SP 2014-02
July 2014

Staff Paper

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Pricing of Options with Stochastic Volatilities: Application to Agricultural Commodity Contracts

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Preface

The original version of this paper was presented at a conference of the Northeastern Agricultural and Resource Economics Association in June 2001, and the abstract was printed in the October 2001 issue of *Agricultural and Resource Economics Review*. Subsequently, a slightly edited version of the paper was presented at a conference in St. Louis in April 2002. But, neither conference resulted in publication.

The paper reports on a thoughtful analysis of the pricing of options on corn and soybean futures, and it gives a sense of a portion of Lordkipanidize's (2004) PhD dissertation research in a relatively few pages. It is still appropriate, therefore, to establish an accessible historical record of her work, by placing it in the Dyson School's Staff Paper Series.

The text of this staff paper is nearly identical to the 2001 and 2002 presentations, except that some parameter estimates have been changed to reflect revised results contained in her dissertation and a few minor editorial changes have been made. The dissertation contains tables, figures, additional discussion, and an application to S & P 500 futures and options markets' data.

I regret that her research results were not made more widely available via journal publication.

WGT

Pricing of Options with Stochastic Volatilities: Application to Agricultural Commodity Contracts

Nasibrola Lordkipanidze and William G. Tomek

Abstract. The empirical evidence in this paper supports the existence of seasonality, time-to-maturity, and long-memory effects in the volatility of prices, but not in the returns themselves, in corn and soybean futures markets. This volatility is modeled as an Orenstein-Ulenbeck process driven by fractional Brownian motion. The inclusion of long-memory stochastic volatility is found to have a significant impact upon the term structure of implied volatilities, and should be able to provide better estimates of in- and out-of-the money options' prices.

Acknowledgement. We gratefully acknowledge the insights and suggestions provided by Robert Jarrow.

At the time this paper was written, Lordkipanidze was a PhD candidate and Tomek an emeritus professor in the Department of Agricultural, Resource and Managerial Economics, now the Dyson School of Applied Economics and Management, at Cornell University.

1. Introduction and Background

Understanding and modeling the stochastic behavior of spot and futures prices is important for pricing contingent claims on commodities. A good empirical model for valuing such claims should capture the observed statistical characteristics of actual market prices. The well-known Black-Scholes options pricing model is widely used for pricing contingent claims. It relies on the assumption that the logarithms of price changes are independently and identically distributed Gaussian variables and can be modeled as geometric Brownian motion – the continuous counterpart of a random walk. Even though the Black-Scholes model provides a reasonable first approximation to options prices, it results in systematic biases across maturities and moneyness. One possible explanation for these inaccuracies is that the statistical properties of price series differ from those implied by the model. In fact, empirical evidence suggests that commodity price dynamics exhibit non-trivial statistical characteristics: substantial skewness and leptokurtosis, non-integer (fractional) dimension of the probability distribution of the underlying process, a short-time memory of only a few minutes in price changes, and long-range correlations in volatilities.

The question of deviations of commodity price behavior from random walk and Gaussian distribution assumptions was first raised in a series of papers by Mandelbrot

(1963, 1966, 1967). Studying cotton prices, Mandelbrot observed the following discrepancies between Brownian motion and the empirical facts: apparent non-stationarity of the underlying process, repeated instances of discontinuous price changes, concentration of high price variability, cyclic behavior of price series, leptokurtosis, and long-term dependence in the variances of price changes. Since then, these characteristics have been repeatedly observed in many speculative price series.

Early studies of the behavior of market prices implied that asset returns of many markets do not follow a random walk and that they may have long-term dependencies in price correlations. Existence of a stochastic memory in market returns implies that prices may not be efficient in quickly arbitraging new information; it may be possible to obtain abnormal profits on the basis of past information (Irwin, Zulauf, and Jackson, 1996). This would contradict the efficient market hypothesis. But, studies (Cheung and Lai, 1993; Crato, 1994; Fung and Lo, 1993), using new statistical tools, have found that financial returns in major liquid markets have no significant memory; i.e., a correlation function of price increments is zero for time periods longer then 15 minutes. Also, short-time correlations in price changes may be consistent with the efficient market hypothesis because transaction costs, associated with arbitrage, are likely important in commodity markets. Furthermore, correlations in spot price changes for commodities can occur in rational markets (Tomek 1994, 2000).

