Competing Through Information Provision*

Jean Guillaume Forand†

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Abstract

This paper studies the symmetric equilibria of a two-buyer, two-seller model of directed search in which sellers commit to information provision. More informed buyers have better differentiated private valuations and extract higher rents from trade. When sellers cannot commit to sale mechanisms, information provision is higher under competition than under monopoly, yet partial information is provided when sellers are price-setters. In contrast, when sellers commit to both information provision and sale mechanisms, I identify simple conditions under which sellers post auctions and provide full information in every equilibrium, ensuring that all equilibrium outcomes are constrained efficient. Sellers capture the efficiency gains from increased information and compete only over non-distortionary rents offered to buyers.

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> *(Christie's and Sotheby's) embarked on cutthroat competition to get goods for sale (... and) provide ever more luxurious services. Catalogues became ever fatter, printed in colour, on glossy art paper. (...) On the inside page of Sotheby's catalogue of the Old Master paintings sale held in London on Dec. 13 (2001), six "specialists in charge" are listed. (...) They identify the paintings, research them, know which world specialist on this or that painter needs to be contacted, and, more mundanely, which client is most likely to be interested in what painting, etc.*[1]

What leads buyers to visit particular sellers is more than simply the terms of trade on offer. In particular, since the quality of buyers' information about goods affects their gains from trade, sellers may try to attract buyers by offering better information. This paper considers a market in which sellers post levels of information provision and buyers sort into selling

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†Room 131, Department of Economics, University of Waterloo, Hagey Hall of Humanities, Waterloo, Ontario, Canada N2L 3G1. Office phone: 519-888-4567 x. 33635. Email: jgforand@uwaterloo.ca.

[1]International Herald Tribune, 12/01/2002.

sites ex ante, drawn by promises of being better informed once on-site. Competition through information provision can be likened to providing higher quality customer service. A buyer's information about his private valuation for a good has two elements: private knowledge of some personal attributes, along with an understanding of how these characteristics relate to the good's properties. By controlling the information about their goods through, say, the quality and knowledge of their sales staff, sellers do not affect or acquire information about the buyers' private tastes. Instead, they shape the precision of the buyers' understanding of how the good matches these tastes. In the art auction market described in the quote above, the clients of Christie's and Sotheby's know their own tastes and would know how they value the objects on offer at these firms were they to have all relevant information about them. However, as this information is specialised and difficult to acquire, these buyers rely on the information provided by the firms' experts to guide their choices.

Privately informed buyers gain informational rents through trade and, as noted by Bergemann and Pesendorfer (2007), by providing less information to buyers *before* trading, sellers give out fewer informational rents *during* the exchange process. A monopolist's choice of information provision trades off informational rents against efficiency, since more information provision better identifies the buyers that most value the goods. However, and this is the novel insight of this paper, if sellers compete for buyers, the latter may shun low-information selling sites. Competing sellers still face the post-sorting efficiency-rents trade-off but also face a pre-sorting trade-off between market share and the rents promised to buyers.

I show that the effect of information provision depends on its role in competition. First, when sellers cannot commit to sale mechanisms and propose ex post optimal terms of trade, competition is channelled only through the equilibrium level of information provision, which depends on the characteristics of the sale mechanisms. If sellers are restricted to offering ex post optimal prices, in which information about buyers' private valuations is not used to efficiently allocate goods, then competition in information provision is softened. If sellers can offer any ex post optimal mechanism, which make better use information to screen buyer types, this increases sellers' benefits from information and leads to more intense competition and higher equilibrium information provision. Second, when sellers can commit to both sale mechanisms and information provision, they channel competition away from inefficient restrictions on information and into redistributive rent transfers to buyers. They provide full information and allocate goods efficiently based on that information, leading to constrained efficient equilibrium allocations.

Competition between the auction houses of Christie's and Sotheby's, in which the services surrounding a sale are crucial for buyers to establish an object's worth to them, provides a good example of competitive information provision. In the early 1990's, competition between the auction houses stiffened considerably, and expanding the services that provide information to buyers became an important competitive tool. Furthermore, later in the decade Phillips, a minor auction house, tried to break the Christie's-Sotheby's duopoly. It did so by

providing high guarantees to sellers who consigned objects there, but it also tried to match the bigger auction houses' superior capacity to inform buyers by luring away some of their teams of experts.[2] However, eventually Phillips became "less willing to provide lavish guarantees and loans. It emerged that Phillips's cash, rather than its expertise, had lured sellers of high-quality art; they returned to Christie's and Sotheby's."[3]

I present a model of directed search in which two sellers with unit supplies compete for the unit demands of two buyers.[4] Sellers commit to information structures and may or may not commit to sale mechanisms, buyers choose which seller to visit and sales take place. With information provision interpreted as quality of customer service, my assumption that sellers can credibly commit to information structures captures the fact that the number, training and availability of sales staffs is observed by potential buyers. Terms of trade, on the other hand, can either be proposed by sellers after buyers have interacted with their sales staff or credibly posted beforehand. As in Peters and Severinov (1997), sorting occurs ex ante; buyers obtain their private information only once they choose a seller. If fully informed, buyers either have (independent and private) high or low valuations for either sellers' objects.

Once at a selling site, buyers' information is mediated by the information structures offered by sellers, which map signals controlled by sellers into buyers' inferences about their valuations for goods.[5] By providing more information, sellers release private signals that allow buyers to differentiate their private valuations from the public expectation, that is, that pool of public knowledge about the goods' ex ante characteristics accessed by any potential buyer. Sellers cannot observe signals' effects on buyers' estimates of their valuations. As in Damiano and Li (2007), Ganuza and Penalva (2010), Johnson and Myatt (2006) and Ivanov (2008), I consider information structures ordered by the precision with which they allow buyers to access their true private valuations. For simplicity and tractability, I assume that information structures have a symmetric correlated structure; sellers commit to a randomisation between two information states for their site: informed or uninformed. The realisation of this information state is commonly known. While ex post all buyers visiting a particular seller are informed or uninformed, ex ante sale sites are differentiated by the probability with which all buyers get access to their private valuations for the goods upon visiting.

In the subgame following the sellers' announcements, I assume that buyers sort into sale sites according to that subgame's (in most cases) unique symmetric mixed strategy equilibrium. Buyers compete for the good when both visit the same seller and this selection,

[2]The Economist, 01/03/2001.

[3]The Economist, 21/02/2002.

[4]Following Moen (1997), see Burdett et al. (2001), Coles and Eeckhout (2003), Peters (2010), Shi (2001) and Shimer (2005). See Shi (2006) for a recent survey of directed search and Delacroix and Shi (2007) for a model in which posted prices act as informative signals about good quality. For competing auctioneers, see Burguet and Sákovics (1999), McAfee (1993), Hernando-Veciana (2005), Peters and Severinov (1997) and Virág (2010).

[5]See Bergemann and Pesendorfer (2007).

common in directed search, rules out equilibrium coordination among buyers and ensures smooth responses in sellers' profits to changes in their announcements. In equilibrium, sellers face a random demand, whose distribution they affect through their choice of information provision and sale mechanisms. In both cases, I restrict attention to symmetric equilibria of the game between the sellers.

In Section 3, I assume that sellers cannot commit to sale mechanisms and can attract buyers only by promising more information. Terms of trade are set optimally by sellers once buyers have sorted into selling sites and information has been provided (or not). I consider both ex post optimal mechanisms and ex post optimal prices. In both cases, sellers (i) would not provide any information as monopolists in the absence of competition and (ii) they always make take-it-or-leave-it offers to buyers visiting uninformed sites that capture all the gains from trade. On the one hand, ex post optimal prices do not screen buyer types that visit informed selling sites; the optimal price is the low valuation irrespective of how many buyers visit the seller. On the other hand, the ex post optimal mechanism screens buyer types by delivering informational rents to high-type buyers. I show that the unique symmetric equilibrium in information provision under price-setting involves partial information provision while the unique symmetric equilibrium under optimal mechanisms involves full information provision. Competition generates a complementarity between information provision and the efficiency with which the terms of trade exploit heterogeneity in buyers' private valuations. Sale mechanisms that better screen buyer types intensify competition and lead to higher equilibrium information provision.

In Section 4, I assume that sellers commit to both sale mechanisms and information provision. This allow sellers to disentangle their rent and information provision decisions. Under a condition guaranteeing that a monopolist seller would serve low-valuation buyers, I fully characterise the model's symmetric equilibria. In these equilibria, sellers provide full information, hold auctions and compete over the rents offered to buyers by setting appropriate (non-distortionary) reserve prices. Closely related to Coles and Eeckhout (2003), who present a two-buyer, two-seller model of directed search with sale mechanisms under perfect information, a continuum of symmetric equilibria exist that are differentiated by the sharing of a fixed level of surplus between buyers and sellers. In all equilibria, competition drives the marginal buyer's rents to its contribution to site surplus.

The full information result exploits the fact that sellers post their offers of information provision and sale mechanisms before buyers sort into selling sites. I show that profiles in which sellers do no offer full information are vulnerable to deviations in which they provide more information, adjust buyers' rents through transfers to keep their visit decisions fixed and pocket the extra surplus generated by the additional information. The intuition that a seller can exploit efficiency gains through ex ante offers is very general. The key to my result is that this arises as a competitive outcome. Sellers endogenously harness the complementarity between information provision and efficiency by channelling all competition for buyers

through non-distortionary transfers.

Recent work in mechanism design, auctions and optimal pricing has found that monopolists have incentives to manipulate their customers' access to information about their private valuations. In a model in which a seller designs a sale mechanism ex post, Bergemann and Pesendorfer (2007) characterise optimal information structures, which take a discrete monotone partitional form. Ganuza and Penalva (2010) study information provision in second-price auctions when buyers' ex post distributions of valuations are ordered by dispersion[6] and show that the seller's incentive to limit buyers' information vanishes as the number of buyers grows and the competition between them for the good wipes out their informational rents.[7] In a model of monopoly pricing, Johnson and Myatt (2006) have information provision order buyers' ex post distributions of valuations by sequences of rotations[8] and in a result recalling that of Lewis and Sappington (1994), they find conditions under which a seller will always optimally release either all or none of the available signals.[9]

In contrast, when a monopolist designs a mechanism ex ante and can 'sell' information to buyers, Esö and Szentes (2007) show that the seller can capture all rents accruing from the information it controls by setting appropriate entry fees and hence provides full information. Their result shows that sellers will have an incentive to manipulate information only in those environments in which they cannot charge entry fees before any information about the goods is revealed. I impose that all buyer participation decisions are made ex post and hence my full information result when sellers can commit to mechanisms does not rely on entry fees but on sellers' ability to channel rents to buyers through means other than information provision. My interpretation of information provision as quality of customer service is consistent with ex post participation constraints as buyers typically discuss terms of trade only after they have received the sales staff's input about a product.

The question of how the incentives to provide information extend to a competitive market has received little attention to date. A later paper by Valverde (2011) studies a model related to mine in which sale mechanisms are restricted to auctions, but in which sellers provide information prior to buyers making their sorting decisions. In that case, while information provision can reduce traffic from low-valuation bidders, Valverde (2011) provides conditions that guarantee the existence of a full-information equilibrium. Damiano and Li (2007) present a model of two-seller competition with information provision and ex post price competition which generalises that of Moscarini and Ottaviani (2001).[10] With a single buyer and price competition, information does not enhance surplus and in equilibrium sellers provide information to differentiate goods ex post and soften competition. Ivanov (2008)

[6]For random variables X and Y with distribution functions F and G, Y is said to be *more dispersed* than X if $F^{-1}(\beta) - F^{-1}(\alpha) \leq G^{-1}(\beta) - G^{-1}(\alpha)$ for all $0 < \alpha \leq \beta < 1$. See Shaked and Shanthikumar (2007).

