1
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	0.1
More Frequent Cost Channel Regime	$p = 0.5$	0.1
More Frequent Cost Channel Regime	$p = 0.25$	0.2^*
More Persistent Cost Channel Regime	$q = 0.25$	0.1
More Persistent Cost Channel Regime	$q = 0.5$	1.0^*

Notes: This table is constructed by fixing $\phi_\mu = 1.5$, $\phi_x = 0$, and searching a grid of values of ϕ_μ starting from from 0 and increasing by 0.1 steps. The first value of ϕ_μ which yields a unique MSS equilibrium is reported in the third column. All other parameters are fixed at the baseline euro area calibration from Table (4).
* Setting $\phi_\mu = 0.134$, the point estimate from Table (2), also yields indeterminacy under this calibration.

The estimated reaction function of the ECB results in a unique MSS equilibrium for the majority of parameter values considered in Table (3). In two instances, the ECB's current policy rule fails to result in determinacy. If the cost channel becomes much more frequent or much more persistent, the ECB would need to increase the aggressiveness with which it targets nominal money growth.[11] Interestingly,

[9] All of the GMM estimates fail to reject the null that the over-identifying restrictions are satisfied.

[10] Including an interest rate smoothing-term in the policy rule yielded discouraging results for the inflation reaction parameter. In particular, the estimates of ϕ_π were small and insignificant when the policy rule was specified as $r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_\mu \mu_t + \phi_y \Delta y_t)$.

[11] In these cases, the ECB could also add a reaction to the output gap to eliminate the indeterminacy as suggested by Table (1).

if the ECB were to drop the second pillar of its price stability framework, the economy would be pushed into the indeterminate region of the parameter space for all of the parameter variations considered.

6.3 The Trade-Off between Output and Inflation

The ECB's continued use of monetary aggregates in its two-pillar approach to price stability has drawn considerable criticism on the grounds that there is very little theoretical support in sticky-price models for using money as an intermediate target to stabilize inflation. Rudebusch and Svensson (2002) and Woodford (2008a,b) among others argue the ECB should drop the second pillar of monetary analysis. A central bank stabilizing money growth will not accommodate an exogenous money-demand increase, causing nominal rates to rise and inflation and output to fall. Therefore, the "Achilles heel" of monetarism as Poole (1970), Bernanke and Blinder (1988) and Ireland (2000) highlight, is that targeting money exposes the economy to money-demand shocks. Along these lines, there is good reason to think that deemphasizing the role of monetary aggregates by only occasionally using money growth as an intermediate target would result in better macroeconomic outcomes. The column on the left in Figure (4) plots the inflation and output gap volatilities under both a constant parameter and a switching parameter money growth response in the Taylor rule.

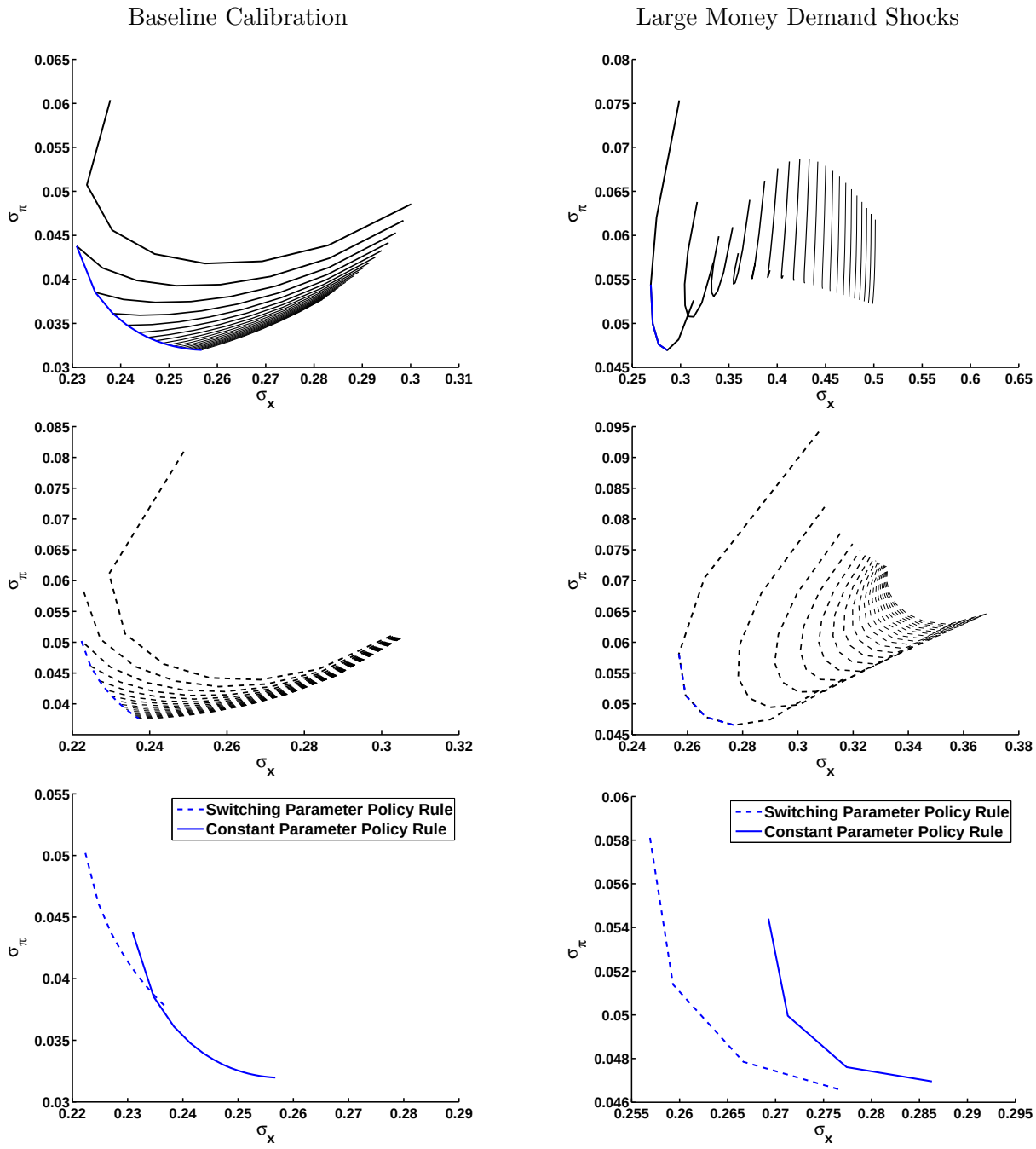
The switching rule does not lead to generally lower inflation and output volatility compared to a constant parameter policy rule. Instead, it generates higher inflation volatility. This seems counter-intuitive since the switching rule should partially shield the economy from money-demand shocks. However, inflation depends directly on the nominal interest in the cost-channel regime. The switching rule introduces volatility into the nominal rate which, by the cost-channel Phillips Curve, introduces volatility into marginal cost and inflation. Therefore, switching in the policy rule has the tendency to increase inflation volatility while lowering the volatility of the output gap. The decision of whether to always or occasionally target money growth comes down to the central bank's preferences for output versus inflation stability.

