University of Zurich

Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 121

Floor Systems for Implementing Monetary Policy: Some Unpleasant Fiscal Arithmetic

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May 2013

Floor Systems for Implementing Monetary Policy: Some Unpleasant Fiscal Arithmetic*

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March 16, 2013

Abstract

An increasing number of central banks implement monetary policy via a channel system or a floor system. We construct a general equilibrium model to study the properties of these systems. We find that the optimal framework is a floor system *if and only if* the target rate satisfies the Friedman rule. Unfortunately, the optimal floor system requires either transfers from the fiscal authority to the central bank or a reduction in seigniorage payments from the central bank to the government. This is the unpleasant fiscal arithmetic of a floor system. When the central bank faces financing constraints on its interest expense, we show that it is optimal to operate a channel system.

JEL Codes: E52, E58, E59

Key words: monetary policy, floor system, channel system, standing facilities

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1 Introduction

Central banks typically implement monetary policy by influencing a particular short-term interest rate. Over the last decades, the most common policy framework was the so-called channel system or corridor system. More recently, and often in response to the financial crisis, several central banks have modified their operational framework to a so-called floor system.[1] Despite the growing use of channel or floor systems, only a few theoretical studies on their use exist. In particular, there are no formal studies that compare the two systems. This paper is an attempt to close this gap.

In a channel system, a central bank offers two facilities: a lending facility and a deposit facility. At the lending facility, it is ready to supply money overnight at a given borrowing rate against collateral. At the deposit facility, banks can make overnight deposits to earn interest. The spread between the borrowing rate and the deposit rate is called the interest rate corridor or interest rate channel. The interest rate corridor is chosen to keep the money market interest rate close to its target, which typically is in the middle of the corridor. A change in policy is implemented by shifting the interest-rate corridor.[2] Since market participants prefer to trade amongst themselves rather than to access the standing facilities, a channel system allows the central bank to control the money market rate, while incurring very little operating cost (it reduces the interest paid on deposits and the monitoring costs associated with lending).

A floor system is similar to a channel system except that the deposit rate is set equal to the target rate (the money market rate). How can a central bank, which in practice cannot directly control the money market rate, achieve this? There are two possibilities. First, a floor system can be implemented by fully satiating the demand for money. By providing ample liquidity, the central bank can drive the money market

[1] Both systems are widely used in practice, see Bernhardsen and Kloster (2010). Versions are, for example, operated by the Bank of Canada, the Bank of England, the European Central Bank, the Reserve Bank of Australia, the Swiss National Bank, the Reserve Bank of New Zealand and the U.S. Federal Reserve System.

[2] In theory, there is no need for direct central bank intervention to control the market rate of interest, since money market participants will never mutually agree to trade at an interest rate that lies outside the interest rate corridor. In practice, central banks still conduct open market operations to adjust the quantity of central bank money in circulation. In "normal" times, they do so to accommodate, for example, seasonal fluctuations in the demand for central bank money. In "exceptional" times, in response to severe aggregate shocks they do so to restore the functioning of money markets.

rate to the deposit rate. Under this policy, agents hold so much liquidity that they do not need to borrow, and so the central bank's borrowing rate is irrelevant for the allocation. This possibility is used in practice. Second, it can be implemented by setting the borrowing rate equal to the deposit rate. In this case, by arbitrage, the money market rate is equal to both the borrowing rate and the deposit rate. We will discuss both possibilities in the paper.

Recently, the floor system has been advocated over a channel system for several reasons. First, it allows the central bank to perfectly control the money market rate, whereas in a channel system the interest rate fluctuates within the interest-rate corridor. Second, Cúrdia and Woodford (2011) show that the floor system eliminates any inefficiencies associated with economizing reserves in the banking system. Third, Keister et al. (2008) advocate a floor system, because it decouples liquidity provision from monetary policy. By this, they mean that the central bank can change the interest rate floor without causing a reallocation of reserves, since banks are satiated with reserves.

All of these studies ignore a critical issue: How are the interest payments on deposits financed by the central bank? Since in a floor system banks are satiated with reserves, they do not borrow from the central bank. Consequently, the central bank 1) must print money to finance the interest payments on deposits or 2) has capital income to cover these expenses or 3) it receives transfers from the fiscal authority. The existing literature implicitly assumes that one or more of these income sources are operating in the background.

In this paper, we construct a general equilibrium model where a central bank chooses to conduct monetary policy either via a floor system or a channel system. Unlike the existing literature, we explicitly take into account the financial implications of paying interest on deposits.

The following results emerge from our analysis. First, the optimal framework is a floor system *if and only if* the target rate satisfies the Friedman rule. So, printing money to make the interest payments does not solve the problem, since it is *not* optimal. Second, implementing the *optimal* floor system is costly for the central bank. It requires that the central bank either has sufficient capital income to incur the interest expense or receives transfers from the fiscal authority. In either case, fewer resources are available to the government to finance its other priorities, which may lead to a political backlash and restrictions on the central bank's ability to pay interest on reserves. This is the unpleasant fiscal arithmetic of a floor system. Third, if the central bank is constrained by the fiscal authority regarding the size of its interest expense, a channel system is optimal. This last result does not mean that the central bank cannot implement *a* floor system.[3] It means that it cannot implement the *optimal* floor system, in which case, it should implement a channel system.[4]

Our paper provides a rationale for operating a channel system as opposed to a floor system. Our explanation rests on the idea that central banks may be unable, or are unwilling for political reasons, to incur the interest expense required by the optimal floor system. Is this a compelling explanation for what we observe in reality, or is it simply a theoretical result that may or may not be relevant? We argue that it is relevant for the following reasons:

(i) Using taxes to finance interest payments to banks may not be politically acceptable, since other areas of government spending may be affected. As Feinman (1993) documents, the Federal Reserve long requested the power to pay interest on reserves only to be denied this on budgetary grounds. To illustrate the political opposition, consider the following Congressional testimony by a U.S. Treasury official on the proposal to pay interest on reserves:

> "As a general matter we are sympathetic to many of the arguments put forth by the proponents, particularly with regards to monetary policy. At the same time, however, we are also mindful of the budgetary costs associated with this proposal which would be significant. The President's budget does not include the use of taxpayer resources for this purpose. At this time, then, the Administration is not prepared to endorse that proposal."[5]

[3] The central bank can always choose a target rate that does not satisfy the Friedman rule and set the deposit rate equal to it. Such a floor system, however, is suboptimal and the central bank can always do better by choosing a channel system instead.

[4] Throughout the paper, we assume that the government has access to non-distortionary taxation. However, if the government only has access to distortionary taxation, then a floor system may not be optimal, since the benefits of a floor system may be outweighed by the costs of raising tax revenue via distortionary methods. In general, this is a quantitative issue which is beyond the focus of this paper.

[5] March 13, 2001: Special House Hearing related to H.R. 1009. Statement by Donald V. Hammond, Acting Under Secretary for Domestic Finance, Department of the Treasury. The proposal was not approved.

(ii) Interest payments on reserves are quantitatively important. The Federal Reserve's Large Scale Asset Purchases (LSAP) generated \$1.5 trillion in reserves at the end of 2012, and they are projected to be over \$2.5 trillion if the latest LSAP continue through 2013. Analysis of the Fed's balance sheet by Federal Reserve economists suggest that the interest expense for locking up reserves in the banking system could top \$60 billion for a couple of years under a plausible scenario of rising interest rates.[6] In this scenario, the analysis also shows that remittances to the Treasury would be zero for more than five years. To highlight the potential political backlash from such large payments, note that according to FDIC data, the combined net income of the top U.S. 10 banks in 2010 was less than \$55 billion. Furthermore, Federal Reserve H.8 data at the end of 2012 shows that nearly half of all reserves are held by foreign banks, which suggests a transfer of U.S. taxpayer resources to foreign banks in the neighborhood of \$30 billion. In the current populist environment confronting U.S. politicians, it is not unreasonable to conjecture that Congress could respond to these large payments to domestic and foreign banks by suspending or eliminating the Fed's power to pay interest on reserves. This would complicate the Fed's strategy for shrinking its balance sheet while attempting to keep inflation under control.

(iii) It is relevant for central bank independence and the conduct of monetary policy. It is widely acknowledged that central bank independence is strengthened by having budgetary independence, since a classic way for politicians to control an agency is through budgetary threats (see e.g., Stella, 2005 and Goodfriend, 1994). Ize and Oulidi (2009) show that if a central bank depends on transfers from the government, the latter has the opportunity to influence the central bank's policy by attaching conditions to the transfers. Hawkins (2003) finds that a central bank's independence can be affected if it has low capital stock, since a low capital stock increases the probability of a recapitalization, which is often used by government to influence central bank decisions.[7]

(iv) Central banks have experienced severe capital losses such that they had to be recapitalized. For example, due to bad asset positions, Chile and Costa Rica have required annual transfers from the government over the last 20 years.[8]

It is important not to confuse the (steady state) results of our model with current

[6] See Carpenter et al (2013).

[7] See Stella (2005) for several country examples, and Vergote et al. (2010) for a discussion of the main factors that affected the ECB's balance sheet and profit and loss account since 1999.

[8] See Stella and Lonnberg (2008) for more discussion on central bank recapitalization.

short-run policies. In response to the financial crisis, several central banks have moved from a corridor system towards a floor system, at least temporarily (see e.g., Bernhardsen and Kloster, 2010). Short-term interest rates are currently at a record low. With the deposit rate close to zero, the fiscal implications of paying interest on reserves are largely irrelevant. However, once the economy recovers and short-term interest rates rise, the fiscal implications of a floor system will become relevant again, particularly if central banks choose not to drain reserves prior to raising its policy rate.

1.1 Related Literature

Despite the growing use of channel or floor systems to implement monetary policy, only a few theoretical studies on their use exist. The earlier literature on channel systems or aspects of channel systems were conducted in partial equilibrium models.[9] Except for some non-technical discussions (e.g., Goodfriend, 2002, Keister et al., 2008, and Bernhardsen and Kloster, 2010), there are no papers that compare floor versus corridor systems in a general equilibrium model.

General equilibrium models of channel systems are Berentsen and Monnet (2008), Curdia and Woodford (2011), Martin and Monnet (2011), and Chapman et al. (2011), where the latter two build on Berentsen and Monnet (2008). Our model also builds on Berentsen and Monnet (2008), who analyze the optimal interest-rate corridor in a channel system. In Berentsen and Monnet (2008), the central bank requires a real asset as a collateral at its borrowing facility. Due to its liquidity premium, the social return of the asset is lower than the private return to market participants. From a social point of view, this results in an over-accumulation of the asset if the central bank implements a zero interest rate spread. It is, therefore, socially optimal to implement a strictly positive interest-rate spread to discourage the wasteful over-accumulation of collateral. In contrast, in our model the collateral is nominal government bonds, and there is no waste involved in producing nominal government bonds.[10] Our result, therefore, that the constrained-efficient monetary policy involves a strictly positive interest-rate spread is due to a very different mechanism than the one proposed in

[9] See, e.g., Campbell (1987), Ho and Saunders (1985), Orr and Mellon (1961), Poole (1968), Furfine (2000), Woodford (2001) and Whitesell (2006).

