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Abstract

Carlin and Soskice (2005) advocate a 3-equation model of stabilization policy. One equation is a monetary reaction rule *MR* derived by assuming that governments have performance objectives, but are constrained by a Phillips curve *PC*. Central banks attempt to implement these objectives by setting interest rates along an *IS* curve. They label this the *IS-PC-MR* model. Observing that governments have more tools than just the interest rate, we drop the *IS* equation, simplifying their model to 2 equations. Adding a random walk model of the unobserved potential growth, we develop their *PC-MR* model into a state space specification of the short-run political economy. This is an appropriate econometric method because it incorporates recursive forecasts of unobservable state variables based on contemporaneous information measured with real-time data. Our results are generally consistent with US economic history. One qualification is that governments are more likely to target growth rates than output gaps. Another is that policy affects outcomes after a single lag. This assumption fits the data better than an alternative double-lag timing: one lag for output, plus a second for inflation has been proposed. We also infer that inflation expectations are more likely to be backward rather than forward looking.

Keywords: new Keynesian stabilization, policy targets, microfoundations, real-time data
JEL Classification: E3, E6

1. Introduction

Central to the new Keynesian conception of stabilization policy is the assumption that governments actively lean against the macroeconomic wind. Carlin and Soskice derive their monetary rule as rational behavior for a government constrained by a Phillips curve.[1] A number of alternative assumptions are consistent with this approach. One of these relates to the functional form of the government's objective function. Starting with a quadratic form involving an inflation target, we highlight the differences implied by substituting an output growth target for the conventional an output gap target.

Another issue is the timing of policy reactions, and thus the lag structure of the model. How quickly do policy makers respond to nominal and real shocks? Is there a delay before policy initiatives have an impact? Carlin and Soskice's assumption is an inherent policy lag of one period, but plausible alternatives include an immediate response or a two period lag, one for real output and a second for inflation. We develop econometric specifications to distinguish among these possibilities.

We apply the state space methodology to specify a coherent model of stabilization and estimate its parameters. This is an appropriate methodology because this theory includes unobserved state variables: the output gap and potential growth rate. We model potential growth as a random walk. By formalizing the relation between observables and

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[1] The original insight for this literature dates to Kalecki (1943); also see Nordhaus (1975). Modern versions begin with Kydland and Prescott (1977) who introduced the logic of rational expectations; Barro and Gordon (1983) further develop this logic.

unobservables, we obtain Bayesian forecasts of the unobservables conditioned on available information.

A third modeling issue concerns how agents and governments make inflation forecasts; we explore several possibilities. For agents we begin with simple backward-looking expectations, and develop an extension to forward-looking ones using on a two-step estimation of a new Keynesian sticky-price model. Forward-looking expectations are appealing because they cohere with the notion of well-informed rational agents. We find, however, that this new Keynesian Phillips curve does not improve the statistical fit of these econometric models.

2. Endogenous stabilization

Phillips' original idea is an inverse relation between wage inflation and the unemployment rate. This empirical regularity has been generalized to account for the related tradeoff between price inflation and the output gap (defined as $x_t = \ln(Y_t) - \ln(Y_t^*)$ where Y_t is real aggregate output and Y_t^* is potential output). Friedman and Phelps augment the equation by added an adjustment for inflation expectations, so that

$$\pi_t = E_{t-1}^a \pi_t + \psi x_t + \varepsilon_t, \tag{1}$$

where π_t is the inflation rate, $E_{t-1}^a \pi_t$ denotes expected inflation based on information available to a representative market agent in the previous period and ε_t an exogenous shock. Beginning with Fischer (1977) the literature has focused on explanations of this macroeconomic regularity founded in microeconomic mechanisms, including overlapping nominal wage contracts, stochastic price resetting, costly price adjustment

and stochastic updating of information. Calvo's (1983) "sticky price" model assumes that firms are uncertain whether they will be able to adjust its price in the each period. An important result is that the resulting new Keynesian Phillips curve is forward looking, as contrasted to the backward-looking interpretation of expectations given to (1). We explore this refinement empirically in Section 6.

We assume an activist stabilization policy. A simple possibility supposes that the government's goals are given as a quadratic function of the output gap and inflation,

$$U = -\left(x^2 + \left(\pi - \hat{\pi}\right)^2\right),$$

where $\hat{\pi}$ is the inflation target, not necessarily the announced target.[2] Quadratic forms are tractable because they result in linear solutions.[3] Within the quadratic family, a variety of alternatives are plausible. Ours has circular indifference curves, but these can be made elliptical by adding a parameter to reflect the relative weight of inflation versus output goals. Some studies consider parabolic indifference curves.[4] Differing targets for inflation could account for ideological differences. Often the output target exceeds zero.[5] Kiefer (2008) estimates several different quadratic forms. He confirms the conventional wisdom that it is not possible to statistically separate goal weights from inflation and output

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[2] The modeling of collective objectives is controversial. Textbooks often define social welfare as an aggregation of individual preferences. Woodford (2003) establishes microfoundations for several close relatives of this function form as an approximation to the utility of a representative consumer.

