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What floats your boat? Preference revelation from lotteries over complex goods*

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Abstract

A lottery that provides participants the opportunity to choose among a set of simple gambles over multi-attribute goods results in an endogenous distribution of win rates over gambles that reflects tradeoffs between the relative desirability of the available goods and the probability of winning. These win rates provide sufficient information to estimate hedonic prices, marginal utility, and marginal rates of substitution among attributes. We develop a model for mapping preferences from this information set and apply it to Idaho's Four Rivers Whitewater Recreation Lottery. This lottery structure shows promise as a foundation for economic experiments for preference revelation.

Keywords: lotteries, complex goods, hedonic pricing, choice under uncertainty.

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1 Introduction

Idaho's Four Rivers Lottery for Whitewater Rafting allows people to apply for a permit to raft one of Idaho's four most popular wilderness rivers. An applicant applies for one particular river and day among many possible river-day combinations. The choice among rivers and days is consequential because these river sections vary in their characteristics, and because water and weather characteristics differ substantially through the permit season. Some river/days are far more preferred than others, resulting in a distribution over the probability of winning a permit that ranges from less than 4% to nearly 100%.

This lottery structure provides applicants a choice among a set of simple gambles, each of which provides the possibility of winning one associated complex (multi-attribute) good. The result of this lottery is a set of equilibrium probabilities of winning, one for each good, determined by applicants' willingness to accept a lower chance of winning for a more preferable good. These equilibrium probabilities can be interpreted as relative prices that reflect the hedonic values of the goods' attributes. Because these prices are probabilities, an expected utility perspective on applicant choice allows the characterization of marginal rates of substitution among characteristics, marginal utilities, and complete von Neumann-Morgenstern (vNM) utility maps.

We develop a model of applicant choice that provides a foundation for mapping applicant preferences. We then apply the model to several year's worth of data from the Four Rivers Lottery. Our estimation strategy proceeds by (1) estimating applicant expectations over river characteristics for the upcoming river rafting season, (2) estimating application rates over the available river/date alternatives, and 3) using the structure of the model of application rates to characterize preference relations.

Evans, Vossler, and Flores (2009) highlight the use of lotteries to provide public goods, which has included goods such as publicly distributed flu vaccines, public school choice, hunting and rafting permits, public market space for vendors, artisans, and performers, public land in Georgia, and wait lists public housing. However, the structure of the Four Rivers Lottery is such that applicants' choice endogenously determines the probability of winning. This type of lottery is often used to distribute recreational permits and even seats in public school classrooms.

The structure of this lottery is at the intersection of several extensive literatures: decision-making under risk or uncertainty and expected utility theory, endogenous risk in decision-making, general equilibrium models, demand for multi-attribute goods, nonmarket valuation methods such as hedonic pricing and conjoint analysis. Though a great deal of literature has covered one or more of the characteristics of this lottery structure, direct examination of it is limited to a useful but methodologically narrow empirical literature focused on valuation methods. We explain below how our approach contributes to this literature.

The literature spawned by the expected utility framework of von Neumann and Morgenstern (1947) and predecessors has focused almost universally on decisions over gambles with exogenous probabilities associated with prospective outcomes. Indeed, the now standard treatment of Arrow-Debreu general equilibrium with uncertainty is based on exogenous probabilities over state-contingent goods (Arrow and Debreu, 1954; Debreu, 1959; Radner, 1968; Radner, 1970; Mas-Colell, Whinston, and Green, 1995). The endogenous distribution of win rates distinguishes the lottery structure we examine from much of this literature, and generates the preference revelation for lottery prospects.

Another extensive literature focuses on preferences over and demand for multi-attribute goods. Lancaster (1966) formalizes the relationship between the utility received from the activity of consuming complex goods and the attributes embodied in them, and Keeney and Raiffa (1976) develop an extensive examination of expected utility theory as applied to multi-attribute goods. Again, however, Keeney and Raiffa (1976) rely on fixed exogenous outcome probabilities throughout, and focus largely on individual, multi-outcome gambles.[1] In contrast, Rapoport et al. (2002) develops a model of multiple simple lotteries, and like the case we examine, it provides the foundation for Nash-equilibria probabilities of winning. However, they focus on simple goods, so their model, unlike our approach, does not facilitate examination of tradeoffs among attributes of complex goods.[2]

[1] Keeney and Raiffa (1976) cover a great many lottery and gamble structures, including several extensive examples at the end of the book. We found no examination of endogenous outcome probabilities in that book.

[2] In addition, most of the attention in Rapoport et al. (2002) is on fixed group sizes. They examine the case of distributions over group sizes, but do not examine the economic process of selection into an applicant pool.

Another literature focuses on endogenous risk due to self-protection against environmental risk (Shogren, 1991; Shogren and Crocker, 1991). This literature primarily focuses on single gambles with multiple (sometimes continuous) and unidimensional outcomes, and while closely related to our problem, we have not found a treatment in this literature of the lottery structure we examine. A related literature focuses on choice among complex goods as a function of environmental risks, and relates these risks to option value, other welfare measures, and hedonic prices (Freeman, 1993). For example, Bin, Kruse, and Landry (2008) examine the effect on housing prices and flood insurance premiums of flood risk. In this set of research, risk as probability of harm is exogenously associated with each potential outcome, and represents one attribute of the set of goods from which people choose. In the lottery structure we examine, the probability of any given outcome is itself dependent on the demand for the alternatives, which in turn are a function of participant preferences over the full set of available gambles.

While a general treatment of this lottery structure is at best sparse in the literature, a series of papers has developed a useful but methodologically narrow treatment of this lottery type to estimate demand and willingness to pay for lottery goods and their characteristics.[3] Nickerson (1990) uses data most similar to ours, and develops the basic foundations of aggregate demand for lottery applications; a foundation upon which we build. We then extend this analysis to make inferences about preferences over the attributes of the goods.[4]

[3] Evans, Vossler, and Flores (2009) and some recent related literature develop theoretical models of hybrid allocation systems of the form of sequential or joint auction/lottery systems. Evans, Vossler, and Flores (2009) focus on unidimensional homogeneous goods, and the interaction between auctions and a lottery. As such, while this is an interesting development, it is not very useful to pursue our objectives.

[4] See also precursors Akabua et al. (1999); Boxall (1995); Loomis (1982a); Loomis (1982b) relating to valuation of hunting using lottery data, and Loomis (1980) using rafting data for valuation. These papers are all travel cost models, with the latter ones using various forms of Random Utility models. Several papers focus on Public school systems that allocate classroom seats via lotteries. Parents apply for entry in their preferred schools, and thus affect the probability of winning by this choice. While the parents' application choice may reflect their values for academic standards, convenience in transportation, classroom type, and ethnic composition of a school, no research on public school lotteries assess demand for school characteristics as we do here, but instead look at differences in schooling outcomes as a function of competition in schools. For example, Cullen, Jacob, and Levitt (2006) show that winning such lotteries do not necessarily lead to improved test scores or attendance rates. Cullen, Jacob, and Levitt (2005) find that some of the value of attending a highly sought after school is captured in the increased graduation probability.

Three related papers, (Scrogin, Berrens, and Bohara, 2000; Scrogin and Berrens, 2003; Scrogin, 2005) (we refer to this group of papers as "SB") examine hunting demand New Mexico's big game lottery system. These papers have goals similar to ours, but they all differ from our approach in one crucial way. Contrasting these approaches with ours is useful to clarify our contribution and to motivate our approach. As described above, applicants choose among gambles based on *ex ante* perceptions of the full set of success probabilities (one for each gamble, each of which corresponds to one final complex good). In all three SB papers, estimates of *ex ante* success probabilities are based primarily on application rates from past lotteries and current lottery quotas. In Scrogin and Berrens (2003) and Scrogin (2005), these estimated success probabilities are then used as explanatory variables in a second stage random utility model for applicant choice over lottery alternatives. The second stage model provides the structure to estimate compensating surplus relating to changes in quality attributes.

Scrogin (2005, p. 200) recognizes correctly that application rates and success probabilities are dependent on the characteristics of lottery alternatives, and includes one attribute of the goods (harvest rates) in a regression to estimate success probabilities.[5]

As Scrogin (2005) rightly points out, past equilibrium application rates are determined by the characteristics of all available lottery goods (and non-lottery alternative goods). While SB use a second stage to characterize demand and willingness to pay for the attributes of lottery goods, we (like Nickerson, 1990) estimate application demand (and success rates) as a direct function of the attributes of the alternatives. We then utilize the application demand functions in conjunction with expected utility theory to infer preference relations directly.[6] In doing so, we extend the demand analysis of Nickerson (1990) and provide a more direct alternative to the estimation strategy applied in SB.

[5] Scrogin and Berrens (2003) mention in a footnote that they also estimated success probability models that included applicant characteristics, but no characteristics of the lottery alternatives. It is also not clear in Scrogin (2005) how or if the indirect effects of changes in quality through changes in success rates are incorporated into the welfare simulations.

[6] There is an extensive experimental/empirical literature suggesting that the expected utility hypothesis does not seem to be reflected in human decisions in some circumstances, and numerous theoretical models of behavior have therefore developed Starmer (2000). Nonetheless, in this paper (or at least in this draft) we rely on expected utility theory largely for tractability and clarity and because no alternative non-expected utility approach dominates, from our perspective, to achieve the objectives of this paper.

Additionally, while the approach taken by Scrogin and Berrens (2003) and Scrogin (2005) to characterize *ex ante* perceptions of win probabilities among applicants is reasonable when lottery characteristics do not change, it becomes problematic when lottery structure or goods change from one year to the next, because changes can affect equilibrium outcomes.[7] For example, if additional units of one of the lottery goods (corresponding to just one of the simple gambles), or if an improvement in a set of goods is made from year to the next, it may draw applicants away from the other lottery alternatives, making last year's distribution of win probabilities inaccurate predictors of this years' probabilities.

Given that public goods are often allocated in a non-market setting (i.e. waiting-line auction, lottery, need-based, or merit-based), understanding the value of these goods and their characteristics can be valuable to policymakers. Our approach to this lottery structure has several merits. It can be used to characterize preferences over complex goods from aggregate application data if data on individual applicants are unavailable. Estimation and inference is relatively straightforward compared to the binary response models (generally motivated by a random utility framework) often used for valuation when data on individual applicant decisions is available (Scrogin and Berrens, 2003; Scrogin, 2005, e.g.), and in some cases reliance on aggregate data may provide better estimates (Kling, 1988; Hellerstein, 1995). Our approach can also be used in place of the first-stage estimation strategy used in Scrogin and Berrens (2003) and Scrogin (2005), thereby using available application demand information more fully, and avoiding the pitfalls of relying directly on past win rates as a predictor of future win rates if lottery characteristics change.

We have not seen this specific lottery structure used in the experimental economics literature, but we suspect that it might provide a practical and convenient environment for preference and valuation analysis. It allows researchers to offer a set of real gambles over complex goods in a relatively simple way, and our approach to preference elicitation is relatively direct and analytically straightforward.