[7]On this see also Board (2009).

[8]Continuous distribution function G is said to obtained from distribution F *by (clockwise) rotation around* z if $F(x) \leq G(x)$ for all $x \leq z$ and $F(x) \geq G(x)$ for all $x \geq z$.

[9]The survey of Bergemann and Välimäki (2006) provides more references to the related literature.

[10]See Huang (2010) for a related model extended to a directed search framework.

studies a related model with any number of sellers and continuous type distributions and shows that as the number of sellers increases there is a unique symmetric equilibrium with full information provision.

1 Model

1.1 Setup

Two sellers, a and b, have a single good for sale and two buyers have unit demands. Informed buyers have private valuation for either seller's good of either θ_H or θ_L, with $\theta_H > \theta_L$. Buyers are initially uninformed about their private valuations for both sellers' goods, which are ex ante identical. The prior distribution of buyer valuations for either good is (p_H, p_L) and their expected valuations are $\bar{\theta} = p_L\theta_L + p_H\theta_H$.

In the first stage of the game, sellers commit to information provision and may simultaneously commit to sale mechanisms, although I also consider the case without commitment in which sellers propose ex post optimal terms of trade. At all selling sites, information provision consists of private signals received by buyers about their private valuations for the good at that site. Information provision is symmetric across buyers that visit a particular seller, that is, it cannot be tailored to individual buyers. As a tractable parametrisation, I assume that sellers commit to a randomisation between two information regimes, full information and no information. That is, seller k posts a probability π_k with which all buyers that attend site k learn their private valuations for the good. The realisation of the information state, although not of the private signals received by the buyers, is commonly known once buyers visit a selling site. Ex post, either all buyers at site k are informed and have private valuations in $\{\theta_H, \theta_L\}$, or all are uninformed, and have a known expected valuation of $\bar{\theta}$.

In the second stage of the game, buyers simultaneously sort into selling sites, either receive information about the good or not (according to (π_a, π_b)), learn how many other buyers are also present and take part in the sale mechanism at that site. Let $\eta \in \{1, 2\}$ denote the *demand state* of a sale site and $\tau \in \{i, u\}$ its *information state*, where i stands for informed and u for uninformed. An *ex post incentive compatible direct mechanism* at site k specifies allocation probabilities $x_k^{\eta,\tau}$ and transfers $y_k^{\eta,\tau}$ as functions of reported ex post types for all information and demand states of the market and are constrained to be anonymous.[11] For $j \in \{H, L\}$, let $X_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} x_k^{2,i}(\theta_j, \theta_{-j})$ and $Y_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} y_k^{2,i}(\theta_j, \theta_{-j})$. Sale mechanisms satisfy the familiar sets of state-contingent incentive and participation constraints, which for completeness are reproduced in the Appendix. The class of incentive compatible direct mechanisms is denoted by Γ, and a particular mechanism at site k by γ_k. Any mechanism $\gamma_k \in \Gamma$ at site k induces *ex ante rents for buyers in state* (η, τ), $R_k^{\eta,\tau}$, which are computed

[11] Note that no reports are necessary when buyers are uninformed.

before buyers receive information. Given mechanism γ_k, these rents are given by

$$\begin{aligned}
R_k^{\eta,u} &= x_k^{\eta,u}\bar{\theta} - y_k^{\eta,u} \quad \text{for } \eta \in \{1,2\},\\
R_k^{1,i} &= \mathbf{E}_\theta\left[x_k^{1,i}(\theta)\theta - y_k^{1,i}(\theta)\right],\\
R_k^{2,i} &= \mathbf{E}_\theta\left[X_k^{2,i}(\theta)\theta - Y_k^{2,i}(\theta)\right].
\end{aligned}$$

Denote the *ex ante surplus at site k in state (η,τ) under mechanism γ_k* as $\mathcal{S}_k^{\eta,\tau}$, which is given by

$$\begin{aligned}
\mathcal{S}_k^{1,u} &= x_k^{1,u}\bar{\theta},\\
\mathcal{S}_k^{2,u} &= 2x_k^{2,u}\bar{\theta},\\
\mathcal{S}_k^{1,i} &= p_H x_k^{1,i}(\theta_H)\theta_H + p_L x_k^{1,i}(\theta_L)\theta_L,\\
\mathcal{S}_k^{2,i} &= 2\left[p_H X_k^{2,i}(\theta_H)\theta_H + p_L X_k^{2,i}(\theta_L)\theta_L\right].
\end{aligned}$$

1.2 Strategies and Equilibrium

To focus on competition in information provision only, Section 3 considers the case in which sellers cannot commit to sale mechanisms and a strategy for seller k is a probability $\pi_k \in [0,1]$. In Section 4, sellers commit to both information and mechanisms, and a strategy for seller k is $(\pi_k,\gamma_k) \in [0,1]\times\Gamma$, a probability π_k along with a mechanism γ_k. To unify notation, I will also denote seller k's strategy when there is no commitment to mechanisms as (π_k,γ_k), where γ_k is understood to be an ex post optimal sale mechanism. A strategy for a buyer is $q : ([0,1]\times\Gamma)^2 \to [0,1]$, where q denotes the probability with which the buyer visits seller a. The buyers' subgame has a large number of equilibria both on and off the equilibrium path and I restrict attention to its symmetric mixed strategy equilibrium. Throughout the paper, I consider symmetric equilibria in the sellers' strategies.

Given strategy (π_a,γ_a) for seller a and a visit probability q for buyers, a buyer attending site a expects rents $\mathcal{R}_a(\pi_a,\gamma_a,q)$ given by

$$\begin{aligned}
\mathcal{R}_a(\pi_a,\gamma_a,q) &= \mathbf{E}_\eta\mathbf{E}_\tau R_a^{\eta,\tau}\\
&= q\Big[\pi_a R_a^{2,i} + (1-\pi_a)R_a^{2,u}\Big]\\
&\quad + (1-q)\Big[\pi_a R_a^{1,i} + (1-\pi_a)R_a^{1,u}\Big]. \qquad (1)
\end{aligned}$$

Similarly to (1), given strategy (π_b,γ_b) for seller b and visit probability q, a bidder attending auction site b expects rents $\mathcal{R}_b(\pi_b,\gamma_b,q)$. Given strategy profile $(\pi_k,\gamma_k,\pi_{-k},\gamma_{-k})$, the profits of seller k can be expressed as surplus less rents as

$$\mathcal{P}_k(\pi_k,\gamma_k,\pi_{-k},\gamma_{-k}) = \mathbf{E}_\eta\mathbf{E}_\tau\left[\mathcal{S}_k^{\eta,\tau} - \eta R_k^{\eta,\tau}\right].$$

Buyers' visit decisions depend on whether or not sellers' mechanisms generate *congestion effects*, that is, whether their rents at a given site decrease when the other buyer visits it more

frequently. Site k's mechanism generates congestion effects if $\pi_k R_k^{1,i} + (1-\pi_k) R_k^{1,u} \geq \pi_k R_k^{2,i} + (1-\pi_k) R_k^{2,u}$. When this is the case, in the unique symmetric mixed strategy equilibrium of the buyers' subgame the visit probability must satisfy[12]

$$q \begin{cases} = 0 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \geq \mathcal{R}_b(\pi_b, \gamma_b, 0), \\ = 1 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \leq \mathcal{R}_b(\pi_b, \gamma_b, 0), \end{cases}$$

while if both $\mathcal{R}_a(\pi_a, \gamma_a, 1) < \mathcal{R}_b(\pi_b, \gamma_b, 0)$ and $\mathcal{R}_a(\pi_a, \gamma_a, 1) > \mathcal{R}_b(\pi_b, \gamma_b, 0)$, $q \in (0,1)$ is the unique solution to

$$\mathcal{R}_a(\pi_a, \gamma_a, q) = \mathcal{R}_b(\pi_b, \gamma_b, q). \tag{2}$$

Natural sales mechanisms, such as posted prices and auctions, always generate congestion effects and hence (2) pins down buyer behaviour uniquely for these mechanisms. The mechanisms considered in this paper, either ex post optimal prices and mechanisms in the absence of commitment or the equilibrium mechanisms with commitment, generate congestion effects. However, as in Coles and Eeckhout (2003), since off the equilibrium path sellers can commit to mechanisms that do not generate congestion effects, it is necessary to determine buyers' behaviour in these cases. The details for such mechanisms, where a symmetric mixed strategy equilibrium satisfying (2) is again selected, are relegated to the Appendix.

1.3 Discussion of Key Assumptions

Two buyers, two sellers. The two-seller, two-buyer setup is restrictive but counters well-known equilibrium existence and tractability issues in finite directed search and competing auctions,[13] complicated in my model since I focus on both information provision and general sale mechanisms. While many of the insights of my results are more general, their details depend on the simplicity of my restriction on the number of buyers. Otherwise, the complexity of buyers' sorting decisions, as expressed by (2), presents many technical difficulties. In that case, determining how buyers' visit decisions vary with sellers' offers, along with sellers' profit functions, rapidly becomes intractable. An important benefit of my setup is that it allows a complete a characterisation of its symmetric equilibria.

Two ex post information states. Information structures are usually modelled as signals that map buyers' ex ante into ex post distributions of types, where the latter are ordered by a suitable notion of precision. However, since I model ex ante competition, buyers

[12] To lessen notation, the visit probability generated by $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ will simply be denoted by q, with its dependence on information provision and mechanisms understood.

[13] This explains why Peters and Severinov (1997), following McAfee (1993), focus on large economies in which a seller's impact on market conditions vanishes. Burguet and Sákovics (1999) prove existence of a symmetric equilibrium in a 2-seller, n-buyer model. See also Hernando-Veciana (2005) and Virág (2010) for existence results in finite competing auctions, and Galenianos and Kircher (2012) along with Galenianos et al. (2011) for directed search equilibria in finite markets.

make their sorting decisions before any information is provided and the essential feature is that choices of (π_a, π_b) allow for a continuous differentiation of the selling sites with respect to information ex ante. More general information structures would not affect any of the central trade-offs faced by sellers. However, having sellers commit to the probability of providing information to all buyers simplifies the model by reducing the ex post information states to two; informed and uninformed.[14]

Symmetric equilibrium of the buyers' subgame. It has been argued, notably by Levin and Smith (1994) in the context of a single auction with entry and by Burdett et al. (2001) for directed search models, that the equilibria of the buyers' subgame with symmetric mixed strategies by buyers and induced random demand for the sellers' goods are more appealing than asymmetric pure strategy equilibria which generate fixed demand. Burdett et al. (2001) show that there exist many equilibria with pure actions on the equilibrium path in which sellers' equilibrium offers are supported by buyers' threats to revert to the mixed strategy equilibrium in the buyers' subgame. In such equilibria coordination improves buyers' payoffs relative to the mixed strategy equilibrium but yields sophisticated behaviour that is hard to interpret. Symmetry requires a plausible anonymity in buyers' visit decisions.

1.4 Preliminaries: Characterising Incentive-Compatible Mechanisms

Note that buyers' sorting decisions, as expressed by (2), depend only on information provision and expected rents $R_k^{\cdot,\cdot}$. In particular, buyers' decisions are not affected by how rents are shared between types conditional on being informed. This ex ante feature of sellers' rent promises allows a useful characterisation of incentive-compatible mechanisms, which simplifies sellers' strategy sets.