[10] Like money, government bonds are essentially pieces of paper that are costless to produce and so there is no social waste in their use.

Berentsen and Monnet (2008). Furthermore, several aspects of our environment, such as, for example, ex-post heterogeneity of money demand, differs substantially from Berentsen and Monnet (2008).

Martin and Monnet (2011) compare the feasible allocations that one can obtain when a central bank implements monetary policy either with a channel system or via open market operations. The focus of our paper is the floor system and the fiscal implications of the optimal floor system. We also have a more complex structure of liquidity shocks, which allows us to study how policy affects the distribution of overnight-liquidity in a general equilibrium model. In Chapman et al. (2011) the value of the collateral is uncertain. The focus in their paper is on the optimal haircut policy of a central bank.

Cúrdia and Woodford (2011) study optimal policy in a New Keynesian framework with financial intermediation. They also find that the floor system is optimal, because it eliminates any inefficiencies associated with economizing reserves in the banking system. Unlike our paper, they do not study the fiscal implications of paying interest on reserves. Our paper differs from Cúrdia and Woodford (2011) in several other dimensions. First, we do not have sticky prices. Second, we do not have inefficiencies in the financial intermediation process that give rise to a need for reserves. Our framework works via a different mechanism – a combination of risk sharing and collateral requirements. Third, Cúrdia and Woodford (2011) are not explicit about the frictions in their model that make financial intermediation and monetary exchange essential. As stated in their working paper (Cúrdia and Woodford, 2010 p.12), their goal is not "in illuminating the sources of the frictions, but in exploring their general-equilibrium consequences". We are explicit about these frictions, since our environment is characterized by anonymity, lack of communication and absence of record keeping, which make monetary exchange essential.

The structure of the paper is the following. Section 2 describes the environment. Optimal decisions by market participants are characterized in Section 3. Section 4 studies symmetric stationary equilibria. Section 5 identifies the optimal policy and discusses its fiscal implications. Section 6 characterizes the second-best policy. Section 7 contains some extensions of our model, and Section 8 concludes. All proofs are in the Appendix.

2 Environment

Our framework is motivated by the functioning of existing channel systems. For example, as discussed in Berentsen and Monnet (2008), the key features of the ECB's implementation framework and of the euro money market are the following. First, at the beginning of the day, any outstanding overnight loans at the ECB are settled. Second, the euro money market operates between 7 a.m. and 5 p.m. Third, after the money market has closed, market participants can access the ECB's facilities for an additional 30 minutes. This means that after the close of the money market, the ECB's lending facility is the only possibility for obtaining overnight liquidity. Also, any late payments received can still be deposited at the deposit facility of the ECB.

To capture the above sequence of trading in the money market and at the central bank's standing facilities, we assume that in each period two markets open sequentially. The first market is a money market, where market participants can trade money for bonds and where all claims from the previous day are settled. The second market is a goods market where market participants trade goods for money. Its purpose is to generate a well defined demand for money.[11] At the beginning of the goods market, agents receive an idiosyncratic liquidity shock which generates a role for the central bank's standing facility as explained below.[12]

In practice, only qualified financial intermediaries have access to the money market and the central bank's standing facilities. Nevertheless, these intermediaries act on the behalf of their customers: households and firms. We simplify the analysis by assuming that the economy is populated by agents who have direct access to the money market and the central bank's standing facilities. This simplifies the analysis and focuses on the varying liquidity needs of agents in the economy rather than the process of intermediation.

Time is discrete and the economy is populated by infinitely-lived agents. There is a generic good that is non-storable and perfectly divisible. Non-storable means that it cannot be carried from one market to the next. There are two types of agents: buyers and sellers. Each type has measure 1. Buyers can consume in the goods market and

[11] In Section 2.2, we discuss the necessary assumption imposed on the exchange process that makes the use of money as a medium of exchange essential in the goods market.

[12] Our environment builds on Berentsen and Monnet (2008) and Lagos and Wright (2005). The framework by Lagos and Wright (2005) is useful, because it allows us to introduce heterogeneous preferences while still keeping the distribution of money balances analytically tractable.

can produce in the money market. Sellers can produce in the goods market and can consume in the money market.

In the goods market, a buyer gets utility $\varepsilon u(q)$ from consuming q units of the good, where $u(q) = \log(q)$, and ε is a preference shock which affects the liquidity needs of buyers. The preference shock ε has a continuous distribution $F(\varepsilon)$ with support $(0, \infty]$, is iid across buyers and serially uncorrelated. Sellers incur a utility cost $c(q_s) = q_s$ from producing q_s units. The discount factor across periods is $\beta = (1+r)^{-1} < 1$, where r is the time rate of discount.

The preference shock creates random liquidity needs among buyers. The buyers learn the realization of the preference shock ε after the money market has closed, but before the goods market opens. This generates a role for the central bank's standing facilities, since the money market is already closed.

In the money market, buyers have a constant returns to scale production technology, where one unit of the good is produced with one unit of labor generating one unit of disutility. Thus, producing h units of goods implies disutility $-h$. For the sellers, we assume the utility of consuming x units of goods yields utility x. As in Lagos and Wright (2005), these assumptions yield a degenerate distribution of portfolios at the beginning of the goods market.[13]

2.1 Information frictions, money and bonds

There are two perfectly divisible financial assets: money and one-period, nominal discount government bonds. Both are intrinsically useless, since they are neither arguments of any utility function, nor are they arguments of any production function. New bonds are issued in the money market. They are payable to the bearer and default-free. One bond pays off one unit of currency in the money market of the following period. The central bank is assumed to have a record-keeping technology over bond trades, and bonds are book-keeping entries – no physical object exists. This implies that households are not anonymous to the central bank. Nevertheless, despite having a record-keeping technology over bond trades, the central bank has no record-keeping technology over goods trades.

[13] The idiosyncratic preference shocks in the goods market play a similar role to that of random matching and bargaining in Lagos and Wright (2005). Due to these shocks, buyers spend different amounts of money in the goods market. Then, without quasilinear preferences and unbounded hours in the money market, the preference shocks would generate a non-degenerate distribution of money holdings, since the money holdings of individual buyers would depend on their history of shocks.

Note that the money market is not a market where agents borrow from each other. The money market rate ρ is defined as the yield rate needed for agents to hold the outstanding stock of government bonds. This is quite different from Berentsen and Monnet (2008), where people can borrow from each other in the money market before they borrow from the central bank. In an earlier version of the paper, we allowed for such private borrowing, which complicated the analysis considerably. Since the results in the two settings are essentially the same, we have chosen to exclude private borrowing in the money market.[14]

Private households are anonymous to each other and cannot commit to honor inter-temporal promises. Since bonds are intangible objects, they are incapable of being used as media of exchange in the goods market, hence they are illiquid.[15] Since households are anonymous and cannot commit, a household's promise in the goods market to deliver bonds to a seller in the money market of the following period is not credible.

To motivate a role for fiat money, search models of money typically impose three assumptions on the exchange process (Shi 2008): a double coincidence problem, anonymity, and costly communication. First, our preference structure creates a single-coincidence problem in the goods market, since households do not have a good desired by sellers. Second, agents in the goods market are anonymous, which rules out trade credit between individual buyers and sellers. Third, there is no public communication of individual trading outcomes (public memory), which, in turn, eliminates the use of social punishments in support of gift-giving equilibria. The combination of these frictions implies that sellers require immediate compensation from buyers. In short, there must be immediate settlement with some durable asset, and money is the only durable asset. These are the micro-founded frictions that make money essential for trade in the goods market. Araujo (2004), Kocherlakota (1998), Wallace (2001), and Aliprantis et al. (2007) provide a more detailed discussion of the features that generate an essential role for money. In contrast, in the money market all agents can

[14]For example, we could add a market whereby agents search for trading partners as in Afonso and Lagos (2012). The outcome would be that if the agents were appropriately matched they would trade with each other at a rate in between the ceiling and the floor. However those who are not matched would go to the central bank to deposit or borrow. For simplicity, we exclude this matching market and focus on borrowing and lending directly from the standing facilities.

[15]The beneficial role of illiquid bonds has been studied by Kocherlakota (2003), Shi (2008) and Berentsen and Waller (2011). More recent models with illiquid assets include, Lagos and Rocheteau (2008), Lagos (2010b), Lester et al. (2011), and many others.

produce for their own consumption or use money balances acquired earlier. In this market, money is not essential for trade.[16]

2.2 Standing Facilities

At the beginning of the goods market, after all preference shocks are observed, the central bank offers a borrowing and a deposit facility. The central bank operates at zero cost and offers nominal loans ℓ at an interest rate i_ℓ and promises to pay interest rate i_d on nominal deposits d with $i_\ell \geq i_d$. Let $\rho_d = 1/(1+i_d)$ and $\rho_\ell = 1/(1+i_\ell)$. Since we focus on facilities provided by the central bank, we restrict financial contracts to overnight contracts. An agent who borrows ℓ units of money from the central bank in the goods market repays $(1+i_\ell)\,\ell$ units of money in the money market of the following period. Also, an agent who deposits d units of money at the central bank in the goods market receives $(1+i_d)\,d$ units of money in the money market of the following period.

2.3 Consolidated Government Budget Constraint

Let $\mathcal{S}$ denote the central bank's surplus ($\mathcal{S} > 0$) or deficit ($\mathcal{S} < 0$) at time t. It satisfies

$$\mathcal{S} = M^{+} - M + i_\ell L - i_d D, \tag{1}$$

where M is the stock of money at the beginning of the current-period money market and M^{+} the stock of money at the beginning of the next-period money market. Since in the money market total loans, L, are repaid and total deposits, D, are redeemed, the difference $i_\ell L - i_d D$ is the central bank's revenue from operating the standing facility.

The central bank's balance sheet (1) has no capital income for the central bank. In Section 7, we amend the central bank's balance sheet along two lines. First, we endow the central bank with a stock of real assets that provides a stream of revenue in each period. Here, we show that our analysis is unaffected if the real return on these assets is not too high. Second, we endow the central bank with a stock of government bonds that pay interest. This second alternative is simply another way to transfer tax revenue from the government to the central bank, since the government has to

[16] One can think of agents as being able to barter perfectly in this market. Obviously in such an environment, money is not needed.

levy taxes to finance interest payments on the government bonds which it then hands over to the central bank.