The ECB's primary objective is defined in the Treaty on the Functioning of the European Union: "The primary objective of the European System of Central Banks [...] shall be to maintain price stability." This is in contrast to the Federal Reserve which was given its "goals of maximum employment, stable prices and moderate long-term interest rates" in the amended Federal Reserve Act. This is typically referred to as the *dual mandate* since stable inflation, and hence inflation expectations, will in most cases lead to moderate long-run interest rates. The Federal Reserve's decision to only occasionally use money as an intermediate target can be rationalized by its intent to prevent money demand shocks from destabilizing the output gap, even if this comes at the cost of higher inflation volatility.[12] Similarly, the ECB's decision to always use money as an intermediate inflation target can be rationalized by its single mandate to stabilize prices.

Another possible reason why the ECB and the Federal Reserve use money differently could be that money demand is significantly more volatile in the U.S. than the euro area. If money demand shocks are large, then the switching policy rule dominates the constant parameter policy rule. The column on the right in Figure (4) shows that shielding the economy from money demand shocks becomes paramount for limiting the volatility of inflation and the output gap. The higher volatility of inflation induced by introducing switching into the policy rule is more than offset by the decreased exposure of both inflation and output to the highly volatile demand for money. This explanation does not appear to be empirically relevant since money demand shocks are not significantly larger in the U.S. relative to the euro area according to structural estimates (see for example Andres, Lopez-Salido, and Valles (2006), Ireland (2004b) and Andres, Lopez-Salido, and Nelson (2009)).

[12]Here I am interpreting the output gap as proportional to the unemployment gap, using Okun's law for example.

Figure 4: Efficiency Frontiers for Constant and Switching Parameter Taylor Rules



Notes: These efficiency frontiers are computed by solving for the MS-DSGE policy rules for different parameters of ϕ_π on a grid from 1.5 to 5 by 0.5 steps and ϕ_μ(or $\phi_\mu(s_t)$) from 0.1 to 2.0 by 0.1 steps. The first row plots the output and inflation volatilities for the constant parameter policy rule, the second row shows the same plots for the switching parameter policy rule, and the third row shows both efficiency frontiers on the same plot for comparison purposes. In the first two rows, lines are plotted by varying ϕ_π for a given value of ϕ_μ where thicker lines denote smaller values of ϕ_μ. The volatilities are measured as the square root of the discounted sum of conditional variances (i.e. $\sigma_{\mathcal{F}} = \sqrt{\sum \beta^j \mathbb{E}_t \mathcal{F}^2_{t+j}}$) using the closed form expressions derived by Bianchi (2013). The large money demand shocks calibration multiplies σ_v by 150.

7 Conclusion

Real wage rigidities, when combined with the cost channel, shrink the determinacy regions of interest rate rules targeting inflation. These smaller determinacy regions pose no challenge to central banks who seek to stabilize the economy by following interest rate rules with coefficients of the magnitude specified in the Taylor (1993) rule. However, when the cost channels varies according to a two-state Markov-Switching process, interest rate rules which target inflation often make the economy indeterminate. Targeting nominal money growth can alleviate this indeterminacy whether the central bank makes the money-growth target a constant feature of their reaction function or only targets money growth in the cost-channel regime. This analysis reveals that the ECB's decision to use a two-pillar approach to stabilizing prices is well justified by their single price stability mandate. If the ECB were to drop their second pillar of monetary analysis and switch to a U.S. style rule, which is modeled as a Taylor rule with a switching money growth reaction, then they could achieve lower output gap volatility, but at the cost of higher inflation volatility.

Although Woodford's (2003) critique that money contains no information beyond that already contained in inflation, output and interest rates applies to this model, central banks in practice have used money when setting monetary policy. Therefore, the results appeal to this observation and study how introducing a constant or switching money growth reaction affects the determinacy regions and the inflation output trade-off in this model. Despite the empirical appeal of this interpretation, theoretically speaking the results presented in this paper regarding money-growth targets apply to nominal GDP growth targets in interest rate rules with smoothing. Which of these two policy approaches would lead to better macroeconomic outcomes likely depends on whether money-demand shocks are larger than the measurement errors associated with measuring nominal GDP at a monetary policy meeting frequency. This is an empirical question which may deserve renewed attention in light of the findings that an intermediate target is, at times, critical to anchor inflation expectations.

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A DSGE Model

This section describes the DSGE model used in the paper in detail. The model motivates the working capital channel through a timing mismatch between when firms pay their wage bill and receive payment for their output. This timing can be described by dividing period t into 2 separate sub-periods: first a production and trading period and then a settlement period.