[3] Ruge-Murcia (2003) questions the conventional linearity assumption. He develops an alternative where the government's inflation preferences are asymmetrical around its target.

[4] See, for example, Alesina et al. (1997).

[5] Barro and Gordon (1983) assume a zero inflation target and an unemployment target below the natural rate.

targets.[6] Thus, the inflation target parameter is a composite measure of weights and targets. More generally the government's objective may include a discounted stream of expected future outcomes. Our function can be interpreted as including only the period in which current policy initially influences outcomes, ignoring periods further in the future as second order.[7]

Governments have limited options in this model. Although it may be able to exploit information advantages implied in (1) to lean against the macroeconomic wind, nevertheless its goals $\left(x = 0 \text{ and } \pi = \hat{\pi}\right)$ are usually unattainable in the short run. Following Carlin and Soskice, we assume that policymaking is only effective after a one-period delay. Although they take periods as years, we extend the analysis to three-month, six-month and yearlong periods. They explain this delay as a lag in the *IS* relation between interest rate and output gap.[8] Governments have more tools than just the interest rate, we assume that similar lags apply to other policy instruments. Accordingly, we add an expectations operator and date the objective as

$$E_t^g U = -E_t^g\left(x_{t+1}{}^2 + \left(\pi_{t+1} - \hat{\pi}\right)^2\right), \tag{2}$$

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[6] Also see Ireland (1999).

[7] See Carlin and Soskice (2005) for an elaboration of this simplification. See Svensson (1997) for monetary policy based on multi-period objective functions. Along the same line of reasoning governments might plan for its current term of office only, or it might plan to be in office for several terms, discounting the future according to the probability of holding office. Furthermore, it might weigh pre-election years more heavily. These ideas are pursued in Kiefer (2000) who finds little evidence that governments have long-term stabilization goals.

[8] Although plausible, such policy lags conflict with conventional consumer choice derivations of the *IS* curve which do not show any lag; for example see Gali (2008).

which defines the government's expectation of next period's welfare. Subject to the Phillips curve constraint, the government's preferred inflation is $\frac{E_t^a \pi_{t+1} + \psi^2 \hat{\pi}}{1+\psi^2}$, assuming that the government cannot forecast the inflation shock, $E_t^g \varepsilon_{t+1} = 0$. To the extent that agents are rational and well informed they would expect this inflation rate, however if expectations are inertial the government has an informational advantage.

Adding a random inflation shock and lagging by one period, gives inflation as

$$\pi_t = \frac{E_{t-1}^a \pi_t + \psi^2 \hat{\pi}}{1+\psi^2} + \varepsilon_t \tag{3}$$

Using (1) and adding another unpredictable shock, the resulting output gap is

$$x_t = -\psi\left(\frac{E_{t-1}^a \pi_t - \hat{\pi}}{1+\psi^2}\right)^2 + \xi_t .$$

We take the inflation shock ε_t and the output shock ξ_t to be exogenous and unpredictable. The output gap and the growth rate are equivalent measures because the growth rate can be defined as

$$g_t = \ln(Y_t) - \ln(Y_{t-1}) = g_t^* - x_{t-1} + x_t ,$$

where $g_t^* = \ln(Y_t^*) - \ln(Y_{t-1}^*)$ is the unobserved growth of potential output. We thus rewrite output in terms of the growth rate as

$$g_t = g_t^* - x_{t-1} - \psi\left(\frac{E_{t-1}^a \pi_t - \hat{\pi}}{1+\psi^2}\right) + \xi_t \tag{4}$$

This has the advantage of putting an observable variable on the left-hand-side. Equations (3) and (4) imply that observed inflation and growth depend on shocks, conditions

inherited from the past, expectations and policy targets.[9] We assume that the government implements policy through fiscal, monetary and other policies, and that the various agencies (budget office, central bank and treasury) pursue this common policy.[10]

The output gap is an unobserved variable. Conventional measures of the output gap suffer from the shortcoming that they are often defined by exogenously detrending observed real GDP. We endogenize the output gap by assuming that real potential growth follows a random walk,

$$g_t^* = g_{t-1}^* + \upsilon_t. \tag{5}$$

This model recognizes that the underlying growth rate changes over time, and that its next turning point is unpredictable. We assume that the potential growth shocks follow the normal distribution $\upsilon_t \sim N(0,\sigma_\upsilon^2)$, that they are serially independent, $\mathrm{cov}(\upsilon_t,\upsilon_{t\ s}) = 0$ and independent of inflation or output shocks $\mathrm{cov}(\upsilon_t,\varepsilon_t) = 0$ and $\mathrm{cov}(\upsilon_t,\xi_t) = 0$. The level of potential GDP can be defined recursively,

$$\ln\left(Y_t^*\right) = \ln\left(Y_{t-1}^*\right) + g_t^*. \tag{6}$$

Altogether this defines a state space model including unobserved state and observed variables. Our state equations are (5) and (6); substituting the