Lemma 1. *Given any strategy profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$ for sellers, there exists a mechanism $\tilde{\gamma}_k \in \Gamma$ in which incentive constraints of θ_H-types in states $(1,i)$ and $(2,i)$ are binding and allocations are as in γ_k. Furthermore, under profile $(\pi_k, \tilde{\gamma}_k, \pi_{-k}, \gamma_{-k})$, buyers' rents and sellers' profits are the same as under profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$.*[15]

Any incentive-compatible mechanism at site k that achieves rents $R_k^{\cdot,\cdot}$ with non-binding θ_H-type incentive constraints can be replaced by an incentive compatible mechanism that achieves the same levels of expected rents with the same allocations, but in which these constraints bind. Under this new mechanism, profits are unchanged and all traffic and

[14] Furthermore, the information structures of my model can be seen to be discrete examples of those of Johnson and Myatt (2006). Consider ex post distribution of valuations F^π for a single buyer over valuation space $\{\theta_L, \bar{\theta}, \theta_H\}$ generated by the information structure of my model with probability π. $\bar{\theta}$ is a rotation point for the family of distributions $\{F^\pi\}$ since for $\pi > \pi'$, $F^\pi(x) \geq F^{\pi'}(x)$ for all $x < \bar{\theta}$ and $F^\pi(x) \leq F^{\pi'}(x)$ for all $x \geq \bar{\theta}$.

[15] All proofs are in the Appendix.

information provision incentives are preserved. The proof is simple: given an incentive compatible mechanism in which the incentive constraint of θ_H-types in state (η, i) does not bind, we can increase θ_L-type rents and decrease θ_H-type rents through transfers until the constraint binds, while ensuring that the expected rents in demand state (η, i) are unchanged.

Denote by $\tilde{\Gamma}$ the set of incentive compatible mechanisms with binding θ_H-type incentive compatibility constraints. Restricting sellers to offering mechanisms in $\tilde{\Gamma}$ does not alter the set of equilibrium outcomes of the game: information provision, allocations, rents and visit probabilities. Denote *low-type rents under mechanism* γ_k *in state* (η, τ) by $r_k^{\eta,\tau}$. These are the rents offered to θ_L-types in informed states and to the uninformed otherwise. Lemma 6 in the Appendix states the familiar result that mechanisms $\gamma_k \in \tilde{\Gamma}$ are characterised by monotone allocation probabilities, low-type rents $r^{\eta,\tau} \geq 0$ for all states (η, τ) and the 'envelope' condition for high-type rents.

2 Example: Second-Price Auctions

I start with an example that explores competitive information provision by auctioneers. That is, sellers hold second-price auctions without reserve prices irrespective of how many buyers visit them.[16] As Board (2009) and Ganuza and Penalva (2010) derive the optimal information structures for monopolists in a second-price auction with two buyers, this example constitutes a useful benchmark to gauge the effects of competition on information provision.

With second-price auctions, buyers obtain the good for free in the one-buyer state, and capture the full surplus $\bar{\theta}$. In the two-buyer state, to bid their best estimate of their true value is a weakly dominant strategy for buyers. When uninformed, this best estimate is $\bar{\theta}$. A buyer that attends site a, given π_a and q, expects rents

$$\mathcal{R}_a(\pi_a, q) = q\pi_a p_H p_L(\theta_H - \theta_L) + (1-q)\bar{\theta},$$

while a bidder attending site b, given π_b and q, expects rents

$$\mathcal{R}_b(\pi_b, q) = (1-q)\pi_b p_H p_L(\theta_H - \theta_L) + q\bar{\theta}.$$

In the mixed strategy equilibrium of the buyers' subgame, the probability with which buyers visit site a is given by

$$q = \frac{\bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}{\bar{\theta} - \pi_a p_H p_L(\theta_H - \theta_L) + \bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}. \tag{3}$$

[16] This is a partial illustration of the version of my model with commitment to sale mechanisms, in that sellers are committed to holding second-price auctions. However, with competition in information provision, these are not equilibrium mechanisms, as I show in Proposition 4.

The profits of seller a, given (π_a, π_b) and the resulting q, are given by

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= q^2\left[\pi_a\Big(p_H^2\theta_H + (1-p_H^2)\theta_L\Big) + (1-\pi_a)\bar{\theta}\right] \\ &= q^2\big[\bar{\theta} - \pi_a p_H p_L(\theta_H - \theta_L)\big]. \end{aligned} \tag{4}$$

The term in the brackets of (4) is the expected price paid by the buyer who obtains the good in the two-buyer state. This price decreases in π_a, since the seller then gives away a higher share of the surplus as informational rents. Denote this price by $w_a(\pi_a)$. Suppose a single second-price auctioneer faced a fixed set of two buyers, then its profits given information provision π would be $w(\pi)$. This immediately leads to the following result, known from Board (2009) and Ganuza and Penalva (2010).

Proposition 1. *A second-price auctioneer with no reserve price facing two buyers maximises profits by setting $\pi = 0$.*

Returning the model with competition, note that (3) can be rewritten as

$$q = \frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}. \tag{5}$$

Since buyers get all the surplus if alone, q depends only on how much profits sellers get from demand states with two buyers. Thus (4) becomes

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= \left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}\right]^2 w_a(\pi_a) \\ &= w_b(\pi_b)\left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)} \cdot \frac{w_a(\pi_a)}{w_a(\pi_a) + w_b(\pi_b)}\right] \\ &= w_b(\pi_b)q(1-q). \end{aligned} \tag{6}$$

Clearly, seller a's choice of information influences profits in (6) only through its effect on $q(1-q)$, which attains a maximum when $q = \frac{1}{2}$. Seller a can attain this maximum by setting $\pi_a = \pi_b$. This leads to the following surprising result.

Proposition 2. *When the sale mechanism is a second-price auction with no reserve price, (π_a, π_b) is an equilibrium if and only if $\pi_a = \pi_b$.*

A seller's best-response to any information offer by an opponent is to match that commitment. Proposition 2 relies on the expression for traffic in (5), which in turn allows the representation of profits in (6), which follows since the sale mechanism is a second-price auction with no reserve price.[17]

[17]Proposition 2 does not depend on my assumptions about buyers' types. Suppose buyers' true valuations were instead given by some continuous random variable Y with mean $\bar{\theta}$. Denote by $Y_{1:2}$ and $Y_{2:2}$ the expected values of the first and second order statistics of Y, then $Y_{1:2} + Y_{2:2} = 2\bar{\theta}$. Rewriting traffic as in (5) uses the discrete version of this identity. Similarly, this result is not due to my special correlated information structures. If instead π indexed ex post valuations Y^π with $\mathbf{E}Y^\pi = \bar{\theta}$ for all π, then $Y^\pi_{1:2} + Y^\pi_{2:2} = 2\bar{\theta}$ for all π, and the result of Proposition 2 still follows.

3 No Commitment to Sale Mechanisms

In this section, sellers commit to levels of information provision but cannot commit to sale mechanisms. This is in fact an extension to competing sellers of the framework of Bergemann and Pesendorfer (2007). Once buyers have chosen sale sites and information states have been realised, sellers deliver their good through each state's ex post optimal mechanism. This constrains the rent offers sellers can extend to buyers through their choice of information provision. To highlight how sale mechanisms affect sellers' trade-off between attracting traffic and handing over more informational rents, and through this equilibrium information provision, I consider two classes of ex post optimal terms of trade, (i) the case in which sellers are restricted to proposing ex post optimal prices in any demand and information state and (ii) the case in which sellers can propose any ex post optimal mechanism.

Under both ex post optimal prices and mechanisms, it must be that $R^{1,u} = R^{2,u} = 0$. That is, in uninformed states sellers make take-it-or-leave-it offers of $\bar{\theta}$ and capture all gains from trade. When buyers are informed, the ex post optimal prices or mechanisms for both the one and two-buyer states depend on whether or not sellers prefer to exclude θ_L-types and sell only to θ_H-types. When θ_L-types are excluded, sellers extract all informational rents from θ_H-types. In that case, buyers expect no rents from any demand state regardless of the level of information provision and their sorting decisions are trivial. The interesting case is when informed θ_H-types obtain rents.

Assumption 1. $\frac{\theta_H}{\theta_L} < \frac{1}{p_H}$.

Under ex post optimal mechanisms, sellers prefer to sell to θ_L-types in both demand states whenever Assumption 1 is satisfied. Furthermore,this is also the condition under which sellers set a price of θ_L in the one-buyer state.

Assumption 2. $\frac{\theta_H}{\theta_L} \in (\frac{2-p_L}{1-p_L}, \frac{1}{1-p_L^2})$.

Under ex post optimal prices, sellers prefer to set a price of θ_L in the two-buyer state (and hence also in the one-buyer state) if $\theta_L > (1 - p_L^2)\theta_H$. Assumption 2 ensures that this condition holds. However, it is more stringent as it also guarantees that under ex post optimal prices partial information provision occurs in equilibrium.

In the Appendix, I show that under both ex post optimal prices (under Assumption 2) and mechanisms (under Assumption 1), sellers' profits in the one and two-buyer informed states are decreasing in information provision. Hence, as with second-price auctions, a monopolist would never provide information under both these ex post optimal terms of trade. Hence, any information provision achieved in equilibrium is due to competition. Furthermore, no equilibrium with $\pi = 0$ can exist, as uninformed buyers get no rents under ex post optimality and any deviation by some seller from such a profile to any $\pi' > 0$ would attract all buyers. Hence relative to monopoly, competition always improves informational efficiency.

Proposition 3. *Under Assumption 1 and ex post optimal mechanisms, the unique symmetric equilibrium has full information provision. Under Assumption 2 and ex post optimal prices, the unique symmetric equilibrium has partial information provision.*

When buyers face optimal mechanisms once sorted, expected rents are low. This increases the sensitivity of their sorting decisions to shifts in information provision and enhances sellers' traffic-stealing incentives. Sellers achieve their favoured ex post outcomes, yet competition leads them to make their most costly ex ante information commitments. Under ex post prices, competition is dampened and for the cases covered by Assumption 2, partial information is provided in equilibrium.[18]

The intensity of the competition between the sellers also depends on the use that the mechanisms being offered ex post make of whatever information is provided. Under ex post prices, information provision has no effect on efficiency, since pricing mechanisms do not discriminate in favour of θ_H-types. However, information provision is beneficial to buyers because it increases their informational rents. In this case, information provision is a tool for sellers to commit to a sharing rule applied to a fixed level of surplus. On the other hand, under optimal ex post mechanisms, information provision also increases efficiency. This illustrates a more general principle; information provision is more attractive to sellers under mechanisms that exploit the information that is generated. This leads to increased competition between sellers and more information provision.[19] The complementarity that competition generates between mechanisms' allocative efficiency and information provision has a more negative implication; if sellers could collude and commit to sales mechanisms while anticipating future competition in information they would protect themselves against its effects by selecting mechanisms with inefficient allocations.

4 Commitment to Sale Mechanisms

In this section, sellers commit jointly to information provision and sale mechanisms. The main result of the paper, Proposition 4, is a characterisation of symmetric equilibria under the no-exclusion Assumption 1 that shows that they all have full information. The proof of Proposition 4 follows from a sequence of lemmas concerning equilibrium information provision, allocations and rents that are presented in the remainder of the paper. Before stating this result, I require the following definition.

[18]Note that since Assumption 1 is satisfied under Assumption 2, there exist parameter values for which full information is provided under ex post optimal mechanisms but only partial information is provided under ex post optimal prices.

