

Trend-Cycle Correlation, Drift Break and the Estimation of Trend and Cycle in Canadian GDP

by

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Abstract:

This paper argues, using Monte Carlo experiments, that bivariate correlated unobserved components (UC) framework delivers more accurate estimation results for trend and cycle parameters than the univariate framework. The paper estimates stochastic trend and cyclical fluctuations in Canada from a bivariate, correlated UC model with drift breaks. In contrast to the univariate results, bivariate estimation shows a fairly volatile stochastic trend after accounting for the drift break along with a negative trend-cycle shock correlation. The estimated cyclical component is moderately large, persistent and consistent with ECRI denoted Canadian recessions.

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Key Words: Stochastic trend; Inflation; GDP; Unobserved components model.

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1. Introduction

Estimation of cyclical fluctuations using unobserved components (UC) model, introduced by Harvey (1985), Watson (1986), Clark (1987), has the advantage of not imposing apriori smoothness (like time trend or Hodrick-Prescott (hereafter HP) (1997)) on the stochastic trend component. However, these models impose a restriction of zero correlation between trend and cycle shocks. In a recent paper, Morley, Nelson and Zivot (2003) (hereafter MNZ) showed that the estimates of the cycle component is sensitive to the correlation of the trend and cycle shocks. A highly negative correlation of the shocks, as they estimate from the data, results in small cycles that resemble the cyclical component from univariate Beveridge and Nelson (1981) decomposition. Recent research by Perron and Wada (2005) (hereafter PW) shows that the MNZ result of a volatile stochastic trend disappears if one allows for a drift break in the trend.

The idea of using inflation along with output in the UC models for getting better estimates of the cycle was introduced by Kuttner (1994), followed by Apel and Jansson (1999) in the US context, Frank and Smets (1997) in a multiple country context, and Kichian (1999) in the Canadian context. Multivariate modeling of the output gap with Canadian data has been used by St. Amant and Van Norden (1997). Multivariate UC models aiming to estimate US cycles with correlated shocks have been used by Basistha and Nelson (2005), and Sinclair (2005).

Using the two new developments in the field — correlated trend-cycle shocks and drift breaks in the stochastic trend — this paper estimates trend and cycle in Canadian GDP building on Kuttner's bivariate inflation–output framework. It argues, using Monte Carlo experiments, that bivariate correlated UC framework delivers more accurate

estimates of trend and cycle parameters than the univariate framework. Estimates using Canadian data produce moderately large and persistent cycles with negative trend-cycle correlation and a volatile stochastic trend even after allowing for drift breaks.

The rest of the paper is organized into four sections. The next section uses Monte Carlo experiments to examine the advantages of bivariate modeling. Section 3 lays out and estimates the bivariate model with Canadian data. Section 4 introduces a random walk model of the drift instead of structural breaks in the drift and re-estimates the UC model of Section 3 for robustness. Section 5 concludes.

2. The Monte Carlo Simulations to Examine the Usefulness of a Bivariate Set-up

Monte Carlo simulations is one way to examine how useful an additional indicator is for estimating an unobserved components model. Before setting up the bivariate model of measuring cyclical fluctuations in Canada, Monte Carlo experiments are used to simulate and examine a similar model with artificial data. It assumes there are two observed variables, y_1 and y_2 . The variable y_1 at time t is a sum of two unobserved components, x_1 and z .

$$1. \quad y_{1,t} = x_{1t} + z_t$$

The variable y_2 also depends on two unobserved components; x_2 and the common factor z , with ϕ as the factor loading parameter.

$$2. \quad y_{2,t} = x_{2,t} + \phi z_t$$

Let x_1 be the stochastic trend component following a random walk with a constant drift μ .

$$3. \quad x_{1,t} = \mu + x_{1,t-1} + \varepsilon_{1,t}$$

Let z be a mean-zero second order autoregressive process – resembling a cycle.

$$4. \quad z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + \varepsilon_{z,t}$$

In specifying the last component, x_2 , two specifications are used. One in which it depends on the first lag of y_2 (the more commonly used inflation specification)

$$5. \quad x_{2t} = \beta_0 + \beta_1 y_{2,t-1} + \varepsilon_{2,t}$$

and another in which it follows a non-zero mean autoregressive process of order one.

$$6. \quad x_{2t} = \beta_0 + \beta_1 x_{2,t-1} + \varepsilon_{2,t}$$

In generating the data using equations 1 – 5 or equations 1 - 4 and equation 6, the correlation coefficients of the inflationary shock with the other two shocks were zero, i.e.

$\rho_{1,2} = \rho_{z,2} = 0$. Three values of the trend-cycle correlation are used, strongly negative as in MNZ ($\rho_{1z} = -0.79$), zero as in Watson (1986) and Clark (1987), and strongly positive ($\rho_{1z} = 0.79$). However, in estimating the model, whether univariate or bivariate, all the shocks are allowed to be correlated.