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[9] Rational agents come to understand that a policy of $\hat{\pi} = 0$ implies inflation. In the absence of shocks or uncertainty, the time-consistent equilibrium inflation rate should occur where inflation is just high enough so that the government is not tempted to spring a policy surprise. This equilibrium is the potential output, potential growth and the inflation target, $x = 0, g = g^*, \pi = \hat{\pi}$.

[10] Because our goal is to model inflation and gap expectations, it is not necessary to treat the instrument question. As such our model can be seen to be the first two equations of Carlin and Soskice's (2005) three-equation model, ignoring the *IS* equation.

definition $x_t = \ln(Y_t) - \ln(Y_t^*)$ into (3) and (4) gives our observation equations.[11] The observation equations are reduced forms determined by $E_{t-1}^a \pi_t, x_{t-1}, g_t^*, \xi_t$ and ε_t; these are linear in the variables, but nonlinear in coefficients.

3. Alternative timing assumptions

Some authors assume that the government implements effective policy remedies for inflation and output shocks without any lag. For example, Clarida et al. (1999) specify an *IS* curve in which current interest rates determine current outputs. If this is possible, the relevant objective function has the current date,

$$E_t^g U = -E_t^g \left(x_t^2 + (\pi_t - \hat{\pi})^2 \right)$$

Solving by the same method, recognizing that $E_t^g \varepsilon_t = \varepsilon_t$, gives a no-lag model,

$$\begin{aligned} \pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \hat{\pi}}{1+\psi^2} + \frac{\varepsilon_t}{1+\psi^2} \\ g_t &= g_t^* - x_{t-1} - \psi \left(\frac{E_{t-1}^a \pi_t - \hat{\pi}}{1+\psi^2} \right) - \frac{\psi \varepsilon_t}{1+\psi^2} \end{aligned} \tag{7}$$

Output shocks do not appear in (7); this is consistent with the theoretical result that optimal policy perfectly accommodates any output shifts, either temporary or potential.[12] Except for the error structure, (7) is identical to (3) and (4), where we assumed that the government can only implement policy with a one-period delay.

A third possibility stipulates a double-lag: the output impact is delayed by one period as before, but the inflation impact is delayed by two periods. Svensson (1997) hypothesizes that output is affected by the interest rate instrument after one period, and

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[11] See Hamilton (1994) for a textbook presentation of the Kalman filter methodology.
[12] This is Clarida's baseline result; they also extend their analysis to policy lags and imperfect information.

inflation effects are delayed an additional period due to the lagging of output gap in the Phillips curve,

$$\pi_t = E^a_{t-1}\pi_t + \psi x_{t-1} + \varepsilon_t, \tag{8}$$

although he offers no theoretic foundations for these lags. Now the relevant objective includes only the arguments that are initially be affected by policy,[13]

$$E^g_t U = -E^g_t\left(x_{t+1}{}^2 + \left(\pi_{t+2} - \hat{\pi}\right)^2\right)$$

Solving by the same method, lagging appropriately and adding random shocks to both the inflation and output solution gives

$$\begin{aligned} \pi_t &= \frac{E^g_{t-2}\left(E^a_{t-1}\pi_t\right) + \psi^2\hat{\pi}}{1+\psi^2} + \varepsilon_t \\ g_t &= g^*_t - x_{t-1} - \psi\left(\frac{E^g_{t-1}\left(E^a_t\pi_{t+1}\right) - \hat{\pi}}{1+\psi^2}\right) + \xi_t \end{aligned} \tag{9}$$

where $E^g_{t-2}\left(E^a_{t-1}\pi_t\right)$ denotes the government's expectation in $(t\text{-}2)^{th}$ period of the private sector's forecast to be formed in the $(t\text{-}1)^{th}$ period. This double-lag timing assumption implies that inflation is affected by the government's two-period forecast of inflation. A two-period government forecast also affects growth, but here it is only one period old; output policy is looking ahead to influence future inflation. Carlin and Soskice favor the double-lag as being more realistic, and for facilitating the derivation a Taylor rule. Of course other timing assumptions are possible. Comparing our three different cases, (3)-

[13] For simplicity we do not discount the inflation term even though that it would be appropriate for this dating.

(4), (7) and (9), is consistent with the notion that policy under imperfect information can be characterized as the certainty equivalent of the perfect information policy.