Section 2 develops a model of the decision calculus over a set of simple lot-

[7] Using past win rates to forecast future win rates is a concise and potentially low-cost way for applicants to forecast future probabilities, and therefore this approach might in fact reflect actual forecasting behavior when past win rates are available to applicants. See for example Conlisk (1988) and Pingle (1995).

teries over complex goods, and characterizes preference relations among their attributes. Section 3 describes the empirical application and data, Section 4 describes our estimation strategy and methods, Section 5 discusses results, and Section 6 concludes.

2 Theory

A lottery authority announces a set of goods of varying quality as lottery prizes. Each applicant is allowed to specify only one of these goods in hopes of winning it. The most desirable goods will tend to draw many applications, but more applications leads to a lower probability of winning. Applicants therefore choose among the goods by trading off the quality and the probability of winning a good. The following model formally characterizes this environment and tradeoff, and the connection between equilibrium outcomes and applicant preferences.

Define a multi-attribute good $\mathbf{x} \in \mathbf{X}$, where $\mathbf{X} = \begin{bmatrix}\mathbf{x}_0 & \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_N\end{bmatrix}'$ includes lottery goods $\mathbf{x}_{i>0}$ as well as $\mathbf{x}_0$, which is a reservation good consumed by non-applicants and non-winning applicants. Each complex good is defined by J characteristics: $\mathbf{x}_i = \begin{bmatrix}x_i^1 & x_i^2 & \cdots & x_i^J\end{bmatrix}'$.

A winning lottery application for good $\mathbf{x}_{i\neq 0}$ provides a permit, which in turn provides the right to consume a corresponding specific lottery good. We assume that each individual can apply for only one permit. The applicant faces a set of probabilities $\boldsymbol{\Pi} = \begin{bmatrix}\pi_0 & \pi_1 & \cdots & \pi_N\end{bmatrix}'$, where each element $\boldsymbol{\pi}_i$ is the probability of winning good $\mathbf{x}_i$. The probability of winning is the ratio of the number of units of the good available and the number of applicants for the good:

$$\pi_i = Q_i/A_i \qquad \forall\, i \in (0, 1, \cdots N), \tag{1}$$

where $\mathbf{A} = \begin{bmatrix}A_0 & A_1 & \cdots & A_N\end{bmatrix}'$ is the vector of application rates for each good $i \in (1, \cdots, N)$, and $\mathbf{Q} = \begin{bmatrix}Q_0 & Q_1 & \cdots & Q_N\end{bmatrix}'$ is the set of available permits for each corresponding good $\mathbf{x}_i$. $\mathbf{A}$ is determined by an application demand function, which is derived in the next section. A_0 can be thought of as a residual non-applicant set, and Q_0 is unrestricted by the lottery authority and is assumed available for consumption for non-applicants and for non-winners. Thus, a potential applicant who chooses not to apply is counted in A_0 and receives good $\mathbf{x}_0$ with certainty: $\boldsymbol{\pi}_0 = Q_0/A_0 = 1$.

It is worth mentioning for context that each observation in our empirical examination includes the number of applications for one complex good among $i > 0 \in N$, so we have a dataset with N observations (one for each river/day) on the number of applications $A_{i>0}$, the number of available permits $Q_{i>0}$, and various river/day characteristics $\mathbf{x}_{i>0}$.

The number of permits allotted for $\mathbf{x}$ will affect the probability of winning a permit, but it might also directly affect the utility of consuming $\mathbf{x}$. For example, if a good is rivalrous in final consumption, then consumption by one permit winner can affect the utility of another. In the case of permits for hunting a scarce game animal for example, a successful hunt reduces the chances of success by another hunter. For river use and other recreational activities, one permit winner may compete for limited campsites, or cause congestion at the launch site or on the river (Rollins, Dumitras, and Castledine, 2008). One would expect direct negative utility impacts in these cases. Conversely, the permitted goods may be club goods for which "more is merrier," such that increases over a range of Q increases utility for each permit winner. We therefore allow Q_i to enter utility directly. For notational convenience, define Q the J^{th} quality characteristic, so that $x_i^J \equiv Q_i$ for a lottery good $\mathbf{x}_i$.[8]

As is the case in our empirical application, individuals must apply in the face of uncertainty about the actual state of the good or environment at consumption time. We therefore define $\mathbf{X}$ as a set of expectations of the goods' characteristics evaluated at the time of application. These expectations may relate not only to the physical characteristics of the goods themselves, but the environment, timing of use, and in principle even personal characteristics of the consumers. Furthermore, while the true probabilities of winning are the ratio of permits per application, the actual *ex post* number of applications is not necessarily known at application time either under imperfect information and variation in applicant preferences and opportunity costs. Applicants must therefore make their decisions based on predicted application rates and therefore predicted win probabilities. We will denote expected application rates and probabilities for a good i as $\hat{A}_i \equiv E[A_i]$ and $\hat{\pi}_i \equiv E[\pi_i] = Q_i/\hat{A}_i$ to facilitate model development; however, we will not augment the notation $\mathbf{x}$ to represent expected characteristics because it is not worth the extra clutter.

[8] For good $\mathbf{x}_0$, the number of permits x_0^J can be thought of as sufficient to assure acquisition of $\mathbf{x}_0$ if one of the lottery goods is not won or an application is not submitted.

The application process amounts to choosing among $N+1$ simple gambles g_i, $i \in (0, 1, \cdots N)$, each corresponding to the possible consumption of one of $N+1$ complex goods $\mathbf{x}$ (including the reservation good):

$$g_i \equiv \{\hat{\pi}_i \circ \mathbf{x}_i, (1-\hat{\pi}_i) \circ \mathbf{x}_0 | \, i \in (0, 1, \cdots, N)\}, \tag{2}$$

where "$\circ$" implies correspondence between probabilities and outcomes.

2.1 The applicant's problem

Define the utility function an individual $l \in L$ receives from the consumption of a good $\mathbf{x}$ is $U_l(\mathbf{x}) = \{U_l : \mathbb{R}^J \to \mathbb{R}\}$ maps $\mathbf{x}$ into utility at application time. Reservation utility is received from not applying (which corresponds to gamble g_0 with $\pi_0 = \hat{pi}_0 = 1$), and is also received if an applicant does not win a permit. For each applicant, $\mathbf{x}_0 \preceq \mathbf{x}_i$ and $U_l(\mathbf{x}_0) \leq U_l(\mathbf{x}_i)$. Choosing to apply for any alternative i effectively sets sets $\pi_j = \hat{\pi}_j = 0$, $j \neq i, j > 0$ for that individual. Choosing not to apply effectively sets $\pi_j = \hat{\pi}_j = 0 \; \forall \; j > 0$ for that individual (though $\hat{\pi}_j$ remains positive for any individual applying for that good). To differentiate among individuals, define personal utility from consumption of good i as[9]

$$U_l(\mathbf{x}_i) = U(\mathbf{x}_i) e^{\varepsilon_{li}}, \tag{3}$$

where $\varepsilon_{li} \sim_{iid} (0, \sigma^2)$ is an element of utility uncorrelated with $\mathbf{X}$ and unobservable in the data. The distribution of ε_{li} implies that $E[e^{\varepsilon_{li}}] = 1$.[10] We assume that at application time each individual knows their own utility $U_l(\mathbf{x}_i)$ and the distribution of ε_{li}, but the actual values of ε_{li} for other individuals are unknown to l. This information structure provides a convenient representative agent concept defined by $E_l[U_l(\mathbf{x}_i)] = U(\mathbf{x}_i)$, which each agent can deduce $U(\mathbf{x}_i)$ at application time given available information. The variation in utility provides a direct link to the empirical model developed in Section 4.

Individual's ls application for good $\mathbf{x}_i$ (that is, the act of choosing gamble g_i)

[9] A comment on notation. Expected utility could be defined as $E[U_l(\mathbf{x}_i)] = \pi_i U_l(\mathbf{x}_i)$, where π_i is known with certainty. Then one could denote the expectation of expected utility as $E[E[U_l(\mathbf{x}_i)]]$. We shorten expected utility as $EU_l(\mathbf{x}_i) \equiv E[U_l(\mathbf{x}_i)]$ and $\widehat{EU}(\mathbf{x}_i) \equiv E[E[U_l(\mathbf{x}_i)]]$ to notationally distinguish between the conventional definition of expected utility (EU_l) and the case in which expected utility is evaluated using an expected value for an element of EU (in this case $\hat{\pi}$).

[10] Note that the distribution of ε_{li} is assumed constant over i for all $i \in (N+1)$.

provides expected utility

$$\widehat{EU}_l(\mathbf{x}_i) = \hat{\pi}_i U_l(\mathbf{x}_i) = \hat{\pi}_i U(\mathbf{x}_i) e^{\varepsilon_{li}}. \tag{4}$$

Assuming that the Expected Utility Theorem holds (Mas-Colell, Whinston, and Green, 1995, p. 175 +), an expected-utility maximizing agent l chooses the gamble g_i for $i \in (0, 1 \cdots N)$ that maximizes personal expected utility by choosing one simple gamble corresponding to one of the $N + 1$ alternatives $\mathbf{X}$. The choice among the alternatives depends on the characteristics of all alternatives:

$$\widehat{EU}_l(\mathbf{X}) = \sup\{\hat{\pi}_i U_l(\mathbf{x}_i) : i = 0, 1 \cdots N\} \tag{5}$$

At application time, each agent l knows the full set of permit allotments $\mathbf{Q}$ (and therefore $\mathbf{q}$); $U_l(\mathbf{X})$ (and $u_l(\mathbf{X})$); and holds estimates of all expected win rates $\hat{\boldsymbol{\Pi}}$. Because application decisions are based in part of win rates and win rates are determined by application decisions, win rates and application rates $\hat{\mathbf{A}}$ are endogenously determined by the expected choices of all L agents based on known $\mathbf{Q}$, $\mathbf{X}$. We now describe an equilibrium for this lottery defined in terms of equilibrium application rates, which in turn defines equilibrium win rates for each available good i.

2.2 Equilibrium application demand

We need to determine $\hat{\mathbf{A}}$. The equilibrium choices of the representative agent provide all the necessary information. To see why, substitute $\frac{Q_i}{\hat{A}_i}$ for $\hat{\pi}_i$ in Equation 5 and take the expectation of $\widehat{EU}_l(\mathbf{x}_i)$ over all agents $l \in L$:

$$\begin{aligned} E_L[\widehat{EU}_l(\mathbf{x}_i)] &= \frac{Q_i}{\hat{A}_i} U(\mathbf{x}_i) \\ &= \widehat{EU}(\mathbf{x}_i), \end{aligned} \tag{6}$$

which is the expected utility for gamble i for the representative agent. Rearranging provides

$$\hat{A}_i = \frac{Q_i U(\mathbf{x}_i)}{EU(\mathbf{x}_i)}, \tag{7}$$

which holds as an expected application rate for any good i by all agents $l \in L$. All that remains to define endogenous $\hat{A}_i$ is $EU(\mathbf{x}_i)$ for the representative agent.

