Dynamic Tax Competition under Asymmetric Productivity of Public Capital

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Dynamic Tax Competition under Asymmetric Productivity of Public Capital*

－A Simulation Analysis Using an Overlapping Generations Model within Two Large Regions－

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Abstract

We here expand the static tax competition models in symmetric small regions, which were indicated by Zodrow and Mieszkowski (1986) and Wilson (1986), to a dynamic tax competition model in large regions, taking consideration of the regional asymmetry of productivity of public capital and the existence of capital accumulation. The aim of this paper is to verify how the taxation policy affects asymmetric equilibrium based on a simulation analysis using an overlapping generations model in two regions.

It is assumed that the public capital as a public input is formed on the basis of the capital tax of local governments and the lump-sum tax of the central government. As demonstrated in related literature, the optimal capital tax rate should become zero when the lump-sum tax is imposed only on older generations, however, the optimal tax rate may become positive when it is imposed proportionally on younger and older generations. In the asymmetric equilibrium, several cooperative solutions can possibly exist which can achieve a higher welfare standard than the actualized cooperative solution either in Region1 or 2.

JEL classification ： H21; H42; H71; H77; R13; R53

Keywords ： Tax competition, Capital taxation, Capital accumulation, Public inputs, Infrastructure

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1. Introduction.

The optimal theory of public investment is crucially important, which determines its level from the point of the efficiency of resource allocation, assuming that public investment is maintained as the public capital and focusing on the stock effects improving productivity or enhancing utility. Many researches in normative analyses of public investment, such as Arrow-Kurz (1970), Sandmo-Dreze (1970), Ogura-Yohe (1977), Burgess (1988), or Yakita (1993), have shown their interest in the social discount rate for an optimal level.

The government structure, as a supply body, is one of the key issues when considering an optimal provision of public investment as well as supply rules such the discount rate. In other words, we need to verify theoretically and empirically which is more desirable for an efficient provision of public capital. Centralized or decentralized financial system.

We can say that the necessity has been increasing to examine how the efficient level of public capital is determined under the Japanese circumstances where decentralization has been accelerating. The Fiscal Competition theory clarifies that a decentralized decision on the tax rate or expenditure level may possibly bring inefficiency in an economy where tax-related regional migration such as capital or labor occurs. It can be a theory which meets the needs. However, only a few theoretical considerations of fiscal competition have been made in relation to the optimal provision of public capital.

Upon analyzing the optimal provision of public capital within the framework of fiscal competition supposing the regional migration of private capital, the following two points may be considered important: Firstly, when public investment is assumed to contribute to the productivity improvement as a production input, it is necessary to give an analysis to consider the difference of the productivity effects among regions. Specifically, an analysis with a model based on regional differences in productivity should be made, since there is a consensus that significant differences exist among regions on the productivity effects of public capital.

Secondly, an analysis to model the accumulation process of private capital is needed in order to examine the stock effects of public capital. It would be possible to make a deep analysis about the effects that not only the regional allocation of private capital but also its fluctuation in the whole economy gives on welfare, by adopting a dynamic model based on capital accumulation. To examine how policy variables such as the tax rate or expenditure level of a local government under fiscal competition give impact on capital mobility or capital accumulation will lead to a possible analysis of the dynamic impact of governmental policy choice on welfare.

In this paper, we extend a static framework with constant capital supply, based on regional symmetry which has been used in the general fiscal competition model, to a dynamic framework incorporating a capital accumulation process in consideration of relevance to the optimal provision of public capital. Concretely, we expand the static capital competition model of Zodrow and Mieszkowski (1986) and Wilson (1986) to an overlapping generations model. We make an analysis on the policy consequence of dynamic tax competition incorporating asymmetry of productivity effects of public capital, which has not fully explained by previous studies.

We adopt an approach to conduce to a quantitative outcome by making a simulation analysis of an overlapping generations model within two regions. We do not use an approach to lead to a qualitative outcome based on a theoretic analysis because our aim in this paper is to examine the capital tax rate, public capital level, and also complicated impacts on the social welfare in consideration of capital accumulation and non-homogeneity of productivity.

In Chapter 2, we review related literature about theoretical analyses on fiscal competition to clarify our position in this paper. In Chapter 3, we describe the households in each region and the optimal action of firms for the overlapping generations model within two regions, and the object function and budget constraint of local governments and the central government. Then, we give a simulation analysis on a policy consequence of capital tax competition for symmetric and asymmetric cases of productivity effects in Chapter 4. Finally, we summarize our conclusion and describe pending issues.

2. Related literature.

Many researches on Fiscal Competition theory have been done since late 1980s. The theory has evolved to a framework which clarifies the consequence of competition over various policy variables among local governments, including tax competition, expenditure competition, and redistribution competition. It has now been recognized as one of the major research area in public economics. Zodrow and Mieszkowski (1986) and Wilson (1986) marked the beginning on the basis of capital tax competition. Their studies were focused on tax-related regional migration such as capital or labor, and on what impacts competitive and uncooperative policy decisions by local governments might give on the regional public goods provision.

Zodrow and Mieszkowski (1986) and Wilson (1986) theoretically clarified that the capital tax competition among local governments under capital flow could induce the underprovision of public goods, and consequently the reduction of resident welfare.

Wildasin (1988) expanded the policy competition, which Zodrow and Mieszkowski (1986) and Wilson (1986) discussed only in relation to the capital tax rate, to a framework encompassing competition over the standards of public goods provision. It showed that the equilibrium point would differ when the tax rate or public expenditure was chosen for policy variables. It further indicated that there would be differences in the deviation range from the optimal point.

In addition to the issues of policy variables, many researches have modified or expanded the capital tax competition model of Zodrow and Mieszkowski (1986) and Wilson (1986). Their main interests have been to clarify theoretical consequences of the capital tax competition in the following three cases: 1) when public capital (public goods) contributes to the improvement of local productivity as a public input; 2) when the regions are heterogeneous or their sizes are large; 3) The capital amount of the whole economy fluctuates.

Noisit (1995) and Noisit and Oakland (1995) gave a theoretical consideration on the possibility whether the capital tax competition could cause the overprovision of public goods. They emphasized that lowering capital tax rate could prevent private capital

from outflowing on one hand, but it could induce decrease of public capital (public goods) contributing as a public input on the other hand, which would bring on decrease in the marginal productivity of private capital and would eventually promote outflowing of private capital. They also maintained that the cut-down in capital tax rate would not necessarily induce any increase of tax-related capital depending on the size of the effects.

Bucovetsky (1991) and Wilson (1991) made a theoretical analysis on the capital tax rate in relation to two heterogeneous regions with different population size. They indicated that asymmetric equilibrium in heterogeneous regions would be actualized unlike symmetric equilibrium heterogeneous regions, however, the resident welfare would become higher in less populated regions which chose relatively lower tax rate than in populated regions.

Piekkola (1995), Wildasin (2003), Kellermann (2006) and Kellermann (2007) gave theoretical consideration on dynamic capital tax competition in reflection of the consumption and saving choices at different time points, when easing the assumption that the capital amount of the whole economy was constant. Piekkola (1995) analyzed a consequence of dynamic capital tax competion based on an overlapping generations model, on the contrary Wildasin (2003) made an analysis on the basis of the Ramsey model. Kellermann (2006) and Kellermann (2007) suggested that there would be a possibility for capital tax competition to induce inefficiency in resource allocation, based on the assumption that public capital was incorporated as a public input in small homogeneous regions.

The research on capital tax competition, which was started by Zodrow and Mieszkowski (1986) and Wilson (1986) has extended from an analysis of symmetric equilibrium in homogeneous regions to that of asymmetric equilibrium in heterogeneous regions, and moreover from a static framework without the issue of consumption and saving choices to a dynamic framework. Nevertheless, there are few researches using dynamic capital tax competition model, which explicitly treated public capital as a public input, other than Kellermann (2006) and Kellermann (2007).

Furthermore, Kellermann (2006) and Kellermann (2007) as well as Zodrow and Mieszkowski (1986) and Wilson (1986) were based on an assumption of small and homogeneous regions where the fluctuation of capital tax rate would not give any influence on the rate of return on capital. There is no research that dealt with a dynamic capital tax competition within large and heterogeneous regions which have a difference in contribution to the productivity of public capital.