The values of the standard deviation of the three shocks for data generation are $\sigma_1 = 0.9$, $\sigma_z = 0.71$, $\sigma_2 = 1.0$. The trend drift (μ) is 0.9, the common factor loading coefficient (ϕ) is 0.4. The autoregressive coefficients, θ_1 and θ_2 , are 1.6 and -0.75 respectively. Finally, the parameters β_0 and β_1 are 1.5 and 0.5, in both equations 5 and 6. The values fairly closely resemble the estimates for Canada and the US as reported in Kuttner (1994), MNZ, Kichian (1999).

The data is generated using one specific bivariate model, then estimated the state space model twice – once using equations 1, 3 and 4 (the univariate framework) and

again using equations 1 – 5 or equations 1 - 4 and 6 as appropriate (the bivariate framework). The sample size for each Monte Carlo iteration is 200 – roughly the size of post war quarterly data. The number of Monte Carlo iterations is 3000.

The results of the Monte Carlo experiments are in Table 1. Panel A shows the results with negative trend-cycle correlation in the artificial data – first using equation 5 and then 6. Using equation 5, the univariate mean estimates of the standard deviation of the trend shock, standard deviation of the cycle shock and the correlation of the trend and the cycle shocks are moderately close to the actual data. But these are fairly imprecise – the standard deviations (reported in the parentheses) are fairly large. The bivariate model, on the other hand, yields a striking improvement over the univariate model. The mean estimates of the standard deviation of the trend shock, standard deviation of the cycle shock and the correlation of the trend and the cycle shocks are very close to actual data – and much more precise than the univariate model as measured by the standard deviations.

This is also true for models using equation 6 instead of equation 5. The univariate model has very similar results as in equation 5. This is expected since it uses similar information as before. The bivariate results are much more accurate and precise. In comparison with equation 5 bivariate results, the equation 6 bivariate results are mildly inaccurate and imprecise.

Panel B and Panel C report the results of the same experiment with different values of the trend cycle correlation. We see a similar pattern as in Panel A. The bivariate results are always more accurate and precise than their univariate counter parts in correctly estimating the true value of the correlation coefficient – a key parameter in estimating the cyclical component, as MNZ demonstrates.

A related issue at this point is determining when the bivariate model will not yield a better result than the univariate model. A trivial answer to this question is when there are no common factors ($\phi = 0$). The second case is when the non-gap inflationary component of the second variable provides very little independent information over the stochastic trend component of the first variable. The non-gap inflationary component is different than the trend component due to the difference in the stochastic processes and due to the independence of the inflationary shocks.

Using equation 6 as the non-gap inflationary process, increase in the persistence of the process decreased the precision of the results. However, using equation 5 instead of equation 6, the results are generally more accurate at higher level of persistence. Additionally, if one assumes a high value of the trend - inflation shock correlation and the cycle-inflation shock correlation is assumed to close enough to the trend-cycle shock correlation, the results become less precise. In both cases, when the non-gap inflationary component is allowed to closely resemble the trend component, some iterations in the Monte Carlo experiments showed instability in convergence. The results are not reported due to the instability concerns and space considerations.

3. The Inflation-GDP Unobserved Components Model of Canada and Its Estimates

Based on the results of the previous section, this section sets up a bivariate unobserved components model for estimating cyclical fluctuations in Canada. In this model, as in Watson (1986), Clark (1987), MNZ among others, the log GDP at time t , y_t , is a sum of a stochastic trend component, T_t and a cyclical component c_t .

$$7. \quad y_t = T_t + c_t$$

The trend is assumed to follow a random walk with constant drift but recent evidence on trend productivity growth (e.g. see Kahn and Rich (2004), Roberts (2001) for the U.S. evidence and Kichian (1999) for Canada) show that a constant drift term may not be appropriate. Kichian (1999) and PW both use dummy variables to capture a drift term with breaks in the stochastic trend. Allowing the drift term to have multiple breaks within the sample period, the appropriate number of breaks in the drift is determined by Bai and Perron (1997, 2001, 2003, hereafter BP) multiple structural break tests on GDP growth. The dummies, $D_{T,i}$ capture these break dates in the drift term μ_0 .

$$8. \quad T_t = \mu_0 + \sum_{i=1}^{B_t} \mu_i D_{T,i} + T_{t-1} + \varepsilon_{T,t}$$

The cyclical component of output follows an autoregressive process with zero mean.

$$9. \quad \theta_c(L)c_t = \varepsilon_{c,t}$$

Following Watson (1986), Harvey and Jaeger (1993) and MNZ, an autoregressive process of order two is assumed.

