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Foreign Direct Investments and trade in agriculture: an incomplete contracts approach*

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Abstract.

Despite the attention given in recent years to the growth of foreign land acquisitions, there have been few studies investigating the pattern of recent Foreign Direct Investment (FDI) in agriculture and the ones that have are mostly focused on the locational drivers of FDI. This paper explores how the contractual features of transactions of agricultural products affect the "internalization" decision of firms, that is, the choice trade/FDI. The paper develops a partial equilibrium model incorporating incomplete contracts and asset specificity, which is used to address a number of questions: What is the impact of the quality of the institutions on the choice trade/FDI? How may the bargaining power of the downstream and upstream firms affect the outcome? How is the choice FDI/trade affected by the presence of a state–owned firm? The model provides some uncommon results, such as the finding that when the investor is private, weak institutions may promote FDI.

Keywords: Foreign Direct Investment, foreign land acquisitions, agriculture, firm organization, bargaining power

JEL codes: Q15, F23, L23, Q17.

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1. Introduction

Media reports about the rapid growth in the number of foreign land acquisitions in Africa, Latin America, Central Asia and South East Asia have brought the phenomenon to the attention of the international community. Early estimates reported the acquisition of millions of hectares of land in few years by private firms, private equity or governments (e.g. Von Braun, Meinzen-Dick, 2009). The rise in commodity prices and the lack of attractive alternative investment opportunities during the 2008 financial crisis led to marked increases in the demand for agricultural land by foreign firms. Foreign Direct Investment (FDI) in agriculture, however, is not a new phenomenon. At the beginning of the 20th century, agri-food firms from developed countries started to produce raw materials and cash crops in developing countries abundant in natural resources (sugar, rubber, bananas, coffee, cotton, tea and so on); agricultural commodities were then exported to the home markets. After the Second World War, FDI in agriculture progressively decreased, mainly because of the restrictive policies adopted by many developing countries on foreign ownership of land. As a consequence, in the final part of the century, multinational traders/processors progressively disinvested from plantations and purchased agricultural raw materials mostly through contracts with local farmers (Unctad, 2009). The recent wave of FDI in agriculture has reversed this trend and, more importantly, changed the pattern shown in the past century. Recent studies emphasize the distinguishing features of recent foreign land acquisitions: the emergence of South-South flows, with new investors from developing countries as well as the Gulf states; the rapid expansion of North-South FDI in new sectors (e.g, biofuels); the concentration of FDI in least developed countries and, more generally, in areas where governance is weak (e.g. Von Braun, Meinzen-Dick, 2009; Unctad, 2009; Cotula et al, 2009; World Bank, 2011; Arzeki et al, 2014; Anseew et al 2012).

This paper addresses a different research question from those addressed by previous studies on the recent foreign land acquisitions: why do some firms choose to vertically integrate by means

of FDI, while others acquire raw materials from local farmers through contractual arrangements? In other words, what factors drive the choice between FDI and trade in agriculture – the so-called "internalization" motives for FDI? To explain the recent growth of intra-firm trade in intermediate products, a number of studies have investigated why some firms choose outsourcing, while others maintain tight control over foreign production operations (i.e. FDI), by means of international trade models which incorporate various elements of firm organization theories. [2] Of particular interest for the issue addressed here are the papers by Antras (2003) and Antras, Helpman (2004, 2008) who apply the Grossman and Hart (1986) model of allocation of property rights to international trade. The basic idea is that contract incompleteness significantly influences the international trade of intermediate products. The input producer and the downstream firms cannot sign ex-ante enforceable contracts, specifying all the details involved. This is because a number of aspects may be unknown at time of signing or there may be too many possible contingencies to be stipulated in a contract; such contingencies may be particularly relevant in international transactions because of the time lags between order and delivery. Monitoring and enforcing an international contract may be costly, also because of the different legal frameworks and institutions in the various countries. These trading problems could be exacerbated by the presence of relation specific investments; this happens if one party, after making the investments necessary for the production of the good, finds it difficult to find other buyers/sellers if the other party breaches the contract. If this is the case, even though the upstream and downstream firms may select ex-ante from a set of competitive suppliers and buyers, they end up forming an ex-post bilateral monopoly, in that they have an incentive to trade with each other instead of with outsiders. Contract incompleteness and relation-specific investments prevent both parties from optimal production and investments (the hold-up problem). The firm chooses the optimal organization before investments by taking into account this contractual environment. There are two possible options: trade if parties remain unaffiliated, and FDI if the downstream firm takes the control of the asset of the upstream firm.

The key assumptions of the Grossman and Hart (1986) model - i.e. specific investments and contract incompleteness – appear realistic in the case of the international transactions of agricultural products. Several sources of lock-in between buyers and sellers can be found in such

[2] A review of this literature can be found in Helpman (2006) and Antras (2014).

transactions. For example, in order to maintain their quality, perishable agricultural goods, such as sugar, need processing within a tight time frame, hence the need for the downstream firm to locate processing plants close to the area of production. After the farmer has grown the crop and the processing firm has built the plant, i.e. having committed themselves in terms of investment, they are locked into a bilateral monopoly. Also for non-perishable products similar issues arise. Plantations, such as jatropha used to produce biofuels, imply initial investments by farmers and by processors, who need to locate processing plants close to plantations. Ex-post, they are tightly bound to each other because, although seeds can be easily stored, the transportation cost is too high to make it sustainable for farmers and the downstream firm to choose another buyer or seller ex post. In these cases, penalties for breach of contract may be desirable because they bind both parties to each other and prevent opportunism, but the contracts are likely to be incomplete for a number of reasons. The time span between the signing and the enforcement of the contract maybe even longer than in the case of the manufactured intermediate products; for instance, the time lag between plantation and harvesting can be more than one year for sugar and 3-4 years for jatropha. Contingencies unforeseen at the time of signing are, thus, likely to occur. The high degree of volatility and uncertainty in international agricultural markets makes such contingencies even more likely.

This paper assumes the same contractual and bargaining environment as that developed by Antras (2003) and Antras, Helpman (2004, 2008), but develops a different market set-up to capture the distinctive features of agriculture. International transactions in natural resources differ from those in manufacturing for two main reasons (Venables, Ruta, 2012): i) natural resources are immobile and countries where such resources are lacking or scarce seek access to these natural resources either by FDI or trade in countries where they abound; ii) state-owned firms are particularly active in a number of transition and developing countries, and provide important sources of FDI in primary industries.

The model developed herein considers one downstream firm originating from the land-scarce food/energy importing country, and one agricultural producer located in the land-abundant food/energy exporting country. Because the agricultural products involved in recent foreign land acquisitions are mostly undifferentiated goods, the product exported to the final market is assumed to be homogeneous. In addition, different objective functions of investing firms are considered to take into account the various types of firms (private or state-owned) involved in

recent FDI in agriculture. Finally, because of the highly concentrated structure of international commodity markets, market power is considered; more specifically, the downstream firm is assumed to be a monopolist on the final market. Hence, the incomplete contracts theory is incorporated in a simple homogeneous product-monopoly-partial equilibrium framework.
The model is used to address relevant research questions for the understanding of the drivers of FDI in agriculture: do better institutions and strong legal protection in the host country promote FDI? How does the higher bargaining power of the downstream firm affect the choice FDI/trade? Is the state-owned firm's choice FDI/trade driven by different factors from those of the private investor? The model offers a number of insights into these issues and shows how consideration of the contractual environment of international transactions may contribute to explain the empirical findings. For example, the model predicts that, under certain circumstances, weak legal protection in the target country may exert a positive impact on FDI, thus offering one possible explanation for the negative relation between agricultural FDI and governance indicators found by empirical studies.
The paper is organized as follows. Building on the available literature, the next section reviews a few stylized facts about FDI in agriculture. The third section illustrates the model, while the fourth investigates on the main differences between the private and the state owned firm. The fifth section explores the relationship between the bargaining power, the quality of the institutions and the choice of the firms, while the sixth offers some concluding remarks.