Let $\mathcal{D} = G - T$ denote nominal expenditure by the government, G, minus the government's nominal tax collection, T. If $\mathcal{D} < 0$ ($\mathcal{D} > 0$), the government has a primary surplus (deficit). The government's budget constraint satisfies

$$\mathcal{D} = \rho B^{+} - B + \mathcal{S}, \tag{2}$$

where B is the stock of of bonds at the beginning of the current-period money market, B^{+} the stock of bonds at the beginning of the next-period money market and $\rho = 1/(1+i)$ the price of bonds in the money market, where i denotes the nominal interest rate on government bonds.[17] The government budget constraint simply requires that any primary deficit $\mathcal{D} > 0$ must be financed by either issuing additional debt $\rho B^{+} - B$, central bank surplus $\mathcal{S}$, or both. If $\mathcal{S} < 0$, there is a transfer of funds from the government to the central bank. If $\mathcal{S} > 0$, the transfer is reversed.

The consolidated government budget constraint at time t is given by

$$\mathcal{D} = M^{+} - M + i_{\ell} L - i_d D + \rho B^{+} - B. \tag{3}$$

Equation (3) states that the consolidated deficit must be financed by issuing some combination of money and government bonds as in Sargent and Wallace (1981).[18] In addition, the difference $i_{\ell} L - i_d D$ is the central bank's profit from operating the standing facility.

In what follows, we simplify our analysis by assuming that $G = 0$, which implies that $\mathcal{D} = -T$. Furthermore, we assume that $T = \tau M$ are lump-sum taxes ($T > 0$) or lump-sum subsidies ($T < 0$). This simplification avoids distortionary taxation, which we do not want to be the focus of this paper.[19] Accordingly, if $\mathcal{D} = -\tau M > 0$, the households receive a lump-sum money subsidy from the government, and if $\mathcal{D} = -\tau M < 0$, the households pay a lump-sum tax to the government.

[17] Throughout the paper, the plus sign is used to denote the next period variables.

[18] Sargent and Wallace (1981) study the interactions of monetary and fiscal policy. In particular, they argue that if fiscal policy dominates monetary policy, then monetary policy might not be able to control inflation. We use the same consolidated government budget constraint as they do but our focus is different. We derive the fiscal implications of implementing monetary policy either via a channel system or a floor system.

[19] See Aruoba and Chugh (2010) who study optimal fiscal and monetary policies when only distortionary taxes are available.

2.4 First-best allocation

In this section, to obtain a reference allocation, we derive the optimal planner allocation. The optimal planner allocation requires neither money nor bonds, since the planner can dictate the consumption and production quantities. We assume without loss in generality that the planner treats all sellers symmetrically. He also treats all buyers experiencing the same preference shock symmetrically. Furthermore, he cares for everyone's utility equally. Given these assumptions, the weighted average of the expected steady state lifetime utility of buyers and sellers at the beginning of the money market, $\mathcal{W}$, can be written as follows

$$(1-\beta)\,\mathcal{W}=\int_0^{\infty}\left[\varepsilon u\left(q_{\varepsilon}\right)-h_{\varepsilon}\right]dF\left(\varepsilon\right)+x-q_s. \tag{4}$$

where h_ε is hours worked by an $\varepsilon-$buyer in the money market, q_ε is consumption of an $\varepsilon-$buyer in the goods market, x is consumption of a seller in the money market, and q_s is production of a seller in the goods market. The planner maximizes (4) subject to the feasibility constraints

$$\begin{aligned}\int_0^{\infty}q_{\varepsilon}dF\left(\varepsilon\right)-q_s &\leq 0 &(5)\\ x-\int_0^{\infty}h_{\varepsilon}dF\left(\varepsilon\right) &\leq 0. &(6)\end{aligned}$$

The first constraint requires that the quantity of goods produced by sellers in the goods market is at least as large as the quantity of goods consumed by buyers. The second constraint has a similar interpretation for the money market. The first-best allocation satisfies

$$\begin{aligned}q_{\varepsilon}^{*} &= \varepsilon \text{ for all } \varepsilon &(7)\\ q_s^{*} &= \bar{\varepsilon}\equiv\int_0^{\infty}\varepsilon dF\left(\varepsilon\right). &(8)\end{aligned}$$

These are the quantities chosen by a social planner who could dictate production and consumption in the goods market.

3 Household decisions

In this Section, we study the decision problems of buyers and sellers in the money market and the goods market. For this purpose, denote P the price of goods in the money market and let $\phi \equiv 1/P$. Furthermore, let p be the price of goods in the goods market.

3.1 Money market

$V_M(m, b, \ell, d)$ denotes the expected value of entering the money market with m units of money, b bonds, ℓ loans, and d deposits. $V_G(m, b)$ denotes the expected value from entering the goods market with m units of money and b collateral. For notational simplicity, we suppress the dependence of the value function on the time index t.

In the money market, the problem of the representative buyer is:

$$
\begin{aligned}
V_M(m, b, \ell, d) &= \max_{h,m\prime,b\prime} -h + V_G\left(m', b'\right) \\
s.t. \quad \phi m' + \phi\rho b' &= h + \phi m + \phi b + \phi\left(1 + i_d\right) d - \phi\left(1 + i_\ell\right)\ell - \phi\tau M.
\end{aligned}
$$

where h is hours worked in the money market, m' is the amount of money brought into the goods market, and b' is the amount of bonds brought into the goods market. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$
\begin{aligned}
V_G^{m'} &\leq \phi \ (= \text{ if } m' > 0\ ) && (9) \\
V_G^{b'} &\leq \phi\rho \ (= \text{ if } b' > 0\ ) && (10)
\end{aligned}
$$

$V_G^{m'} \equiv \frac{\partial V_G(m',b')}{\partial m'}$ is the marginal value of taking an additional unit of money into the goods market. Since the marginal disutility of working is one, $-\phi$ is the utility cost of acquiring one unit of money in the money market. $V_G^{b'} \equiv \frac{\partial V_G(m',b')}{\partial b'}$ is the marginal value of taking additional bonds into the goods market. Since the marginal disutility of working is 1, $-\phi\rho$ is the utility cost of acquiring one unit of bonds in the money market. The implication of (9) and (10) is that all agents enter the goods market with the same amount of money and the same quantity of bonds (which can be zero).

The envelope conditions are

$$V_M^m = V_M^b = \phi; V_M^d = \phi\left(1+i_d\right); V_M^\ell = -\phi\left(1+i_\ell\right) \tag{11}$$

where V_M^j is the partial derivative of $V_M(m, b, \ell, d)$ with respect to $j = m, b, \ell, d$.

3.2 Goods Market

We first consider the problem solved by sellers and then the one solved by buyers. During the goods market, the central bank operates a borrowing facility and a deposit facility, which allows households to borrow at rate i_ℓ and deposit unspent money at rate i_d.

Decisions by sellers Sellers produce goods in the goods market with linear cost $c\left(q\right) = q$ and consume in the money market, obtaining linear utility $U(x) = x$. It is straightforward to show that sellers are indifferent as to how much they sell in the goods market if

$$p\beta\phi^+\left(1+i_d\right) = 1 \tag{12}$$

where ϕ^+ is the price of money in the next period money market. Since we focus on a symmetric equilibrium, we assume that all sellers produce the same amount. With regard to bond holdings, it is straightforward to show that, in equilibrium, sellers are indifferent to holding any bonds if the Fisher equation holds and will hold no bonds if the yield on the bonds does not compensate them for inflation or time discounting. Thus, for brevity of analysis, we assume sellers carry no bonds across periods.

It is also clear that sellers always deposit their proceeds from sales at the deposit facility, since they can earn the interest rate i_d. Furthermore, they will never bring money into the next period and so for them $m' = 0$.

Decisions by buyer The indirect utility function of an $\varepsilon-$buyer in the goods market is

$$V_G(m, b|\,\varepsilon) = \max_{q_\varepsilon, d_\varepsilon, \ell_\varepsilon} \varepsilon u\left(q_\varepsilon\right) + \beta V_M\left(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon\right)$$

$$\text{s.t.} \quad m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon \geq 0, \text{ and } \frac{b}{1+i_\ell} - \ell_\varepsilon \geq 0$$

where d_ε is the amount of money an ε-buyer deposits at the central bank, and ℓ_ε is the loan received from the central bank. The first inequality is the buyer's budget constraint. The second inequality is the collateral constraint. Let $\beta\phi^+\lambda_\varepsilon$ denote the Lagrange multiplier for the first inequality, and $\beta\phi^+\lambda_\ell$ denote the Lagrange multiplier of the second inequality. Then, using (11) to replace V_M^m, V_M^ℓ and V_M^d, the first-order conditions for q_ε, d_ε, and ℓ_ε can be written as follows:

$$\begin{aligned} \varepsilon u'(q_\varepsilon) - \beta p\phi^+(1+\lambda_\varepsilon) &= 0 \\ i_d - \lambda_\varepsilon &\leq 0 \qquad (= 0 \text{ if } d_\varepsilon > 0) \\ -i_\ell + \lambda_\varepsilon - \lambda_\ell &\leq 0 \qquad (= 0 \text{ if } \ell_\varepsilon > 0) \end{aligned} \tag{13}$$

Lemma 1 below characterizes the optimal borrowing and lending decisions by an $\varepsilon-$buyer and the quantity of goods obtained by the $\varepsilon-$buyer:

Lemma 1 *There exist critical values ε_d, ε_ℓ, $\varepsilon_{\bar{\ell}}$, with $0 \leq \varepsilon_d \leq \varepsilon_\ell \leq \varepsilon_{\bar{\ell}}$, such that the following is true: if $0 \leq \varepsilon < \varepsilon_d$, a buyer deposits money at the central bank; if $\varepsilon_\ell < \varepsilon \leq \varepsilon_{\bar{\ell}}$, he borrows money and the collateral constraint is nonbinding; if $\varepsilon_{\bar{\ell}} \leq \varepsilon$, he borrows money and the collateral constraint is binding; and if $\varepsilon_d \leq \varepsilon \leq \varepsilon_\ell$, he neither borrows nor deposits money. The critical values solve:*

$$\varepsilon_d = (1+i_d)\beta\phi^+ m,\ \varepsilon_\ell = (1+i_\ell)\beta\phi^+ m, \text{ and } \varepsilon_{\bar{\ell}} = (1+i_\ell)\beta\phi^+ m + \beta\phi^+ b. \tag{14}$$