Sub-Period 1: Production and Trading Period

- All shocks are realized.
- The intermediate goods firms hire labor to produce their differentiated output. The final goods firm purchases inputs from the intermediate goods firms. A fraction of these purchases are paid for on the spot, the remaining fraction are bought on zero-interest firm credit. The intermediate goods producers finance a portion of their wage bill $\alpha(s_t)$ with a working-capital loan from the bank. All wages are paid. The household buys consumption goods and carries out financial transactions.

Sub-Period 2: Settlement Period

- The fraction of the intermediate goods that haven't yet been paid for by the final goods producers, receive payment from the final goods producing firms allowing these firms to pay-off their working-capital loans with interest. The household receives their deposits with interest from the bank and dividend payments from the intermediate goods firms.

A.1 The Household

The representative household enters any period $t = 0, 1, 2, ...$ with a portfolio consisting of maturing bonds B_{t-1} and monetary assets totaling A_{t-1}. The household faces a sequence of budget constraints in any given period. In the securities trading session the household can buys and sells bonds, receives wages W_t for hours worked H_t during the period, purchases consumption goods C_t and allocates their monetary assets between currency N_t and deposits D_t. Any loans L_t^{hh} needed to finance these transactions are made at this time. This is summarized in the constraint below.

$$N_t + D_t = \frac{B_{t-1}}{\Pi_t} + \frac{A_{t-1}}{\Pi_t} - \frac{B_t}{R_t} + W_t H_t - C_t + L_t^{hh} \tag{A.1}$$

At the end of the period, the household receives dividends $F_{i,t}$ from intermediate goods firms and receives interest on deposits $i_t^D D_t$, repays loans $i_t^L L_t^{hh}$ and receives any residual assets of bank F_t^b. Any remaining funds are combined with central bank transfers τ_t and are carried over in the form of monetary assets A_t into the next period.

$$A_t = N_t + \int_0^1 F_{i,t} di + R_t^D D_t - R_t^L L_t^{hh} + F_t^b + \tau_t. \tag{A.2}$$

The household seeks to maximize their lifetime utility, discounted at rate β, subject to these constraints. The period flow utility of the household takes the following form.

$$U_t = \zeta_t \left[ln(C_t) - \xi \frac{H_t^{1+\varphi}}{1+\varphi} - H_t^s \right]$$

The household receives utility from consumption and dis-utility from working and shopping. Time spent shopping increases with aggregate consumption C_t^A (i.e. long lines) but is reduced with higher

liquidity services.[13] Therefore the time spent shopping takes the following form:

$$H_t^s = \frac{1}{\chi}\left(\frac{\upsilon_t C_t^A}{M_t}\right)^{\chi}. \tag{A.3}$$

The time-varying preference parameter ζ_t enters the linearized Euler equation as an IS shock and similarly, υ_t enters the linearized money demand equation as a money demand shock. Both of these processes are assumed to follow an AR(1) (in logs).

$$ln(\zeta_t) = \rho_\zeta ln(\zeta_{t-1}) + \varepsilon_t^\zeta \qquad \varepsilon_t^\zeta \sim \mathcal{N}(0, \sigma_\zeta) \tag{A.4}$$

$$ln(\upsilon_t) = \rho_\upsilon ln(\upsilon_{t-1}) + \varepsilon_t^\upsilon \qquad \varepsilon_t^\upsilon \sim \mathcal{N}(0, \sigma_\upsilon) \tag{A.5}$$

The monetary aggregate, M_t, which enters the shopping-time function takes a rather general CES form,

$$M_t = \left[\nu^{\frac{1}{\omega}}(N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}}(D_t)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{A.6}$$

where ν calibrates the relative expenditure shares on currency and deposits and ω calibrates the elasticity of substitution between the two monetary assets. Given these parameters, χ is left free to calibrate the interest semi-elasticity of money demand.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting $\mathcal{C}_t = [C_t, H_t, M_t, N_t, D_t, L_t^{hh}, B_t, A_t]$ denote the vector of choice variables, the household's problem can be recursively defined using Bellman's method:

$$\begin{aligned} V_t\Big(B_{t-1}, A_{t-1}\Big) = \max_{\mathcal{C}_t}\Bigg\{ & \zeta_t\left[ln(C_t) - \xi\frac{H_t^{1+\varphi}}{1+\varphi} - \frac{1}{\chi}\left(\frac{\upsilon_t C_t^A}{M_t}\right)^{\chi}\right] \\ & -\lambda_t^1\Big(D_t + N_t + C_t - L_t^{hh} - W_t H_t - A_{t-1}/\Pi_t + B_t/R_t - B_{t-1}/\Pi_t - \tau_t\Big) \\ & -\lambda_t^2\Big(M_t - \left[\nu^{\frac{1}{\omega}}(N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}}(D_t)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}\Big) \\ & -\lambda_t^3\Big(A_t - N_t - \int_0^1 F_{i,t}di - R_t^D D_t + R_t^L L_t^{hh} - F_t^b\Big) + \beta\mathbb{E}_t\Big[V_{t+1}\Big(B_t, A_t\Big)\Big]\Bigg\}. \end{aligned}$$

The first order necessary conditions are given by the following equations:

$$\frac{\zeta_t}{C_t} = \beta\mathbb{E}_t\left[\frac{\zeta_{t+1}}{C_{t+1}}\frac{R_t}{\Pi_{t+1}}\right] \tag{A.7}$$

$$MRS_t = \xi H_t^\varphi C_t \tag{A.8}$$

$$W_t = W_{t-1}^{\rho_W} MRS_t^{(1-\rho_w)} \tag{A.9}$$

$$\frac{\upsilon_t C_t}{M_t} = \left(\frac{\lambda_t^2}{\lambda_t^1}\upsilon_t\right)^{\frac{1}{\chi+1}} \tag{A.10}$$

$$N_t = \nu M_t\left[\frac{\lambda_t^2/\lambda_t^1}{(R_t-1)/R_t}\right]^{\omega} \tag{A.11}$$

$$D_t = (1-\nu)M_t\left[\frac{\lambda_t^2/\lambda_t^1}{(R_t-R_t^D)/R_t}\right]^{\omega} \tag{A.12}$$

$$R_t = R_t^L. \tag{A.13}$$

[13]The assumption that aggregate consumption enters the shopping time specification as opposed to household consumption prevents the real-balances effect (i.e. real balances appearing in the log-linearized IS and Phillips Curve) which is not well supported by either U.S. or European Data (See for example Ireland (2004a); Andres et al. (2009).)