Here in this paper, we focus on this issue. We use the overlapping generations model of Diamond (1965) and construct a dynamic capital tax competition model incorporating public capital as a production function. We, on the basis of simulation analysis, clarify how much impact the change of policy variables gives to asymmetric equilibrium.

3. Theoretical model.

In this paper, we give an analysis on tax competition based on the case where two regions procure public investment funds through capital tax, given that two regions receive fiscal transfer from the central government. Public capital, provided for by public investment, is used as a public input. Representative firms in each region produce goods with public inputs of labor, private capital and public capital based on their own production techniques. Population, utility function of a representative household, and goods produced in both regions are identical. The two regions are differentiated only by the production technique.

We use the overlapping generations model of Diamond (1965) for a dynamic process of public as well as private capital accumulation, and expand the model to two regions by putting in public capital from governments as a public input. Each region has younger generations born in time t and older generations born in time t-1. When their populations are respectively L_{it} and L_{it-1} and the population growth rate is n, the equation $L_{it} = (1+n)L_{it-1}$ is formed. The population in each region is equal and there is no regional migration. In what follows, we describe behaviors of firms, households, governments, and then the market equilibrium.

3-1. Firms.

Firms in Region i (i=1,2) produce goods (Y_{it}) using the linear homogeneous production function ($F_i(L_{it}, K_{it}, G_{it})$) by means of labor ($L_{it}$), private capital ($K_{it}$), and public capital ($G_{it}$) as productive inputs [1]。 Firms solve the following profit maximization problem with public capital (G_{it}) and production technique:

$$\underset{L_{it}, K_{it}}{MAX} \quad \Pi_{it} = Y_{it} - w_{it} L_{it} - r_{it} K_{it} \tag{1}$$

$$s.t \quad Y_{it} = F_i(L_{it}, K_{it}, G_{it}) \tag{2}$$

w_{it} means the wage rate and r_{it} means the rate of return on capital. Due to the first order condition of profit maximization, $w_{it} = \partial F_i / \partial L_{it}$ and $r_{it} = \partial F_i / \partial K_{it}$ are derived. Accordingly, the profit shall be $\Pi_{it} = (\partial F_{it} / \partial G_{it}) G_{it}$. It is assumed that the profit should be distributed to the capital K. The rate of return of capital ρ shall be $\rho_{it} = r_{it} + \Pi_{it} / K_{it}$. Income distribution shall be $Y_{it} = w_{it} L_{it} + \rho_t K_{it}$.

Similarly to the Diamond model, we assume that the labor supply is fixed. The production volume, private capital and public capital shall be described as 1 unit of labor. The rate of return on capital and income distribution shall be:

$$\rho_{it} = r_{it} + \frac{(\partial f_i / \partial g_{it}) g_{it}}{k_{it}} \tag{3}$$

$$y_{it} = w_{it} + \rho_{it} k_{it} \tag{4}$$

For this purpose, $f = F/L$, $g = G/L$, $y = Y/L$, $k = K/L$. When the production function is specified as $F_i(L_{it}, K_{it}, G_{it}) = (L_{it})^{\beta_i^L} (K_{it})^{\beta_i^K} (G_{it})^{1-\beta_i^L-\beta_i^K}$, a Cobb-Douglas function, the capital demand and wage rate shall be expressed as follows for g_{it} and ρ_{it} functions:

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[1] $Y_{it} = \frac{\partial F_i}{\partial L_{it}} L_{it} + \frac{\partial F_i}{\partial K_{it}} K_{it} + \frac{\partial F_i}{\partial G_{it}} G_{it}$ shall be formed based on the linear homogeneousness.

$$k_{it} = (\rho_{it})^{\frac{-1}{1-\beta_i^K}} (1-\beta_i^L)^{\frac{1}{1-\beta_i^K}} (g_{it})^{\frac{1-\beta_i^L-\beta_i^K}{1-\beta_i^K}} \quad (5)$$

$$w_{it} = (\rho_{it})^{\frac{-\beta_i^K}{1-\beta_i^K}} \beta_i^L (1-\beta_i^L)^{\frac{\beta_i^K}{1-\beta_i^K}} (g_{it})^{\frac{1-\beta_i^L-\beta_i^K}{1-\beta_i^K}} \quad (6)$$

For the purpose of this paper, it is assumed that labor has no regional migration whereas capital freely moves between the regions. When the rate of return on capital differs between the regions, capital moves to the region with a higher rate of return on capital, under the condition that tax shall be imposed on the return of capital at the rate of τ_i. Assuming that the rate of return on capital is θ after tax, θ shall be equal between the regions due to the capital movement, the following equation shall be derived:

$$\theta_t = (1-\tau_{it})\rho_{it} \quad (7)$$

Based on equation (7), the capital demand and wage rate shall be represented as follows for the functions θ_t and τ_{it}:

$$k_{it} = \left(\frac{\theta_t}{1-\tau_{it}}\right)^{\frac{-1}{1-\beta_i^K}} (1-\beta_i^L)^{\frac{1}{1-\beta_i^K}} (g_{it})^{\frac{1-\beta_i^L-\beta_i^K}{1-\beta_i^K}} \quad (5)'$$

$$w_{it} = \left(\frac{\theta_t}{1-\tau_{it}}\right)^{\frac{-\beta_i^K}{1-\beta_i^K}} \beta_i^L (1-\beta_i^L)^{\frac{\beta_i^K}{1-\beta_i^K}} (g_{it})^{\frac{1-\beta_i^L-\beta_i^K}{1-\beta_i^K}} \quad (6)'$$

3-2. households.

Assuming that households in region i (i=1,2) consume c_{it}^y during earlier life (time t) and c_{it+1}^o during older life (time t+1), they are facing the following utility

maximization problem under budget constraints:

$$\underset{c_{it}^{y},\,c_{it+1}^{o}}{MAX} \quad u_i(c_{it}^{y}, c_{it+1}^{o}) = \frac{(c_{it}^{y})^{1-1/\mu} - 1}{1 - 1/\mu} + \frac{1}{1+\delta}\frac{(c_{it}^{o})^{1-1/\mu} - 1}{1 - 1/\mu} \tag{8}$$

$$s.t \qquad c_{it}^{y} = w_{it} - s_{it} - t_{it}^{y} \tag{9}$$

$$c_{it+1}^{o} = (1+\theta_{t+1})s_{it} - t_{it+1}^{o} \tag{10}$$

The utility function μ means the elasticity of substitution between different time points, δ means the subjective rate of time preference. s represents savings, t_{it}^{y} and t_{it+1}^{o} are the lump-sum taxes respectively imposed during early life and older life. The following consumption function and savings function are derived by solving this maximization problem:

$$c_{it}^{y} = \alpha_{t+1}(w_{it} - t_{it}^{y} - \frac{t_{it+1}^{o}}{1+\theta_{t+1}}) + \frac{t_{it+1}^{o}}{1+\theta_{t+1}} \tag{11}$$

$$\begin{aligned} s_{it} &= (1-\alpha_{t+1})(w_{it} - t_{it}^{y} - \frac{t_{it+1}^{o}}{1+\theta_{t+1}}) + \frac{t_{it+1}^{o}}{1+\theta_{t+1}} \\ &= (1-\alpha_{t+1})(w_{it} - t_{it}^{y}) + \frac{\alpha_i t_{it+1}^{o}}{1+\theta_{t+1}} \end{aligned} \tag{12}$$

where α is marginal propensity of consumption and $\alpha_{t+1} = \left\{1+\left(\frac{1+\theta_{t+1}}{1+\delta}\right)^{\mu}(1+\theta_{t+1})^{-1}\right\}^{-1}$.

When the parameter μ of the utility function is 1, the marginal propensity of consumption shall be the invariable $\alpha = \left(1+\frac{1}{1+\delta}\right)^{-1}$.

3-3. Government budget constraints.

