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Stability of long-run relationships for countries in transition: A Hansen test study

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Abstract

The results of a Monte Carlo research for the Hansen *Lc*, *MeanF* and *SupF* stability tests for long-run relationships are reported. The tests are related to the Phillips-Hansen and Hansen semiparametric methods, which involve kernel density estimation of the long-run covariance matrix. We compare the effect of the choice of kernel on the performance of the tests, and also check the effect of misspecification of the model (lags for the disturbances) on the behaviour of test statistics, and behaviour of percentiles. The results indicate that the best – both in terms of efficiency and robustness to misspecification error – is the Parzen kernel.

JEL codes:C01,C02,C12,C14,C46

Key words: Hansen stability tests, Phillips-Hansen estimators, long-run covariance matrix, spectral analysis, semiparametric estimates

1. Introduction

The aim of this research is to compare properties of the Hansen L_c, *MeanF* and *SupF* tests.[1] Those tests allow for testing null hypothesis of stability of parameters of the model versus alternatives of one structural break with unknown timing in the case of *SupF* test, and parameters behaving as a martingale, as it is for the L_c and *MeanF* tests. The tests were developed in a framework of the Phillips-Hansen and Hansen semiparametric methods of estimation. The estimation of a long-run covariance matrix employs kernel density estimation. This step includes the choice of window lag and window width (bandwidth). Procedures for automatic choice of an optimal bandwidth exist (see Hansen (1992)), so we concentrate here on the question of effect of the choice of a kernel on the performance of the tests statistics. Our results show that the best choice seems to be the Parzen kernel.

Economic transformations in the Central European countries can be treated as structural change from enforced quasi-equilibrium, corresponding to the period of centrally planned economy, to the new market equilibrium established as a result of fundamental institutional and economic reforms of early 1990's. The modelling of such changes is of great importance. Intuitively, one should be able, with proper econometric tools, to describe the original equilibrium, and to detect the emergence of a new equilibrium.

Usefulness of the Hansen test for checking the presence of structural break of the type of economic reform in transforming countries stems from several reasons. First, for a model with time varying parameters there is no Error Correction Mechanism, so other methods of testing, based on ECM, are not appropriate in this case. Second, the tests are quite flexible in that they may be used as tests of stability of parameters as well as tests of cointegration. Third, they do not require choice of timing of a structural break. Fourth, the Hansen methods of estimation take into account several features characteristic of the

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transforming economies: the influence of distant past and the long memory nature of the process of establishing a new mechanisms.

The Hansen method is a generalisation of the Phillips-Hansen method for non-constant structural parameters. It is, therefore, well suited for modelling the reform process.

The Phillips-Hansen and the Hansen methods of estimation are described briefly in section 2. Section 3 addresses question of the choice of bandwidth and presents results on the efficiency of various kernels. Section 4 compares the performance of Hansen tests for several kernels. It presents the results of the Monte Carlo study. Section 5 concludes.

2. Estimation and testing: Phillips-Hansen and Hansen methods.

Semiparametric models have a finite-dimensional parameter of interest - the parametric component, and an infinite dimensional nuisance parameter - the nonparametric component. A semiparametric model combines a parametric form for some component of the data generating process, usually the relation between the dependent and explanatory variables, and weak nonparametric restrictions on the remaining component, usually distribution of the unobservable errors (see Powell (1994)). The term "semiparametric estimator" is also used to describe a statistic which involves a preliminary estimator of a nonparametric component, for example a density estimation.

Single-equation semiparametric least squares and instrumental variables methods of estimation proposed by Phillips and Hansen (1990) permit direct estimation of long-run relationships. They involve a two-step method, and as a first step the data are filtered using a nonparametric correction. *The Hansen method* is a generalisation of the Phillips-Hansen method for the case of non-constant structural parameters (see Hansen (1993)). Both in the case of OLS estimation, and in the case of Fully Modified estimation method, the L_c, *SupF* and *MeanF* statistics can be defined and used for testing stability of parameters of the model. The methods involve estimation of the long-run variance-covariance matrix Ω of the unobservable errors with use of the spectral density estimation.

2.1. Fully Modified Estimation Method: Phillips-Hansen and Hansen

Fully modified estimation method, introduced by Phillips and Hansen, is a single-equation approach based on OLS with semiparametric corrections for serial correlation and endogeneity, and has the same asymptotic behaviour as the full system MLE (see Phillips and Loretan (1990), who give the detailed analysis of asymptotic results for u_t = iid(0, Σ)).

For a typical cointegration system we assume that the data generating process may be described as in Phillips and Loretan (1990):

(1) $$y_t = \beta^T x_t + u_{1t},$$

(2) $$\Delta x_t = u_{2t}$$

where $u_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$ is a martingale difference sequence or iid(0, Σ), Σ > 0. By assumption there is a cointegrated relationship to estimate.

The idea of the Phillips - Hansen method is that the limit variate for $\beta^* - \beta$, where β^* is the OLS estimate of the parameters β, may be written as sum of three terms: a Gaussian mixture, a matrix unit root distribution and a bias term arising from contemporaneous correlation of u_{1t} and u_{2t}, i.e. from the endogeneity of the x_t. In the Phillips-Hansen procedure the third term is removed by employing a serial correlation correction, and the endogeneity of x_t is removed by the construction of additional correction.

The Hansen method is a generalization of the Phillips-Hansen method for the case of non-constant parameters. Hansen (1992) describes in detail the two steps of the method, for changing parameters A_t in the first equation and slightly different formulation of eq.(2):

(1') $$y_t = A_t x_t + u_{1t}, \qquad t = 1,2,...,n,$$

$$x_{1t} = k_{1t},$$

(2') $x_{2t} = \Pi_1 k_{1t} + \Pi_2 k_{2t} + x^{o}_{2t}$

$x^{o}_{2t} = x^{o}_{2t-1} + u_{2t}$

where Π_1 and Π_2 are matrices of parameters, A_t is a changing matrix of parameters, $u_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$, and the elements of $k_t = \begin{bmatrix} k_{1t} \\ k_{2t} \end{bmatrix}$ are nonnegative powers of time: k_{1t} contains trends and a constant, and is placed directly in the regression equation, k_{2t} determines the behaviour of the stochastic regressors x_{2t} , and is excluded from the regression.

The long-run covariance matrix is defined as

(3) $$\Omega = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{n} E(u_j \, u_t^T)$$

Matrix Λ represents bias due to endogeneity of regressors:

(4) $$\Lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{t} E(u_j \, u_t^T) .$$

The first step of the method is to estimate (1') and obtain the parameter estimates and residuals, say $\tilde{u}_{1t}$. The second step is to estimate (2') by OLS in differences, which yields the residuals $\tilde{u}_{2t}$. Let $\mathbf{u_t} = \begin{bmatrix} \tilde{u}_{1t} \\ \tilde{u}_{2t} \end{bmatrix}$. Then the covariance matrices Ω and Λ are estimated from the residuals $\mathbf{u_t}$ via a kernel; Hansen (1992) suggests that in moderate samples, before calculating the long-run covariance parameters, the residuals should be prewhitened, as proposed by Andrews and Monahan (1992), i.e., in the case of serial correlation of u_t the AR(1) for u_t is estimated

$$u_t = \rho u_{t-1} + v_t$$

and v_t is used instead of u_t in subsequent steps of the method. The use of prewhitened residuals is preferable because if the cointegrating residuals $\tilde{u}_{1t}$ are highly correlated, as is the case on most applications, the direct estimate is either biased or inaccurate due to the high variance of the estimator.

The next step involves spectral density estimation with the application of a kernel, using a plug-in bandwidth suggested by Andrews (1991). A kernel estimator is applied to $\tilde{e}_t$, which represent respectively the residuals u_t of the model or the prewhitened residuals v_t :

$$(5) \qquad \tilde{\Lambda}_e = \sum_{j=0}^{N} w(j/M) \frac{1}{N} \sum_{t=j+1}^{N} \tilde{e}_{t-j} \tilde{e}_t^T$$

and for the matrix Ω

$$(6) \qquad \tilde{\Omega}_e = \sum_{j=-N}^{N} w(j/M) \frac{1}{N} \sum_{t=j+1}^{N} \tilde{e}_{t-j} \tilde{e}_t^T ,$$

where $w(.)$ is a weight kernel and M is a window width (bandwidth), (see Hansen (1992a), p. 323), and $\tilde{\Omega}_e$ can be seen as a scaled estimate of a spectral density of the residuals.

These estimates require the choice of a kernel and a bandwidth parameter. Below we show results of comparisons of performance of stability tests for different choice of kernels. Computations were performed with use of GAUSS package (COINT2.0), in which choice of bandwidth is performed automatically.

Hansen stability tests

In testing parameter stability for a model, a severe problem is the need to select the timing of the structural break which occurred under the alternative hypothesis (see Hansen (1992)). Under the null of parameter constancy the date of break is not defined. Neither selecting the timing in an arbitrary way nor selecting it conditionally on the data is a good solution. Quandt (1960) proposed a test, in which for every possible breakpoint tests for structural change are calculated, and the largest test statistic examined. In this approach, extended by Davies (1977) to general models with parameters unindentified, the inference is based on the LR statistic, which is the maximal F_T statistic over a range of break dates $r_0,\ldots, r_1$; and F_T is the test for a break at fraction r/T through the sample. If the sum of squared residuals from the estimation of the regression on observations $m, m+1,\ldots,n$ is denoted by SSR(m, n), and k is number of regressors, then F_T is expressed as:

$$F_T\left(\frac{r}{T}\right) = \frac{SSR(1,T) - [SSR(1,r) + SSR(r+1,T)]}{[SSR(1,r) + SSR(r+1,T)]/(T-2k)}.$$

This yields the Quandt likelihood ratio statistic:

$$QLR = \max_{r=r_0,\ldots,r_1} F_T\left(\frac{r}{T}\right)$$

which has a power against a change on parameter values even through the break date is unknown (Stock (1994)).

Hansen (1990) presented a distributional theory for Quandt type tests, including L_c, *MeanF* and *SupF* tests. Hansen (1995) presents computationally convenient approximations to the asymptotic *p*-value functions, found by Andrews for the distributions of tests for structural change in econometric models. The Hansen tests can be used for investigating the behaviour of cointegration relationship between macroeconomic variables. Examples of applications can be found in Haug (1992) and in Quintos and Phillips (1993).

For all the three tests the null hypothesis is of parameter stability. The alternative for the L_c test is that the parameters of the model follow a martingale, and this incorporates simple structural breaks of unknown timing as well as random walk parameters (see Hansen (1992b)). The L_c test checks instability caused by relatively stable changes of parameters. This test is a development of Lagrange Multiplier tests for linear model with Gaussian error term introduced by Gardner (1969), Pagan and Tanaka (1981), Nyblom and Makelainen (1983) and King (1987).

The two other tests differ slightly in formulation of the alternative. The *SupF* test can be used for detecting a change of regime of cointegrating vector. The *MeanF* checks instability of model due to gradual change of parameters in time. The Hansen tests can be also treated as tests of cointegration.

The L_c Hansen test for OLS estimation

The L_c test for OLS estimation of standard linear model is described in Hansen (1992b) for a model:

$$
\begin{aligned}
& y_t = \beta_1 x_{1t} + \beta_2 x_{2t} + ... + \beta_3 x_{3t} + e_t, \\
& E(e_t | x_t) = 0, \\
& E(e_t^2) = \sigma_t^2, \\
& \lim \frac{1}{n} \sum_{i=1}^{n} \sigma_t^2 = \sigma^2
\end{aligned}
\tag{7}
$$

The test of stability of parameters, i.e. of vector β and parameter σ^2 , is performed in the following way:

1) After estimating eq.(3) by OLS on a whole sample, „first order terms" f_{it} for OLS estimates $\tilde{\beta}$ and $\tilde{\sigma}^2$ are computed:

$$
f_{it} = \begin{cases} x_{it}\, \tilde{e}_t, & i = 1,2,...,m \\ \tilde{e}_t^2 - \tilde{\sigma}^2, & i = m+1 \end{cases}
\tag{8}
$$

where $\tilde{e}_t$ are residuals of the model.

The test statistics depends on cumulative sums of f_{it} :

$$
S_{it} = \sum_{j=1}^{t} f_{ij}.
\tag{9}
$$

Testing stability of individual parameters is based on the statistic (see Hansen (1992), equations (6)-(10)):

$$
L_i = \frac{1}{n V_i} \sum_{i=1}^{n} S_{it}^2,
\tag{10}
$$

where

$$
V_i = \sum_{i=1}^{n} f_{it}^2.
$$

The joint stability test statistic is:

$$L_C = \frac{1}{n}\sum_{i=1}^{n} S_t^T V^{-1} S_t, \tag{11}$$

with matrix V defined as:

$$V = \sum_{t=1}^{n} f_t \, f_t^T,$$

and vectors f_t and S_t equal to

$$f_t = [\, f_{1t}, f_{2t}, \ldots, f_{m+1,t} \,]^T,$$
$$S_t = [\, S_{1t}, S_{2t}, \ldots, S_{m+1,t} \,]^T.$$

Under the null hypothesis the cumulative sums tend to wander around zero. Under the alternative of parameter instability the cumulative sums will have nonzero mean in parts of the sample and the test statistic tends to be large. Hence for large values of L_c the null hypothesis of stability is rejected.

