FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

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August 2005

Working Paper 2006-02
http://www.frbsf.org/publications/economics/papers/2006/wp06-02bk.pdf

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Monetary Policy Shocks, Inventory Dynamics, and Price-setting Behavior*

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Abstract

In this paper, we estimate a VAR model to present an empirical finding that an unexpected rise in the federal funds rate decreases the ratio of sales to stocks available for sales, while it increases finished goods inventories. In addition, dynamic responses of these variables reach their peaks several quarters after a monetary shock. In order to understand the observed relationship between monetary policy and finished goods inventories, we allow for the accumulation of finished goods inventories in an optimizing sticky price model, where prices are set in a staggered fashion. In our model, holding finished inventories helps firms to generate more sales at given their prices. We then show that the model can generate the observed relationship between monetary shocks and finished goods inventories. Furthermore, we find that allowing for inventory holdings leads to a Phillips curve equation, which makes the inflation rate depend on the expected present-value of future marginal cost as well as the current period's marginal cost and the expected rate of future inflation.

JEL classification: E22; E30; E52

Keywords: Monetary Policy; Inventory Dynamics; Sticky Price; Phillips Curve

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* We appreciate helpful comments from Mark Bils, YongSung Chang, Dale Henderson, Jinill Kim, and Andrew Levin. All errors are our own.

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1 Introduction

Much of business cycles literature has emphasized that inventory behavior is an important factor in understanding the character of the aggregate business fluctuations. An important research topic on inventory behavior is why inventory investment is procyclical. Recent works in the literature on inventory behavior also have stressed that it is important to explain why inventory investment is not more procyclical over phases of business cycles. For example, Bils and Kahn (2000) have shown that manufacturer's finished goods inventories are less cyclical than shipments.

In this paper, we analyze the role of monetary policy shocks in generating the observed sluggish adjustment of inventory stocks. To this end, we estimate a vector autoregression for a set of selected aggregate variables, which includes the ratio of sales to the stock available for sales as well as real GDP, the inflation rate, and the federal funds rate. We then show that real GDP and the ratio of sales to stocks increase in response to an expansionary monetary shock. Besides, the expansionary monetary shock decreases the stock of finished goods inventories measured at the end of each period, while its dynamic responses reach their minimum several quarters after the monetary shock. In an attempt to understand such behaviors of inventories in response to monetary shocks, we present a dynamic stochastic general equilibrium model in which firms set prices as in the staggered price-setting model of Calvo (1983) and hold finished goods inventories to facilitate sales. Hence, our model incorporates the partial equilibrium model of Bils and Kahn (2000) into a complete dynamic general equilibrium model with nominal price rigidity, which permits quantitative analysis on the effect of monetary shocks.

The reason for the inclusion of nominal price rigidity is associated with the cyclical behavior of real marginal cost over business cycles. It has been known in the recent literature on business cycles that nominal price rigidity can induce procyclical movements of the real marginal cost in response to demand shocks, as discussed in Rotemberg and Woodford (1999). Besides, the observed sluggish behavior of finished goods inventories requires procyclical movements of marginal costs to make production more expansive during booms than during recessions. The price stickiness is therefore included into our model as a mechanism to generate procyclical movements of real marginal costs in response to monetary policy shocks.

Our findings can be summarized as follows. First, it has been emphasized in the recent literature on the Phillips curve that the current period's inflation depends the aggregate real marginal cost as well as the expectation about the next period's inflation in a canonical closed-economy version of the classic Calvo model. In relation to this, we show that the current period's inflation rate depends on the next period's expected present-value of

marginal cost as well as the current period's marginal cost in the forward-looking Phillips curve equation, when a fraction of firms make decision on their prices and inventories at the same time. This is because when firms hold inventories, the opportunity cost of selling one unit in the current period is the expected present-value of the next period's marginal cost.

Second, we find that under both of a high depreciation of the inventory stock and a high elasticity of demand with respect to the stock available for sales, our model can generate the observed inventory dynamics in responses to a monetary policy shock. The reason for this is that up to the first-order approximation, a sufficiently high depreciation rate helps to avoid excessive fluctuations of finished goods inventory in the model with only sale-expansion benefits from inventory holdings.

Third, we allow for adjustment costs, which take place when sales-stock ratio deviates from its fixed target ratio. We then demonstrate that the inclusion of such adjustment costs in the model helps to match the observed variability of finished goods inventories, if one wants to assume a small depreciation of the inventory stock. In particular, a key specification of the linear-quadratic cost function approach to inventory behavior is the inclusion of quadratic costs for deviations of sales-stock ratio from its fixed target ratio, as discussed in Ramey and West (1999). Besides, the quadratic adjustment costs described above reflect that holding inventories allows firms to satisfy their demands, which cannot be backlogged. Our findings therefore indicate that a joint specification of stock-out costs and sales-expansion benefits as incentives for holding finished goods inventories helps to understand the observed behavior of finished goods inventories, for those goods that have small depreciation.

It is now worthwhile to compare our model with a set of existing studies on inventory behavior. For example, Hornstein and Sarte (2001) and Chang, Hornstein, and Sarte (2004) have studied optimizing sticky price models with inventories. However, our analysis differs from theirs in two aspects. First, their models have used the staggered price-setting of Taylor (1980), while our model builds on the staggered price setting of Calvo (1983). Second, the analysis of Chang, Hornstein, and Sarte (2004) relies on a partial equilibrium industry model, whereas our analysis is based on a dynamic stochastic general equilibrium model. In addition, the primary concern of Hornstein and Sarte (2001) is the production smoothing of inventory in the presence of increasing short-run marginal cost, while we focus on sales-expansion benefits of inventory holding without relying on upward-sloping marginal cost. Christiano (1988) also studies a real business cycle model with inventories, in which inventory stocks are included in production functions as a production input. In our model, however, changes in inventories shift demand curves of differentiated goods,

given their prices.

The rest of our paper is organized as follows. In section 2, we discuss how a monetary policy shock is identified on the basis of a VAR for a set of selected aggregate variables. In section 3, we develop a sticky price model in which a fraction of firms accumulate their finished goods inventories. In this section, we discuss the effect of holding finished goods inventories on the Phillips curve equation on the basis of log-linearized equilibrium conditions. In section 4, we report simulation results from the model and compare them with observed effects of monetary shocks on inventory dynamics. Section 5 concludes.

2 Effects of a Monetary Policy Shock on Inventory Dynamics

We begin our analysis by estimating effects of a monetary policy shock on inventory dynamics. In order to identify a monetary policy shock, we estimate a unrestricted VAR for selected aggregate variables and then impose a set of structural restrictions on the variance-covariance matrix of its residual vector, which has been widely used in the literature since the work of Sims (1980).

The choice of variables included in the VAR is made to reflect the requirement that one can see effects of monetary policy on the key aggregate variables and inventory dynamics at the same time. Hence, the sample we use in this paper consists of the U.S. quarterly time series on real GDP, the GDP deflator inflation rate, the ratio of sales to stocks, the finished goods inventory stock measured at the end of period, and the federal funds rate over the period 1967:1 - 1996:4.[1] The unrestricted VAR we estimated in this paper can be then written as

$$X_t = \Gamma_0 + \sum_{k=1}^{4} \Gamma_k X_{t-k} + u_t, \tag{2.1}$$

where X_t is $p \times 1$ vector and $\{\Gamma_k\}_{k=0}^{4}$ is a set of $p \times p$ matrices, u_t is a $p \times 1$ residual vector, and p is the number of variables in the VAR.

In order to identify monetary policy shocks, we follow the identification strategy employed in Christiano, Eichenbaum, and Evans (2001). More explicitly, the monetary policy of the central bank is described as follows:

$$r_t = f(\Omega_t) + e_{rt}, \tag{2.2}$$

[1]The finished goods inventory stock is the real (manufacturing) finished inventories (end of period, chained 2000, B.E.A.) divided by population. The ratio of sales to stock is constructed by using real inventory-sales ratio for finished goods inventories (chained 2000, B.E.A.). Real GDP, sales-stock ratio, and inventory stock are logged for the purpose of comparison with numerical solutions to the theoretic model analyzed in this paper.

where r_t is the federal funds rate, f is a linear function, Ω_t is the information set at period t, and e_{rt} is the monetary policy shock. Here, we identify the federal funds rate as the monetary policy instrument. As an example of such an identification scheme, we can choose an ordering of the variables in X_t of the form:

$$X_t = [\pi_t, \log Y_t, \log \frac{S_t}{A_t}, \log L_t, r_t]', \tag{2.3}$$

where π is the inflation rate, Y is output, $\frac{S}{A}$ is the sales to stock ratio, and L is the inventory stock. The relationship between X_t and the vector of true shocks, denoted by e_t, is assumed to satisfy

$$D^{-1}X_t = \tilde{\Gamma}_0 + \sum_{k=1}^{4} \tilde{\Gamma}_k X_{t-k} + De_t, \tag{2.4}$$

where D is a 5×5 lower triangular matrix with diagonal terms equal to 1, $\{\tilde{\Gamma}_k\}_{k=0}^4$ is a set of 5×5 matrices, and e_t is a 5×1 vector of serially uncorrelated shocks with mean zero and diagonal variance-covariance matrix. The fifth element of e_t is then identified as a monetary policy shock, which is denoted by e_{rt}.

For a concrete example of the identification of elements in the matrix D, we can use the variance-covariance matrix of the residual vector u_t. Suppose that variances of elements of e_t are assumed to be one. Comparing (2.1) with (2.4), one can see that the following relationship holds:

$$\Omega = DD', \tag{2.5}$$

where Ω ($= E[u_t u_t']$) denotes the variance-covariance matrix of residuals.[2] Since D is assumed to be a lower triangular matrix, we can apply a Cholesky decomposition to the variance covariance matrix of u_t in order to identify elements of the matrix D.

Figure 1 reports impulse responses to an expansionary monetary shock. An expansionary monetary shock increases the rate of inflation, real GDP, and the ratio of sales to stocks, while it decreases the inventory stock of finished goods measured at the end of period. Besides, one can see that the sales-stock ratio as well as the inflation rate and real GDP show hump-shaped responses to an expansionary monetary policy shock, while the finished goods inventory stock displays U-shaped responses to the same shock.

3 The Model

This section presents a dynamic stochastic general equilibrium model with nominal price rigidity and inventory holding. Money is assumed to play only a role of unit of account,

[2]Refer to Christiano, Eichenbaum, and Evans (1999) for detailed explanation about identification procedures for monetary shocks.

following recent literature on sticky price models.[3] It is also assumed that it takes one period for private agents to observe monetary shocks. We do this to make the information set of households and firms consistent with the identification strategy for monetary shocks described in the previous section. Specifically, when I_t denotes the information set at period t. we assume that I_t includes all the past monetary policy shocks other than the monetary policy shock at period t. Hence, private agents do not observe the realization at period t of the monetary policy shock, when they form their expectation about X_{t+1} based on I_t, which is denoted by $E_t[X_{t+1}]$,

3.1 Firms

We assume that there are two classes of goods, depending upon whether to accumulate finished goods inventories. In what follows, goods that require accumulating their inventories are called "inventory goods", while goods that do not hold inventories are called "non-inventory goods".