The above equations are quite standard with a few exceptions. Notice that equations (A.8) and (A.9) can be combines to yield the typical condition that the real wage equals the marginal rate of substitution when $\rho_w = 0$. However, when $0 < \rho_w < 1$ this condition only holds in steady state and there may be short-run deviations from this optimality condition due to real wage rigidity as in Hall (2005) and Blanchard and Gali (2007). Also, I have imposed $C_t^A = C_t$ in the shopping-time function after optimizing.

A.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by $i \in [0, 1]$ who produce a differentiated product. The final goods firm produces Y_t combining inputs $Y_{i,t}$ using the production technology,

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

in which $\theta > 1$ governs the elasticity of substitution between inputs. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem:

$$\max_{Y_{i,t} \in [0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di,$$

subject to the above constant returns to scale technology. The resulting first order condition defines the demand curve for each intermediate goods producing firm's product:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t. \tag{A.14}$$

Intermediate Goods Producing Firm

Given the downward sloping demand for its product in (A.14), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. To permit aggregation and allow for the consideration of a representative firm, I assume all such firms have the same constant returns to scale technology:

$$Y_{i,t} = z_t H_{i,t}. \tag{A.15}$$

The z_t term in (A.15) is an aggregate technology shock that follows an AR(1),

$$ln(z_t) = \rho_z ln(z_{t-1}) + \varepsilon_t^z \qquad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z). \tag{A.16}$$

The term $H_{i,t}$ in the production function denotes the level of employment chosen by the intermediate goods firm. Given the linear production function, the intermediate goods producing firm's real marginal cost takes the same functional form in the effective factor prices:

$$MC_t = v \left(\frac{R_t^L}{\bar{R}^L} \right)^{\alpha(s_t)} \frac{W_t}{P_t z_t},$$

which shows the effective wage rate differs from the real wage according to the share of the wage bill which must be financed. Reasons for using this functional form to specify the cost channel in the non-linear model is discussed further in Section (A.6). A production subsidy, v, is introduced to make

the steady state price of goods equal to the social marginal cost of production. Without the subsidy, the monopolist would set prices higher than the marginal social benefit.

The price setting ability of each firm is constrained as in Calvo (1983). In this staggered price-setting framework, the price level P_t is determined in each period as a weighted average of a fraction of firms $1-\omega$ are able to re-optimize their price and a fraction ω must leave their prices unchanged. Therefore, each firm maximizes the present value of its current and future discounted profits, taking into account the possibility that the firm may not be able to re-optimize for sometime:

$$\max_{\{P^*_{i,t}\}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \frac{\lambda^3_{t+j}}{\lambda^1_t} \left[P^*_{i,t} Y_{i,t+j} - MC_{t+j} P_{t+j} Y_{i,t+j} \right]$$

subject to

$$Y_{i,t+j} = \left(\frac{P^*_{i,t}}{P_{t+j}} \right)^{-\theta} Y_{t+j}.$$

The firm's first order condition is given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \frac{\lambda^3_{t+j}}{\lambda^1_t} Y_{i,t+j} \left(\frac{P^*_{i,t}}{P_{t-1}} - \frac{\theta}{\theta-1} MC_{t+j} \prod_{k=0}^{j} \Pi_{t+k} \right) = 0. \tag{A.17}$$

Finally, in the symmetric equilibrium, the aggregate price dynamics are determined by the following price aggregate:

$$P_t = \left[\omega \left(P_{t-1} \right)^{1-\theta} + (1-\omega) \left(P^*_t \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{A.18}$$

where P^*_t is the optimal price firms choose who re-optimize in period t.

A.3 The Financial Firm

The financial firm performs the intermediation process of accepting household's deposits and in turn loaning these funds to firms and households. The financial firm must satisfy the accounting identity which specifies assets (loans to firms plus reserves) equal liabilities (deposits),

$$L^{hh}_t + L^f_t + rr D_t = D_t. \tag{A.19}$$

Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans. Therefore, instead of assuming the central bank controls the reserve ratio rr, I assume it is exogenously fixed and represents the average ratio of deposits banks hold for regulatory and liquidity purposes.

The financial firm chooses $L_t = L^{hh}_t + L^f_t$ and D_t in order to maximize period profits

$$\max_{L_t, D_t} R^L_t L_t - R^D_t D_t - L_t + D_t$$

subject to the balance sheet constraint (A.19). Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero profits results in the loan-deposit spread,

$$R^L_t - R^D_t = (R^L_t - 1) rr. \tag{A.20}$$

This expression describes the loan deposit spread as a function of the foregone revenue of making loans when deposits are held as reserves instead of being loaned out.