Although evidence of correlations in price changes for futures contracts is not strong, autocorrelations in the variance of price changes appear to be positive and decay very slowly, implying long-range dependence in volatility (Streeter and Tomek, 1992;

Crato and Ray, 2000). Extremely slow power law decay provides a measure of the well-documented clustering or persistence of volatility observed in market data.

In addition to negligible correlations of price increments and the long-range correlation of volatilities, another striking feature observed in many markets is that marginal distributions of price changes have 'fat tails', i.e., are leptokurtic, and possibly skewed to the right. If this is true, price variations usually decay more slowly than a Gaussian distribution would imply. Corazzo, Malliaris, and Nardelli (1997) investigated the structure of six agricultural futures markets by estimating parameters of the Pareto-Levy stable probability distribution and found that returns have long-memory and fractal structure. That is, they are characterized by nonstandard properties such as fine structure, local and global irregularities, self-similarity, and non-integer dimension of probability distribution of the underlying process. The Hurst exponent estimates for various agricultural futures indicate evidence of long-term memory for the entire daily returns time series.

The existence of volatility smiles and skews in option prices is also evidence of the failure of Gaussian assumptions. In theory, the volatility should be constant for all strikes, but predictions based on the Black-Scholes model find that out-of-the-money options are priced too low, while in-the-money options are priced too high. This suggests that a log-normal distribution and a constant volatility assumption do not adequately describe price behavior in commodity markets.

Attempts to reconcile theory with the empirical evidence have led to a rapidly expanding class of models that modify the volatility specification to make it stochastic. Stochastic volatility allows reproducing more realistic returns distribution. In particular, it

generates kurtosis in an otherwise Gaussian process. Introducing correlations between noise sources allows for asymmetry of the tails of the probability density function of the returns. Stochastic volatility causes options to display both smile and term structure effects.

Accuracy of stochastic volatility models, however, depends on correctly specifying the volatility process itself. Many alternative models have been developed to fit the data. One hypothesis, conceived by Clark (1973) and developed further by Engle (1982) and Bollerslev et. al (1992) in classes of models, is specified as GARCH models. GARCH processes have fat-tailed unconditional distributions and conditional distributions whose second moments are changing through distributed lag or ARMA processes. Using a GARCH model for corn, pork bellies, soybeans, sugar, wheat, and gold daily spot price changes, Yang and Brorsen (1992) confirmed that the variance of the price changes is not constant.

The GARCH specification reproduces many characteristics of non-Gaussian distributions, such as fat tails, clustering and the presence of autocorrelation in volatility. Some studies have shown, however, that the autocorrelation function of the variance has a much longer time memory than GARCH models suggest. The empirical presence of long memory is found in the persistence of autocorrelation. Studying high frequency price series, Bredit, de Lima and Crato (1993) and Bollerslev and Mikkelsen (1996) found that volatility of many U.S. stock return series exhibits long-memory behavior, that is, autocorrelation of functional moments of return series decays very slowly. Scaling analysis of market indexes and exchange rates shows that the volatility autocorrelations

decline hyperbolically with time, in contrast with the GARCH models where they decay exponentially.

To allow for the effects of long-memory in option pricing models, several authors propose a fractional Brownian motion (fBm) as a source of noise for returns. In contrast to standard and geometric Brownian motions, fBm allows long-range dependence for the whole history of the process. But, the difficulty of this approach is that fractional Brownian motion it is not a semi-martingale, and can not be arbitraged to become one (Rogers, 1997). Therefore, in this case, no equivalent martingale measure exists, and the first fundamental asset pricing theorem (the absence of arbitrage is equivalent to the existence of equivalent martingale measure) cannot be applied. As Rogers explains, fBm is a convolution of the Brownian increments with the power-law kernel, and arbitrage is happening because of the behavior of this kernel near zero. Long-range behavior is happening because of the behavior of the kernel at infinity. A way of overcoming this difficulty is to change the kernel into a kernel smoother in the neighborhood of zero.