3.1.1 Demand Functions of Firms

Households and government purchase both of two classes of goods for their consumption in each period $t = 0, 1, \cdots, \infty$. Specifically, an index of the two classes of goods is defined as

$$S_t = (\gamma^{\frac{1}{\phi}} \bar{S}_t^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}} \tilde{S}_t^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}}, \quad \phi > 0, \tag{3.1}$$

where $\bar{S}_t$ denotes the aggregate sales at period t for firms that hold inventories, and $\tilde{S}_t$ denotes the aggregate sales at period t for firms that do require inventories. During each period, households minimize the total cost of obtaining S_t, which in turn leads to the following demand curves:

$$\bar{S}_t = \gamma(\frac{\bar{P}_t}{P_t})^{-\phi} S_t; \quad \tilde{S}_t = (1-\gamma)(\frac{\tilde{P}_t}{P_t})^{-\phi} S_t, \tag{3.2}$$

where $\bar{P}_t$ denotes the price index for goods holding their inventories and $\tilde{P}_t$ denotes the price index for goods that do not inventories. The aggregate price index, denoted by P_t, is now given by

$$P_t = (\gamma \bar{P}_t^{1-\phi} + (1-\gamma)\tilde{P}_t^{1-\phi})^{\frac{1}{1-\phi}}. \tag{3.3}$$

Furthermore, there is a continuum of differentiated goods for each type of goods classes. Households purchase differentiated goods in retail market and combines them into composite goods using a Dixit-Stiglitz (1977) aggregator. More explicitly, $\bar{S}_t$ is defined as an

[3]See, for example, Woodford (2003) for cashless economy in models with nominal rigidities.

index of differentiated goods:

$$\bar{S}_t = (\int_0^1 (\frac{A_{jt}}{A_t})^{\frac{\theta}{\epsilon_i}} (\bar{S}_{jt})^{\frac{\epsilon_i - 1}{\epsilon_i}} dj)^{\frac{\epsilon_i}{\epsilon_i - 1}}; \quad \epsilon_i > 1; \quad 0 \leq \theta \leq 1; \tag{3.4}$$

where $\bar{S}_{jt}$ denotes differentiated goods of type j in the inventory goods class and A_{jt} is the stock of firm j available for sales at period t. In addition, the parameter θ measures the elasticity of demand with respect to the amount of the stock available for sales and ϵ_i is the elasticity of demand for an individual firm with respect to its own price. In particular, as the parameter θ takes a higher value between 0 and 1, holding inventory stock creates a larger effect on sales at given prices of goods.

Before proceeding, it is worth discussing two features of the specification of the aggregator described in (3.4). The reason for the inclusion of the stock available for sale in the aggregator is that holding finished inventories helps firms to generate greater sales at a given price, following Bils and Kahn (2000). The key difference from their model, however, is that holding finished goods inventory facilitates sales only when its stock available for sale is higher than the average level in the economy.

Households minimize the total cost of obtaining differentiated goods indexed by a unit interval $[0,\ 1]$, taking as given their nominal prices $\bar{P}_{jt}$. The cost-minimization then gives a demand curve of the form:

$$\bar{S}_{jt} = (\frac{A_{jt}}{A_t})^{\theta} (\frac{\bar{P}_{jt}}{\bar{P}_t})^{-\epsilon_i} \bar{S}_t, \tag{3.5}$$

where the price index for differentiated goods in non-inventory goods class, denoted by $\bar{P}_t$, is defined to be

$$\bar{P}_t = (\int_0^1 (\frac{A_{jt}}{A_t})^{\theta} (\bar{P}_{jt})^{1-\epsilon_i} dj)^{\frac{1}{1-\epsilon_i}}. \tag{3.6}$$

Similarly, demand curves of differentiated goods in the non-inventory goods class is

$$\tilde{S}_{jt} = (\frac{\tilde{P}_{jt}}{\tilde{P}_t})^{-\epsilon_n} \tilde{S}_t, \tag{3.7}$$

where output and price indices of the non-inventory goods class, respectively, are defined as

$$\tilde{S}_t = (\int_0^1 (\tilde{S}_{jt})^{\frac{\epsilon_n - 1}{\epsilon_n}} dj)^{\frac{\epsilon_n}{\epsilon_n - 1}}; \quad \tilde{P}_t = (\int_0^1 (\tilde{P}_{jt})^{1-\epsilon_n} dj)^{\frac{1}{1-\epsilon_n}}, \quad \epsilon_n > 1. \tag{3.8}$$

3.1.2 Cost Function of a Representative Firm

Consider a firm that purchases material inputs and labor services to produce differentiated goods of type j. It produces output using the following production technology:

$$Q_{jt} = \min\{\frac{M_{jt}}{s_M}, \frac{Z_t H_{jt}}{1 - s_M}\}, \tag{3.9}$$

where S_M is the share of material, ν is the elasticity of value-added output with respect to capital, Q_{jt} denotes the output level at period t of firm j, H_{jt} denotes the number of hours hired by the firm, and M_{jt} is the real amount of material inputs[4]. In addition, the fixed coefficient technology specified in (3.9) implies that the value-added production of firm j is

$$Y_{jt} = Z_t H_{jt}. \tag{3.10}$$

The logarithm of the aggregate technology process is also assumed to follow an AR(1) process:

$$z_t = \rho_z z_{t-1} + e_{zt}; \quad 0 \leq \rho_z < 1, \tag{3.11}$$

where z_t (=$\log Z_t$) denotes the logarithm of the aggregate productivity at period t and e_{zt} denotes an i.i.d. white noise with mean zero and standard deviation σ_z.

Furthermore, labor services are free to move across individual firms and between two sectors. Factor prices are also assumed to be fully flexible. Individual firms therefore have an identical cost function for producing one unit of value-added output, as long as they have a constant returns to scale technology for value-added output. In particular, note that constant returns to scale production technologies (3.9) and (3.10), along with full flexibility of factor prices in perfectly competitive factors market, make the unit cost for the value-added output independent of output levels of individual firms. Specifically, letting V_t denote the unit cost for the value-added output, the unit cost for value-added output is given by

$$V_t = \frac{W_t}{Z_t}, \tag{3.12}$$

where W_t denotes the real wage rate at period t.

So far, we have made no distinction between firms holding inventories and firms not holding inventories. From now on, we assume that production technologies for individual firms have an identical functional form, except the share parameter of material inputs denoted by s_M. Specifically, the share parameter s_M is set to be $s_M = \bar{s}_M$ for firms that hold inventories and $s_M = \tilde{s}_M$ for firms that do not hold inventories. Hence, real marginal costs at period t of real gross output can be written as

$$\bar{MC}_t = \bar{s}_M + (1 - \bar{s}_M)V_t; \quad \tilde{MC}_t = \tilde{s}_M + (1 - \tilde{s}_M)V_t, \tag{3.13}$$

where $\bar{MC}_t$ denotes the real marginal cost of firms that hold inventories and $\tilde{MC}_t$ denotes the real marginal cost of firms that do not hold inventories.

[4]See, for example, Rotemberg and Woodford (1995), Bils and Kahn (2000) and Woodford (2005) for the use of the fixed coefficient technology of the type we use in this paper.

3.1.3 Price Setting of Firms Holding Inventories

Having described demand and cost functions of firms, we now discuss pricing decisions of firms that hold inventories. In doing so, we first specify the evolution of the stock available for sales and profit flows of individual firms. The stock of firm j that is available for sales evolves over time according to

$$A_{jt} = (1 - \delta_a)(A_{jt-1} - \bar{S}_{jt-1}) + Q_{jt}, \tag{3.14}$$

where A_{jt} denotes the stock of firm j and δ_a denotes the depreciation rate of stocks, which reflects storage costs. The realized profit flow at period t of firm j holding its inventories is therefore given by

$$\Phi_{jt} = (\frac{A_{jt}}{A_t})^{\theta}(\frac{\bar{P}_{jt}}{\bar{P}_t})^{1-\epsilon}\bar{S}_t - \bar{MC}_t Q_{j,t}, \tag{3.15}$$

where S_t is the aggregate amounts of real sales at period t, and $\bar{S}_t$ is defined as

$$\bar{S}_t = \gamma(\frac{\bar{P}_t}{P_t})^{-\phi} S_t. \tag{3.16}$$

Furthermore, we assume that the price-setting of firms follows a variant of staggered price-setting of Calvo (1983), which allows for indexation. Specifically, during each period, a fraction of firms, $(1 - \alpha_i)$, are allowed to re-optimize, while the other fraction of firms, α_i, do not. In particular, those firms that do not optimize at period t determine their prices at period t by multiplying a common indexation factor to their previous period's prices. Thus, the price at period t of firms that re-optimize at period $t - k$ can be written as

$$\bar{P}_{t-k,t} = \bar{\Upsilon}_{t-k,t}\bar{P}^*_{t-k}, \tag{3.17}$$

where $\bar{P}_{t-k,t}$ is the price at period t of firms that re-optimize at period $t - k$, $\bar{P}^*_{t-k}$ is the price at period $t - k$ of firms that re-optimized at period $t - k$. In addition, $\bar{\Upsilon}_{t-k,t}$ is the indexation factor at period t, which is determined by an indexation rule.[5] Here, firms that hold inventories update their indexation factor by multiplying the previous period's inflation to their previous period's indexation factor:

$$\bar{\Upsilon}_{t-k,t} = (\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}})^{\xi}\bar{\Upsilon}_{t-k,t-1}, \tag{3.18}$$

for $k = 1, \cdots, \infty$ and where $\bar{\Upsilon}_{t-k,t-k} = 1$. The parameter ξ measures the degree of indexation, which takes a constant value between 0 and 1. Then, combining equations

[5] Refer to Christiano, Eichenbaum and Evans (2001), Smets and Wouters (2002), and Woodford (2003) for the modified Calvo-type staggered price-setting, which allows for indexation of prices to past inflation.

(3.6), (3.17), and (3.18), we can see that the price level at period t under the Calvo type staggered price-setting can be written as

$$\bar{P}_t^{1-\epsilon} = (1-\alpha_i)\sum_{k=0}^{\infty}\alpha_i^k(\frac{A_{t-k,t}}{A_t})^{\theta}(\bar{\Upsilon}_{t-k,t}\bar{P}_{t-k}^*)^{1-\epsilon_i}, \tag{3.19}$$

where $A_{t-k,t}$ denotes the stock at period t of finished inventories of firms that set their price at period $t-k$.

Next, we formulate profit maximization problems of firms as dynamic programming problems. Since firms make pricing and investment decisions at the same time, firms that re-optimize at period t may have different value functions, depending on the most recent time period that they re-optimized. In order to allow for such a possibility, let $V^{0,k}(A_{t-k,t-1}, \bar{P}_{t-k}^*, \bar{X}_t)$ denote the value function at period t of firms, which re-optimize their prices at period t and had their previous price changes at period $t-k$. Here, $\bar{X}_t$ denotes the aggregate state vector at period t. Then, firms that re-optimize at period t solve the following profit maximization problem:

$$\left(\begin{array}{c} V^{0,k}(A_{t-k,t-1}, \bar{P}_{t-k}^*, \bar{X}_t) = \max_{A_{t,t}, Q_{t,t}, \bar{P}_t^*}(\frac{A_{t,t}}{A_t})^{\theta}(\frac{\bar{P}_t^*}{\bar{P}_t})^{1-\epsilon_i}\bar{S}_t - \bar{MC}_t Q_{t,t} \\ +\omega_{t,t}((1-\delta_a)(A_{t-k,t-1} - (\frac{A_{t-k,t-1}}{A_{t-1}})^{\theta}(\frac{\bar{\Upsilon}_{t-k,t-1}\bar{P}_{t-k}^*}{\bar{P}_{t-1}})^{-\epsilon_i}\bar{S}_{t-1}) + Q_{t,t} - A_{t,t}) \\ +E_t[d_{t,t+1}(\alpha_i V^1(A_{t,t+1}, \bar{P}_t^*, \bar{X}_{t+1}) + (1-\alpha_i)V^{0,1}(A_{t,t+1}, \bar{P}_t^*, \bar{X}_{t+1}))], \end{array}\right) \tag{3.20}$$

where $\omega_{t,t}$ denotes the Lagrange multiplier for (3.13) and $A_{t-k,t-1}$ is the real amounts of stocks at period $t-1$ of firms that set prices at period $t-k$.