This model requires the central government and two local governments. The

central government imposes $\left(t_{it}^{y}, t_{it}^{o}\right)$ as a lump-sum tax on households in the two regions, and uses it as IG_{it}^{C}, or the funds for fiscal transfer. On the other hand, the local government collects capital taxes from firms of its own region, and uses them as IG_{it}^{L}, or the funds for public investment. Public investment of each region is $IG_{it}^{C} + IG_{it}^{L}$. The budget constraint equation of the government shall be described as follows:

$$G_{it+1} = G_{it} + \left(IG_{it}^{C} + IG_{it}^{L}\right) \tag{13}$$

$$IG_{it}^{C} = L_{t} t_{it}^{y} + L_{t-1} t_{it}^{o} \tag{14}$$

$$IG_{it}^{L} = \tau_{it} \rho_{it} K_{it} \tag{15}$$

3-4. Market equilibrium.

Produced goods and capital in the two regions are transferable within the regions. The equilibrium of the capital market can be formed when the capital demand of both regions is equivalent to their capital supply. Therefore, it shall be described as follows:

$$\sum_{i=1}^{2} L_{it} s_{it} = \sum_{i=1}^{2} K_{it+1} \tag{16}$$

When the equilibrium of the capital market is established, the balance equation shall be derived as follows, by assigning equation (15) to equation (16) from equation (6):

$$\sum_{i=1}^{2} Y_{it} = \sum_{i=1}^{2}\left(L_{it} c_{it}^{y} + L_{it-1} c_{it}^{o}\right) + \sum_{i=1}^{2}\left(K_{it+1} - K_{it}\right) + \sum_{i=1}^{2}\left(G_{it+1} - G_{it}\right) \tag{17}$$

The private capital demand shall be the function of $\left(\theta_{t+1}, \tau_{it}, g_{it+1}\right)$ with equation (5)'.

The capital supply shall be the function of $\left(w_{it}, \theta_{t+1}, t_{it}^{y}, t_{it+1}^{o}\right)$ in accordance with equation (12). w represents the function of $\left(\theta_t, \tau_{it}, g_{it}\right)$ according to equation (6)', accordingly capital supply shall be the function of $\left(\theta_t, \tau_{it}, t_{it}^{y}, t_{it+1}^{o}, g_{it+1}\right)$. When the government policy variable $\left(\tau_{it}, t_{it}^{y}, t_{it+1}^{o}, g_{it}, g_{it+1}\right)$ is given, the capital market equilibrium of equation (16) can be represented as a dynamic system of $\left(\theta_{t,}\theta_{t+1}\right)$. In what follows, it is assumed that the government policy variable is $\left(\tau_i, t_i^{y}, t_i^{o}, g_i\right)$ and constant all the time, and θ holds steady state for a long term. Also, w_i and ρ_i are constant as well as the per capita variables c_i^{y}, c_i^{o}, s_i and k_i.

3-5. Policy objectives in a long-term steady state.

Here, we summarize the policy objectives, policy instruments and constrained conditions of the central and local governments. The local governments are given the fiscal transfer dG_i^{C} from the central government, the taxes $\left(t_i^{y}, t_i^{o}\right)$ from their own residents, and the capital tax rate $\tau_{\neq i}$ from the other region. They decide the capital tax rate τ_i based to provide the public investment $dG_i^{C} + dG_i^{L} = dG_i$. The policy objective of the local governments here is to determine the standard of τ_i so that the utility of their own residents becomes highest. When using the consumption function of equation (11), the indirect utility function $v_i\left(w_i, \theta, t_i^{y}, t_i^{o}\right)$ shall be derived. Therefore, the objective function of the local governments shall be formulated as:

$$\underset{\tau_i}{MAX} \quad v_i\left(w_i, \theta, t_i^{y}, t_i^{o}\right) \tag{18}$$

$$s.t. \quad ng = t^{y} + \frac{t_o}{1+n} + \tau_i \rho_i k_i \tag{19}$$

Equation (19) is a representation of the steady state amount per capita from equations (13), (14) and (15), which show the government budget constraints. Each local government $\tau_{\neq i}$ is given the capital tax rate of the other region, and compete against each other over the standard of such tax rate τ_i.

As for the measures how the central government involves in the capital tax competition, a mediation or assignation pertaining to the capital tax rate can be as an option other than the lump sum tax of equation (14). In this case, the objective function of the central government is assumed to be the following social welfare function:

$$W = \frac{(v_1^*)^\gamma + (v_2^*)^\gamma}{\gamma} \tag{20}$$

4. Simulation analysis.

In this chapter, we give a simulation analysis on the relevance between the standard of policy variables, for the lump-sum as well as the capital taxes, and the economic welfare. For this purpose, we look at the cases that the local governments form public capital using the lump-sum and capital taxes on the basis of the overlapping generations model within two regions constructed in the previous chapter. For the purpose of shorthand, it is assumed that $n_1 = n_2 = 1, \delta_1 = \delta_2 = 1, \mu_1 = \mu_2 = 1$, and also γ, the social significance between the regions for the social welfare function, is 1[2].

In what follows, we first make a simulation analysis on the standard of a lump-sum tax and the capital tax rate, which realize the maximization of social welfare for the case where the central government can choose an instrument for financial resources procurement without any constraints in Subchapter 4-1. Next in 4-2, we give a simulation analysis on the capital tax competition in the case where the productivity is symmetric between the two regions, assuming that the central government can only choose the lump-sum tax. Finally in 4-3, we make another simulation analysis on the capital tax rate in the case where the labor and productivity of public capital are asymmetric between the regions based on the same assumption as 4-2.

[2] This means that the social welfare function is a Benthamite type.

4-1. Only the central government does financial resources procurement.

Firstly, we look at the case where only the central government does financial resources procurement for public capital on the condition that it can choose any procuring instrument without constraints. The central government does fiscal transfer, or subsidies in other words, imposes lump-sum taxes on younger and older generations and also collects capital taxes. On the other hand, the local governments form their public capital, or local public goods in other words, based on the financial resources from capital taxes and fiscal transfer from the central government. It is assumed that the productivity is symmetric between the two regions ($\beta_1^L = \beta_2^L = 0.5, \beta_1^K = \beta_2^K = 0.3, \beta_1^G = \beta_2^G = 0.2$). Figures 4-1-1 and 4-1-2 show the standards of social welfare which are determined by the combination of the capital tax rate and the standard of fiscal transfer. A circle with vertically-striped pattern indicates the grand optimum which maximizes social welfare. In the case where a lump-sum tax is imposed only on older generations ($t^y = 0$), the fiscal transfer standard to maximize social welfare shall be 670, and the capital tax rate shall be 0. In this case, the impact on capital accumulation is almost neutralized and distortion from the capital tax occurs. Accordingly, such simulation result is derived which follows the conclusion of related literature based on a static framework, or the optimal capital tax rate is 0.

Meanwhile, in the case where a lump-sum tax is imposed on younger and older generations ($t^y = t^o$), the fiscal transfer standard to maximize social welfare shall be 0, and the capital tax rate shall be 0.65. This result is different from that of related literature showing that the optimal capital tax rate is 0. We will further discuss the mechanism why the capital tax rate is not zero in Subchapters 4-2 and 4-3. Briefly, it can be explained as follows: The optimal rate is determined based on a standard which achieves a balance between the effects eliminating any negative impact on capital accumulation from the lump-sum tax on younger generation by elevating the capital tax rate, which leads to the decrease of capital tax, and any welfare loss caused by the distortion from capital tax. Thus, the tax rate shall not become zero.