4. Growth targets

Returning to the single-lag timing assumption, we consider a related objective function parameterized on growth rates, rather than output levels,

$$E_t^g U = -E_t^g\left(\left(g_{t+1} - g_{t+1}^*\right)^2 + \left(\pi_{t+1} - \hat{\pi}\right)^2\right).$$

Although this specification is uncommon, it is arguably the better form if voters and governments are more concerned about the growth rate than the level of output. Woodford (2003) derives a similar form from microfoundations under the assumption that the representative citizen's utility exhibits habit persistence. Deriving government policy as before we find that

$$\begin{aligned} \pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \hat{\pi}}{1+\psi^2} + \frac{\psi x_{t-1}}{1+\psi^2} + \varepsilon_t \\ g_t &= g_t^* - x_{t-1} - \psi\left(\frac{E_{t-1}^a \pi_t - \hat{\pi}}{1+\psi^2}\right) + \frac{x_{t-1}}{1+\psi^2} + \xi_t \end{aligned} \tag{10}$$

Comparing the solutions, (3) and (4) versus (10), we see that the only differences involve the lagged value of the output gap, which now enters the inflation equation. The lagged gap still influences growth, but its impact is reduced as compared to (4).

The growth-target function also can be used to derive no-lag and double-lag versions. The no-lag solution is identical to the single-lag version (10) except for error terms. When the government can react at once, it accommodates the output shock and

leans against the inflation shock. And under the double-lag timing assumption, the growth-target objective results in

$$\begin{aligned} \pi_t &= \frac{E^g_{t-2}\left(E^a_{t-1}\pi_t\right)+\psi^2\hat{\pi}}{1+\psi^2}+\frac{\psi x_{t-2}}{1+\psi^2}+\varepsilon_t \\ g_t &= g^*_t - x_{t-1} - \psi\left(\frac{E^g_{t-1}\left(E^a_t\pi_{t+1}\right)-\hat{\pi}}{1+\psi^2}\right)+\frac{x_{t-1}}{1+\psi^2}+\xi_t \end{aligned} \tag{11}$$

Here the inflation solution depends on a two-period inflation forecast and a two-period lag of the output gap.

5. Econometric results

For expected inflation, we initially use an inertial approximation,

$$E^a_{t-1}\pi_t = \frac{1}{T}\sum_{\tau=1}^{T}\pi_{t-\tau}\,. \tag{12}$$

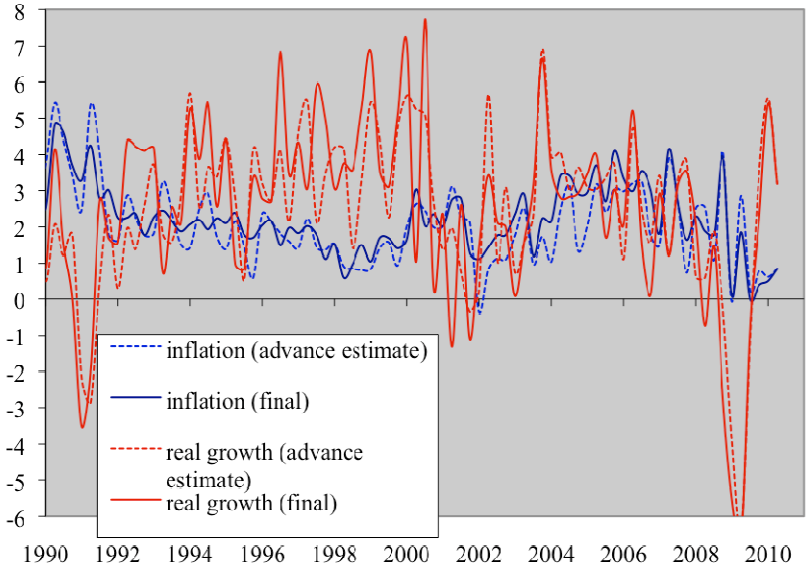
We refer to this moving average of lagged rates covering the *T* previous periods as the *MA*(*T*) specification. Although many economists view such backward-looking models with suspicion because they lack microfoundations and because their forecasts can be irrational, they are well known to provide a good empirical fit. Taking the period to be a quarter, we select the averaging length *T* empirically.

It is appropriate to use contemporaneously available data to specify these models. The real-time data published by the Philadelphia Federal Reserve Bank accurately reports availability; it is constructed as dated histories from public information.[14] These data are quarterly cohorts, each of which is revised over time. The most recent national account entries in each cohort are the Bureau of Economic Analysis' (BEA) "advance estimates."

[14] See Croushore and Stark, (2001).