2. FDI in agriculture: a few stylized facts

The sharp rise in the number of foreign land acquisitions after 2007 has captured the attention of media, NGOs and international organizations and become in recent years a hot issue in the international debate about development and land governance. Evidence about the size, types and features of this phenomenon at the global level, however, is still scarce and controversial. Nowadays, a considerable number of case studies are available, and significantly contribute to improving our knowledge of the determinants and the impact of foreign land acquisitions in specific areas and industries; so far, however, there have been few analyses of the phenomenon at the global and cross-country level. The main barrier to research in this field is the lack of official cross-country data. Governments of target and investing countries in some cases do not make data public because they are concerned about the consequences that the dissemination of

this kind of information could have, given that recent foreign land acquisitions are a highly sensitive issue politically. In other cases, official data are not available because local and/or national administrations find it difficult to collect, store and maintain data on land deals; local administrations themselves do not have updated and consolidated information on the acquisitions of land in their own territory (Cotula *et al*, 2009).

As a consequence, most studies so far have based their analyses on information collected by media or from other unofficial sources and their conclusions have been challenged because of the poor quality of the data on which they are based. [3] Among the main critiques, it has been argued that these data significantly overestimate the global amount of land involved in foreign land acquisitions; the role of the new investors - such as China, Gulf and East Asian countries – with respect to the more traditional ones (US and EU countries) is overvalued and too much emphasis is given to the acquisitions of marginal and idle land compared to agricultural land. Efforts to improve the quality of the data have recently been carried out by many organizations and researchers. As a result, the latest estimates have considerably reduced the overall amount of hectares involved in foreign land acquisitions and emphasized the still dominant role of traditional investing countries and of the acquisitions of agricultural land (e.g. Anseeuw et al 2013; Schoneveld, 2014; Messerli *et al*, 2014).

Recent data from the Land Matrix [4] report almost 1,000 transnational deals involving about 60 million ha of land. One third of this land is used to produce raw materials for the biofuel industry, another third to produce food and about 27% is forestry. The majority of deals involve large scale acquisitions in developing countries in South East Asia (31%), Eastern Africa (21%), South America (15%) and Western Africa (12%). The top ten investors involved include developed countries that traditionally invest in the agri-food industry abroad (US, Great Britain, Netherlands and Canada), but also many new investors coming from the South, such as Malaysia, Singapore, United Arab Emirates, India, South Arabia, Hong Kong, Republic of Korea and China.

[3] For a critical review of the available dataset and analyses see, among others, Oja (2013), Schoneveld (2014) and Arezki *et al* (2014).

[4] The Land Matrix Global Observatory is a database compiled by NGOs and research institutes coordinated by the International Land Coalition (Anseeuw et al 2013). Data are verified by the partners, and include deals starting from 2000, although deals concluded before 2007 are a very small percentage of the total (Arezki *et al* 2014).

The literature on foreign land acquisitions for agricultural production has highlighted a number of stylized facts that are of particular interest for this paper (e.g. Von Braun, Meinzen-Dick, 2009; UNCTAD, 2009; Cotula *et al*, 2009; World Bank, 2011; Anseew *et al* 2012; Arzeki *et al*, 2014; Schoneveld, 2014).

Unlike manufacturing industries, a considerable part of land investments concerns countries differing in natural resources endowments. Indeed, most deals involve firms coming from countries relatively scarce in natural resources (land, water, forestry) which acquire land in countries abundant in natural resources, with the primary objective of exporting agricultural materials back to the home country. This one-way natural-resource driven FDI includes the large scale investments in land by firms from food insecure developing countries, such as Northern African and Middle East countries, in African countries, where land and water is abundant; the long-standing food security concern of the former has been reinforced in the last decade by the expected shortages in the staple food supply associated with the high international food prices (Von Braun, Menzen-Dick, 2009; Unctad, 2009; World Bank, 2011). A considerable part of the investments by developed food secure countries has also been driven by the relative abundance of natural resources in the host countries. This is the case of the European firms involved in the production of biofuels, which have acquired land in land-abundant countries to produce raw materials for the EU biofuel industry. Available evidence also indicates that a part of recent FLA has not been fueled by food or energy security concerns. Examples are the large-scale "speculative" acquisitions of cheap land often made by non-agrifood firms, which in some cases do not even start producing on the acquired land (e.g. Schoneveld, 2014; Cotula et al 2009). These types of FDI have received much attention and fueled the accusation of a new "land grab", in that they imply, by and large, the dispossession of the local population from their land (Borras, Franco, 2012).

Another distinguishing feature of foreign investment in agriculture is that, much more than in other sectors, foreign governments play a significant role in several ways (Venables, Ruta, 2012; Cotula et al 2009). In few cases the government itself, for example the Ministry of Agriculture, is involved in the direct acquisition of land in the target countries. More frequently, however, governments are indirectly involved because investing firms are owned by the state. This is the case with the acquisitions made by firms from countries where state-owned firms are of

particular importance, such as Saudi Arabia, China, Egypt, Djibouti, Lybia and Quatar. The few available data provide a rather different quantification of the role of state-owned firms in the acquisition of foreign land, depending also on the definition of the private/public nature of the firm. According to Schoneveld (2014), at least 10% of the projects reviewed in Africa involved state-owned firms; according to other data, reported by Arezki *et al* (2014), state-owned firms account for a share of the deals ranging from 7% to 26%. As highlighted by Cotula *et al* (2009) the role of state-owned investing firms is certainly higher in terms of the percentage of land acquired. Governments play a key role also in a number of target countries, where the land is owned by the State; this is the case of most African countries, while in South America and Asia deals tend to involve private firms.

Finally, different contractual forms are used to regulate the foreign firm's control over the natural resource in the host country. In Africa, as most of the land is owned or managed by the State, foreign firms agree on long-term land leases with the local government, commonly of one century (Cotula, 2011). The leasing contracts include payments of land fees and/or commitment by the investor to contribute capital and develop infrastructure. Also for the extraction of other natural resources, such as hydrocarbons, long-term contracts between governments and foreign firms are often put in place (Venables, Ruta, 2012). Because of the long-term duration of the lease and the frequent commitment of foreign firms to make investments, this type of arrangement is considered equivalent to FDI. In other countries where land is mainly owned and managed by the private sector, such as Latin American or Asian countries, foreign land acquisitions more frequently imply the full transfer of ownership from a local to a foreign firm. While long-term leases and land ownership transfers clearly qualify for (vertical) FDI, this is not the case with other forms of vertical control used in agriculture, such as contract farming or contracts specifying standards (Unctad, 2009), that are often used by (foreign) agri-food firms to reduce uncertainties about timing of the delivery and the quality of the raw materials purchased from local farmers. These contractual arrangements do not involve direct control by the foreign firm over land use, and are actually the most common types of arm's length international transactions of agricultural products.