The MeanF, SupF and L_c tests for FM estimation

In this section we follow Hansen (1992a). The *SupF* test checks the null hypothesis of constancy of parameters of the model (1') versus one structural break with unknown timing. The other two tests, *MeanF* and L_c, model the parameter as a martingale process:

$$A_t = A_{t-1} + \varepsilon_t: \quad E(\varepsilon_t | \Im_{t-1}) = 0, \quad E(\varepsilon_t \, \varepsilon_t^T) = \delta^2 G_t \tag{12}$$

The null hypothesis for *MeanF* and for L_c tests is that the variance of the martingale differences is zero, while the alternative is that the variance is nonzero, equal to δ^2 G_t , with G_t being a positive definite matrix depending on the variance-covariance matrix Ω, defined with (3), and matrix Λ, representing bias due to endogeneity of regressors, defined as in (4). The alternatives for the two tests differ in that G_t has a different form. The exact formulae for G_t in both cases, as given in Hansen (1992)), are

derived in a following way. The matrices Ω and Λ are partitioned in conformity with u_t, i.e.:

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix},$$

and next matrices $\Omega_{1\cdot 2} = \Omega_{11} - \Omega_{12}\Omega_{22}^{T}\Omega_{21}, \quad \Lambda_{21}^{+} = \Lambda_{21} - \Lambda_{22}\Omega_{22}^{T}\Omega_{21}$ are defined. $\tilde{\Omega}_e$ and $\tilde{\Lambda}_e$ are partitioned and $\tilde{\Omega}_{1\cdot 2}$ and $\tilde{\Lambda}_{21}^{+}$ defined in the same way as for original matrices. The transformed dependent variable is defined as:

$$y_t^{+} = y_t - \tilde{\Omega}_{12}\tilde{\Omega}_{22}^{-1}u_{2t}$$

The FM estimator of A, A^{+}, gives the residuals $\tilde{u}_{1t}^{+} = y_t^{+} - A^{+}x_t$. The scores are the variables : $\tilde{s}_t = x_t\left(\tilde{u}_{1t}^{+}\right)^{T} - \frac{1}{N}\sum_{t=1}^{N} x_t\left(\tilde{u}_{1t}^{+}\right)^{T}$.

Let S_{nt} denote the partial sum of scores:

$$S_{nt} = \sum_{i=1}^{n} \tilde{s}_i .$$

Define $M_{nt} = \sum_{i=1}^{t} x_i x_i^{T}$ and $V_{nt} = M_{nt} - M_{nt}M_{nn}^{-1}M_{nt}$.

Then the tests statistics for stability of parameters A_t of equation (1') are the following:

(13) $$F_{nt} = tr\left\{S^{T}{}_{nt}V_{nt}^{-1}S_{nt}\tilde{\Omega}_{1\cdot 2}^{-1}\right\}$$

is used for test of H_0: A_t *constant* against H_1 : $A_1 \neq A_2$, *one structural break at known time t.*

The *SupF* assumes that the timing of break is unknown, and $[t/n]$ is an integer in a compact subset T of $(0,1)$:

(14) $$SupF = \sup_{[t/n]\in T} F_{nt} .$$

For both the *MeanF* and L_c tests the null hypothesis can be written as H_0: $\delta^2 = 0$. The alternative for the *MeanF* test is $\delta^2 > 0$, $G_t = (\tilde{\Omega}_{1\cdot 2} \otimes V_{nt})^{-1}, [t/n] \in T$, and the test statistic is:

(15) $$MeanF = \frac{1}{n^*} \sum_{[t/n] \in T} F_{nt} \quad \text{where } n^* = \text{number of elements in } T.$$

The alternative hypothesis for the L_c test is: $\delta^2 > 0$, $G_t = (\tilde{\Omega}_{1\cdot 2} \otimes M_{nn})^{-1}$, and the test statistics is:

(16) $$L_c = \mathrm{tr}\left\{ M_{nn}^{-1} \sum_{t=1}^{n} S_t \tilde{\Omega}_{1\cdot 2}^{-1} S_t^T \right\}$$

Asymptotic theory for the test distribution was given by Nyblom (1989) and Hansen (1990). Hansen gives critical values for L_c, however only asymptotic. Recent studies are devoted also only to an asymptotic behaviour of test statistics: Hansen (1991) examines asymptotic local power of tests for parameter instability, under assumption that parameters are generated by a martingale process, and in 1995 paper presents numerical approximations to the asymptotic distributions of tests for structural change.

3. Hansen Tests: Kernels and Bandwidth Parameter.

One step in the Phillips- Hansen method is the estimation of the long-run variance-covariance matrix Ω in a spectral framework, with use of spectral window or kernel (see (6) - (7)). Several spectral windows (kernels) can be applied. GAUSS COINT 2.0 package, used for computations presented here, contains the Bohman, Fejer, Tukey-Hamming, Tukey-Hanning, Parzen and Quadratic Spectral kernels. B.E.Hansen used for his 1992 paper the procedure in GAUSS containing the Quadratic Spectral, Bartlett and Parzen kernels. We compare here the performance of the Hansen tests for those kernels, and present critical values computed for 10000 replication, and additionally critical values of the L_c test for 50000 replications.

3.1. The Efficiency of Kernels

The kernel used in spectral density estimation, is a symmetric function satisfying the following conditions (Silverman (1987) p. 38):

(17) $$\int K(t)dt = 1, \int tK(t)dt = 0, \quad \text{and } \int t^2 K(t)dt = k_2 \neq 0.$$

When a probability density function is used as a kernel, then k_2 is its variance. For the optimally chosen bandwidth M the approximate value of the mean square error is:

(18) $$\tfrac{5}{4} C(K) \left\{ \int f''(x)^2 dx \right\}^{0.2} N^{-0.8}$$

where f is the unknown density, N – the sample size and the constant $C(K)$ is given by (Silverman (1987)):

(19) $$C(K) = k_2^{0.4} \left\{ \int K(t)^2 dt \right\}^{0.8}.$$

For univariate kernel density estimation several kernels are used. The QS kernel was constructed to minimise the mean square error of density estimation and in that sense it is the most efficient kernel.

The formula for the QS kernel is:

(21) $$k(u) = \frac{25}{12\pi^2 u^2} \left(\frac{\sin\left(6\pi u/5\right)}{6\pi u/5} - \cos\left(6\pi u/5\right) \right)$$

(see Hansen (1992a, p.323). Priestley (1994) gives a slightly different formula for Quadratic-Spectral or Bartlett-Priestley kernel:

(20') $$k(u) = \frac{3}{\pi^2 u^2} \left(\frac{\sin(\pi u)}{\pi u} - \cos(\pi u) \right)$$

Its properties were studied by Epanechnikov (1969), (see Priestley (1994)), who proved its optimality for a wide class of kernels used for univariate density estimation. However Silverman (1987) gives slightly different formula for Epanechnikov kernel, namely:

(21)

$$k(t) = \begin{cases} \frac{3}{4\sqrt{5}}(1-0.2t^2) & \text{if} \quad -\sqrt{5} \le t \le \sqrt{5} \\ 0 & \text{otherwise} \end{cases}.$$

Silverman (1987) gives the following formula for comparing *efficiency of a kernel with comparison to the Epanechnikov* (*21*) *kernel:*

(22) $\text{eff}(K) = \left\{C(K_e)/C(K)\right\}^{5/4}$

$$= \frac{3}{5\sqrt{5}}\left\{\int t^2 K(t)dt\right\}^{-0.5}\left\{\int K(t)^2 dt\right\}^{-1}$$

The smaller $C(K)$ the better is the performance of the kernel for univariate density estimation. We checked the results given by Silverman for the Epanechnikov, rectangular, biweight and triangular kernels, and applied the same formula for kernels not covered by Silverman. The results are following:

Triangular kernel $k(u)$ = (1 - $|u|$ for $|u|<1$, 0 otherwise): **0.98590060**

Parzen kernel:

(23) $$k(u) = \begin{cases} 1-6u^2+6|u|^3 & |u| \le 0.5, \\ 2(1-|u|)^3 & 0.5 \le |u| \le 1, \\ 0 & \text{otherwise} \end{cases}$$

Its efficiency equals **0.90932667.**

Epanechnikov kernel: **1.0000000**

Biweight kernel : This kernel is defined as 15/16 $(1-u^2)^2$ for $|u|<1$, 0 otherwise. Efficiency equals **0.99390140**.

Rectangular kernel (0.5 for $|u|<1$, 0 otherwise): **0.92951600**

Daniell window is defined as $k(u) = \sin(\pi u)/(\pi u)$ (Priestley(1994) p.447): eff= **0.66023470.**

General **Tukey window** is defined as:

(24) $k(u) = 1-2a+2a\cos(\pi u)$ if $|u| \le 1$, 0 if $|u|>1$.

The constant a is an additional parameter lying in the range (0,0.25] (Priestley (1994) p.447).

a) Tukey originally suggested taking $a = 0.23$, in which case

(25) $k(u) = 0.54+0.46 \cos(u)$ if $|u| \leq 1$, 0 otherwise

(see Priestley, pp.442-443). This is known as the **Tukey-Hamming** window, and named after R.W. Hamming (Chatfield (1989)). Its efficiency equals **0.16457210** .

b) The **Tukey-Hanning** window:

A slightly more convenient computational form of the lag window is given by taking a=0.25, this form is known as the Tukey-Hanning window (Priestley, p.443); named after Julius von Hann (Chatfield (1989)):

(26) $k(u)=0.5*\{1+\cos(u)\}$ if $|u| \leq 1$; 0 otherwise.

Its efficiency equals **0.16758281** .

Bartlett window $k(u) = 1 - |u|$ for $|u| < 1$: **0.98590060.** Chatfield (1989) argues that its properties are inferior to the Tukey and Parzen windows.

Quadratic Spectral window (20), has efficiency equal to **0.52437780** .

3.2. Bandwidth parameter.

The choice of bandwidth, window width or truncation parameter M is an important issue for kernel density estimation. In the 1992 paper Hansen advocates using of QS, Bartlett and Parzen kernels for the estimation of the matrix Ω, and the use of plug-in bandwidth estimator, based on Andrews (1991). For the Bartlett, Parzen and the Quadratic Spectral kernels the choices are:

$$M = 1.1147(a\,(1)\,n)^{1/3}$$

$$M = 2.6614\,(a\,(2)\,n)^{1/5}$$

$$M = 1.3221(a\,(2)\,n)^{1/5},$$

respectively, where a's are computed on the base of e_t (see formulae in Hansen (1992)). It removes the arbitrariness associated with the use of bandwidth and improves the mean square error of semiparametric estimates of cointegrating relationship. This choice was

applied in the calculation of percentiles on base of 50 000 replications for the L_c test with Parzen kernel, given in the Appendix B.

4. Comparison of the Hansen tests for different kernels

To compare performance of the Hansen tests for a small model, estimated by use of the Hansen method with different choice of kernels, we have performed a Monte Carlo study. The data generating process consists of one equation with two exogenous variables, and changing vector of parameters

$$y_t = \beta_{1t}\, x_{1t} + \beta_{2t}\, x_{2t} + \varepsilon_t,$$

$$x_{it} = x_{i,t-1} + \varepsilon_{it} \text{ for } i = 1,2,$$

where the parameter β_1 changed linearly from value of 1 for the first 30 observations to value of 2 for last 30 observations; $\varepsilon_t \sim N\left(0,\sigma^2\right)$ and $\varepsilon_{it} \sim N\left(0,\sigma_i^2\right)$ are identically independently distributed, with no cross-correlation.

For this data generating process, under the null hypothesis of the parameter stability, the model estimated with use of Hansen method is overspecified. In the process of estimation prewhitening was performed, with the number of lags equal to 0,1,2…,5.

The aim of the simulations performed was to compare behaviour of the test statistics of all the three Hansen tests, when different possible kernels are applied in the process of estimation, to check robustness of test statistic with respect to misspecification error, and to perform this comparison for the relatively simple formulation of the model. The results help to make the best choice of kernel in a sense of robustness and performance of the tests. Computations were repeated for 10000 replications, with the number of observations fixed at 100. The model was estimated by the Phillips-Hansen method for seven possible kernels: the Bohman, Parzen, Fejer, Tukey-Hamming, Tukey-Hanning, Riesz and Quadratic Spectral kernel.

Both simulated statistic values and percentiles were computed. A comparison of graphs of the values of the L_c test statistic for fixed number of lags and different choice of the kernel shows that dependence of results on the choice of kernel is not too strong. It is worth pointing out that the values of percentiles do not change monotonically with an

increase of the number of lags, and behaviour for different kernels is not the same, although practically similar.

In the appendix for all the three test statistics there are tables containing percentiles find in Monte Carlo study for 10000 replications and 100 observations each. Additional tables contain values of percentiles for the L_c test statistic computed with use of the Parzen kernel, with no prewhitening and no trends, computed for 50000 replications.

The inferences drawn are the following.

4.1. Comparison of test statistics for Bohman, Fejer, Riesz, Parzen, Tukey-Hamming, Tukey-Hanning and Quadratic Spectral kernels.

The correlation between values of statistics simulated for several lags is very high for all kernels. The performance of tests does not change much with a choice of different kernel. Test statistics are highly correlated especially when no lags are included for error terms (see Table 1). In the Appendix we give tables of the Hansen test statistic percentiles. The table A.1 contains 99%, 97%, 95%, 90%, 75%, 50%, 25%, 10% , 5% and 2.5% percentiles for the L_c test statistic. The values are the same for all the kernels, with the only exception of the Quadratic Spectral kernel. With increase of number of lags the values tend to differ.

Graphs of the percentiles for the seven kernels (see Fig.1) show that although differences between critical values for kernels are not big, yet their behaviour is slightly different. Fig.2 shows graphs of sorted simulated values of the L_c statistic for fixed number of lags. Fig. 3a-3g show, for fixed kernel, graphs of sorted simulated values with 0,1,...,5 lags. Fig.4 shows the graphs of percentiles for the *MeanF* test. The percentiles themselves and the differences between them for the seven kernels have greater values than in case of the L_c test. Fig.5 shows the sorted simulated values of the test statistics for fixed number of lags. Fig.6 shows the graphs of percentiles of the *SupF* test statistic, for which the values and the spread between the maximum and minimum are even higher than for the *MeanF* test statistic, otherwise pattern of the graph is similar.

Table 1. Correlation matrices

With no lags, correlations between the generated Lc test statistics for different kernel are equal to 1.