Another possibility, employed by Comte and Renault (1998), is to maintain standard Brownian motion as a source of noise for the returns process and to use fBm as a source of noise for the volatility process. In this case, since volatility itself is not a traded asset, it is impossible to construct arbitrage, and therefore, the underlying asset price process and the price process of the option are conformable to a necessary and sufficient condition for the existence of equivalent martingale measure. The volatility is modeled as an Ornstein-Ulenbeck process driven by a fBm, which is independent of the underlying Brownian motion. This model should be able to account for both a long

memory in volatility and a fractional dimension of the underlying probability distribution of futures returns.

Given this background, the objective of this paper is to implement a fractional stochastic volatility model of Comte and Renault (1998) to price options on corn and soybean futures contracts. The rest of this paper is organized as follows. In section 2 we describe the fractional stochastic volatility model and provide the associated option pricing formulas. Section 3 outlines the estimation techniques and statistical inference issues. Section 4 describes the data and empirical findings, and Section 5 concludes the paper.

2. The Fractional Stochastic Volatility Model

In this section we describe the stochastic volatility model for pricing a European call option on a futures contract with strike price X and maturity T_C. $F(t)$ is a futures price level and $\sigma(t)$ is an instantaneous volatility at time t. For convenience, the instantaneous interest rate is assumed constant, so that a traded money market account earns interest at this rate. Then the stochastic differential equations for a price process $F(t)$ with a constant drift μ and the stochastic volatility $\sigma(t)$ can be written as (1) and (2) respectively.

$$dF(t) = \mu F(t)dt + \sigma(t)F(t)dW_1(t) \quad (1)$$

$$X(t) = \ln\sigma(t)$$

$$dX(t) = k(\theta - X(t))dt + \gamma dW_2^{\alpha}(t) \quad (2)$$

where $\{W_1(t), W_2^{\alpha}(t)\}$ are the standard and fractional Brownian motions respectively. The volatility function is an exponential function of an Ornstein-Ulenbeck process, with constant rate of mean-reversion k and constant long-run mean level θ. It is possible to replace the fractional Brownian motion with a standard Brownian motion in equation (2)

by differentiating it at the fractional order α. Now the stochastic differential equation for the fractional volatility process can be written as

$$dX^{\alpha}(t) = k(\theta - X^{\alpha}(t))dt + \gamma dW_2(t) \tag{3}$$

where

$$W_2^{a}(t) = \int_0^t \frac{(t-s)^{\alpha}}{\Gamma(1+\alpha)} d\, W_2(s)$$

Note, the volatility coefficients k, θ, γ remain invariant to this transformation (see proof in Comte and Renault, 1996).

The stochastic process specified by equations (1) and (3) admits an equivalent martingale representation. Since the volatility is not a traded asset and nonstandard fractional properties are set only on the volatility process and not directly on the underlying futures price process, a riskless arbitrage cannot be constructed. Thus, there exists a probability distribution under which the discounted futures price process is a martingale. However, this martingale probability is not unique. The addition of the stochastic volatility, which is not directly tradable, makes the market incomplete in the sense that trading in the money market account and the underlying futures contracts can not replicate all contingent claims based on the underlying futures contract.