3. Set-up of the model

The model considers two firms. Firm *M* is the agricultural producer in a land-abundant country, while firm *H* is the provider of the downstream services necessary to export a final product *x* in a land-scarce country. These services may include international trading services and/or physical processing of the agricultural raw material. The service provided by *H* is *h*, while firm *M* provides the agricultural raw material *m*. The production of *m* and *h* requires some initial investment by the firms, which sustain cost c_h and c_m for each unit of *h* and *m* provided. [5] Only *H* knows how to trade and/or process the final product *x,* which is assumed to be homogeneous, on the export market. As aforementioned, recent foreign land acquisition aims at producing agricultural products – such as sugar, oilseed, rubber, flour - which are not likely to be perceived by users as differentiated. The downstream firm is assumed to be a monopolist on the final market. This assumption significantly helps in terms of analytical tractability of the model, although it could be considered too simplistic; much of the literature has emphasized the prevalence of oligopoly in international agricultural markets. However, in the specific context of the developed model here, analyzing the choice FDI/trade of a single downstream firm, the assumption of monopoly may be considered as less naïve. Indeed, it is not so unusual to find just one downstream firm importing an agricultural product in the country of origin: examples are the many food-dependent countries whose imports are managed, by and large, by one State Trading Enterprise.

As for the assumptions on contracting and bargaining, the structure of the model follows Grossman and Hart's property rights approach, as applied by Antras (2003, 2014) and Antras, Helpman (2004, 2008) to the international trade of intermediate products for backward integration. The basic idea is that the input's producer and the downstream firm cannot sign ex-ante enforceable contracts specifying all the details involved in the trade of the input at a certain point of time, for a certain price and quality.

Further, the production of the input and the supply of the downstream services necessary to place the final product on the export market involve specific investments by firms; hence, although the two firms may select from a set of competing suppliers and buyers ex-ante, ex-post once they

[5] It is worth noting that in the Antras and Helpman papers, the ownership of the assets is associated with the ownership of the inputs produced with those assets (Antras, 2014).

have made their investment they form a bilateral monopoly because, as a consequence of investment specificity, they have an incentive to trade with each other rather than with outside parties. Under a bilateral monopoly each party bargains to obtain the surplus generated ex-post by that relationship, but only after the input and downstream services have been produced. Contract incompleteness, specific investments, ex-post bargaining and bilateral monopoly prevent both parties from optimal investments (the hold-up problem).

The choice of the ownership structure is made before investment and takes into account this contractual environment. [6] There are two possible ownership structures: outsourcing, if parties remain unaffiliated, and backward integration if the downstream firm takes the control of the upstream firm. In this specific context, outsourcing includes all the contractual relationships that do not involve direct control by the foreign firm over the use of land (e.g. contracts specifying standards, contract farming); while integration (FDI) involves taking control over the use of land by a foreign downstream firm (e.g. long-term land lease, acquisition of land).

At time $t = 0$ the two parties negotiate a contract determining the ownership structure. At $t = 1$ both firms make investments and their decisions are taken simultaneously but not cooperatively. At $t = 2$ the two inputs, h and m, are produced; once they are produced, they are useless or incompatible with other alternatives; in that sense, investments made by both parties to produce h and m, are specific to that relationship and lock-in both of them. Firms at this stage negotiate the terms of the transaction; it is assumed that the ex-post agreement is fully enforceable, that parties have symmetric information and that the negotiation outcome can be approximated by a Nash bargaining solution. At the final date $t = 3$ firm H sells the final good on the final consumption market.

During bargaining, parties negotiate the distribution of surpluses arising from the transaction. Under the generalized Nash bargaining solution parties obtain from bargaining the outside option plus a share of the surplus generated by the agreement, which depends upon their bargaining power. The higher is the outside option, the greater the share of the surplus. If R is the surplus

[6] Indeed, in the Grossman and Hart (1986) theory, the allocation of the residual rights over the assets, that is the ownership structure, is the only contractible at the initial stage. Grossman and Hart (1986) define the residual right of control as all rights over another party's assets except those specifically mentioned in the contract.

generated by the agreement, the distribution of surplus for each firm and for ownership structure is that reported in Table 1.

Under outsourcing, the outside option for both *H* and *M* is zero. Hence, *H* gets *B R* and *M* obtains *(1 - B) R*. Under integration, the outside option of *M* is always zero, and the outside option of *H* is greater than zero. The Grossman and Hart model assumes that under integration *H* cannot use input *m* as effectively as with *M*; firing *M* implies a loss 1 - δ in sales revenues. Hence, under integration *M* gets its outside option (zero) plus a share *(1 - B)* of the surplus generated by the agreement, which is (1 – δ*)* *R*. H gets its outside option δ *R* plus a share of the surplus created by the agreement, which is β (1 – δ) *R*. Overall, this is equal to δ *R* + *B* (1 – δ*)* *R,* with 0 < *B* < 1 and 0 < δ < 1.

Let $\beta_i = \delta + B(1-\delta)$ be the share of revenues of *H* under vertical integration and $\beta_o = B$ the share of revenues of *H* under outsourcing; it is worth noting that, given that 0 < *B* < 1 and 0 < δ < 1, $\beta_i > \beta_o$.

Parameter δ is of particular interest here, as it captures the cost of a contractual breach. The higher δ, the lower the losses in sales revenues of a vertical integrated firm due to a breach of contract. Hence, δ is expected to be high in countries with better contract enforcement and legal protection, while it is likely to be low in countries where contract enforcement and legal protection are weak (Antras, Helpman, 2008).

At time *t* = 1 the parties choose, simultaneously but non-cooperatively, the amount of *m* and *h* that maximizes their payoff. We consider two types of downstream firms: a private and a state owned firm.

3.1 The private-private deal

We first examine the case of a private-to-private deal. Both *M* and *H* maximize their profits. The maximization problem at time *t* = 1 for a given *k* ownership structure can be written as follows:

$$\begin{cases} \max_h \beta_k R - c_h\, h & (1) \\ \max_m (1-\beta_k) R - c_m\, m & (2) \end{cases}$$

With $k = \{i, o\}$, $h > 0$, $m > 0$.

At time $t = 0$ they agree on the ownership structure by maximizing their joint payoff for the given quantities decided at $t = 1$ and $t = 2$.

There are also fixed organizational costs F_k that vary with the organizational structure.

The joint maximized profit is:

$$\Pi_k = \beta_k R_k - c_h h_k + (1 - \beta_k) R_k - c_m m_k - F_k \qquad (3)$$

with R_k, h_k, m_k resulting from the maximization problems (1) and (2).

The final optimal ownership structure is the one that maximizes the joint profit.

In the context of the international trade of raw or intermediate goods, the surplus generated by the agreement R is the revenue coming from the sale of the final product x on the export market. Functional forms for the demand and the production functions are hence needed to solve the maximization problems. Previous papers have mostly assumed the functional forms commonly used in international trade models under monopolistic competition, that is, the CES preferences function and the Cobb-Douglas production function (Antras, 2014). We here explore a different combination of functional forms, in that we assume that preferences are quadratic while maintaining the assumption of a Cobb-Douglas technology.