[19]This follows from comparative statics results derived in an earlier version of this paper. Details are available upon request.

Definition 1. A mechanism $\gamma_k \in \Gamma$ has *partial allocative efficiency* (PAE) if and only if

$$x_k^{1,i}(\theta_H) = x_k^{1,u} = 1,$$
$$x_k^{2,u} = \frac{1}{2},$$
$$x_k^{2,i}(\theta_H, \theta_L) = 1, \text{ and } x_k^{2,i}(\theta_H, \theta_H) = \frac{1}{2}.$$

Furthermore, a mechanism $\gamma_k \in \Gamma$ has *full allocative efficiency* (FAE) if and only if it has partial allocative efficiency and also

$$x_k^{1,i}(\theta_L) = 1,$$
$$x_k^{2,i}(\theta_L, \theta_L) = \frac{1}{2}.$$

A mechanism has partial allocative efficiency whenever the good is always sold to some buyer in uninformed states, and to a θ_H-type in informed states if such a type is present, while it has full allocative efficiency whenever it has partial allocative efficiency and the good is always sold to a θ_L-type in informed states if no θ_H-type is present. Under FAE, the surplus in state $(2, i)$ is maximized and denoted it by $\bar{\mathcal{S}}^{2,i}$. A mechanism with PAE may exclude θ_L-types.

Proposition 4. *Under Assumption 1,* $(\pi, \gamma, \pi, \gamma) \in ([0,1] \times \Gamma)^2$ *is a symmetric equilibrium if and only if* $\pi = 1$*,* γ *has full allocative efficiency,* $R^{2,i} \leq R^{1,i}$ *and* $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$*.*

Proposition 4 characterises symmetric equilibria under Assumption 1. While this assumption guarantees allocative efficiency in monopoly, efficient mechanisms also lead monopolists to not provide information in order to restrain informational rents. With competition, as shown in Section 4.1, sellers manage to disentangle information and rent provision decisions even in the presence of competition. As shown in Section 4.2, sellers post auctions and take advantage of their allocative efficiency by providing full information. Competition then determines equilibrium rents, which are delivered to buyers through non-distortionary transfers.

There is a continuum of equilibria that are ranked from the most favourable to sellers (with rents $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and $r^{2,i} = 0$) to the most favourable to buyers (with rents $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and $R^{2,i} = R^{1,i}$). All mechanisms have congestion effects and, as seen in Section 4.3, the condition that $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ has the interpretation that the seller equates the rents owed the marginal buyer to its contribution to site surplus. The equilibria differ in how the surplus is shared between buyers and sellers, yet all equilibrium outcomes are constrained efficient. In my setup, constrained efficiency requires that (i) there is full information provision, so that θ_H-type buyers, if present at a sale site, can be identified, that (ii) the sale mechanisms satisfy FAE, so that goods are allocated first to θ_H-types and then to θ_L-types, and that (iii) the equilibrium between the sellers be symmetric, since while the efficient distribution

of buyers across sale sites has one of them with each seller, in the absence of coordination, efficiency requires maximising the likelihood of having one buyer at each site, which happens when $q = \frac{1}{2}$. Profits are not driven to zero in any equilibrium.[20] In the one-buyer state, profits are positive since they are given by $\bar{\theta} - \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and it is the case that $2\bar{\theta} > \bar{\mathcal{S}}^{2,i}$. In the two-buyer state, profits are $\bar{\mathcal{S}}^{2,i} - 2R^{2,i}$, which is positive except in the equilibrium most favourable to buyers.

The continuum of rent levels supported in equilibrium is closely related to Coles and Eeckhout (2003). Adjusting for the fact that with high and low-type buyers surplus levels vary across demand states and that incentive constraints imply that buyers cannot be made to expect zero rents, the equilibrium rent levels pinned down by Proposition 4 mirror theirs. In their paper with known valuations, a mechanism consists of demand state-dependent prices which are all equally efficient. In my model, information provision, allocations and rent levels are interdependent and must be determined simultaneously. The benefits of screening between types imply that in my model auctions have an efficiency advantage. A by-product of my model's setup is that it yields a clear interpretation of why competition fixes rents only in the one-buyer state, which is simply a consequence of equating marginal rents to marginal contributions to site surplus.

4.1 Equilibrium Information Provision

This section derives necessary conditions for full information provision in symmetric equilibrium with endogenous mechanisms.

Lemma 2. *Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is an equilibrium, that $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_a^{\eta,\tau}$ is strictly increasing (decreasing) in π_a, and that it is not the case that γ_a and γ_b are the ex post optimal mechanisms. Then $\pi_a = 1$ ($\pi_a = 0$).*

Intuitively, as information provision increases the potential size of the surplus, it allows Pareto-improving deviations for sellers from any profile with less than full information. Fixing a profile of mechanisms, information provision also has a distributive effect through rents as it shifts probability among information states within and across demand states. However, since sellers commit ex ante to both state-contingent rents and information, consider a deviation from a strategy profile with less than full information in which a seller increases information provision and offsets its effect on buyer rents through transfers. In this way, buyers' sorting decisions are unaffected and sellers pocket the newly generated surplus. There are two provisos to the above argument. First, the initial mechanisms must be such that more information actually increases the expected surplus at site k, $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_k^{\eta,\tau}$. In equilibrium,

[20] That sellers do not compete away all profits in the presence of traffic effects has been noted in the literature on competing auctions (see Peters and Severinov (1997) and Burguet and Sákovics (1999)). Congestion effects and mixed strategies by buyers smooth out jumps in demand induced by changes in rent offers and competition between sellers is not as fierce as in Bertrand competition.

since the sellers will post auctions, this will be true but if some buyer types are excluded this need not be the case. However, in this case, reduced information provision will generate constrained efficiency gains that the seller can capture through transfers. Second, it cannot be that buyers' rents are at a minimum, i.e., at the ex post optimal mechanisms.[21]

Full information provision arose in Section 3 for ex post optimal mechanisms. However, as sellers could not commit to sale mechanisms, increasing information provision was profitable only if the increase in traffic generated compensated the seller for the higher rents now offered to buyers. Sellers provided full information because in my setup ex post optimal mechanisms generate sufficient incentives for traffic-stealing. With ex ante commitments to mechanisms, Lemma 2 shows that sellers can deviate to a full information profile and capture its efficiency benefits without concerning themselves with traffic effects, since they directly control state-contingent rents. This discussion makes it clear that the logic of the full information result is general and goes beyond my two-by-two setup with simple information structures. If sellers (i) compete ex ante through both information provision and (ii) sale mechanisms and equilibrium mechanisms generate higher surplus when buyers have better information, then sellers must provide full information.

4.2 Equilibrium Allocations

This section presents results on the efficiency of symmetric equilibrium allocations with endogenous mechanisms. The first result shows that holding auctions is weakly dominant.[22]

Lemma 3. *A strategy (π_k, γ_k) for seller k in which γ_k does not have partial allocative efficiency is weakly dominated.*

More specifically, for any profile in which seller k posts a mechanism that does not have PAE, an alternative mechanism with PAE can be found that leaves buyer rents and hence visit decisions unchanged and yields strictly higher profits to seller k, whenever buyers visit seller k with positive probability. Stated this way, the result states not only that equilibrium mechanisms have PAE, but that it is without loss of generality when searching for equilibria to consider deviations from candidate profiles that have PAE. The proof deals with θ_H-type and uninformed allocations separately, and mirrors analogous results in the monopoly framework. It shows that profits can be increased while relaxing the incentive constraints of θ_L-types if seller k increases θ_H-type allocations and transfers simultaneously, keeping θ_H-types at the same level of rents.[23] Similarly, a profile in which uninformed buyers are

[21] Hence, my results differ from those of Bergemann and Pesendorfer (2007) since ex post optimal mechanisms are not included in any of the equilibria of Proposition 4.

[22] This is as in McAfee (1993), where, however, the focus is on large markets. This shows that the logic of the result is more general. In small markets, arguments must consider the effect of a change in any seller's mechanism on market-wide rents and profits. See also Pai (2009) and Virág (2007).

[23] In the two-buyer state, it may be the case that θ_L-types receive the good even in the presence of θ_H-types, and that the feasibility constraint (that allocation probabilities for both buyers sum up to less than

excluded with positive probability is vulnerable to a deviation where a seller increases both allocation probabilities and transfers, keeping buyers at the same level of rents.

In my model, the classic arguments from the monopoly case that determine θ_L-type allocations cannot be applied directly due to their competitive effects on traffic across sale sites. In the monopoly case, Assumption 1 determines whether sellers exclude θ_L-types in either demand state, since given any mechanism in which θ_L-types are excluded with some probability, the seller can increase profits by increasing both θ_L-types' allocation probabilities and transfers, keeping their rents unchanged, even if this increases θ_H-type rents (through θ_H-types' binding incentive compatibility constraint). This increases rents expected over informed types. The problem with this argument when sellers compete is that an increase in rents in any state increases traffic but may decrease the likelihood of the one-buyer state (when $q > \frac{1}{2}$), and hence its effect on total profits may depend on the relation between profits in the one-buyer and two-buyer states. The next result, unlike Lemma 3, presents only a necessary condition on θ_L-type allocations in symmetric equilibria.

Lemma 4. *Under Assumption 1, if $(\pi, \gamma, \pi, \gamma)$ is a symmetric equilibrium, then γ has full allocative efficiency.*

From Lemma 3, PAE is necessary for any equilibrium in which both sellers are visited with positive probability and in a symmetric equilibrium $q = \frac{1}{2}$. To show that under Assumption 1, θ_L-types always receive the good in the absence of θ_H-types in a symmetric equilibrium, the proof applies the argument for the monopoly case outlined above to find a deviation from any symmetric equilibrium that violates FAE. The difficulty mentioned above is dealt with by the fact that at a symmetric profile small increases in traffic have a negligible effect on the probability of the one-buyer state. The proof of Lemma 4 also guarantees that profits in the two-buyer state are nonegative.

Without Assumption 1, a seller wants to exclude θ_L-types to depress θ_H-type rents. Marginally, whether this is profitable depends on whether the increased profits from θ_H-types compensate the drop in traffic in the two-buyer state. This traffic-rents trade-off will also involve the level of information provision. Without Assumption 1, it is difficult to derive a simple necessary condition on θ_L-type allocations which, as above, does not depend on information provision.

4.3 Equilibrium Rents

This section derives necessary conditions on equilibrium rents under Assumption 1.

Lemma 5. *Under Assumption 1, if $(\pi, \gamma, \pi, \gamma)$ is a symmetric equilibrium, then $R^{2,i} \leq R^{1,i}$ and $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$.*

one) binds, so that the seller cannot allocate the good more often to θ_H-types without allocating it less often to θ_L-types. But then the seller can simply 'free up' allocation probabilities by delivering the good less often to θ_L-types and keep their rents constant by decreasing their transfers.

In Section 3, under ex post optimal prices or mechanisms, buyers faced congestion effects and preferred being alone at a sale site when informed. Lemma 5 confirms that a seller will always impose congestion effects in a symmetric equilibrium when it commits to sale mechanisms. The intuition for this is as follows. Rewrite a buyer's expected rents at site a from a symmetric profile with $\pi = 1$ as

$$R^{1,i} + q(R^{2,i} - R^{1,i}). \tag{7}$$

That is, it is as though a buyer is charged an 'attendance fee' of $R^{1,i}$ along with a 'bonus' ('congestion charge') of $R^{2,i} - R^{1,i}$ when another buyer attends and $R^{2,i} > R^{1,i}$ ($R^{2,i} \leq R^{1,i}$). If $R^{2,i} > R^{1,i}$, decreasing $R^{2,i}$ lowers the bonus, but buyers remain indifferent between attending sites a and b only if this bonus is handed out more often, i.e., if q increases. As sellers can decrease rents while increasing traffic, profiles with $R^{2,i} > R^{1,i}$ admit a profitable deviation.