The second observable variable, inflation rate (π_t), depends on expected inflation, supply shocks (s_t) and the cyclical output. The expected inflation term can be modeled in several ways. One way is to simply assume lagged inflation as the expected inflation¹. Another way is to model it in levels with appropriate number of lags and MA terms. The advantage of the second method is that the mean breaks in inflation (e.g. see Levin and Piger (2003), Rapach and Wohar (2003)) can be modeled as intercept shifts

¹ Gerlach and Smets (1997) uses lag inflation as the expected inflation in Canadian data – specifying the inflation equation in first differences. Kichian (1999) experiments with both first differenced inflation data and levels with mean breaks. She found the inflation level data with mean breaks to be a better fit. This paper found very similar results but does not report them here.

represented by dummies ($D_{\pi,j}$). The number of intercept breaks is to be determined by BP break tests.

$$10. \quad \pi_t = \beta_{\pi,0,0} + \sum_{j=1}^{B_j} \beta_{\pi,0,j} D_{\pi,j} + \sum_{k=1}^{B_k} \beta_{\pi,1,k} \pi_{t-k} + \phi_c c_t + \sum_{m=1}^{B_m} \gamma_m s_{m,t} + \varphi_{\pi}(L) \varepsilon_{\pi,t}$$

Quarterly time series is obtained from International Financial Statistics maintained by the IMF, ranging from 1961:1 to 2004:4. The logarithm of real GDP, seasonally adjusted, times 100, is used as the output series. The inflation rate is based on seasonally adjusted CPI. The two supply shocks used here are the (natural logs of) demeaned nominal effective exchange rate change and the demeaned world average crude price change.

The empirics start out by performing a series of BP multiple structural break tests on inflation level and GDP growth². The results are in Table 2. The UDMax and WDMMax tests do suggest the existence of at least one break for the inflation rate. The sequential SupF tests imply 3 breaks in the mean. The result of 3 mean breaks and the corresponding estimates of the break dates confirm the findings reported by Demers (2003), Rapach and Wohar (2003). The tests for GDP growth show one break in the fourth quarter of 1973.

Three dummy variables for the inflation equation (eqn. 10) and one dummy variable for the break in the drift term of GDP stochastic trend (eqn. 8) are generated. The parameters are estimated using maximum likelihood method and then the Kalman filter is used to get the estimates of the unobserved state variables. The number of own lags and

² See BP and Rapach and Wohar (2003) for a complete description of all the test statistics. The tests were performed on just the mean without including any lags.

the MA lags in the inflation equation is chosen according to the significance of the last lag. One lag of inflation and one MA lag proved to be adequate.

The parameter estimates for univariate correlated UC models, useful for comparison purposes, are in Table 3. The MNZ stochastic trend (eqn. 8 without break) is highly volatile but the PW (eqn. 8 with break) trend is almost non-stochastic after the break. These are very much in accordance with the US results. However, the MNZ cycle proved to be large for Canada (much larger than the Frank and Smets (1997) and Kichian (1999) estimates), although with fairly low persistence. The PW cycle is large and persistent. Figure 1 illustrates these results. The trend cycle shock correlation estimates yield sharply divergent results – almost perfectly negatively correlated to perfectly positively correlated³. So, the MNZ and PW methods provide very different results regarding the volatility of stochastic trend, persistence of cycle and trend-cycle correlation.

The results from bivariate model, eqns. 7 – 10 with break and correlated shocks, are in Table 4. The first result is that the estimate of standard deviation of the trend shock is fairly large and precise even after accounting for drift break. This is in contrast to the PW result that appropriately accounted drift breaks in a correlated trend cycle model produces non-stochastic trend. However, the point estimate of the standard deviation of the stochastic trend shock is lower than the MNZ estimates. The estimate of the standard deviation of the cyclical shock is moderate, lower than both MNZ and PW, but mildly imprecise. The cycle shows a fairly persistent movement; the sum of the autoregressive coefficients is 0.89. It is more persistent than MNZ cycle but less persistent than PW.

³ Interestingly, PW also found perfectly positively correlated trend-cycle shock in their unrestricted univariate model.

The trend-cycle correlation is highly negative and significant – thereby confirming the MNZ results. The rest of the correlations of the shocks are very close to zero and imprecise. The lagged inflation coefficient in the Phillips curve indicates a stationary process. The output gap coefficient in the Phillips curve is low but significantly different from zero. All the above three factors confirm the role of inflation in estimating trend and cycle from GDP data. The break in the drift term is negative and significant, confirming the PW estimates and the general observation of a slowdown in Canadian productivity from the mid-Seventies.

In Figure 2, the two – sided estimates of the stochastic trend and the cyclical fluctuations are shown. The trend is volatile before and after 1973. The shaded quarters are Economic Cycle Research Institute (ECRI) peak to trough movement in Canadian economic activity⁴. The cycle estimates nicely pick up all the three recessions. This illustrates the finding that bivariate estimation of output and inflation generates persistent economic cycles that describes the Canadian cycles quite well even after allowing for trend-cycle correlation, a stochastic trend and a drift break. One does not need to impose a priori smoothness on the trend or zero correlation of trend cycle shocks when the trend is volatile. This confirms Kuttner's (1994) (also used by Frank and Smets (1997) and Kichian (1999) in the Canadian context) findings that inflation can improve the estimates of cyclical fluctuations.