Correlations for 1 lag

Kernel	*B*	*F*	*R*	*P*	*THM*	*THN*	*QS*
Bohman	1.0000	0.9999	1.0000	0.9996	0.9999	0.9999	0.9996
Fejer	0.9999	1.0000	0.9999	0.9998	1.0000	1.0000	0.9998
Riesz	1.0000	0.9999	1.0000	0.9996	0.9998	0.9999	0.9995
Parzen	0.9996	0.9998	0.9996	1.0000	0.9998	0.9998	0.9999
Tukey-Hamming	0.9999	1.0000	0.9998	0.9998	1.0000	1.0000	0.9998
Tukey-Hanning	0.9999	1.0000	0.9999	0.9998	1.0000	1.0000	0.9998
Quadratic Spectral	0.9996	0.9998	0.9995	0.9999	0.9998	0.9998	1.0000

Correlations for 2 lags

Kernel	*B*	*F*	*R*	*P*	*THM*	*THN*	*QS*
Bohman	1.0000	0.9999	1.0000	0.9995	0.9998	0.9998	0.9996
Fejer	0.9999	1.0000	0.9998	0.9997	0.9999	0.9999	0.9998
Riesz	1.0000	0.9998	1.0000	0.9994	0.9997	0.9998	0.9995
Parzen	0.9995	0.9997	0.9994	1.0000	0.9998	0.9998	0.9999
Tukey-Hamming	0.9998	0.9999	0.9997	0.9998	1.0000	1.0000	0.9999
Tukey-Hanning	0.9998	0.9998	0.9998	0.9998	1.0000	1.0000	0.9998
Quadratic Spectral	0.9996	0.9998	0.9995	0.9999	0.9998	0.9998	1.0000

Correlations for 3 lags

Kernel	*B*	*F*	*R*	*P*	*THM*	*THN*	*QS*
Bohman	1.0000	0.9999	1.0000	0.9998	0.9999	0.9999	0.9998
Fejer	0.9999	1.0000	0.9999	0.9998	0.9999	0.9999	0.9999
Riesz	1.0000	0.9999	1.0000	0.9997	0.9998	0.9998	0.9998
Parzen	0.9998	0.9998	0.9997	1.0000	0.9999	0.9999	0.9999
Tukey-Hamming	0.9999	0.9999	0.9998	0.9999	1.0000	1.0000	1.0000
Tukey-Hanning	0.9999	0.9999	0.9998	0.9999	1.0000	1.0000	0.9999
Quadratic Spectral	0.9998	0.9999	0.9998	0.9999	1.0000	0.9999	1.0000

Correlations for 4 lags

Kernel	*B*	*F*	*R*	*P*	*THM*	*THN*	*QS*
Bohman	1.0000	0.9999	1.0000	0.9993	0.9999	1.0000	0.9999
Fejer	0.9999	1.0000	0.9999	0.9995	0.9999	0.9999	0.9999
Riesz	1.0000	0.9999	1.0000	0.9993	0.9999	0.9999	0.9999
Parzen	0.9993	0.9995	0.9993	1.0000	0.9994	0.9994	0.9995
Tukey-Hamming	0.9999	0.9999	0.9999	0.9994	1.0000	1.0000	0.9999
Tukey-Hanning	1.0000	0.9999	0.9999	0.9994	1.0000	1.0000	0.9999
Quadratic Spectral	0.9999	0.9999	0.9999	0.9995	0.9999	0.9999	1.0000

Correlations for 5 lags

Kernel	*B*	*F*	*R*	*P*	*THM*	*THN*	*QS*
Bohman	1.0000	0.9998	1.0000	0.9990	0.9999	1.0000	0.9997
Fejer	0.9998	1.0000	0.9998	0.9994	0.9999	0.9999	0.9999
Riesz	1.0000	0.9998	1.0000	0.9989	0.9999	0.9999	0.9997
Parzen	0.9990	0.9994	0.9989	1.0000	0.9993	0.9992	0.9997
Tukey-Hamming	0.9999	0.9999	0.9999	0.9993	1.0000	1.0000	0.9999
Tukey-Hanning	1.0000	0.9999	0.9999	0.9992	1.0000	1.0000	0.9998
Quadratic Spectral	0.9997	0.9999	0.9997	0.9997	0.9999	0.9998	1.0000

Source: own computations in GAUSS.

4.2. The test statistics for different kernels and the same number of lags.

As it can be seen from the critical values tables, and from visual inspection of the whole sample of statistics values, when there are no lags in the model, the values of the three tests statistics do not differ, with exclusion of the Quadratic Spectral kernel case, for which however the differences are small. For one lag the L_c test statistics differs slightly between kernels. For 2,3,..,5 lags there are also slight differences. Behaviour of the *MeanF* and *SupF* test statistics shows that differences between kernels in the case of misspecification (lags) are bigger than for the L_c statistic.

4.3. The test statistics for each of the kernels, and different lags.

The next question is whether performance of the test for a given kernel is robust to a misspecification error. Graphs show that for Parzen, Bohman and Quadratic Spectral kernels values of statistics seem to be more concentrated, for other kernels slightly more dispersed. As for proper specification the values of statistics do not differ for the seven kernels, to compare the test statistics we applied a simple measure: the Euclidean distance between vector b_i of sorted simulated values computed for i lags (i.e. for the case of misspecification) and vector b_0 computed when no lags were included. The results shown in Table 2 are as follows: For the Parzen and Bohman kernel the average distance between vector of the L_c test statistics for no misspecification and for misspecified model is smallest. It means that they are relatively „safer" than others to apply in practice, when we may not be sure what the actual number of lags is.

Table 2b compares the distances for the *MeanF* test statistic. The values of the MeanF statistics, and hence the values of percentiles, are higher than the values for the Lc statistics. Here again the Bohman and Parzen kernels give values the most concentrated around values for properly specified model. The Parzen kernel performs slightly better than others in this case.

As it can be seen from the Table 2c, the distances between the vectors of simulated values of the *SupF* test statistics are even higher than for the *MeanF* test statistics. Quadratic Spectral kernel does not perform particularly well in this case. The best choice

again seems to be the Parzen kernel, the second best the Bohman kernel, as they give the most „concentrated" values of test statistic.

5. Summary

Long-run dynamic relationships can be described and studied by using cointegration techniques and stability tests. The tests and methods proposed by Hansen are useful in that they allow not only for the estimation of a cointegration relationship, but also for testing stability of parameters of a model. The Hansen and Phillips-Hansen methods seem to be well suited for modelling relationships describing transforming economies, because those methods make corrections for such characteristics of the relationship as long run variation changes and correlation with past values of errors. The Hansen methods of estimation and testing are flexible and allow for a different choice of kernels for the spectral window correction. The Monte Carlo results presented here show that the behaviour of the test statistics for fixed number of lags does not change much with choice of kernel used for correction. The percentiles computed for the kernels differ in their behaviour. For some of them the value of, say, 95% percentile with increase of number of lags first increases, and then diminishes; but for some kernels, after this decrease the value of percentile for greater number of lags increases again. Also the spread between minimum and maximum value of a particular percentile differs for different kernels.

Hansen (1992) studied asymptotic values of tests; he showed that asymptotic properties of tests did not differ much for different kernels, and that they did depend on choice of bandwidth parameter. This is in agreement with the views established in the statistical literature on kernel density estimation (cf. e.g. Koronacki (1984) or Devroye and Gyorfi (1985)), that the choice of particular kernel is not too important as far as it „behaves well" in the sense of quite general properties.

Presented here results of the Monte Carlo study for samples of 100 observations for several kernels - the Bohman, Fejer, Parzen, Tukey-Hamming, Tukey-Hanning and Quadratic Spectral kernel - show that the most robust versus misspecification of too much lags are the Parzen and Bohman kernel. The Parzen kernel is also one of the most efficient in the sense of definition given by Silverman (1989). The Quadratic Spectral test as implemented in the COINT 2.0 package of GAUSS proved not to be so efficient as it

should be in view of its optimality (i.e. the property of minimising the mean square error for univariate density estimation.

Table 2a. Euclidean distance between lagged and unlagged Lc-statistic vectors.

Kernel	1	2	3	4	5	Average
Bohman	9.3479	16.8866	14.5604	9.5629	7.7822	11.6280
Fejer	16.5109	11.5789	6.5317	10.0945	15.7037	12.0839
Parzen	7.0583	15.8479	15.6711	11.2520	7.8302	11.5319
Riesz	30.8126	16.1319	7.9709	14.9746	21.5288	18.2838
Tukey-Hamming	18.4101	17.2341	9.4640	8.9877	14.7030	13.7598
Tukey- Hanning	16.5489	18.3799	11.2627	7.9681	12.3908	13.3101
Quadratic-Spectral	18.6630	10.3894	9.2247	17.6811	24.6425	16.1201

Table 2b. Euclidean distance between lagged and unlagged MeanF-statistic vectors.

Kernel	1	2	3	4	5	Average
Bohman	76.7984	138.2533	121.5307	90.1896	87.0147	102.7573
Fejer	135.3217	96.8429	72.3748	104.6866	147.3695	111.3191
Parzen	57.9594	129.7824	129.3927	99.5261	83.5013	100.0324
Riesz	251.6723	135.0844	89.4592	141.3224	187.3562	160.9789
Tukey-Hamming	151.0652	142.2108	91.0532	97.7357	139.8419	124.3814
Tukey-Hanning	135.8475	150.9809	101.2276	89.0060	123.1389	120.0402
Quadratic - Spectral	152.5336	93.4170	99.9312	162.6535	212.2169	144.1505

Table 2c. Euclidean distance between lagged and unlagged SupF-statistic vectors.

Kernel	1	2	3	4	5	Average
Bohman	126.1224	234.0011	220.5916	181.0892	170.2419	186.4092
Fejer	223.0511	175.8479	144.5053	182.3611	236.8789	192.5288
Parzen	94.9778	217.8498	228.8778	192.9977	169.6942	180.8795
Riesz	419.5194	252.9582	171.4161	218.3217	301.3494	272.7129
Tukey-Hamming	250.0871	252.8174	184.4440	179.2505	221.0737	217.5345
Tukey-Hanning	224.5620	264.0019	199.5678	173.4820	203.2153	212.9658
Quadratic Spectral	259.0574	186.2282	179.6572	249.7374	309.7566	236.8874

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Appendix A: The Results of Simulations.

B - Bohman kernel
F - Fejer kernel
P - Parzen kernel
R - Riesz kernel
THM - Tukey-Hamming kernel
THN - Tukey-Hanning kernel
QS - Quadratic-Spectral kernel

A. THE RESULTS FOR THE Lc TEST STATISTIC

Table A.1: The Lc Statistic Percentiles for 0,1,2,...,5 Lags

99% Percentiles:

B	2.4928	2.6357	2.7183	2.5642	2.3888	2.2171
F	2.4928	2.7636	2.5275	2.3131	2.1183	1.9676
P	2.4928	2.6093	2.7124	2.6137	2.4533	2.2869
R	2.4928	3.0199	2.5528	2.2096	1.9873	1.8539
THAM	2.4928	2.7978	2.6262	2.3762	2.1418	1.9776
THAN	2.4928	2.7590	2.6716	2.4470	2.2104	2.0500
QS	2.5626	2.8243	2.5289	2.2047	1.9912	1.8582

97.5% Percentiles:

B	2.1840	2.3250	2.4290	2.2935	2.1335	1.9928
F	2.1840	2.4573	2.2393	2.0654	1.8949	1.7421
P	2.1840	2.2880	2.4219	2.3442	2.1986	2.0500
R	2.1840	2.7176	2.2927	1.9737	1.7547	1.6309
THAM	2.1840	2.4845	2.3488	2.1212	1.9250	1.7557
THAN	2.1840	2.4558	2.3892	2.1682	1.9815	1.8126
QS	2.2492	2.5301	2.2507	1.9896	1.7561	1.6315

95% Percentiles:

B	1.9078	2.0506	2.1188	2.0411	1.9076	1.7745
F	1.9078	2.1466	2.0045	1.8249	1.6709	1.5661
P	1.9078	2.0186	2.1163	2.0693	1.9561	1.8340
R	1.9078	2.3525	2.0455	1.7590	1.5717	1.4571
THAM	1.9078	2.1715	2.0931	1.8973	1.7149	1.5755
THAN	1.9078	2.1459	2.1129	1.9474	1.7708	1.6253
QS	1.9778	2.2162	2.0049	1.7672	1.5892	1.4698

90% Percentiles:

B	1.6244	1.7586	1.8454	1.7854	1.6607	1.5505
F	1.6244	1.8550	1.7446	1.5955	1.4681	1.3707
P	1.6244	1.7232	1.8359	1.8103	1.7088	1.5938
R	1.6244	2.0635	1.7823	1.5336	1.3742	1.2655
THAM	1.6244	1.8816	1.8234	1.6492	1.5021	1.3813
THAN	1.6244	1.8556	1.8484	1.6938	1.5436	1.4244
QS	1.6840	1.9317	1.7378	1.5416	1.3879	1.2768

75% Percentiles:

B	1.2009	1.3065	1.3918	1.3596	1.2759	1.1938
F	1.2009	1.3844	1.3212	1.2138	1.1178	1.0366
P	1.2009	1.2799	1.3811	1.3782	1.3055	1.2303
R	1.2009	1.5531	1.3714	1.1819	1.0469	0.9549
THAM	1.2009	1.4066	1.3908	1.2726	1.1523	1.0556
THAN	1.2009	1.3850	1.4102	1.3023	1.1867	1.0908
QS	1.2448	1.4604	1.3457	1.1889	1.0630	0.9704

50% Percentiles:

B	0.7927	0.8836	0.9636	0.9547	0.9073	0.8520
F	0.7927	0.9490	0.9186	0.8541	0.7883	0.7330
P	0.7927	0.8599	0.9510	0.9619	0.9269	0.8761
R	0.7927	1.0805	0.9768	0.8466	0.7466	0.6776
THAM	0.7927	0.9667	0.9769	0.9075	0.8283	0.7539
THAN	0.7927	0.9502	0.9865	0.9248	0.8526	0.7801
QS	0.8315	1.0130	0.9543	0.8507	0.7592	0.6896

25% Percentiles:

B	0.5100	0.5750	0.6502	0.6543	0.6245	0.5869
F	0.5100	0.6309	0.6296	0.5858	0.5448	0.5081
P	0.5100	0.5586	0.6381	0.6568	0.6364	0.6030
R	0.5100	0.7350	0.6706	0.5841	0.5193	0.4667
THAM	0.5100	0.6455	0.6719	0.6231	0.5707	0.5220
THAN	0.5100	0.6315	0.6746	0.6385	0.5862	0.5394
QS	0.5360	0.6919	0.6558	0.5878	0.5248	0.4775

10% Percentiles:

B	0.3443	0.3911	0.4427	0.4509	0.4366	0.4130
F	0.3443	0.4296	0.4360	0.4088	0.3803	0.3545
P	0.3443	0.3795	0.4323	0.4512	0.4432	0.4241
R	0.3443	0.5009	0.4672	0.4107	0.3664	0.3335
THAM	0.3443	0.4378	0.4629	0.4372	0.4026	0.3691
THAN	0.3443	0.4302	0.4645	0.4476	0.4132	0.3814
QS	0.3643	0.4734	0.4579	0.4136	0.3702	0.3386

5% Percentiles:

B	0.2715	0.3158	0.3621	0.3655	0.3569	0.3384
F	0.2715	0.3495	0.3521	0.3344	0.3145	0.2939
P	0.2715	0.3019	0.3562	0.3680	0.3603	0.3471
R	0.2715	0.4037	0.3786	0.3370	0.2994	0.2753
THAM	0.2715	0.3568	0.3745	0.3577	0.3291	0.3029
THAN	0.2715	0.3500	0.3763	0.3659	0.3388	0.3121
QS	0.2884	0.3791	0.3751	0.3364	0.3038	0.2794

2.5% Percentiles:

B	0.2226	0.2552	0.2919	0.3016	0.2975	0.2805
F	0.2226	0.2796	0.2863	0.2786	0.2599	0.2427
P	0.2226	0.2480	0.2851	0.3010	0.3004	0.2891
R	0.2226	0.3219	0.3090	0.2800	0.2504	0.2323
THAM	0.2226	0.2882	0.3094	0.2990	0.2713	0.2556
THAN	0.2226	0.2808	0.3087	0.3026	0.2796	0.2602
QS	0.2360	0.3101	0.3115	0.2817	0.2552	0.2375

B. THE RESULTS FOR THE MeanF TEST STATISTIC

Table B.1: The MeanF Statistic Percentiles for 0,1,2,...,5 Lags

99% Percentiles:

B	17.2840	18.1309	18.4104	17.3445	16.1247	15.1053
F	17.2840	18.8699	17.2456	15.5039	14.4295	13.4933
P	17.2840	17.9659	18.4487	17.6828	16.5369	15.5654
R	17.2840	20.6236	17.3735	15.1170	14.0210	13.5092
THAM	17.2840	19.0378	17.8092	16.0335	14.6475	13.7676
THAN	17.2840	18.8811	18.1187	16.4500	15.0945	14.1635
QS	17.6934	19.1336	16.9648	14.9909	13.8738	13.3591

97.5% Percentiles:

B	15.6462	16.7135	17.0101	16.0927	14.8783	13.9059
F	15.6462	17.3363	15.7949	14.3438	13.2289	12.3849
P	15.6462	16.3687	16.9951	16.3912	15.3022	14.3607
R	15.6462	18.9983	15.9545	13.7991	12.5708	12.1179
THAM	15.6462	17.4991	16.4417	14.7560	13.5608	12.5257
THAN	15.6462	17.3285	16.7578	15.1169	13.8372	12.8706
QS	16.1009	17.7169	15.6872	13.8130	12.6105	12.0916

95% Percentiles:

B	14.5339	15.3507	15.7399	14.9701	13.8652	12.8558
F	14.5339	15.9657	14.7188	13.3740	12.3118	11.5229
P	14.5339	15.1719	15.7307	15.2616	14.2672	13.2892
R	14.5339	17.5588	14.8609	12.7990	11.6363	11.0260
THAM	14.5339	16.1856	15.2840	13.7102	12.4902	11.6322
THAN	14.5339	15.9684	15.5111	14.1113	12.7886	11.9000
QS	14.8948	16.3473	14.5520	12.8101	11.6835	11.0777

90% Percentiles:

B	12.9114	13.7890	14.2300	13.6040	12.7202	11.8032
F	12.9114	14.4370	13.3929	12.2147	11.1843	10.4375
P	12.9114	13.5723	14.1933	13.8488	12.9925	12.1936
R	12.9114	15.9392	13.5588	11.6416	10.4569	9.8100
THAM	12.9114	14.5959	13.9251	12.6106	11.4275	10.4915
THAN	12.9114	14.4365	14.1577	12.9275	11.7653	10.8040
QS	13.3184	14.8597	13.3385	11.7222	10.5338	9.7963

75% Percentiles:

B	10.4399	11.2413	11.8198	11.4047	10.6445	9.8962
F	10.4399	11.8705	11.1379	10.1683	9.3352	8.6631
P	10.4399	11.0527	11.7736	11.5942	10.9090	10.1829
R	10.4399	13.1198	11.3808	9.8250	8.7296	8.0652
THAM	10.4399	12.0648	11.6840	10.5709	9.5648	8.7812
THAN	10.4399	11.8858	11.8520	10.8497	9.8572	9.0508
QS	10.8164	12.3684	11.1715	9.8232	8.8345	8.1508

50% Percentiles:

B	7.7655	8.5879	9.1943	8.9621	8.4788	7.9091
F	7.7655	9.1509	8.7106	8.0469	7.4131	6.8679
P	7.7655	8.3908	9.0981	9.0853	8.6753	8.1580
R	7.7655	10.3034	9.0769	7.8417	6.9437	6.3190
THAM	7.7655	9.2822	9.1973	8.4358	7.6410	7.0126
THAN	7.7655	9.1476	9.3103	8.6357	7.8911	7.2425
QS	8.1264	9.6681	8.9093	7.8673	7.0293	6.4257

25% Percentiles:

B	5.1894	5.9056	6.6729	6.7648	6.4535	6.0807
F	5.1894	6.5000	6.4751	6.0556	5.5930	5.1938
P	5.1894	5.7153	6.5368	6.7812	6.5764	6.2459
R	5.1894	7.5622	7.0145	6.0321	5.3498	4.8650
THAM	5.1894	6.6524	6.9214	6.4616	5.9008	5.3747
THAN	5.1894	6.5032	6.9738	6.6309	6.0845	5.5734
QS	5.4671	7.1301	6.8214	6.0694	5.4132	4.9457

10% Percentiles:

B	3.4484	4.0760	4.7372	4.9556	4.8161	4.5561
F	3.4484	4.5135	4.6300	4.4526	4.1673	3.8972
P	3.4484	3.8970	4.6126	4.9199	4.8824	4.6854
R	3.4484	5.4851	5.2136	4.5626	4.0675	3.6913
THAM	3.4484	4.6465	5.0890	4.8542	4.4417	4.0473
THAN	3.4484	4.5170	5.0951	4.9328	4.5872	4.2005
QS	3.6971	5.1206	5.1034	4.5855	4.0986	3.7342

5% Percentiles:

B	2.7086	3.2089	3.8629	4.0958	4.0247	3.8546
F	2.7086	3.6397	3.8732	3.7366	3.5354	3.3046
P	2.7086	3.0588	3.7441	4.0536	4.0551	3.9485
R	2.7086	4.4583	4.3837	3.8891	3.4988	3.1226
THAM	2.7086	3.7400	4.2098	4.0684	3.7986	3.4858
THAN	2.7086	3.6482	4.2234	4.1305	3.8760	3.6049
QS	2.8901	4.1617	4.2264	3.8868	3.5002	3.2135

2.5% Percentiles:

B	2.2705	2.6967	3.1935	3.4544	3.4728	3.3896
F	2.2705	2.9829	3.1895	3.1619	3.0066	2.8313
P	2.2705	2.5838	3.0876	3.4145	3.4607	3.4323
R	2.2705	3.7061	3.8097	3.3999	3.0194	2.7187
THAM	2.2705	3.0616	3.5488	3.5254	3.3390	3.0449
THAN	2.2705	2.9844	3.5320	3.5817	3.3958	3.1536
QS	2.4277	3.5050	3.6890	3.4054	3.0761	2.7882

C. THE RESULTS FOR THE SupF TEST STATISTIC

Table C.1: The SupF Statistic Percentiles for 0,1,2,...,5 Lags

99% Percentiles:

B	26.1285	27.1286	27.2754	25.7289	24.0113	23.2182
F	26.1285	28.2026	25.4925	23.4868	22.6487	21.9326
P	26.1285	26.8726	27.3198	26.1297	24.5511	23.4826
R	26.1285	30.4739	25.6580	24.1100	23.8330	25.7152
THAM	26.1285	28.4864	26.2325	23.8845	23.0274	22.7677
THAN	26.1285	28.2546	26.5407	24.4400	23.2331	22.7999
QS	26.6966	28.2434	25.2001	23.3691	23.1332	23.4450

97.5% Percentiles:

B	24.2289	25.3397	25.5909	24.1145	22.4570	21.1904
F	24.2289	26.2657	23.9066	21.7362	20.4315	19.7001
P	24.2289	25.0122	25.6923	24.6479	23.0293	21.7169
R	24.2289	28.5352	24.0562	21.4831	20.6571	21.1488
THAM	24.2289	26.5091	24.6381	22.3933	20.7482	20.0568
THAN	24.2289	26.2803	25.0729	22.7788	21.2000	20.2444
QS	24.7042	26.6604	23.4157	21.1821	20.2455	20.7152

95% Percentiles:

B	22.5455	23.8125	24.1781	22.7841	21.1354	19.9054
F	22.5455	24.7101	22.4962	20.4457	19.0015	18.1253
P	22.5455	23.4667	24.1697	23.2294	21.7422	20.3696
R	22.5455	26.8422	22.5180	19.9135	18.6210	18.5390
THAM	22.5455	24.9730	23.3142	21.0041	19.3729	18.4381
THAN	22.5455	24.7223	23.6325	21.4676	19.7944	18.7734
QS	23.1067	24.9513	22.1158	19.8132	18.5558	18.2286

90% Percentiles:

B	20.5228	21.6474	22.2088	21.0750	19.6582	18.3848
F	20.5228	22.5848	20.7410	18.9175	17.4917	16.5125
P	20.5228	21.3834	22.1715	21.5256	20.1603	18.9537
R	20.5228	24.7531	21.0443	18.3768	16.8620	16.3323
THAM	20.5228	22.8877	21.5758	19.5441	17.8816	16.7764
THAN	20.5228	22.5886	21.9366	20.0142	18.3601	17.1239
QS	21.0753	23.1316	20.5722	18.3470	16.9058	16.1093

75% Percentiles:

B	17.2400	18.4798	19.3156	18.5828	17.3763	16.1396
F	17.2400	19.4222	18.2142	16.5993	15.2586	14.2415
P	17.2400	18.1776	19.2163	18.8518	17.8172	16.6473
R	17.2400	21.4946	18.5897	16.0706	14.5131	13.5356
THAM	17.2400	19.6900	19.0364	17.2431	15.6298	14.5098
THAN	17.2400	19.4396	19.2454	17.6999	16.1037	14.9076
QS	17.8112	20.1307	18.2210	16.0774	14.5563	13.5703

50% Percentiles:

B	13.6564	15.0179	16.0702	15.7444	14.8685	13.8970
F	13.6564	15.9914	15.2781	14.0823	12.9699	12.0660
P	13.6564	14.6800	15.9226	15.8816	15.2154	14.3400
R	13.6564	17.8340	15.9310	13.8031	12.2787	11.3111
THAM	13.6564	16.2343	16.1110	14.8425	13.4326	12.3394
THAN	13.6564	15.9991	16.2555	15.1767	13.8731	12.7348
QS	14.2316	16.8310	15.6368	13.8401	12.4068	11.4459

25% Percentiles:

B	9.5891	11.0229	12.5817	12.7419	12.2621	11.5314
F	9.5891	12.1279	12.1816	11.4013	10.6330	9.9227
P	9.5891	10.6624	12.2765	12.7733	12.4814	11.8555
R	9.5891	14.2032	13.1448	11.4872	10.2661	9.3811
THAM	9.5891	12.4357	13.0810	12.2689	11.2108	10.3010
THAN	9.5891	12.1309	13.1364	12.5437	11.5611	10.6531
QS	10.1901	13.4307	12.8666	11.5491	10.3960	9.5197

10% Percentiles:

B	6.7533	7.9406	9.4260	9.9680	9.7517	9.3194
F	6.7533	8.9607	9.3771	8.9806	8.4956	8.0039
P	6.7533	7.6449	9.1345	9.8264	9.8796	9.5129
R	6.7533	10.7634	10.6108	9.4342	8.4468	7.6901
THAM	6.7533	9.2183	10.2383	9.8188	9.1664	8.4668
THAN	6.7533	8.9818	10.2124	10.0382	9.3816	8.7606
QS	7.2345	10.1795	10.3140	9.4381	8.4991	7.8193

5% Percentiles:

B	5.7041	6.5842	7.8690	8.3975	8.3986	8.1773
F	5.7041	7.3414	7.8210	7.6835	7.3544	7.0120
P	5.7041	6.3561	7.5675	8.2506	8.4182	8.2858
R	5.7041	9.0162	9.1179	8.3072	7.4943	6.8753
THAM	5.7041	7.6247	8.5954	8.5279	8.0626	7.4605
THAN	5.7041	7.3615	8.5274	8.6732	8.2707	7.6743
QS	6.0731	8.4694	8.8209	8.2987	7.5371	6.9728

2.5% Percentiles:

B	4.8958	5.6769	6.7125	7.3110	7.3420	7.2090
F	4.8958	6.3328	6.8490	6.8035	6.5491	6.2742
P	4.8958	5.5426	6.4826	7.1565	7.3246	7.2403
R	4.8958	7.5415	7.8068	7.2586	6.6760	6.1819
THAM	4.8958	6.5123	7.4112	7.4430	7.1454	6.7066
THAN	4.8958	6.3328	7.3749	7.5272	7.2683	6.8292
QS	5.2314	7.1787	7.6871	7.3150	6.7564	6.1989

Appendix B.

Percentiles for the Lc test with Parzen kernel on base of 50 000 replications.

The author is grateful to W.W. Charemza and B.E. Hansen for permission to use their computer programs. The results presented here resulted from application (50000 replications) of modified procedure by W. W. Charemza for computing and smoothing percentiles, and procedures for the Hansen methods of estimation and testing, written by B. E. Hansen for his 1992a paper.

