REAL OPTIONS APPROACH AND STOCHASTIC PROGRAMMING IN FARM LEVEL ANALYSIS: THE CASE OF SHORT-ROTATION COPPICE CULTIVATION

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2016

Paper prepared for presentation at the 56th annual conference of the GEWISOLA (German Association of Agricultural Economists) „Agricultural and Food Economy: Regionally Connected and Globally Successful“
Bonn, Germany, September 28 – 30, 2016

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REAL OPTIONS APPROACH AND STOCHASTIC PROGRAMMING IN FARM LEVEL ANALYSIS

The Case of Short-Rotation Coppice Cultivation

Abstract

The Real Options Approach (ROA) is often considered as being more plausible compared to the Classical Investment Approach, since the latter ignores the possibility to postpone investments and, more generally, state contingent decisions. At the same time, simulation approaches based on stochastic programming (SP) are convenient instruments for solving the ROA, when no closed-form solution is available. The combination of both can be applied for many different investment problems, though has been hardly used in agricultural economics. Our paper aims to fill that gap by illustrating the application of a multi-stage ROA and SP to model investment decisions with respect to short-rotation coppice (SRC) plantation in the context of competition for different farm endowments. Beyond the methodological contribution, we show that farm-level constraints, often being ignored in the literature on SRC, can substantially influence results. Furthermore, we consider flexibility in harvesting in our analysis.

Keywords

Perennial Energy Crops, Farming Investment Decision, New Investment Theory, Multi-stage Option.

1 Introduction

The Real Option Approach (ROA) is often sketched as taking into account the possibility to postpone investment decisions. More generally, it acknowledges flexibility in future management, depending on how the decision environment evolves, where waiting is just one example. The direct counterpart in a programming model is state contingency. In both cases the analysis not only takes into account that the future is uncertain and might unfold in different directions, but also that decisions can be adjusted to some degree, depending on these future states-of-nature. Closed form solutions to real option problems are elegant from a scholarly perspective, but often require assumptions, e.g., about stochastic processes that are too restrictive for complex real world examples. The simulation approaches based on stochastic programming (SP) are usually more suitable. The aim of this paper is to show the close relation between ROA and SP, based on an investment project analysis in agriculture. The paper is organized as follows. We first motivate why programming approaches are often useful for investment decision at farm level; and review the state of the art with respect to stochastic programming and the real options. Next, we sketch the steps of designing a model based on the ROA and solve with SP, illustrating our example of SRC. Finally, we present the results of our model and conclude.

2 Programming models for investment planning in agriculture

Larger investment projects in agriculture lead typically to manifold changes in the way the farm is managed. An extension of a farm operation not only requires physical capital, but also has effects on farm production and input use, which in turn require expansion of the farm's endowments or changes in its management. Without considering these consequences, a proper evaluation of the investment project is not feasible. Formalized analytical tools, such as mathematical programming approaches, depict the relations between different physical and

economic aspects in farm management in a quantitative way, while being relatively transparent.
In the presence of production and market risk, the ROA is favored over the classical deterministic analysis for investment projects in agriculture, since farmer can adjust their management to future states-of-nature. Should, for instance, output prices increase or decrease, farmers might adjust their crop portfolio, herd sizes, planned (re)investments or even close down their operation. Moreover, farmers consider the potential for future flexibility already when making investment decisions. Hence, considering future flexibility not only contributes to a better understanding of farm level decisions, but also typically leads to different, often more plausible economic indicators of interest, such as the expected net present value (NPV) and its distribution, when investment projects are analyzed.
The combination of the ROA and stochastic programming approach is required for depicting real-world complex decisions, and has been successfully applied for different types of investment problems, such as managing project portfolio (BERALDI et al. 2013), natural resources extraction (ALONSO-AYUSO et al. 2014) or problems in energy economics (SAGASTIZÁBAL 2012; FENG and RYAN 2013; SIMOGLOU et al. 2014; VAN ACKOOIJ and SAGASTIZÁBAL 2014). A recent example of applying ROA combined with SP is provided by TEE et al. (2014) who model harvesting decision in forestry management in New Zealand, based on the model developed by GUTHRIE (2009, pp.347–360).