To derive equilibrium $EU(\mathbf{x}_i)$, consider a tâtonnement process of sequential application of a sufficiently large set of replicant representative agents $r = 1 \cdots \mathbb{R}_+$, and a Walrasian auctioneer who announces the outcome of each sequential application decision. For exposition, rank available goods such that $U(\mathbf{x}_1) > U(\mathbf{x}_2) > \cdots > U(\mathbf{x}_N) > U(\mathbf{x}_0)$, where again $\mathbf{x}_0$ is the reservation good received if the agent does not apply.

Representative agent 1 applies first and chooses good $\mathbf{x}_i$ (the outcome of which is announced). Agent $r = 2$ applies for either good 1 or 2 depending on which provides higher expected utility given agent 1s decision. In general, applicant r applies for good i if

$$\widehat{EU}_r(\mathbf{x}_i) \geq \widehat{EU}_r(\mathbf{x}_{i+1}) \quad i \in (0, 1, \cdots N), \tag{8}$$

The number of applications for any good $\mathbf{x}_i$ at any stage r of the application process is simply the sum of applicants for that good to that point:

$$A_{ir} = \sum_{k=1}^{r} I_{ik\succ}, \tag{9}$$

where

$$I_{ik\succ} = \begin{cases} 1 & \text{if } \widehat{EU}_k(\mathbf{x}_i) > \widehat{EU}_k(\mathbf{x}_j) \; \forall k \in (0, 1, \cdots r) \text{ and } j \neq i \in N+1, \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Equation 8 can be reorganized as

$$A_{ir} \leq A_{(i+1)r} \frac{Q_i}{Q_{i+1}} \frac{U_r(\mathbf{x}_i)}{U_r(\mathbf{x}_{i+1})}. \tag{11}$$

As long as this inequality holds, each successive applicant r adds $I_{ik\succeq} = 1$ to A_{ir} (on the left) while the right side remains constant. At some point the inequality in Equation 7 switches, which implies that $\mathbf{x}_{i+1}$ provides a higher expected utility and will be chosen over $\mathbf{x}_i$. Each successive applicant will chose among alternatives, driving down expected utility and choosing less and less desirable alternatives because the probability of winning the more desirable alternatives

is shrinking. For example, $\mathbf{x}_1$ will be chosen by all until $EU_r(\mathbf{x}_1) \leq EU_r(\mathbf{x}_2)$. Then, $\mathbf{x}_1$ and $\mathbf{x}_2$ will be chosen alternately until $EU_r(\mathbf{x}_1) \approxeq EU_r(\mathbf{x}_2) \leq EU_r(\mathbf{x}_3)$, at which point $\mathbf{x}_3$ will begin to be chosen. At some point a potential applicant finds the expected utility of the least preferred lottery good to be less than the value of the reservation good:

$$EU_{R+1}(\mathbf{x}_N) < U(\mathbf{x}_0). \tag{12}$$

This representative agent will not apply. The subscript $R = \sum_{i=1}^{N} A_i$ in equation 12 is the total number of lottery applicants. In the limit (or with continuous A and Q), the equilibrium application rates provide a fixed point such that

$$\widehat{EU}(\mathbf{x}_i) = \widehat{EU}(\mathbf{x}_j) = \cdots = U(\mathbf{x}_0). \tag{13}$$

Figure 1 shows how the expected value of two alternatives proceeds with additional applicants. The process begins for some r such that $\widehat{EU}(\mathbf{x}_i) > \widehat{EU}(\mathbf{x}_{i+1})$. Applications are added for $\mathbf{x}_i$ until expected utility for the two alternatives are equal. Applications to each (sequentially) continue until the expected utility for each equal reservation utility.[11]

[Figure 1 about here.]

Substituting U_0 from Equation 13 into equation 7 for $EU(\mathbf{x}_i)$ provides

$$\hat{A}(\mathbf{x}_i) = Q_i \frac{U(\mathbf{x}_i)}{U(\mathbf{x}_0)}. \tag{14}$$

The full set of application demands are

$$\hat{\mathbf{A}}(\mathbf{X}) = \begin{bmatrix} \hat{A}(\mathbf{x}_1) & \hat{A}(\mathbf{x}_2) & \cdots & \hat{A}(\mathbf{x}_N) \end{bmatrix}'. \tag{15}$$

While expected application rates are now established, actual application rates depend not only on expected application rates (which are equivalent to the application rates of a representative agent), but also on the random disturbances ε_{li}. We use the statistical distribution of these application rates as

[11] Equation 13 implies that $\pi(\mathbf{x}_i)/\pi(\mathbf{x}_j) = U(\mathbf{x}_j)/U(\mathbf{x}_i)$, so the equilibrium probabilities make up the relative prices associated with two goods, reminiscent of the treatment by Bruno De Finetti on subjective probabilities as conceptually identical to relative prices vis-à-vis choices among lotteries (Nau, 2001), but in this case, these prices are endogenous as in a general equilibrium setting.

the foundation of the empirical model developed in Section 4.2. Before that however, we use application demands $\hat{\mathbf{A}}(\mathbf{X})$ in Equation 14 to recover marginal utility, hedonic prices and marginal rates of substitution among the attributes in $\mathbf{X}$.

2.3 Marginal utility and substitution

Equation 13 implies that for the representative agent, expected utility for any $\mathbf{x} \in \mathbf{X}$ equals $U(\mathbf{x}_0)$. Equations 6 and 13 imply that for any $\mathbf{x} \in \mathbf{X}$,

$$U(\mathbf{x}_i) = U(\mathbf{x}_0)\frac{\hat{A}(\mathbf{x}_i)}{Q_i}. \tag{16}$$

To examine utility from and substitution among attributes of the complex goods $\mathbf{x}$, we recast the model developed in the previous section. Assume continuous functions for application rates and permit numbers, and redefine $\mathbf{x}$ to exclude $\mathbf{Q}$ to highlight its importance in the implied tradeoffs.[12] Above we represented the number of permits (known at application time) for lottery good i as Q_i. Now let $\mathbf{Q}(\mathbf{x})$ denote the correspondence between $\mathbf{Q}$ and $\mathbf{x}$. This notation clarifies the fact that an applicant jointly chooses $\mathbf{x}$ and $\mathbf{Q}$, and that $\mathbf{Q}$ can vary over $\mathbf{x}$ according to the decisions of the lottery authority. Utility associated with the reservation good, $U(\mathbf{x}_0)$, acts as a reference point in this economic environment and $\mathbf{x}_0$ is unobserved. We therefore treat it as a constant and denote it $U_0 \equiv U(\mathbf{x}_0)$. For additional clarity given that we are assuming continuity in all variables, we suppress the index i. After these changes in notation, the continuous-space version of 4 for the representative agent (with $\varepsilon_{li} = 0$ for all i) is

$$\begin{aligned} U_0 &= \pi(\mathbf{x}, Q(\mathbf{x}))U(\mathbf{x}, Q(\mathbf{x})) \\ &= \frac{Q(\mathbf{x})}{\mathbf{A}(\mathbf{x}, Q(\mathbf{x}))}U(\mathbf{x}, Q(\mathbf{x})). \end{aligned} \tag{17}$$

It is convenient to proceed by working in logarithms. Taking logs and rearranging provides the gain in log expected (indirect) utility from choosing $\mathbf{x}$:

$$u(\mathbf{x}, q(\mathbf{x})) - u_0 = \hat{a}(\mathbf{x}, q(\mathbf{x})) - q(\mathbf{x}), \tag{18}$$

[12] This means that there are $J - 1$ characteristics in $\mathbf{x}$, but this is inconsequential for the purposes of this section.

where $u = \ln(U)$, $\hat{a} = \ln(\hat{A})$, and $q = \ln(Q)$. By choosing $\mathbf{x}$ the applicant is indirectly choosing both utility and the probability of winning, so the risk of winning nothing is endogenously determined. There are $J-1$ first-order necessary conditions for maximization; one for each attribute $\mathbf{x}^j \in \mathbf{x}$. These first order conditions can be written as[13]

$$\left(\frac{\partial u(\mathbf{x})}{\partial x^j} + \frac{\partial u(\mathbf{x})}{\partial q}\frac{\partial q(\mathbf{x})}{\partial x^j}\right) = \frac{\partial \hat{a}(\mathbf{x})}{\partial x^j} + \left(\frac{\partial \hat{a}(\mathbf{x})}{\partial q} - 1\right)\frac{\partial q(\mathbf{x})}{\partial x^j}, \qquad \forall j \in J-1; \quad (19)$$

where $u(\mathbf{x}) \equiv u(\mathbf{x}, q(\mathbf{x}))$ and $\hat{a}(\mathbf{x}) \equiv \hat{a}(\mathbf{x}, q(\mathbf{x}))$ are used to reduce clutter. Holding q constant so $\frac{\partial q(\mathbf{x})}{\partial x^j} = 0$, the indirect effects of $\mathbf{x}$ through q on utility drop out, implying that the marginal log utility of a characteristic x^j holding q constant is

$$\frac{\partial u(\mathbf{x})}{\partial x^j} = \frac{\partial \hat{a}(\mathbf{x})}{\partial x^j}. \quad (20)$$

Given Equation 20, it follows that the (direct) marginal log utility of q is[14]

$$\frac{\partial u(\mathbf{x})}{\partial q}\frac{\partial q(\mathbf{x})}{\partial x^j} = \left(\frac{\partial \hat{a}(\mathbf{x})}{\partial q} - 1\right)\frac{\partial q(\mathbf{x})}{\partial x^j}, \text{ or}$$
$$\frac{\partial u(\mathbf{x})}{\partial q} = \frac{\partial \hat{a}(\mathbf{x})}{\partial q} - 1. \quad (21)$$

The terms $\frac{\partial \hat{a}(\mathbf{x})}{\partial x^j}$ and $\frac{\partial \hat{a}(\mathbf{x})}{\partial q}$ are estimable with a regression of $\hat{\mathbf{A}}(\mathbf{x}, \mathbf{Q})$ (or a log transformation of it). The term $\frac{\partial q(\mathbf{x})}{\partial x^j}$ is observable as the lottery authority's policy function $\mathbf{Q}(\mathbf{x})$.[15]

Given Equations 20 and 21, the Marginal Rate of Substitution (MRS) between two characteristics j and k, and between characteristics j and q, can be

[13] Maximizing the logarithm of a function is equivalent to maximizing the original function.

[14] It may seem superfluous to keep track of a logarithmic utility transformation, but because $U(\mathbf{x}, \mathbf{Q})$ is equivalent to a von Neumann-Morgenstern Utility function, which is unique up to an affine transformation, not a non-affine transformation such as a natural logarithm even though it is monotonic.