For further comparison of the estimates to univariate detrending of GDP, a cubic time trend and a HP trend are also used. In Table 5, the standard deviations of the different cycle estimates and their correlations are shown. The graphs of the four

⁴ The February 2005 version of ECRI Canadian recession dates have only two recessions, 1981-82 and 1991-92. However, Bodman and Crosby (2000) reports three recessions from ECRI that also included 1974-75. All three are included in this paper.

univariate cycle estimates along with the bivariate cycle are in Figure 3. The MNZ cycle is the largest – though not persistent as one can see from the bottom left panel of Figure 3. The PW cycle is almost similar to time trend cycle and the HP cycle is the smallest. The bivariate cycle is larger than the HP cycle but smaller than the rest. To sum up, the bivariate model delivers a moderately large and persistent cycle that picks up the Canadian recessions quite well after accounting for all the shock correlations and breaks.

4. Using Random Walk Drift in the Bivariate Unobserved Components Model and Its Estimates

This section relaxes the assumption of just one break in the drift term by letting the drift term be a random walk, replacing eqn. 8 by the following stochastic trend equation.

$$11. \quad T_t = d_{t-1} + T_{t-1} + \varepsilon_{T,t}$$

Where the drift term d_{t-1} follows a simple random walk:

$$12. \quad d_t = d_{t-1} + \varepsilon_{d,t}$$

However, estimation of eqns. 7, 9 - 12 results in the well known ‘pile-up’⁵ problem of variance of the drift shock – even when it is uncorrelated with all other shocks. To avoid the ‘pile-up’ problem Stock and Watson’s (1998) median unbiased estimation of the variance of the drift term is used.

The growth rate of stochastic trend estimates from the previous section is used to compute the exponential Wald statistics (Andrews and Ploberger (1994)) to test for one

⁵ The problem is when the variance of the shock to a random walk component is small in an UC model, its maximum likelihood estimates are biased towards zero. See Stock (1994), Stock and Watson (1998) for discussions on this problem.

break in mean at an unknown break point. The computed Exp-Wald statistics is in Panel A of Table 6. Based on the Exp- Wald statistics, Table 3 of Stock and Watson (1998) is used to compute the median unbiased estimate of λ ⁶. Equations 7 and 9 – 12 is then re-estimated using maximum likelihood with $\sigma_d = (\lambda / N) * \sigma_T$, where N is the sample size. The drift shock is also assumed to be uncorrelated with any other shock as the Stock and Watson (1998) method requires.

The results are in Panel B of Table 6. The point estimates are very similar to the ones in the previous section. The trend shock component is still large. The log likelihood value dropped due to two less parameters and also due to another diffused starting value for the random walk drift component. Addition of the random walk drift also resulted in a decline in the precision of the parameter estimates. The estimates of the cycle in Figure 4 are persistent and picks up the three ECRI recessions nicely. It has a correlation of 0.67 with the estimates of the bivariate cycle in Section 3.

The estimates of the drift component are presented in Figure 5. It depicts a productivity growth decline in Canada from early 1970s and a moderate recovery from the early 1990s. Overall, this section does a robustness check on the previous results by introducing a random walk drift term in the stochastic trend. The point estimates confirm previous findings even with the random walk drift. The trend is still volatile suggesting most shocks are in the trend level and the cycle is moderate and persistent.

5. Conclusion

⁶ The mean-Wald and QLR statistics also produce the same results.

This paper presents three results on using the UC models to estimate business cycles, especially in the Canadian context. The Monte Carlo experiments show that the bivariate models are capable of delivering more accurate results than the univariate models in a correlated UC framework. The stochastic trend is volatile even after drift breaks or random walk drifts. Estimates of Canadian business cycle, estimated from inflation and output, are moderately large and persistent. The estimates also confirm the empirical result of negative trend-cycle correlation in the Canadian context.

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Table 1: The Monte Carlo Simulation Results

<i>Addl. eqn.</i>	<i>Univariate results</i>			<i>Bivariate results</i>		
Panel A: $\sigma_1 = 0.90, \sigma_z = 0.71, \rho_{1,z} = -0.79$						
Using eqn. 5	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.73 (0.29)	0.64 (0.17)	-0.37 (0.68)	0.89 (0.12)	0.70 (0.15)	-0.76 (0.14)
Using eqn. 6	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.73 (0.29)	0.64 (0.17)	-0.37 (0.67)	0.87 (0.16)	0.69 (0.16)	-0.72 (0.23)
Panel B: $\sigma_1 = 0.90, \sigma_z = 0.71, \rho_{1,z} = 0$						
Using eqn. 5	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.77 (0.31)	0.71 (0.24)	0.36 (0.65)	0.87 (0.13)	0.69 (0.15)	0.07 (0.29)
Using eqn. 6	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.77 (0.31)	0.71 (0.24)	0.37 (0.64)	0.86 (0.20)	0.70 (0.19)	0.12 (0.42)
Panel C: $\sigma_1 = 0.90, \sigma_z = 0.71, \rho_{1,z} = 0.79$						
Using eqn. 5	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.82 (0.34)	0.81 (0.30)	0.70 (0.49)	0.88 (0.13)	0.71 (0.13)	0.80 (0.17)
Using eqn. 6	σ_1	σ_z	$\rho_{1,z}$	σ_1	σ_z	$\rho_{1,z}$
	0.83 (0.34)	0.81 (0.30)	0.70 (0.48)	0.88 (0.19)	0.74 (0.18)	0.73 (0.27)