Eisenberg and Jarrow (1994) showed that the introduction of an additional traded asset, imperfectly correlated to the underlying asset, can complete the market. Following this approach, we complete the market with another call option $A(t)$ written on the futures $F(t)$ with the same strike price and the closest maturity T_A. The time t price of this call option satisfies the following stochastic differential equation:

$$dA(t) = \eta_0(t)A(t)dt + \eta_1(t)A(t)dW_1(t) + \eta_2(t)A(t)dW_2(t) \quad (4)$$

where

$$\eta_0(t) = \frac{\partial A(t)}{\partial t}F(t)dt + \frac{\partial A(t)}{\partial F(t)}\mu(t)F(t)dt + \frac{1}{2}\frac{\partial^2 A(t)}{(\partial F(t))^2}F^2(t)\sigma^2(t)dt$$

$$+ \frac{\partial^2 A(t)}{(\partial\sigma(t))}k(\theta - X(t))\sigma(t)dt + \frac{1}{2}\frac{\partial^2 A(t)}{(\partial\sigma(t))^2}\gamma^2(t)dt$$

$$\eta_1(t) = \frac{\partial A(t)}{\partial F(t)}\sigma(t)F(t)dt \qquad \eta_2(t) = \frac{\partial A(t)}{\partial\sigma(t)}\gamma(t)dt$$

The second Brownian motion influences futures price dynamics through a nonzero volatility coefficient η_2. With the two sources of risk $\{W_1(t), W_2(t)\}$ and two imperfectly correlated traded risky assets, the market is complete and the equivalent martingale measure can be determined uniquely.

Now Girsanov's theorem can be used to convert the discounted futures price $e^{-rt}F(t)$ and discounted options price $e^{-rt}A(F(t), t)$ into martingales. The two market prices of risk, $\lambda_1(t)$ - the risk premium associated with the futures contract - and $\lambda_2(t)$ - the volatility risk premium - can be found using the following transformations:

$$\tilde{W}_1(t) = W_1(t) + \int_0^t \lambda_1(u)du \qquad \tilde{W}_2(t) = W_2(t) + \int_0^t \lambda_2(u)du \quad (5)$$

where $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are independent standard Brownian motions adapted to the filtration ($\mathcal{F}_t$) in the probability space (Ω, $\mathcal{F}$, Q). Using (5) it can be shown that the martingale property of the discounted prices $e^{-rt}F(t)$ and $e^{-rt}A(t)$ under Q implies that

$$\lambda_1(t) = -\frac{\mu(t) - r}{\sigma(t)} \quad (6)$$

$$\lambda_2(t) = -\frac{\eta_0(t) - r}{\eta_2(t)} - \frac{\eta_1(t)\lambda_1(t)}{\eta_2(t)}. \quad (7)$$

Given the above relations, the dynamics of the fractional volatility under the equivalent martingale measure *Q* can be expressed as

$$dX^{\alpha}(t) = k(\theta - X^{\alpha}(t))dt + \gamma\lambda_2(t)dt + \gamma d\tilde{W}_2^{\alpha}(t) \tag{8}$$

where

$$\mathrm{X}^{\alpha}(t) = \frac{d}{dt}\int_0^t \frac{(t-s)^{-\alpha}}{\Gamma(1-\alpha)} X(s)ds. \tag{9}$$

The price of the call option is given by

$$C(F(t),K,T_C) = e^{-r(T_C-t)}E^Q\left(\max[F(t)-K,0] \,\middle|\, \mathcal{F}_t\right). \tag{10}$$

Since the volatility premium in (7) depends only on the current level of the volatility, the expectation in (10) can be computed by conditioning on the volatility path, i.e.

$$C(F(t),K,T_C) = e^{-r(T_C-t)}F(t)\left\{E^Q\left[\Phi\left(\frac{m_t}{U_{t,T_c}}+\frac{U_{t,T_c}}{2}\right)\right] - e^{-m_t}E^Q\left[\Phi\left(\frac{m_t}{U_{t,T_c}}-\frac{U_{t,T_c}}{2}\right)\right]\right\}$$

where (11)

$$m_t = \ln\left(\frac{F(t)}{KB(t,T_c)}\right),\quad U_{t,T_c} = \sqrt{\int_t^{T_c}\sigma(u)^2du},\quad \Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}e^{-t^2/2}dt\,.$$

This is an option pricing formula under the Hull and White stochastic volatility model, where the option value is a weighted average of the Black-Scholes values, and the conditional probability distribution is computed with respect to the conditional probability distribution of $U_{t,T}$ given the instantaneous volatility $\sigma(t)$.