Figure 1 shows that the BEA's measurement errors can be considerable for both inflation as calculated from real-time GDP deflator and growth; large revisions are especially apparent for growth.[15]

Figure 1. Comparing advance estimates with final values over the past two decades



In the middle of the t^{th} quarter agents are aware of current public information, the advance estimates of the $(t\text{-}1)^{st}$ quarter. On the right-hand-side of our observation equations we take real-time estimates of inflation. The backward-looking sum (12) is an indicator of what agents currently know about recent inflation. Likewise, we use the real-time estimate of the real GDP in the first term in $\ln(Y_{t-1}) - \ln(Y^*_{t-1}) = x_{t-1}$ on the right-hand-side. We use final statistics to measure left-hand-side variables; this is appropriate to our study of how available information affects actual outcomes.

[15] We take the final values as the 2010Q2 data cohort.

The likelihood statistics reported in Table 1 assume the single-lag structure of policy effectiveness; all assume independent errors.[16] Comparing the goodness of fit of the output gap objective assumption, (3) and (4), on the left with the growth objective assumption (10) on the right, the growth version clearly fits the data better. Although similar, the likelihood statistics favor *MA* expectations averaged over about the past four quarters.

Table 1. Comparing log likelihood statistics:
single-lag new Keynesian models, 171 observations, 1967Q3-2010Q1

	gap target			growth target		
	$\sigma_v^2 = 0.01$	$\sigma_v^2 = 0.04$	$\sigma_v^2 = 0.25$	$\sigma_v^2 = 0.01$	$\sigma_v^2 = 0.04$	$\sigma_v^2 = 0.25$
MA(1)	-775	-770	-764	-742	-742	-742
MA(2)	-760	-756	-749	-718	-718	-718
MA(3)	-758	-753	-748	-706	-706	-706
MA(4)	-758	-754	-748	-696	-697	-696
MA(5)	-768	-763	-757	-699	-699	-699
MA(6)	-777	-773	-767	-709	-709	-710
MA(7)	-787	-782	-777	-713	-712	-713
MA(8)	-793	-789	-783	-718	-717	-718

In our model potential growth is smoothed by restricting the variance on the random-walk steps. The first and fourth columns of Table 1 reports results for a strongly smoothed restriction that $\sigma_v^2 = 0.01$ (a standard deviation of 1/10% per quarter), and repeats the analysis with restriction that $\sigma_v^2 = 0.04$ (a standard deviation of 1/5% per

[16] By comparison we find that generalizing the specification of ε_t and ξ_t to *VAR*(1) for model (e) reported in Table 2 improves the fit. Its log likelihood statistic increases to –686. A further generalization to *VAR*(2) errors increases the log likelihood to -684. Neither the estimate of the slope of the Phillips curve nor the inflation target change markedly. Although these generalizations improve the statistical fit, they introduce a modeling inconsistency in that the government would rationally forecast nonzero errors, violating our modeling assumption that shocks are unpredictable.

quarter) and for the more volatile $\sigma_v^2 = 0.25$ (a standard deviation of 1/4% per quarter).[17] Although our results favor the more volatile potential growth versions, we prefer the $\sigma_v^2 = 0.04$ smoothing for reasons discussed below.

Table 2 reports detailed results for some of the more likely specifications, shaded in Table 1, along with estimates of double-lag specifications, (9) and (11). Attempted estimation of the no-lag model (7) does not converge for either the gap-target or growth-target versions. This supports an inference that the no-lag policy timing is unrealistic. Our single-lag and double-lag estimates of the output variance ξ_t are large compared to those for ε_t, suggesting that the perfect accommodation of output shocks (so that ξ_t drops out) implied by the no-lag specification explains our non-convergence results.

Table 2. Estimation details for selected backward-looking models, *MA*(4) expectations, 171 observations, 1967Q3-2010Q1 (*z* statistics in parentheses)

	(a)	(b)	(c)	(d)	(e)	(f)
model	gap target, single-lag	gap target, single-lag	gap target, double-lag	growth target, single-lag	growth target, single-lag	growth target, double-lag
Phillips curve slope ψ	0.346	0.372	0.410	0.324	0.265	0.328
	(5.928)	(6.640)	8.477	(9.256)	(9.099)	(10.244)
inflation target $\hat{\pi}$	4.461	4.389	4.162	3.606	3.418	4.085
	(4.945)	(5.489)	6.770	(4.644)	(3.514)	(5.563)
potential growth variance σ_v^2	1.998	0.040 (imposed)	0.040 (imposed)	0.426	0.040 (imposed)	0.040 (imposed)
price shock variance σ_ϑ^2	1.439	1.443	1.601	0.925	0.989	1.329
growth shock variance σ_ξ^2	6.798	13.037	13.695	8.756	10.218	24.544
log likelihood	-739	-754	-766	-696	-697	-811

[17] By comparison the CBO's estimate to the variance of quarter-to-quarter change in potential real GDP is only 0.008 over this sample period.