Note: The standard deviations from the Monte Carlo simulations are in the parentheses.
The sample size for each Monte Carlo simulation is 200 and the number of Monte Carlo simulations is 3000.

Table 2: Bai-Perron Multiple Structural Breaks Tests for Canada

	UDMax	WDMax	SupF(1 0)	SupF(2 1)	SupF(3 2)	SupF(4 3)	SupF(5 4)
Inflation	54.93*	79.25*	20.25*	17.14*	39.49*	2.05	-
GDP Growth	12.43*	12.43*	12.43*	1.28	1.84	1.68	1.12
Break Dates:	Inflation			GDP Growth			
	1972:2	1982:3	1991:1			1973:4	

Note: The critical value for WDMax at the 5 percent level is 9.91. The 5 percent critical values for the following tests are: UDMax - 8.88, SupF(1|0) - 8.58, SupF(2|1) - 10.13, SupF(3|2) - 11.14, SupF(4|3) - 11.83 and SupF(5|4) - 12.25. The asterisked numbers are significant at the 5 percent level.

Table 3: Maximum Likelihood Estimates from Univariate UC Models for Canada

	MNZ		PW	
Standard	σ_T	σ_c	σ_T	σ_c
Deviations	1.49 (0.35)	1.96 (0.50)	0.05 (0.48)	0.74 (0.47)
Autoregressive	$\theta_c(1)$	$\theta_c(2)$	$\theta_c(1)$	$\theta_c(2)$
Parameters	0.80 (0.25)	-0.04 (0.06)	1.22 (0.19)	-0.28 (0.19)
Drift and	μ_0		μ_0	μ_1
Break	0.88 (0.11)		1.28 (0.07)	-0.57 (0.09)
Correlation and	$\rho_{T,c}$	$LogL$	$\rho_{T,c}$	$LogL$
Likelihood	-0.99 (0.01)	-215.85	1.00 (0.00)	-211.07

Note: The numbers in the parentheses are standard errors computed using delta method.

Table 4: Maximum Likelihood Estimates from the Bivariate UC Model

The Standard Deviations and the Correlations of the Shocks					
σ_T	σ_c	σ_π	$\rho_{T,c}$	$\rho_{T,\pi}$	$\rho_{c,\pi}$
0.93 (0.18)	0.51 (0.30)	1.35 (0.18)	-0.77 (0.26)	-0.08 (0.12)	-0.05 (0.20)
The AR terms and the Drift terms					
$\theta_c(1)$	$\theta_c(2)$		μ_0	μ_1	
1.61 (0.19)	-0.72 (0.17)		1.24 (0.14)	-0.51 (0.16)	
The Phillips Curve Coefficients and the Log Likelihood					
$\beta_{\pi,1,1}$	$\varphi_\pi(1)$		ϕ_c	$LogL$	
0.75 (0.04)	-1.23 (0.21)		0.11 (0.05)	-389.95	

Note: The numbers in the parentheses are standard errors computed using delta method.

Table 5: Standard deviation and Correlation Matrix with Different Univariate Estimates

	Time	HP	MNZ	PW	Bivariate
Time	2.33				
HP	0.85	1.38			
MNZ	0.37	0.50	3.07		
PW	0.90	0.72	0.31	2.59	
Bivariate	0.53	0.69	0.43	0.55	1.64

Note: The diagonal terms are standard deviations of the cycle estimates from the respective methods. The off-diagonal terms are correlation of the respective cyclical measures.

Table 6: Parameter Estimates from the Bivariate Model with a Random Walk Drift

Panel A: The Exponential-Wald Test on Trend GDP Growth

Exp-Wald = 6.122

Panel B: Maximum Likelihood Estimates from the Bivariate UC Model With RW Drift

The Standard Deviations and the Correlations of the Shocks

σ_T	σ_c	σ_π	$\rho_{T,c}$	$\rho_{T,\pi}$	$\rho_{c,\pi}$
0.80 (0.33)	0.45 (0.81)	1.52 (0.32)	-0.52 (0.88)	-0.03 (0.08)	-0.14 (0.47)

The AR terms

$\theta_c(1)$	$\theta_c(2)$
1.59 (0.58)	-0.69 (0.50)

The Phillips Curve Coefficients and the Log Likelihood

$\beta_{\pi,1,1}$	$\varphi_\pi(1)$	ϕ_c	LogL
0.76 (0.08)	-1.01 (0.37)	0.12 (0.05)	-400.60

Note: In Panel B, the numbers in the parentheses are standard errors.