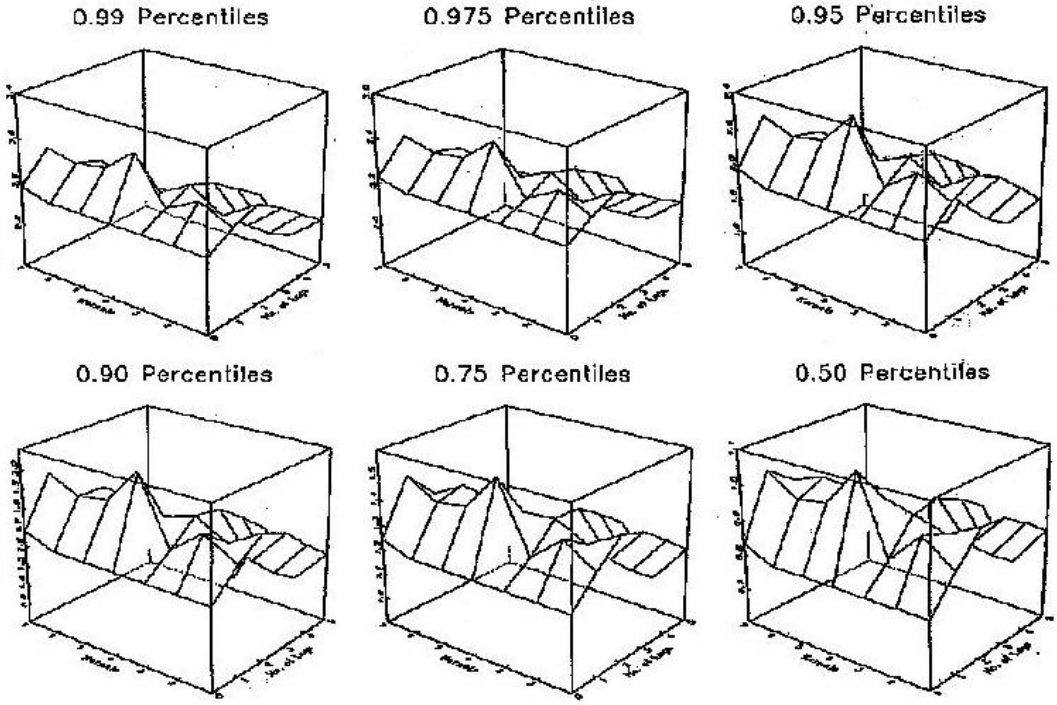
	Percentile:	**0.99**		**Percentile:**	**0.95**
No of obs.	*Lower limit*	*Upper limit*	*No of obs.*	*Lower limit*	*Upper limit*
15	0.72288765	1.1981571	15	0.53910106	0.69082844
20	0.70003127	1.1753007	20	0.52788050	0.67960787
25	0.68511599	1.1603854	25	0.52055834	0.67228572
30	0.67438212	1.1496515	30	0.51528891	0.66701629
40	0.65960326	1.1348727	40	0.50803374	0.65976111
50	0.64963676	1.1249062	50	0.50314102	0.65486839
75	0.63416373	1.1094331	75	0.49554506	0.64727244
100	0.62483784	1.1001073	100	0.49096683	0.64269421
125	0.61836943	1.0936388	125	0.48779138	0.63951876
150	0.61351203	1.0887814	150	0.48540681	0.63713418
175	0.60967270	1.0849421	175	0.48352202	0.63524940
200	0.60652751	1.0817969	200	0.48197799	0.63370537
225	0.60388198	1.0791514	225	0.48067926	0.63240664
250	0.60161112	1.0768805	250	0.47956446	0.63129183
275	0.59963026	1.0748997	275	0.47859202	0.63031940
300	0.59787969	1.0731491	300	0.47773264	0.62946001

	Percentile:	**0.90**		**Percentile**	**0.10**
No. of obs.	*Lower limit*	*Upper limit*	*No. of obs.*	*Lower limit*	*Upper limit*
15	0.44525092	0.55120355	15	0.11067384	0.15282513
20	0.43342866	0.53938129	20	0.10120805	0.14335935
25	0.42571386	0.53166649	25	0.095031003	0.13718230
30	0.42016187	0.52611450	30	0.090585656	0.13273695
40	0.41251764	0.51847027	40	0.084465110	0.12661641
50	0.40736255	0.51331518	50	0.080337561	0.12248886
75	0.39935927	0.50531190	75	0.073929530	0.11608083
100	0.39453553	0.50048816	100	0.070067287	0.11221858
125	0.39118981	0.49714244	125	0.067388446	0.10953974
150	0.38867736	0.49462999	150	0.065376791	0.10752809
175	0.38669150	0.49264413	175	0.063786766	0.10593806
200	0.38506468	0.49101731	200	0.062484209	0.10463551
225	0.38369630	0.48964893	225	0.061388585	0.10353988
250	0.38252172	0.48847435	250	0.060448123	0.10259942
275	0.38149714	0.48744977	275	0.059627768	0.10177907
300	0.38059167	0.48654430	300	0.058902781	0.10105408

	Percentile:	**0.05**		**Percentile:**	**0.025**
No. of obs.	*Lower limit*	*Upper limit*	*No. of obs.*	*Lower limit*	*Upper limit*
15	0.090929093	0.13806116	15	0.074321874	0.13032329
20	0.081979845	0.12911191	20	0.0658164306	0.12181786
25	0.076139875	0.12327194	25	0.060266081	0.11626750
30	0.071937107	0.11906917	30	0.056271736	0.11227316
40	0.066150554	0.11328262	40	0.050772149	0.10677357
50	0.062248242	0.10938031	50	0.047063360	0.10306478
75	0.056189892	0.10332195	75	0.041305454	0.097306875
100	0.052538407	0.099670471	100	0.037835054	0.093836475
125	0.050005749	0.097137812	125	0.035427995	0.091429415
150	0.048103868	0.095235932	150	0.033620432	0.089621853
175	0.046600609	0.093732673	175	0.032191723	0.088193143
200	0.045369132	0.092501195	200	0.031021316	0.087022737
225	0.044333294	0.091465358	225	0.030036848	0.086038269
250	0.043444153	0.090576217	250	0.029191801	0.085193222
275	0.042668564	0.089800627	275	0.028454674	0.084456095
300	0.041983139	0.089115202	300	0.027803241	0.083804662

	Percentile:	**0.01**
No. of obs.	*Lower limit*	*Upper limit*
15	0.054310641	0.12735277
20	0.046267397	0.11930953
25	0.041018655	0.11406079
30	0.037241366	0.11028350
40	0.032040632	0.10508276
50	0.028533383	0.10157552
75	0.023088368	0.096130500
100	0.019806553	0.092848685
125	0.017530296	0.090572428
150	0.015820958	0.088863090
175	0.014469886	0.087512018
200	0.013363081	0.086405213
225	0.012432110	0.085474242
250	0.011632984	0.084675116
275	0.010935913	0.083978045
300	0.010319879	0.083362012

Fig.1: The Lc Test Statistics Percentiles



On right axis: number of lags;
On left axis: kernels: 1– Bohman, 2–Fejer, 3– Riesz, 4–Parzen, 5–Tukey-Hamming, 6– Tukey-Hanning, 7–Quadratic Spectral