The first order conditions for the dynamic programming problem (3.20) can be summarized as follows. The first-order condition for output is given by

$$\omega_{t,t} = \bar{MC}_t. \tag{3.21}$$

The first-order condition for the stock can be written as

$$\omega_{t,t} = \theta\frac{\bar{S}_{t,t}}{A_{t,t}} + E_t[d_{t,t+1}(\alpha_i V_1^1(A_{t,t+1}, P_t^*, \bar{X}_{t+1}) + (1-\alpha_i)V_1^{0,1}(A_{t,t+1}, P_t^*, \bar{X}_{t+1}))], \tag{3.22}$$

where $\bar{S}_{t,t}$ denotes the real sales at period t of firms that re-optimize at period t. The first-order condition for price-setting is

$$(\epsilon_i - 1)\frac{\bar{S}_{t,t}}{\bar{P}_t} = E_t[d_{t,t+1}(\alpha_i V_2^1(A_{t,t+1}, \bar{P}_t^*, \bar{X}_{t+1}) + (1-\alpha_i)V_2^{0,1}(A_{t,t+1}, \bar{P}_t^*, \bar{X}_{t+1}))]. \tag{3.23}$$

We now move on to envelop conditions. It follows from (3.20) that differentiating $V^{0,k}(A_{t-k,t-1}, \bar{P}_{t-k}^*, \bar{X}_t)$ with respect to $\bar{P}_{t-k}^*$ yields

$$V_2^{0,k}(A_{t-k,t-1}, \bar{P}_{t-k}^*, \bar{X}_t) = \epsilon_i(1-\delta_a)\frac{\bar{MC}_t\bar{S}_{t-k,t-1}}{\bar{P}_{t-k}^*}. \tag{3.24}$$

Besides, the value function at period t of firms that re-optimized at period $t-k$ can be written as follows:

$$\left(\begin{array}{c} V^k(A_{t-k,t-1},\bar{P}^*_{t-k},\bar{X}_t)=\max_{A_{t-k,t},Q_{t-k,t}}(\frac{A_{t-k,t}}{A_t})^{\theta}(\frac{\bar{\Upsilon}_{t-k,t}\bar{P}^*_{t-k}}{\bar{P}_t})^{1-\epsilon_i}\bar{S}_t-\bar{MC}_tQ_{t-k,t} \\ +\omega_{t-k,t}((1-\delta_a)(A_{t-k,t-1}-(\frac{A_{t-k,t-1}}{A_{t-1}})^{\theta}(\frac{\bar{\Upsilon}_{t-k,t-1}\bar{P}^*_{t-k}}{\bar{P}_{t-1}})^{-\epsilon_i}\bar{S}_{t-1})+Q_{t-k,t}-A_{t-k,t}) \\ +E_t[d_{t,t+1}(\alpha_iV^{k+1}(A_{t-k,t},\bar{P}^*_{t-k},\bar{X}_{t+1})+(1-\alpha_i)V^{0,k}(A_{t-k,t},\bar{P}^*_{t-k},\bar{X}_{t+1}))], \end{array}\right) \tag{3.25}$$

where $V^k(A_{t-k,t-1},\bar{P}^*_{t-k},\bar{X}_t)$ is the value function at period t of firms that re-optimized at period $t-k$. Differentiating (3.25) with respect to $\bar{P}^*_{t-k}$ and then setting $\omega_{t-k,t}=\bar{MC}_t$, we also have the following envelop condition;

$$\begin{array}{l} V^k_2(A_{t-k,t-1},\bar{P}^*_{t-k},\bar{X}_t)=\epsilon_i(1-\delta_a)\frac{\bar{MC}_t\bar{S}_{t-k,t-1}}{\bar{P}^*_{t-k}}-(\epsilon_i-1)\frac{\bar{S}_{t-k,t}}{\bar{P}_t}+ \\ E_t[d_{t,t+1}(\alpha_iV^{k+1}_2(A_{t-k,t},\bar{P}^*_{t-k},\bar{X}_{t+1})+(1-\alpha_i)V^{0,k}_2(A_{t-k,t},\bar{P}^*_{t-k},\bar{X}_{t+1}))]. \end{array} \tag{3.26}$$

Having derived the first-order and envelope conditions for price-setting described above, we will combine them to yield the optimization condition for the price-setting at period t of firms that hold finished goods inventories. First, substituting equation (3.24) evaluated at $t+1$ into (3.26) and then rearranging, we can obtain a difference equation for partial derivatives of value functions with respect to the price re-optimized at period $t-k$:

$$\begin{array}{l} V^k_2(A_{t-k,t-1},\bar{P}^*_{t-k},\bar{X}_t)=-\frac{\bar{S}_{t-k,t}\Gamma_{t-k}(t)}{\bar{P}^*_{t-k}}+\frac{\epsilon_i((1-\alpha_i)MS_t\bar{S}_{t-k,t}+(1-\delta_a)\bar{MC}_t\bar{S}_{t-k,t-1})}{\bar{P}^*_{t-k}}+ \\ \alpha_iE_t[d_{t,t+1}V^{k+1}_2(A_{t-k,t},\bar{P}^*_{t-k},\bar{X}_{t+1})], \end{array} \tag{3.27}$$

where $\Gamma_{t-k}(t)$ is defined as

$$\Gamma_{t-k}(t)=(\epsilon_i-1)\frac{\bar{\Upsilon}_{t-k,t}\bar{P}^*_{t-k}}{\bar{P}_t}. \tag{3.28}$$

Then, a successive forward iteration of equation (3.27) leads to

$$\begin{array}{c} V^k_2(A_{t-k,t-1},\bar{P}^*_{t-k},\bar{X}_t)= \\ -E_t[\sum_{j=0}^{\infty}\alpha_i^jd_{t,t+j}\frac{\bar{S}_{t-k,t+j}}{\bar{P}^*_{t-k}}(\Gamma_{t-k}(t+j)-\epsilon_iMS_{t+j})]+\frac{\epsilon_i(1-\delta_a)\bar{MC}_t\bar{S}_{t-k,t-1}}{\bar{P}^*_{t-k}}, \end{array} \tag{3.29}$$

where $\lim_{t\to T}E_t[\alpha_i^Td_{t,t+T}V^T_2(A_{t-k,t+T},\bar{P}^*_{t-k},\bar{X}_{t+T})]=0$. It then follows from (3.29) that the partial derivative of the value function at period $t+1$ with respect to the price re-optimized at period t can be written as

$$\begin{array}{c} V^1_2(A_{t,t+1},\bar{P}^*_t,\bar{X}_{t+1})= \\ -E_{t+1}[\sum_{j=1}^{\infty}\alpha_i^jd_{t+1,t+j}\frac{\bar{S}_{t,t+j}}{\bar{P}^*_t}(\Gamma_t(t+j)-\epsilon_iMS_{t+j})]+\frac{\epsilon_i(1-\delta_a)\bar{MC}_{t+1}\bar{S}_{t,t}}{\bar{P}^*_t}. \end{array} \tag{3.30}$$

Furthermore, substituting (3.24) into (3.23) and then rearranging, the first-order condition for the price-setting at period t described in (3.23) can be rewritten as

$$\alpha_iE_t[d_{t,t+1}V^1_2(A_{t,t+1},\bar{P}^*_t,\bar{X}_{t+1})]=\frac{\bar{S}_{t,t}}{\bar{P}^*_t}(\Gamma_t(t)-\epsilon_i(1-\alpha_i)MS_t). \tag{3.31}$$

As a result, substituting (3.30) into (3.31) and then rearranging, one can see that the optimization condition for the price-setting at period t can be written as

$$\sum_{k=0}^{\infty} \alpha_i^k E_t[d_{t,t+k}\bar{S}_{t,t+k}(\frac{\bar{\Upsilon}_{t,t+k}\bar{P}_t^*}{\bar{P}_{t+k}} - \frac{\epsilon_i}{\epsilon_i - 1}MS_{t+k})] = 0, \tag{3.32}$$

where MS_{t+k} is the expected present-value of the next period's real marginal cost:

$$MS_{t+k} = (1-\delta_a)E_{t+k}[d_{t+k,t+k+1}\bar{MC}_{t+k+1}]. \tag{3.33}$$

It is noteworthy that MS_{t+k} is included in the profit maximization condition (3.32), which reflects that holding finished goods inventories leads firms to take into account the expected present value of the next period's marginal cost, rather than the current period's marginal cost.

We now discuss the optimization condition for the stock available for sales at period t. It follows from the two value functions described above that the partial derivatives of value functions with respect to the previous period's stocks can be written as

$$V_1^k(A_{t-k,t-1}, P_{t-k}^*, \bar{X}_t) = \omega_{t-k,t}(1-\delta_a)(1-\theta\frac{S_{t-k,t-1}}{A_{t-k,t-1}}), \tag{3.34}$$

$$V_1^{0,k}(A_{t-k,t-1}, \bar{P}_{t-k}^*, \bar{X}_t) = \omega_{t,t}(1-\delta_a)(1-\theta\frac{S_{t-k,t-1}}{A_{t-k,t-1}}), \tag{3.35}$$

for $k = 1, 2, \cdots, \infty$. Next, substituting period $(t+1)$ versions of equations (3.34) and (3.35), with $k = 1$, into (3.22) and then setting $\omega_{t,t+1} = \omega_{t+1,t+1} = \bar{MC}_{t+1}$ in the resulting equation, we find that the optimization condition for the stock of firms that re-optimize at period t can be written as

$$\bar{MC}_t = \theta\frac{\bar{S}_{t,t}}{A_{t,t}} + MS_t(1-\theta\frac{\bar{S}_{t,t}}{A_{t,t}}). \tag{3.36}$$

Similarly, optimization conditions for stocks of firms that re-optimized at period $t-k$ are

$$\bar{MC}_t = \theta\frac{\bar{S}_{t-k,t}}{A_{t-k,t}} + MS_t(1-\theta\frac{\bar{S}_{t-k,t}}{A_{t-k,t}}), \tag{3.37}$$

for $k = 1, 2, \cdots, \infty$. To the extent that the real marginal cost is independent of output levels of individual firms, one can see from (3.36) and (3.37) that sales-stocks ratios are identical across individual firms in each period $t = 0, 1, \cdots, \infty$. Hence, the following equation holds for the aggregate sales-stock ratio:

$$\bar{MC}_t = \theta\frac{\bar{S}_t}{A_t} + MS_t(1-\theta\frac{\bar{S}_t}{A_t}). \tag{3.38}$$

3.1.4 Price Setting of Non-Inventory Goods Producing Firms

Having described the price-setting of firms that hold inventories, we will discuss the profit maximization of individual firms that do not hold their finished goods inventories. As we did in the previous section, this section assumes a variant of the staggered price setting of Calvo(1983), which allows for indexation. Specifically, during each period, a fraction of firms, $(1-\alpha_n)$, are allowed to re-optimize, while the other fraction of firms, α_n, do not. In the absence of inventories, the price level at period t under the Calvo-type staggered price-setting can be written as

$$\tilde{P}_t^{1-\epsilon_n} = (1-\alpha_n)(\tilde{P}_t^*)^{1-\epsilon_n} + \alpha_n(\tilde{\Upsilon}_{t-1,t}\tilde{P}_{t-1})^{1-\epsilon_n}, \tag{3.39}$$

where $\tilde{P}_t^*$ denotes the optimal price at period t of firms resetting prices at period t in the non-inventory goods sector and the indexation factor $\tilde{\Upsilon}_{t-1,t}$ is defined as $\tilde{\Upsilon}_{t-1,t} = (\frac{\tilde{P}_{t-1}}{\tilde{P}_{t-2}})^{\xi}$.