Figure 1: Estimates of Trend and Cyclical Fluctuations in Canada from the Uniivariate Models

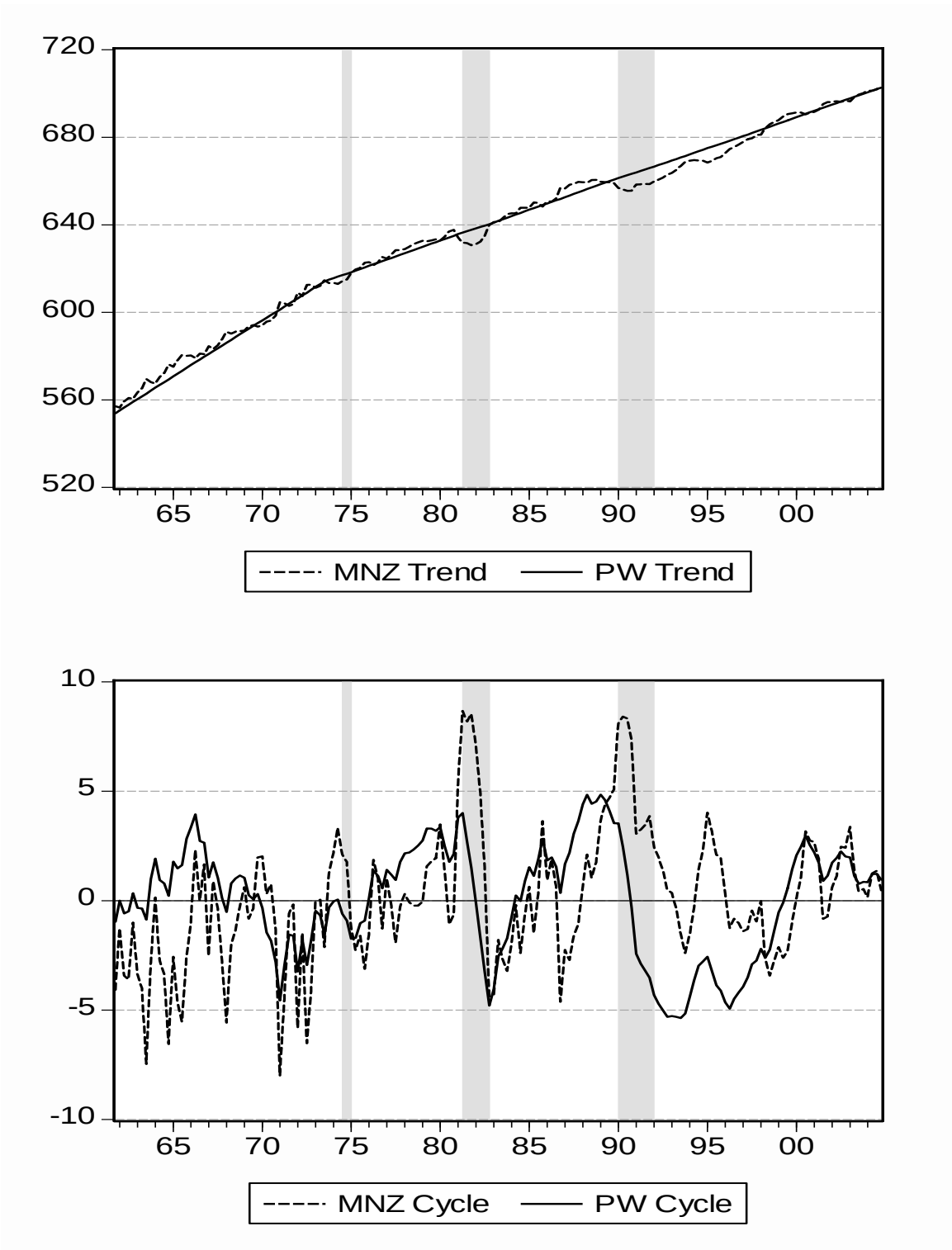
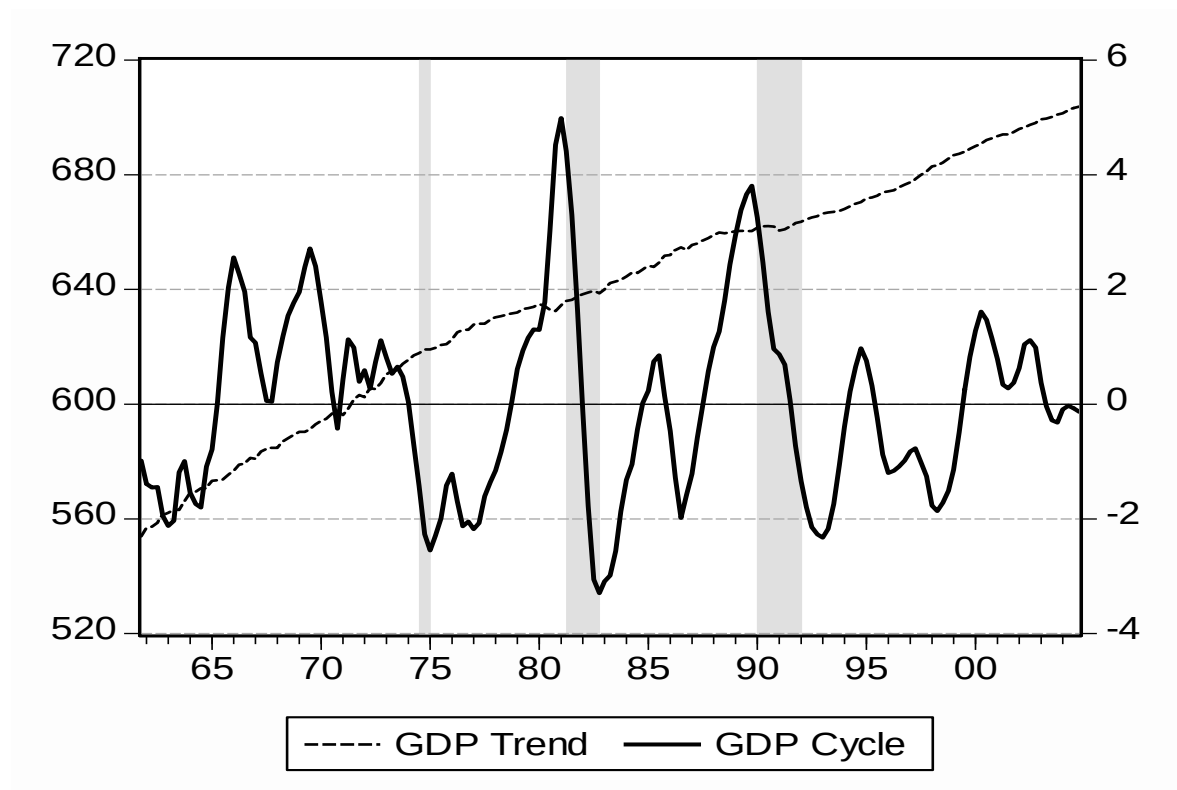