3 Farm-level modelling of short-rotation coppice cultivation

We focus on short rotation coppice (SRC) cultivation in Germany as an illustrating example to show how the elements of the ROA for an agricultural investment project can be implemented into a SP approach. SRC uses fast growing trees[1] that are harvested in relatively short intervals – typically between two or five years[2] – to produce biomass for energy production. SRC can be highly attractive both from an environmental and economic perspective, as it is characterized by low-input production comparing with alternative crops (FAASCH and PATENAUDE 2012). This, however, means that a large share of the costs are sunk, once the plantation is set-up: typically, about 2/3 of the costs of a SRC plantation relate to its set-up and final re-conversion (LOWTHE-THOMAS, SLATER, and RANDERSON 2010). The typical lifetime of a plantation is twenty years, such that the future prices for the harvested biomass can be assumed to be uncertain. Planting and harvesting are usually outsourced to a contractor, such that little or no on-farm labor is required (MUSSHOFF 2012, p.77).
Setting up a SRC plantation competes with alternative use of farm land, such as arable cropping. However, that competition has been reduced under the latest CAP reform, which requires that larger arable farms devote 5% of their farm land to "Ecological Focus Area" (EFA), for which SRC is to a certain degree eligible. In Germany, for instance, one hectare of SRC counts with a factor of 0.3 towards the EFA requirement (BUNDESMINISTERIUM FUER ERNAEHRUNG UND LANDWIRTSCHAFT 2015). Thus, a farm with 100 ha cropping[3] 15 ha of SRC would fulfill the 5% EFA requirement. Assuming that the alternative would be to let 5 ha idle with no economic returns, competition for land for SRC would only occur for 10 ha, and not for the full 15 ha devoted to SRC. At the same time, as SRC requires no or very little on-farm labor, the farm labor necessary on the 10 non-idling hectares could be devoted to another use (e.g., leisure, on- or off-farm work). We analyze that problem in a SP approach, acknowledging that (1) the plantation might be set-up not only immediately, but also in the three following years, (2) harvesting can be undertaken between two and five years after the initial set-up or the latest harvest, and (3) the plantation might be closed down and

[1] Common plants suitable for SRC in Europe include poplar (populus spp.) and willow (salix ssp.).
[2] Flexibility in harvesting interval depends on the end product. We restrict ourselves to the most common end product in Germany, namely wood chips, and therefore to the harvesting interval from two to five years.
[3] Hereinafter "ha" stays for "hectare".

reconverted to arable land also before its maximal lifetime of twenty years is reached. The points (1) - (3) are, hence, state contingent decisions in our SP approach. Considering different harvest intervals allows the plantation to work as storage for biomass, such that temporal arbitrage can be applied: one might let the trees grow if prices are currently low and expected to increase again.
In order to model the competition for land and labor we consider – next to SRC – two alternative crops, of which one is more labor intensive and has also a higher gross margin per hectare. Finally, we consider the EFA requirements and thus introduce set-aside as an alternative to SRC in order to fulfill them. That also implies, that the acreage of the three other potential uses of land (less labor intensive cropping, more labor intensive cropping and set-aside) are also state contingent.

3.1 Conversion into a stochastic programming approach

In order to model our SP investment problem quantitatively, we first set up a deterministic programming approach that maximizes the net present value (NPV). Specifically, the farm interactions are depicted by three types of equations:

1. *Endowment constraints*

   They depict the competition for land and labor, based on a classical mathematical programming approach with fixed input-output coefficients at given farm endowments:

$$\sum_{c} \bar{a}_{c,i} * l_c \leq \bar{b}_i \tag{1}$$

   where
   c – index for crops including SRC;
   i – index for endowments;
   a – fixed input-output coefficients;
   b – farm endowments;
   l – crop levels measured in hectares.
   Time indices are left out for simplicity.

2. *The EFA requirements*

$$l_{seta} + 0.3 l_{src} \geq 0.05 \bar{b}_{land} \tag{2}$$

   where
   l_{seta} - area of set aside land;
   l_{src} – area of land under SRC.

3. *The objective function*

   The NPV of the discounted gross margins.

Depicting the SRC plantation by equations is more complex and is based on a Mixed Integer Programming (MIP) approach, considering different plots that might differ in size. The reason for MIP is threefold: (1) we can only have SRC after the initial set-up of the plantation; (2) the plantation can only grow and be harvested, if there was SRC on that plot in the year(s) before; and (3) harvesting must be done in certain intervals. In addition, there are costs of closing down the plantation and the maximum lifetime needs to be considered. The latter is assumed as a necessary part of a SRC project, to prevent that the plantation becomes legally a permanent forest with a strong protection[4].