Furthermore, the profit maximization problem of firms resetting prices at period t is given by

$$\sum_{k=0}^{\infty} \alpha_n^k E_t[d_{t,t+k}((\frac{\tilde{\Upsilon}_{t,t+k}\tilde{P}_t^*}{P_{t+k}})^{1-\epsilon_n} - \tilde{MC}_{t+k}(\frac{\tilde{\Upsilon}_{t,t+k}\tilde{P}_t^*}{P_t})^{-\epsilon_n})\tilde{S}_{t+k}], \tag{3.40}$$

where $\tilde{S}_t$ denotes the aggregate sales at period t of the non-inventory goods sector. The optimization condition for the optimal price at period t can be then written as

$$\sum_{k=0}^{\infty} \alpha_n^k E_t[d_{t,t+k}\tilde{S}_{t,t+k}(\frac{\tilde{\Upsilon}_{t,t+k}\tilde{P}_t^*}{\tilde{P}_{t+k}} - \frac{\epsilon_n}{\epsilon_n - 1}\tilde{MC}_{t+k})] = 0. \tag{3.41}$$

3.2 Phillips Curve Equation

In this section, we consider a Phillips curve equation for the aggregate inflation rate on the basis of log-linear approximations to the pricing equations of firms.[6] In doing so, we first log-linearize (3.19) around the steady state with constant prices to yield

$$\bar{p}_t = (1-\alpha_i)\sum_{k=0}^{\infty} \alpha_i^k(\bar{p}_{t-k}^* + \xi(\bar{p}_{t-1} - \bar{p}_{t-k-1}) + (a_{t-k,t} - a_t)), \tag{3.42}$$

where $\bar{p}_t$ and $\bar{p}_{t-k}^*$ are log deviations of $\bar{P}_t$ and $\bar{P}_{t-k}^*$ from their steady state levels, respectively. The second-term of the right-hand side of (3.42) results from log-linearizing the indexation factor (3.18). Besides, note that the sum of individual stocks leads to the aggregate stock:

$$a_t = (1-\alpha_i)\sum_{k=0}^{\infty} \alpha_i^k a_{t-k,t}. \tag{3.43}$$

[6] Refer to King, Plosser and Rebelo (1988a, 1988b) for the log-linear approximation technique used in this paper.

Hence, substituting (3.43) into (3.42) and then subtracting the resulting equation's period $(t-1)$ version from its period t version, we have

$$\bar{p}_t^* - \bar{p}_t = \frac{\alpha_i}{1-\alpha_i}(\bar{\pi}_t - \xi\bar{\pi}_{t-1}), \tag{3.44}$$

where $\bar{\pi}_t$ (= $\log \bar{P}_t$ - $\log \bar{P}_{t-1}$) denotes the inflation rate of the inventory goods sector. In addition, log-linearizing (3.32) around the steady state with constant prices leads to

$$\sum_{k=0}^{\infty}(\alpha_i\beta)^k E_t[\bar{p}_t^* - \bar{p}_{t+k} + \xi(\bar{p}_{t+k-1} - \bar{p}_{t-1}) - \bar{m}s_{t+k}] = 0, \tag{3.45}$$

It then follows from (3.45) that we can obtain a linear difference equation of the form:

$$\bar{p}_t^* - \bar{p}_t = -\alpha_i\beta\xi\bar{\pi}_t + (1-\alpha_i\beta)ms_t + \alpha_i\beta E_t[\bar{\pi}_{t+1} + (\bar{p}_{t+1}^* - \bar{p}_{t+1})], \tag{3.46}$$

where ms_t denotes the log-deviation of MS_t from its steady state value. Thus, substituting (3.44) into (3.46), one can obtain a linear difference equation for the inflation rate of the inventory goods sector of the form:

$$\bar{\pi}_t - \xi\bar{\pi}_{t-1} = \bar{\kappa}ms_t + \beta E_t[\bar{\pi}_{t+1} - \xi\bar{\pi}_t], \tag{3.47}$$

where $\bar{\kappa} = \frac{(1-\alpha_i)(1-\alpha_i\beta)}{\alpha_i}$.

Next, we turn to the derivation of the Phillips curve for the inflation rate for the non-inventory goods. In the similar way as we did above, log-linearizing (3.39) and (3.41) and then rearranging leads to a linear difference equation for the inflation rate of the non-inventory goods sector:

$$\tilde{\pi}_t - \xi\tilde{\pi}_{t-1} = \tilde{\kappa}\tilde{m}c_t + \beta E_t[\tilde{\pi}_{t+1} - \xi\tilde{\pi}_t], \tag{3.48}$$

where $\tilde{\pi}_t$ (= $\log \tilde{P}_t$ - $\log \tilde{P}_{t-1}$) is the inflation rate of the non-inventory goods sector, and $\tilde{\kappa} = \frac{(1-\alpha_n)(1-\alpha_n\beta)}{\alpha_n}$.

In order to obtain a Phillips curve equation for the aggregate inflation rate, we add up two linear difference equations (3.47) and (3.48) so that the Phillips curve can be written as

$$\pi_t = \frac{\xi}{1+\xi\beta}\pi_{t-1} + \frac{\gamma\bar{\kappa}}{1+\xi\beta}ms_t + \frac{(1-\gamma)\tilde{\kappa}}{1+\xi\beta}\tilde{m}c_t + \frac{\beta}{1+\xi\beta}E_t[\pi_{t+1}], \tag{3.49}$$

where the aggregate inflation rate, denoted by π_t, is defined as a weighted average of inflation rates of inventory and non-inventory goods:

$$\pi_t = \gamma\bar{\pi}_t + (1-\gamma)\tilde{\pi}_t. \tag{3.50}$$

Before going further, recall that real marginal costs of gross outputs can be expressed in terms of real unit costs of value-added outputs. In order to express the aggregate Phillips cure equation in terms of the unit cost of the aggregate value-added output, we log-linearize real marginal costs of firms. Specifically, log-linearizing (3.13) around the steady state with constant prices then implies that log-deviations of the real marginal cost of gross output can be expressed in terms of the real unit cost of the value-added output:

$$\bar{m}c_t = (1 - \bar{\mu}\bar{s}_M)v_t; \quad \tilde{m}c_t = (1 - \tilde{\mu}\tilde{s}_M)v_t, \tag{3.51}$$

where $\bar{m}c_t$ (= $\log \bar{MC}_t$ - $\log \bar{MC}$), $\tilde{m}c_t$ (= $\log \tilde{MC}_t$ - $\log \tilde{MC}$), and v_t = $\log V_t$ - $\log V$. In addition, $\bar{\mu}$ (= $\frac{1}{\bar{MC}}$) and $\tilde{\mu}$ (= $\frac{1}{\tilde{MC}}$) denote steady state markups for inventory goods and non-inventory goods firms. It also follows from (3.33) that the log-deviation of the expected present-value of the next period's real marginal cost is

$$ms_t = E_t[\lambda_{t+1} - \lambda_t + \bar{m}c_{t+1}], \tag{3.52}$$

where λ_t (= $\log \Lambda_t$ - $\log \Lambda$) is the log-deviation of the marginal utility of consumption. Substituting (3.52) into (3.48) and then (3.51) into the resulting equation, we now express the Phillips curve equation in terms of real unit cost of the aggregate value-added output:

$$\pi_t = \frac{\xi}{1+\xi\beta}\pi_{t-1} + \kappa_0 E_t[v_{t+1}] + \kappa_1 E_t[\lambda_{t+1} - \lambda_t] + \kappa_2 v_t + \frac{\beta}{1+\xi\beta}E_t[\pi_{t+1}], \tag{3.53}$$

where v_t denotes the log-deviation of the real unit cost of the aggregate value-added output. Here, coefficients κ_0, κ_1, and κ_2 are defined as

$$\kappa_0 = \frac{\gamma\bar{\kappa}(1 - \bar{\mu}\bar{s}_M)}{1+\xi\beta}; \ \kappa_1 = \frac{\gamma\bar{\kappa}}{1+\xi\beta}; \ \ \kappa_2 = \frac{(1-\gamma)\tilde{\kappa}(1 - \tilde{\mu}\tilde{s}_M)}{1+\xi\beta},$$

where the parameter γ denotes the output share of the inventory goods sector. Similarly, the Phillips curve for the non-inventory goods can be written as

$$\tilde{\pi}_t = \frac{\xi}{1+\xi\beta}\tilde{\pi}_{t-1} + \tilde{\kappa}_0 v_t + \frac{\beta}{1+\xi\beta}E_t[\tilde{\pi}_{t+1}], \tag{3.54}$$

where $\tilde{k}_0$ is defined as $\tilde{k}_0 = \frac{\tilde{\kappa}(1-\tilde{\mu}\tilde{s}_M)}{1+\xi\beta}$. In sum, we can find that a joint decision on price-setting and inventory holding has the current period's inflation rate depend on the expected present value of the next period's real unit cost, given the expected rate of the next period's inflation.[7]

[7]Refer to Gali and Gertler (1999) and Sbordone (2002) for the specification of the New Keyensian Phillips curve and its empirical tests.

3.3 Inventory Dynamics

In order to derive a linearized law of motion for the inventory stock, we first consider log-linear approximations to the optimization condition for the stock-sales ratio. Specifically, log-linearizing (3.38) leads to

$$\bar{s}_t - a_t = \frac{1}{1-(1-\delta_a)\beta}\bar{mc}_t - \frac{(1-\delta_a)\beta(\bar{\mu}-1)}{(1-(1-\delta_a)\beta)(\bar{\mu}-(1-\delta_a)\beta)}ms_t, \tag{3.55}$$

where $\bar{s}_t$ and a_t are logarithmic deviations of the aggregate sales $\bar{S}_t$ and the aggregate stock A_t from their steady state values, respectively. In addition, $\bar{mc}_t$ (= $\log \bar{MC}_t$ - $\log \bar{MC}$) denotes the log deviation of the real marginal cost at period t, while ms_t (= $\log MS_t$ - $\log MS$) denotes the log-deviation of MS_t from its steady state value.[8] Then, substituting (3.52) into (3.55) and then plugging (3.51) into the resulting equation, one can express the ratio of sales to stocks in terms of the aggregate real unit cost and changes in the marginal utility of consumption:

$$\bar{s}_t - a_t = b_0 v_t - b_1 E_t[v_{t+1}] - b_2 E_t[\lambda_{t+1} - \lambda_t], \tag{3.56}$$

where b_0, b_1, and b_2 are defined as

$$b_0 = \frac{1-\bar{\mu}\bar{s}_M}{1-(1-\delta_a)\beta}; b_1 = \frac{(1-\delta_a)\beta(\bar{\mu}-1)}{(\bar{\mu}-(1-\delta_a)\beta)}b_0; b_2 = \frac{(1-\delta_a)\beta(\bar{\mu}-1)}{(\bar{\mu}-(1-\delta_a)\beta)(1-(1-\delta_a)\beta)}.$$

Furthermore, note that $\bar{Y}_t = \gamma(\frac{\bar{P}_t}{P_t})^{-\phi}Y_t$. Equation (3.3) also implies that $\gamma(\frac{\bar{P}_t}{P_t})^{1-\phi} = \gamma + (1-\gamma)P_{rt}^{-(1-\phi)}$, where P_{rt} $(=\frac{\bar{P}_t}{P_t})$ is the ratio between price levels of inventory and non-inventory goods sectors. Thus, we have $\bar{Y}_t = \gamma[\gamma + (1-\gamma)P_{rt}^{-(1-\phi)}]^{\frac{\phi}{1-\phi}}Y_t$. As a result, log-linearizing this equation leads one to express the aggregate value-added output of the inventory goods sector in terms of the aggregate value-added output and relative price:

$$\bar{y}_t = y_t - (1-\gamma)\phi p_{rt}. \tag{3.57}$$

We now turn to the discussion of how to obtain a law of motion for the aggregate inventory stock. The aggregate inventory stock measured at the end of period t, denoted by L_t, is defined as $L_t = A_t$ - $\bar{S}_t$. It also follows from (3.9) and (3.14) that the aggregate stock available for sales is $A_t = (1-\delta_a)L_{t-1} + \frac{1}{1-s_M}\bar{Y}_t$. Thus, combining these two equations, we have the following equation:

$$L_t = ((1-\delta_a)L_{t-1} + \frac{1}{1-s_M}\bar{Y}_t)(1 - \frac{\bar{S}_t}{A_t}). \tag{3.58}$$

[8] It follows from (3.24) and (3.27) that the steady state equilibrium condition can be written as $\bar{MC}$ - $MS = \theta\frac{\bar{S}}{A}(1-MS)$. Since $MS = (1-\delta_a)\beta\bar{MC}$, it implies that $\theta\frac{\bar{S}}{A} = \frac{1-(1-\delta_a)\beta}{\bar{\mu}-(1-\delta_a)\beta}$. This steady state relation is used to calculate the coefficient for ms_t in (3.55).