Fig. 2: The Lc Test Statistics, Influence of Lags

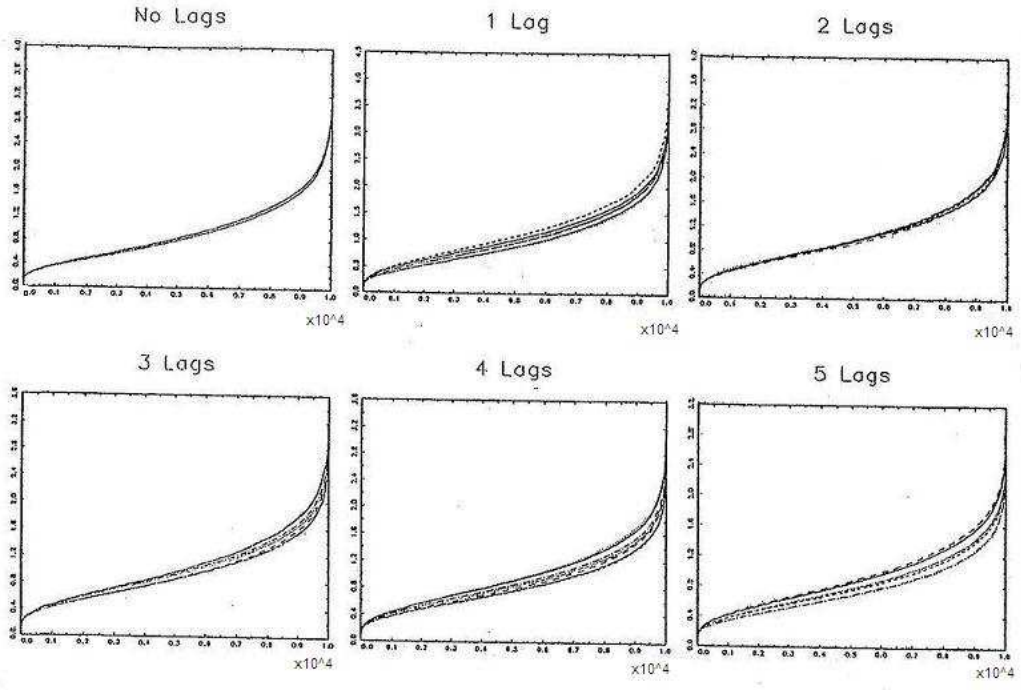


Fig.3a. Lc: The Bohman Kernel

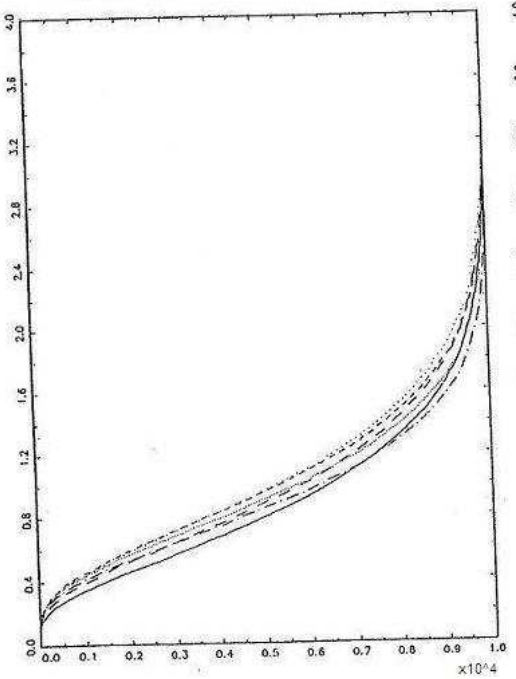


Fig.3b. Lc: The Fejer Kernel

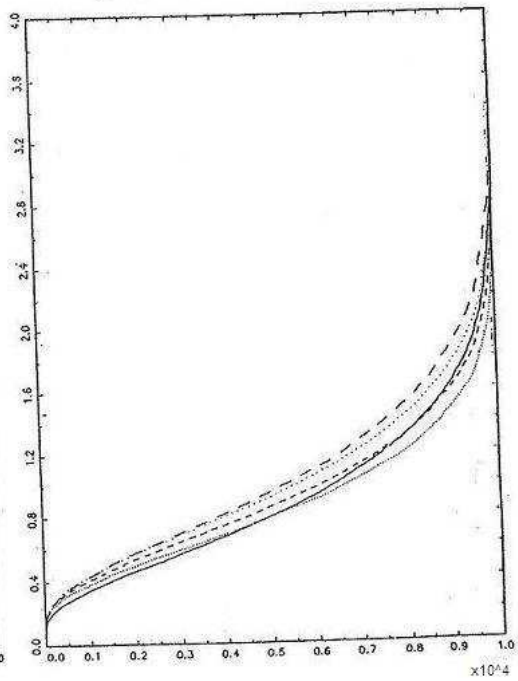


Fig.3c. Lc: The Parzen Kernel

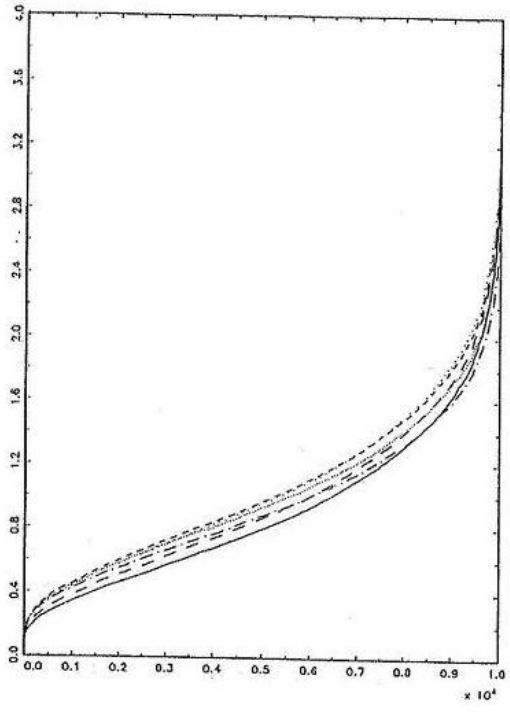
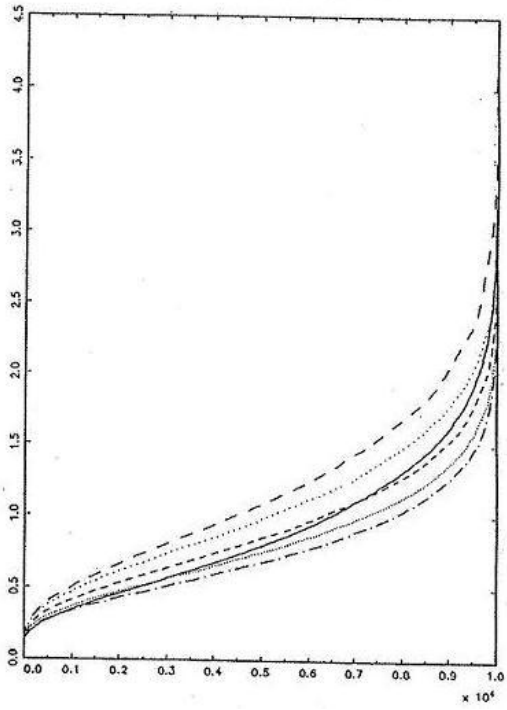
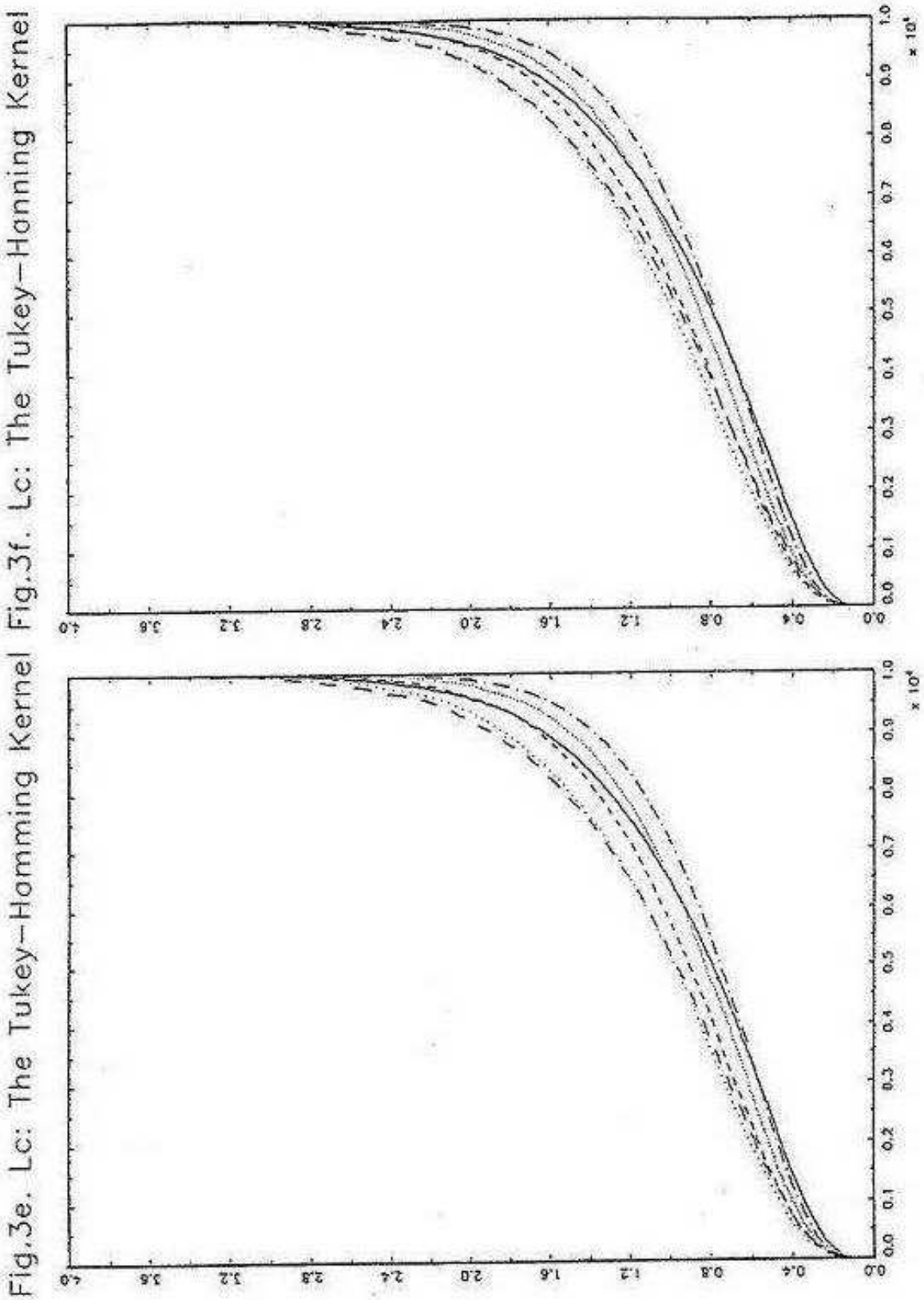
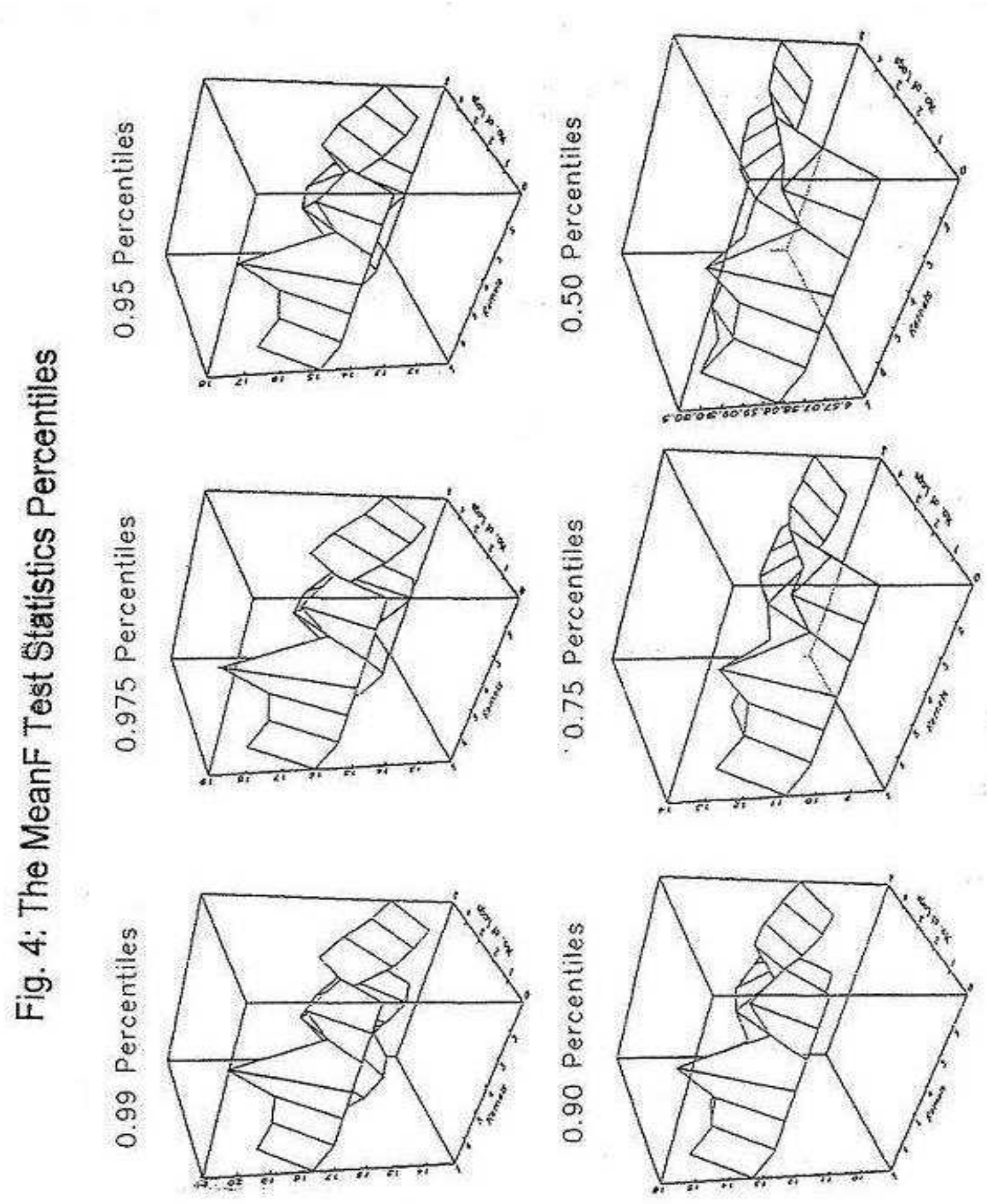


Fig.3d. Lc: The Riesz Kernel




Fig.3e. Lc: The Tukey–Homming Kernel
Fig.3f. Lc: The Tukey–Hanning Kernel



On right axis: number of lags;
On left axis: kernels: 1– Bohman, 2–Fejer, 3– Riesz, 4–Parzen, 5–Tukey-Hamming, 6– Tukey-Hanning, 7–Quadratic Spectral