[4] According to the FEDERAL FORESTS ACT (1975), short rotation coppice or any perennial crop, rotated longer than 20 years and intended for logging, is recognized as forest.

Given these relations, allowing for fractional size of the SRC plantation would lead to a complex quadratic mixed integer programming approach. We therefore opt for a simplification by only taking certain pre-defined plantation sizes into account, assuming that the farmer would typically convert existing plots into a SRC plantation.
We thus consider three competing farm activities: (1) SRC cultivation; (2) More labor intensive and more profitable traditional agriculture (alternative crop 1); and (3) Less labor intensive and less profitable traditional agriculture (alternative crop 2). The two activities for traditional agriculture are characterized by their different labor requirements and annual gross margins, assumed to be deterministic for the sake of simplicity. The first equation of relating to SRC cropping defines a biomass growth function on each plot:

$$stock_{p,t} + harvQuant_{p,t} = stock_{p,t-1}\overline{gwl}_p + \overline{gwc}_p \quad (3)$$

where
p – index for land plots;
t – index for time period;
$stock$ - standing biomass;
$harvQuant$ - biomass removal by harvesting;
$\overline{gwl}$ - growth multiplier, relating to last year's stock;
$\overline{gwc}$ - growth constant, depicting yearly biomass growth independent from previous biomass.

The link between the decision to harvest in a specific year and the harvested biomass is reflected by a binary indicator inequality and a maximal bound:

$$harvQuant_{p,t} \geq harvest_{p,t} * \overline{minHarvQuant}_p \quad (4)$$

$$harvQuant_{p,t} \leq \overline{maxHarvQuant}_p \quad (5)$$

where
$harvest$ - indicates if a plot is harvested (=1) or not (=0);
$\overline{minHarvQuant}$ – constant, defining the minimal harvesting quantity;
$\overline{maxHarvQuant}$ - constant, defining the maximal harvesting quantity.

Using the biomass growth function, the maximal harvest quantities at any point in time can be calculated based on the last harvest or the initial set-up. These data can be introduced in the two equations above to reflect a minimal and maximal waiting time between subsequent harvests.
In order to prevent that only a part of the biomass is removed when harvesting – which is not considered feasible – an additional equation ensures that last year's standing stock is completely removed in a harvest year:

$$stock_{p,t} \leq (1 - harvest_{p,t}) * \overline{maxHarvQuant}_p \quad (6)$$

A similar equation ensures that the plot must be devoted to SRC if there is biomass:

$$stock_{p,t} \leq src_{p,t} * \overline{maxHarvQuant}_p \quad (7)$$

where
src - binary variable, indicating that the plot is used in that year for SRC (=1) or not (=0).

The maximal lifetime of the plantation is depicted by a year counter in combination with an upper bound:

$$age_{p,t} = age_{p,t-1} + src_{p,t} \quad (8)$$

$$age_{p,t} \leq \overline{maxage} \quad (9)$$

where

age – natural variable, reflecting the age of the plantation;
$\overline{maxage}$ – constant upper bound of plantation's age.

Finally, two equations describe set-up costs, linked to a positive change in SRC on a plot, and reconversion costs, linked to a negative change:

$$iniCost_{p,t} \geq \left(src_{p,t} - src_{p,t-1}\right) * \overline{costIni}_p \tag{10}$$

$$reconvCost_{p,t} \geq \left(src_{p,t-1} - src_{p,t}\right) * \overline{costReconv}_p \tag{11}$$

where
$iniCost$ – actual set-up costs per plot;
$costIni$ – coefficient of set-up costs per plot;
$\overline{reconvCost}$ – actual reconversion costs per plot;
$\overline{costReconv}$ - coefficient of reconversion costs per plot.

The set-up and reconversion costs also ensure that a plot is permanently used for SRC during the whole rotation period.
The settings above depict the deterministic version of the programming model where neither different future outcomes (states) nor state contingent decision variables are depicted. In order to convert the deterministic version into a SP equivalent, four additional elements are needed. First, the decision variables need to carry an additional index for the decision node (i.e. state). Second, an ancestor matrix, reflecting the order of the nodes in the decision tree, has to be introduced. That matrix is used where a lag operator $(t-1)$ is found in an equation. Next, outcomes for the stochastic parameters for each state need to be defined. And finally, the probabilities for each node should be assigned. We will next discuss how the outcomes and related probabilities can be constructed.