Figure 4-1-1: Optimal fiscal transfer (subsidies) and optimal capital tax rate ($t^y = 0$)

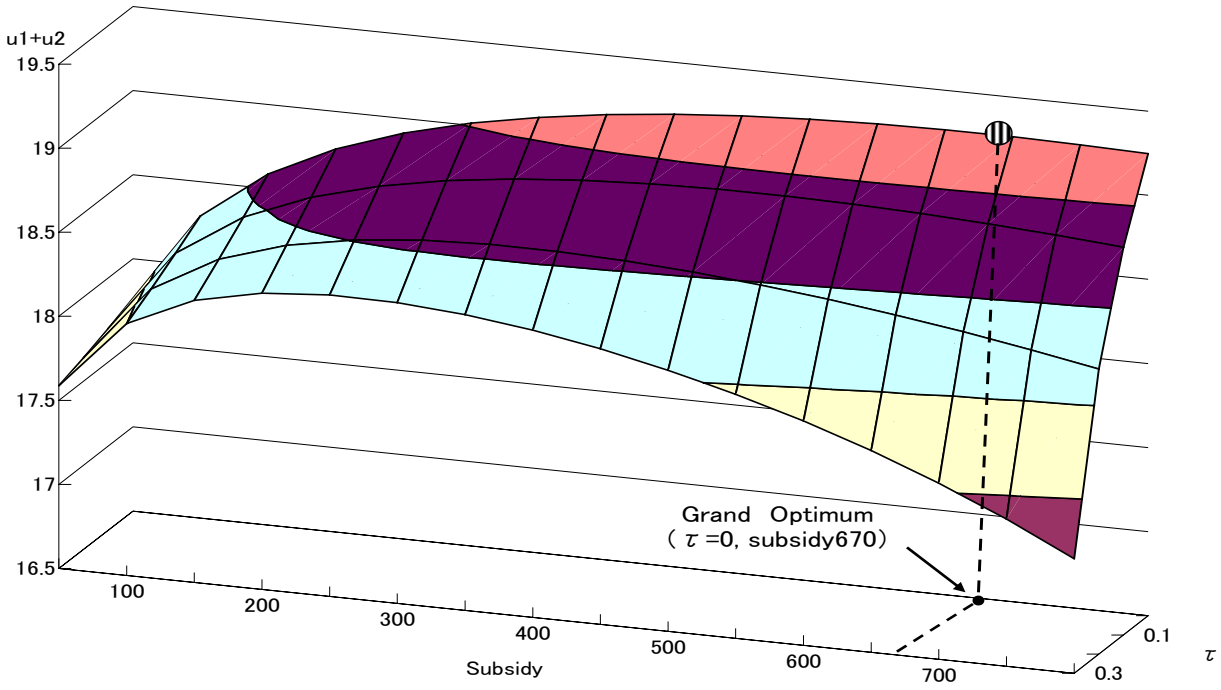
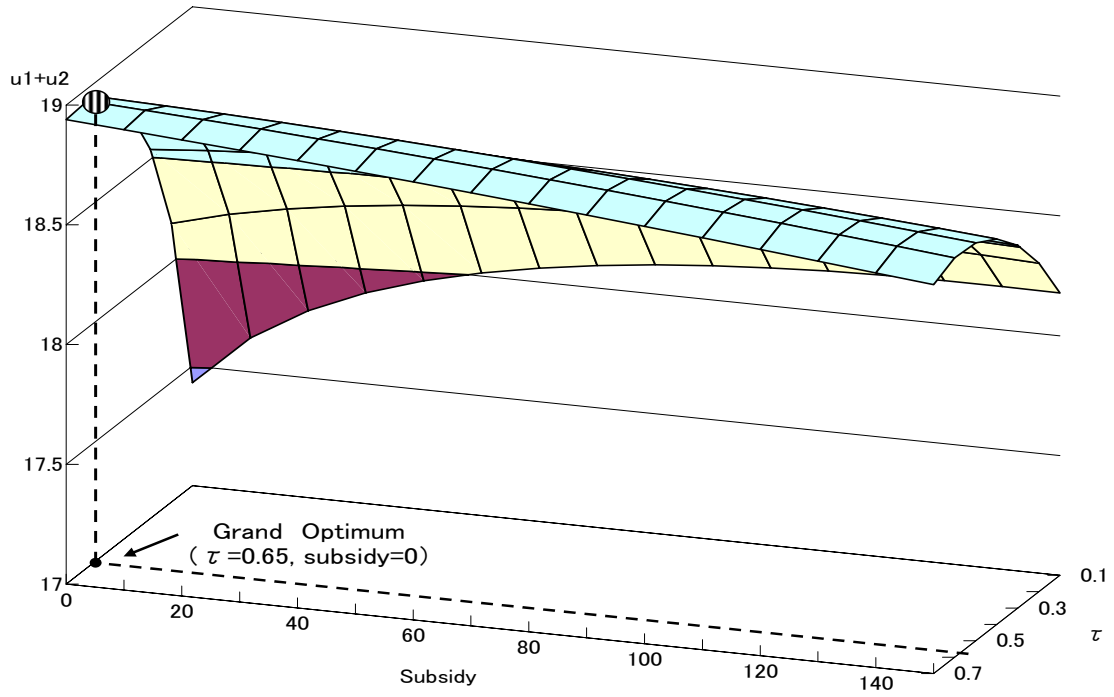


Figure 4-1-2: Optimal fiscal transfer (subsidies) and optimal capital tax rate ($t^y = t^o$)



Note: Circles with vertically-striped pattern indicate the grand optimums which maximize social welfare.

4-2. Capital tax competition within the symmetric regions.

Next, we look at the case where the central government procures financial resources for public capital from lump-sum taxes whereas the local governments do from the capital taxes. It is assumed that the productivity is symmetric between the two regions and the regions compete against each other over the capital tax rate based on the condition that each government is given the other's actions. The central government does fiscal transfer to local governments using lump-sum taxes as its financial resources. It has an option to impose tax only on older generation ($t^y = 0$) or to equally impose on both older and younger generations ($t^y = t^o$). Also, the public capital standard made available by the fiscal transfer from the central government is 100.

Figures 4-2-1 and 4-2-2 show Nash equilibrium solution, cooperative solutions and optimal solutions (or social welfare maximization point) in symmetric regions with the same productivity. It is based on an assumption of symmetric regions, and the cooperative solution and the optimal solution is identical. Also, it leads to the same consequence whether the local governments realize policy coordination or the central government intervene in the capital tax competition between the local governments. Social welfare becomes higher than the Nash equilibrium solution when the cooperative solution is equal to the optimal solution, in both cases where only older generations are imposed a lump-sum tax (figure 4-2-1) and where both generations are equally imposed lump-sum taxes (figure 4-2-2).

The capital tax rates, , of the Nash equilibrium solution and the cooperative solution equaling the optimal solution, are respectively 0.4 and 0.55 in the cases where the tax is imposed only on older generations whereas they are 0.4 and 0.65 in the cases where the tax is equally imposed on both generations. The standards are not the same, however, the capital tax rate of the Nash equilibrium solution becomes lower than that of the cooperative solution equaling the optimal solution. It can be said that a simulation result is obtained consistent with the theory of the capital tax competition, which has shown the tax rate would become lower the optimal standard due to the

Figure 4-2-1: Nash equilibrium solution, cooperative solution and optimal solution in symmetric regions (in the case of $t^y = 0$).

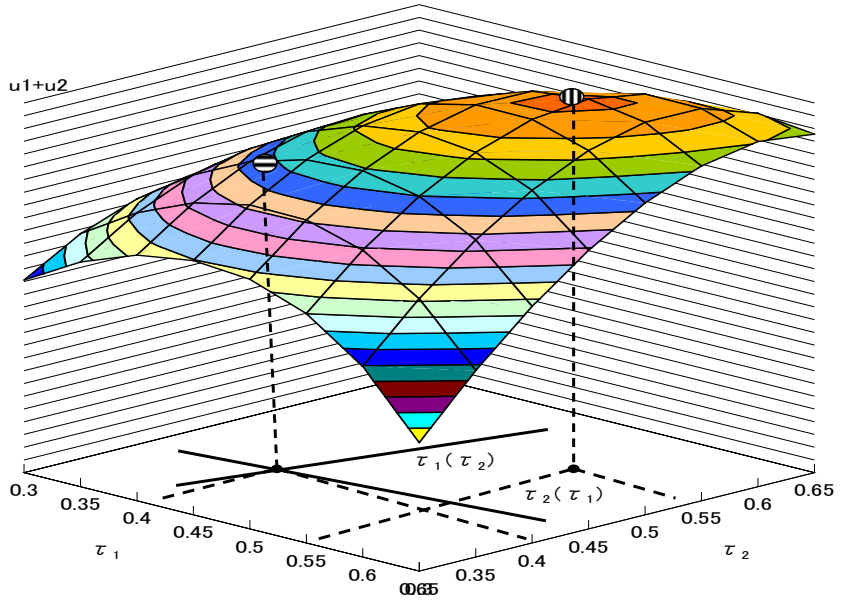
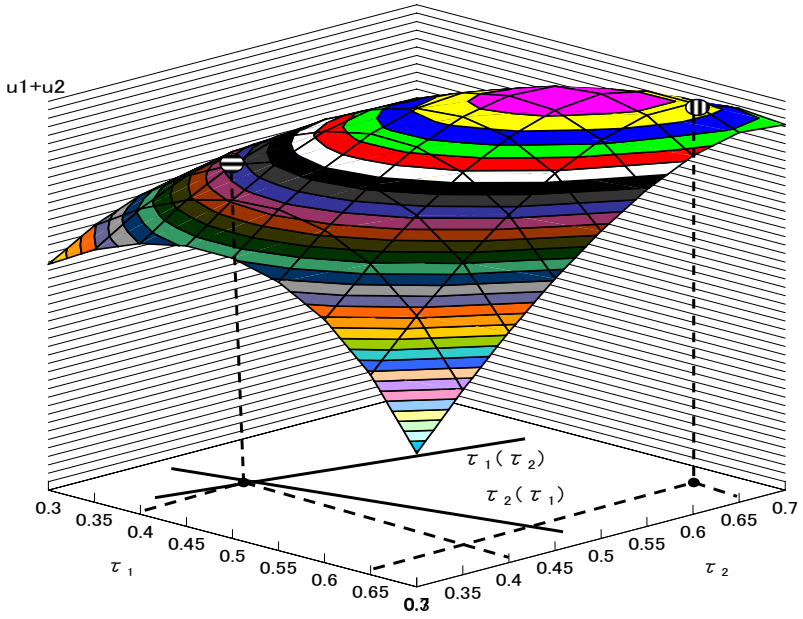