The condition $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ states that the marginal buyer attending a site is awarded his marginal contribution to site surplus.[24] This follows since seller a's profits at symmetric profiles with FAE are marginally increasing in $R^{1,i}$ (or $R^{2,i}$) whenever $R^{1,i} < \frac{\bar{\mathcal{S}}^{2,i}}{2}$.[25] A marginal buyer drawn to site a by a marginal change in rents receives $R^{1,i}$, its 'attendance fee', from seller a. On the other hand, this marginal buyer brings its share of the surplus when another buyer is also present, $\frac{\bar{\mathcal{S}}^{2,i}}{2}$, to site a. Since the probability of the one-buyer state is unaffected by small changes in q at a symmetric profile, a marginal buyer brings nothing to that state. A seller will want to attract a marginal buyer whenever his contribution exceeds the cost of luring him. Similarly, if $\frac{\bar{\mathcal{S}}^{2,i}}{2} < R^{1,i}$, a seller can gain by shedding a marginal buyer through a decrease in rents.

4.4 Sufficiency

The proof of Proposition 4 follows from the results of the previous sections. The necessity of FAE for symmetric equilibrium has been established in Lemma 4. Under FAE, information provision increases the surplus available at a selling site since two buyers generate more surplus when informed than when uninformed, as $\mathcal{S}^{2,i} = \bar{\mathcal{S}}^{2,i} > \bar{\theta}$ and $\mathcal{S}^{1,i} = \bar{\theta}$, and hence Lemma 2 states that $\pi = 1$ is necessary for symmetric equilibrium unless both sellers commit to the ex post optimal mechanisms. The necessity of full information under Assumption 1 for ex post optimal mechanisms follows from Proposition 3. Lemma 5 provides the necessary conditions for equilibrium rents. Note that $R^{2,i} \leq R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ implies that $2R^{2,i} \leq \bar{\mathcal{S}}^{2,i}$ and hence that profits in the two-buyer state are nonegative. The sufficiency argument is direct; taking a profile satisfying the conditions of the proposition, I show that no deviation can be profitable.

[24] Interpret the marginal buyer as the mass of visits involved in a marginal increase in q.
[25] See (16) in the Appendix.

5 Conclusion

This paper has analysed the strategic interactions of sellers who compete for buyers by committing to information provision. When sellers cannot commit to sale mechanisms and compete solely through offers of information, they may prefer to compete in environments in which ex post optimal terms of trade offer higher rents to buyers, as these lessen the intensity of competition and lead to lower information provision. Furthermore, as higher surplus mechanisms increase sellers' competitive incentives to provide information, they prefer to compete in environments with low allocative efficiency, and hence low information provision. When sellers commit to both information provision and mechanisms, all symmetric equilibria have full information provision and are constrained efficient. However, a variety of rent levels are supported in equilibrium as a result of different equilibrium offers of mechanisms. One interpretation of this result is that sellers prefer to channel competition through sale mechanisms rather than through restrictions on information provision. By doing so they maximize the available surplus, while competition determines the equilibrium share of this surplus going to buyers.

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A Appendix

A.1 Incentive Compatibility Constraints

Let $\mathcal{W}^{\eta,\tau}$ denote the set of *report profiles* that can be received by the seller in state (η,τ), where

$$\mathcal{W}^{\eta,\tau}=\begin{cases}\{(\theta_m,\theta_n)\}_{(m,n)\in\{L,H\}^2} & \text{if } \eta=2 \text{ and } \tau=i,\\ \{\theta_m\}_{m\in\{L,H\}} & \text{if } \eta=1 \text{ and } \tau=i,\\ \emptyset & \text{if } \tau=u.\end{cases}$$

An *anonymous direct mechanism* for seller k is a collection of functions

$$\left\{\left\{x_k^{\eta,\tau}:\mathcal{W}^{\eta,\tau}\to[0,1],y_k^{\eta,\tau}:\mathcal{W}^{\eta,\tau}\to\mathbf{R}\right\}\right\}_{\substack{\eta\in\{1,2\}\\ \tau\in\{i,u\}}},$$

where $x_k^{\eta,\tau}(w)$ and $y_k^{\eta,\tau}(w)$ are, respectively, the probability a buyer obtains the good and the transfer he must pay to seller k when the report profile is $w\in\mathcal{W}^{\eta,\tau}$ in state (η,τ). Since no report is necessary when buyers are uninformed, I write probabilities and transfers as $x_k^{\eta,u}$ and $y_k^{\eta,u}$ for $\eta\in\{1,2\}$. Also, since mechanisms are anonymous, define $x_k^{2,i}(\theta_m,\theta_n)$ as the probability that a buyer reporting θ_m obtains the good when the other buyer reports θ_n. A similar remark holds for the transfer $y_k^{2,i}(\theta_m,\theta_n)$. The allocation probabilities satisfy feasibility restrictions

$$\begin{aligned}&x_k^{1,\tau}(w)\le 1 \quad \text{for } w\in\mathcal{W}^{1,\tau} \text{ and } \tau\in\{i,u\},\\ &x_k^{2,u}\le\frac{1}{2},\\ &x_k^{2,i}(\theta_m,\theta_n)+x_k^{2,i}(\theta_n,\theta_m)\le 1 \quad \text{for } (m,n)\in\{L,H\}^2.\end{aligned}$$

When no information is released at site k, no incentive constraints apply. The relevant participation constraints are

$$x_k^{1,u}\bar{\theta}-y_k^{1,u}\ge 0, \tag{$\mathrm{PC}_k^{1,u}$}$$
$$x_k^{2,u}\bar{\theta}-y_k^{2,u}\ge 0. \tag{$\mathrm{PC}_k^{2,u}$}$$

In state $(1,i)$ at site k, the set of constraints is given by

$$x_k^{1,i}(\theta_H)\theta_H-y_k^{1,i}(\theta_H)\ge x_k^{1,i}(\theta_L)\theta_H-y_k^{1,i}(\theta_L), \tag{$\mathrm{IC}_k^{1,i}(\theta_H)$}$$
$$x_k^{1,i}(\theta_L)\theta_L-y_k^{1,i}(\theta_L)\ge x_k^{1,i}(\theta_H)\theta_L-y_k^{1,i}(\theta_H), \tag{$\mathrm{IC}_k^{1,i}(\theta_L)$}$$
$$x_k^{1,i}(\theta_L)\theta_L-y_k^{1,i}(\theta_L)\ge 0. \tag{$\mathrm{PC}_k^{1,i}(\theta_L)$}$$

As is well known, the participation constraint of the θ_H-type, $(\mathrm{PC}_k^{1,i}(\theta_H))$, is satisfied whenever $(\mathrm{IC}_k^{1,i}(\theta_H))$ and $(\mathrm{PC}_k^{1,i}(\theta_L))$ hold.

The constraints that need to be satisfied in state $(2,i)$ at site k are given by

$$X_k^{2,i}(\theta_H)\theta_H - Y_k^{2,i}(\theta_H) \geq X_k^{2,i}(\theta_L)\theta_H - Y_k^{2,i}(\theta_L), \qquad (\mathrm{IC}_k^{2,i}(\theta_H))$$
$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq X_k^{2,i}(\theta_H)\theta_L - Y_k^{2,i}(\theta_H), \qquad (\mathrm{IC}_k^{2,i}(\theta_L))$$
$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq 0. \qquad (\mathrm{PC}_k^{2,i}(\theta_L))$$

Again, the participation constraint of the θ_H-type, $(\mathrm{PC}_k^{2,i}(\theta_H))$, is satisfied whenever $(\mathrm{IC}_k^{2,i}(\theta_H))$ and $(\mathrm{PC}_k^{2,i}(\theta_L))$ hold.

A.2 Buyers' Subgame Equilibrium with no Congestion Effects

If some sellers' sale mechanisms does not generate congestion effects, visit probability q satisfies

$$q \begin{cases} = 0 & \text{if } \mathcal{R}_a(\pi_a,\gamma_a,0) < \mathcal{R}_b(\pi_b,\gamma_b,0) \text{ and } \mathcal{R}_a(\pi_a,\gamma_a,1) < \mathcal{R}_b(\pi_b,\gamma_b,1), \\ = 1 & \text{if } \mathcal{R}_a(\pi_a,\gamma_a,0) > \mathcal{R}_b(\pi_b,\gamma_b,0) \text{ and } \mathcal{R}_a(\pi_a,\gamma_a,1) > \mathcal{R}_b(\pi_b,\gamma_b,1). \end{cases}$$

However, if either $\mathcal{R}_a(\pi_a,\gamma_a,0) \geq \mathcal{R}_b(\pi_b,\gamma_b,0)$ and $\mathcal{R}_a(\pi_a,\gamma_a,1) \leq \mathcal{R}_b(\pi_b,\gamma_b,1)$ or $\mathcal{R}_a(\pi_a,\gamma_a,0) \leq \mathcal{R}_b(\pi_b,\gamma_b,0)$ and $\mathcal{R}_a(\pi_a,\gamma_a,1) \geq \mathcal{R}_b(\pi_b,\gamma_b,1)$, then both $q=1$ and $q=0$ are equilibria, along with any q satisfying (2). That is, when mechanisms do not generate congestion effects, buyers have an incentive to coordinate onto a common site, and the strategies allowing for this coordination are symmetric. Hence symmetry alone does not yield a unique equilibrium. I assume that in such cases the equilibrium selected is the mixed strategy equilibrium satisfying (2), the equilibrium that allows the least coordination between the buyers. As noted in the text, sale mechanisms without congestion effects never occur on the equilibrium path. However, it is possible to argue in favour of this selection off the equilibrium path by noting that in the symmetric pure strategy equilibrium one seller receives no visits and makes no profits, and hence has an incentive to offer a different mechanism at the offer stage.[26]

A.3 Characterisation of Incentive-Compatible Mechanisms

Proof of Lemma 1. Consider an incentive compatible mechanism γ_k at site k such that $(\mathrm{IC}_k^{1,i}(\theta_H))$ is slack. In particular, say

$$x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + C,$$

[26] Coles and Eeckhout (2003) give a different justification for ignoring pure strategy symmetric coordination equilibria. They note that since the mixed strategy equilibrium is always determined by (2), a seller that wishes to induce the mixed strategy outcome can always change his mechanism to induce congestion effects without varying rents and hence have the mixed equilibrium be the unique symmetric equilibrium of the subgame.

with $C > 0$. Consider an alternative mechanism $\tilde{\gamma}_k$ identical to γ_k except that

$$\tilde{y}_k^{1,i}(\theta_H) = y_k^{1,i}(\theta_H) + p_L C$$
$$\tilde{y}_k^{1,i}(\theta_L) = y_k^{1,i}(\theta_L) - p_H C.$$

In that case,

$$\begin{aligned}\tilde{x}_k^{1,i}(\theta_H)\theta_H - \tilde{y}_k^{1,i}(\theta_H) &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - p_L C \\ &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - C + p_H C \\ &= x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + p_H C \\ &= \tilde{x}_k^{1,i}(\theta_L)\theta_H - \tilde{y}_k^{1,i}(\theta_L).\end{aligned}$$

Thus, $\widetilde{IC}_k^{1,i}(\theta_H)$ binds. Since under $\tilde{\gamma}_k$ the transfer of type θ_L has been decreased, $\widetilde{PC}_k^{1,i}(\theta_L)$ is satisfied. Since both $\widetilde{IC}_k^{1,i}(\theta_H)$ and $\widetilde{PC}_k^{1,i}(\theta_L)$ hold, then so does $\widetilde{PC}_k^{1,i}(\theta_H)$. Finally, under $\tilde{\gamma}_k$ θ_H-types are worse off and θ_L-types are better off, so that $\widetilde{IC}_k^{1,i}(\theta_L)$ holds. Hence $\tilde{\gamma}_k$ is incentive compatible.