[15] While the policy function may be endogenous in the long run, it is predetermined and therefore exogenous from the perspective of an applicant in any given year.

written as

$$\text{MRS}^{jk} \equiv \frac{U(\mathbf{x})/\partial x^j}{U(\mathbf{x})/\partial x^k} = \frac{\partial \hat{a}(\mathbf{x})/\partial x^j}{\partial \hat{a}(\mathbf{x})/\partial x^k}, \qquad j \neq k \neq q, \tag{22}$$

$$\text{MRS}^{jq} \equiv \frac{U(\mathbf{x})/\partial x^j}{U(\mathbf{x})/\partial q} = \frac{\partial \hat{a}(\mathbf{x})/\partial x^j}{\partial \hat{a}(\mathbf{x})/\partial q - 1}, \qquad j \neq q. \tag{23}$$

Multiplying the right hand side of Equation 22 by q/q provides $\text{MRS}^{jk} = \frac{\partial \hat{\pi}(\mathbf{x})/\partial x^j}{\partial \hat{\pi}(\mathbf{x})/\partial x^k}$, implying that in this case, the ratio of hedonic prices for attributes j and k represent the equilibrium relative price of attributes j and k. As with Equations 20 and 21, the elements of Equations 22 and 23 are estimable.[16]

2.4 vNM utility index

An expected utility maximizer chooses alternatives under a lottery according to Equation 5. Given the structure of this choice problem and the assumption of expected utility maximization, the utility function $U(\mathbf{x})$ can be interpreted as a von Neumann-Morgenstern (vNM) utility index.

The vNM utility index $U(\mathbf{x})$ is a unique representation of preferences up to an affine transformation, and it can be normalized such that the utility provided by most preferred alternative is $U(\mathbf{x}_1) = 1$ and the least preferred alternative, which we take to be no application, is be normalized to $U(\mathbf{x}_0) = 0$ (Laffont, 1990). Based on Equation 13, $\pi(\mathbf{x}_1) \cdot 1 = \pi(\mathbf{x}_i)U(\mathbf{x}_i)$, and the vNM utility index value for the i^{th} ranked alternative is

$$U(\mathbf{x}_i) = \frac{\pi(\mathbf{x}_1)}{\pi(\mathbf{x}_i)} = \frac{A(\mathbf{x}_i)}{A(\mathbf{x}_1)} \frac{Q_1}{Q_i}, \tag{24}$$

which is estimable with an econometric model of application demand, and therefore mappable over $\mathbf{X}$.[17]

[16] It is straightforward to show that these marginal rates of substitution (and indeed their associated indifference curves) are invariant to the log transformation, so even though we derive MRS based on marginal log-utility $\ln(U) = u$, the shape of the indifference curves apply to U.

[17] To see that this structure is consistent with vNM, note first that for $i = 1$, $U(\mathbf{x}_1) = \frac{\pi(\mathbf{x}_1)}{\pi(\mathbf{x}_1)} = 1$. To see that it is consistent with our model as well, note that Equation 17 implies that $U(\mathbf{x}_i) = \frac{U_0}{\pi(\mathbf{x}_i)}$, so U_0 in $U(\mathbf{x}_i)/U(\mathbf{x}_1)$ drops out leaving Equation 24.

3 Empirical Application and Data

In this section, we describe our empirical application in more detail, and describe and summarize the data used in the empirical model.

3.1 Idaho's Four Rivers Lottery

The U.S. Forest Service (USFS) implements the Four Rivers Lottery for whitewater rafting permits for portions of the Selway River, Snake River, Main Fork of the Salmon River, and Middle Fork of the Salmon ("Middle Fork") (United States Forest Service, 2012). Figure 2 provides a map of river locations in Idaho. The USFS ostensibly limits use during the summer to reduce congestion on limited camping beaches and the river corridor in general, and to reduce pressure on wildlife habitat and other management objectives (USFS, 2008b).

[Figure 2 about here.]

For the period covered by our data, an individual can submit one application to the lottery per year during the application period (usually December through January) for the upcoming summer rafting season. Private (noncommercial) lottery applicants pay a $6 non-refundable application fee, and permit winners (and their groups) pay $4 per boater for launching (United States Forest Service, 2012).

Rafting permits won in the lottery are nontransferable, and USFS personnel verify that the permit winner accompanies each rafting group at river launch sites USFS (2008b). If permit transfer were allowed, a secondary market for these permits would likely exist because the market value of these permits would be quite high. Almost ten thousand people applied for a noncommercial permit to float the Middle Fork in 2010, and under four percent of applicants won permits USFS (2008a). Four commercial launches are allowed per day, and commercial guide companies charge between $1,500 and $2,000 per person for a six-day raft trip. This fee covers food and services provided by commercial outfits, but probably embodies substantial scarcity rents as well.

3.2 Data

We begin by describing the data used to estimate *ex ante* applicant expectations of river and weather characteristics, and then describe the data used in the

application demand model. Descriptions and summary statistics for all data used in this analysis are provided in Tables 1 and 2 and are discussed in more detail below.

[Table 1 about here.]

[Table 2 about here.]

3.2.1 River and weather data

The characteristics of a river trip are shaped by river and weather conditions, and applicants must apply for specific river/dates before they know the actual river and weather conditions on launch day. We hypothesize that applicants form expectations of likely river and weather characteristics for each river/day, and use these expectations to help them choose among lottery dates and rivers.

At application time, applicants have information on the past history of daily river flows, as well as a general river flow forecasts for the upcoming season based on known snow pack in the the river drainages. Daily data on river flows are available online. For the first stage regressions to generate expected flows, the regressand is the mean daily discharge (United States Geological Survey, 2012) each day and each river sections (1 January 1993 to 29 January 2012). The annual river flow forecasts are available from U.S. Natural Resource Conservation Service (1993-2011.). As a practical matter, river conditions and difficulty tend to be reflected better by the water "level" than by volume, and water level is more closely related to the natural log of volume due to the typical bowl-shape of a riverbed. For this reason we use the natural log of discharge in our analysis, defined as $w = \ln(\text{feet}^3/\text{second})$.

Figure 3 shows these data, as well as predictions discussed in Section 5. The Snake River through Hell's Canyon is the only section of these four that has dams upriver of the permit section, and water levels are managed throughout the season by the Idaho Power Company, in part for hydroelectric power generation.

[Figure 3 about here.]

Temperatures, rainfall, and cloud cover are among the weather factors that affect a river trip, but they are all quite highly correlated in this region during

the permit seasons. After considering available data on these variables with both correlation analysis and preliminary regressions, we use daily maximum temperature as a proxy for all weather characteristics. Figure 4 shows temperature data and predictions discussed in Section 5.

[Figure 4 about here.]

3.2.2 Lottery data

Four Rivers Lottery data were provided in various forms by the U.S. Forest Service for the years 2000 through 2011, for the four sections of river covered by the lottery: the Main Salmon (MS), the Middle Fork of the Salmon (MF), the Selway river, (SE), and the Snake river (SN) (United States Forest Service, 2011). The application process changed in 2010. From then on, an individual can submit one application per river, each with their top four ranked dates. Due to the complications with this change in sampling, we do not utilize the data for 2010 onward, but we instead limit our analysis to the years 2000-2009.

The application form requires the applicant to rank the top four preferred river and date combinations. For any given river/day, a name is drawn from the pool of names for which that river/day was listed as their first choice. The drawing moves to the pool of applicants who list that day as a second choice only if the number of permits exceeds the number of first choice applicants for that day. There are fewer permits than applicants in the first-choice pool for almost all river/days, so the second, third, and fourth choice pool provide very low probabilities of winning. While accounting for these additional rankings would provide additional information (and therefore probably would increase estimation efficiency), we focus on first choices only. [18]

Figures 5 and 6 show the number of (first-choice) applicants per river/day and the number of applicants per permit, and the probability of winning, respectively.

[18] We have developed a theoretical model of a 4-choice lottery such as this (available from the authors). The model suggests a multivariate regression framework with one regression for each choice (e.g. Seemingly Unrelated Regression). However, rather than pursuing this more complicated approach in this present draft we estimate a single equation model without loss of consistency. However, it may be worthwhile to investigate the application of ranked choices to more general non-expected utility models such as rank-dependent expected utility theory. (Quiggin, 1982; Quiggin, 1991).

[Figure 5 about here.]

The Selway is allocated only one private launch permit per day, while the others generally allow more: 3-4 on the Middle Fork, and even more on the other two rivers. While the Selway has relatively few applications per day, the number of applications per permit is highest for the Selway and the Middle Fork. Indeed, these two wilderness river sections are internationally known for the quality of the wilderness experience as well as their length ($\tilde{5}$0 miles of wilderness river for the Selway and 100 miles for the Middle Fork).

The number of applicants per permit is the inverse of the probability of winning, and Figure 6(a) shows that the Middle Fork and the Selway tend to be the most preferred. This means that people are willing to accept a much lower probability of winning for a (relatively small) chance of running these rivers, especially around early July, when river flows are at an intermediate level and the weather is generally sunny and warm.

[Figure 6 about here.]

A visual comparison of the application data in Figure 6(b) to the flow and temperature data in Figures 3 and 4 suggest a tradeoff between temperature, river flow, and the probability of winning.

Early season is characterized by high water and cold air temperatures. High spring water is exciting: big rapids, big waves and fast travel. However, it also tends to be more difficult and dangerous. Cold spring temperatures are uncomfortable, and the combination of the two creates more dangerous conditions because both the likelihood and the consequences of error are usually more severe. Hypothermia can be a major concern in the event of flipped rafts or extended swims. Spring rain or snow may be a nuisance especially in camp, and overcast skies more common in the spring reduce sun radiation.

Late in the season, water levels are so low that navigation through rocks on the Selway and Middle Fork become difficult (rafts may need to be drug across rocks in places with very low water). Waves are smaller and fewer, and Furthermore, completing these river section takes longer (on the order of days longer) compared to early season because the water is moving much more slowly, imposing a higher opportunity cost of time.

As can be inferred by a comparison of Figures 3,4, and 6(b) mid-season river trips are generally preferred by most prospective applicants. Water levels are

high enough to provide more waves than late season, but flows are slower (but not too slow), making river navigation relatively easy, and safer. Air temperatures are also higher than during the early season, and days are more likely to be sunny.

The next section describes the estimation methods used to predict river and weather characteristics, and for modeling the application choice among river/date combinations.

4 Estimation methods

Estimation proceeds in two steps. River flow characteristics and weather predictions for each river/day are generated with an initial round of regressions. These river and weather predictions are then used as explanatory variables in a second stage regression model of application choices.

4.1 First stage: Water and weather expectations

We hypothesize that applicants form expectations of river flow for each river/day based on the past history of daily river flows as well as the December river flow forecasts for the each upcoming season, which is available at application time. Expectations about weather (as proxied by maximum daily temperature), are also based on past weather history for each river/day.