Figure 4-2-2: Nash equilibrium solution, cooperative solution and optimal solution in symmetric regions (in the case of $t^y = t^o$).



Note 1: Circles with vertically-striped pattern indicate the optimal solutions, and those with lateral-striped pattern indicate the Nash equilibrium solutions.

Note 2: Reaction functions are shown on the τ_1-τ_2 plain.

capital tax competition within symmetric regions, and has also indicated that decreased welfare by underprovision of public goods due to the cut down of the capital tax rate could be improved by collaboration between the local governments or intervention by the central government.

Interestingly, the case of equal taxation on both generations shows a bigger divergence between the capital tax rates of the Nash equilibrium solution and the cooperative solution equaling the optimal solution, in comparison with the case where the tax is imposed only on older generations. We here discuss the factors based on Tables 4-2-3 and 4-2-4. These tables show the variables in simulation.

In the case of equal taxation on both older and younger generations, because capital accumulation is blocked by the lump-sum tax on younger generations, savings S and private capital K become smaller, compared to that where the tax is imposed only on older ones. The capital tax rate of the Nash equilibrium solution is 0.4 in both cases, and the capital tax revenue ($\tau\rho K$) becomes smaller in the case of equal taxation on both generations. Moreover, the standard of public capital (G_L) supplied by the local governments becomes smaller, whose financial resources are based on the capital tax revenue, and the entire public capital (G), including the public capital (G_C) which is supplied by the financial resources based on fiscal transfer from the central government, also becomes smaller.

Table 4-2-3: Economic variables (symmetric regions, $G=100, t^y=0$)

	G	Gl	τ	ρ	θ	w	K	S	cy	co
Optimal	536.47	436.47	0.55	5.21	2.35	793.58	152.18	304.37	489.21	818.59
	536.47	436.47	0.55	5.21	2.35	793.58	152.18	304.37	489.21	818.59
Nash	387.36	287.36	0.40	5.29	3.18	718.39	135.69	271.39	447.00	933.46
	387.36	287.36	0.40	5.29	3.18	718.39	135.69	271.39	447.00	933.46

Table 4-2-4: Economic variables (symmetric regions, $G=100, t^y=t^o$)

	G	Gl	τ	ρ	θ	w	K	S	cy	co
Optimal	592.75	492.75	0.65	6.20	2.17	758.08	122.24	244.49	446.92	708.47
	592.75	492.75	0.65	6.20	2.17	758.08	122.24	244.49	446.92	708.47
Nash	359.63	259.63	0.40	6.38	3.83	649.08	101.67	203.34	379.08	915.57
	359.63	259.63	0.40	6.38	3.83	649.08	101.67	203.34	379.08	915.57

Note: Upper rank shows figures of Region 1 while lower rank indicates those of Region 2, for both of the optimal solution and the Nash equilibrium solution.

Meanwhile, the capital tax rate and public capital (G_L) become larger in the case of equal taxation on both generations when the cooperative solution equals to the optimal solution. This can be explained that the blocking effect of private capital accumulation by imposing a lump-sum tax on younger generations is stronger than the promotion effect by the cut down of the rate of return on capital (ρ) through lowering the capital tax rate, and that leads to an increased provision of public capital, or the elevation of the capital tax rate, in order to improve the deteriorated social welfare.

Therefore, it can be construed that the divergence between the capital tax rate of the Nash equilibrium solution and the cooperative solution equaling the optimal solution becomes bigger, comparing between lump-sum taxation only on older generations and equal taxation on both generations, when productivity is symmetric.

4-3. Capital tax competition within the asymmetric regions.

Now we look at the case where the local governments are competing against each other over the capital tax rate based on the condition that each government is given the other's actions, when the labor and public capital productivity are asymmetric between the two regions. Like the case in 4-2, it is assumed that the central government procures financial resources for public capital from lump-sum taxes whereas the local governments do from the capital taxes. The central government has an option to impose tax only on older generation ($t^y = 0$) or to equally impose on both older and younger generations ($t^y = t^o$).

Also, the public capital standard made available by the fiscal transfer from the central government is 100, just like the case in 4-2. However, the parameters of production function in Region 1 are $\beta_1^L = 0.5, \beta_1^K = 0.3, \beta_1^G = 0.2$ whereas those in Region 2 are $\beta_2^L = 0.55, \beta_2^K = 0.3, \beta_2^G = 0.15$. It means that the value of elasticity for RDP of public capital is relatively high, or that of labor is relatively low in Region1. Region 1 would be a metropolitan area and Region 2 would be a country area in

Japanese context[3].

Figures 4-3-1 and 4-3-2 show the Nash equilibrium solution, the cooperative solution and optimal solution (or social welfare maximization point) between asymmetric regions with a different productivity level. For both cases, one that the capital tax rate is imposed only on older generations (figure 4-3-1) and the other that a lump-sum tax is imposed equally on younger and older generations (figure 4-3-2), the social welfare is higher in the optimal solution than in the Nash equilibrium solution. The simulation result is the same as that of 4-2, which is based on symmetric regions.

There are two distinctions in the simulation on the assumption of asymmetric regions. Firstly, different tax rates are chosen between Region 1 and 2 for the capital tax in the Nash equilibrium solution as well as taxation equations for such tax ($t^y = 0, t^y = t^o$). More specifically, the rate is 0.4 in Region 1 and 0.2 in Region 2.

It is considered that there are two effects by lowering the capital tax rate. One of them is to promote capital accumulation by attracting private capital to the region, and the other is to hinder capital accumulation by decreasing the provision of public capital.

The reason why the capital tax rate in the Nash equilibrium solution becomes lower in Region 2 than in Region 1 can be explained that the productivity of public capital is relatively low in Region 2, and promoting capital accumulation by lowering the capital tax rate may improve the welfare more effectively than elevating the rate to increase public capital, which accordingly leads to the promotion of private capital accumulation. The result is consistent with related literature such as Bucovetsky (1991) and Wilson (1991) which theoretically clarified the tax competition between asymmetric regions[4].

The second distinction in the simulation on the assumption of asymmetric regions is that the optimal solution corresponds to one of the cooperative solutions in the simulation of asymmetric region, whereas the cooperative solution is perfectly matching the optimal solution in that of symmetric region. By looking at each social

[3] According to Homma and Tanaka (2004), the value of elasticity for RDP of public capital was 0.22 in a metropolitan area whereas that in a country area was 0.06. The result was based on a positive analysis using prefectural panel data between 1977 and 2000.

[4] Asymmetry between regions is described as not the difference of productivity but the difference of population in Bucovetsky (1991) and Wilson (1991).

Figure 4-3-1: Nash equilibrium solution, cooperative solution and optimal solution in asymmetric regions (in the case of $t^y = 0$).