The production function is:

$$x = \left(\frac{h}{\eta}\right)^{\eta} \left(\frac{m}{1-\eta}\right)^{1-\eta} \qquad (4)$$

With $0 < \eta < 1$ being a technological parameter measuring downstream services intensity.

On the final consumption market, firm H faces a linear demand curve:

$$p = a - bx \qquad (5)$$

Solving the maximization problems (1) and (2) for h and m, using functional forms (4) and (5), provides the following first order conditions:

$$\frac{\delta(\beta_k R - c_h h)}{\delta h} = \beta_k \left(a - b\left(\left(\frac{h}{\eta}\right)^{\eta}\left(\frac{m}{1-\eta}\right)^{1-\eta}\right)\left(\frac{h}{\eta}\right)^{\eta-1}\left(\frac{m}{1-\eta}\right)^{1-\eta} - b\beta_k\left(\frac{h}{\eta}\right)^{2\eta-1}\left(\frac{m}{1-\eta}\right)^{2-2\eta} - c_h = 0 \qquad (6)$$

$$\frac{\delta((1-\beta_k)R - c_m m)}{\delta m} = (1-\beta_k)\left(a - b\left(\left(\frac{h}{\eta}\right)^{\eta}\left(\frac{m}{1-\eta}\right)^{1-\eta}\right)\left(\frac{h}{\eta}\right)^{\eta}\left(\frac{m}{1-\eta}\right)^{\eta} - b(1-\beta_k)\left(\frac{h}{\eta}\right)^{2\eta}\left(\frac{m}{1-\eta}\right)^{1-2\eta} - c_m = 0$$

(7)

In order to solve the equations system (6) and (7) for *h* and *m*, the value of the technology parameter has to be assigned. Assuming $= 1/2$, yields the following closed form equations for *h* and *m*:[7]

$$h_k = \frac{c_h c_m + 2a\sqrt{c_h c_m(1-\beta_k)\beta_k}}{4\, b c_h c_m(1-\beta_k)} \tag{8}$$

$$m_k = \frac{c_h c_m + a\sqrt{c_h c_m(1-\beta_k)\beta_k}}{4\, b c_m(1-\beta_k)\beta_k} \tag{9}$$

Plugging (8) and (9) into equation (3), yields Π_i and Π_o as function of fixed and variable costs, of demand parameters and of β_k.
If $\Pi_i > \Pi_o$ then *H* and *M* will agree at *t* = 0 the acquisition of control over *m* by firm *H*; If $\Pi_i < \Pi_o$the optimal ownership structure is outsourcing.

3.2. The state owned-private deal

Consider the case of a firm *H* owned by the State of the importing country. State-owned firms differ in three main respects from private ones (De Fraja, DelBono, 1989; Mc Corriston, Mac Laren 2013): a) the state-owned firm has a different objective function, in that it maximizes the social welfare (producers surplus plus consumers surplus); b) the state-owned firm may have inbuilt problems, such as organizational inefficiencies, due to the conflicting instructions by policy-makers, changes in government and so on; c) the state-owned firm may operate at a loss.

In our specific context, because the importing country is assumed not to produce, the payoff to the state-owned firm is given by the consumer surplus, *CS*, plus the share of the surplus generated by the agreement:

$$P_k^{so} = CS + \beta_k R \tag{10}$$

with $\beta_k R$ being the share of the revenues obtained by the state-owned firm.
With the assumed linear demand curve (6) the *CS* is equal to:

[7] The choice of the value 0.5 for the technological parameter is aimed at reducing the algebraic complexity of solving the system for different values. How the results here obtained may be affected by this assumption is discussed in the next sections.

$$CS = \int (a - bx)dx - (a - bx)x \qquad (11)$$

The maximization problem (1) in the case of a state-owned maximizing consumer surplus changes as follows:

$$\max_h CS + \beta_k R - c_h h \qquad (12)$$

Assume $\eta = 1/2$; solving the system of equations (12) and (2) yields the following:

$$h_k^{so} = \frac{c_h c_m + a^2\theta_k + a\sqrt{\theta_k(2c_h c_m\, \varphi_k + a^2\theta_k)}}{4bc_h\theta_k} \qquad (13)$$

$$m_k^{so} = \frac{2a\beta_k\sqrt{(a^2\theta_k - 2c_h c_m\, \varphi_k)\theta_k} + c_h c_m \varphi_k - 2a^2\beta_k\theta_k}{2bc_m\varphi_k^2} \qquad (14)$$

With $\theta_k = 1 - \beta_k$ and $\varphi_k = 1 - 2\beta_k$.

The joint maximized payoff when *H* is a state-owned firm is:

$$\Pi_k^{SO} = CS_k + \beta_k R_k^{so} - c_h h_k^{so} + (1 - \beta_k) R_k^{so} - c_m m_k^{so} - F_k \qquad (15)$$

Plugging (13) and (14) into equation (15) yields Π_i^{SO} and Π_o^{SO} as a function of fixed and variable costs, of demand parameters and of β_k. If $\Pi_i^{SO} > \Pi_o^{SO}$ then the state-owned firm and *M* agree at $t = 0$ on vertical integration; if $\Pi_i^{SO} < \Pi_o^{SO}$ the optimal ownership structure is outsourcing.

4. FDI *versus* trade: a comparison between the private and the state-owned firm

Because of the different objective function, the optimal ownership structure differs according to the nature of the investing firm. We start by comparing the behavior of the two firms at time $t = 1$, when they choose the amount of input to produce.

From equations (8) and (13) it is straightforward to verify that $h_k < h_k^{so}$, whatever the values of other parameters and variables. Hence, at $t = 1$ the public firm invests more in downstream services than the private one. This result is the consequence of the different objective function of firms. Since the state-owned firm aims at maximizing consumers' welfare, it invests more than

the private one to obtain a higher production of the final good to be exported to the domestic market.

A private firm always produces more *h* under vertical integration, than under outsourcing. From equation (8), it is straightforward to verify that $h_i > h_o$. The reason for this is that under vertical integration *H* share of revenues is always higher than under outsourcing ($\beta_i > \beta_o$, see Table 1).

This is not always the case for the state-owned firm. Whether $h_i^{so} \lessgtr h_o^{so}$ depends upon the value of the parameters B and δ. The following proposition holds (proof is in the Appendix):

Proposition 1: if $B \geq \frac{1-\delta}{2-\delta}$ then $h_o^{so} > h_i^{so} > h_i > h_o$.

As shown in the Appendix, a sufficient condition for $h_i^{so} > h_o^{so}$ is that *H* has a lower bargaining power, that is, $B < 1/2$. Hence, unlike the private firm, if $B \geq 1/2$ the state-owned firm produces fewer downstream services under vertical integration, than under outsourcing.

As for *m*, from equations (9) and (14) it can be shown that if $B \geq \frac{1-\delta}{2-\delta}$, then firm *M* produces a higher quantity of input *m* under outsourcing. The following proposition holds (proof is in the Appendix):

Proposition 2: if $B \geq \frac{1-\delta}{2-\delta}$ then $m_o > m_i$. A sufficient condition for $m_o > m_i$ is $B \geq 1/2$.

The reason is that under vertical integration firm *M* 's share of revenues is lower than with outsourcing; hence, *M* has an incentive to produce more when unaffiliated, than when controlled by firm *H*.