In any equilibrium, the amount of borrowing and depositing by a buyer with a taste shock ε and the amount of goods purchased by the buyer satisfy:

$$\begin{array}{llll} q_\varepsilon = \varepsilon, & d_\varepsilon = p(\varepsilon_d - \varepsilon), & \ell_\varepsilon = 0, & \text{if } 0 \leq \varepsilon \leq \varepsilon_d \\ q_\varepsilon = \varepsilon_d, & d_\varepsilon = 0, & \ell_\varepsilon = 0, & \text{if } \varepsilon_d \leq \varepsilon \leq \varepsilon_\ell \\ q_\varepsilon = \varepsilon\rho_\ell/\rho_d, & d_\varepsilon = 0, & \ell_\varepsilon = p(\varepsilon\rho_\ell/\rho_d - \varepsilon_d), & \text{if } \varepsilon_\ell \leq \varepsilon \leq \varepsilon_{\bar{\ell}}, \\ q_\varepsilon = \varepsilon_{\bar{\ell}}\rho_\ell/\rho_d, & d_\varepsilon = 0, & \ell_\varepsilon = \rho_\ell b, & \text{if } \varepsilon_{\bar{\ell}} \leq \varepsilon. \end{array} \tag{15}$$

The optimal borrowing and lending decisions follow the cut-off rules according to the realization of the taste shock. The cut-off levels, ε_d, ε_ℓ, and $\varepsilon_{\bar{\ell}}$ partition the set of taste shocks into four regions. For shocks lower than ε_d, a buyer deposits money at the standing facility; for shocks higher than ε_ℓ, the buyer borrows at the standing facility. For values between ε_d and ε_ℓ, the buyer does not use the central bank's standing facility. Finally, the cut-off value $\varepsilon_{\bar{\ell}}$ determines whether a buyer's collateral constraint is binding or not.

4 Equilibrium

We focus on symmetric stationary equilibria with strictly positive demands for nominal government bonds and money. Such equilibria meet the following requirements: (i) Households' decisions are optimal, given prices; (ii) The decisions are symmetric across all sellers and symmetric across all buyers with the same preference shock; (iii) The goods and bond markets clear; (iv) All real quantities are constant across time; (v) The consolidated government budget constraint (3) holds in each period.

Market clearing in the goods market requires

$$q_s - \int_0^\infty q_\varepsilon dF(\varepsilon) = 0, \tag{16}$$

where q_s is aggregate production by sellers in the goods market.

Let $\gamma \equiv M^+/M$ denote the constant gross money growth rate, $\eta \equiv B^+/B$ denote the constant gross bond growth rate, and let $\mathcal{B} \equiv B/M$ denote the bonds-to-money ratio. We assume there are positive initial stocks of money M_0 and government bonds B_0.[20]

Lemma 2 *In any stationary equilibrium, the bond-to-money ratio $\mathcal{B}$ has to be constant, and this can be only achieved when the growth rates of money and bonds are equal.*

According to Lemma 2, in any stationary equilibrium the stock of money and the stock of bonds must grow at the same rate. This result follows from the budget constraints of the buyers. By definition, in a stationary equilibrium, all real quantities are constant. Consider a buyer who, according to Lemma 1, does not use the central bank's standing facilities. His budget constraint satisfies $q_\varepsilon = (1+i_d)\beta\phi^+ m$. Symmetry requires that $m = M^+$, which implies that the real stock of money $\phi^+ M^+$ must be constant. Consider, next, a buyer who, according to Lemma 1, is constrained by his bond holdings. His budget constraint satisfies $q_\varepsilon = \left(\beta\phi^+ m + \beta\rho_\ell\phi^+ b\right)/\rho_d$. Then, since $\phi^+ m$ is constant, the real quantity of bonds $\phi^+ b$ must be constant, since in

[20] Since the assets are nominal objects, the government and the central bank can start the economy off by one-time injections of cash M_0 and bonds B_0.

a symmetric equilibrium $b = B^{+}$. The result that $\phi^{+}M^{+}$ and $\phi^{+}B^{+}$ are constant implies $\gamma = \eta$. Finally, note that the gross inflation rate p^{+}/p in the goods market is equal to γ. This follows from the seller's first-order condition (12).

The result of Lemma 2 raises an interesting question. Since the growth rate of bonds η is chosen by the government and the growth rate of money γ by the central bank, the question is: Who is in charge? A related issue is discussed in Sargent and Wallace (1981). They show that if fiscal policy is dominant (chosen first), then the central bank may lose control over the inflation rate. In our context, if the government chooses η, then the central bank must follow by setting $\gamma = \eta$. Conversely, if the central bank chooses γ, then government must choose $\eta = \gamma$. Even though these considerations are interesting, for the optimal policy that we will present below it does not matter which agency is dominant. We, therefore, assume that the government chooses η, which forces the central bank to choose $\gamma = \eta$. It then follows that the remaining policy variables of the central bank are the interest rates i_d and i_ℓ (or equivalently ρ_d and ρ_ℓ).

Proposition 3 *A symmetric stationary equilibrium with a positive demand for money and bonds is a policy (i_d, i_ℓ) and endogenous variables $(\rho, \varepsilon_d, \varepsilon_\ell, \varepsilon_{\bar{\ell}})$ satisfying*

$$\frac{\rho_d \eta}{\beta} = \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) \tag{17}$$

$$\frac{\rho \eta}{\beta} = \int_0^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} dF(\varepsilon) \tag{18}$$

$$\varepsilon_\ell = \varepsilon_d \frac{\rho_d}{\rho_\ell} \text{ and } \varepsilon_{\bar{\ell}} = \varepsilon_d \frac{\rho_d}{\rho_\ell} (1 + \rho_\ell \mathcal{B}) \tag{19}$$

In the proof of Proposition 3, we also show that for a given policy (i_d, i_ℓ), a stationary equilibrium exists and is unique if $\rho_d \eta / \beta > 1$.

Equation (17) is obtained from the choice of money holdings (9). Equation (18) is obtained from (9) and (10); in any equilibrium with a strictly positive demand for money and bonds, $\rho V_G^m(m, b) = V_G^b(m, b)$. We then use this arbitrage equation to derive (18). Finally, equations (19) are derived from the budget constraints of the buyers.

The system of equations (17)-(19) is block recursive. To see this, note that the critical values ε_ℓ and $\varepsilon_{\bar{\ell}}$ are functions of ε_d only. If we use (19) to replace them in (17), equation (17) becomes a function of ε_d only. Once one has solved (17) for ε_d, one can derive ρ from equation (18). The critical values are then obtained from (19). All remaining endogenous variables can then be calculated as follows: The amount of borrowing and depositing by a buyer and the amount of goods purchased by the buyer are obtained from (15); from (14), the real stock of money is $\phi M = \rho_d \varepsilon_d / \beta$, and the real stock of bonds is $\phi B = (\varepsilon_{\bar{\ell}} - \varepsilon_\ell)/\beta$; aggregate production in the goods market is obtained from (16); from the consolidated government budget constraint (3) one obtains the value of τ that is consistent with the policy choice (i_d, i_ℓ) and the initial stocks of money and bonds.

5 Optimal policy

The central bank chooses (i_d, i_ℓ) to maximize (4) subject to (17) - (19).

Proposition 4 *The optimal policy is to set $\rho_d = \beta/\eta$. This policy implements the first-best allocation. Under the optimal policy, the money market rate satisfies $\rho = \rho_d$.*

As in Cúrdia and Woodford (2011), it is optimal to set the deposit rate equal to the operating target rate for the policy rate (the money market rate i in our model) in each period. Note that the optimal policy (i_d, i_ℓ) is not unique, since under the optimal policy the lending rate is irrelevant. The reason is that under the optimal policy the buyers never borrow. Accordingly, any value of $i_\ell \geq i_d$ is consistent with the optimal policy. Therefore, under the optimal policy, the central bank operates a floor system instead of a channel system.

The optimal policy makes holding money costless and therefore satiates money demand as described by the Friedman rule. In Cúrdia and Woodford (2011) the same policy satiates the demand for central bank reserves. Note that such a policy means that the money market rate and the central bank's deposit rate exactly compensate market participants for their impatience and for inflation. To see this, define $r \equiv (1-\beta)/\beta$ and $\pi \equiv \eta - 1$ and rewrite $\rho_d = \beta/\eta$ to get $1 + i_d = (1+\pi)(1+r)$. Under this policy, the rate of return on money is the same as the rate of return on government bonds. Hence, they have the same marginal liquidity value, which is zero.[21]

[21] Since the first-best quantities are $q_\varepsilon = \varepsilon$ with the support of ε being unbounded, the real value of money approaches infinity; i.e., the price level approaches zero. Any finite upper bound would

In summary, the optimal monetary policy satisfies the Friedman rule, and takes the form of the central bank paying interest on deposits of central bank money. Intuitively, when the central bank pays interest on central bank money (to offset the effects of time preference and inflation), people will hold enough money to meet any transaction needs, and their money balance will never be binding, and so any inefficiency associated with a binding money balance will be eliminated. Under this policy, the interest rate on government bonds will be the same as the deposit rate. To see this, note that no borrowing will ever happen, so bonds will never be used as collateral to borrow money from the central bank, the only return from holding bonds is the yield on bonds. As a result, the price (equivalently, the interest rate) of the government bond that clears the market is the one that gives the same interest rate as the deposit rate on money.

We are clearly not the first to point out that the Friedman rule can take the form of paying interest on deposits. For example, in Section 2.4.1, the textbook of Walsh (2010) states that the Friedman rule can be achieved by paying interest on money. Another example for this result can be found in chapter 6 of the book by Nosal and Rocheteau (2011). Finally, Andolfatto (2010), Lagos (2010a), and Williamson (2012) derive results on the optimality and implementation of the Friedman rule in search theoretical models of money. The novel results of our paper are now presented below.

For what follows, let $i_d^* = \eta/\beta - 1$ be the optimal deposit rate. Under the optimal policy, no one borrows from the standard facility, but there are deposits. The question is how are these interest payments on deposits financed?

Proposition 5 *The optimal policy requires that $\mathcal{D} < 0$.*

The optimal policy requires that the government runs a primary surplus ($\mathcal{D} < 0$). Therefore, it must collect taxes and hand them over to the central bank to finance the interest payments on deposits. To see this, from (1) the central bank's surplus is

$$\mathcal{S} = M^+ - M + i_\ell L - i_d D.$$

Under the optimal policy (see the proof of Proposition 4), $L = 0$ and $D = 0$. Accordingly, the central bank's surplus is

$$\mathcal{S} = M^+ - M - i_d M.$$

yield a finite strictly positive price level.