Log-linearizing this equation and then substituting (3.57) into the resulting equation leads to a linear difference equation for the inventory stock of the form:

$$l_t = \rho l_{t-1} + \frac{1}{(1-\bar{s}_M)s_a} y_t - \frac{(1-\gamma)\phi}{(1-\bar{s}_M)s_a} p_{rt} - (\frac{s_s}{s_l})(\bar{s}_t - a_t), \tag{3.59}$$

where $s_a = \frac{A}{Y}$, $s_l = \frac{L}{Y}$, $\rho = (1-\delta_a)\frac{s_l}{s_a}$, $\bar{s}_t = \log \frac{\bar{S}_t}{\bar{S}}$, $l_t = \log \frac{L_t}{L}$, and $a_t = \log \frac{A_t}{A}$. It is then clear from (3.59) that it is necessary to have a law of motion for p_{rt}. Hence, substituting $P_{rt} = \frac{\bar{P}_t}{\bar{P}_t}$ into the definition of P_t specified in (3.3), we have

$$p_{rt} = p_{rt-1} + \frac{1}{\gamma}(\pi_t - \tilde{\pi}_t). \tag{3.60}$$

3.4 Households

The preference at period t of the representative household is represented by

$$E_t \sum_{k=0}^{\infty} \beta^k \, [\frac{(C_{t+k} - bC_{t+k-1})^{1-\sigma} - 1}{1-\sigma} - \frac{H_{t+k}^{1+\chi}}{1+\chi}], \quad \sigma > 0, \quad \chi > 0, \tag{3.61}$$

where $0 < \beta < 1$ denotes the discount factor, C_t is an index of consumption goods and H_t is the number of hours worked at period t. We assume that the parameter b takes a positive value, in order to allow for habit formation in consumption preferences. The flow budget constraint at period t of the representative household can be therefore written as

$$C_t + E_t[d_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] = \frac{B_t}{P_t} + W_t H_t + \Phi_t - T_t, \tag{3.62}$$

where B_{t+1} denotes a portfolio of nominal state contingent claims in the complete contingent claims market, $d_{t,t+1}$ is the stochastic discount factor for computing the real value at period t of one unit of consumption goods at period $t+1$, W_t is the real wage rate, T_t is the real lump-sum tax, and Φ_t is the real dividend income. Then, the first-order conditions for consumption and labor supply can be written as

$$\Lambda_t = E_t[(C_t - bC_{t-1})^{-\sigma} - \beta b(C_{t+1} - bC_t)^{-\sigma}], \tag{3.63}$$

$$H_t^{\chi} = W_t \Lambda_t, \tag{3.64}$$

where Λ_t is the Lagrange multiplier of the budget constraint (3.62). The optimization condition for bonds holdings is

$$d_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}. \tag{3.65}$$

Hence, if R_t represents the risk-free (gross) nominal rate of interest at period t, the absence of arbitrage at an equilibrium gives the following Euler equation:

$$\beta E_t[R_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}}] = 1. \tag{3.66}$$

Having described the optimization conditions of the household's utility maximization, their log-linear approximations are discussed. Log-linearizing the Euler equation around the steady state with a zero inflation rate leads to

$$E_t[\lambda_{t+1} - \lambda_t + r_t - \pi_{t+1}] = 0, \tag{3.67}$$

where λ_t (= $\log \Lambda_t$ - $\log \Lambda$) denotes the log-deviation of the marginal utility of consumption at period t from its steady state level, r_t is the log deviation of the nominal interest rate from its steady state level, and π_t denotes the inflation rate at period t. The marginal utility equation under the habit formation of consumption specified in (3.63) gives an intertemporal equation:

$$c_t = -\frac{(1-b)(1-\beta b)}{\sigma(1+\beta b)}\lambda_t + \frac{b}{1+\beta b^2}c_{t-1} + \frac{\beta b}{1+\beta b^2}E_t[c_{t+1}], \tag{3.68}$$

where c_t (= $\log C_t$ - $\log C$) denotes the logarithmic deviation of consumption from its steady state value.

3.5 Social Resource Constraint

In order to obtain the aggregate market clearing condition, we begin with the aggregate dividend income, which is defined as the aggregate sales minus the aggregate costs of both production and holding inventories. Specifically, the aggregate dividend income is defined as Φ_t = S_t - $W_t H_t$ - M_t. In addition, the aggregate inventories at the end of period t, denoted by L_t, can be written as $L_t = (1-\delta_a)L_{t-1} + Q_t$ - S_t, where Q_t is the aggregate real gross output at period t. Substituting this equation into the definition of the aggregate dividend and then setting Y_t = Q_t - M_t in the resulting equation, we find that the aggregate dividend income can be rewritten as

$$\Phi_t = -(L_t - (1-\delta_a)L_{t-1}) + Y_t - W_t H_t, \tag{3.69}$$

In the meanwhile, the government's flow budget constraint at period t is defined as

$$E_t[d_{t,t+1}\frac{B_{t+1}}{P_{t+1}}] = \frac{B_t}{P_t} + G_t - T_t. \tag{3.70}$$

Then, substituting (3.69) and (3.70) into the period budget constraint of the representative household (3.62), one can see that the aggregate market clearing condition can be written as

$$Y_t = C_t + G_t + L_t - (1-\delta_a)L_{t-1}. \tag{3.71}$$

Having described the aggregate market clearing condition, we now discuss the relationship between the aggregate value-added output and the aggregate production inputs. First, note that demand curves for value-added outputs of individual firms are $\bar{Y}_{jt} = (\frac{A_{jt}}{A_t})^{\theta}(\frac{\bar{P}_{jt}}{\bar{P}_t})^{-\epsilon}\bar{Y}_t$ and $\tilde{Y}_{jt} = (\frac{\tilde{P}_{jt}}{\tilde{P}_t})^{-\epsilon}\tilde{Y}_t$, where $\bar{Y}_t = \gamma(\frac{\bar{P}_t}{P_t})^{-\phi}Y_t$ and $\tilde{Y}_t = (1-\gamma)(\frac{\tilde{P}_t}{P_t})^{-\phi}Y_t$[9]. In addition, the demand for the value-added output of firm j should be equal to its output in an equilibrium. Hence, adding up market clearing conditions across firms yield $\frac{Z_t}{\bar{\Delta}_t}\bar{H}_t = \gamma(\frac{\bar{P}_t}{P_t})^{-\phi}Y_t$ and $\frac{Z_t}{\tilde{\Delta}_t}\tilde{H}_t = \gamma(\frac{\tilde{P}_t}{P_t})^{-\phi}Y_t$. Here, $\bar{\Delta}_t$ and $\tilde{\Delta}_t$ denote measures of relative price distortion for inventory goods and non-inventory goods respectively:

$$\bar{\Delta}_t = \int_0^1 (\frac{A_{jt}}{A_t})^{\theta}(\frac{\bar{P}_{jt}}{\bar{P}_t})^{-\epsilon}dj; \quad \tilde{\Delta}_t = \int_0^1 (\frac{\tilde{P}_{jt}}{\tilde{P}_t})^{-\epsilon}dj. \tag{3.72}$$

Thus, market clearing conditions for each class of goods can be added to yield the following aggregate production function:

$$Y_t = \frac{Z_t}{\Delta_t}H_t, \tag{3.73}$$

where $H_t = \tilde{H}_t + \bar{H}_t$ and the aggregate relative price distortion is defined as

$$\Delta_t = \gamma\bar{\Delta}_t(\frac{\bar{P}_t}{P_t})^{-\phi} + (1-\gamma)\tilde{\Delta}_t(\frac{\tilde{P}_t}{P_t})^{-\phi}. \tag{3.74}$$

We now log-linearize the aggregate production function to express the aggregate real unit cost in terms of value-added output. Here, it should be noted that up to its first-order log-linear approximation, measures of relative price distortions turn out to be zero, following the literature.[10] Thus, log-linearizing the aggregate production function yields $y_t = z_t + h_t$, when h_t and y_t denote the log-deviation of the aggregate hours and real GDP from their steady state values, respectively. In addition, substituting (3.12) into (3.64) and then log-linearizing the resulting equation leads to $\chi h_t = v_t + z_t + \lambda_t$. Hence, combining these two equations, the aggregate unit cost can be expressed in terms of the aggregate value-added output as follows:

$$v_t = \chi y_t - (1+\chi)z_t - \lambda_t. \tag{3.75}$$

[9]The amount of demand for the value-added output of each firm differs from its sales. But $\bar{M}_{jt}$ is proportional to $\bar{Y}_{jt}$ because of the fixed coefficient technology in (3.9). Thus, the demand function of each firm in inventory goods sector has the same functional form as that of its total sales.

[10]Refer to Christiano, Eichenbaum and Evans (2001) and Schmitt-Grohé and Uribe (2004) for detailed explanations about log-linear approximation to measures of relative price distortion under the Calvo pricing.

Besides, log-linearizing the aggregate market clearing condition (3.71) leads to

$$c_t = \frac{1}{s_c} y_t - \frac{s_g}{s_c} g_t - \frac{\delta_a s_l}{s_c}(l_t - (1-\delta_a) l_{t-1}), \tag{3.76}$$

where s_c is the share of consumption in real GDP, s_g is the share of government expenditures in real GDP, and s_l is the steady state ratio of inventory to real output.

Finally, the monetary policy rule is assumed to follow a variant of Taylor (1993) rule with partial adjustment of the form:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_\pi \pi_t + \phi_y y_t) + e_{rt}, \tag{3.77}$$

where ρ_r is the partial adjustment parameter, ϕ_π measures the responsiveness of the policy interest rate with respect to inflation rate, and ϕ_y measures the responsiveness of the policy interest rate with respect to real output.

4 Simulation Results

4.1 Calibration and Estimation

The computation of a numerical solution to the model requires assigning numbers to parameters of the model. Specifically, parameters of the model are partitioned into three classes. For the first class of parameters, we simply choose their values. For example, we set $\sigma = 1$ and $\chi = 1$, which imply a logarithmic utility function for consumption and a quadratic function for hours worked. We also set $\beta = 0.99$, which assumes that the average yearly real interest rate is 4 %. The other parameter values that belong to the first class are specified in Table 1.

The parameter values of the second set are chosen to match the impulse responses of the key variables estimated from the VAR we discussed in the previous section. In order to see this, let ζ be the vector of the model parameters that we estimate, while $\Psi(\zeta)$ is a mapping from ζ to the model impulse responses. $\hat{\Psi}$ is the corresponding impulse responses, which are estimated from the VAR we have discussed. Then, estimates of ζ, denoted by $\hat{\zeta}$ are the solution to the following minimization problem:

$$\min_{\zeta} (\hat{\Psi} - \Psi(\zeta))' V^{-1} (\hat{\Psi} - \Psi(\zeta)),$$

where V is a diagonal matrix whose diagonal elements are sample variances of the elements of $\hat{\Psi}$[11]. In sum, the first and second sets contain σ, χ, b, β, α_i, α_n, ξ, θ, $\bar{s}_M$, δ_a, s_r $(= \frac{\bar{S}}{A})$,

[11] Refer to Christiano, Eichenbaum and Evans (2001) and Altig, Christiano and Eichenbaum (2004) for a detailed discussion about how to estimate a subset of parameter values minimizing a measure of the gap between model and estimated impulse responses.