3.2 Scenario tree construction and deriving option values

Having defined the basic parameters of the deterministic problem and the equation structure of the SP model, we need to construct the nodes of the scenario tree and to assign probabilities to them.
We assume the natural logarithm of the output price for SRC[5] to follow a mean-reverting process (MRP), in particular an Ornstein-Uhlenbeck process[6]. The choice is motivated by the assumption that the farmer is a price-taker in a market where the price fluctuates around a constant long-term level due to market forces, for instance, under the assumption of no monopolistic power (METCALF and HASSETT 1995, p.1472) and/or of constant technology (SONG, ZHAO, and SWINTON 2011, p.775). The choice of a MRP process is also in line with the usual assumptions for SRC (MUSSHOFF 2012; DE OLIVEIRA et al. 2014). However, an option usually cannot be solved analytically under the assumption of a MRP (DIXIT and PINDYCK 1994, p.161). Instead, one of several numerical-approximate option valuation methods, such as the binominal tree method or the stochastic simulation, should be applied (HULL 2008).
By converting the MRP into a scenario tree, we have to restrict the number of final leaves. Otherwise, even assuming two output price's scenarios only at each node, and considering the maximal rotation period of twenty years and the possibility to postpone the set-up for x years, we would get $2^{20+x} > 10^6$ final leaves in the tree. This is computationally impossible, since we would end up with a MIP with thousands of binaries that might not be solvable to any

[5] Note that in this paper we assume one stochastic process only, because the primary issue of the study is to develop a methodology, combining ROA with SP. Further research might introduce several stochastic processes, as well as correlations between them.
[6] See also NICOLATO and VENARDOS (2003) and PÉREZ-ABREU (2010) for an overview of Ornstein-Uhlenbeck process.

acceptable optimality tolerance without requiring days of processor time. Furthermore, price developments in a binary tree with chained relative ups and downs in each node lead after a few times points to unrealistic prices: already rather conservative assumptions about the variance at any node (e.g. +/-10%) would quickly lead to branches with exploding prices or prices close to zero. Such a set-up is hence hardly useful for the analysis of a long-lasting investment project.

Restricting the number of leaves in the tree to speed up the solution of the SP problems runs the risk to not correctly represent the distribution. That holds especially for our problem where we need outcomes for more than 20 years and introduce temporal arbitrage with flexible harvesting intervals, such that price differences between time points also matters[7]. In order to advance here, we use the tree reduction and construction algorithm by HEITSCH and RÖMISCH (2008). Similar to a Gaussian quadrature, which describes a probability density function with a few characteristic values and their probability mass, this algorithm picks representative nodes and assigns probabilities to them to capture approximately the distribution in original trees. Graphically, one could imagine the algorithm as lumping together neighboring nodes and branches in the tree to bigger ones, where the thickness represents probability mass. For our example, we opt to use a pre-defined number of final leaves and let the algorithm decide which nodes to maintain. Fig. 1 provides an example of a constructed scenario tree with 50 final leaves. The generated tree features 10 decision nodes in the second year with probabilities between 1.4% and 20%. Far less branching can be observed in later time points, which reflects the mean reversion characteristic of the in-going fan: the later the decision point, the more uniform the price expectations