A.4 Equilibrium and the Output Gap

Here I define the equilibrium conditions which close the model. Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t \tag{A.21}$$

holds. Equilibrium in the money market, bond market and loan market requires that at all times:

$$\begin{aligned} A_t &= \frac{A_{t-1}}{\Pi_t} + \tau_t \\ B_t &= B_{t-1} = 0 \\ L_t &= L_t^{hh} + \int_0^1 L_{t,t}^f di \end{aligned}$$

respectively. Market clearing in the labor market requires that labor supply equals labor demand:

$$H_t = \int_0^1 H_{i,t} di = \int_0^1 \frac{Y_{i,t}}{z_t} di = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} di \frac{Y_t}{z_t}$$

where the second equality uses the firm's production function (A.15) and the third equality uses the demand for the intermediate goods product (A.14). Therefore, aggregate output is related to aggregate labor supply and technology by:

$$Y_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} di H_t z_t. \tag{A.22}$$

To facilitate the analysis of monetary policy rules which feature a reaction to the real economy, I will log-linearize the model in terms of the efficient, or first-best, output gap $X_t = \frac{Y_t}{Y_t^*}$. In this economy, Y_t^* this is the level of output arising from the frictionless problem:

$$\max_{C_t, H_{i,t}} \zeta_t [ln(C_t) - \xi \frac{\left(\int_0^1 H_{i,t} di\right)^{1+\varphi}}{1+\varphi}] \qquad \text{subject to} \qquad C_t = z_t \left[\int_0^1 H_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}.$$

The resulting first order condition yields the efficient level of output: $Y_t^* = C_t^* = z_t/(\xi)^{1/(1+\varphi)}$, which yields the follow expression for the efficient output gap, denoted by X_t:

$$X_t = \frac{Y_t}{Y_t^*} = \xi^{1/(1+\varphi)} \frac{Y_t}{z_t}. \tag{A.23}$$

Finally, the production subsidy v to the intermediate goods producers is set so that in steady state the price of goods equals the social marginal cost of production. The monopolistically competitive firm in the economy sets their price in steady-state:

$$P = (1-v)\frac{\theta}{\theta-1} PW.$$

The social marginal cost of production is the marginal rate of substitution between labor and consumption for the household, which in steady state, equals the real wage rate. Therefore, setting $W = 1$ and solving for $1 - v$ implies:

$$1 - v = \frac{\theta - 1}{\theta}. \tag{A.24}$$

A.5 The Non-Linear Model

This section lists the full set of equilibrium conditions from the non-linear model.

$$\frac{\zeta_t}{C_t} = \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] \tag{A.25}$$

$$MRS_t = \xi C_t H_t^{1+\varphi} \tag{A.26}$$

$$W_t = W_{t-1}^{\rho_w} MRS_t^{(1-\rho_w)} \tag{A.27}$$

$$\frac{\upsilon_t C_t}{M_t} = (u_t \upsilon_t)^{\frac{1}{\chi+1}} \tag{A.28}$$

$$N_t = \nu M_t \left[\frac{u_t}{(R_t - 1)/R_t} \right]^{\omega} \tag{A.29}$$

$$D_t = (1-\nu) M_t \left[\frac{u_t}{(R_t - R_t^D)/R_t} \right]^{\omega} \tag{A.30}$$

$$M_t = \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \tag{A.31}$$

$$\Lambda_t^1 = \frac{\zeta_t}{C_t} \tag{A.32}$$

$$u_t = \Lambda_t^2 / \Lambda_t^1 \tag{A.33}$$

$$\Lambda_t^1 = \Lambda_t^3 R_t^L \tag{A.34}$$

$$R_t^L = R_t \tag{A.35}$$

$$Y_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} di z_t H_t \tag{A.36}$$

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \frac{\Lambda_{t+j}^3}{\Lambda_t^1} Y_{i,t+j} \left(\Pi_t^* - \frac{\theta}{\theta - 1} MC_{t+j} \prod_{k=0}^{j} \Pi_{t+k} \right) \tag{A.37}$$

$$\Pi_t^{1-\theta} = \omega + (1-\omega) \left(\Pi_t^* \right)^{1-\theta} \tag{A.38}$$

$$MC_t = \frac{\theta - 1}{\theta} \left(\frac{R_t^L}{\bar{R}^L} \right)^{\alpha(s_t)} W_t / z_t \tag{A.39}$$

$$L_t = D_t (1 - rr) \tag{A.40}$$

$$R_t^L - R_t^D = (R_t^L - 1) rr \tag{A.41}$$

$$X_t = \xi^{1/(1+\varphi)} \frac{Y_t}{z_t} \tag{A.42}$$

$$Y_t = C_t \tag{A.43}$$

$$ln(\zeta_t) = \rho_\zeta ln(\zeta_{t-1}) + \varepsilon_t^\zeta \tag{A.44}$$

$$ln(\upsilon_t) = \rho_\upsilon ln(\upsilon_{t-1}) + \varepsilon_t^\upsilon \tag{A.45}$$

$$ln(z_t) = \rho_z ln(z_{t-1}) + \varepsilon_t^z \tag{A.46}$$

A.6 The Log-Linear Model

In this section I provide a linear representation of the model by taking a first order Taylor expansion of the relevant equations around the the symmetric equilibrium with no trend in inflation or technology. All lower case variables denote log deviations from the steady-state: $g_t = ln(\hat{g}_t) - ln(\bar{g})$, where $\bar{g}$ is the steady state value of g_t.

The log-linear Euler equation, expressed in terms of the efficient output gap, can be derived from combining (A.25), (A.42) and (A.43):

$$x_t = \mathbb{E}_t x_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) + (1-\rho_\zeta)\zeta_t - (1-\rho_z)z_t, \tag{A.47}$$

so that in equation (1) $\varepsilon_t^x = (1-\rho_\zeta)\zeta_t - (1-\rho_z)z_t$.

The evolution of the real wage is determined by (A.27) which can be combined with (A.26), (A.36), (A.42) and (A.43) to express the real wage as a function of last periods real wage and the output gap (and technology):

$$w_t = \rho w_{t-1} + (1-\rho)(1+\varphi)(x_t + z_t). \tag{A.48}$$

The above expression eliminates H_t from MRS_t using the relationship between aggregate output and hours supplied, technology and price dispersion. The price dispersion term is zero to a first-order approximation around the zero inflation steady state. This expression shows that in equation (2) $\varepsilon_t^w = (1-\rho)(1+\varphi)z_t$.