$\tilde{\mu}$, ρ_r, ϕ_π, and ϕ_y, whose values are summarized in Table 1.

The third set of parameter values are then determined by using steady state equilibrium conditions, given the first two sets of parameter values. Hence, we now briefly discuss how one can use steady state equilibrium conditions to assign numbers to the second set of parameters. The first-order condition for the stock becomes $\theta s_r = \frac{1-\beta}{\bar{\mu}-(1-\delta_a)\beta}$ at the steady state with constant prices, which in turn implies

$$\bar{\mu} = (1-\delta_a)\beta(1-\frac{1}{\theta s_r}) + \frac{1}{\theta s_r};\ \epsilon_i = \frac{\bar{\mu}}{\bar{\mu}-(1-\delta_a)\beta}, \tag{4.1}$$

where the second equation is obtained from the steady state version of the pricing equation for firms that hold inventories, $(1-\delta_a)\beta\epsilon_i = (\epsilon_i - 1)\bar{\mu}$. Besides, note that the real unit cost of the value-added output, denoted by V, satisfies the following steady state relations:

$$V = \frac{1-\bar{\mu}\bar{s}_M}{\bar{\mu}(1-\bar{s}_M)} = \frac{1-\tilde{\mu}\tilde{s}_M}{\tilde{\mu}(1-\tilde{s}_M)}, \tag{4.2}$$

where $\tilde{\mu}$ is the steady state markup for firms that do not hold inventories.[12] The first equality of this equation is then used to compute a value of V. We also use the second equality to compute the share of material inputs for firm that do not holding inventory goods:

$$\tilde{s}_M = \frac{1-V\tilde{\mu}}{(1-V)\tilde{\mu}}. \tag{4.3}$$

Furthermore, the law of motion for the aggregate stock at the steady state with constant prices turns out to be $\frac{1}{1-\bar{s}_M} = \delta_a s_a + (1-\delta_a)s_s$. Given that $s_s = s_r\ s_a$, we solve this equation to yield

$$s_s = \frac{s_r}{(1-\bar{s}_M)(\delta_a + (1-\delta_a)s_r)};\ s_a = \frac{1}{(1-\bar{s}_M)(\delta_a + (1-\delta_a)s_r)}. \tag{4.4}$$

The steady state ratio of inventory to value-added output therefore can be written as

$$s_l = \frac{L}{Y} = s_a - s_s. \tag{4.5}$$

The steady state share of consumption in output, denoted by s_c, is given by

$$s_c = 1 - s_g - \delta_a s_l. \tag{4.6}$$

In sum, we can use equations (4.1) - (4.6) to compute values of V, $\bar{\mu}$, ϵ_i, $\tilde{s}_M$, s_a, s_s, s_l, and s_c given the first set of parameter values, while the first set of parameter values is reported in Table 1.

[12]Note that the second equality results from the assumption of labor's free movement across sectors. The next equation then implies that $0 \leq \tilde{s}_M < 1$ requires $1 < \tilde{\mu} \leq \frac{1}{V}$. We therefore use this restriction when we choose a value $\tilde{\mu}$.

4.2 Results

In this section, we report quantitative implications of the model we described.[13] In particular, we ask if the model can generate observed dynamic responses of sales-stock ratio and finished goods inventories in response to a monetary policy shock. Furthermore, we do this for models with and without adjustment costs. Here, adjustment costs take place when sales-stock ratio deviates from its fixed target, while the equilibrium conditions for the model with adjustment costs are summarized in Appendix A.

Figure 2 demonstrates impulse responses to an expansionary monetary policy shock from the model without adjustment costs and compare them with estimated impulse responses from the VAR discussed in section 2. Figure 2 indicates that one needs a high level of depreciation in order to match the estimated impulse responses when there are no adjustment costs. It also shows that an exogenous fall in the interest rate increases real output, the inflation rate and the ratio of sales to stocks but decreases the inventory stock of finished goods, which is consistent with the observed impulse responses from the VAR.

Figures 3 and 4 report impulse responses of models with adjustment costs, responding to an expansionary monetary policy shock[14]. Figures 3 and 4 are constructed under the constraint that the depreciations rate of inventories is small. Specifically, we restrict the depreciation rate in an interval between 0 and 0.01 for Figure 2 and set the depreciation rate equal to zero for Figure 4. Figures 3, in particular, does not show hump-shaped

[13] In order to obtain a numerical solution, we define a 10×1 column vector $k_t = [\ \pi_t,\ y_t,\ \bar{s}_t - a_t,\ l_t,\ r_t,\ c_t,\ v_t,\ \tilde{\pi}_t,\ \lambda_t,\ p_{rt}\ ]'$. we then use 10 equilibrium conditions to yield $\tilde{E}_t\ [\ m_0 k_{t+1} + m_1 k_t + m_2 k_{t-1} + c_0 \omega_{t+1} + c_1 \omega_t\ ] = 0$. Here, m_0, m_1, and m_2 are 10×10 matrices and c_0 and c_1 are 10×2 matrices. The order of equations is (3.53), (3.68), (3.56), (3.59), (3.77), (3.76), (3.75), (3.54), (3.67), (3.60). We employ a method of undetermined coefficients to solve the model, given the informational restriction that the monetary policy shock at period t is observable only for 5th equation. See, for example, Christiano, Eichenbaum, and Evans (2001) and Christiano (2001) for the method of undetermined coefficients in the presence of informational differences across equilibrium conditions.

[14] Figures 2 and 3 use monetary policy parameter values, which are obtained by minimizing the gap between model and estimated impulse responses as we discussed. Figure 4, however, uses the policy parameters, which are estimated on the basis of U.S. data. Specifically, we use GMM to estimate a variant of Taylor rule (1993), which allows for partial adjustment. The sample covers U.S. time series on real GDP, GDP deflator, 10 year T-bonds yield, commodity price index, and the federal funds rate over the period 1960:1 - 1997:4, which are all taken from the Citibase data set. In addition, real GDP has been logged and detrended by HP filter, while the inflation rate is identified with logarithmic difference of GDP deflator. The result of our estimation of a monetary policy rule can be written as

$$\begin{array}{rcccccccc} r_t & = & 0.905 & r_{t-1} + & (1 - 0.905) & (1.694 & \pi_t + & 0.614 & y_t) + e_{rt}, \\ & & (0.034) & & & (0.435) & & (0.266) & \end{array}$$

where numbers in parenthesis are standard errors. The set of instruments is $\{\ r_{t-2},\ r_{t-3},\ r_{t-4},\ \pi_{t-2},\ \pi_{t-3},\ \pi_{t-4},\ y_{t-2},\ y_{t-3},\ y_{t-4},\ r_{t-2},\ r_{t-3},\ r_{t-4},\ ls_{t-2},\ ls_{t-3},\ ls_{t-4},\ p_{ct-2},\ p_{ct-3},\ p_{ct-4}\ \}$, where ls_t is the difference between 10 year treasury-bonds yield and federal funds rate and p_{ct} is the logarithmic difference of commodity price index.

responses of real output to an expansionary monetary shock. Hence, we assume a certain degree of consumption habit persistence when we generate model impulse responses in Figure 4. Figure 4 then indicates that the model with adjustment costs can generate dynamic responses of the selected variables consistent with the estimated ones from the VAR.

Furthermore, it should be noted that one needs a very high level of nominal price rigidity to match the estimated impulse responses of the aggregate inflation rate. For example, we set $\alpha_n = \alpha_i = 0.93$, 0.97, and 0.96 in order to match the observed variability of the dynamic responses of the aggregate inflation rate.

5 Conclusion

We have investigated if a sticky price model with inventories can explain the observed dynamic responses of finished goods inventories in response to a monetary policy shock. We have demonstrated two alternative modeling strategies to match the observed variability of finished goods inventory dynamics in response to monetary policy shocks. One is to assume a large depreciation of the inventory stock in the absence of any additional mechanism to avoid excessive responses of inventories. The other is to assume a combination of a small depreciation rate and adjustment costs from deviations of sale-stock ratio from its target.

Next, we discuss future research directions associated with the present paper. In this paper, we analyze only a variant of Calvo-type staggered price-setting. A reason for this is to investigate the effect of holding inventories on the specification of the forward-looking Phillips curve equation. However, it would be interesting to analyze inventory dynamics in a sticky price model with a staggered price-setting other than the Calvo pricing used in this paper and then compare it with the one presented in this paper. An example is to analyze the inventory dynamics in a sticky price model with the Taylor-type staggered price-setting as in the model of Chang, Hornstein and Sarte (2004). Furthermore, as we noted earlier, the model in this paper requires a large degree of nominal price rigidity in order to match the observed variability of the aggregate inflation rate. Hence, it would be interesting to put additional mechanisms into the model, in order to match the observed variability of the inflation rate under a smaller degree of nominal price rigidity than the one used in this paper. An apparent remedy for this would be the inclusion of firm-specific capital in the model, as discussed in Altig, Christiano and Eichenbaum (2004) and Woodford (2005). In particular, they show that when firm-specific capital is introduced into sticky models with the Calvo pricing, a lower level of nominal price rigidity is required to match the Phillips curve relation observed in the US data. Another candidate would be the inclusion of nominal wage rigidity as in the model of Erceg, Henderson and Levin (1999). Finally,

we have focused on the cyclical behavior of finished goods inventories. It is, however, noteworthy that finished goods inventories takes only a small share of total inventories, as discussed in Ramey and West (1999). In addition, finished goods inventories and work-in-progress inventories may respond to an interest rate shock in a different way. Hence, it would be interesting to extend our analysis to models with various types of inventory holdings.

Table 1
Parameter Values

Parameter	Values			Description and definitions
	Fig. 1	Fig. 2	Fig. 3	
Subset of parameters whose values are chosen				
σ		1		Inverse of inter-temporal substitution
χ		1		Inverse of elasticity of labor supply
β		0.99		Time discount factor
s_M		0.5		Share of material inputs in gross output
ξ		0.99		Degree of indexation
γ		0.5		Output share of inventory goods sector
ϕ		0.5		Substitution elasticity of two sectors
$\frac{\bar{S}}{A}$		0.63		Average sales-stock ratio
$\tilde{\mu}$	1.8	1.8	1.04	Markup of non-inventory goods
b	0	0	0.76	Degree of habit persistence
Subset of parameters whose values are estimated				
δ_a	0.58	0.01	0	Depreciation rate of inventory
	(0.03)	(0.02)		
τ	0	0.66	5.80	Share of adjustment costs in total costs
		(0.03)	(0.0002)	
ρ_r	0.85	0.85	0.91	Partial adjustment coefficient
	(0.01)	(0.01)	(0.03)	in interest rate rule
ϕ_π	1.50	1.00	1.69	Responsiveness to inflation rate
	(0.04)	(0.04)	(0.44)	in interest rate rule
ϕ_y	0.60	0.70	0.61	Responsiveness to output gap
	(0.01)	(0.02)	(0.27)	in interest rate rule
α	0.93	0.97	0.96	Fraction of firms that re-optimize
	(0.01)	(0.01)		in each period
θ	0.89	0.37	0.80	Elasticity of demand for goods with
	(0.03)	(0.03)	(0.0002)	respect to stocks

Note: Fig. 1 is a model with large depreciation. Fig. 2 is a model with small depreciation and adjustment costs. Fig. 3 is a model with zero depreciation and adjustment costs. Fig. 3 assumes habit persistence in consumption, while Fig. 1 and Fig 2. do not. Numbers in parenthesis are standard errors.