To parsimoniously account for the cyclical nature of river flows and temperature over the course of the year and the permit season in particular, we apply trigonometric regression (Eubanks and Speckman, 1990), also known as Fourier flexible functional forms.[19]

Let y_i represent one observation (river/day) i, for either daily water level w or maximum daily temperature t. The general form for the trigonometric

[19]Trigonometric regression and Fourier decomposition has been used for a wide variety of applications. It is used widely for modeling cyclical and directional relationships (Mardia and Jupp, 2000; Gebert, Calkin, and Yoder, 2007), for time-series analysis (Hamilton, 1994; Greene, 2011), and in the business and economics literature estimating flexible cost functions (Chalfant and Gallant, 1985; Skolrud and Shumway, 2012), and in banking literature (Das and Das, 2007).

regressions is

$$y_i = \alpha_0 + \boldsymbol{\alpha}_z' \mathbf{z}_i + \sum_{\phi \in \Phi} \left[\alpha_\phi^s \sin\left(2\pi \frac{d_i \phi}{365}\right) + \alpha_\phi^c \cos\left(2\pi \frac{d_i \phi}{365}\right) \right] + v_{yi}, \qquad i = 1 \cdots M, \tag{25}$$

where $\mathbf{z}_i$ are explanatory variables in addition to the trigonometric functions, and α_ϕ^s, α_ϕ^c, $\boldsymbol{\alpha}_z$ are associated parameters, $\mathbf{v}_i$ is a random disturbance and M is the number of available observations. The choice of $\phi \in \Phi$ determines the frequency (or the inverse, the wavelength) of a given sine/cosine pair. In particular, a sine/cosine pair has frequency ϕ per year given this specification such that there are ϕ complete sine/cosine waves per year.[20]

The set of $\phi \in \Phi$ frequencies in Equation 25 could be chosen on an entirely empirical basis with the use of test statistics such as χ^2-tests of joint significance of sine-cosine pairs, or with the use of information criteria (e.g. Akaike or Schwartz) or risk criteria (e.g. Generalized Cross-Validation (GCV) or the Unbiased Risk criterion (Eubanks and Speckman, 1990). In our case, however, a question arises about the domain of frequencies that are realistically used by lottery applicants in making predictions about river flows and temperature over the course of a permit season. Including large values of ϕ implies that applicants use high frequency fluctuations in water discharge from past years to predict future discharge. It seems intuitive to think that very high frequency cycles that may show up in a regression on a 10-year sample of flows are discounted for ex ante predictions of future seasons. We therefore limit our regressions to include a set of lower frequency functions ($\phi \leq 10$). However, preliminary regressions show substantial robustness to changes in the set Φ on the outcomes of the second stage application demand model.

4.2 Second stage: Application rates

Section 2.1 introduced heterogeneity in agents, but until now we have examined only the outcomes for a representative agent. Now we utilize the heterogeneity as a foundation for an empirical model to characterize preferences. Based on

[20] This general structure applies to both $\mathbf{y} \in (w, t)$, but specifics (e.g. parameter estimates, observation numbers, specification of Φ, and the other elements differ across the w and t regressions.

Equations 6 and 13, the expected utility for any agent $l \in L$ is

$$\widehat{EU}_l(\mathbf{x}_i) = U_0 e^{\varepsilon_{li}}. \tag{26}$$

Thus, while expected application rates and therefore expected success probabilities are deterministically defined, expected utility varies among heterogeneous agents for any good $\mathbf{x}$, and actual application rates (as opposed to expected application rates) have a random component.

Using Equation 26, the probability that agent l chooses $\mathbf{x}_i$ over $\mathbf{x}_j$ is

$$\begin{aligned} \text{Prob}[EU_l(\mathbf{x}_i) > EU_l(\mathbf{x}_j)] &= \text{Prob}[U_0 e^{\varepsilon_{li}} > U_0 e^{\varepsilon_{lj}}] && (27) \\ &= \text{Prob}[\varepsilon_{li} > \varepsilon_{lj}]. && (28) \end{aligned}$$

Recalling Equation 9, A_i is the sum of all applications for whom $I_{ik\succ} = 1$:

$$A_i = \sum_{l=1}^{L} I_{li\succ}, \tag{29}$$

These application counts correspond to *ex post* application outcomes and, to restate the obvious, are count data. Recasting $\hat{A}(\mathbf{X})$ in terms of the probability of agent l choosing i over an other alternative provides

$$\hat{A}(\mathbf{x}_i) = E[A_i|\mathbf{X}] = \sum_{l=1}^{L} I_{li\succ} \cdot \text{Prob}[I_{li\succ} = 1|\mathbf{X}], \tag{30}$$

where $I_{li\succ}$ is defined in Equation 2.2. The difference between $\hat{A}(\mathbf{x}_i)$ and A_i is a function of the unobserved variation in utility ε_{li}. Define this compound random disturbance process

$$v_i|\mathbf{X} = A_i(\mathbf{x}_i) - A_i(\mathbf{x}_i), \tag{31}$$

Now, solving Equation 18 for $a(\mathbf{x})$ (and reinstituting alternative-specific subscript) provides

$$\begin{aligned} a(\mathbf{x}_i) &= \hat{a}_i + \varepsilon_i \\ &= (u(\mathbf{x}_i, q_i) - u_0) + q_i + \varepsilon_i, && (32) \end{aligned}$$

where $\varepsilon_i = \ln(v_i)$. To reinterpret this function as an estimable regression rela-

tionship, define

$$g(\mathbf{x}_i, q_i) \equiv (u(\mathbf{x}_i, q_i) - u_0). \tag{33}$$

It follows that

$$a(\mathbf{x}_i) = g(\mathbf{x}_i, q_i) + q_i + \varepsilon_i, \tag{34}$$

Given that A_i are count data, this regression relationship is in the form of a compound Poisson regression model (Cameron and Trivedi, 1986, See Equation (3) on p. 32). If ε_i follows a Gamma distribution, then $a(\mathbf{x}_i)$ is distributed as a Negative Binomial.[21] We use a canonical (and convenient) log-link index function

$$g(\mathbf{x}_i) \equiv \boldsymbol{\beta}'\mathbf{x}_i, \tag{35}$$

which implies the estimable regression equation

$$a_i = \boldsymbol{\beta}'\mathbf{x}_i + \beta_q q_i + \varepsilon_i \qquad i = 1 \cdots N, \tag{36}$$

where, based on Equations 14and 35,

$$\beta_q = \frac{\partial u(\mathbf{x}, q)}{\partial q} + 1. \tag{37}$$

This formulation is essentially a rearrangement of Equation 21.

This regression approach is consistent with the existing literature using data on lotteries such as this. Nickerson (1990) uses a log-linear index similar to Equation 36, but estimates the regression in log-linear form using Ordinary Least Squares, which does not account for the count-data nature of applications. In our data, there are no applications for about 4% of river/day alternatives, and one, two, and three application for 9%, 14%, and 18% of river/days respectively.

In addition to Generalized Poisson form we derived above (see Equation 34), we find evidence of overdispersion relative to a simple Poisson specification, so we estimate a standard Negative Binomial with a mean-based dispersion (Cameron and Trivedi, 1986; Hellerstein and Mendelsohn, 1993; Englin and Shonkwiler, 1995; Hilbe, 2011). Scrogin, Berrens, and Bohara (2000) pursue several specifications of Negative Binomial regression, and settle on a generalized Negative Binomial functional form. However, they a larger component of

[21] Coincidentally, Cameron and Trivedi (1986) motivate this form of model on the basis of interpersonal heterogeneity, which is the foundation for unobserved heterogeneity in our model.

zeros on their application data.

4.3 Preference maps

To capture curvature, and in particular diminishing marginal utility, a restricted quadratic of $\mathbf{x}$ is applied in estimation. Let $\boldsymbol{\beta}'\mathbf{x}_i = \beta_0 + \sum_j^J \beta_j x_i^j + \sum_j^J \sum_k^J \beta_{jk} x_i^j x_i^k$. Based on Equations 20 and 21, marginal (log) utility is

$$\frac{\partial u(\mathbf{x}_i)}{\partial x^j} = \beta_j + 2\beta_{jj} x_j^i + \sum_{k \neq j}^{J} \beta_{jk} x_k^i, \quad j, k \in J \neq q \tag{38}$$

$$\frac{\partial u(\mathbf{x})}{\partial q} = \beta_q - 1. \tag{39}$$

The marginal rates of substitution between variables x^j, x^k, and q are simply the ratios of the respective marginal (log) utilities based on Equations 22, 23, 38, and 39.

Mapping (predicted) vNM utility over $(\mathbf{X}, \mathbf{Q})$ is relatively straightforward. Equation 24 can be estimated as

$$\hat{U}(\mathbf{x}_i) = \frac{\hat{\pi}(\mathbf{x}_1, Q_1)}{\hat{\pi}(\mathbf{x}_i, Q_i)} = \frac{\hat{A}(\mathbf{x}_i, q_i)}{\hat{A}(\mathbf{x}_1, q_1)} \frac{Q_1}{Q_i}, \tag{40}$$

where $\hat{A}(\mathbf{x}_i, q_i) = e^{\hat{a}(\mathbf{x}_i, q_i)}$ as defined by Equation 32. The values $\hat{\pi}(\mathbf{x}_1, Q_1)$ and $\hat{A}(\mathbf{x}_1, q_1)$ are those values associated with the river/day with the lowest predicted probability of winning a given lottery, which implies, given our theory, that it is the most desirable river/day available in the lottery. Because we limit curvature to a quadratic form, we impose relatively well-behaved indifference curves. We shall see, however, that it is possible for a river/day to have too much of a given characteristic from an applicant's perspective, indicating peaked utility in attribute space.

5 Results

Results from the first-stage regressions to generate predicted river flows and predicted surface temperature are shown in Table 3. River forecasts are based on separate regressions for each river (columns 2-5). Temperatures in the region are highly correlated for a given day of the year, so The temperature fore-

casts are generated with a single regression (column 6 of Table 3), using river-specific intercept dummies to allow for differences in altitude of the proximate weather stations (see Table 1).

[Table 3 about here.]

The included sine/cosine pairs are generally jointly significant, and provide a reasonable foundation to represent pre-season expectations of river flows and weather for any given river/date. Note that $\ln(f)$, which is the natural log of the January 1 river flow forecast for April through July of the rafting year is positive and statistically significant at the 1% level for each river. Predictions from these regressions are shown as the darker dots (which look like dark lines) in Figures 3 and 4, using 2007 as an example year. The predictions follow the scatter patterns quite well.

The point of predicting temperature and flows is to use them to represent expectations about river characteristics for application decisions made prior to the rafting season. The predicted values from these regressions are therefore included as regressors to explain the variation in the number of applicants for each river/date in our sample.

Calculating the marginal rates of substitution and characterizing vNM utility functions requires at least second-order curvature among regressors. Including a full set of second-order interactions leads to a large number of insignificant parameter estimates. Based on preliminary regressions and a desire for parsimony, we have settled on a limited set of second-order interaction effects that show the primary empirical relationships of interest clearly.

Table 4 shows the structure and results from the second-stage regression. As discussed in Section 4.2. The log of the mean dispersion parameter α is statistically different from zero, indicating that a Negative Binomial is warranted as a generalization of a Poisson distribution. A quadratic in temperature is included, as well as a river-specific quadratic for river flows, because river flows patterns vary substantially across the four river sections. Annual dummy variables are included to account for differences in lottery conditions across years.

[Table 4 about here.]