The log-linear money demand equation can be derived in two steps. First, I combine equations (A.29), (A.30) and (A.31) to show that in equilibrium:

$$\begin{aligned} u_t &= \left[\nu\left(\frac{R_t - 1}{R_t}\right)^{1-\sigma} + (1-\nu)\left(\frac{R_t - R_t^D}{R_t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \frac{R_t - 1}{R_t}\left[\nu + (1-\nu)\left(rr\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

where the second equality follows from (A.41). Then I use this expression for u_t in (A.28) to arrive at the following expression for real money balances:

$$m_t = x_t - \frac{\beta^2}{1-\beta}\frac{1}{1+\chi}r_t + \frac{\chi}{1+\chi}\upsilon_t + z_t, \tag{A.49}$$

so that in equation (3) $\eta = \beta^2/(1-\beta)(1+\chi)$ and $\varepsilon_t^m = \frac{\chi}{1+\chi}\upsilon_t + z_t$.

Finally, the Phillips Curve can be derived in two steps. First, log-linearizing (A.37):

$$\begin{aligned} \pi_t^* &= (1-\beta\omega)\mathbb{E}_t \sum_{j=0}^{\infty}(\beta\omega)(w_{t+j} - z_{t+j} + \alpha(s_{t+j})r_{t+j} + \sum_{k=0}^{j}\pi_{t+k}) \\ &= (1-\beta\omega)\mathbb{E}_t \sum_{j=0}^{\infty}(\beta\omega)(w_{t+j} - z_{t+j} + \alpha(s_{t+j})r_{t+j}) + \mathbb{E}_t\sum_{j=0}^{\infty}(\beta\omega)\pi_{t+j} \\ &= \beta\omega\pi_{t+1}^* + (1-\beta\omega)(w_t - z_t + \alpha(s_t)r_t) + \pi_t. \end{aligned}$$

The first relationship follows from linearizing the firms pricing decision around the zero-inflation steady-state. The second equality follows after some algebra and the third equality rewrites the infinite sum as a recursive formula. In this derivation, I follow the *Partition Principle* of Foerster et al. (2013), and therefore I do not linearize around $\alpha(s_t)$ so that the log-linear regime switching model maintains

the inherent non-linearity in the switching parameters. The fact that I don't linearize around $\alpha(s_t)$ and that it doesn't affect the model's steady state, allows me to use this same linearization to study local determinacy in the regime switching model. Therefore, this specification of the cost channel produces a log-linear approximation in the constant parameter model that is isomorphic to standard log-linear cost channel model like those specified by Rabanal (2007) and Christiano et al. (2010) and estimated in Ravenna and Walsh (2006) while, at the same time, this specification introduces switching in the coefficient matrices in the MS-DSGE model. Next, I linearize equation (A.38), around the zero inflation steady state and use the resulting expression $\pi_t = (1-\omega)\pi_t^*$ to eliminate π_t^* above:

$$\pi_t = \frac{(1-\omega)(1-\beta\omega)}{\omega}(w_t - z_t + \alpha(s_t)r_t) + \beta\mathbb{E}_t\pi_{t+1}, \tag{A.50}$$

so that in equation (4) $\kappa = (1-\omega)(1-\beta\omega)/\omega$ and $\varepsilon_t^\pi = -\kappa z_t$.

A.7 Baseline Calibration

Table 4: Baseline Model Calibration

Parameter		U.S	Euro Area
Discount Rate	β	0.99	0.99
Real Wage Adjustment	ρ	0.90	0.90
Inverse Frisch Elasticity	φ	0	0
Interest Semi-Elasticity of Money Demand	η	2	3.2
Calvo Probability	ω	0.6	0.75
Share of Cost Channel Firms	α	1	1
Inflation Reaction in Policy Rule	ϕ_π	1.5	1.5
Output Gap Reaction in Policy Rule	ϕ_x	0.125	0
Money Growth Reaction in Policy Rule	ϕ_μ	0	0.134
Standard Deviation of Preference Shock	σ_ζ	0.0405	0.0120
Standard Deviation of Technology Shock	σ_z	0.0109	0.0042
Standard Deviation of Money Demand Shock	σ_υ	0.0012	0.0044

The model is calibrated so that each period represents one quarter. I set $\beta = 0.99$ which implies an annualized nominal bond rate equal to 4%. There is little empirical evidence on the real wage adjustment parameter, so I follow the baseline calibration in Blanchard and Gali (2007) and set $\rho = 0.9$. Also supporting this calibration, this parameter is large enough to generate an inertial inflation response to a monetary policy shock. I set the elasticity of hours worked, $\varphi = 0$, following much of the macro literature (Hansen, 1985; Rogerson, 1988; Ireland, 2004a). Ravenna and Walsh (2006) estimate α to be statistically indistinguishable from 1 using U.S. data. Furthermore, Chowdhury et al. (2006) find similar evidence for many euro area countries. The multi-equation decompositions of inflation by (Tillmann, 2008) also finds that $\alpha = 1$ is difficult to statistically reject for the U.S. and euro area.

For the U.S. model, I set $\omega = 0.6$ so the average duration of a price is about 7 months, as found by Bils and Klenow (2004) from micro level data. Meanwhile, for the euro area calibration I set $\omega = 0.75$ as found by Dhyne, Alvarez, Le Bihan, Veronese, Dias, Hoffmann, Jonker, Lunnemann, Rumler, and Vilmunen (2006) who also use micro level data. Following Ireland (2009), I set $\eta = 2$ for the U.S. Meanwhile, for the euro area, I use the estimate from Andres et al. (2009) of $\eta = 3.2$. Both of these

studies also find support for the unit income elasticity of money demand specified in this equilibrium model. The policy rule parameters for the U.S. are taken from the benchmark work of Taylor (1993). for the euro area I use the estimates from Section (6.2). The values used for the structural shocks follow from Ireland (2004b) for the U.S. and Andres et al. (2006) for the euro area. I calibrate all of the shocks as white noise due to the computational difficulties associated with solving the model with more state variables. I relax many of these parameter assumptions when analyzing the robustness of the results in Section (6).