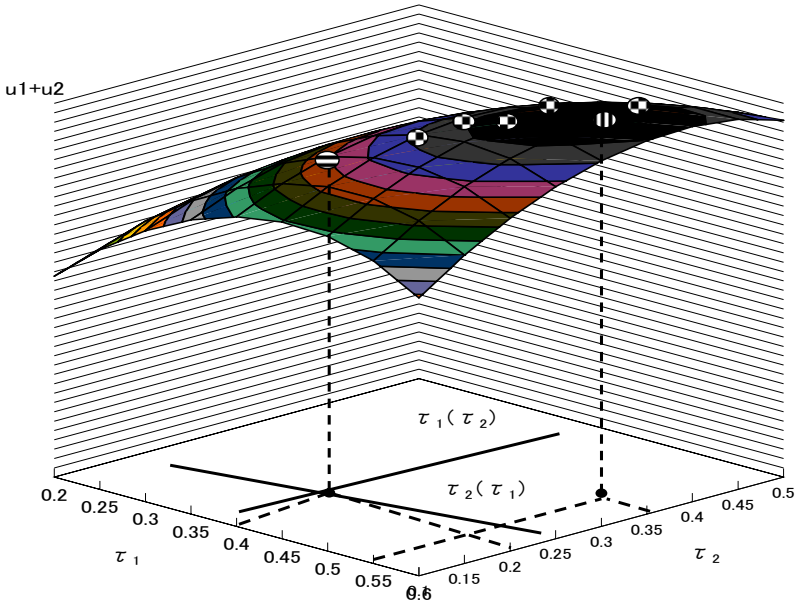
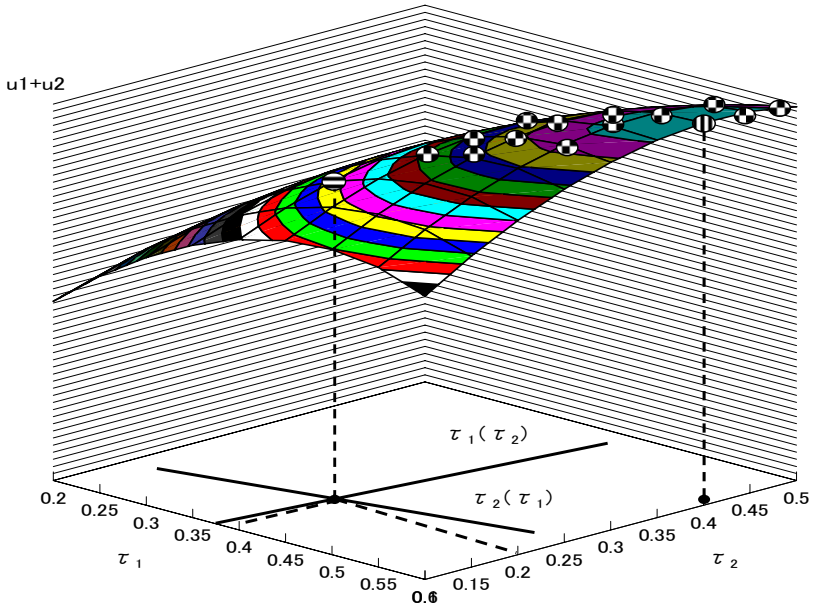


Figure 4-3-2: Nash equilibrium solution, cooperative solution and optimal solution in asymmetric regions (in the case of $t^y = t^o$).



Note 1: Circles with vertically-striped pattern indicate the optimal solutions, those with lateral-striped pattern indicate the Nash equilibrium, and those with checked pattern indicate the cooperative solutions.

Note 2: Reaction functions are shown on the τ_1- τ_2 plain.

welfare standard of Region 1 and 2, it is apparent that there are several cooperative solutions more favorable to Region 1 and several cooperative solutions more favorable to region2 as you can see in figures 4-3-1 and 4-3-2. Any cooperative solutions show Pareto improvement than the Nash equilibrium solution, and policy coordination between the local governments under the capital tax competition can improve the welfare.

The question is which cooperative solution can actualize such policy coordination. The local governments would have incentives to move from the Nash equilibrium solution to a cooperative solution given that policy coordination can make a Pareto improvement possible. But another cooperative solution may exist which can possibly achieve a higher social welfare standard than the actualized cooperative solution either in Region 1 or 2. It is, thus, realistic to consider that the cooperative solutions are dependent on the magnitude correlation of the bargaining power between the local governments.

The optimal solution can be predominant over other cooperative solutions when the equity standard of the central government is a Benthamite type. There is no predominance in welfare among cooperation solutions in the light of the efficiency standard[5]. Under the circumstances where the central government involves, one cooperative solution among others shall be chosen as the optimal solution only when the central government has both of the efficiency and equity standards. More specifically, the position of the optimal solution in simulation shall be correspondently determined when the social welfare function of the central government is defined as a Benthamite type.

Similarly to the symmetric regions, from tables 4-3-3 and 4-3-4 showing each variable in the simulation, it can be said that both saving S and private capital K are smaller in the case of equal taxation on both generation in the optimal solution and the Nash equilibrium solution than the case where the tax is imposed only on older generation because the lump-sum tax on younger generation hinders capital

[5] Accordingly, it is not possible to set priorities among cooperative solutions from the perspective of the efficiency of resource allocation, since the utilities of Region 1 and 2 are on the utility possibility frontier in any cooperative solutions.

accumulation. As we mentioned in 4-2, the reason can be explained that equal taxation on both generation has a bigger blocking effect.

Table 4-3-3: Economic variables (asymmetric regions, $G=100, t^y=0$)

	G	Gl	τ	ρ	θ	w	K	S	cy	co
Optimal	525.42	425.42	0.55	5.46	2.46	773.50	141.67	296.40	477.09	824.63
	248.45	148.45	0.35	3.78	2.46	518.39	112.21	211.37	307.03	530.68
Nash	385.52	285.52	0.40	5.36	3.21	713.80	133.24	269.57	444.23	936.03
	176.83	76.83	0.20	4.02	3.21	469.52	95.61	188.14	281.37	592.88

Table 4-3-4: Economic variables (asymmetric regions, $G=100, t^y=t^o$)

	G	Gl	τ	ρ	θ	w	K	S	cy	co
Optimal	517.26	417.26	0.60	6.93	2.77	695.43	100.42	221.38	407.39	767.96
	256.83	156.83	0.40	4.62	2.77	479.19	84.92	149.30	263.23	496.21
Nash	352.29	252.29	0.40	6.73	4.04	630.72	93.67	196.84	367.22	925.36
	168.98	68.98	0.20	5.05	4.04	421.57	68.30	127.12	227.79	574.01

Note: Upper rank shows figures of Region 1 while lower rank indicates those of Region 2, for both of the optimal solution and the Nash equilibrium solution.

When looking at the public capital standard (G) in both regions at the optimal solution, G of Region 1 is greater in the case of taxation only on older generations (525.42) than that of equal taxation on both generations (517.26). This means that the accumulation level of public capital is relatively low in the case of equal taxation. This can be because the sensitivity of private capital for the capital tax rate τ differs between Region 1 and 2[6]. Capital flight from Region 1 by the elevation of τ exceeds that in Region 2, and the extent of decrease of private capital in Region 1 becomes larger. Therefore, the capital tax revenue in the case of equal taxation becomes smaller than that of taxation only on older generations. It is accordingly considered that G in Region 1 becomes relatively small, which leads to lower the welfare of Region 1.

5. Conclusion.

In this paper, we have shown some important policy implications on the capital tax competition as financial resources of public investment, by considering the effects of

[6] In other words, the gradient of the capital demand function is different between Region 1 and 2.

accumulation of public and private capitals over the production and by explaining such accumulation using an overlapping generations model. We have also confirmed from our dynamic model that the Nash equilibrium solution of capital tax competition, which has been indicated by the traditional static model, choose a lower capital tax rate than the optimal solution. Now, we summarize our contribution in connection with model expansion.

The first distinction is that we have given an analysis taking the productivity effects of public capital into consideration based on a dynamic mode. Public capital contributes not only to public input in itself, but also to increase the private capital demand. This means that it leads to the increase of capital tax revenue.