Based on equation 36, the number of permits enters loglinearly as q. We do not interact q with river-specific dummy variables, because the number of per-

mits per day are constant for the Selway (one permit per day) and Snake (three permits per day). For the Negative Binomial regression, the conditional predicted number of applications is $e^{\boldsymbol{\beta}'\mathbf{x}+\beta_q q}$, so the coefficient of $\hat{\beta}_q = 0.257$ is the elasticity of permit applications with respect to permits. It is significantly different from zero at $p < 0.001$, and indicates that the percentage of applications increases by less than one percent as the percentage of permits increases.

The null hypothesis that congestion does not affect utility is $\partial u/\partial q = 0$ (Equation 21). The testable null hypothesis for this implied by Equation 39 is $(\beta_q - 1) = 0$.[22] The estimated congestion elasticity is $(0.257 - 1) = -0.743$, and the associated z statistic for the two-sided test is 9.63 ($p < 0.001$). This suggests that congestion is viewed negatively and is a statistically significant factor in application choice among river sections.[23]

Figures 7 provide illustrations of the estimated marginal utility and marginal rates of substitution over the course of the year 2007. We will focus this discussion on the Middle Fork and the Selway, which are the most sought-after sections and have the most extreme fluctuations in river characteristics and associated consequences over the course of a season. At the beginning of the rafting season, average river levels are high and temperatures are cold, leading to discomfort, more difficulty, and more danger. This is especially true for the Selway, which is infamous for its difficulty and danger during the highest flows. Subfigure 8(a) shows that the marginal utility of water level for the rivers (except the Snake) is near zero early in the season, and increases as water levels drop over the course of the summer.[24] Temperature has a higher marginal utility early and declines as the summer heats up, but the effect is relatively minor compared to changes in the marginal utility of water levels.

Although marginal utility is everywhere positive in Figure 7, these values are based on average flows for the year 2007, which itself was more or less an average water year. Consider increasing the January flow forecast (known at application time) by 25%, which increases predicted flows for roughly the first half of the permit season. This case is shown in Figure 8. Marginal utility of

[22] Notice that this null hypothesis also implies that the hypothesis that the elasticity of A with respect to Q should equal 1 if Q does not enter utility directly.

[23] While the congestion effect with regard to campsites is unclear because campsite availability varies across rivers, this result suggests that congestion at the launch and take-out sites and on the river are viewed negatively.

[24] There is a slight u-shape to the Selway's marginal utility curve. This is because the season starts slightly ahead of the average peak water runoff in the spring.

water level is now negative early in the year for three of the four rivers. These rivers are big and intimidating at these higher levels, and many experienced boaters will not risk these types of water levels. By mid-season, marginal utility becomes positive on the Middle Fork, but remains negative well into the Selway season. This too is not inconsistent with river characteristics. The Middle fork is a long trip: 100 miles. Many groups take 8-10 days when the water is low, navigation and getting stuck on rocks and over boulder gardens can be time consuming and annoying. The Selway, on the other hand, while it can also be difficult in low water, does not take as long and not so "sticky" in general. The marginal utility of additional water in the Snake is declining almost throughout. The Snake River through Hell's canyon has relatively few rapids, and they tend to be relatively big but not difficult to maneuver through. The water level simply does not matter as much as in the other rivers. The regression results in Table 4 suggest this too.[25]

In contrast, higher temperature is always better (though with diminishing utility) in the range of *ex ante* expected temperatures in our sample. The optimal high temperature according to our results is $112^{\circ}F$, which is quite high but arguably plausible when boating on rivers whose late-summer water temperature remain quite cool or even cold, and given that swimming between rapids on late summer low-water days is a common activity.

[Figure 7 about here.]

[Figure 8 about here.]

Given that the swings in marginal utility of water level dominates the changes in marginal utility of temperature, and for our sample the marginal utility of q in terms of congestion is constant, it is not surprising to see in Figures 7 and 8 that the marginal rate of substitution looks much like the marginal utility of water. Interestingly, the marginal rate of substitution between water levels and temperature on the Snake River's Hell's canyon first climb, then fall, as summer temperatures heat up. It's known as Hell's Canyon for a reason.

[25] A test that the effect of quadratic effect of water on utility is significant at $p < 0.0001$ for each river section except the snake, which shows an insignificant effect at conventional levels ($p = 0.401$). The narrow sample space of Snake River flows certainly affects the variance of the parameter estimates, but there is effectively no little evidence that differences in water levels matter much within the sample space of flows on the Snake.

Figure 9 shows utility levels as defined by Equation 24, for the lottery year 2007. The Selway, right about July 1, is the most sought-after river-day in 2007. Only one private party per day is allowed to launch on the Selway during the permit season, so while more people apply for the Middle Fork, fewer people win on the Selway. From year to year, the Selway or the Middle Fork always are the most competitive, and this is reflected in our estimated utility measures.

[Figure 9 about here.]

The fact that the marginal utility of water levels can be negative and that air temperature can be too hot means that we can expect to have a peaked utility function in water/temperature space. Figure 10 maps the contours of utility.

[Figure 10 about here.]

Comparing the subfigures for each river section, applicants' "bliss point" for the water level tends to be a bit lower for the Middle Fork than for the Main or the Selway. Note that the Utility map for the Snake indicates a preference for ever-increasing water levels. However, these results are suspect due to the insignificance of the second-order water parameter for the Snake. Figure 3 shows little variation in flows due to the fact that this section is dam controlled, so it is not so surprising that these results are weak.

6 Conclusion

The choice among a set of simple binary lotteries over complex goods provides a useful environment from which to elicit preferences for the characteristics of the goods. Our empirical example is Idaho's Four River's Lottery, in which applicants can choose one river/day out of an entire season on four different river sections to apply for in hopes of winning a permit.

The probability of winning a permit for any given river/day amounts to a relative price over the set of available lottery-allocated goods, and because these are complex goods, the probabilities of winning contains hedonic price information about the value of the individual characteristics of these goods, and these relative values are a reflection of the marginal rates of substitution and their underlying marginal utility. These four river sections differ in their characteristics in terms of length, difficulty, and in the number of permits issued per day

(among other differences). They also differ substantially in terms of water levels and weather over the course of the rafting season. Every year, applicants for these lotteries reveal their relative preference for mid-season intermediate water levels and warm weather over powerful and cold spring runoff or late season low water by accepting very low equilibrium probabilities of winning a river/day with the most preferred conditions.

We develop a model of application demand for simple binary lotteries over a set of complex goods that provides a foundation for estimating hedonic prices for characteristics, marginal utility and marginal rates of substitution among characteristics, and the ability to fully map von Neumann-Morgenstern utility over the characteristics of goods.

Plausible results emerge from our estimation strategy. When river water levels are very high from spring runoff, the marginal utility of water levels are low or (for high-water years) negative, and they vary across rivers in accordance with the consequences of high water for whitewater rafters. The bliss point for maximum daily temperatures on these cool-water rivers in the summer is $112^{\circ}F$ — perfect for being splashed and for the occasional swim between rapids on cold rivers. As the number of permits increases for a given river/day, the number of applicants increases because the probability of winning would otherwise be higher. However, application rates do not increase in proportion to an increase in the number of permits, which we interpret as evidence of disutility from congestion at launch and exit sites, on the river, and at the limited number of campsites.

These types of lotteries and this analytical framework is useful as part of a foundation for non-market valuation of complex publicly held resources or non-marketed complex goods in general. For goods prone to congestion in use, this framework is also useful in principle for informing decisions about the number of permits to issue for a given public or congestible good. However, we are not able to examine this function in detail due to data limitations.

While there have been several empirical examinations based on actual game hunting and river lotteries such as this one in the literature, we have not seen this lottery structure used in the experimental economics literature. We suspect that there are likely to be a relatively broad range of goods for which this type of application might be practical and convenient for preference and valuation analysis. Unlike conjoint analysis or contingent ranking exercises, a multiple-

gamble lottery such as this utilizing real multi-attribute goods which will be one by experiment participants will, in principle, avoid hypothetical bias and perhaps other problems as well. By having lottery prizes given away as the end product and not necessarily requiring any expenditures by participants, preferences and hedonic prices can be elicited in a relatively simple way.

We also suggest that using our approach for modeling application rates would be a useful first step as a foundation for modeling individual applicant choice as is pursued by Scrogin and Berrens (2003) and Scrogin (2005), and others, and careful examination of how these two steps fit together might provide some useful insights. In general, the application rate data is useful for characterizing the aggregate impact of the characteristics of the lottery-allocated goods, while the individual choice component (e.g. as estimated by conditional logit or related models) are useful for looking at differences in preferences across individuals if individual characteristics are available in the data.

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Table 1: Data descriptions.

	Description and source
w	Natural log of discharge, $\ln(\text{feet}^3/\text{second})$. Daily means of real-time flow data, 1990–Latest available. Hydrologic measurement site numbers: 13307000 [MS]; 13309220 [MF]; 13336500 [SE]; 13290450 [SN]. (United States Geological Survey, 2012)
t	Daily Maximum temperature, ($F°$) [NOAA abbrev: TMAX]. Site IDs (COOPID): 105897 [MF]; 108246 [SE]; 109963 [MS]; 107706 [SN]. (National Oceanic and Atmospheric Administration, 2012)
f	January 1 annual State Basin Outlook reports for Idaho, river flow forecast % of normal (average) for April-July. Line items: "MF Salmon R at MF Lodge", [MF]; "Selway R nr Lowell" [SE]; "Salmon R at White Bird" [MS]; "SNAKE RIVER at Hells Canyon Dam" [SN]. (U.S. Natural Resource Conservation Service, 1993-2011.)
$A; a$	# first-choice applicants by river/date; $a = \ln(A)$. (United States Forest Service, 2011)
$Q; q$	# available permits by river/date; $q = \ln(Q)$. (United States Forest Service, 2011)
d	Day of year.
y	Year.

Table 2: Summary statistics. The variable s is generated from first stage regression. Others represent original data applied to the first or second stage regressions as applicable.

Var.	# Obs	Mean	S.D.	Min	Max
w	25,033	8.317	1.369	5.075	11.502
t	25,033	63.41	20.62	-9.00	113.00
f	25,033	90.7	27.5	50.0	201.0
A	3,458	43.78	64.97	0	523
Q	3,458	3.010	1.134	1	6
d	3,458	195.9	30.16	135	253
y	3,458	2004.5	2.864	2000	2009

Table 3: River flow (by river) and temperature (with river dummies, Main Salmon=base) regressions. j is the frequency of sine/cosine cycles per year (see Equation 25).