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Appendix

A Model with Adjustment Costs of Deviations of Sales-Stock Ratio from its Fixed Target

In this section, we briefly highlight changes of equilibrium conditions induced by the inclusion of adjustment costs in the model. When we include adjustment costs for sales-stock ratio into the model, the realized profit flow at period t of firm j that holds inventories can be written as

$$\Pi_{jt} = (\frac{A_{jt}}{A_t})^{\theta}(\frac{\bar{P}_{jt}}{\bar{P}_t})^{1-\epsilon}\bar{S}_t - \bar{MC}_t Q_{j,t} - \frac{\tau}{2}(A_{j,t} - \psi(\frac{A_{jt}}{A_t})^{\theta}(\frac{\bar{P}_{jt}}{\bar{P}_t})^{1-\epsilon}\bar{S}_t)^2 A_t^{-1}, \tag{A.1}$$

where τ is coefficient of adjustment costs and ψ is the inverse of steady state sales-stock ratio. The third term of this equation corresponds to the adjustment costs of sales-stock ratio. Hence, the profit maximization problem at period t for firms that re-optimize at period t is given by

$$\begin{gathered} V^{0,k}(A_{t-k,t-1}, \bar{P}^*_{t-k}, \bar{X}_t) = \max_{A_{t,t}, Q_{t,t}, \bar{P}^*_t} \\ \left(\begin{array}{c} (\frac{A_{t,t}}{A_t})^{\theta}(\frac{\bar{P}^*_t}{\bar{P}_t})^{1-\epsilon_i}\bar{S}_t - \bar{MC}_t Q_{t,t} - \frac{\tau}{2}(A_{t,t} - \psi(\frac{A_{t,t}}{A_t})^{\theta}(\frac{\bar{P}^*_t}{\bar{P}_t})^{-\epsilon_i}\bar{S}_t)^2 A_t^{-1} \\ +\omega_{t,t}((1-\delta_a)(A_{t-k,t-1} - (\frac{A_{t-k,t-1}}{A_{t-1}})^{\theta}(\frac{\Upsilon_{t-k,t-1}\bar{P}^*_{t-k}}{\bar{P}_{t-1}})^{-\epsilon_i}\bar{S}_{t-1}) + Q_{t,t} - A_{t,t}) \\ +E_t[d_{t,t+1}(\alpha_i V^1(A_{t,t+1}, \bar{P}^*_t, \bar{X}_{t+1}) + (1-\alpha_i)V^{0,1}(A_{t,t+1}, \bar{P}^*_t, \bar{X}_{t+1}))]. \end{array} \right) \end{gathered} \tag{A.2}$$

Besides, the value function at period t of firms that re-optimize at period $t-k$ can be written as follows:

$$\begin{gathered} V^{k}(A_{t-k,t-1}, \bar{P}^*_{t-k}, \bar{X}_t) = \max_{A_{t-k,t}, Q_{t-k,t}} \\ \left(\begin{array}{c} (\frac{A_{t-k,t}}{A_t})^{\theta}(\frac{\Upsilon_{t-k,t}\bar{P}^*_{t-k}}{\bar{P}_t})^{1-\epsilon_i}\bar{S}_t - \bar{MC}_t Q_{t-k,t} - \frac{\tau}{2A_t}(A_{t-k,t} - \psi(\frac{A_{t-k,t}}{A_t})^{\theta}(\frac{\Upsilon_{t-k,t}\bar{P}^*_{t-k}}{\bar{P}_t})^{-\epsilon_i}\bar{S}_t)^2 \\ +\omega_{t-k,t}((1-\delta_a)(A_{t-k,t-1} - (\frac{A_{t-k,t-1}}{A_{t-1}})^{\theta}(\frac{\Upsilon_{t-k,t-1}\bar{P}^*_{t-k}}{\bar{P}_{t-1}})^{-\epsilon_i}\bar{S}_{t-1}) + Q_{t-k,t} - A_{t-k,t}) \\ +E_t[d_{t,t+1}(\alpha_i V^{k+1}(A_{t-k,t}, \bar{P}^*_{t-k}, \bar{X}_{t+1}) + (1-\alpha_i)V^{0,k}(A_{t-k,t}, \bar{P}^*_{t-k}, \bar{X}_{t+1}))]. \end{array} \right) \end{gathered} \tag{A.3}$$

As we did in the text, we can show that the optimal pricing equation from the profit maximization problem (A.1) can be written as

$$\sum_{k=0}^{\infty} \alpha_i^k E_t[d_{t,t+k}\bar{S}_{t,t+k}(\frac{\Upsilon_{t,t+k}\bar{P}^*_t}{\bar{P}_{t+k}} + \frac{\tau\psi\epsilon_i}{\epsilon_i - 1}\frac{A_{t,t+k}}{A_{t+k}}(1 - \psi\frac{\bar{S}_{t,t+k}}{A_{t,t+k}}) - \frac{\epsilon_i}{\epsilon_i - 1}MS_{t+k})] = 0. \tag{A.4}$$

The optimization conditions for stocks can be written as

$$\bar{MC}_t = \theta\frac{\bar{S}_{t-k,t}}{A_{t-k,t}} + MS_t(1 - \theta\frac{\bar{S}_{t-k,t}}{A_{t-k,t}}) - \tau(\frac{A_{t-k,t}}{A_t} - \psi\frac{\bar{S}_{t-k,t}}{A_t})(1 - \theta\psi\frac{\bar{S}_{t-k,t}}{A_{t-k,t}}), \tag{A.5}$$

for $k = 0, 1, \cdots, \infty$.

Having described optimization conditions of firms, we derive a Phillips curve equation as we did in the text. The Phillips curve equation then can be written as

$$\pi_t = \frac{\xi}{1+\xi\beta}\pi_{t-1} + \kappa_0 E_t[v_{t+1}] + \kappa_1 E_t[\lambda_{t+1} - \lambda_t] + \kappa_2 v_t + \frac{\beta}{1+\xi\beta}E_t[\pi_{t+1}], \tag{A.6}$$

where κ_0, κ_1, and κ_2 are defined as

$$\kappa_0 = \frac{\gamma\rho_0\bar{\kappa}(1-\bar{\mu}\bar{s}_M)}{1+\beta\xi}; \quad \kappa_1 = \frac{\gamma\rho_0\bar{\kappa}}{1+\beta\xi}; \quad \kappa_2 = \frac{(1-\gamma)\tilde{\kappa}(1-\tilde{\mu}\tilde{s}_M) + \gamma\bar{\kappa}\rho_1(1-\bar{\mu}\bar{s}_M)}{1+\beta\xi}.$$

In addition, coefficients ρ_0 and ρ_1 are defined as

$$\rho_0 = 1 - \frac{\bar{\mu}\tau\psi\beta(1-\delta_a)(\bar{\mu}-1)}{(1-(1-\delta_a)\beta+\bar{\mu}(1-\theta)\tau)(\bar{\mu}-(1-\delta_a)\beta)}; \rho_1 = \frac{\bar{\mu}\tau\psi}{1-(1-\delta_a)\beta+\bar{\mu}(1-\theta)\tau}.$$

Besides, log-linearizing (A.5) for each firm and then summing up resulting equations leads to

$$\bar{s}_t - a_t = \frac{\bar{m}c_t}{1-\beta(1-\delta_a)+\bar{\mu}(1-\theta)\tau} - \frac{\beta(1-\delta_a)(\bar{\mu}-1)ms_t}{(1-(1-\delta_a)\beta+\bar{\mu}(1-\theta)\tau)(\bar{\mu}-(1-\delta_a)\beta)}. \tag{A.7}$$

Given the relationship between marginal costs of gross output and unit costs of value-added output, this equation can be rewritten as follows:

$$\bar{s}_t - a_t = b_0 v_t - b_1 E_t[v_{t+1}] - b_2 E_t[\lambda_{t+1} - \lambda_t], \tag{A.8}$$

where b_0, b_1, and b_2 are defined as

$$b_0 = \frac{1-\bar{\mu}\bar{s}_M}{1-(1-\delta_a)\beta+\tau(1-\theta)\bar{\mu}}; \; b_1 = \frac{\beta(\bar{\mu}-1)b_0}{\bar{\mu}-(1-\delta_a)\beta}; \; b_2 = \frac{b_1}{1-\bar{\mu}\bar{s}_M}.$$

As a result, equations (3.53) and (3.56) are replaced by (A.6) and (A.8) respectively, when we take into account adjustment costs of sales-stock ratio.