In all cases the estimated slopes of the Phillips curve are positive and statistically significant. The estimated target variable implies equilibrium inflation rates of around 4%.[18] As a benchmark we find that a *VAR*(1) model on the same two dependent variables achieves a log likelihood of -731.

The literature identifies other methods of estimating unobserved potential growth. Figure 2 compares our Kalman filter estimates, from models (d) and (e), with two alternatives: the Hodrick-Prescott filter and an estimate published by the Congressional Budget Office (2001). Both alternatives are omniscient in the sense that they are based on future as well as past observations. The popularity of the HP filter may be due to its simple agnostic formula.[19] The CBO estimate is more complicated, using a growth accounting method inspired by the Solow growth model.[20] All these estimates illustrate the conclusion that the underlying growth rate of the US economy has changed over time.

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[18] These inferences are limited by the restrictions and approximations assumed in our model. For example, our estimates of ψ and $\hat{\pi}$ maximize each model's likelihood function. A shortcoming of this procedure is that it assumes that governments know ψ and $\hat{\pi}$, and that this knowledge is based on the entire sample, not just available information. We investigate the seriousness of this limitation by redefining the target as a random coefficient, $\hat{\pi}_t = \hat{\pi}_{t-1} + \omega_t$, where $\omega_t \sim N(0,\sigma_\omega^2)$. This evolving target generalization can be specified by the addition of another state variable, namely $\hat{\pi}_t$. As a plausible prior we specify $\hat{\pi}_{1|0} = 4\%$ with a variance of 4. We arbitrarily restrict $\sigma_\omega^2 = 0.01$. Starting at 4, the Kalman estimate of the target rises to almost 6 by 1979, and declines to around 3 near the end of the sample period. Since model (e) is slightly more likely to have generated the data than this random coefficient model (-697 log likelihood), we conclude that the fixed-and-known target assumption is not an important limitation.

[19] It estimates of the potential rate series by minimizing the expression

$$\sum_{t=2}^{T-1}\left\{\left(g_t - g_t^*\right)^2 + \lambda\left[\left(g_{t+1}^* - g_t^*\right) + \left(g_t^* - g_{t-1}^*\right)\right]^2\right\},$$

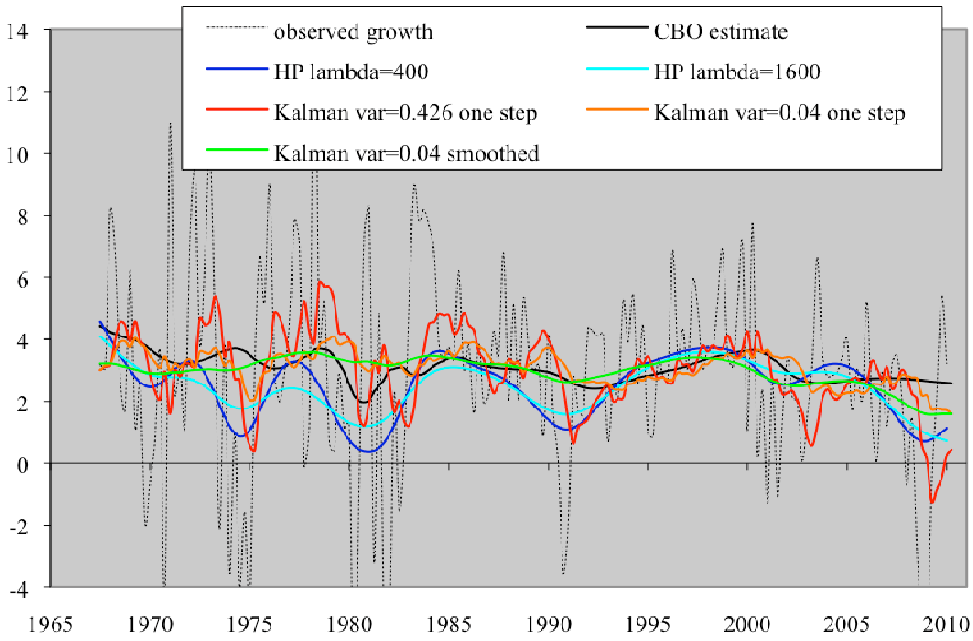
where λ is an arbitrary smoothness parameter that penalizes sharp curves in the g_t^* series. It is conventional to set λ=1600 for quarterly data; as a comparison, Figure 2 also shows the estimate using λ=400. See Roberts (2005) for application in macroeconomics.

[20] This method combines estimates of the trends in the labor force, the capital stock and technological progress. Cyclical components of the labor supply and productivity are removed from observed statistics using the CBO's estimate of the non-accelerating inflation rate of unemployment, constraining potential labor and productivity growth rates to be constant over the business cycle.