Thus, ignoring money creation, the central bank incurs a loss, $-i_d M$, from operating the standing facility. The remaining question is whether the central bank can finance this loss by printing money. Since in steady state $M^+/M = \eta$ and under the optimal policy $i_d = \eta/\beta - 1$, the central bank's surplus is

$$\mathcal{S} = (\eta/\beta)(\beta - 1) M < 0. \tag{20}$$

Under the optimal policy, the central bank makes a deficit which requires a transfer of funds equal to $(\eta/\beta)(\beta - 1) M$ from the government to the central bank in each period. From the consolidated budget constraint, it then requires that the government has a primary surplus. One can think of two ways out of this problem. First, the central bank is endowed with a stock of real assets that provide sufficient revenue in each period to cover the losses that occur under the optimal policy. Second, the central bank can be endowed with a stock of government bonds that pay sufficient interest to cover the losses described above. We discuss these possibilities in Section 7.

Cúrdia and Woodford (2011) never discuss how the central bank finances interest on reserves. Implicitly, they must assume that the central bank can directly tax households or receives tax revenue from the treasury. In practice, central banks have no fiscal power to levy taxes and therefore would rely on the treasury to provide the funds necessary to run the optimal policy. In the following section, we assume that the central bank does not receive sufficient funds from the treasury to implement the optimal policy and derive the constrained optimal policy for this case.

So far, we have assumed that the government chooses the growth rate of bonds η. The central bank then is forced to choose the growth rate of the money supply such that $\gamma = \eta$. Since the first-best allocation can be attained for any η, this assumption does not matter for the optimal policy. Suppose, however, that the central bank is the dominant player and dislikes receiving funds from the treasury. In this case, setting $\gamma = \beta$ (which forces the government to choose $\eta = \gamma$) minimizes the transfers received from the government, but $\mathcal{S}$ remains negative.[22]

Finally, there could be an equilibrium, where agents do not bring money into the goods market, but only bonds. They then use bonds to borrow money at the lending facility and then use the money to purchase goods. Sellers accept money in

[22] Note that no equilibrium exists for $\gamma < \beta$.

the goods market, because they want to purchase general goods in the money market and buyers demand money in the money market to reimburse their loans. The amount of money deposited at the deposit facility by sellers is exactly the amount of cash that is borrowed at the central bank by buyers. So, when the lending rate is set equal to the deposit rate, the revenue of the central bank equals its expenditures. In the Appendix, we show that such an equilibrium cannot exist.

6 Constrained-optimal policy

In the previous section, we have shown that under the optimal policy the central bank makes a deficit; i.e., $\mathcal{S}^o \equiv \frac{\eta}{\beta}(\beta - 1) M < 0$. This requires a transfer of funds from the government to the central bank in each period. In this section, we assume that the central bank does not receive enough funds to run the optimal policy. Receiving less funds implies that $\mathcal{S} > \mathcal{S}^o$.

Proposition 6 characterizes the optimal policy under the constrained $\mathcal{S} > \mathcal{S}^o$.[23]

Proposition 6 *If $\mathcal{S} > \mathcal{S}^o$, the constrained-optimal policy is to choose a strictly positive interest rate spread. Furthermore, under the constrained-optimal policy the money market rate satisfies $i > i_d$.*

According to Proposition 6, the constrained optimal policy deviates from the optimal policy along two dimensions. First, it is optimal to choose a strictly positive interest-rate spread. Second, the constrained-optimal policy is to set the deposit rate strictly below the money market rate. Consequently, when the central bank receives insufficient transfers from the government, it chooses to operate a channel system ($i_\ell > i > i_d$) rather than a floor system ($i = i_d$).

In the proof of Proposition 6, we also show that the constrained optimal policy requires that $\rho_d \eta / \beta > 1$. This inequality immediately implies that the deposit rate under the constrained-optimal policy is strictly smaller than the one under the optimal policy, i_d^*. Moreover, we also show that increasing i_d is strictly welfare improving. The reason is that paying interest on "idle" money holdings improves economic efficiency (see e.g., Berentsen et al. (2007)). Thus, fiscal considerations are the reason why the central bank chooses $i_d < i_d^*$, since without sufficient funds it is not able to set $i_d = i_d^*$.

[23] Note that this constraint includes the case where the central bank makes a surplus ($\mathcal{S} > 0$) as it is the case for most central banks.

Note that the condition $\mathcal{S} > \mathcal{S}^o$ does not mean that the central bank is unable to run a floor system. It can always choose $i_\ell = i = i_d < i_d^*$. Rather, it means that it is not optimal to do so. Intuitively, suppose the central bank does not receive enough income to pay the optimal interest rate on deposits; i.e., $i_d < i_d^*$. In this case, buyers will not hold enough money to meet transaction needs in all cases. And when a buyer has a high shock, he will need to borrow from the central bank. In this case, to implement the floor system, the central bank must set the borrowing rate equal to the deposit rate, so that by arbitrage, the equilibrium yield rate on government bonds will equal the deposit rate (and also the borrowing rate). Proposition 6 states that it is not optimal to make the borrowing rate equal the deposit rate in this case. Instead, it is better to set the borrowing rate strictly higher than the deposit rate. The reason is that, a slightly higher borrowing rate will reduce the consumption of buyers whose borrowing constraint is not binding, but it will also increase the consumption of buyers whose borrowing constraint is binding (see Figure 1), because the expected higher borrowing cost will cause buyers to carry more money into the goods market, so they will have more money to buy goods. Marginally, the second effect is higher than the first effect.

This argument is shown more formally below. In the proof of Proposition 6, we show that if $\mathcal{S} > \mathcal{S}^o$ and if the central bank sets $i_d = i_\ell$, then increasing the loan rate marginally is strictly welfare improving. This result is driven by the reallocation of consumption that occurs from increasing the loan rate above the deposit rate as depicted in Figure 1.

– Insert Figure 1 here –

Figure 1 graphically illustrates why a strictly positive interest rate spread is welfare improving. The black dotted linear curve (the 45–degree line) plots the first-best consumption quantities. The red curve (labelled *zero band*) plots the consumption quantities when $i_\ell = i = i_d$. Up to some critical value for ε, $\tilde{\varepsilon}$, the buyer receives the first-best consumption quantities after which the collateral constraint is binding, as indicated by the consumption quantities that are independent of ε. The blue curve (labelled *positive band*) plots the quantities for a strictly positive spread; i.e., $i_\ell > i > i_d$. Up to the critical value ε_d, the buyer consumes the first-best quantity; i.e., $q_\varepsilon = \varepsilon$. For $\varepsilon \in [0, \varepsilon_d]$ he deposits any excess money at the deposit facility. For

$\varepsilon \in [\varepsilon_d, \varepsilon_\ell]$ he neither deposits nor borrows money. He simply spends all the money brought into the period and consumes $q_\varepsilon = \varepsilon_d$. For $\varepsilon \in [\varepsilon_\ell, \varepsilon_{\bar{\ell}}]$ the buyer borrows, but his collateral constraint is non-binding. Finally, for $\varepsilon > \varepsilon_{\bar{\ell}}$ the collateral constraint is binding.

As indicated by Figure 1, the welfare gain from increasing the borrowing rate i_ℓ marginally rises, because it lowers the consumption of medium ε-buyers and increases the consumption of high-ε buyers. The first effect lowers welfare, while the second increases welfare. In the proof of Proposition 6, we show that, starting from $i_d = i_\ell$, the net gain is always positive.

The mechanism works as follows. By marginally increasing i_ℓ, the central bank makes it relatively more costly to turn bonds into money and hence consumption. This affects the portfolio choice of agents in the money market. The demand for money and hence its value increases. For those who are not borrowing-constrained, i.e., for buyers with $\varepsilon \in [\varepsilon_d, \varepsilon_{\bar{\ell}}]$, the higher marginal borrowing cost lowers their consumption at the margin. However, starting from $i_d = i_\ell$, this welfare loss is of second order. For those who are borrowing-constrained, i.e., for buyers with $\varepsilon > \varepsilon_{\bar{\ell}}$, the marginal higher borrowing cost has no effect on their consumption, yet their higher real balances allow them to consume more. Again, starting from $i_d = i_\ell$, this welfare gain is of first order.

We have shown that if $\mathcal{S} > \mathcal{S}^o$, increasing the spread by increasing the loan rate is always welfare improving starting from $i_d = i_\ell$. Alternatively, one could consider lowering i_d starting from $i_d = i_\ell$. However, lowering i_d lowers the demand for money and hence its value, which reduces consumption for all constrained buyers and does not increase the consumption of unconstrained buyers. This is clearly welfare reducing (see the proof at the end of the Appendix).

7 Discussion

One can think of two ways out of this problem. First, the central bank is endowed with a stock of real assets that provide sufficient revenue in each period to cover the losses that occur under the optimal policy. In practice, many central banks have such capital and it is often argued that the benefit of being well capitalized is that it helps preserve the independence of a central bank. Second, the central bank is endowed with a stock of government bonds that pay sufficient interest to cover the losses described above.

In what follows we amend the central bank's balance sheet with capital and bonds.

Bonds Let us assume the central bank holds government bonds. Let B_C be the stock of government bonds held by the central bank, and let B be the stock of bonds held by private agents. Then, the total stock of bonds in circulation is $B_G = B_C + B$. The bond-augmented central bank's surplus is therefore

$$\mathcal{S} = M^{+} - M + i_{\ell}L - i_d D - \rho B_C^{+} + B_C.$$

Substituting $\mathcal{S}$ into (2) yields

$$\mathcal{D} = M^{+} - M + i_{\ell}L - i_d D + \rho B^{+} - B$$

which is identical to the consolidated government budget constraint (3). Consequently, the result in Proposition 3 is not affected. What matters for the consolidated budget constraint is the total stock of bonds in circulation and not the stock of bonds in the hands of the central bank.

Endowing the central bank with government bonds is simply a way to hide transfer payments from the government to the central bank. In this case, the government has to levy taxes to finance interest payments on the government bonds which it then hands over to the central bank. The central bank, then uses these funds to pay interest on reserves. In practice, this means it pay considerable sums to private sector banks which can be politically problematic as argued in the introduction.

The literature on paying interest on reserves is largely not concerned with this point. For example, Goodfriend (2002, p.5) argues "Suppose a central bank such as the Fed confines its asset purchases mainly to Treasury securities. In that case, interest on the increase in reserves will be self-financing if there is a positive spread between longer term Treasury securities and the rate of interest on reserves. Reserve balances at the central bank paying market interest are like one-day Treasury securities. Hence, interest rate spreads between longer term Treasury securities and overnight deposits at the central bank should exhibit term premia ordinarily reflected in the Treasury yield curve. Therefore, a central bank such as the Fed should be able to self-finance interest on the enlarged demand for reserves in the new regime. In fact, the net interest spread earned on new assets acquired in the interest-an-reserves regime would raise additional revenue for the central bank."