How does the different behavior of firms at time *t* =1 affect the choice FDI/outsourcing at time *t* = 0? Figure 1 reports the gap in joint profits, $\Pi_i - \Pi_o$ of the private firm as a function of the gap in the fixed costs $\Delta F = F_i - F_o$, i.e. the fixed cost incurred with vertical integration, with respect to those incurred under outsourcing. When $\Delta F < 0$, organizational costs are lower under vertical integration than under outsourcing; this may be due to the economies of scope in the management of diverse activities that reduce the organizational costs of a vertically integrated firm with respect to outsourcing. If $\Delta F > 0$, then the costs linked to the control of an integrated firm, such as the costs of supervision of the production of *m*, are higher than the economies of

scope. Not surprisingly, the greater ΔF, the smaller the gap in joint profits $\Pi_i - \Pi_o$. When the gap in fixed costs is higher (lower) than the critical value ΔF^*, then outsourcing (integration) becomes the optimal ownership structure.

Everything else being equal, the same function for the state owned firm is different. Assume that the costs of producing *m* and *h* are low enough with respect to the demand parameter *a*. Under this assumption the curve for the state owned firm shifts to the right. The critical value of $\Delta {F_I}^{*so}$ above which outsourcing prevails is higher in the case of the state-owned firm; this means that, everything else being equal, if firm *H* is state-owned FDI is expected to be more likely (i.e. FDI becomes the optimal ownership structure, even with relatively high ΔF^*), than when the firm is private. On the contrary, when the cost of producing *m* and *h* are high with respect to parameter *a*, then the critical value of $\Delta {F_{II}}^{*so}$ above which outsourcing prevails is lower for the state-owned firm; this means that, everything else being equal, if firm *H* is public we should expect outsourcing to be more likely than when the firm is private. The following proposition holds:[8]

Proposition 3: A sufficient condition for $\Delta\Pi^{*so} > \Delta\Pi^*$ is $c_m, c_h < \frac{2\sqrt{2}a}{9}$.

For an insight into the mechanism driving these results, it is worth analyzing how the overall equilibrium changes with the state-owned firm. Table 2 provides the value of the changes with a state-owned firm, compared to a private firm, in a number of key variables and for different values of the bargaining power *B* under the two costs scenarios. [9] Because the state-owned firm produces a higher quantity of *h* than the private firm, its output always incorporates more *h* and less *m*. The final output is higher with a state-owned firm and the price lower. Revenues of the state owned firm are higher under vertical integration. This is because under integration *m* is lower and, as a result, the final output is smaller, prices and revenues are higher. Although consumer surplus is lower under integration, overall vertical integration is relatively more sustainable. In other words, maximization of consumer welfare in the first stage results in an overproduction of *h*, which reduces overall revenues; vertical integration, by reducing *m*, limits this overproduction and, *ceteris paribus*, becomes a more sustainable ownership structure.

[8] Due to space constraints, proof of proposition 3 is not included in the Appendix, but it is available upon request.

[9] Table 2 is based on numerical simulations run by assuming arbitrary values for demand parameters (a=10, b=10) and that δ=0.5, c_m=c_h=1, F_i = F_o. While the values reported in the Table obviously change with the assigned values, signs do not depend upon the latters. It is worth noting that these results hold under the assumption $\eta = 0.5$.

Conversely, if the costs of producing *m* and *h* are high with respect to demand parameter *a*, then both public and private downstream firms produce less *h*. What really differs is the behavior of firm *M*. Indeed, while with a private firm high costs imply the reduction of both *h* and *m*, with a state owned firm higher costs, by severely limiting the production of *h,* make it profitable to use a higher amount of *m* in the production process; and as *M* has a greater incentive to provide the input under outsourcing, outsourcing turns out to be relatively more feasible for the state owned firm.

So far, the objective function has been the sole difference between the private and the public firm. Yet what if the public firm is also less efficient than the private one? If c_h of the state owned firm is higher the curve shifts to the left and, as a consequence, the critical value of the gap in fixed costs above which outsourcing becomes the optimal structure is lower. This means that the inefficiencies of the state-owned firm make outsourcing more sustainable. The driving mechanism is that higher costs limit the production of *h* by the state-owned firm and increase the use of *m* in the production process; this decreases the critical point above which outsourcing becomes the optimal ownership structure.

A further issue is how a fixed subsidy to the state-owned firm (for instance, the government underwriting of losses can be equivalent to a fixed subsidy) could affect the public firm's choice with respect to the private firm. A fixed subsidy does not influence the decisions of firms at time $t = 1$ on the amount of *h* and *m*, but it may influence the joint profits (equation 15) and hence the optimal ownership structure. If the government grants a subsidy to a firm investing abroad, this shifts the curve in Figure 1 to the right, and vertical integration becomes feasible even for relatively higher values of the gap in fixed costs.

Overall, the findings suggest that: a) the decision of the state owned firm on FDI/ trade, everything else being equal, is different from that of a private firm; b) compared to a private firm, a state owned firm is expected to procure raw materials abroad more through FDI, and less through outsourcing, if the costs of production of inputs are low in relation to the size of the final market; c) inefficiencies of state owned firms make outsourcing relatively more feasible than vertical integration; d) a subsidy granted to state-owned firms investing abroad increases the feasibility of vertical integration.

5. Institutional quality, bargaining power and the choice FDI/outsourcing

The choice of the optimal ownership structure is significantly affected by the distribution of the revenues which, in turn, depends upon the bargaining power *B* of firm *H*, and on parameter δ. Before analyzing the impact of δ and *B*, it is worth investigating how parameter β_i as a whole influences the optimal structure.

Figure 2 reports the gap in joint profits, $\Pi_i - \Pi_o$ as a function of β_i, that is, the *H* share of revenues under vertical integration. If the investor is private, for relatively small values of β_i an increase in *H* share of revenues makes integration more sustainable (i.e. $\Pi_i - \Pi_o$increase). On the contrary, when β_i is sufficiently high, an increase in *H*'s share of revenues reduces the gap in joint profits. As regards the private firm, the findings can be summarized in the following proposition (proof is in the Appendix):

Proposition 4: if $\beta_i \geq 1/2$, then β_i negatively affects $\Pi_i - \Pi_o$.

When the investor is state-owned, the impact of β_i on the choice FDI/outsourcing is rather different. In this case, $\Pi_i - \Pi_o$turn out to monotonically increase with β_i. In other words, whatever the bargaining power of *H*, the higher (lower) is the share of revenues of *H*, the higher (lower) the likelihood that vertical integration is the optimal ownership structure.

Parameters δ and *B* play a crucial role in determining these results. One of the issues that can be addressed by using the model developed in section 3 is how the quality of the institutions affects the firm's choice of integrating in the foreign country. In this model, the quality of the institutions is captured by parameter δ. Indeed, the higher δ, the lower the cost of a contractual breach with a vertical integrated firm, the higher the fraction of the surplus generated by the agreement captured by firm *H*. So, the better the legal protection, generally ensured by effective institutions, the higher is δ.

Figure 3 reports the gap in joint profits, $\Pi_i - \Pi_o$ as a function of δ under the assumption that *M* and *H* have the same marginal costs ($c_m = c_h$) and that the two parties have the same bargaining power ($B = 1/2$).[10]

As the figure shows, the curve is decreasing and above the critical value δ^* outsourcing becomes the optimal ownership structure. Because *H* cannot use input *m* as effectively as with *M*, the lower (higher) δ, the greater (lower) the loss of *H* revenues due to a breach of the agreement, and the higher (lower) the share of revenues obtained by *M*. If δ is sufficiently low, the amount of input *m* produced under vertical integration is higher and vertical integration turns out to be the optimal structure.