Recently, all show a peak during the late 1990s followed by a decline over the past decade.

Clearly the HP and CBO estimates are smoother. The difference reflects different assumptions about potential growth as well as different methods of estimation. Conventionally, the potential level changes over time as technology advances, as capital is accumulated and as the labor force grows. Assuming that these influences evolve slowly and independently of business cycles, both the HP filter and CBO estimate impose a gradually evolving process, without large shifts. On the other hand, our generating process is typified by small random shifts that can be occasionally large. Relaxing the smoothing restriction by letting σ_v^2 be an estimated parameter, models (a) and (d) estimate even greater potential growth volatility. Although model (d) attains the maximum likelihood in Table 2, it is nevertheless questionable due to the volatility of its $\hat{g}_t^*$ series. Its sensitivity to the business cycle is questionable; notice the implausibly large drop in the $\sigma_v^2 = 0.426$ series in 2009. The series estimated under the restricted model (e) looks more plausible in Figure 2.

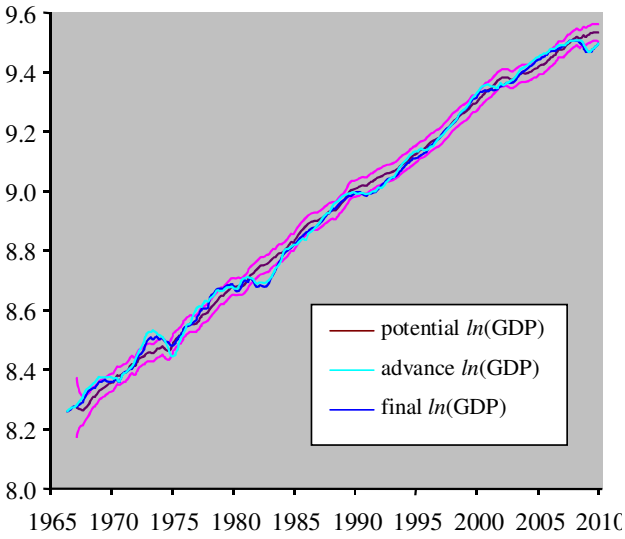
Figure 2. Alternatives estimates of the US potential growth rate g_t^*



An appealing feature of the state space approach is that potential growth is a one-step forecast based on only contemporaneous information.[21] On the other hand the HP and CBO methods are omniscient in the sense that they include both past and future observations; they are more comparable to the "smoothed" Kalman estimate of potential growth conditioned on the entire data set. Figure 2 shows the evident differences between one-step and smoothed Kalman forecasts. Although the smoothed estimate is not always closer to the HP and CBO estimates, it is obviously less volatile. The one-step forecast is a more appropriate indicator of available information.

[21] As a plausible prior for the potential growth we specify $g_{110}^* = 3$ with a variance of 9, and set $\ln\left(Y_{110}^*\right)$ equal to the observed value of $\ln(Y)$ in 1967Q2 with a variance of 0.0025.

Figure 3. Observed real GDP and Kalman predictions of its potential level $\ln(Y_t^*)$, model (c)



Conditional on current observations and our model specification, the Kalman filter defines recursive estimates of the unobserved state variables. Figure 3 compares the model (e) one-step estimate of the potential output and its 95% confidence interval with the BEA's advance and final estimates. Although Figure 1 shows substantial measurement error in the growth rate series, Figure 3 shows smaller errors in the output series. Usually they are also smaller than our confidence interval for potential output. The plot also indicates how quickly experience comes to dominate our prior assumptions for potential output. We prefer model (e) because of its goodness-of-fit and its relatively smooth potential growth series.

Our double-lag timing solutions involve the government's forecast of an expectation by agents in the future. Logically the government should use its knowledge

of the Phillips curve. We substitute the double-lag version of the Phillips curve (8) into the *MA* specification (12), as a forecast of π_{t-1} based on information available in *t*-2,

$$E^g_{t-2}\left(E^a_{t-1}\pi_t\right) = \frac{1}{T}\left(E^g_{t-2}\pi_{t-1} + \pi_{t-2} \cdots + \pi_{t-T}\right)$$
$$= \frac{1}{T}\left(E^g_{t-2}\left(E^a_{t-2}\pi_{t-1} + \psi x_{t-2} + \varepsilon_{t-1}\right) + \pi_{t-2} \cdots + \pi_{t-T}\right)$$
$$= \frac{1}{T}\left(\frac{1}{T}\sum_{\tau=2}^{T+1}\pi_{t-\tau} + \psi x_{t-2} + \sum_{\tau=2}^{T}\pi_{t-\tau}\right)$$

This approximation applies our *MA* approximation twice. Substituting this expression into the double-lag models above, adds the twice lagged output gap and changes the weights on the lagged inflation terms. Estimation results in Table 2 suggest that the single-lag mechanism is more likely than the double-lag one to have generated these data.