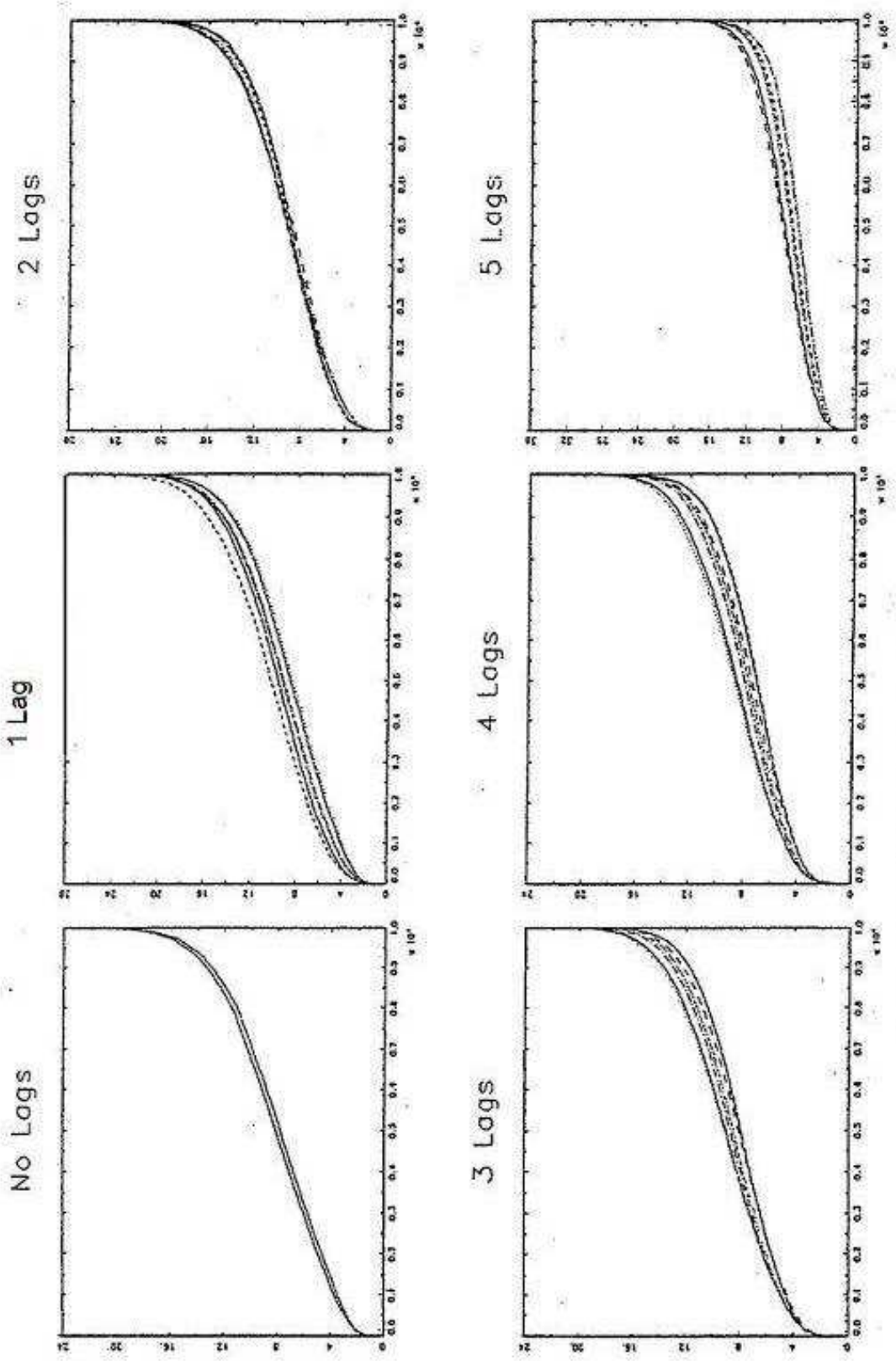


Fig. 5: The MeanF Statistics, Influence of Lags

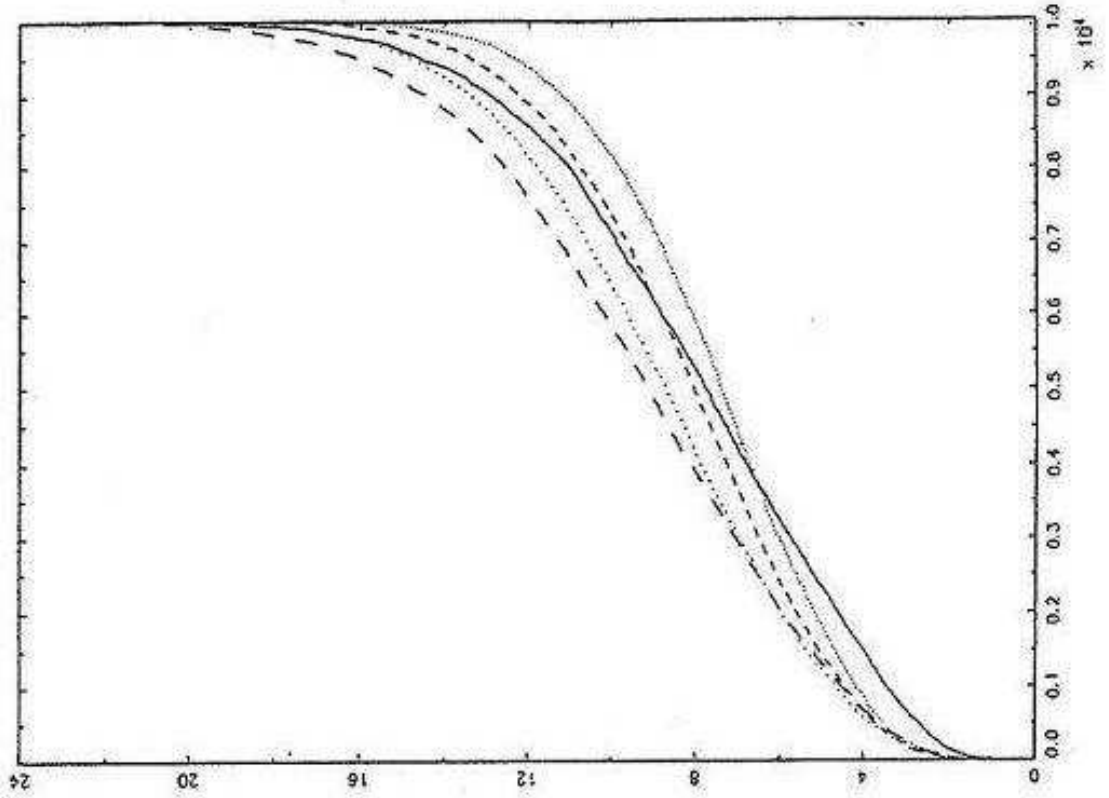


Fig.6b. MeanF: The Fejer Kernel

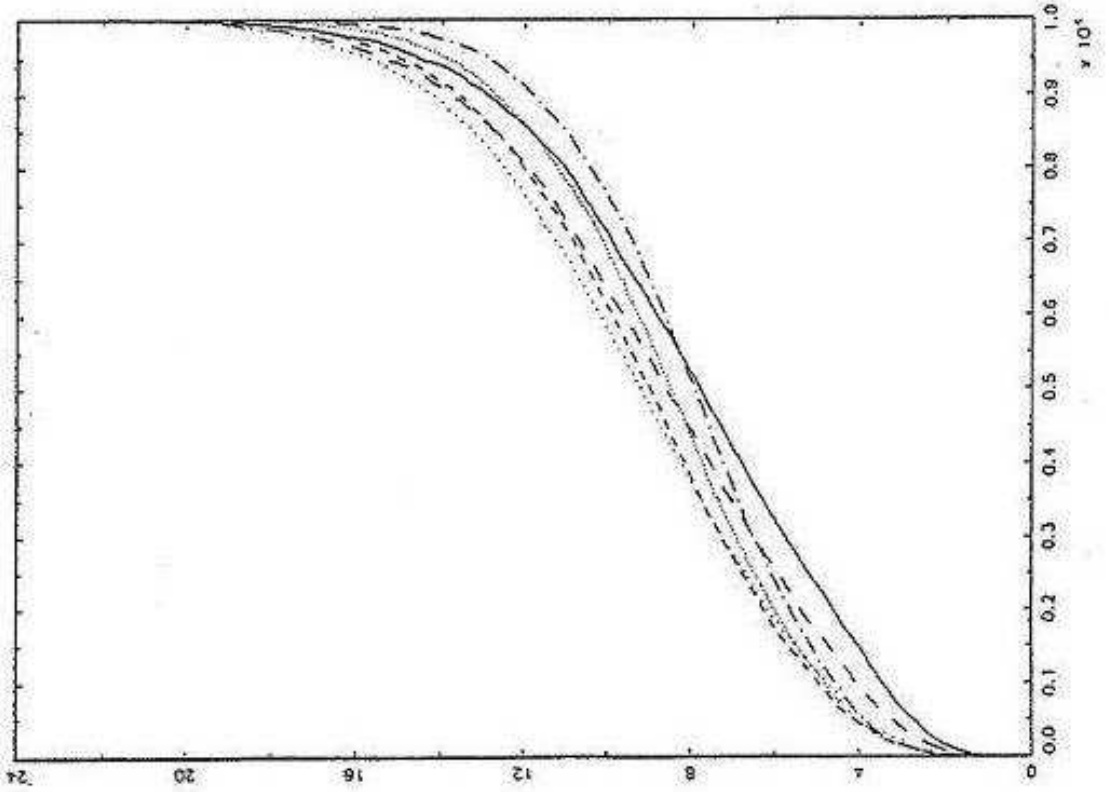


Fig.6a. MeanF: The Bohman Kernel

Fig.6c. MeanF: The Parzen Kernel

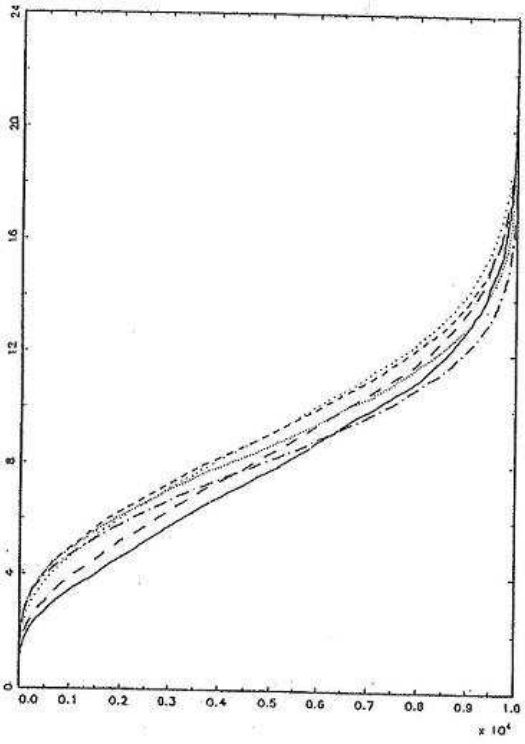


Fig.6d. MeanF: The Riesz Kernel

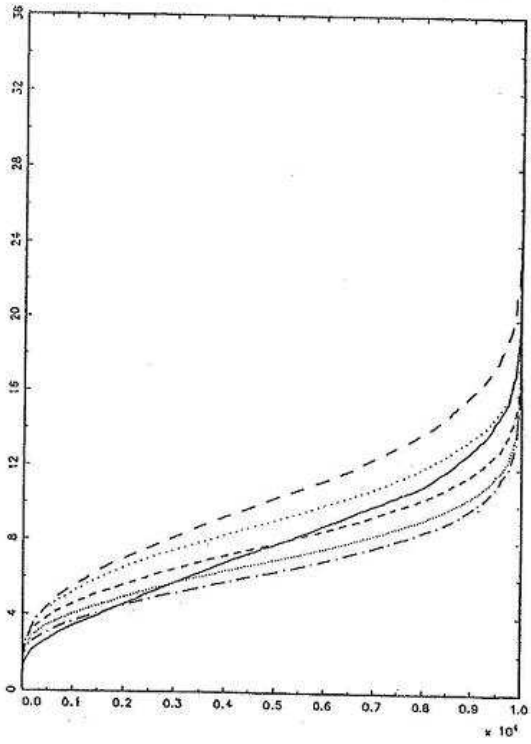


Fig.6e. MeanF: Tukey–Hamming

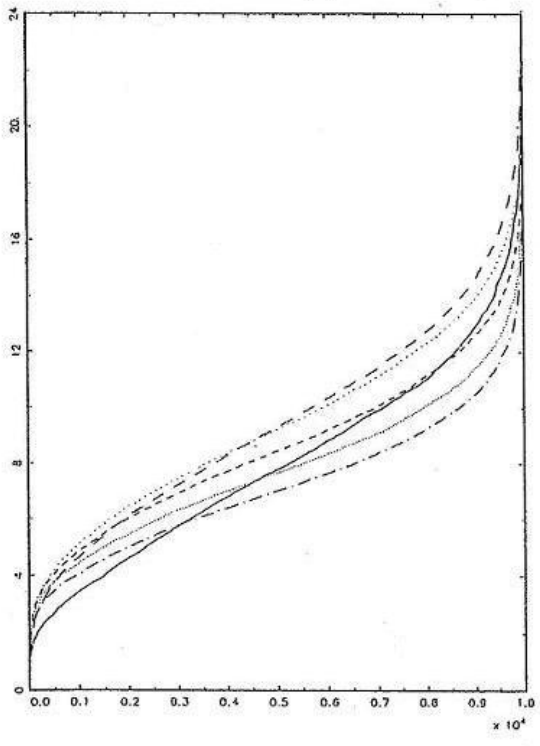


Fig.6f. MeanF: Tukey–Hanning

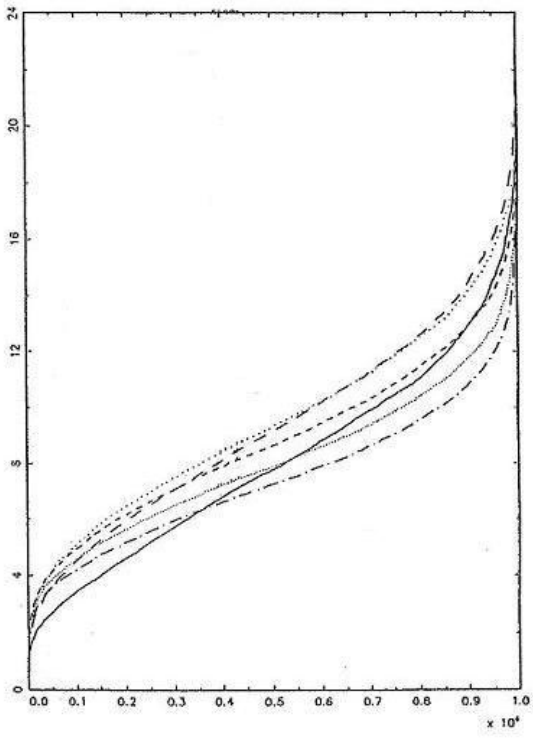
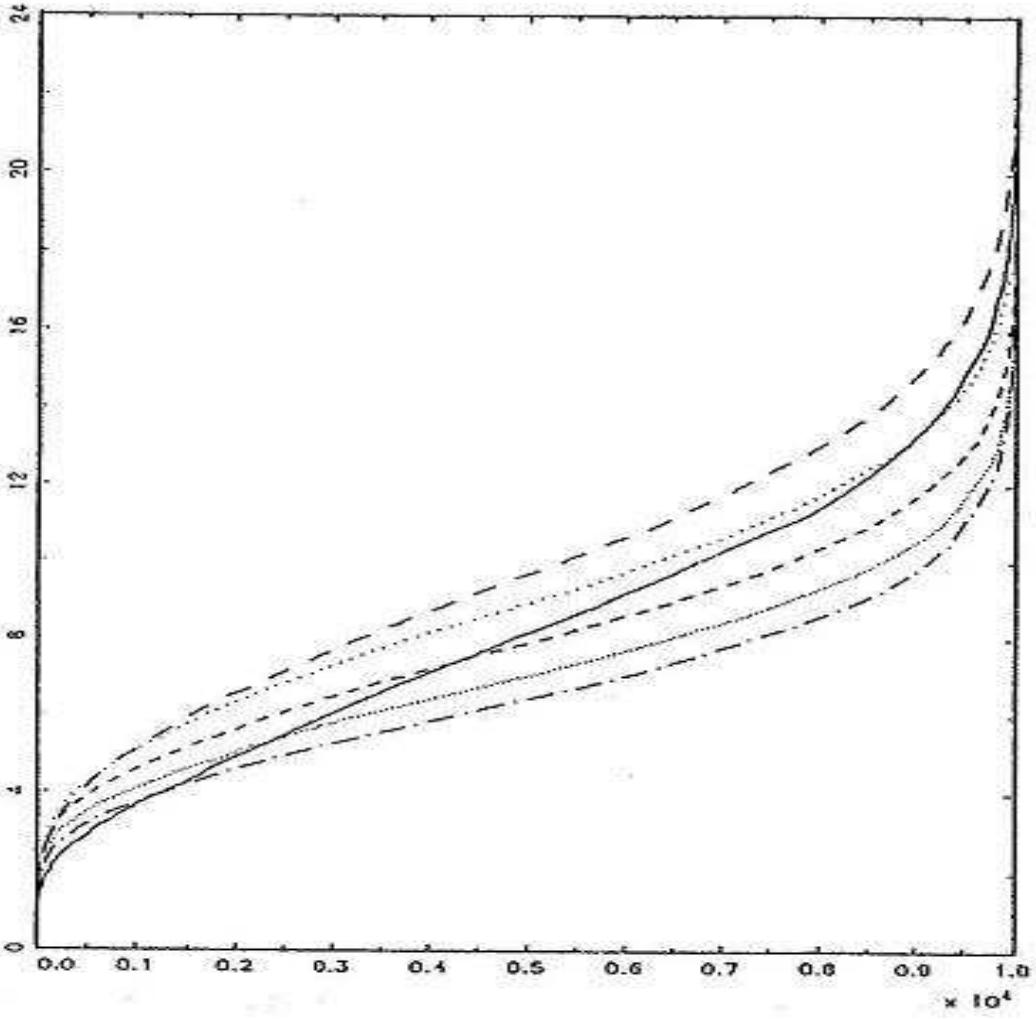


Fig.6g. The Quadratic–Spectral Kernel



O

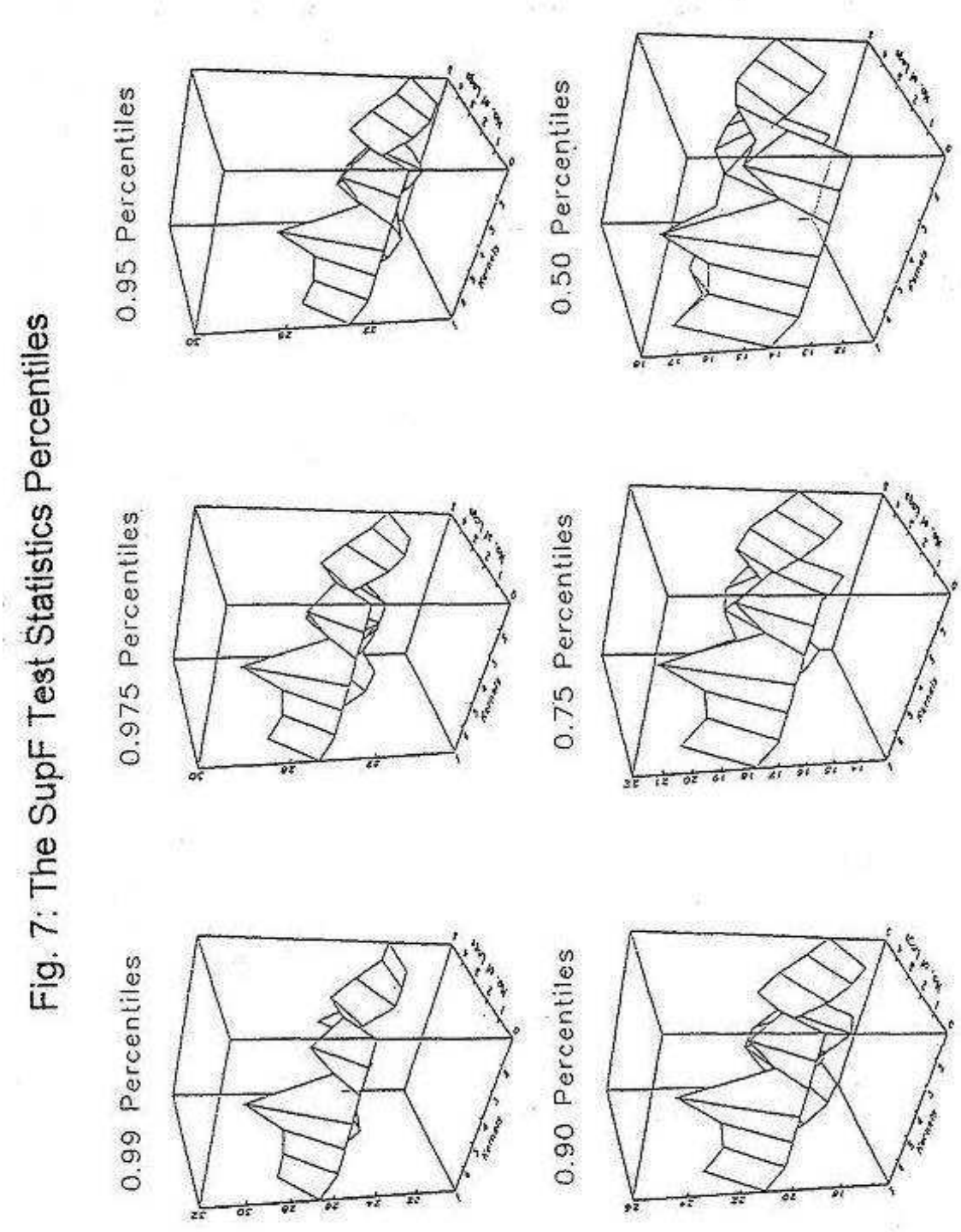


Fig. 7: The SupF Test Statistics Percentiles

On right axis: number of lags;
On left axis: kernels: 1– Bohman, 2–Fejer, 3– Riesz, 4–Parzen, 5–Tukey-Hamming, 6– Tukey-Hanning, 7–Quadratic Spectral.