Profits for seller k in state $(1,i)$ under mechanism $\tilde{\gamma}_k$ are given by

$$\begin{aligned}p_H\tilde{y}_k^{1,i}(\theta_H) + p_L\tilde{y}_k^{1,i}(\theta_L) &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L) + p_H p_L C - p_L p_H C \\ &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L),\end{aligned}$$

where the last line is profits under γ_k in state $(1,i)$. Profits in other states are also unaffected. The proof for the case in which $IC_k^{2,i}(\theta_H)$ is slack is identical, with reduced-form mechanisms replacing the mechanisms. To that end, note that in state $(2,i)$, profits under mechanism γ_k are given by

$$\begin{aligned}&p_H^2\left[2y_k^{2,i}(\theta_H,\theta_H)\right] + 2p_L p_H\left[y_k^{2,i}(\theta_H,\theta_L) + y_k^{2,i}(\theta_L,\theta_H)\right] + p_L^2\left[2y_k^{2,i}(\theta_L,\theta_L)\right] \\ &= 2\left[p_H Y_k^{1,i}(\theta_H) + p_L Y_k^{1,i}(\theta_L)\right].\end{aligned}$$

□

As the proof manipulates mechanisms in different demand states independently, given an original profile where the incentive compatibility constraints of θ_H-types in both demand states are slack, one could find a rent and profit-equivalent mechanism with incentive constraints binding in both states by the same procedure.

Denote $\tilde{\gamma}_k$ as the *IC*(θ_H)*-equivalent* of γ_k. Similarly, denote by $\tilde{\Gamma}$ the set of IC(θ_H)-equivalent mechanisms. Given information provision (π_a, π_b), a game with mechanisms $(\gamma_a, \gamma_b) \in (\Gamma \setminus \tilde{\Gamma})^2$ generates the same distribution over outcomes as a game with mechanisms $(\tilde{\gamma}_a, \tilde{\gamma}_b)$, where $\tilde{\gamma}_k$ is the $IC(\theta_H)$-equivalent mechanism of γ_k. That is, excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibria in terms of information provision. On the other hand, when sellers also choose mechanisms, it is not the case that equilibrium

mechanisms must belong to $\tilde{\Gamma}$. However, Lemma 1 states that excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibrium allocations, traffic levels and payoffs. In what follows, incentive compatible mechanisms refers to mechanisms in $\tilde{\Gamma}$.

Given mechanism γ_k at site k, we can rewrite the expected rents promised at site k as

$$\begin{aligned} R_k^{\eta,u} &= r_k^{\eta,u} \text{ for } \eta \in \{1,2\}, \\ R_k^{1,i} &= r_k^{1,i} + p_H x_k^{1,i}(\theta_L)(\theta_H - \theta_L), \\ R_k^{2,i} &= r_k^{2,i} + p_H X_k^{2,i}(\theta_L)(\theta_H - \theta_L). \end{aligned}$$

Furthermore, Lemma 1 justifies the use of the following well-known result, whose proof is standard and omitted.

Lemma 6. *$\gamma_k \in \tilde{\Gamma}$ if and only if $x_k^{1,i}(\theta_H) \geq x_k^{1,i}(\theta_L)$, $X_k^{1,i}(\theta_H) \geq X_k^{1,i}(\theta_L)$, $r_k^{\eta,\tau} \geq 0$ for all $\eta \in \{1,2\}$ and $\tau \in \{i,u\}$ and θ_H-type rents are given by $x_k^{1,i}(\theta_H - \theta_L)$ in state $(1,i)$ and $X_k^{2,i}(\theta_H - \theta_L)$ in state $(2,i)$.*

A.4 Proofs of Main Results

Proof of Proposition 3. The following lemma characterises candidates for optimal information provision in symmetric equilibria with both ex post optimal prices or mechanisms.

Lemma 7. *Under Assumption 2, given either ex post optimal prices or mechanisms, there is a unique candidate profile for symmetric equilibrium in information provision, given by*

$$\pi^* \equiv \begin{cases} \frac{-(R^{1,i}+R^{2,i})\bar{\theta}}{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))} & \text{if } 2R^{1,i} > \bar{\theta} \text{ and } R^{1,i}+R^{2,i} > \frac{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})}{2R^{1,i}-\bar{\theta}}, \\ 1 & \text{otherwise.} \end{cases}$$

Proof. In any mechanism that is ex post optimal, seller a's profits can be written as

$$\mathcal{P}_a(\pi_a,\gamma,\pi_b,\gamma) = q^2\left[\pi_a\mathcal{S}^{2,i} + (1-\pi_a)\bar{\theta} - 2\pi_a R^{2,i}\right] + 2q(1-q)\left[\bar{\theta} - \pi_a R^{1,i}\right]. \tag{8}$$

At symmetric profiles, the market is shared equally between the two sellers, which maximises the probability that a seller is visited by a single buyer ($2q(1-q)$). Hence, marginal shifts in information provision at symmetric profiles have no effect on this probability.[27] This simplifies the expression for marginal profits at symmetric profiles under regular mechanism γ, which is given by

$$\begin{aligned} \left.\frac{\partial \mathcal{P}_a(\pi_a,\gamma,\pi_b,\gamma)}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi} &= \left.\frac{\partial q}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi}\left[\pi\mathcal{S}^{2,i} + (1-\pi)\bar{\theta} - 2\pi R^{2,i}\right] \\ &\quad + \frac{1}{4}\left[\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i}\right] - \frac{1}{2}R^{1,i}. \end{aligned} \tag{9}$$

[27] This observation, often useful in the the rest of the paper, is due to the binomial distribution of demand at sale sites. That is, if $X \sim B(n,q)$ then $\frac{\partial \Pr(X=k)}{\partial q} > 0$ whenever $k > qn$, where qn is the mean state of X. If qn is an integer, then $\frac{\partial \Pr(X=qn)}{\partial q} = 0$. That is, if q is increased marginally, states above the mean state become more likely and states below the mean less likely, while the probability of the mean state is unchanged.

Setting (9) equal to zero and checking the conditions for which $\pi < 1$, we obtain the expression for π^*.

To show uniqueness of the symmetric equilibrium candidate, I establish that both $\pi\mathcal{S}^{2,i} + (1-\pi)\bar{\theta} - 2\pi R^{2,i}$ and $\frac{\partial q}{\partial \pi_a}\big|_{\pi_a=\pi_b=\pi}$ are decreasing in π. First, seller a's profits in the two-buyer state decrease in π_a, since the term in the first brackets of (8) is linear in π_a and

$$\begin{aligned}
\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i} &= \mathcal{S}^{2,i} - \bar{\theta} - 2\Big(r^{2,i} + X^{2,i}(\theta_L)p_H(\theta_H - \theta_L)\Big) \\
&\leq \mathcal{S}^{2,i} - \bar{\theta} - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= \theta_H\left[2p_H X^{2,i}(\theta_H) - p_H\right] + \theta_L\left[2p_L X^{2,i}(\theta_L) - p_L\right] - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= (\theta_H - \theta_L)\left[p_L - 2X^{2,i}(\theta_L)\right] \\
&\leq 0. \qquad (10)
\end{aligned}$$

The second line follows since $r^{2,i} \geq 0$ and the fourth and fifth lines follow by Assumption 2 since $p_H X_k^{2,i}(\theta_H) + p_L X_k^{2,i}(\theta_L) = \frac{1}{2}$ and $X^{2,i}(\theta_L) \geq \frac{p_L}{2}$. Finally, by (2) and using the fact that $R^{1,u} = R^{2,u} = 0$ for ex post optimal prices or mechanisms, we have that

$$q = \frac{\pi_a R^{1,i} - \pi_b R^{2,i}}{(R^{1,i} - R^{2,i})(\pi_a + \pi_b)},$$

and it can be verified that

$$\left.\frac{\partial q}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi} = \frac{R^{1,i} + R^{2,i}}{4\pi(R^{1,i} - R^{2,i})},$$

which is decreasing in π. □

Since the profit function in (8) does not have convenient properties in π (for example, it is not concave), Lemma 7 alone is not sufficient to establish the existence of a symmetric equilibrium. This is done separately for ex post prices and mechanisms in the following two lemmas.

Lemma 8. *Under Assumption 2 and given optimal ex post prices, a symmetric equilibrium exists.*

Proof. I show that seller a's profit function is single-peaked around $\pi_a = \pi^*$ when $\pi_b = \pi^*$ and $\pi^* < 1$. Consider a candidate symmetric profile (π, π) and a deviation by seller a to $\pi + \lambda$ for $\lambda \in (-\pi, 1 - \pi]$, which induces traffic level $q^\lambda \in (0, 1]$. Then we have that

$$\begin{aligned}
q^\lambda &= \frac{\pi(R^{1,i} - R^{2,i}) + \lambda R^{1,i}}{(R^{1,i} - R^{2,i})(2\pi + \lambda)} \\
&= \frac{1}{2} + z, \\
\text{with } z &= \frac{\lambda(R^{1,i} + R^{2,i})}{2(R^{1,i} - R^{2,i})(2\pi + \lambda)}.
\end{aligned}$$

Also,

$$\begin{aligned}
\mathcal{P}_a(\pi+\lambda,\pi)-\mathcal{P}_a(\pi,\pi) &= z(z+1)\left[\pi\mathcal{S}^{2,i}+(1-\pi)\bar{\theta}-2\pi R^{2,i}\right]-2z^2\left[\bar{\theta}-\pi R^{1,i}\right] \\
&\quad +(\frac{1}{2}+z)^2\left[\mathcal{S}^{2,i}-\bar{\theta}-2R^{2,i}\right]-2\lambda(\frac{1}{2}+z)(\frac{1}{2}-z)R^{1,i} \\
&= \frac{\lambda^2}{D}\Bigg[4R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})\Bigg[(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})\bar{\theta} \\
&\quad -2\lambda(R^{1,i})^2(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))\Bigg] \\
&\quad +(R^{1,i}+R^{2,i})^2\bar{\theta}\Bigg[\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})(5R^{1,i}-R^{2,i}) \\
&\quad +(R^{1,i}+R^{2,i})^2\Bigg]\Bigg] \\
&\leq F\Bigg[(\mathcal{S}^{2,i}-\bar{\theta})(4(R^{1,i})^2+R^{1,i}R^{2,i}-(R^{2,i})^2) \\
&\quad -2R^{2,i}(R^{1,i}+R^{2,i})(2R^{1,i}-R^{2,i})\Bigg] \\
&< 0
\end{aligned}$$

Where D, F, $H>0$ are functions of parameters. The second equality follows from setting $\pi=\pi^*$ and rearranging terms. The inequality follows from the fact that $q^\lambda \leq 1$ when $\lambda \leq \frac{\bar{\theta}(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})}{-2R^{1,i}R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))}$. The last equality follows since, under optimal ex post prices, $\mathcal{S}^{2,i}-\bar{\theta}=0$ and

$$\begin{aligned}
2R^{1,i}-R^{2,i} &= \frac{3}{2}R^{1,i} \\
&= \frac{3}{2}p_H(\theta_H-\theta_L) \\
&> 0.
\end{aligned}$$

□

To complete the proof of my results for ex post optimal prices, note that Lemma 7 implies that $\pi^*<1$ as long as $2R^{1,i}-\bar{\theta}>0$, which can be reduced to $\frac{\theta_H}{\theta_L}>\frac{2-p_L}{1-p_L}$.