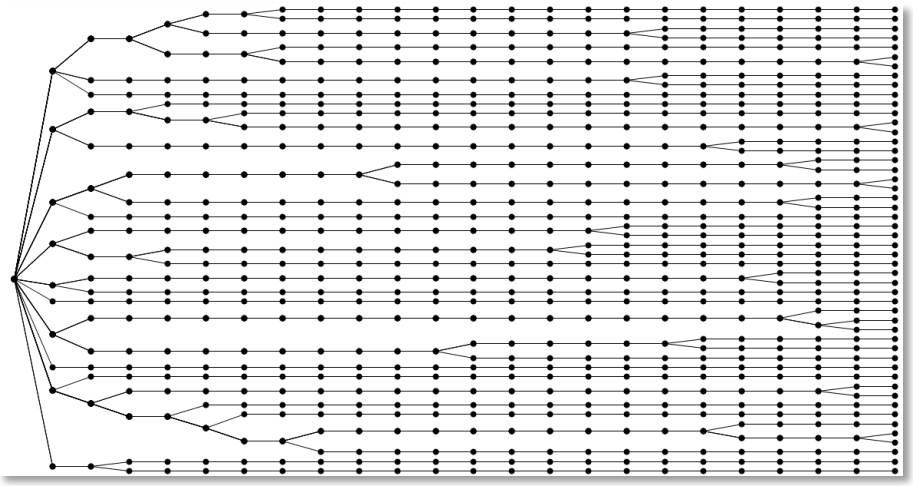

Figure 1. Reduced scenario tree with 50 final leaves

Assume now that the farmer could sign a contract that guarantees the expected mean price over the lifetime of the plantation instead of facing uncertain prices in the future. Signing the contract would also mean that flexibility in planting and harvesting would not be considered any longer. We implicitly aim to derive the costs of choosing that contract, i.e. the lost value of the option to postpone (or refrain altogether from) planting and adjust harvesting in the future. To this end, we consider only one scenario with a fixed expected price in our SP model and compare the simulated NPV from that model with the expected NPV considering uncertain prices and flexibility in plantation set-up and harvesting. The expected mean prices should be identical in average between the deterministic and the SP variants, but the SP

[7] If we would only consider a mix of annual arable crops, the problem with a limited number of Monte-Carlos would be less severe. Still, depending on the stochastic process, a few hundreds draws might still not be representative for the relevant moments of the distributions.

approach should give a higher expected NPV. The difference is the option value (DIXIT and PINDYCK 1994, pp.97–101), which can here be also understood as a risk premium[8].
The technical implementation of such a model using SP for a ROA can hence be summarized with five main steps. First, we define the (state contingent) decision variables of the problem. Second, we define the relations (i.e. equations and constraints) between these decision variables, including lagged relations between time points. Third, we build a deterministic programming model from these two elements, and next introduce node indices for the state contingent decision variables and reflect the ancestor relation between lagged nodes (states). Fourth, we choose an appropriate distribution for the uncertain parameter(s), draw a sufficient number of Monte-Carlo scenarios and construct from them a reduced tree with probabilities. Finally, we solve for the expected NPV and estimate the option values.
Note that assuming a risk-averse farmer would require applying different discount rates to the cash flows from SRC (i.e. stochastic) and from alternative agriculture (i.e. risk-free) (e.g. see FINGER (2016) for further details). We restrict ourselves to a risk-neutral decision and hence apply the market-based risk-free discount rate to all cash flows.

4 Parameters

The data for the model are taken from MUSSHOFF (2012) and selected other sources (see Table 1), referring to cultivation of SRC poplar on a farm in the region Mecklenburg-Western Pomerania (northern Germany) which is characterized, compared to average German conditions, by low soil quality and precipitation, thus also low returns for alternative agriculture. This choice is motivated by the observation that in this region, according to SCHULER et al. (2014, p.69), more than 90% of agricultural lands are suitable for SRC cultivation.

Table 1. Data used as the benchmark and the sources of the data

Parameter	Units	Assumed value
Short-Rotation Coppice		
Labour requirements	Labour units / ha	0
Planting costs	€ / ha	2875.00
Biomass growth function		
Multiplier for last year's biomass	-	1.54
Constant increase per year	t DM / ha	6.68
Harvesting costs		
Fixed costs a farm level	€	66.75
Quasi-fixed costs for each plot	€ / ha	272.13
Variable costs, depending on harvested quantity	€ / t DM / ha	10.67
MRP for logarithmic output price ($\ln P_t$)		
Starting value	-	3.92
Average value	-	3.92
Speed of reversion	-	0.22
Variance of Wiener process	-	0.22
Reconversion costs	€ / ha	1400.00

[8] Note that this is the option value for the first period and for planting decision only, while there is a different option value at each node of the scenario tree and for each following decision. In order to calculate those, we have to repeat the simulation with a fixed expected price for each single node of the scenario tree. For the sake of simplicity, we restrict ourselves to the option value for the planting decision and for the first period only.