3. Estimation Procedures

To estimate the long-memory volatility model described above, we follow the procedure suggested in Comte and Renault (1998). First, the long-memory parameter α is found using the semiparametric method of Robinson (1995) and then the parameters (k, θ, γ) of the Ornstein-Ulenbeck log-volatility process are obtained by estimating an AR(1) process after fractional differentiation at the estimated order α. In contrast to Comte and Renault (1998), our model does not assume that the volatility risk premium is zero. Thus, in addition to the parameters (k, θ, γ) we also need to estimate market prices of risk $\lambda_1(t)$ and $\lambda_2(t)$.

Since the volatility is not directly observable, it must be either estimated from time-series data of underlying futures prices or inferred from options prices. In this paper the latter approach is used. The implied volatility $\sigma_{i,t}$ can be obtained by inverting the Black-Scholes formula, i.e. solving the nonlinear equation

$$C_{t,T_c} = e^{-r(T_C-t)}F(t)\left[\Phi\left(\frac{m_t}{U_{t,T_c}}+\frac{U_{t,T_c}}{2}\right) - e^{-m_t}\Phi\left(\frac{m_t}{U_{t,T_c}}-\frac{U_{t,T_c}}{2}\right)\right]$$

where (12)

$$m_t = \ln\left(\frac{F(t)}{KB(t,T_c)}\right), \quad U_{t,T_c} = \sigma_{i,t}\sqrt{T-t}, \quad \Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-t^2/2}dt\,.$$

The implied volatilities will be numerically evaluated using the Newton-Raphson method.

After obtaining implied volatilities, the market prices of risk $\lambda_1(t)$ and $\lambda_2(t)$ are obtained by discretising equations (1) and (4) and estimating the parameters $\mu, \eta_0, \eta_1, \eta_2$ in

$$F_{t+\Delta t} = (1+\mu)F_t + \sigma_t F_t \sqrt{\Delta t}\,\xi_{t+\Delta t} \quad (13)$$

$$A_{t+\Delta t} = (1+\eta_0)A_t + \eta_1 A_t \sqrt{\Delta t}\,\xi_{t+\Delta t} + \eta_2 A_t \sqrt{\Delta t}\,\varepsilon_{t+\Delta t} \quad (14)$$

where Δt =1. We compute OLS estimates of μ and the residuals $\xi_{t+\Delta t}$ from equation (13), and then substitute these residuals into equation (14) to find parameters η_0, η_1, η_2. Given these parameters the volatility risk premium can be computed from (6) and (7).

To estimate the long-memory parameter α we use a semiparametric method known as a regression on the periodogram first suggested by Geweke and Porter-Hudak (1983) and developed further by Robinson (1995). The periodogram is defined as

$$I(\nu) = \frac{1}{2\pi N}\left|\sum_{j=1}^{N} X_j e^{ij\nu}\right|^2 \quad (15)$$

where ν is the frequency, N is the number of terms in the series, and X_j are the implied volatilities computed in the previous step. At the origin, a fractional Gaussian noise model has a spectral density proportional to $\nu^{2\alpha}$. Since $I(\nu)$ is an estimator of the spectral density, an OLS regression of the logarithm of the periodogram versus the logarithm of the frequency ν should give an estimate of α. The spectral regression equation is

$$\ln(I(\nu_k)) = c - \alpha(2 \ln \nu_k) + \xi_k \qquad k=l+1, \ldots, m \quad (16)$$

and the estimate of α is given by

$$\hat{\alpha} = \mathrm{Y'X(X'X)^{-1}}$$

where $Y = (Y_{l+1}, \ldots, Y_m)$, $X = (X_{l+1}, \ldots, X_m)$, $Y_k = \ln(I(\nu_k))$ and $X_k = -2\ln(\nu_k)$. Since the proportionality to $\nu^{2\alpha}$ holds only for the ν close to the origin, not all the periodogram

frequencies are used in the regression. Robinson (1995) shows that the periodogram for frequencies between $2\pi/k$ and $2\pi/n$ satisfying

$$\begin{aligned} m &= o(N^{4/5}) \\ \frac{l}{m} &+ \frac{m^{1/2}\ln(m)}{l} + \frac{(\ln N)^2}{m} \to 0, \end{aligned}$$

is asymptotically i.i.d. χ^2 and the estimate of α is consistent and asymptotically normal.