Figure 2: Estimates of Trend and Cyclical Fluctuations in Canada from the Bivariate Model



Note: The right scale is for trend output measured in natural logarithm times 100 and the left scale is for cycle measured in percentage points.

Figure 3: Comparison of Cyclical Fluctuations from the Bivariate Model with Univariate Methods

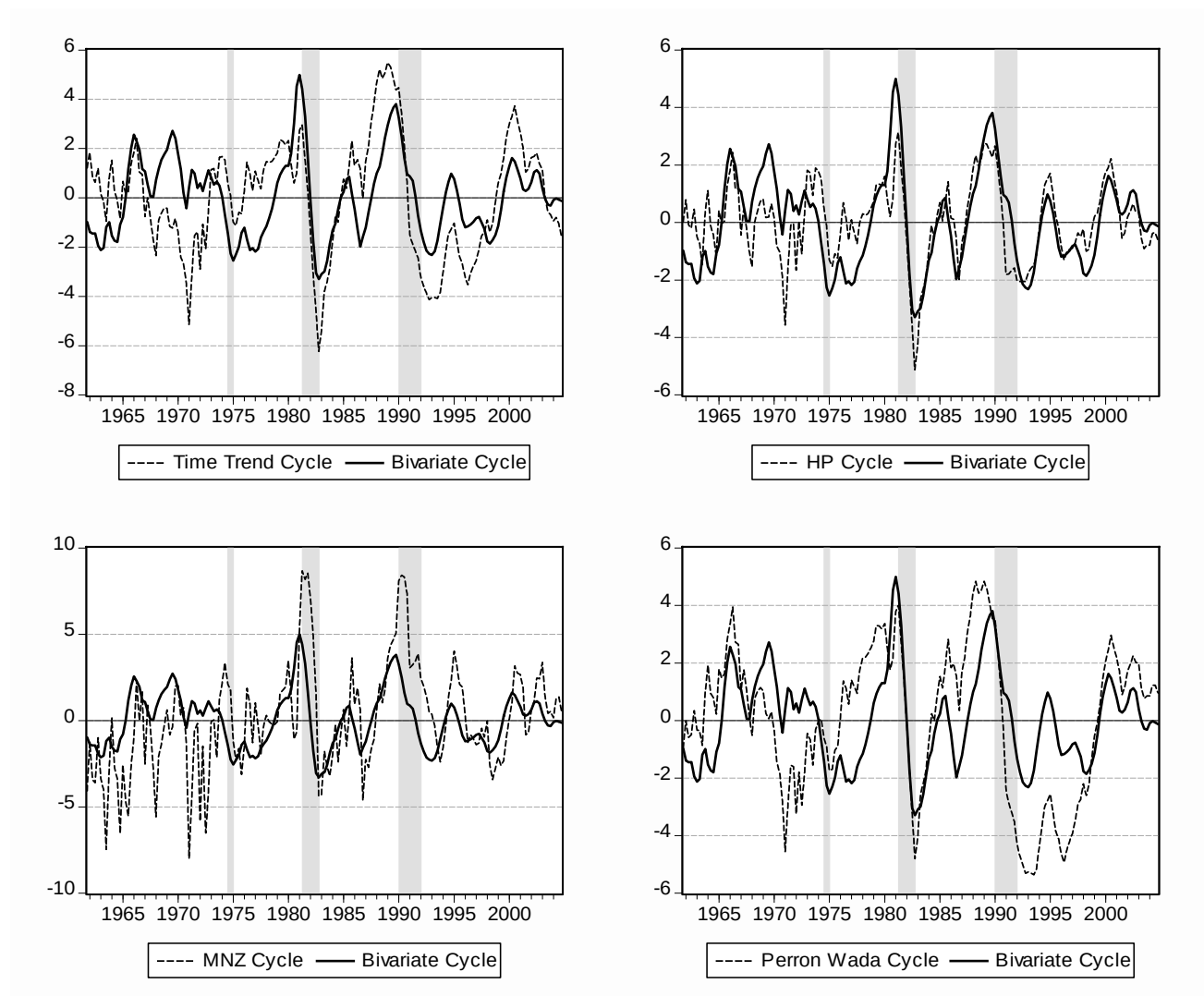