Dep. var. $\rightarrow$	w:MA	w:MF	w:SE	w:SN	t
constant	5.066***	4.767***	4.731***	6.637***	59.647***
$\sin(j=1)$	0.461***	0.387***	0.915***	0.374***	-6.585***
$\sin(j=2)$	-0.509***	-0.522***	-0.631***	0.001	4.563***
$\sin(j=3)$	0.196***	0.224***	0.163***	0.116***	-0.488***
$\sin(j=4)$	-0.069***	-0.040***	-0.056***	-0.013	0.797***
$\sin(j=5)$	0.042***	0.032***	0.069***	-0.002	-0.539***
$\sin(j=6)$	0.008	0.011	0.025**	0.004	0.464***
$\cos(j=1)$	-0.720***	-0.801***	-0.715***	-0.108***	-24.747***
$\cos(j=2)$	0.228***	0.183***	0.230***	-0.018**	-2.182***
$\cos(j=3)$	-0.030***	0.054***	-0.032***	0.043***	-0.653***
$\cos(j=4)$	-0.039***	-0.078***	-0.031***	0.075***	0.039
$\cos(j=5)$	0.036***	0.039***	-0.005	0.025***	0.269***
$\cos(j=6)$	-0.024***	-0.017*	-0.018*	-0.050***	-0.013
$\ln(f)$	0.837***	0.443***	0.624***	0.700***	
MF					2.941***
SE					2.426***
SN					6.701***
R^2	0.84	0.85	0.81	0.47	0.82
$\#Obs$	6,935	4,685	6,935	6,478	37,644

*significant at 10%; **significant at 5%; ***significant at 1%.

Table 4: Negative Binomial regression. Dependent variable: Number of applicants choosing river/day i as their first choice (A). # Observations = 3,458, $\chi^2 = 12,323$ ($p < 0.0001$). Base case: Main Salmon.

var./interaction	β
q	0.257***
MF	8.159
SE	4.792
SN	80.313**
$\hat{w}$	13.301***
$\hat{w}\cdot$ MF	0.984
$\hat{w}\cdot$ SE	-0.220
$\hat{w}\cdot$ SN	-18.510**
$\hat{w}^2\cdot$ MN	-0.648***
$\hat{w}^2\cdot$ MF	-0.808***
$\hat{w}^2\cdot$ SE	-0.654***
$\hat{w}^2\cdot$ SN	0.383
$\hat{t}$	0.896***
$\hat{t}^2$	-0.004***
2001	-0.037
2002	-0.429***
2003	-0.313***
2004	-0.321***
2005	0.023
2006	-0.570***
2007	-0.349***
2008	-0.272***
2009	-0.315***
constant	-110.512***
$\ln(\alpha)$	-1.038***

*10%; **5%; ***1%.

Figure 1: Equilibrium condition for applications over $\mathbf{X}$.

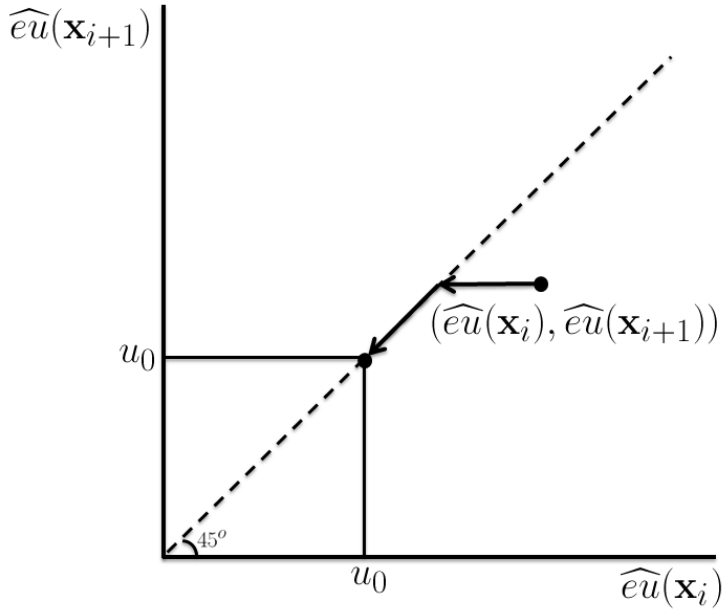


Figure 2: Four Rivers Lottery permit sections. Source: Adapted from http://geology.com/lakes-rivers-water/idaho.shtml.

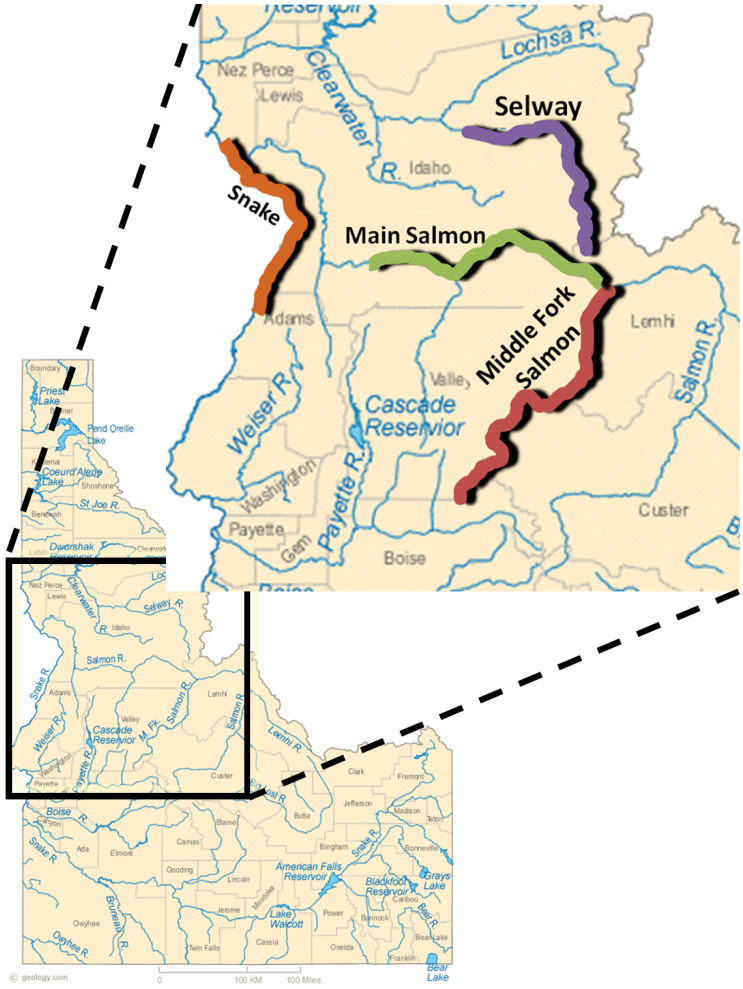


Figure 3: River levels, w ($\ln(\text{feet}^3/\text{second})$), and predictions for 2007 (see Table 3).

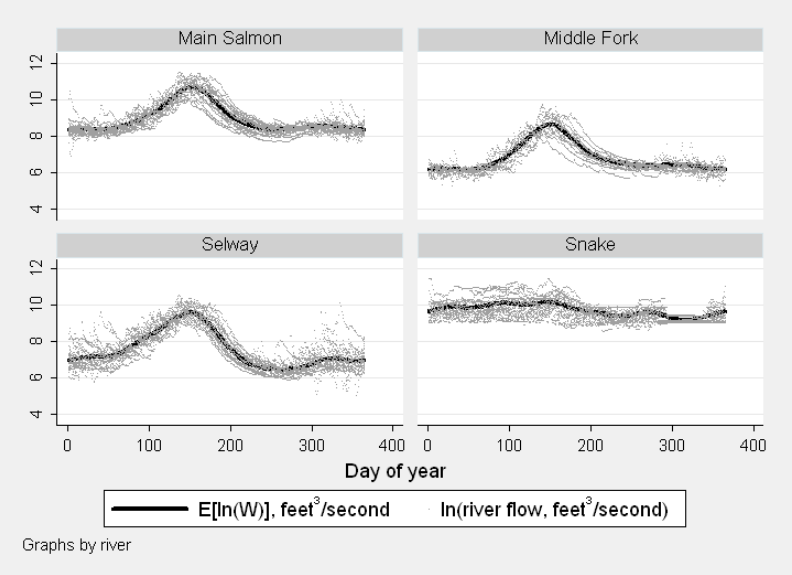


Figure 4: Daily Maximum temperature,(F°), and predictions for 2007 (see Table 3)).

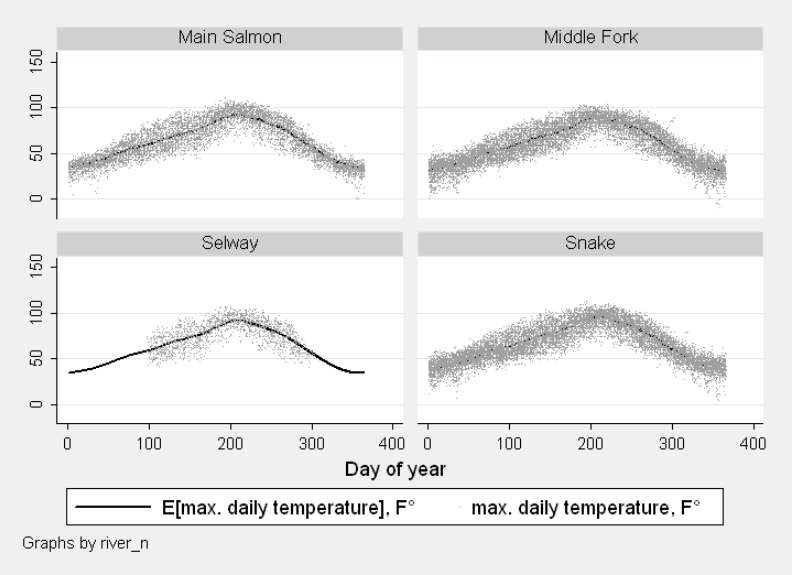
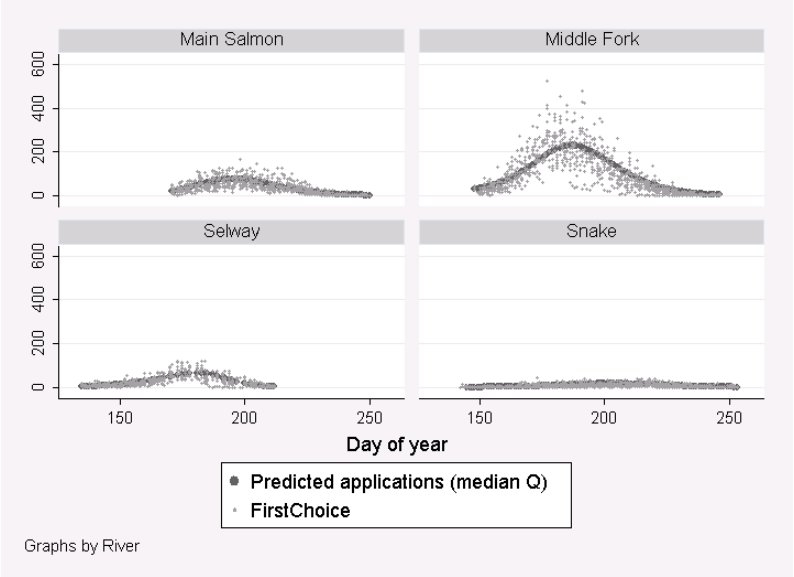
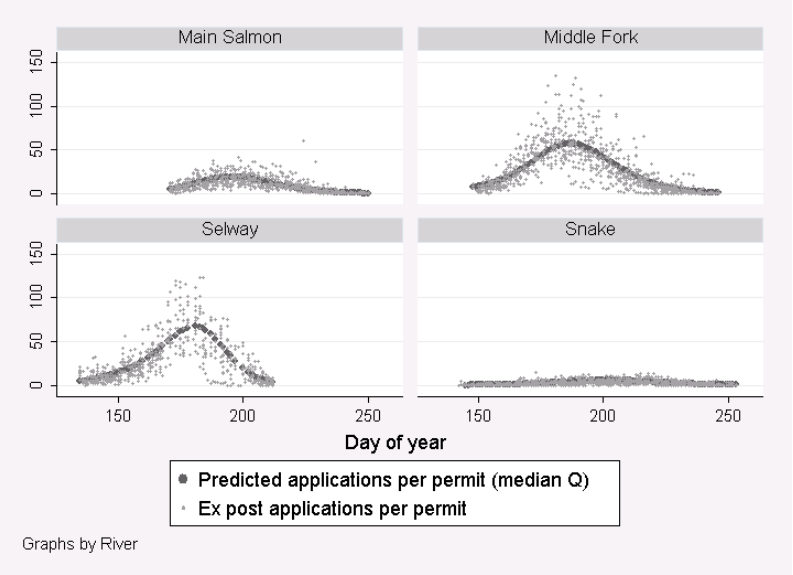


Figure 5: Application rates and predictions for 2007, by river/day.