As specified in the previous section, the log-volatility process dynamics under the martingale measure follows the stochastic differential equation

$$d\ln\sigma^{\alpha}(t) = k(\theta - \ln\sigma^{\alpha}(t))dt + \gamma\lambda_2(t)dt + \gamma d\tilde{W}_2^{\alpha}(t). \tag{17}$$

To estimate the parameters of this equation, we need to fractionally differentiate the log-volatility process, that is to approximate the integral

$$x^{(\alpha)}(t) = \int_0^t \frac{(t-s)^{-\alpha}}{\Gamma(1-\alpha)} dx(s)$$

where $x(t) = \ln\sigma(t)$. This can be done by numerically evaluating integrands on a discrete partition of the interval of observation $[0, t]$: k/n, $k = 0,1,\ldots,[ht]$[1] by step functions:

$$\hat{x}^{(\hat{\alpha}_m)}(t) = \sum_{kh<t} \frac{(t-kh)^{-\alpha}}{\Gamma(1-\alpha)} \Delta x_{kh}. \tag{18}$$

Comte and Renault (1998) showed that $\hat{x}^{(\hat{\alpha}_m)}(t)$ converges to $x^{(\alpha_m)}(t)$ for $kh \to \infty$.

After obtaining the fractional log-volatilities the process can be estimated using an indirect inference method suggested by Pastorello, et al (2000). Equation (17) is an AR(1) process and can be directly integrated, giving

[1] $[z]$ is the integer j such that $j \le z \le j+1$

$$\ln\sigma^{(\alpha)}(t+\Delta t) = \theta(1-e^{-k\Delta t}) + e^{-k\Delta t}\ln\sigma^{(\alpha)}(t) + \gamma\lambda_2(t) + \gamma\sqrt{\frac{1-e^{-k\Delta t}}{2k}}\xi(t+\Delta t) \quad (19)$$

To estimate this equation we need to maximize the log-likelihood function

$$\ln L(a,b,c) = \frac{1}{n}\sum_{t=2}^{n}\left\{-\frac{1}{2}\ln c^2 - \frac{1}{2}\frac{\sigma^{(\alpha)}(t+\Delta t) - a - b\sigma^{(\alpha)}(t) - \gamma\lambda_2(t))}{c^2}\right\},$$

where $a = \theta(1-e^{-k\Delta t})$, $b = e^{-k\Delta t}$, $c = \gamma\sqrt{\frac{1-e^{-k\Delta t}}{2k}}$. Maximization of the log-likelihood function with respect to the parameters *a*, *b*, and *c* is equivalent to running an OLS regression on the volatility $\ln\sigma^{(\alpha)}(t+\Delta t)$, its lagged value $\ln\sigma^{(\alpha)}(t)$, the volatility risk premium $\lambda_2(t)$, and a constant.

Given the estimated parameters of Ornstein-Ulenbeck process and the volatility risk premium, a sample path of the $\ln\sigma^{(\alpha)}(t+\Delta t)$ can be simulated. Then the expectation (10) is computed by Monte Carlo simulations to get the values $C(F(t),K,T_C)$ conformable to the long-memory stochastic volatility model.

4. Data and Estimation Results

The empirical analysis is based on daily futures and options prices for corn and soybeans for the period January 3, 1989 to November 28, 2000. (These contracts are traded on Chicago Board of Trade, now a division of the Chicago Mercantile Exchange.) The 1989 start date provides a sample period where market prices were above government price support levels; also before 1989, the volume of trading in options contracts was small.