B Proofs

In this section I present the proofs to the results stated in the paper. All proofs for determinacy omit the possibility that an eigenvalue is exactly equal to one. In such a case, a log-linear approximation to the non-linear model can not pin down the question of local equilibrium existence and uniqueness. The proofs rely on the results from Woodford (2003) regarding determinacy in 2-dimensional forward looking models and 3-dimensional models with 2 forward-looking variables and 1 predetermined variable. For ease of exposition, I restate the propositions from Woodford (2003) before providing proofs to the various propositions in the paper.

Proposition C.1, Woodford (2003)

Consider a linear rational-expectations model of the form

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A}\mathcal{F}_t + \mathcal{B}\epsilon_t$$

where $\mathcal{F}_t$ is a 2×1 vector of forward-looking variables, ϵ_t is a vector of exogenous disturbance terms, and $\mathcal{A}$ is a 2×2 matrix of coefficients. The rational expectations equilibrium is determinate if and only if the matrix $\mathcal{A}$ has both eigenvalues outside the unit circle. This condition in turn is satisfied if and only if either Case I or Case II below are true.

Case I

$$det\mathcal{A} > 1 \tag{B.1}$$
$$det\mathcal{A} - tr\mathcal{A} > -1 \tag{B.2}$$
$$det\mathcal{A} + tr\mathcal{A} > -1 \tag{B.3}$$

Case II

$$det\mathcal{A} - tr\mathcal{A} < -1 \tag{B.4}$$
$$det\mathcal{A} - tr\mathcal{A} < -1 \tag{B.5}$$

Proposition C.2, Woodford (2003)

Consider a linear rational-expectations model of the form

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A}\mathcal{F}_t + \mathcal{B}\epsilon_t$$

where $\mathcal{F}_t$ is a 3×1 vector with 2 forward-looking variables and 1 predetermined variable, ϵ_t is a vector of exogenous disturbance terms, and $\mathcal{A}$ is a 3×3 matrix of coefficients. The rational expectations equilibrium is determinate if and only if the matrix $\mathcal{A}$ has exactly 2 eigenvalues outside the unit circle. This condition in turn is satisfied if and only if either Case I, Case II or Case III below are true, in which the characteristic equation of the matrix $\mathcal{A}$ is written in the form:

$$\mathcal{P}(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0. \tag{B.6}$$

Case I

$$A_2 + A_1 + A_0 + 1 < 0 \tag{B.7}$$
$$A_2 - A_1 + A_0 - 1 > 0 \tag{B.8}$$

Case II

$$A_2 + A_1 + A_0 + 1 > 0 \tag{B.9}$$
$$A_2 - A_1 + A_0 - 1 < 0 \tag{B.10}$$
$$A_0^2 + A_1 - A_0 - 1 > 0 \tag{B.11}$$

Case III

$$A_2 + A_1 + A_0 + 1 > 0 \tag{B.12}$$
$$A_2 - A_1 + A_0 - 1 < 0 \tag{B.13}$$
$$\mid A_2 \mid > 3 \tag{B.14}$$

Proposition 1

Consider the dynamic system defined by equations (1), (2), (4) and the policy rule $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$,

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A}\mathcal{F}_t + \mathcal{B}\epsilon_t$$

$$\mathcal{A} = \begin{bmatrix} 1 + \frac{\kappa(1+\varphi)}{\beta} & \frac{\phi_\pi(\beta+\alpha\kappa)-1}{\beta} \\ -\frac{\kappa(1+\varphi)}{\beta} & \frac{1-\phi_\pi\alpha\kappa}{\beta} \end{bmatrix}$$

where $\mathcal{F}_t = \begin{bmatrix} x_t, & \pi_t \end{bmatrix}^T$. The system has two non-predetermined variables and therefore the system will have a unique rational expectations equilibrium if, and only if, Case I or Case II is satisfied in Proposition C.1. I will show the conditions in the theorem are necessary and sufficient for Case I to hold.

The conditions for Case I to hold are as follows:

$$det\mathcal{A} > 1 \iff 1 + \kappa\phi_\pi(1 + \varphi - \alpha) > \beta \tag{B.15}$$
$$det\mathcal{A} - tr\mathcal{A} > -1 \iff \kappa(1+\varphi)(1-\phi_\pi) + \beta < \beta \tag{B.16}$$
$$det\mathcal{A} + tr\mathcal{A} > -1 \iff \phi_\pi\kappa(2\alpha - (1+\varphi)) < 2(1+\beta) + \kappa(1+\varphi). \tag{B.17}$$

Condition (B.15) holds so long as $\phi_\pi > 0$, condition (B.16) places the lower bound restriction that $\phi_\pi > 1$ and condition (B.17) always holds when $\alpha \leq (1+\varphi)/2$; but when $\alpha > (1+\varphi)/2$ then condition (B.17) imposes the upper-bound that $\phi_\pi < \frac{2(1+\beta)+\kappa(1+\varphi)}{\kappa(2\alpha-(1+\varphi))}$.

Proposition 2

Consider the dynamic system defined by equations (1) - (4) and the policy rule $r_t = \phi_\mu \mu_t + \varepsilon_t^{mp}$,