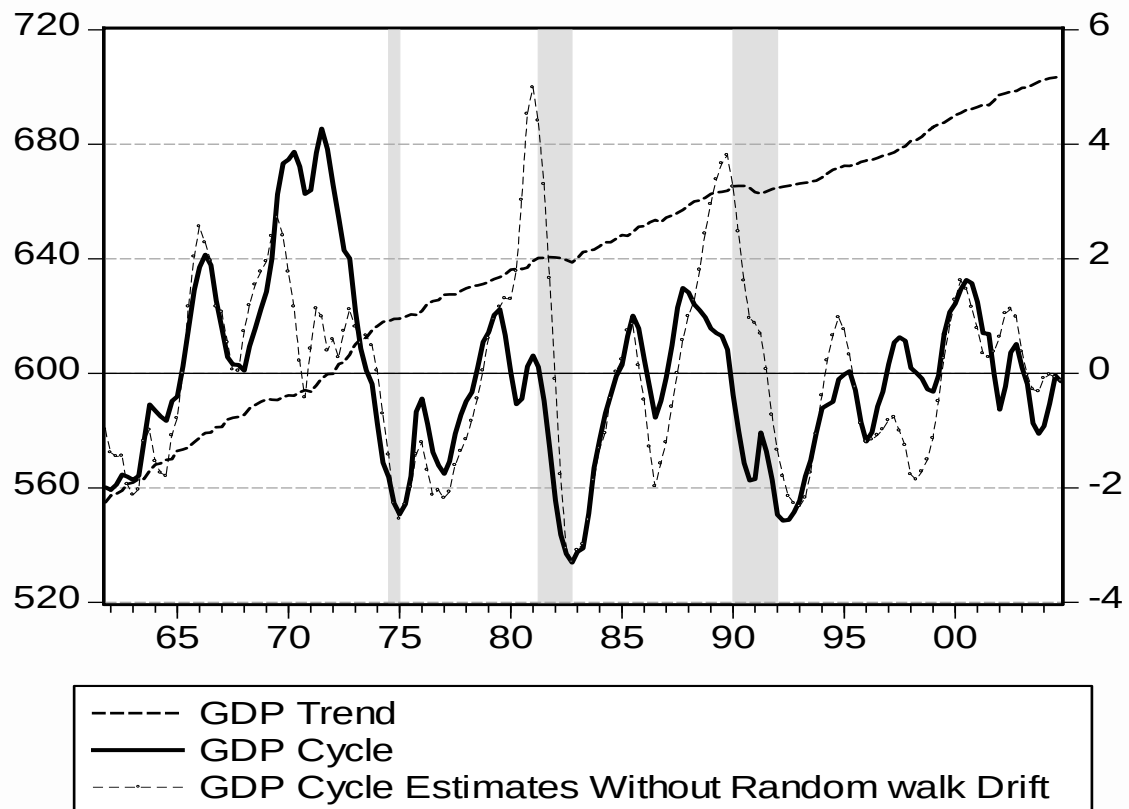


Figure 4: Estimates of Trend and Cyclical Fluctuations in Canada from the Bivariate Model with the Random Walk Drift



Note: The right scale is for trend output measured in natural logarithm times 100 and the left scale is for cycle measured in percentage points.

Figure 5: Estimates of the Random Walk Drift in Canada