Lemma 9. *Under Assumption 1 and given ex post optimal mechanisms, a symmetric equilibrium exists.*

Proof. Under ex post optimal mechanisms $R^{1,i} = p_H(\theta_H - \theta_L)$ and it follows that

$$\begin{aligned} 2R^{1,i} - \bar{\theta} &= p_H\theta_H - p_L\theta_L - 2p_H\theta_L \\ &= p_H\theta_H + p_L\theta_L - 2\theta_L \\ &< \theta_L(p_L - 1) \\ &< 0, \end{aligned}$$

where the first inequality follows from Assumption 1. Hence, by Lemma 7 the only candidate for symmetric equilibrium is $\pi^* = 1$. To establish existence of equilibrium, I show that if γ is the ex post optimal mechanism under Assumption 1, then $\mathcal{P}_a(\pi_a, \gamma, 1, \gamma)$ is increasing in π_a. Given $\pi_a \leq 1$, $q \leq \frac{1}{2}$, and if π_a is such that $q > 0$, then

$$\begin{aligned} \mathcal{P}_a(\pi_a, \gamma, 1, \gamma) &= \left(\frac{\pi_a - \frac{p_L}{2}}{(1+\pi_a)(1-\frac{p_L}{2})}\right)^2 \bar{\theta} + 2\left(\frac{(\pi_a - \frac{p_L}{2})(1 - \frac{\pi_a p_L}{2})}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\big(\bar{\theta} - \pi_a p_H(\theta_H - \theta_L)\big) \\ &= \left(\frac{\pi_a - \frac{p_L}{2}}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\Big(\bar{\theta}(p_H\pi_a + 2 - \frac{p_L}{2}) + 2(1 - \frac{\pi_a p_L}{2})\pi_a p_H(\theta_H - \theta_L))\Big) \\ &\equiv A(\pi_a)\big(B(\pi_a) + C(\pi_a)\big) \end{aligned}$$

Where $B(\pi_a)$ is clearly increasing in π_a, while it can be verified that $A(\pi_a)$ and $C(\pi_a)$ are increasing whenever $\pi_a \leq 1 + p_L$ and $\pi_a \leq \frac{1}{p_L}$, respectively, which is always true. □

This completes the proof of Proposition 3. □

Proof of Lemma 2. Lemma 2, stated in terms of $IC(\theta_H)$-equivalent mechanisms, requires that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$. This condition simply states that it is always possible, for at least one state, to decrease transfers in an incentive compatible way. Any mechanism $\gamma_a \in \Gamma$ that satisfies this last property would have its $IC(\theta_H)$-equivalent mechanism satisfy the property that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ (through Lemma 1). The following proof then applies to all incentive compatible mechanisms that are components of some equilibrium, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is an equilibrium, that $\mathbf{E}_\eta\mathbf{E}_\tau\mathcal{S}_a^{\eta,\tau}$ is increasing in π_a, that it is not the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ and that $\pi_a < 1$. Consider a deviation by seller a to a profile in which

$$\begin{aligned} \hat{\pi}_a &= \pi_a + \lambda \\ \hat{r}_a^{\eta,\tau} &= r_a^{\eta,\tau} - \delta^{\eta,\tau}, \end{aligned}$$

where $\lambda \in (0, 1-\pi_a]$ and $\delta^{\eta,\tau} < r_a^{\eta,\tau}$ for all (η, τ). For this deviant profile not to affect buyers'

visit decisions (or expected rents), we need

$$\begin{aligned} & q\left[(\pi_a+\lambda)\left[r_a^{2,i}-\delta^{2,i}+z_a^{2,i}\right]+(1-\pi_a-\lambda)\left[r_a^{2,u}-\delta^{2,u}\right]\right] \\ & \quad +(1-q)\left[(\pi_a+\lambda)\left[r_a^{1,i}-\delta^{1,i}+z_a^{1,i}\right]+(1-\pi_a-\lambda)\left[r_a^{1,u}-\delta^{1,u}\right]\right] \\ & \quad =q\left[\pi_a\left[r_a^{2,i}+z_a^{2,i}\right]+(1-\pi_a)\left[r_a^{2,u}\right]\right]+(1-q)\left[\pi_a\left[r_a^{1,i}+z_a^{1,i}\right]+(1-\pi_a)\left[r_a^{1,u}\right]\right], \end{aligned}$$

or

$$\begin{aligned} & (\pi_a+\lambda)\left[q\delta^{2,i}+(1-q)\delta^{1,i}\right]+(1-\pi_a-\lambda)\left[q\delta^{2,u}+(1-q)\delta^{1,u}\right] \\ & \quad =\lambda\left[q\left[r_a^{2,i}+z_a^{2,i}-r_a^{2,u}\right]+(1-q)\left[r_a^{1,i}+z_a^{1,i}-r_a^{1,u}\right]\right], \end{aligned} \tag{11}$$

where $z_a^{1,i}=r_a^{1,i}+p_H x_a^{1,i}(\theta_L)(\theta_H-\theta_L)\geq 0$ and $z_a^{2,i}=r_a^{2,i}+p_H X_a^{2,i}(\theta_L)(\theta_H-\theta_L)\geq 0$ are the expected informational rents given the allocations of the original mechanism. The sign of the right-hand side (RHS) of (11) is given by the properties of the mechanism at site a. It is positive if buyers prefer, on average, to be informed at the site, and negative if buyers prefer, on average, to be uninformed.

Suppose $\pi_a>0$. Suppose $RHS(\lambda)>0$. Set $\delta^{\eta,\tau}=0$ for all $(\eta,\tau)\neq(2,u)$ and $\delta^{2,u}>0$. Then

$$\begin{aligned} \lim_{\lambda\to 0} LHS((\delta^{\eta,\tau}),\lambda) &> \lim_{\lambda\to 0} RHS(\delta) \\ &= 0 \end{aligned}$$

since $\pi_a<1$. Also

$$\begin{aligned} LHS((\delta^{\eta,\tau}),1-\pi_a) &= 0 \\ &< RHS(1-\pi_a). \end{aligned}$$

Hence there exists $\hat{\lambda}\in(0,1-\pi_a]$ such that $LHS((\delta^{\eta,\tau}),\hat{\lambda})=RHS(\hat{\lambda})$.

Suppose $RHS(\lambda)<0$. Suppose $\pi_a>0$. By assumption, there exists some $r_a^{\hat{\eta},\hat{\tau}}>0$. Set $\delta^{\eta,\tau}=0$ for all $(\eta,\tau)\neq(\hat{\eta},\hat{\tau})$ and $\delta^{\hat{\eta},\hat{\tau}}$ such that

$$LHS((\delta^{\eta,\tau}),1-\pi_a)>RHS(1-\pi_a) \tag{12}$$

Fix $\hat{\lambda}\in(0,1-\pi_a]$ such that $LHS((\delta^{\eta,\tau}),\hat{\lambda})=RHS(\hat{\lambda})$. Such a $\hat{\lambda}$ exists by (12) and since

$$\begin{aligned} \lim_{\lambda\to 0} RHS(\lambda) &= 0 \\ &> \lim_{\lambda\to 0} LHS(\delta^{\eta,\tau},\lambda) \end{aligned}$$

as $\pi_a>0$ and $\hat{\delta}^{\hat{\eta},\hat{\tau}}<0$.

Suppose $r_a^{2,u}=r_a^{1,u}=0$. Then buyers get no rents from visiting seller a in equilibrium. Either $q=0$ and seller a makes no profits in equilibrium, or buyers get no rents from either

site in equilibrium. The first case cannot occur in equilibrium, as any deviation for sellers that ensure positive profits and visit probabilities is profitable, and such deviations always exist. In the second case, any seller could deviate by offering marginally more rents and capturing all buyer visits, another contradiction. Hence there is some $\hat{\eta}$ with $r_a^{\hat{\eta},u} > 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta,\tau) \neq (\hat{\eta},u)$ and $\delta^{\hat{\eta},u} < 0$ Set $\hat{\lambda}$ such that $LHS((\delta^{\eta,\tau}),\hat{\lambda}) = RHS(\hat{\lambda})$. Such a $\hat{\lambda}$ exists since $LHS((\delta^{\eta,\tau}),1) = 0 > RHS(1)$ and

$$\begin{aligned}\lim_{\lambda\to 0} RHS(\lambda) &= 0 \\ &< \lim_{\lambda\to 0} LHS(\delta^{\eta,\tau},\lambda)\end{aligned}$$

as $\hat{\delta}^{\hat{\eta},u} < 0$.

Finally, if $RHS(\lambda) = 0$, buyers are indifferent between informed and uninformed states at site a and a seller can increase information provision without shifting traffic by setting $\delta^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$.

In all cases, the arguments above yield a deviation for seller a which keeps rent payouts unchanged and strictly increases the surplus available at site a. This implies that $(\pi_a,\gamma_a,\pi_b,\gamma_b)$ is not an equilibrium. □

Proof of Lemma 3. My argument proceeds with mechanisms in $\tilde{\Gamma}$. However, if a mechanism in $\Gamma \setminus \tilde{\Gamma}$ without PAE were a component of an equilibrium, applying the following proof to its IC(θ_H)-equivalent (through Lemma 1) would yield a contradiction, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Consider an incentive compatible mechanism γ_k at site k such that $x_k^{1,i}(\theta_H) < 1$. Consider an alternative mechanism $\hat{\gamma}_k$ identical to γ_k except that

$$\begin{aligned}\hat{x}_k^{1,i}(\theta_H) &= x_k^{1,i}(\theta_H) + \epsilon \\ \hat{y}_k^{1,i}(\theta_H) &= y_k^{1,i}(\theta_H) + \epsilon\theta_H,\end{aligned}$$

where $\epsilon \in (0, 1 - x_k^{1,i}]$. We have $\hat{x}_k^{1,i}(\theta_H) > x_k^{1,i}(\theta_H) \geq \hat{x}_k^{1,i}(\theta_L) > x_k^{1,i}(\theta_L)$ and $\hat{r}^{1,i} = r^{1,i} \geq 0$ since $\gamma_k \in \tilde{\Gamma}$, and so $\hat{\gamma}_k \in \tilde{\Gamma}$. Note that $\hat{R}^{1,i} = R^{1,i}$ and hence buyer rents and visit decisions are unaffected. However, seller k's profits are higher under $\hat{\gamma}_k$ than under γ_k if buyers sometimes visit k since θ_H-type transfers in the one-buyer state are higher.