Hence, the model suggests that when the investor is private, better institutions can result in the prevalence of outsourcing.

Figure 3 reports the same function for the state-owned firm maximizing consumer welfare. As the Figure shows, the relationship between the gap in profits and the quality of institutions is increasing. [11] The model, therefore, does not support the hypothesis that the better the quality of the institutions, the less likely FDI; rather, it concludes that better institutions may imply the prevalence of vertical integration by the state-owned firm. The key reason is the different equilibrium of the two firms in the first stage ($t = 1$). Because with the state-owned firm output incorporates more *h* and less *m*, with higher δ, *m* further reduces, but *h* reduces as well; as a consequence, output is lower and price is higher. As a result, revenues under integration increase with δ, and, above a certain level of institutional quality, integration becomes the optimal strategy.

The impact of the bargaining power on the choice FDI/trade is illustrated in Figure 4. As the Figure shows, the higher the bargaining power of a downstream private firm, the lower the gap in joint profits $\Pi_i - \Pi_o$: if *H*'s bargaining power is higher than a certain critical level, the optimal

[10] Figures 3 and 4 are based on numerical simulations run by using the same values of demand parameters and costs as in Table 2. The shape of the curves holds for a wide range of costs and demand parameters values. On the contrary, changes are relevant if the technological parameter η assumes different values; in the latter case, functions are not monotonic, indicating the lack of a clear-cut relationship between quality of the institutions /bargaining power and the choice integration/outsourcing

[11] It is worth noting that δ^* for the state owned firm can be higher or lower than δ^* for the private firm, depending upon the value of the other parameters. The Figure illustrates one possible case.

structure becomes outsourcing. The mechanism driving this result is the following: if *B* is high, *M* has less incentive to produce *m,* because its share of revenue is low, and the latter is certainly lower under vertical integration; therefore, if *B* increases, m_i decreases more than $m_o$and, if *B* is large enough, outsourcing becomes the optimal structure. On the basis of this result one should expect more FDI in country or industries where the bargaining power of the downstream firm is relatively low.

Conversely, the curve for the state-owned firm is upward sloping and shows that FDI should be expected when the bargaining power of downstream firms is stronger. The reason is that if *B* increases, m_i decreases more than m_o, but in this case the optimal choice for *H* is to decrease the amount of h; and because h_i decreases more than h_o, vertical integration becomes the optimal ownership structure. The key reason for the different behavior of the state owned firm is the over-production of h^{so} with respect to the private firm. If input *m* decreases the state owned firm no longer finds it profitable to increase h, possibly because of a marked increase in costs.

6. Concluding remarks

This paper aims at addressing an issue so far seldom explored by the literature, that is, what the "internalization" drivers of recent FDI in agriculture are. Building on the literature incorporating the Grossmann and Hart (1986) property rights theory in international trade, the model developed in this paper explores how the consideration of contract incompleteness and asset specificity in the international transaction of agricultural products may contribute to offer insights on the reasons why some firms invest abroad in land, while other procure raw materials from food or energy importing countries by means of contractual arrangements with farmers in land abundant countries.

The framework here used stresses a number of features of the contractual environment of the international transactions. The distribution of surpluses under vertical integration – which crucially influences the behavior of the two firms in the first stage - is here the result of the interplay between the bargaining power and the cost of the contractual breach. This interplay drives one of the most striking implications of the model, that is, under certain circumstances better institutions lead firms to choose outsourcing, while weak institutions drive FDI. The reason is that better institutions, by limiting the cost of breach of contract, reduce the upstream

firm's share of revenues and thus there is less incentive for vertical integration. On the contrary, the higher the surplus of the upstream firm obtained under vertical integration, due to a higher cost of contractual breach, the more likely FDI becomes. In other words, the model stresses the importance of the services provided by the upstream agent under vertical integration.

This result contrasts with the common expectation based on the growth and financial literature that a higher quality of the institutions implies more FDI; better institutions are expected to increase productivity, to reduce corruption and uncertainty in target countries and, by and large, to have a positive effect on their growth. By improving the investment climate, better institutions are expected to promote inward FDI. Empirical studies, by and large, confirm this positive impact for the economy as a whole and/or for manufacturing industries (e.g. Wei, 2000; Stein, Daude, 2001; Globerman, Shapiro 2002; Benassy-Quere et al, 2007) albeit with some *caveat* (see Daude, Stein, 2006). However, the evidence available for agriculture seems to suggest the opposite. Arezki *et al* (2014) find a negative relationship between governance and land governance indicators and FDI in agriculture. Models incorporating contracting may provide insights into the underlying factors behind this evidence; good institutions affect FDI, on one hand, through the improvement in overall productivity and the reduction of uncertainty for the foreign firms about the returns on their investment in land; on the other hand, they affect the contractual environment of international transactions in intermediate goods. According to the framework here developed, good governance and strong downstream bargaining power may inhibit FDI because they reduce the upstream agent's incentive to produce raw materials under vertical integration.

A further striking result of the paper is that the optimal ownership structure may differ with the nature of the investing firm. Results suggest that a state-owned firm produces the final output by using a higher amount of downstream services and a lower amount of inputs provided by the upstream party; moreover, a state-owned firm procures raw material abroad more through FDI, and less through outsourcing, if the costs of production of inputs are low relative to the size of the final market; on the contrary, if costs are high and/or the size of final market is small, the state-owned firm is expected to be more oriented toward outsourcing. Finally, bargaining power and institutional quality are expected to influence the choice of a state-owned firm in the opposite direction, in that, they both promote FDI if the firm is state-owned.

Consideration of the contractual environment of international transactions may have potential implications for empirical analyses on FDI in agriculture. The model here developed shows that the drivers of FDI differ depending on the type of investor and that the direction of the impact of key variables may change for different kinds of firms. As a consequence, empirical analyses using country level data may estimate an average (opposite) effect; this issue could be potentially relevant when both private and public firms originate from the same country, as is the case for countries such as China. In these cases, firm level analyses may better capture the impact of important variables on agricultural FDI.

From the methodological point of view, this paper highlights how the choice made on functional forms influences the tractability and the outcomes of the model. Instead of drawing on the monopolistic competition trade model, which assumes CES preferences, here we explore a combination of quadratic preferences and Cobb Douglas production function. As the paper has shown, this required *ad hoc* hypotheses about the technological parameter in order to solve maximization problems. The findings of the paper are thus based on the assumption of a particular Cobb Douglas. Further research efforts could shed lights on how different values of the technological parameter, and different combinations of functional forms could influence the main findings.

Notwithstanding these limitations, the framework developed here shows how consideration of the contractual environment of international transactions may offer a different perspective on the drivers of FDI in agriculture and shed light on a number of key aspects, such as the relationship between governance, bargaining power and FDI.