Density of trees	Number of trees / ha	9000
Alternative agriculture		
Net annual cash flow from traditional agriculture		
Alternative crop 1	€ / ha	487.25
Alternative crop 2	€ / ha	584.70
Set-aside land	€ / ha	-50.00
Labour requirements		
Alternative crop 1	Labour units / ha	15
Alternative crop 2	Labour units / ha	25
Set-aside land	Labour units / ha	1
Farm characteristics		
Land endowment	ha	10
Labour endowment	Labour units	160
Real risk-free discount rate	%	3.87

Source: based on MUSSHOFF (2012), FAASCH and PATENAUDE (2012), and WOLBERT-HAVERKAMP (2012)

Two elements in the parameterization are worth further comments. First, we take the *yield function* from ALI (2009), introduce some required parameters and regress from there a linear function for biomass stock that depends on previous year's stock:

$$stock_{p,t} = 1.651 * stock_{p,t-1} + 3.962 \quad (12)$$

Second, we aim to model *harvesting cost* separated by (a) costs at farm (fixed) and (b) per plot (quasi-fixed) plus (c) costs per ton of harvested biomass (variable), in order to consider possible economy of scale. Based on SCHWEIER and BECKER (2012) and PECENKA and HOFFMANN (2012), we derive the following harvesting costs function:

$$harvCost_{p,t} = 66.75 + 272.13 * l_{src} + 10.67 * l_{src} * harvQuant_{p,t} \quad (13)$$

5 Results

We've illustrated so far how ROA, combined with SP, can be applied to investment problems in agriculture. We see computing time not as an issue[9] if tree reduction is used, even if farm constraints are taken into account and complex binary choices are considered, as in our case with flexible harvest intervals for SRC.

The significant effect of considering farm endowments beyond land can be seen in Fig.2. Considering the EFA under "greening", SRC can be used to fulfill the EFA requirements, but uses more land compared to set-aside as one ha of SRC only count as 0.3 ha EFA. But the labor previously used on the land that is additionally required for SRC, compared of using set-aside to fulfill the EFA, is partly freed. That allows increasing the share of labor intensive crops with a higher gross margin per hectare (alternative crop 2) on the remaining hectares (Fig.2) and thus dampens the impact of competition for land. A similar result would be found if we would assume that the freed labor can be employed off-farm. Due to that effect, the investment trigger is reduced compared to a simpler model where only competition for land is reflected.

[9] Note that the running time might be reduced further by processing in parallel on multiple cores.

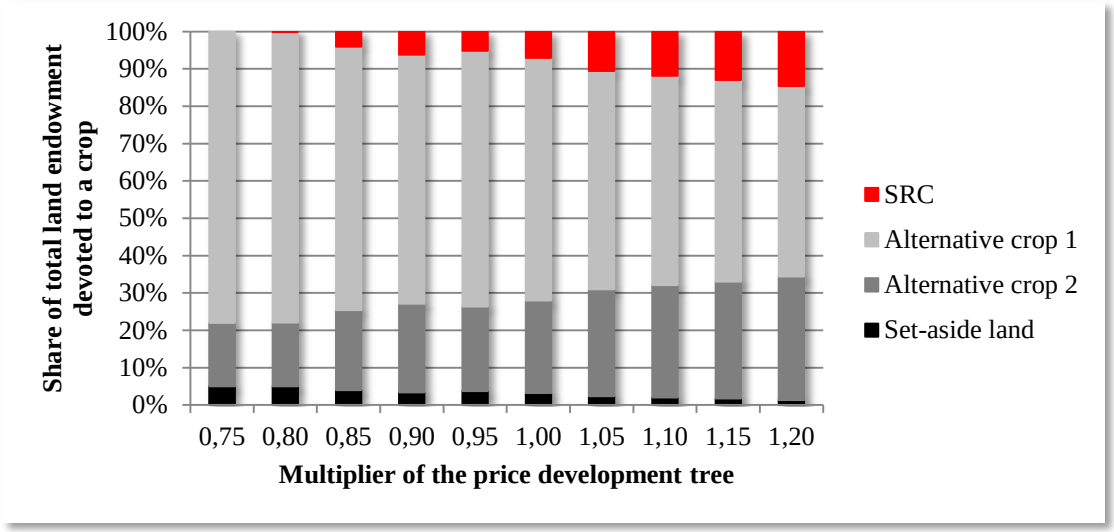

Figure 2. Expected land distribution (average per year) under different multipliers of the price development tree (based on the ROA)

Next, we found that under the benchmark market conditions SRC cannot compete with conventional crops which fits the empirical evidence. It is important to highlight that Fig.2 illustrates the *expected* mean land distribution over the simulation period. Although SRC is expected to be cultivated under the benchmark condition (when the price multiplier is set equals one), it is not planted immediately, but the decision is rather postponed. The land share of SRC observed stems, hence, from future states-of-nature where prices increase above the current level and a SRC plantation is set up in years 2-4. Shifting the price development tree up by 20% doesn't lead to an immediate cultivation either. In contrast, the same model, but based on the Classical Investment Theory[10], states an immediate planting of SRC to be optimal already by the annual output prices being five percent above the benchmark. The observed difference between the Classical and New Investment Theory is in line with the theoretical background. The Classical Investment Theory also provides a lower maximized expected NPV than the ROA (Fig.3) since under the latter the farmer can always adjust the land use and management, depending on the state-of-nature.