Figure 1: Estimated Impulse Responses from a VAR Model

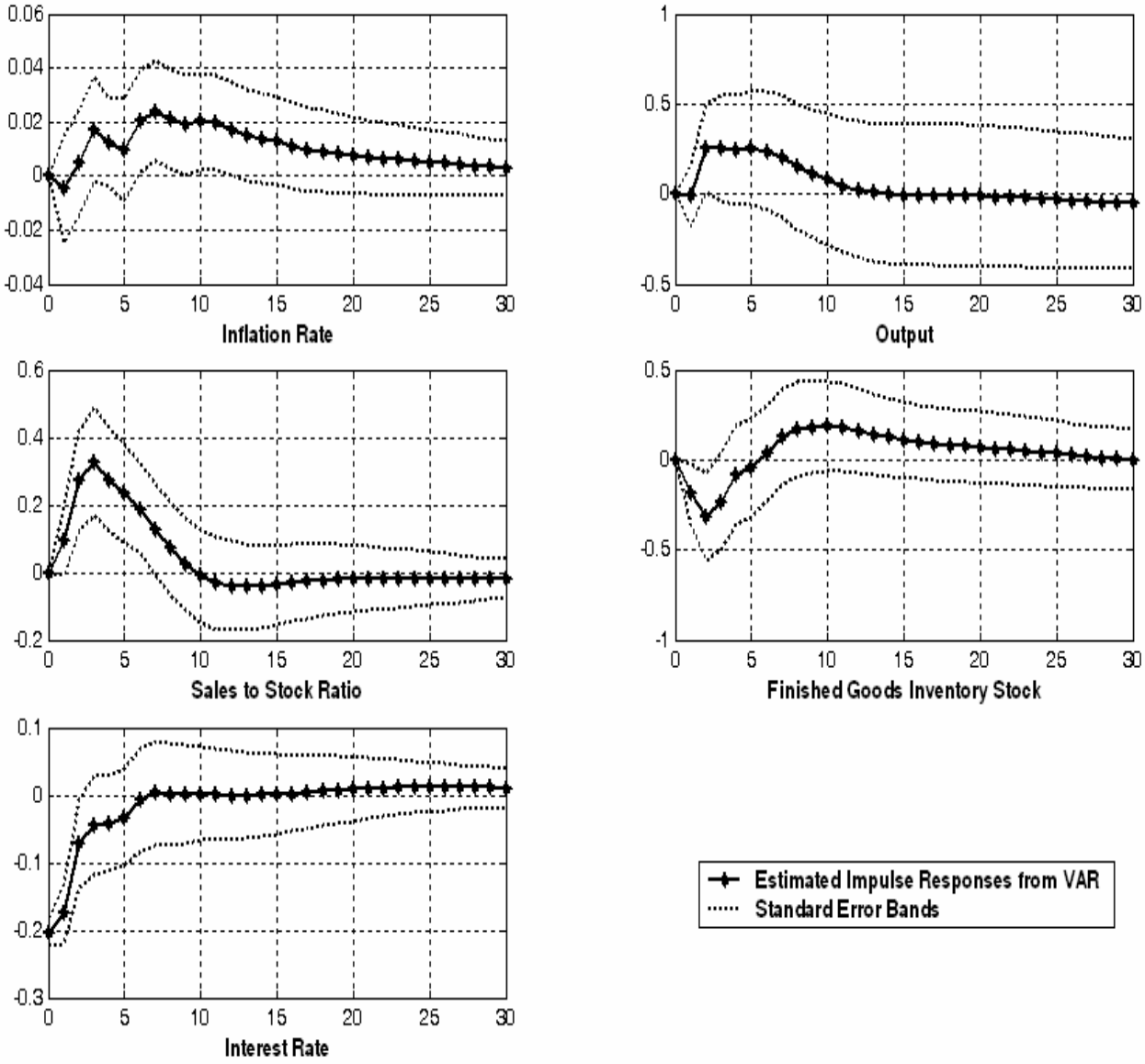
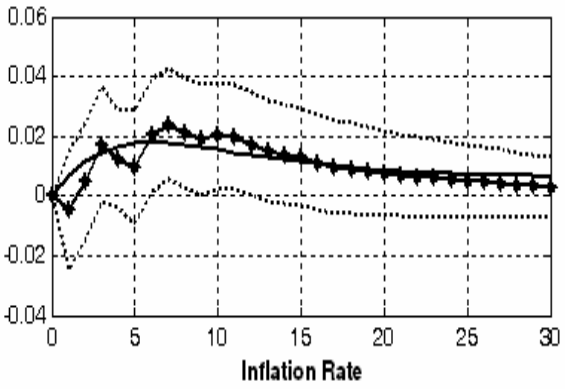
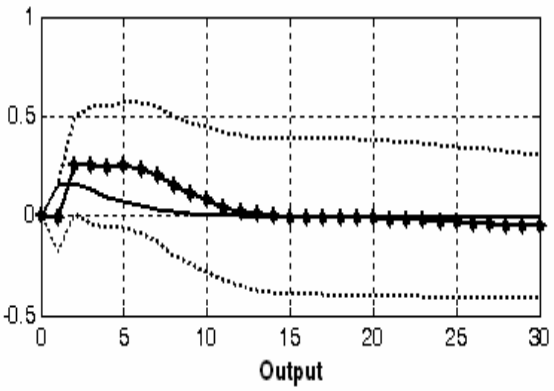
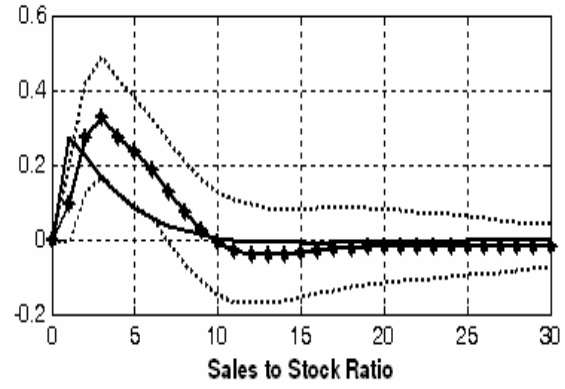
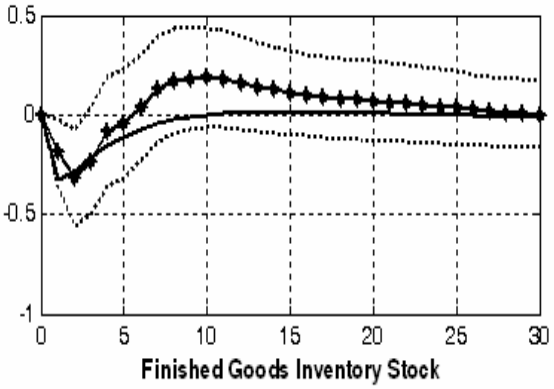


Figure 2: Impulse Responses: High Depreciation Rate without Adjustment Costs









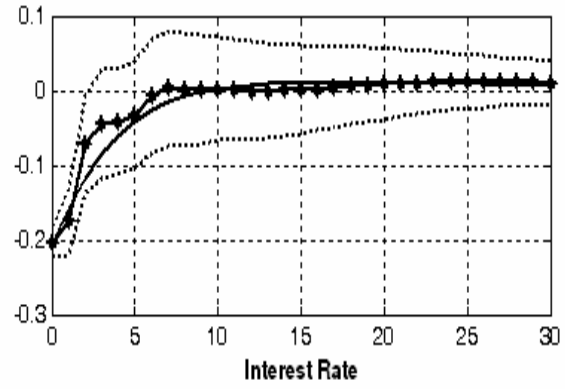
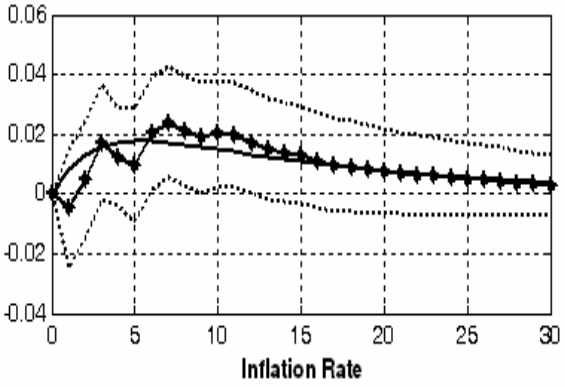
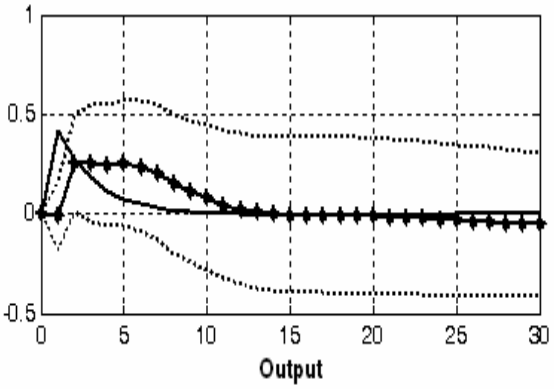
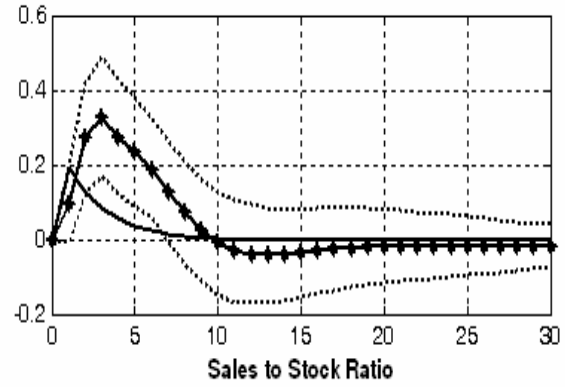
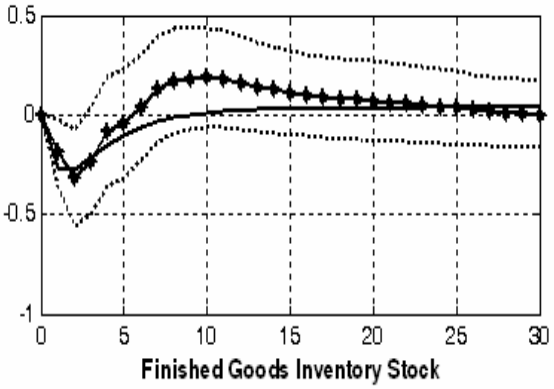


Figure 3: Impulse Responses: Quadratic Adjustment Costs and Low Depreciation Rate









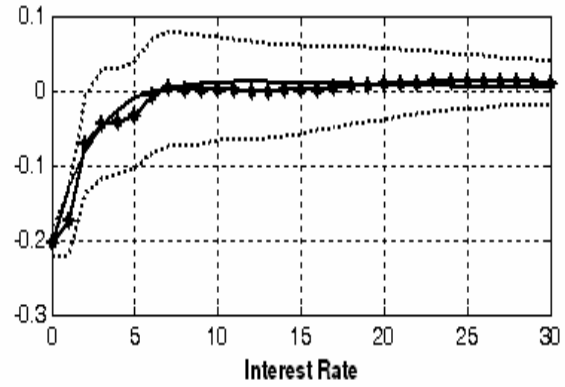
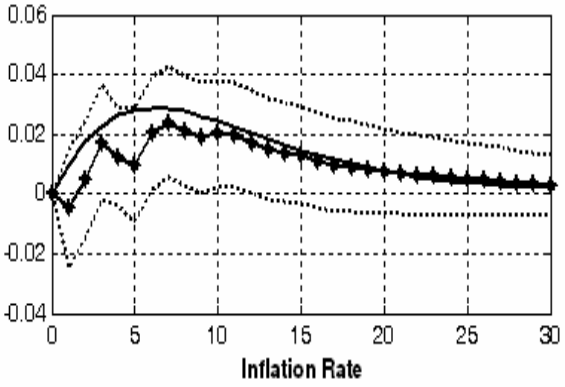
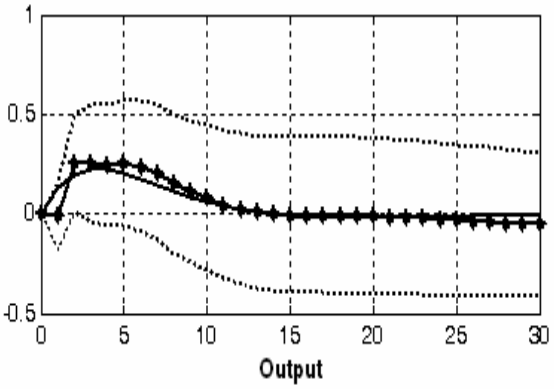
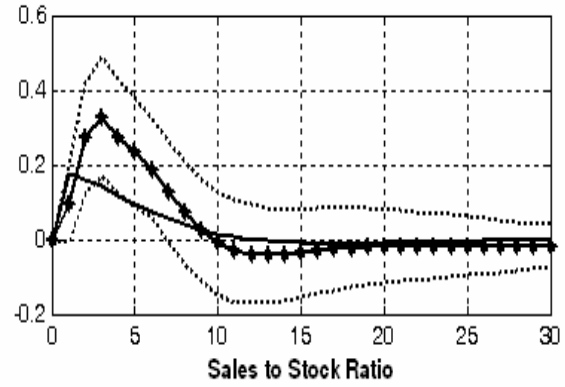
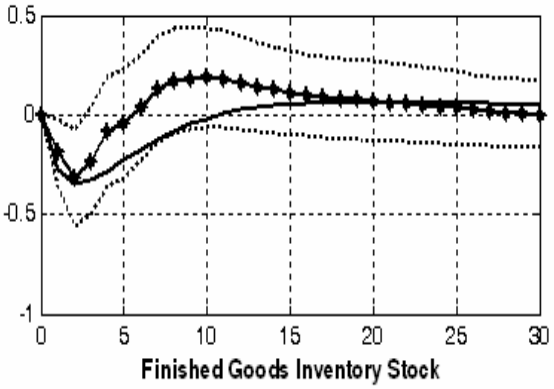
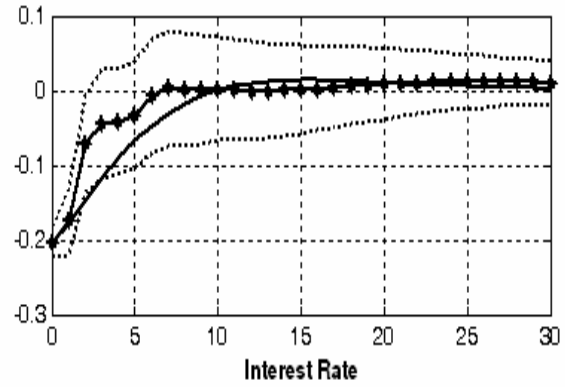



Figure 4: Impulse Responses: Quadratic Adjustment Costs and Zero Depreciation Rate