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Table 1: Outside options and share of revenues of the bargaining game

Ownership structure		**M**	**H**
Integration	*Outside option*	0	δ R
	Share of revenues	$(1 - B)(1 - \delta)R$	$B(1 - \delta)R$
Outsourcing	*Outside option*	0	0
	Share of revenues	$(1 - B)R$	BR

Table 2: State-owned firm: % variation with respect to a private firm ($\eta = 0.5$)

	$c_h, c_m < 2\sqrt{2}\,a/9$			$c_h, c_m > 2\sqrt{2}\,a/9$	
	$B = 0.3$	$B = 0.5$	$B = 0.8$	$B = 0.3$	$B = 0.5$
h_i	17.38	13.69	8.89	18.00	32.34
h_o	37.69	23.26	12.20	49.33	19.60
m_i	-0.66	-0.56	-0.18	3.98	11.53
m_o	-0.83	-0.75	-0.49	2.81	2.41
x_i	1.49	1.55	1.86	8.73	19.44
x_o	1.54	1.47	1.60	12.84	7.38
p_i	-0.98	-0.97	-0.93	-0.77	-0.70
p_o	-0.99	-0.98	-0.96	-0.87	-0.82
R_i	-0.94	-0.92	-0.79	1.27	5.04
R_o	-0.97	-0.96	-0.90	0.77	0.51

Figure 1: Private *versus* state-owned firm

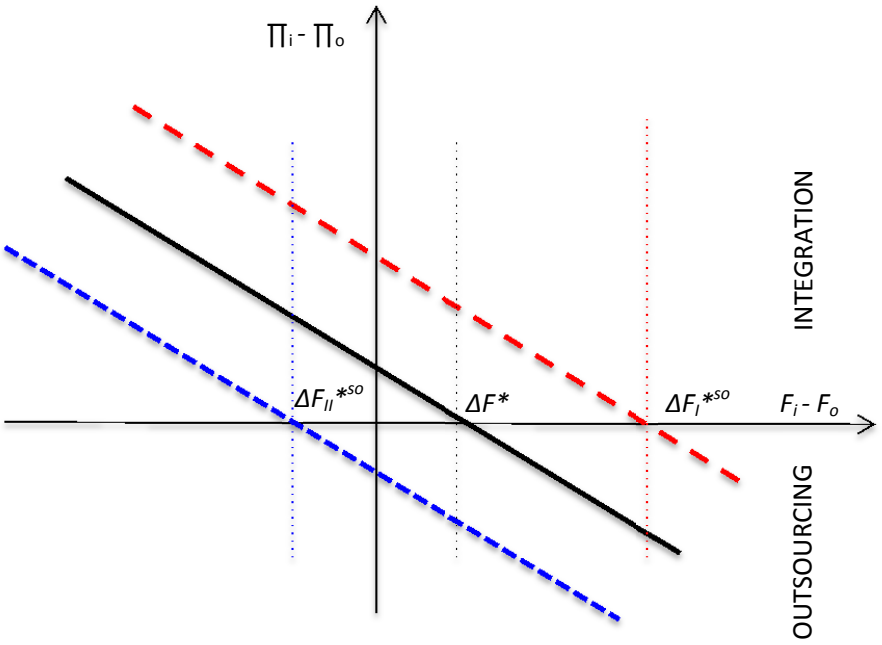


Figure 2. The firms' choice as a function of parameter β_i

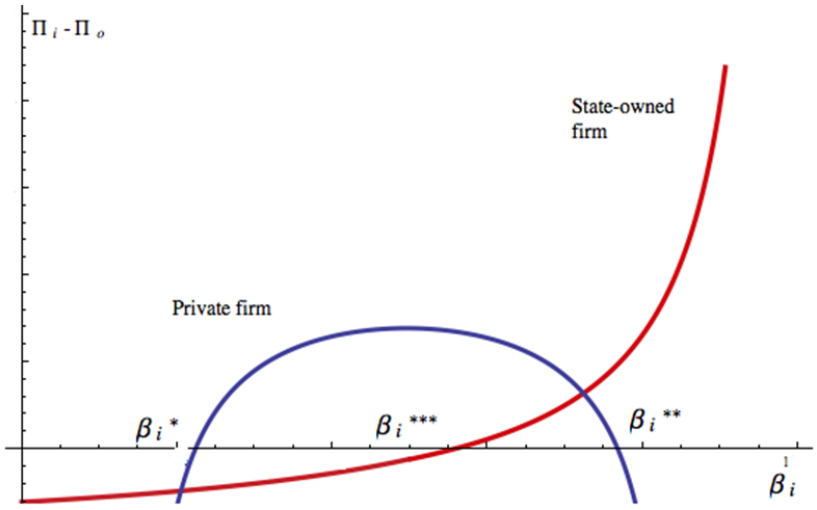


Figure 3. The firms' choice as a function of parameter δ

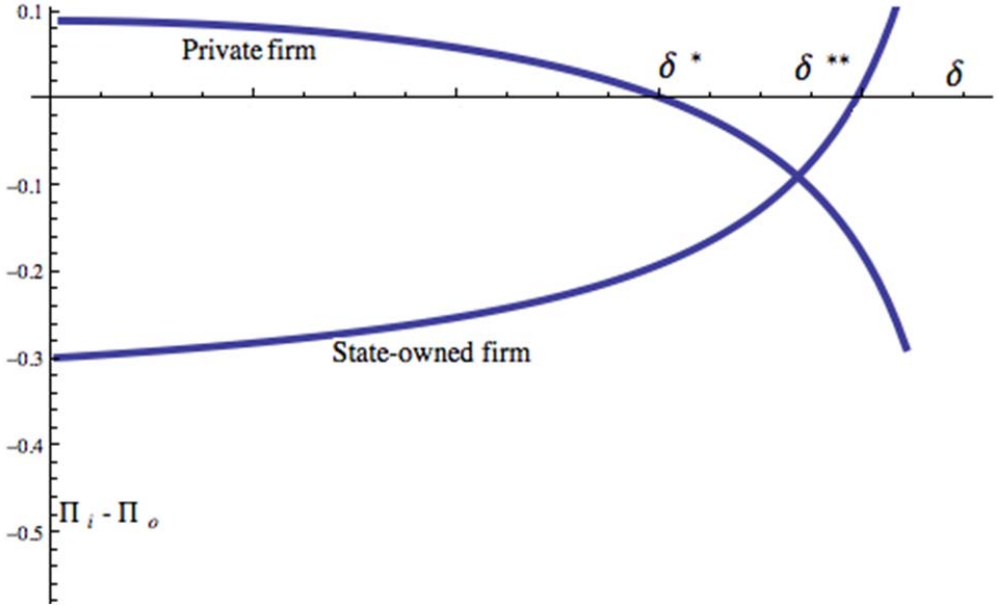
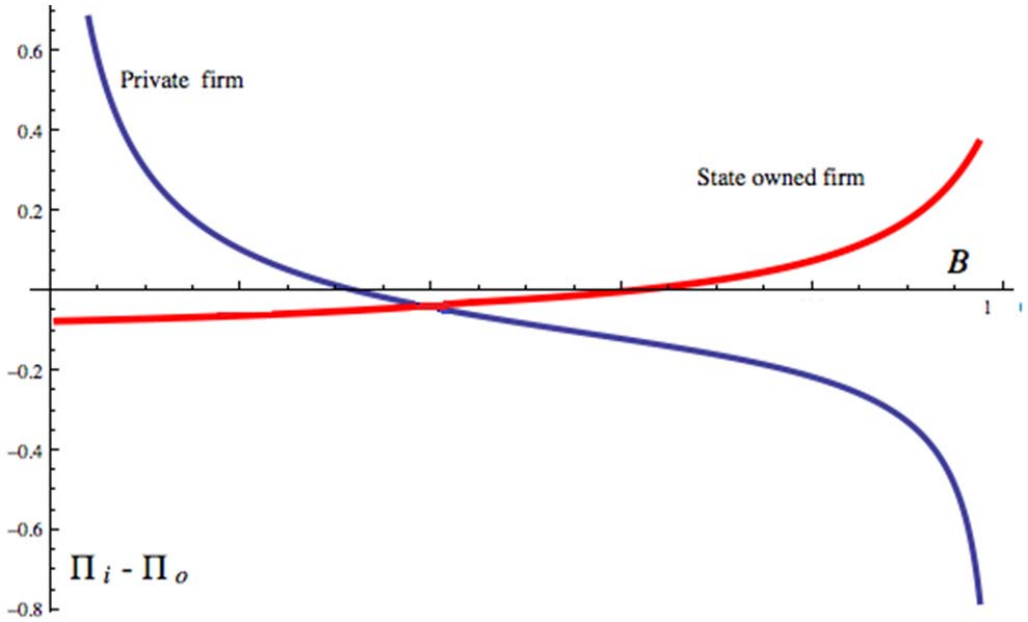