Since futures contracts mature, a continuous sequence of prices was created by rolling over to the next contract when the nearby contract reached its expiration month. Futures prices during the maturity month are excluded to avoid any nonstationarity in price series due to increased volatility associated with unusual market activity near maturity. The options data used for each trade date are the settlement price, exercise price, and time to maturity. The risk free rate of interest associated with each option is derived from the three-month Treasury bill prices obtained from CRSP daily files.

In the first step of the estimation procedure, implied volatilities are obtained from the options that are the closest to being at-the-money. The moneyness of an option is defined as its discounted strike price divided by the corresponding futures price. An option is exactly at-the-money when its moneyness equals one. The Newton-Raphson method is used to compute the implied volatilities, except at maturity (T=0), when they are obtained from the conditional variances at the valuation date.

The term structure of the implied volatilities for at-the-money options for varying times to maturity are summarized by year and on average in the dissertation. The estimates of the term structures for both corn and soybeans indicate that they are relatively low in the early part of the life of a contract, increase gradually, reach their maximum values sometime between seven to eight months to expiration, and then decrease throughout the remaining life of the contract. The standard errors behave in a similar fashion.

The estimates of long-memory parameters for the logarithms of returns and for the volatility for corn and soybeans are reported for varying sample lengths in dissertation Tables 5.4 and 5.5. The estimates of the α for *returns* are near zero, though

are sometimes statistically different than zero. (The significant non-zero estimates may reflect the way that the futures price series were constructed by rolling over the prices from one contract to the next as maturity approaches. This procedure creates a series that approximates the behavior of the underlying spot prices.)

For the *volatility*, the estimates for the varying sample lengths are in the range 0.29 to 0.40 for corn and 0.42 to 0.51 for soybeans and are significantly different than zero. (Time series that have no memory would have α=0.) To test for the sensitivity of the estimates to different sample periods and sample sizes, the data were divided into two relatively equal sub-samples. The results for the two sub-samples show that the estimates are not sensitive to the estimation period and number of observations used. Thus, we find little or no memory in returns and strong evidence of long memory in volatility.

After fractionally differentiating log-volatility at the estimated degree α, we fit the regression equation (19) for the Ornstein-Ulenbeck process (Lordkipanidze 2004, Table 5.6). The estimates of all of the parameters, except the parameter γ, are statistically significant and consistent with respect to the different sample periods. Using these estimates, Monte Carlo simulations are performed to compute the conditional probability distribution of $U_{t,T}$ given the instantaneous volatility $\sigma(t)$. Call option prices conditional on the volatility path are computed from equation (11). The simulated volatility paths over time are computed from the fractional stochastic volatility model and from the Black-Scholes model, and plots of the paths are available in the dissertation. The Back-Scholes implied volatilities differ considerably for options with different maturities, whereas the volatilities implied by fractional stochastic volatility model fluctuate moderately around the mean level. Thus, the inclusion of a long-memory stochastic

volatility has a significant impact upon the term structure of implied volatilities. This finding suggests that the stochastic volatility model should be able to get better estimates of in- and out-of-the-money options prices.

5. Conclusion

The empirical evidence provided in this paper supports the existence of long memory in the volatilities of both corn and soybean futures prices. These volatilities also have strong seasonal and time-to-maturity effects (Lordikpanidze 2004). The findings also suggest that options pricing models should be adjusted to accommodate a long memory in volatility. The fractional stochastic volatility model utilized in this paper offers improvement over the standard Black-Scholes option pricing models by incorporating a long memory in volatility and a fractional dimension of underlying probability distribution of futures returns.

This improvement comes at the expense of considerable complexity. Further research is required to test the performance of the model discussed in this paper relative to alternatives. In particular, it would be interesting to compare the performance of the fractional stochastic volatility model with other models of persistence in volatilities, such as the GARCH and ARFIMA classes of models.

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