As noted in the text, the proof needs to be modified in the two-buyer state if $X_k^{2,i}(\theta_H) < p_L + \frac{1}{2}p_H$ and if the constraint $x^{2,i}(\theta_H,\theta_L) + x^{2,i}(\theta_L,\theta_H) \leq 1$ is binding under the original mechanism γ_k. If this is not the case, then the previous proof applies to the reduced-form mechanisms. If not, it must be that $x^{2,i}(\theta_L,\theta_H) > 0$, that is, a θ_L-type is sometimes allocated the good in the presence of a θ_H-type. Consider an alternative mechanism $\hat{\gamma}_k$ identical to γ_k except that

i. θ_L-types never get preference over θ_H-types, $\hat{x}^{2,i}(\theta_H, \theta_L) = 1$ and $\hat{x}^{2,i}(\theta_L, \theta_H) = 0$, so that

$$\hat{X}^{2,i}(\theta_L) = X^{2,i}(\theta_L) - p_H x_k^{2,i}(\theta_L, \theta_H)$$
$$\hat{X}^{2,i}(\theta_H) = X^{2,i}(\theta_H) + p_L x_k^{2,i}(\theta_L, \theta_H).$$

ii. Transfers are adjusted so that rents to both types are unchanged

$$\hat{Y}^{2,i}(\theta_L) = Y^{2,i}(\theta_L) - \theta_L(X_k^{2,i}(\theta_L) - \hat{X}^{2,i}(\theta_L))$$
$$\hat{Y}^{2,i}(\theta_H) = Y^{2,i}(\theta_H) + \theta_H(\hat{X}_k^{2,i}(\theta_H) - X^{2,i}(\theta_H)).$$

By condition i and since $\gamma_k \in \tilde{\Gamma}$, we have that $\hat{X}^{2,i}(\theta_H) > X^{2,i}(\theta_H) \geq X^{2,i}(\theta_L) > \hat{X}^{2,i}(\theta_L)$. Along with condition ii, this implies that $\hat{\gamma}_k \in \tilde{\Gamma}$.

Profits to seller k in the two-buyer state under $\hat{\gamma}_k$ are given by

$$\begin{aligned} 2\left[p_L\hat{Y}^{2,i}(\theta_L) + p_H\hat{Y}^{2,i}(\theta_H)\right] &= 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H) + p_H p_L(\theta_H - \theta_L)x^{2,i}(\theta_L, \theta_H)\right] \\ &> 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H)\right], \end{aligned}$$

where the last expression is profits to seller k in the two-buyer state under γ_k. The inequality follows since by hypothesis $x^{2,i}(\theta_L, \theta_H) > 0$. Thus seller k gains by offering $\hat{\gamma}_k$ if buyers visit k with positive probability since traffic and one-buyer state profits are unchanged and two-buyer state profits are higher. Furthermore, under $\hat{\gamma}_k$ it is the case that $\hat{X}_k^{2,i}(\theta_H) = p_L + \frac{1}{2}p_H$.

Similarly, for uninformed allocations, consider an incentive compatible mechanism γ_k at site k such that $x_k^{\eta,u} < 1$ for some $\eta \in \{1, 2\}$. Consider an alternative mechanism $\hat{\gamma}_k$, identical to γ_k except that in state (η, u)

$$\hat{x}_k^{\eta,u} = x_k^{\eta,u} + \epsilon$$
$$\hat{y}_k^{\eta,u} = y_k^{\eta,u} + \epsilon\bar{\theta},$$

where $\epsilon \in (0, 1 - x^{\eta,u}]$. Thus buyer rents are the same under both mechanisms but seller k's profits are higher in state (η, u) if buyers visit seller k with positive probability since the good is sold more often at higher prices. □

Proof of Lemma 4. Consider an incentive compatible mechanism γ_k at site k such that $x_k^{1,i}(\theta_L) < 1$ and the level of rents provided to type θ_L is given by $r^{1,i} \geq 0$.[28] Then

$$y_k^{1,i}(\theta_L) = \theta_L x_k^{1,i}(\theta_L) - r^{1,i}, \tag{13}$$

[28] I need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.

and, by Lemmas 1 and 3

$$y_k^{1,i}(\theta_H) = \theta_H - x_k^{1,i}(\theta_L)(\theta_H - \theta_L) - r^{1,i}. \tag{14}$$

By (13) and (14), write seller k's profits conditional on $(\text{IC}_k^{1,i}(\theta_H))$ binding and type θ_L receiving rents $r^{1,i}$ as

$$x_k^{1,i}(\theta_L)(\theta_L - p_H\theta_H) + p_H\theta_H - r^{1,i}. \tag{15}$$

These are increasing in $x_k^{1,i}(\theta_L)$ whenever $\theta_L > p_H\theta_H$. Since $x^{1,i}(\theta_H) = 1$ by Lemma 3, an increase in $x^{1,i}(\theta_L)$ maintains incentives compatibility so seller k can increase profits in state $(1,i)$ by doing so. This increases traffic to site k (since rents to θ_H-types increase). But at a symmetric equilibrium $q = \frac{1}{2}$ and marginal changes in traffic have negligeable effects on the probability of the one-buyer state $(2q(1-q))$, so that profits of seller k increase with marginal changes in $x^{1,i}(\theta_L)$ if profits in the two-buyer state are assumed to be nonnegative. However, note that this argument ensures that profits in the two-buyer state must be nonnegative in a symmetric equilibrium. If not, a seller could marginally increase transfers in the two-buyer state without affecting traffic significantly in the one-buyer state, while both traffic and losses per buyer would decrease in the two-buyer state. □

Proof of Lemma 5. To show that $R^{2,i} \leq R^{1,i}$, consider a symmetric equilibrium with $\pi = 1$, FAE and a mechanism[29] γ such that $R^{1,i} < R^{2,i}$. This last fact implies that $r^{2,i} > 0$. Consider a mechanism $\hat{\gamma}_k$ for seller k identical to γ except that $\hat{r}^{2,i} = r^{2,i} - \Delta$. By (2) and the argument in the text for $\Delta \approx 0$, $\hat{\gamma}_k$ leads to an infinitesimal increase in the number of buyers visiting site k. Locally, moving away from a symmetric profile does not change the probability of the one-buyer state, while it increases that of the two-buyer state, where rents are now lower. This deviation is thus profitable given that profits in the two-buyer state are nonnegative (see proof of Lemma 4).

To show that $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$, consider marginal variations in $R^{1,i}$ and $R^{2,i}$ that leave $\pi = 1$ and allocative efficiency unchanged. Assume for now that $r^{1,i} > 0$ and $r^{2,i} > 0$ to ensure that it is always possible to effect such marginal changes through transfers. Profits for seller a are given by

$$\mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b) = q^2[\bar{S}^{2,i} - 2R_a^{2,i}] + 2q(1-q)[\bar{\theta} - R_a^{1,i}].$$

At a symmetric profile, the marginal changes in the term $q(1-q)$ can be ignored and thus

$$\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{1,i}} = 2q\left[\frac{\partial q}{\partial R_a^{1,i}}(\bar{S}^{2,i} - 2R_a^{2,i}) - (1-q)\right], \tag{16}$$

[29] I need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.

where, at a symmetric profile with $\pi = 1$ we have $q = \frac{1}{2}$ and $\frac{\partial q}{\partial R_a^{1,i}} = \frac{1}{4(R_a^{1,i} - R_a^{2,i})}$. Thus

$$\begin{aligned}\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{1,i}} &= \left(\frac{1}{4}\right) \frac{\bar{S}^{2,i} - 2R_a^{2,i}}{R_a^{1,i} - R_a^{2,i}} - \frac{1}{2} \\ &= 0 \quad \text{only when } R^{1,i} = \frac{\bar{S}^{2,i}}{2}.\end{aligned}$$

In the same way, it can be computed that $\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{2,i}} = 0$ only when $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$. That is, the same condition holds for marginal changes in expected rents in both one-buyer and two-buyer states. Since $\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{2,i}} = 0$ and $\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{1,i}} = 0$ yield the same condition, we need to worry about the existence of derivatives only when $r^{1,i} = r^{2,i} = 0$. But then an argument considering deviations $R_a^{1,i} + \Delta$ or $R_a^{2,i} + \Delta$ yields the result. □

Proof of Proposision 4. Fixing some profile that satisfies the assumptions of the proposition, I will first show that with $\pi = 1$ and FAE, no deviation consisting of either individual or joint shifts (not necessarily local) in $R^{1,i}$ and $R^{2,i}$ can achieve higher profits. Since the candidate profile has full information and FAE, considering changes in rents where surplus in both states is maximized gives an upper bound on the profitability of deviations that involve the same changes in rents but that include a decrease in information provision and/or allocative efficiency.

Consider some profile with $\pi = 1$ and associated rents $R^{1,i} \geq R^{2,i}$. Consider a deviation profile for seller a in which

$$\begin{aligned}\hat{R}_a^{1,i} &= R^{1,i} + \Delta^1 \\ \hat{R}_a^{2,i} &= R^{2,i} + \Delta^2,\end{aligned}$$

where Δ^η for $\eta \in \{1, 2\}$ need not be positive. Clearly, seller a cannot profitably deviate to any mechanism for which $\hat{q} = 0$. Also, the most profitable deviation to some mechanism such that $\hat{q} = 1$ is such that any less generous mechanism leads to $\hat{q} < 1$. Hence we can restrict attention to pairs (Δ^1, Δ^2) such that the level of traffic $\hat{q} \in (0, 1]$ is given by (2). Hence $\hat{q}$ is given by

$$\begin{aligned}\hat{q} &= \frac{(R^{1,i} - R^{2,i}) + \Delta^1}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2} \\ &= \frac{1}{2} + z \\ &\qquad \text{with } z = \left(\frac{1}{2}\right) \frac{\Delta^1 + \Delta^2}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2}.\end{aligned}$$

The difference in profits can be written as

$$\begin{aligned}\mathcal{P}_a(1,\hat{\gamma_a},1,\gamma_b)-\mathcal{P}_a(1,\gamma_a,1,\gamma_b) &= \left[\bar{S}^{2,i}-2R^{2,i}\right](x(x+1))-2\left[m-R^{1,i}\right]x^2 \\ &\quad -2\Delta^2\left(\frac{1}{2}+x\right)^2-2\Delta^1\left(\frac{1}{2}+x\right)\left(\frac{1}{2}-x\right) \\ &= C\Bigg[\left[\bar{S}^{2,i}-2R^{2,i}\right]\left(4((R^{1,i}-R^{2,i}))+3\Delta^1-\Delta^2\right)\left(\Delta^1+\Delta^2\right) \\ &\quad -2\left[\bar{\theta}-R^{1,i}\right]\left(\Delta^1+\Delta^2\right)^2 \\ &\quad -8\left((R^{1,i}-R^{2,i})\right)\left((R^{1,i}-R^{2,i})+\Delta^1\right)\left(\Delta^1+\Delta^2\right)\Bigg],\end{aligned}$$

where $C=\left(\frac{1}{4}\right)\left[\frac{1}{2(R^{1,i}-R^{2,i}))+\Delta^1-\Delta^2}\right]^2>0$. Set the original candidate profile as

$$\begin{aligned}R^{1,\imath} &= \frac{\bar{S}^{2,i}}{2} \\ R^{2,i} &= \frac{\bar{S}^{2,i}}{2}-\epsilon, \text{ for } \epsilon\geq 0.\end{aligned}$$

simplifying the profit difference yields

$$\begin{aligned}\mathcal{P}_a(1,\hat{\gamma_a},1,\gamma_b)-\mathcal{P}_a(1,\gamma_a,1,\gamma_b) &= C\left[(\Delta^1+\Delta^2)^2(-2\epsilon-(2\bar{\theta}-\bar{S}^{2,i}))\right] \\ &< 0 \text{ for any } (\Delta^1,\Delta^2), \text{ since } \epsilon>0 \text{ and } 2\bar{\theta}>\bar{S}^{2,i}.\end{aligned}$$

Thus no deviations are profitable. □