Figure 4. The firms' choice as a function of bargaining power *B*



APPENDIX

Proposition 1: if $B \geq \frac{1-\delta}{2-\delta}$ then $h_o^{so} > h_i^{so} > h_i > h_o$. A sufficient condition for $h_o^{so} > h_i^{so}$ is $B \geq 1/2$.

Proof: The first step of the proof consists in showing that:

$$h_i^{so} - h_i = \frac{c_h c_m + a^2\theta_i + a\sqrt{\theta_i(2c_h c_m\,\varphi_i + a^2\theta_i)}}{4bc_h\theta_i} - \frac{c_h c_m + 2a\sqrt{c_h c_m \theta_i \beta_i}}{4\,bc_h c_m \theta_i} > 0$$

This holds if $c_m < \frac{a^2\beta_i\theta}{c_h}$.
The latter is the necessary condition for:

$$x_i = \frac{1}{4}\sqrt{\frac{a\sqrt{\theta_i\beta_i} - \mathrm{c_h c_m}}{b\mathrm{c_m}\theta_i}}\sqrt{\frac{2a\sqrt{\mathrm{c_h c_m}\theta_i\beta_i} - \mathrm{c_h c_m}}{b\mathrm{c_h}\theta_i}} \in \mathbb{R}$$

Hence, $h_i^{so} > h_i$.

The second step consists in showing that if $B \geq 1/2$, then $h_o^{so} > h_i^{so}$.

$$h_o^{so} - h_i^{so} = \frac{c_h c_m + a^2\theta_i + a\sqrt{\theta_i(2c_h c_m\,\varphi_i + a^2\theta_i)}}{4bc_h\theta_i} - \frac{c_h c_m + a^2\theta_o + a\sqrt{\theta_o(2c_h c_m\,\varphi_o + a^2\theta_o)}}{4bc_h\theta_o} > 0$$

This holds if $B = \beta_o \geq \theta_i$. Providing that:

$\theta_i = 1 - \beta_i = 1 - (\delta + B(1-\delta))$

then $B \geq (1-\beta_i)$ if $B \geq (1 - (\delta + B(1-\delta))$, that is, $B \geq \frac{1-\delta}{2-\delta}$.

Given that $\delta < 1$, this condition holds when $B \geq \frac{1}{2}$. QED

Proposition 2: if $B \geq \frac{1-\delta}{2-\delta}$ then $m_o > m_i$. A sufficient condition for $m_o > m_i$ is $B \geq 1/2$

Proof:

$$m_o - m_i > 0 \text{ if } \frac{c_h c_m + a\sqrt{c_h c_m(1-\beta_o)\beta_o}}{4\,bc_m(1-\beta_o)\beta_o} - \frac{c_h c_m + a\sqrt{c_h c_m(1-\beta_i)\beta_i}}{4\,bc_m(1-\beta_i)\beta_i} > 0$$

This holds if $(1-\beta_o)\beta_o$>$(1-\beta_i)\beta_i$ that is, if $(1-\beta_i) < \beta_o = B$. Likewise proposition 1 proof, this holds if $\geq \frac{1-\delta}{2-\delta}$. As before, sufficient condition for $(1-\beta_i) < B$ is $B \geq 1/2$. QED

In the case of a public firm $m_o^{so} > m_i^{so}$ if $c_h c_m < a$ and $B \geq 1/2$ and $\beta_i < \frac{1}{2} + \frac{1}{2a}\sqrt{a^2 - c_h c_m}$.

Proposition 4: if $\beta_i \geq 1/2$, then β_i negatively affects $\Pi_i - \Pi_o$.

Proof: The proof consists in showing that $\frac{\delta G}{\delta \beta i} < 0$ if $\beta_i \geq 1/2$, with $G = \Pi_i - \Pi_o$.

$$\frac{\delta G}{\delta \beta_i} = (-6a^3 b\theta_i{}^2\beta_i\varphi_o + 4a^2 b\theta_i\sqrt{c_h c_m\theta_i\beta_i}\varphi_o + 6ab^2\theta_i\sqrt{c_h c_m\theta_i\beta_i}\varphi_o\sqrt{\frac{c_h c_m\theta_i + a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_m\theta_i\beta i}}\sqrt{\frac{c_h c_m\theta_i + 2a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_h\theta_i{}^2}} - a^2 b^2\theta_i{}^2\beta_i(6\beta_i - 1)\sqrt{\frac{c_h c_m + a\sqrt{c_h c_m\theta_i\beta_i}}{bc_m\beta_i}}\sqrt{\frac{c_h c_m\theta_i + 2a\beta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_h\theta^2}}) / (32\beta_i{}^2 b^2\theta_i{}^2\sqrt{\frac{bc_h c_m\theta_i + a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_m\theta_i\beta_i}}\sqrt{\frac{c_h c_m\theta_i + 2a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_h(\theta_i)^2}})$$

This is shown to be negative when $\beta_i > 1/2$ because:

a) parameters and variables are all greater than zero, hence, the denominator is always greater than zero;

b)
$-6a^3 b\theta_i{}^2\beta_i\varphi_o + 4a^2 b\theta_i\sqrt{c_h c_m\theta_i\beta_i}\varphi_o +$
$6ab^2\theta_i\sqrt{c_h c_m\theta_i\beta_i}\varphi_o\sqrt{\frac{c_h c_m\theta_i + a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_m\theta_i\beta i}}\sqrt{\frac{c_h c_m\theta_i + 2a\theta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_h\theta_i{}^2}} - a^2 < 0$

if $\beta_i > 1/2$;

c) $-a^2 b^2\theta_i{}^2\beta_i(6\beta_i - 1)\sqrt{\frac{c_h c_m + a\sqrt{c_h c_m\theta_i\beta_i}}{bc_m\beta_i}}\sqrt{\frac{c_h c_m\theta_i + 2a\beta_i\sqrt{c_h c_m\theta_i\beta_i}}{bc_h\theta^2}} < 0$

if $\beta_i > 1/6$. QED.