As to adopting the capital tax for financial resources of public investment, it is necessary to consider the inefficiency from distortion brought by the capital tax as well as the positive effects generated by the increase of public capital. In accordance with our simulation analysis in this paper, it is confirmed that the optimal capital tax rate at the Nash equilibrium solution falls below that at the optimal solution, even in the case where the effect of public capital to uplift the private capital demand.

The second distinction is that we have indicated there is a case where the capital tax rate becomes a positive number at the optimal solution by adopting an overlapping generations model, contrary to some related literature supporting that the optimal capital tax rate becomes zero with a static model, which is a traditional model of the capital tax competition. It can be said that the conclusion, which shows the rate becoming zero with a static model is limited to a case where the lump-sum tax is imposed only on older generations with an overlapping generations model. When taking the productivity effects of public and private capitals into consideration, it is important not to overlook the effects of taxation on capital accumulation. It is essential to model a dynamic process of capital accumulation for this analysis.

Our analysis in this paper shows an expansion from the traditional capital tax competition model. We analyze taxation effects by using an overlapping generations model and demonstrating that savings at early life form private capital. The result of simulation analysis indicates that the lump-sum tax on younger generations has an

effect to decrease their savings, in comparison with the capital tax. Accordingly, the capital taxation has primacy in the aspect of capital accumulation.

Thirdly, we show there is more than one cooperative solution in addition to the Nash equilibrium solution and the optimal solution by modeling a case where the public capital productivity is different in two regions. The existence of plural optimal cooperative solutions gives a new point, such as negotiation between local governments or mediation of the central government, to the issue of which cooperative solution should be chosen. The optimal solution in this paper is one of the cooperative solutions. It is the solution chosen by the central government's adoption of a Benthamite social welfare function.

Now, we would like to note some pending issues. Firstly, it should be noted the analysis result shown in this paper may possibly be dependent on the parameters of specified utility function and production function in the simulation. A simulation analysis makes it possible to analyze with more complicated model. This is one of the advantages of simulation analysis. Our analysis in this paper is a combination of the analyses of related literature and the conclusion obtained here is organically connected with preceding conclusions.

Our conclusion here is a consequence led by a choice of policy variable based on the assumption of a constant parameter, whereas the related literature has uses a theoretic model to lead a qualitative conclusion. Deliberate sensitivity analysis is indispensable in order to improve the generality of analysis.

We can point out as the second issue that the options of policy instruments by the central government and local governments are limited. We discuss the lump-sum tax and capital tax as financial resources for public investment and expanded them in comparison with related literature. Our contribution here is that we have integrated heterogeneity in the sense that the two regions have different production techniques, however, it is based on an assumption that the central government takes a symmetric policy in fiscal transfer and taxation on both regions whereas the local government adopt an asymmetric policy.

According to our analysis, there is a regional disparity, specifically the utility

standard is lower in Region 2 than Region 1. It may suggest that an asymmetric policy should be chosen for the purpose of regional redistribution. The central government applies different capital tax rates by region at the viewpoint of maximizing the social welfare function, to illustrate. It is important to consider the consequences when an asymmetric policy is chosen and to compare them with symmetric cases.

Thirdly, our analysis only makes comparison in the long-term steady state. We have indicated that the Nash equilibrium solution chooses a lower capital tax rate than the optimal solution does. It means that the utility rises at the optimal solution higher than at the Nash equilibrium solution due to the increased amount of public investment and elevation of the capital tax rate. It should be noted, however, that this is based on the comparison of two steady states.

The result may be different in a transition process by any policy changes from that in the steady state. It is necessary to study the role of policy in the transition process besides in the steady state. In that case, such analysis should be made with a social discount rate, a new standard for the social welfare function.

Appendix

Table 4-1-1: Optimal fiscal transfer (subsidies) and optimal capital tax rate ($t^y = 0$)

Subsidy amount	Capital tax rate			
	0	0.05	0.3	0.45
0		16.645	18.541	18.838
400	19.148	19.180	19.237	19.176
600	19.259	19.264	19.184	19.031
670	19.266	19.263	19.139	18.952
700	19.265	19.258	19.116	18.915

Table 4-1-2: Optimal fiscal transfer (subsidies) and optimal capital tax rate ($t^y = t^o$)

Subsidy amount	Capital tax rate			
	0	0.5	0.65	0.7
0		18.892	18.952	18.939
10	16.366	18.884	18.937	18.922
160	17.932	18.638	18.627	18.598
170	17.933	18.615	18.602	18.571
180	17.931	18.591	18.575	18.545

Note: Figures in the tables indicate those of social welfare. Only proximities of the optimal points are shown.

Table 4-2-1: Nash equilibrium solution, cooperative solution and optimal solution in symmetric regions (in the case of $t^y = 0$).

u1

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	9.476	9.490	9.505	9.520	9.536	9.552	9.569	9.585	9.599
0.35	9.487	9.502	9.518	9.535	9.552	9.570	9.589	9.606	9.623
0.4	9.488	9.505	9.522	9.540	9.560	9.579	9.600	9.620	9.639
0.45	9.480	9.497	9.516	9.536	9.557	9.579	9.602	9.624	9.647
0.5	9.460	9.479	9.499	9.521	9.544	9.569	9.594	9.619	9.645
0.55	9.428	9.448	9.470	9.494	9.520	9.547	9.575	9.604	9.634
0.6	9.381	9.403	9.427	9.453	9.481	9.511	9.543	9.576	9.611
0.65	9.316	9.340	9.366	9.394	9.425	9.459	9.495	9.533	9.573
0.7	9.228	9.254	9.282	9.313	9.347	9.384	9.425	9.469	

u2

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	9.476	9.487	9.488	9.480	9.460	9.428	9.381	9.316	9.228
0.35	9.490	9.502	9.505	9.497	9.479	9.448	9.403	9.340	9.254
0.4	9.505	9.518	9.522	9.516	9.499	9.470	9.427	9.366	9.282
0.45	9.520	9.535	9.540	9.536	9.521	9.494	9.453	9.394	9.313
0.5	9.536	9.552	9.560	9.557	9.544	9.520	9.481	9.425	9.347
0.55	9.552	9.570	9.579	9.579	9.569	9.547	9.511	9.459	9.384
0.6	9.569	9.589	9.600	9.602	9.594	9.575	9.543	9.495	9.425
0.65	9.585	9.606	9.620	9.624	9.619	9.604	9.576	9.533	9.469
0.7	9.599	9.623	9.639	9.647	9.645	9.634	9.611	9.573	

u1+u2

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	18.951	18.976	18.993	19.000	18.996	18.980	18.950	18.901	18.828
0.35	18.976	19.004	19.023	19.032	19.031	19.019	18.992	18.946	18.877
0.4	18.993	19.023	19.044	19.056	19.059	19.050	19.027	18.986	18.921
0.45	19.000	19.032	19.056	19.072	19.078	19.073	19.055	19.019	18.959
0.5	18.996	19.031	19.059	19.078	19.089	19.088	19.075	19.045	18.992
0.55	18.980	19.019	19.050	19.073	19.088	19.093	19.086	19.063	19.018
0.6	18.950	18.992	19.027	19.055	19.075	19.086	19.086	19.071	19.035
0.65	18.901	18.946	18.986	19.019	19.045	19.063	19.071	19.066	19.042
0.7	18.828	18.877	18.921	18.959	18.992	19.018	19.035	19.042	

Table 4-2-2: Nash equilibrium solution, cooperative solution and optimal solution in symmetric regions (in the case of $t^y = t^o$).