(a) Applicants (A) and predictions for 2007.



(b) Applicants per permit (A/Q), and predictions for 2007.

Figure 6: Probability and predicted probability of winning first river/day choice.

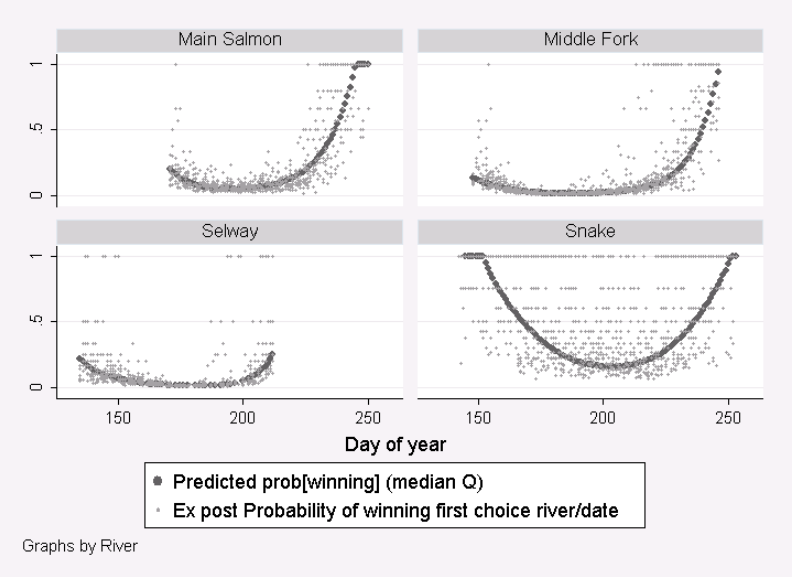
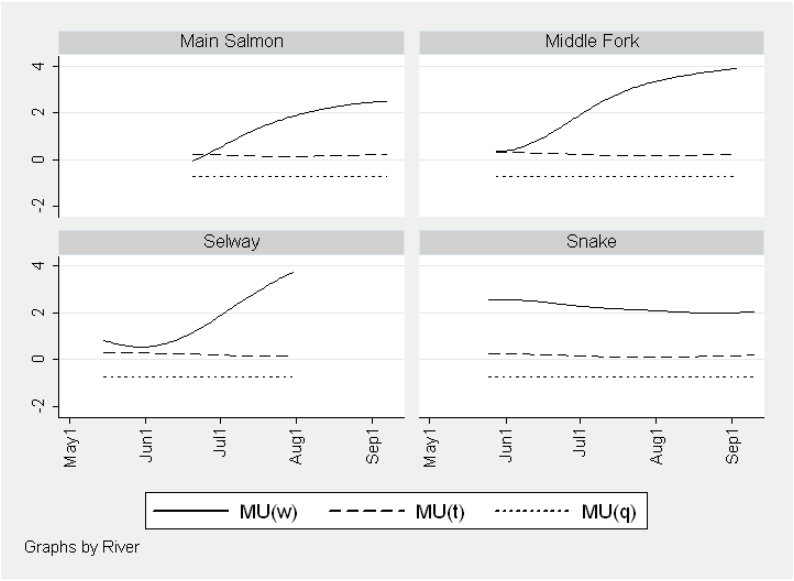
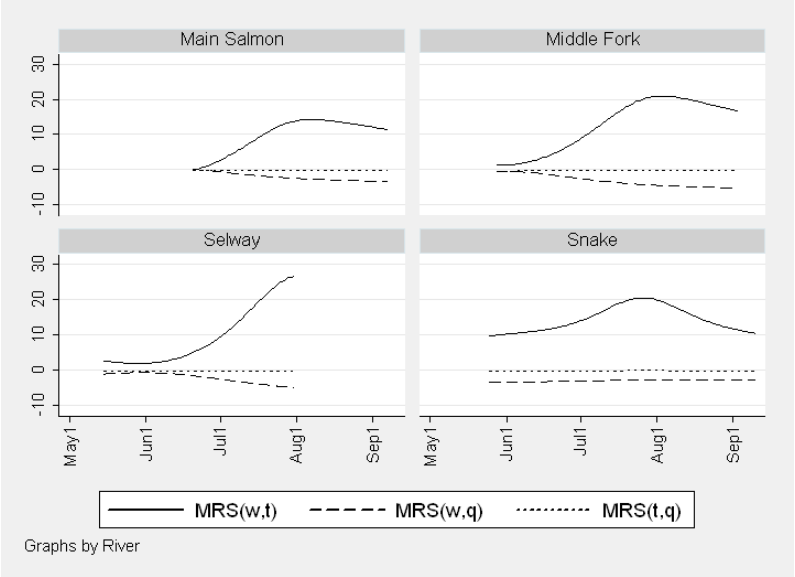


Figure 7: Marginal utility and rates of substitution between w, t, and q. 2007 base.

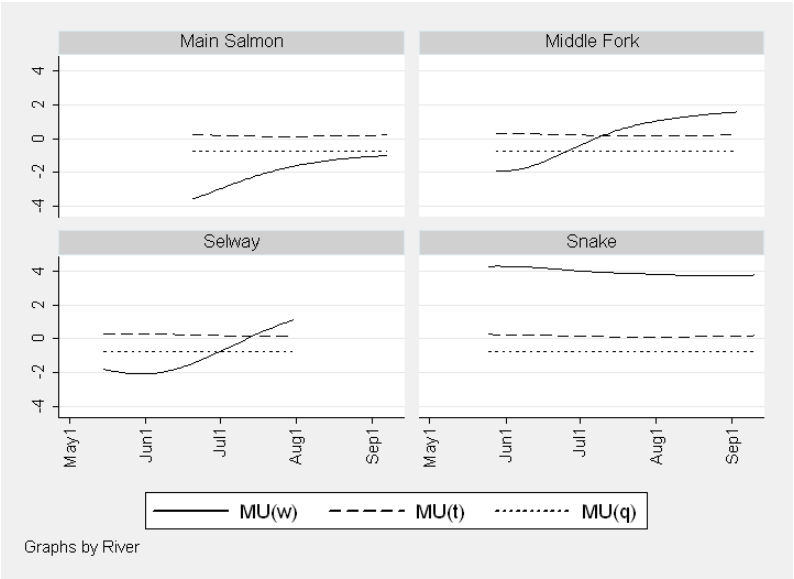


(a) Marginal utility

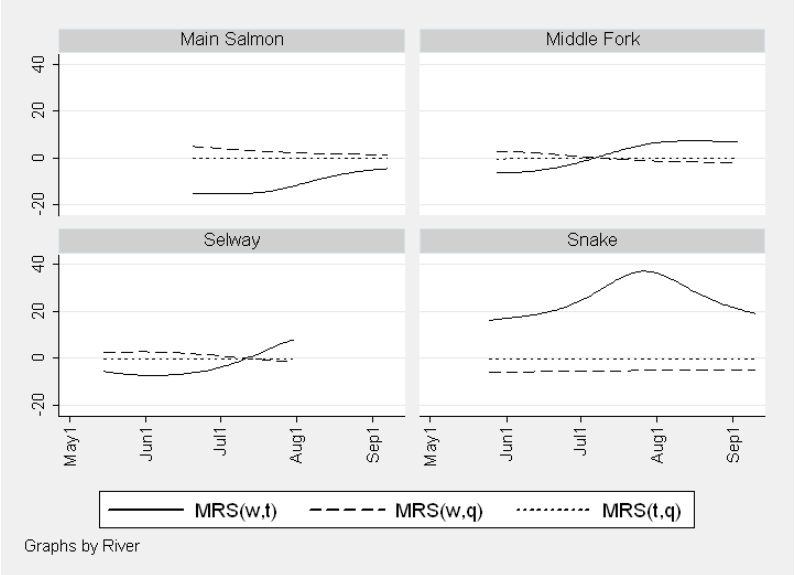


(b) Marginal rate of substitution

Figure 8: Marginal utility and rates of substitution between w, t, and q: January water level forecasts 25% above average.



(a) Marginal utility



(b) Marginal rate of substitution

Figure 9: vNM utility over the course of lottery year 2007.

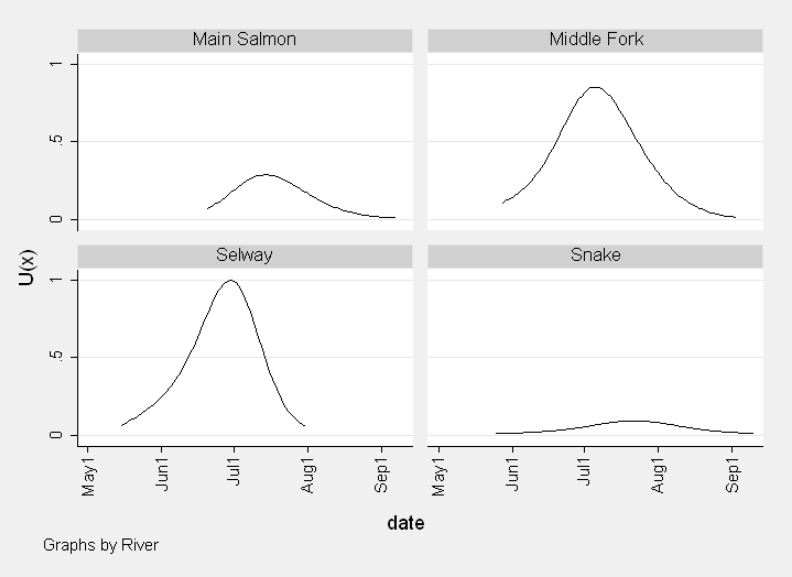


Figure 10: vNM Utility function contour plots for each river, 2007 base.