Goodfriend's (2002) argument is a technical one. It states that a central bank can always run the Friedman rule if it is endowed with a sufficiently large stock of bonds or capital. The argument that we make in this paper is not whether it is technically feasible, but rather that it is not sustainable politically, since at the end of the day the private sector has to be taxed to finance the interest on reserves, and these reserves are mainly held by a few large banks, some of them being foreign banks.

Capital Let K be the nominal stock of capital in the central bank's balance sheet and r the rate of return of capital. Then, the capital-augmented central bank's surplus is

$$\mathcal{S} = M^{+} - M + i_{\ell}L - i_{d}D + rK. \tag{21}$$

Using the last equation, the consolidated government budget constraint (2), is given by

$$\mathcal{D} = M^{+} - M + i_{\ell}L - i_{d}D + \rho B^{+} - B + rK.$$

Let $\mathcal{K} \equiv K/M$ denote the capital-to-money ratio and $\bar{\mathcal{K}} \equiv (\eta/\beta + \mathcal{B})(1-\beta)/r > 0$.

Proposition 7 *The optimal policy generates no losses for the central bank if $\mathcal{K} \geq \bar{\mathcal{K}}$.*

According to Proposition 7, the central bank can operate the optimal floor system if it has a sufficiently large capital stock. The income generated by the capital stock is then used to finance the interest payments on reserves. Note that this condition is more likely to be violated if: 1) r is too low, 2) inflation, η, is too high or 3) the ratio of bonds to money, $\mathcal{B}$, is too large. Thus, having a significant holding of real assets may still not be sufficient to avoid having to receive fiscal transfers under the optimal floor system.[24]

8 Conclusions

Many central banks implement monetary policy via a channel system or a floor system. In this paper, we constructed a general equilibrium model and studied the

[24]Having assets with insufficiently low interest rates is exactly the situation Costa Rica has faced for the past 20 years; its interest-earning assets do not generate enough income to pay the interest on its liabilities. Consequently, the central bank must get annual transfers of revenue from the Treasury.

properties of these systems. The following results emerged from our analysis. First, the optimal framework is a floor system *if and only if* the target rate satisfies the Friedman rule. Second, implementing the optimal policy is costly for the central bank. It requires that the central bank either has sufficient capital income or receives transfers from the fiscal authority. Either way, the fiscal authority has fewer resources for financing its spending priorities. This is the unpleasant fiscal arithmetic of a floor system. Third, if the central bank has insufficient capital income or receives insufficient transfers from the fiscal authority to implement the optimal floor system, a channel system is the constrained-optimal policy.

We have analyzed how the design of the optimal system for implementing monetary policy depends on the central bank's available funds. To make our case, we have assumed that the government has access to non-distortionary taxation. However, if the government only has access to distortionary taxation, then a floor system may not be optimal since the benefits of a floor system may be outweighed by the costs of raising tax revenue via distortionary methods. In general this is a quantitative issue which is left for future research.

Appendix

Proof of Lemma 1. We first derive the cut-off values ε_d and ε_ℓ. For this proof, to the notation of the consumption level of a buyer, we add a subscript d if the buyer deposits money at the central bank, a subscript ℓ if the buyer takes out a loan and the collateral constraint is nonbinding, a subscript $\bar{\ell}$ if the buyer takes out a loan and the collateral constraint is binding, and a subscript 0 if the buyer does neither.

From (13), the consumption level of a buyer who enters the goods market satisfies:

$$q_d(\varepsilon) = \frac{\varepsilon}{p\beta\phi^+(1+i_d)}, \quad q_\ell(\varepsilon) = \frac{\varepsilon}{p\beta\phi^+(1+i_\ell)}. \tag{22}$$

A buyer who does not use the deposit facilities will spend all his money on goods, since, if he anticipated that he would have idle cash after the goods trade, it would be optimal to deposit the idle cash in the intermediary, provided $i_d > 0$. Thus, consumption of such a buyer is:

$$q_0(\varepsilon) = \frac{m}{p}. \tag{23}$$

At $\varepsilon = \varepsilon_d$, the household is indifferent between depositing and not depositing. We can write this indifference condition as:

$$\varepsilon_d u(q_d) - \beta\phi^+(pq_d - i_d d) = \varepsilon_d u(q_0) - \beta\phi^+ pq_0.$$

By using (22), (23), and $d = m - pq_d$, we can write the equation further as

$$\varepsilon_d u\left[\frac{\varepsilon_d}{\beta\phi^+(1+i_d)m}\right] = \varepsilon_d - (1+i_d)\beta\phi^+ m.$$

The unique solution to this equation is $\varepsilon_d = (1+i_d)\beta\phi^+ m$, which implies that $\beta\phi^+ m < \varepsilon_d$.

At $\varepsilon = \varepsilon_\ell$, the household is indifferent between borrowing and not borrowing. We can write this indifference condition as

$$\varepsilon_\ell u(q_\ell) - \beta\phi^+(pq_\ell + i_\ell \ell) = \varepsilon_\ell u(q_0) - \beta\phi^+ pq_0.$$

Using (22), (23) and $\ell = pq_\ell - m$, we can write this equation further as

$$\varepsilon_\ell u\left[\frac{\varepsilon_\ell}{(1+i_\ell)\,\beta\phi^+ m}\right] = \varepsilon_\ell - (1+i_\ell)\,\beta\phi^+ m.$$

The unique solution to this equation is $\varepsilon_\ell = (1+i_\ell)\,\beta\phi^+ m$. Using the expression for ε_d we get

$$\varepsilon_\ell = \varepsilon_d\left(\rho_d/\rho_\ell\right). \tag{24}$$

We now calculate $\varepsilon_{\bar{\ell}}$. There is a critical buyer who enters the goods market and wants to take out a loan, whose collateral constraint is just binding. From (13), for this buyer we have the following equilibrium conditions: $q_{\bar{\ell}} = \frac{\rho_\ell \varepsilon_{\bar{\ell}}}{\beta\phi^+ p}$ and $pq_{\bar{\ell}} = m + \rho_\ell b$. Eliminating $q_{\bar{\ell}}$ we get

$$\varepsilon_{\bar{\ell}} = (1+i_\ell)\,\beta\phi^+ m + \beta\phi^+ b.$$

Using (24) we get

$$\varepsilon_{\bar{\ell}} = \varepsilon_d \frac{\rho_d}{\rho_\ell}\left(1 + \rho_\ell \frac{b}{m}\right).$$

It is then evident that

$$0 \leq \varepsilon_d \leq \varepsilon_\ell \leq \varepsilon_{\bar{\ell}}.$$

■

Proof of Lemma 2. A stationary equilibrium requires that all real quantities are constant and symmetry requires that $m = M^+$ and $b = B^+$. From Lemma 1, there are two critical consumption quantities in our model:

$$\begin{array}{ll} q_\varepsilon = \varepsilon_d = (1+i_d)\,\beta\phi^+ M^+ & \textit{if } \varepsilon_d \leq \varepsilon \leq \varepsilon_\ell \\ q_\varepsilon = \varepsilon_{\bar{\ell}}\rho_\ell/\rho_d = (1+i_d)\left(\beta\phi^+ M^+ + \beta\rho_\ell\phi^+ B^+\right) & \textit{if } \varepsilon_{\bar{\ell}} \leq \varepsilon. \end{array}$$

The first quantity requires that the real stock of money is constant; i.e., $\phi M = \phi^+ M^+$, implying that $\phi/\phi^+ = \gamma$.

Since $\phi^+ M^+$ is constant, the second quantity requires that $\phi^+ B^+$ is constant too; i.e., $\phi B = \phi^+ B^+$. This implies that the stock of bonds has to grow at the same rate as the stock of money. ■

Proof of Proposition 3. The proof contains two steps. First, we derive equations (17), (18) and (19). Second, we show existence and uniqueness.

First step. Equations (19) are derived in the proof of Lemma 1. To derive equation (17), differentiate $V_G(m,b) = \int_0^\infty V_G(m,b|\,\varepsilon)\,dF(\varepsilon)$ with respect to m to get

$$V_G^m(m,b) = \int_0^\infty \left[\beta V_M^m(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon|\,\varepsilon) + \beta\phi^+\lambda_\varepsilon\right] dF(\varepsilon).$$

Then, use (11) to replace V_M^m and (13) to replace $\beta\phi^+\lambda_\varepsilon$ to obtain

$$V_G^m(m,b) = \int_0^\infty \frac{\varepsilon u'(q_\varepsilon)}{p} dF(\varepsilon). \tag{25}$$

Use the first-order condition (12) to replace p to get

$$V_G^m(m,b) = \beta\phi^+(1+i_d)\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon).$$

Use (9) to replace $V_G^m(m,b)$ and replace ϕ^+/ϕ by η to get

$$\frac{\eta}{\beta} = (1+i_d)\int_0^\infty \varepsilon u'(q_\varepsilon)\,dF(\varepsilon).$$

Finally, note that $u'(q) = 1/q$ and replace q_ε using Lemma 1 to get (17) which we replicate here:

$$\frac{\rho_d\eta}{\beta} = \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}}\frac{\rho_d}{\rho_\ell} dF(\varepsilon) \tag{26}$$

where $\varepsilon_\ell = \varepsilon_d\frac{\rho_d}{\rho_\ell}$ and $\varepsilon_{\bar{\ell}} = \left(\varepsilon_d\frac{\rho_d}{\rho_\ell}\right)(1+\rho_\ell\mathcal{B})$. Note that if we replace ε_ℓ and $\varepsilon_{\bar{\ell}}$ by $\varepsilon_d\frac{\rho_d}{\rho_\ell}$ and $\left(\varepsilon_d\frac{\rho_d}{\rho_\ell}\right)(1+\rho_\ell\mathcal{B})$, respectively, then (26) yields ε_d, since no other endogenous variables are contained in (26).