Fig. 8: The SupF Test Statistics, Influence of Lags

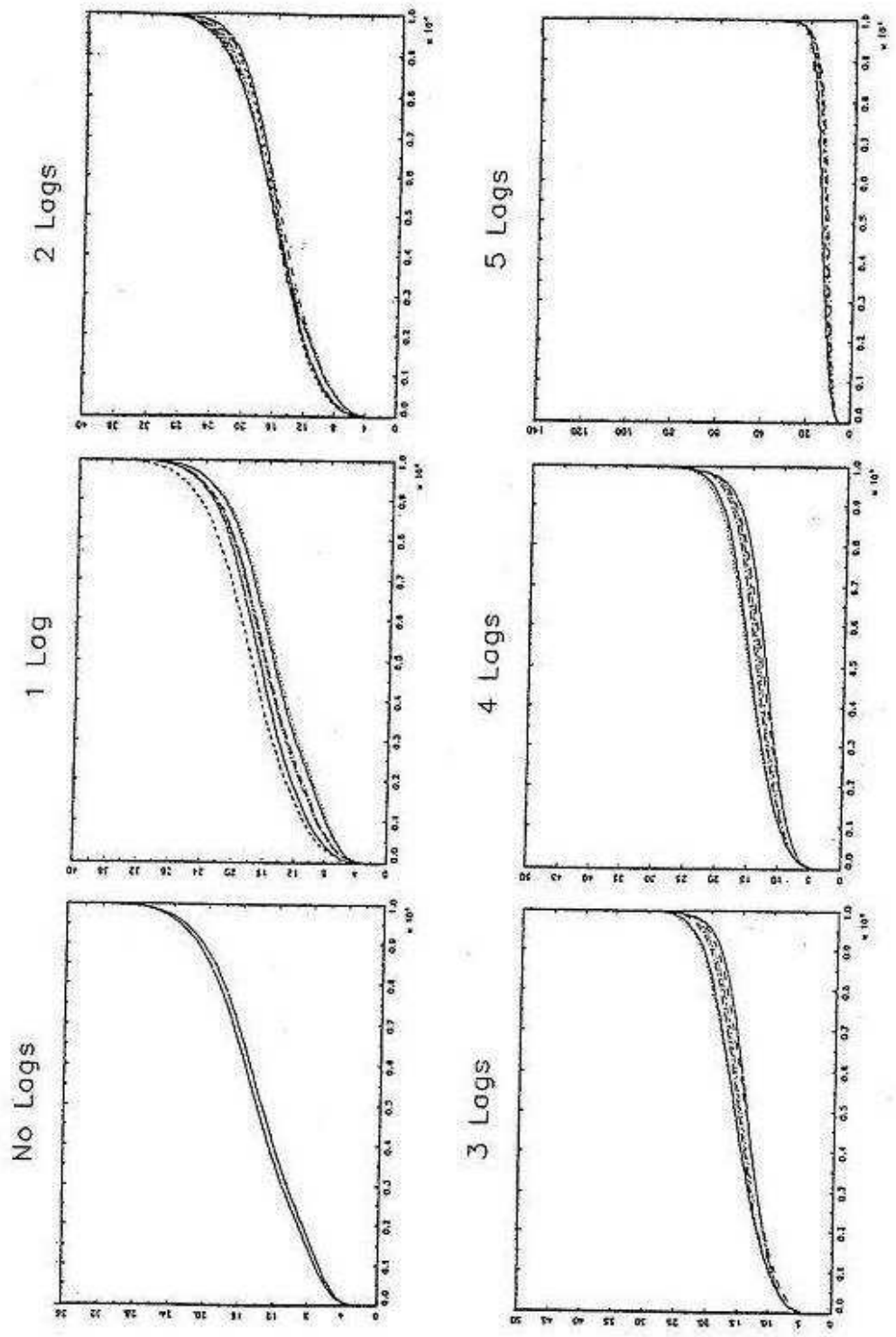


Fig.9a. SupF: The Bohman Kernel

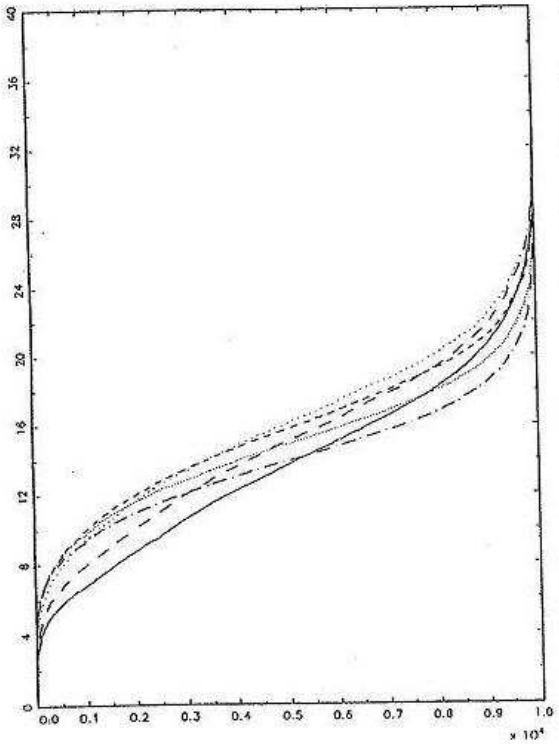


Fig.9b. SupF: The Fejer Kernel

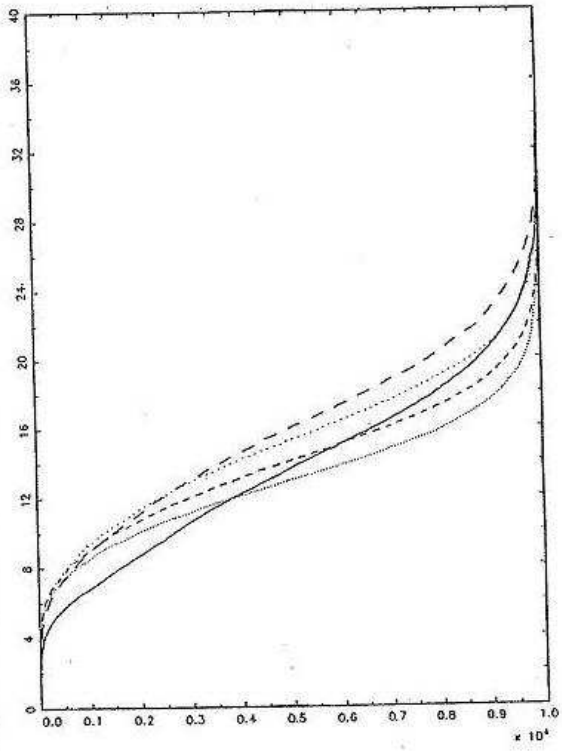


Fig.9c. SupF: The Parzen Kernel

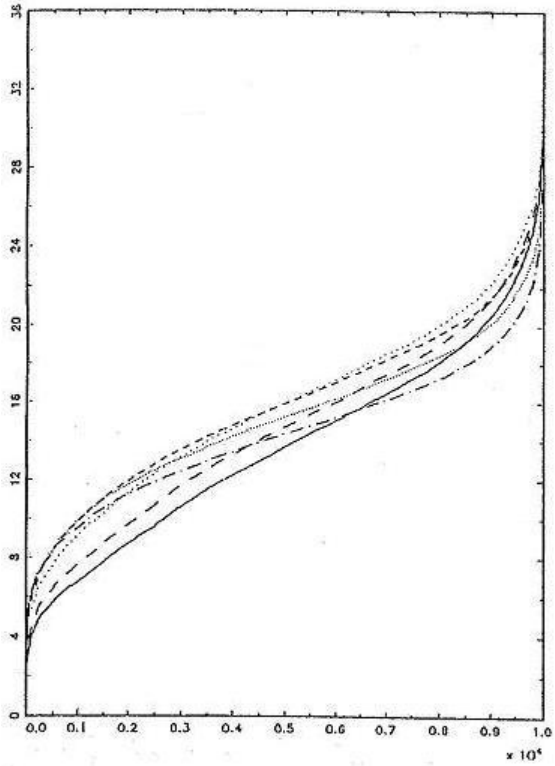


Fig.9d. SupF: The Riesz Kernel

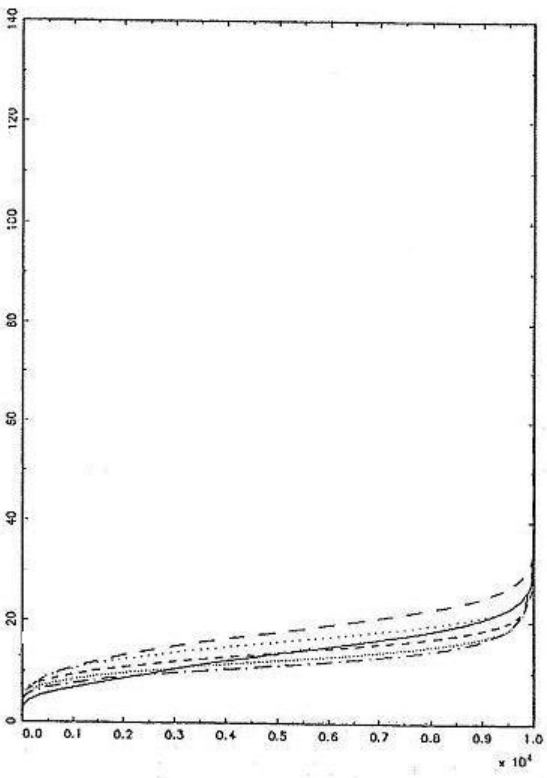


Fig.9e. SupF: Tukey–Hamming

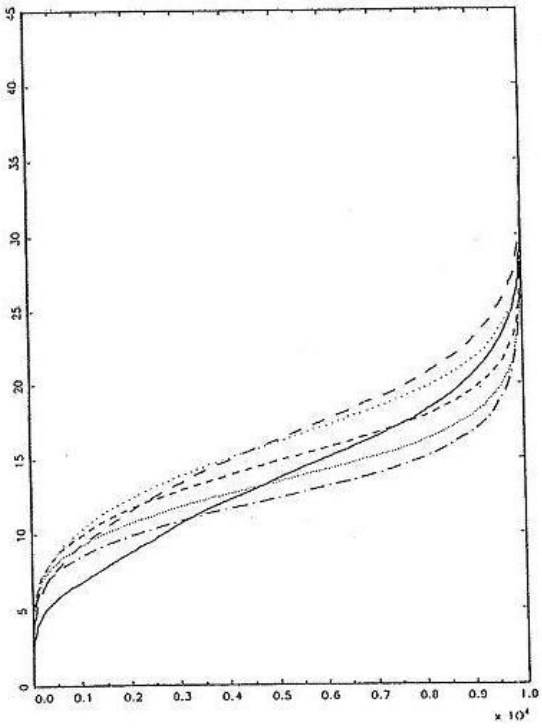
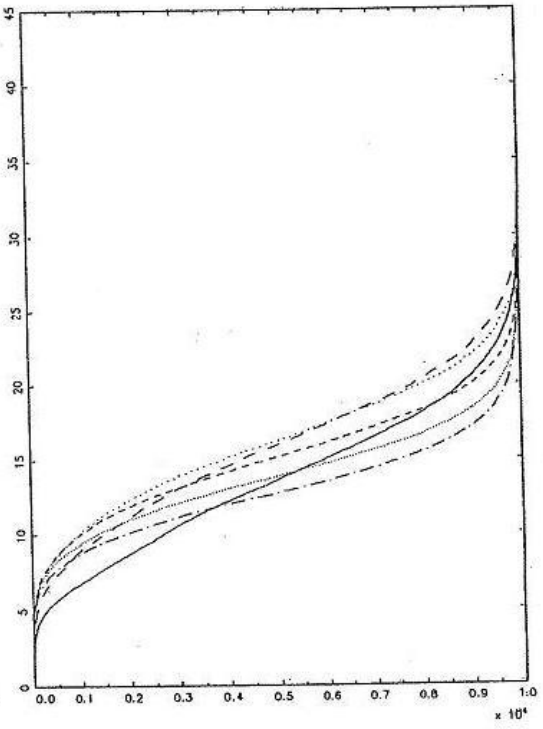


Fig.9f. SupF: Tukey–Hanning



Sup F

Fig.9g. The Quadratic-Spectral Kernel