u1

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	9.289	9.307	9.326	9.345	9.364	9.383	9.402	9.420	9.435
0.35	9.302	9.321	9.342	9.362	9.383	9.404	9.425	9.445	9.463
0.4	9.305	9.326	9.348	9.370	9.393	9.416	9.439	9.461	9.482
0.45	9.297	9.319	9.343	9.367	9.392	9.418	9.443	9.468	9.492
0.5	9.277	9.301	9.327	9.353	9.381	9.409	9.438	9.466	9.493
0.55	9.244	9.270	9.298	9.327	9.357	9.388	9.420	9.453	9.485
0.6	9.195	9.224	9.253	9.285	9.318	9.353	9.389	9.426	9.464
0.65	9.127	9.157	9.190	9.224	9.261	9.300	9.341	9.384	9.428
0.7	9.033	9.066	9.101	9.139	9.180	9.224	9.271	9.320	9.371

u2

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	9.289	9.302	9.305	9.297	9.277	9.244	9.195	9.127	9.033
0.35	9.307	9.321	9.326	9.319	9.301	9.270	9.224	9.157	9.066
0.4	9.326	9.342	9.348	9.343	9.327	9.298	9.253	9.190	9.101
0.45	9.345	9.362	9.370	9.367	9.353	9.327	9.285	9.224	9.139
0.5	9.364	9.383	9.393	9.392	9.381	9.357	9.318	9.261	9.180
0.55	9.383	9.404	9.416	9.418	9.409	9.388	9.353	9.300	9.224
0.6	9.402	9.425	9.439	9.443	9.438	9.420	9.389	9.341	9.271
0.65	9.420	9.445	9.461	9.468	9.466	9.453	9.426	9.384	9.320
0.7	9.435	9.463	9.482	9.492	9.493	9.485	9.464	9.428	9.371

u1+u2

$\tau 1$ \ $\tau 2$	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.3	18.577	18.609	18.630	18.642	18.642	18.628	18.597	18.546	18.468
0.35	18.609	18.643	18.667	18.682	18.685	18.675	18.649	18.602	18.528
0.4	18.630	18.667	18.695	18.713	18.720	18.714	18.692	18.651	18.583
0.45	18.642	18.682	18.713	18.735	18.746	18.745	18.728	18.693	18.632
0.5	18.642	18.685	18.720	18.746	18.762	18.766	18.756	18.727	18.674
0.55	18.628	18.675	18.714	18.745	18.766	18.777	18.774	18.753	18.709
0.6	18.597	18.649	18.692	18.728	18.756	18.774	18.779	18.768	18.734
0.65	18.546	18.602	18.651	18.693	18.727	18.753	18.768	18.768	18.747
0.7	18.468	18.528	18.583	18.632	18.674	18.709	18.734	18.747	18.743

Note 1: Figures in the tables indicate those of social welfare.
Note 2: Figures in thick line frames show the optimal solutions, or the maximization points of social welfare, and those in double line frames show the Nash equilibrium solution.

Table 4-3-1: Nash equilibrium solution, cooperative solution and optimal solution in asymmetric regions (in the case of $t^y = 0$).

u1

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	9.421	9.428	9.436	9.444	9.452	9.460	9.468	9.476	9.484
0.25	9.453	9.462	9.470	9.479	9.488	9.497	9.506	9.515	9.523
0.3	9.477	9.486	9.495	9.504	9.514	9.524	9.534	9.544	9.554
0.35	9.491	9.500	9.510	9.521	9.532	9.543	9.554	9.565	9.576
0.4	9.496	9.506	9.517	9.529	9.540	9.553	9.565	9.577	9.590
0.45	9.491	9.502	9.514	9.527	9.540	9.554	9.567	9.581	9.595
0.5	9.476	9.489	9.502	9.515	9.530	9.545	9.560	9.576	9.592
0.55	9.449	9.463	9.477	9.492	9.508	9.525	9.542	9.560	9.578
0.6	9.409	9.423	9.439	9.456	9.474	9.493	9.512	9.532	9.553

u2

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	8.775	8.782	8.783	8.777	8.765	8.745	8.717	8.681	8.635
0.25	8.784	8.792	8.794	8.789	8.777	8.757	8.730	8.695	8.650
0.3	8.794	8.803	8.806	8.801	8.790	8.772	8.745	8.711	8.666
0.35	8.804	8.815	8.818	8.815	8.805	8.787	8.762	8.728	8.685
0.4	8.816	8.827	8.832	8.830	8.821	8.804	8.780	8.747	8.705
0.45	8.828	8.841	8.847	8.846	8.838	8.822	8.799	8.768	8.727
0.5	8.841	8.855	8.863	8.863	8.856	8.842	8.821	8.791	8.752
0.55	8.854	8.871	8.879	8.881	8.876	8.864	8.844	8.816	8.779
0.6	8.869	8.886	8.897	8.901	8.897	8.887	8.869	8.844	8.809

u1+u2

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	18.195	18.211	18.219	18.221	18.216	18.204	18.185	18.157	18.119
0.25	18.237	18.254	18.264	18.268	18.264	18.254	18.236	18.209	18.173
0.3	18.270	18.289	18.301	18.306	18.304	18.296	18.279	18.255	18.220
0.35	18.295	18.315	18.329	18.336	18.336	18.330	18.316	18.293	18.260
0.4	18.311	18.334	18.349	18.358	18.361	18.357	18.345	18.324	18.294
0.45	18.319	18.343	18.361	18.373	18.378	18.376	18.367	18.349	18.322
0.5	18.317	18.344	18.364	18.378	18.386	18.387	18.381	18.367	18.343
0.55	18.304	18.333	18.357	18.374	18.385	18.389	18.387	18.377	18.357
0.6	18.277	18.310	18.336	18.357	18.371	18.380	18.382	18.376	18.362

Table 4-3-2: Nash equilibrium solution, cooperative solution and optimal solution in asymmetric regions (in the case of $t^y = t^o$).

u1

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	9.209	9.219	9.229	9.240	9.250	9.260	9.270	9.279	9.288
0.25	9.244	9.255	9.267	9.278	9.289	9.300	9.311	9.321	9.331
0.3	9.270	9.282	9.294	9.306	9.319	9.331	9.343	9.354	9.365
0.35	9.286	9.299	9.312	9.326	9.339	9.352	9.365	9.378	9.391
0.4	9.293	9.307	9.321	9.335	9.350	9.365	9.379	9.394	9.408
0.45	9.290	9.304	9.320	9.335	9.351	9.368	9.384	9.400	9.416
0.5	9.275	9.291	9.308	9.325	9.342	9.360	9.378	9.397	9.415
0.55	9.247	9.265	9.283	9.302	9.321	9.341	9.362	9.382	9.403
0.6	9.204	9.224	9.244	9.265	9.286	9.309	9.332	9.355	9.379

u2

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	8.514	8.523	8.524	8.518	8.504	8.482	8.452	8.412	8.362
0.25	8.531	8.541	8.543	8.538	8.525	8.504	8.475	8.437	8.387
0.3	8.549	8.559	8.563	8.559	8.547	8.528	8.500	8.463	8.415
0.35	8.567	8.579	8.583	8.580	8.570	8.552	8.526	8.490	8.443
0.4	8.586	8.599	8.605	8.603	8.594	8.578	8.553	8.519	8.474
0.45	8.605	8.620	8.627	8.627	8.620	8.605	8.582	8.550	8.507
0.5	8.625	8.641	8.650	8.652	8.646	8.633	8.612	8.582	8.542
0.55	8.644	8.662	8.673	8.677	8.673	8.662	8.644	8.616	8.578
0.6	8.664	8.684	8.697	8.702	8.701	8.693	8.677	8.652	8.617

u1+u2

τ1 \ τ2	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.2	17.723	17.741	17.753	17.757	17.754	17.742	17.722	17.692	17.650
0.25	17.775	17.796	17.809	17.815	17.814	17.804	17.786	17.758	17.719
0.3	17.819	17.841	17.857	17.865	17.866	17.858	17.843	17.817	17.780
0.35	17.853	17.878	17.896	17.906	17.909	17.904	17.891	17.868	17.834
0.4	17.879	17.906	17.926	17.939	17.944	17.943	17.932	17.913	17.882
0.45	17.894	17.924	17.947	17.962	17.971	17.973	17.966	17.950	17.923
0.5	17.899	17.932	17.957	17.976	17.989	17.993	17.990	17.979	17.956
0.55	17.892	17.927	17.956	17.979	17.995	18.004	18.005	17.998	17.981
0.6	17.869	17.908	17.940	17.967	17.987	18.001	18.008	18.007	17.996

Note 1: Figures in the tables indicate those of social welfare.
Note 2: Figures in thick line frames show the optimal solutions, or the maximization points of social welfare, shadowed figures indicate the cooperative solutions, and those in double line frames show the Nash equilibrium solution.

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