To derive (18), note that in any equilibrium with a strictly positive demand for money and bonds, we must have $\rho V_G^m(m,b) = V_G^b(m,b)$. We now use this arbitrage equation to derive (18). We have already derived $V_G^m(m,b)$ above. To get $V_G^b(m,b)$

differentiate $V_G(m,b)$ by b to get

$$V_G^b(m,b) = \int_0^\infty \left[\beta V_M^b(m + \ell_\varepsilon - pq_\varepsilon - d_\varepsilon, b, \ell_\varepsilon, d_\varepsilon | \varepsilon) + \rho_\ell \beta \phi^+ \lambda_\ell\right] dF(\varepsilon).$$

Use (11) to replace V_M^b to get

$$V_G^b(m,b) = \beta\phi^+ \int_0^\infty (1 + \rho_\ell \lambda_\ell)\, dF(\varepsilon).$$

Use (13) to replace λ_ℓ and rearrange to get

$$V_G^b(m,b) = \int_0^{\varepsilon_{\overline{\ell}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\overline{\ell}}}^\infty \rho_\ell \frac{\varepsilon u'(q_\varepsilon)}{p} dF(\varepsilon).$$

Use the first-order condition for q_s in (12) to get

$$V_G^b(m,b) = \int_0^{\varepsilon_{\overline{\ell}}} \beta\phi^+ dF(\varepsilon) + \int_{\varepsilon_{\overline{\ell}}}^\infty \beta\phi^+ (\rho_\ell/\rho_d)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon).$$

Equate $\rho V_G^m(m,b) = V_G^b(m,b)$ and simplify to get

$$\int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) = \int_0^{\varepsilon_{\overline{\ell}}} (\rho_d/\rho)\, dF(\varepsilon) + \int_{\varepsilon_{\overline{\ell}}}^\infty (\rho_\ell/\rho)\, \varepsilon u'(q_\varepsilon)\, dF(\varepsilon).$$

Note that $\int_0^\infty \varepsilon u'(q_\varepsilon)\, dF(\varepsilon) = \rho_d \eta/\beta$ and use Lemma 1 to get (18) which we replicate here:

$$\frac{\rho\gamma}{\beta} = \int_0^{\varepsilon_{\overline{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\overline{\ell}}}^\infty \frac{\varepsilon}{\varepsilon_{\overline{\ell}}} dF(\varepsilon).$$

Note that since $\varepsilon_\ell = \varepsilon_d \frac{\rho_d}{\rho_\ell}$ and $\varepsilon_{\overline{\ell}} = \left(\varepsilon_d \frac{\rho_d}{\rho_\ell}\right)(1 + \rho_\ell \mathcal{B})$, ρ depends on ε_d only.

Second step. Equations (17) and (18) are block recursive. We can first solve (17) for ε_d. Such a value exists and is unique, since the right-hand side of (17) is decreasing in ε_d. Furthermore, the right-hand side is approaching infinity for $\varepsilon_d \to 0$

and is approaching 1 for $\varepsilon_d \to \infty$. Accordingly, there exists a unique value $\varepsilon_d \in (0, \infty)$ that solves (17) if $\eta/\beta > 1/\rho_d$. Once we know ε_d, we get ρ from (18). Since the value of ε_d is unique, ρ is unique. Hence, for a given policy (i_d, i_ℓ), an equilibrium exist and it is unique. ■

Proof of Proposition 4. Setting $\rho_d = \beta/\eta$ reduces (17) and (18) as follows

$$1 = \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho}{\rho_\ell} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} \frac{\rho}{\rho_\ell} dF(\varepsilon)$$

This equation holds if and only if $\varepsilon_d \to \infty$. Then, from (19), $\varepsilon_\ell, \varepsilon_{\bar{\ell}} \to \infty$. Thus, from Lemma 1, the first-best allocation $q_\varepsilon = \varepsilon$ for all ε is attained. Moreover, from (18), it is clear that the money market rate must satisfy $\rho = \beta/\eta$. ■

Proof of Proposition 5. We now show that the optimal policy requires that the government has a primary surplus $\mathcal{D} > 0$. In any equilibrium, the sellers' money holdings satisfy

$$pq_s = M + L - \int_0^{\varepsilon_d} (M - pq_\varepsilon)\, dF(\varepsilon).$$

The left-hand side is the aggregate money receipts of sellers. The right-hand side is the beginning of period quantity of money, M; plus aggregate lending of money by the central bank, L; minus deposits by late-buyers at the central bank. These buyers simply deposit any "idle" money to receive interest on it. Furthermore, in any equilibrium aggregate deposits satisfy

$$D = pq_s + \int_0^{\varepsilon_d} (M - pq_\varepsilon)\, dF(\varepsilon),$$

where pq_s is deposits by sellers. These two equations imply that in any equilibrium, total deposits satisfy

$$D = M + L. \tag{27}$$

From Lemma 1, we know that only buyers with a shock $\varepsilon \geq \varepsilon_\ell$ borrow. Thus, aggregate lending is $L = \int_{\varepsilon_\ell}^{\infty} \ell_\varepsilon dF(\varepsilon)$. From Lemma 1, we also know that $\ell_\varepsilon = p[(\rho_\ell/\rho_d)\varepsilon - \varepsilon_d]$ if $\varepsilon_\ell \leq \varepsilon \leq \varepsilon_{\bar{\ell}}$, and $\ell_\varepsilon = b/(1+i_\ell) = p[(\rho_\ell/\rho_d)\varepsilon_{\bar{\ell}} - \varepsilon_d]$ if $\varepsilon \geq \varepsilon_{\bar{\ell}}$. Thus, real aggregate lending is

$$L/p = \Psi, \tag{28}$$

where

$$\Psi \equiv \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \left[(\rho_\ell/\rho_d)\,\varepsilon - \varepsilon_d \right] dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} \left[(\rho_\ell/\rho_d)\,\varepsilon_{\bar{\ell}} - \varepsilon_d \right] dF(\varepsilon) .$$

Divide both sides of (3) by M to get

$$\frac{\mathcal{D}}{M} = \eta - 1 - \frac{i_d D - i_\ell L}{M} - \mathcal{B}(1 - \rho\eta) .$$

Eliminating D and L using (27), respectively (28), and noting that $M/p = \varepsilon_d$, the last expression can be rewritten as follows

$$\frac{\mathcal{D}}{M} = \eta - (1 + i_d) - \frac{(i_d - i_\ell)\,\Psi}{\varepsilon_d} - \mathcal{B}(1 - \rho\eta) . \tag{29}$$

Finally, under the optimal policy $\rho_d = \rho$ we have $\rho = \beta/\eta$. Replacing ρ_d and ρ by β/η and noting that $\Psi = 0$ under the optimal policy yields

$$\frac{\mathcal{D}}{M} = (\eta/\beta + \mathcal{B})(\beta - 1) \leq 0.$$

Thus, the optimal policy requires that the government generates a primary surplus.
■

At the end of Section 5, we propose an equilibrium, where agents do not bring money into the goods market, but only bonds. They then use bonds to borrow money at the lending facility, and then use the money to purchase goods. Sellers accept money in the goods market, because they want to purchase general goods in the money market, and buyers demand money in the money market to reimburse their loans. The amount of demand deposited at the deposit facility by sellers is exactly the amount of cash that is borrowed at the central bank by buyers. So, when the lending rate is set equal to the deposit rate, the revenue of the central bank equals its expenditures.

In what follows we show that such an equilibrium cannot exist.

Proof. From (9) and (10), such an equilibrium requires that $V_G^{m'} < \phi$ and $V_G^{b'} = \phi\rho$.

Following the proof of Proposition 3, one can replace $V_G^{m'}$ and $V_G^{b'}$ to get

$$\begin{aligned}\frac{\rho_d \eta}{\beta} &> \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) \\ \frac{\rho \eta}{\beta} &= \int_0^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} dF(\varepsilon)\end{aligned}$$

In the proposed equilibrium, $\rho_d = \rho_\ell = \rho$ and so we get

$$\begin{aligned}\frac{\rho \eta}{\beta} &> \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} dF(\varepsilon) \\ \frac{\rho \eta}{\beta} &= \int_0^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_{\bar{\ell}}} dF(\varepsilon)\end{aligned}$$

Thus, such an equilibrium requires that

$$\int_{\varepsilon_d}^{\varepsilon_\ell} dF(\varepsilon) > \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon)$$

which is a contradiction. ■

Proof of Proposition 6.

The proof involves to show that if $\mathcal{S} > \mathcal{S}^o$, it is not optimal to operate a floor system with $i_d = i_\ell < i_d^*$, where i_d^* is the deposit rate under the optimal policy. The proof involves two steps. First, we show that the constraint $\mathcal{S} > \mathcal{S}^o$ implies $i_d = i_\ell < i_d^*$. Second, we show that if $i_d = i_\ell < i_d^*$, it is optimal to choose a non-zero corridor by increasing the loan rate i_ℓ marginally.

First step. From (1), in any equilibrium the surplus satisfies

$$S = M^+ - M + i_\ell L - i_d D$$

In the proof of Proposition 5, we show that in any equilibrium, $D = M + L$, which allows us to write the previous equation as follows

$$S = [\eta - 1 + (i_\ell - i_d)\, L/M - i_d]\, M$$

where $M^+/M = \eta$.

Under the optimal policy, we have shown that $i_d = i_d^*$ and $L = 0$ and hence we get

$$\mathcal{S}^o = (\eta - 1 - i_d^*)\, M$$

In any floor system, we have $i_d = i_\ell$ and therefore,

$$S = (\eta - 1 - i_d)M$$

This immediately implies that when $S > S^o$, then $i_d = i_\ell < i_d^*$. In the next step, we show that for any floor system with $i_d = i_\ell < i_d^*$ it is optimal to deviate and increase the loan rate marginally.

Second step. The welfare function is

$$\begin{aligned} \mathcal{W} \;=\; & \int_0^{\varepsilon_d} [\varepsilon u(q_\varepsilon) - q_\varepsilon]\, dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u(q_\varepsilon) - q_\varepsilon]\, dF(\varepsilon) \\ & + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\varepsilon u(q_\varepsilon) - q_\varepsilon]\, dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u(q_\varepsilon) - q_\varepsilon]\, dF(\varepsilon)\,. \end{aligned} \tag{30}$$

To show that it is never optimal to choose a zero band, we calculate $d\mathcal{W}/d\rho_\ell$, evaluate it $\rho_\ell = \rho_d = \rho$, and then show that $d\mathcal{W}/d\rho_\ell|_{\rho_\ell=\rho_d=\rho} < 0$.

Note that ρ_ℓ affects $\mathcal{W}$ directly and indirectly via ε_d; that is

$$\frac{d\mathcal{W}}{d\rho_\ell} = \frac{\partial \mathcal{W}}{\partial \varepsilon_d}\frac{d\varepsilon_d}{d\rho_\ell} + \frac{\partial \mathcal{W}}{\partial \rho_\ell}.$$

We get the term $\frac{d\varepsilon_d}{d\rho_\ell}$ by taking the total derivative of the equilibrium equation (17), which we replicate here for easier reference:

$$\frac{\rho_d \gamma}{\beta} = \int_0^{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon \rho_d}{\varepsilon_{\bar{\ell}} \rho_\ell} dF(\varepsilon)\,.$$

From this equation, we get

$$\frac{d\varepsilon_d}{d\rho_\ell} = -\frac{\int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{(\rho_\ell)^2} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon \mathcal{B}}{\varepsilon_d (1+\rho_\ell \mathcal{B})^2} dF(\varepsilon)}{\int\limits_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{(\varepsilon_d)^2} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{(\varepsilon_d)^2 (1+\rho_\ell \mathcal{B})} dF(\varepsilon)} < 0,$$

since $\varepsilon_\ell = \varepsilon_d \frac{\rho_d}{\rho_\ell}$ and $\varepsilon_{\bar{\ell}} = \varepsilon_d \frac{\rho_d}{\rho_\ell} (1 + \rho_\ell \mathcal{B})$.