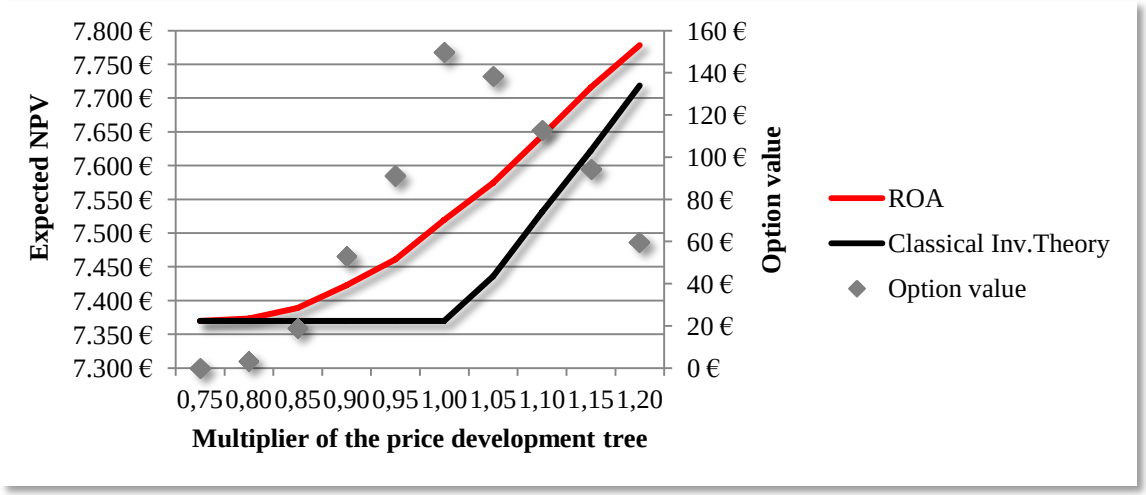

Figure 3. Expected NPVs based on different investment theories and the option value as the difference between two respective NPVs

[10] In order to run the model under the Classical Investment Theory, we force the farmer to make the final planting, harvesting and reconversion decision now, based on the expectations of output price, i.e. we switch from stochastic to deterministic programming. Note that the underlying stochastic process for the output price stays the same.

Finally, we illustrate the relationship between the mean level of the stochastic output price and the option value (Fig.3). Increasing the expected mean has first a positive effect on the option value and then a negative one, converging to zero at the investment trigger price.

6 Conclusion

We present a straightforward approach of combining ROA and SP for the analysis of agricultural investment projects, based on the example of short-rotation coppice cultivation, considering flexibility in planting and harvesting, as well as farm constraints. Based on a deterministic standard mathematical programming model, we derive a stochastic program by rendering the relevant decision variables state contingent and allowing for several states at each decision node. Next, we discuss how the SP approach can be used to derive different option values. We also explain why simple symmetric trees are typically unsuitable for the analysis of investment projects with many states due to the curse of dimensionality, and therefore propose to use scenario tree reduction and construction approaches. Furthermore, we argue that it is often necessary to use Mixed Integer Programming (MIP) approaches to properly reflect investment problems in agriculture. In this regard, it is even more important to use advanced tree construction methods as MIP can be quite hard to solve.
Our example for short rotation coppices is relatively simple in structure, since we aim to focus on the methodology. Still, our model for investment and management of SRC plantation is more detailed than was found currently in literature. The main empirical findings from applying our model are twofold. First, SRC is not viable under the benchmark market condition compared to alternative crops. Second, especially if SRC counts towards the EFA requirement under the latest CAP reform, SRC cultivation might become interesting, as it requires no or very little farm labor and hence allows reallocation of remaining land to a more labor intensive use and thus more profitable agricultural activities.
More generally, the described combination of ROA, SP and tree reduction can be applied to many real world (agricultural) applications, where distributional assumptions and model complexity (e.g. multi-stage problems) prevent closed form solutions.

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