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A}\mathcal{F}_t + \mathcal{B}\epsilon_t$$

$$\mathcal{A} = \begin{bmatrix} 1 + \frac{\kappa(1+\varphi)}{\beta} + \phi_\mu\frac{\alpha\kappa+\beta}{\beta(\phi_\mu\eta+1)} & \phi_\mu + \frac{\phi_\mu\alpha\kappa-1}{\beta} - \frac{\phi_\mu^2(\alpha\kappa+\beta)\eta}{\beta(\phi_\mu\eta+1)} & -\phi_\mu - \frac{\phi_\mu\alpha\kappa}{\beta} + \frac{\phi_\mu^2(\alpha\kappa+\beta)\eta}{\beta(\phi_\mu\eta+1)} \\ -\frac{\kappa(1+\varphi)}{\beta} - \frac{\phi-\mu\alpha\kappa}{\beta(\phi_\mu\eta+1)} & \frac{1-\phi_\mu\alpha\kappa}{\beta} + \frac{\phi_\mu^2\alpha\kappa\eta}{\beta(\phi-\mu\eta+1)} & \frac{\phi_\mu\alpha\kappa}{\beta} - \frac{\phi_\mu^2\alpha\kappa\eta}{\beta(\phi-\mu\eta+1)} \\ \frac{1}{\phi_\mu\eta+1} & -\frac{\phi_\mu\eta}{\phi_\mu\eta+1} & \frac{\phi_\mu\eta}{\phi_\mu\eta+1} \end{bmatrix}$$

where $\mathcal{F}_t = \begin{bmatrix} x_t, & \pi_t, & m_t \end{bmatrix}^T$. The system has two non-predetermined variables and one predetermined variable and therefore the system will have a unique rational expectations equilibrium if, and only if, Case I, Case II, or Case III is satisfied in Proposition C.2. Here, I show that the conditions in Case III are always satisfied if, and only if, $\phi_\mu > 1$.

The conditions for Case III to hold are as follows:

$$
\begin{aligned}
&A_2 + A_1 + A_0 + 1 > 0 \\
&\iff \\
&\frac{\kappa(1+\varphi)(\phi_\mu - 1)}{\beta(\phi_\mu \eta + 1)} > 0
\end{aligned}
\tag{B.18}
$$

$$
\begin{aligned}
&A_2 - A_1 + A_0 - 1 < 0 \\
&\iff \\
&-\frac{2\beta(\phi_\mu\eta+1) + 2\phi_\mu(\eta(2+\beta+\kappa(1+\varphi)) + (1+\beta)) + \kappa(1+\varphi)(\phi_\mu+1) + 2)}{2\beta(\phi_\mu\eta+1)} < 0
\end{aligned}
\tag{B.19}
$$

$$
\begin{aligned}
&\mid A_2 \mid > 3 \\
&\iff \\
&2\beta(\phi_\mu\eta+1) + \phi_\mu\eta(1+\kappa(1+\varphi)) + \phi_\mu\beta + \kappa(1+\varphi) + (1-\beta) > 3\beta(\phi_\mu\eta+1).
\end{aligned}
\tag{B.20}
$$

Condition (B.18) holds if, and only if, $\phi_\mu > 1$, condition (B.19) is always true for positive parameters and condition (B.20) always holds, assuming $\phi_\mu > 1$ as required by condition (B.18).

Proposition 3

Consider the dynamic system defined by equations (1), (2), (4) and the policy rule $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$,

$$
\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A}\mathcal{F}_t + \mathcal{B}\epsilon_t
$$

$$
\mathcal{A} = \begin{bmatrix}
1 + \frac{\kappa(1-\rho)(1+\varphi)}{\beta} & \frac{\phi_\pi(1+\alpha\kappa)+1}{\beta} & \frac{\kappa\rho}{\beta} \\
-\frac{\kappa(1-\rho)(1+\varphi)}{\beta} & \frac{1-\phi_\pi\alpha\kappa}{\beta} & -\frac{\kappa\rho}{\beta} \\
(1-\rho)(1+\varphi) & 0 & \rho
\end{bmatrix}
$$

where $\mathcal{F}_t = \begin{bmatrix} x_t, & \pi_t, & w_t \end{bmatrix}^T$. The system has two non-predetermined variables and one predetermined variable and therefore the system will have a unique rational expectations equilibrium (REE) only if Case I, Case II, or Case III is satisfied in Proposition C.2. Here, I show that the conditions in Case I are never satisfied when $\phi_\pi > 1$. Case I requires that $A_2 + A_1 + A_0 + 1 < 0$, which is true only if $\phi_\pi < 1$. Therefore, active monetary policy equilibria only exist in Case II or Case III. In either Case II or Case III case, a necessary condition for a unique equilibrium to exist is that:

$$
\begin{aligned}
&A_2 - A_1 + A_0 - 1 < 0 \\
&\iff \\
&\phi_\pi\kappa(2\alpha(1+\rho) - (1+\varphi)(1-\rho)) - 2(1+\beta)(1+\rho) + \kappa(1+\varphi)(1+\rho) < 0
\end{aligned}
\tag{B.21}
$$

If $\alpha \leq \frac{(1+\varphi)(1-\rho)}{2(1+\rho)} \equiv \alpha^\rho_{min}$, then condition (B.21) always holds. However, when $\alpha > \alpha^\rho_{min}$, condition (B.21) requires that in any unique rational expectations equilibrium:

$$
\phi_\pi < \frac{2(1+\beta)(1+\rho) + \kappa(1+\varphi)(1+\rho)}{\kappa(2\alpha(1+\rho) - (1+\varphi)(1-\rho))} \equiv \phi^\rho_{\pi,max}.
$$

Notice that when $\rho = 0$, this condition collapses to the upper bound shown in Proposition 1. However, for $0 < \rho < 1$, $\alpha^\rho_{min} = \frac{(1+\varphi)(1-\rho)}{2(1+\rho)} < \frac{(1+\varphi)}{2} = \alpha^0_{min}$. This establishes the first claim of the proposition. As for the second claim, notice that,

$$
\frac{d\phi^\rho_{\pi,max}}{d\rho} = -\frac{2(1+\beta)(1+\varphi)(1-\rho) + \kappa(1+\varphi)(2(1+\beta)(1+\rho) + (\kappa+1)(1+\varphi)(1+\rho))}{(\kappa(2\alpha(1+\rho) - (1+\varphi)(1-\rho)))^2} < 0.
$$

Therefore, in the presence of partial real wage adjustment, the minimum strength of the cost channel necessary to induce an upper bound on the inflation response is decreasing in ρ. Furthermore, the upper bound on the inflation response necessary for a unique REE to exist is decreasing in ρ.