The partial derivative $\frac{\partial \mathcal{W}}{\partial \varepsilon_d}$ is

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \varepsilon_d} &= \int\limits_0^{\varepsilon_d} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) \\ &\quad + \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon). \end{aligned}$$

Using (15), we can write this partial derivative as follows:

$$\frac{\partial \mathcal{W}}{\partial \varepsilon_d} = \int\limits_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u'(q_\varepsilon) - 1] dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] (1 + \rho_\ell \mathcal{B}) dF(\varepsilon). \tag{31}$$

Note that $\frac{\partial \mathcal{W}}{\partial \varepsilon_d}$ is strictly positive.

For $\frac{\partial \mathcal{W}}{\partial \rho_\ell}$ we get

$$\begin{aligned} \frac{\partial \mathcal{W}}{\partial \rho_\ell} &= \int\limits_0^{\varepsilon_d} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\rho_\ell} dF(\varepsilon) + \int\limits_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\rho_\ell} dF(\varepsilon) \\ &\quad + \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\rho_\ell} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\rho_\ell} dF(\varepsilon), \end{aligned}$$

which using (15) can be written as

$$\frac{\partial \mathcal{W}}{\partial \rho_\ell} = \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\varepsilon u'(q_\varepsilon) - 1] \frac{\varepsilon}{\rho_d} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] \varepsilon_d \mathcal{B} dF(\varepsilon),$$

which is strictly positive. This implies that $\frac{d\mathcal{W}}{d\rho_\ell}$ can be either positive or negative. Increasing ρ_ℓ (decreasing i_ℓ) has a positive effect on welfare through its direct effect $\frac{\partial\mathcal{W}}{\partial\rho_\ell}$, but a negative effect through its indirect effect $\frac{\partial\mathcal{W}}{\partial\varepsilon_d}\frac{d\varepsilon_d}{d\rho_\ell}$.

We now evaluate these derivatives at $\rho_\ell = \rho_d = \rho$. We get

$$\begin{aligned}
\left.\frac{d\varepsilon_d}{d\rho_\ell}\right|_{\rho_\ell=\rho_d=\rho} &= -\frac{\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\rho\varepsilon\mathcal{B}}{\varepsilon_{\bar{\ell}}(1+\rho\mathcal{B})} dF(\varepsilon)}{\int_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\rho\varepsilon}{\varepsilon_d\varepsilon_{\bar{\ell}}} dF(\varepsilon)} \\
\left.\frac{\partial\mathcal{W}}{\partial\rho_\ell}\right|_{\rho_\ell=\rho_d=\rho} &= \int_{\varepsilon_{\bar{\ell}}}^{\infty} \left[\varepsilon u'(q_\varepsilon) - 1\right]\varepsilon_d\mathcal{B}dF(\varepsilon) \\
\left.\frac{\partial\mathcal{W}}{\partial\varepsilon_d}\right|_{\rho_\ell=\rho_d=\rho} &= \int_{\varepsilon_{\bar{\ell}}}^{\infty} \left[\varepsilon u'(q_\varepsilon) - 1\right](1+\rho\mathcal{B})\,dF(\varepsilon),
\end{aligned}$$

since $\varepsilon_d = \varepsilon_\ell$ and $\varepsilon_{\bar{\ell}} = \varepsilon_d(1+\rho\mathcal{B})$ at $\rho_\ell = \rho_d = \rho$. Use these expressions to write $\left.\frac{d\mathcal{W}}{d\rho_\ell}\right|_{\rho_\ell=\rho_d=\rho}$ as follows

$$\left.\frac{d\mathcal{W}}{d\rho_\ell}\right|_{\rho_\ell=\rho_d=\rho} = -\int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u'(q_\varepsilon) - 1\right]dF(\varepsilon)\,\frac{(\varepsilon_{\bar{\ell}})^2\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} dF(\varepsilon)}{\rho\int_{\varepsilon_{\bar{\ell}}}^{\infty}\varepsilon dF(\varepsilon)} < 0.$$

Hence, a marginal decrease of ρ_ℓ (marginal increase of i_ℓ) from $\rho_\ell = \rho_d = \rho$ is welfare improving. It follows that a floor system is not optimal if $i_d < i_d^*$. ■

Proof that $\frac{\partial\mathcal{W}}{\partial\rho_d} < 0$. The welfare function is

$$\begin{aligned}
\mathcal{W} &= \int_0^{\varepsilon_d}\left[\varepsilon u(q_\varepsilon) - q_\varepsilon\right]dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell}\left[\varepsilon u(q_\varepsilon) - q_\varepsilon\right]dF(\varepsilon) \qquad (32) \\
&\quad + \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}}\left[\varepsilon u(q_\varepsilon) - q_\varepsilon\right]dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty}\left[\varepsilon u(q_\varepsilon) - q_\varepsilon\right]dF(\varepsilon).
\end{aligned}$$

To show that it is never optimal to lower i_d if $\eta/\beta > 1 + i_d$, we calculate $d\mathcal{W}/d\rho_d$.

Note that ρ_d affects $\mathcal{W}$ directly and indirectly via ε_d; that is

$$\frac{d\mathcal{W}}{d\rho_d} = \frac{\partial \mathcal{W}}{\partial \varepsilon_d}\frac{d\varepsilon_d}{d\rho_d} + \frac{\partial \mathcal{W}}{\partial \rho_d}.$$

We get the term $\frac{d\varepsilon_d}{d\rho_d}$ by taking the total derivative of the equilibrium equation (17) which we replicate here for easier reference:

$$\frac{\rho_d \eta}{\beta} = \int\limits_0^{\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{\rho_d}{\rho_\ell} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{\varepsilon_d (1 + \rho_\ell \mathcal{B})} dF(\varepsilon).$$

From this equation, we get

$$\frac{d\varepsilon_d}{d\rho_d} = -\frac{\frac{\gamma}{\beta} - \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \frac{1}{\rho_\ell} dF(\varepsilon)}{\int\limits_{\varepsilon_d}^{\varepsilon_\ell} \frac{\varepsilon}{(\varepsilon_d)^2} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} \frac{\varepsilon}{(\varepsilon_d)^2 (1 + \rho_\ell \mathcal{B})} dF(\varepsilon)} < 0,$$

since $\varepsilon_\ell = \varepsilon_d \frac{\rho_d}{\rho_\ell}$ and $\varepsilon_{\bar{\ell}} = \varepsilon_d \frac{\rho_d}{\rho_\ell}(1 + \rho_\ell \mathcal{B})$.

The partial derivative $\frac{\partial \mathcal{W}}{\partial \varepsilon_d}$ is

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \varepsilon_d} &= \int\limits_0^{\varepsilon_d} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) \\
&\quad + \int\limits_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] \frac{dq_\varepsilon}{d\varepsilon_d} dF(\varepsilon).
\end{aligned}$$

Using (15), we can write this partial derivative as follows:

$$\frac{\partial \mathcal{W}}{\partial \varepsilon_d} = \int\limits_{\varepsilon_d}^{\varepsilon_\ell} [\varepsilon u'(q_\varepsilon) - 1] dF(\varepsilon) + \int\limits_{\varepsilon_{\bar{\ell}}}^{\infty} [\varepsilon u'(q_\varepsilon) - 1] (1 + \rho_\ell \mathcal{B}) dF(\varepsilon).$$

Note that $\frac{\partial \mathcal{W}}{\partial \varepsilon_d}$ is strictly positive.

For $\frac{\partial \mathcal{W}}{\partial \rho_d}$ we get

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \rho_d} &= \int_0^{\varepsilon_d} \left[\varepsilon u'(q_\varepsilon) - 1\right] \frac{dq_\varepsilon}{d\rho_d} dF(\varepsilon) + \int_{\varepsilon_d}^{\varepsilon_\ell} \left[\varepsilon u'(q_\varepsilon) - 1\right] \frac{dq_\varepsilon}{d\rho_d} dF(\varepsilon) \\
&+ \int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \left[\varepsilon u'(q_\varepsilon) - 1\right] \frac{dq_\varepsilon}{d\rho_d} dF(\varepsilon) + \int_{\varepsilon_{\bar{\ell}}}^{\infty} \left[\varepsilon u'(q_\varepsilon) - 1\right] \frac{dq_\varepsilon}{d\rho_d} dF(\varepsilon),
\end{aligned}$$

which using (15) can be written as

$$\frac{\partial \mathcal{W}}{\partial \rho_d} = -\int_{\varepsilon_\ell}^{\varepsilon_{\bar{\ell}}} \left[\varepsilon u'(q_\varepsilon) - 1\right] \frac{\rho_\ell}{(\rho_d)^2} dF(\varepsilon) < 0.$$

This implies that $\frac{d\mathcal{W}}{d\rho_d}$ is negative. Thus, increasing ρ_d (decreasing i_d) has a negative effect on welfare. It follows that increasing i_d is optimal if $\eta/\beta > \rho_d$. ■

Proof of Proposition 7. Divide both sides of (3) by M to get

$$\frac{\mathcal{D}}{M} = \eta - 1 + \frac{i_\ell L - i_d D}{M} - \mathcal{B}(1 - \rho\eta) + r\mathcal{K}.$$

Eliminating D and L using (27) and (28), respectively, and noting that $M/p = \varepsilon_d$, the last expression becomes

$$\frac{\mathcal{D}}{M} = \eta - (1 + i_d) - \frac{(i_d - i_\ell)\Psi}{\varepsilon_d} - \mathcal{B}(1 - \rho\eta) + r\mathcal{K}. \tag{33}$$

Optimal policy requires $\rho_d = \rho = \beta/\eta$. Replacing ρ_d and ρ by β/η and noting that $\Psi = 0$ under the optimal policy yields

$$\frac{\mathcal{D}}{M} = (\eta/\beta + \mathcal{B})(\beta - 1) + r\mathcal{K}.$$

Thus, the optimal policy generates no losses for the central bank ($\mathcal{D} > 0$) if and only if $\mathcal{K} \geq \bar{\mathcal{K}}$ where $\bar{\mathcal{K}} \equiv (\eta/\beta + \mathcal{B})(1 - \beta)/r$. ■

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