Table 3. Comparing quarterly, semiannual and annual frequencies

	(g)	(h)	(i)	(j)
	gap target, single-lag	gap target, double-lag	growth target, single-lag	growth target, double-lag
171 quarterly observations				
Phillips curve slope	0.372	0.410	0.265	0.318
inflation target	4.389	4.162	3.418	4.221
log likelihood	-754	-766	-697	-802
85 semiannual observations				
Phillips curve slope	0.431	0.444	0.335	0.343
inflation target	4.227	3.958	3.465	3.781
log likelihood	-352	-370	-321	-407
42 annual observations				
Phillips curve slope	0.518	0.215	0.417	6.269
inflation target	4.227	4.606	3.707	4.076
log likelihood	-170	-192	-161	-171

To further explore the question of policy lags, Table 3 re-estimates these models on semiannual and annual observations. We want to vary the assumption about the policy lag length, holding agent expectations unchanged. Thus for the semiannual specification expected inflation is defined as *MA*(2), the average of the preceding two six-month periods; and for the annual one, it is just the preceding year's inflation. The *t*-1 dated variables are taken as the most recent real-time inflation and output information reported in the middle of the preceding quarter. Thus for the annual model, expected inflation is measured by the rates made public in the fourth quarter of the previous year. We define the dependent variables by averaging final inflation and growth rates as measured by the 2010Q2 cohort. In Table 2 we prefer restricting the variance of the potential growth rate to 0.04. Assuming that potential growth is generated as specified, a quarterly random walk, and further assuming that we observe this series only semiannually, the semiannual variance would be 0.08; or if we observe it only annually, it would be 0.16. Thus, we restrict the potential growth variance to 0.08 for the semiannual models and to 0.16 for the annual models.

Table 3 reports only the estimates of the Phillips curve slope and the inflation target; all are statistically significant and have roughly similar magnitudes (except for model (j)'s slope estimate). The log likelihood statistics are not comparable since they are based on different numbers of observations. Nevertheless, comparing across any row, the

growth target, single-lag model (i) fits the data best. Averaging the quarterly and semiannual predictions for model (i) to produce comparable annual predictions, squaring the residuals and adding together unweighted for inflation and growth, we obtain an annual sum of squared errors of 193 for the quarterly specification, for the semiannual 197, and for the annual 235. This result supports a one-quarter policy lag, although it is, of course, easier to obtain accurate forecasts when expectations are based on more recent information. We conclude that although these data do not permit confident inferences about the length of the policy lag, they do call into question the double-lag hypothesis.

6. Forward-looking expectations of inflation

Although backward-looking expectations fit these data well, many may be skeptical of this *ad hoc* specification. The Kalman methodology defines forecasts of inflation, growth and the output gap. Figure 4 shows that our output gap forecast closely parallels an alternative calculated from CBO's potential GDP. Although we present no evidence that firms, workers or policymakers learn according to Bayes rule, these are Bayesian updates, weighted averages of the most recent forecast and observation. We interpret these as estimates what the agents might have rationally thought at the time that decisions were taken, conditional on available information.

Figure 4. Comparing GDP gap x_t estimates: model (c) and CBO

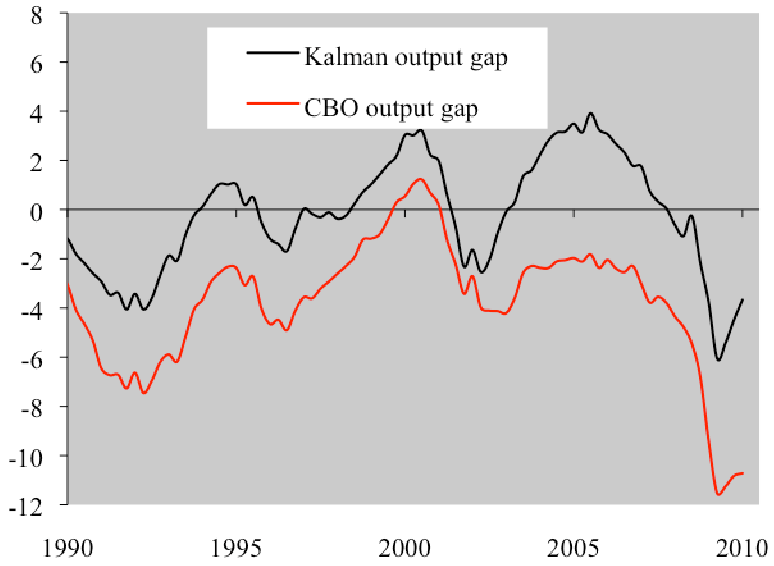


Table 4 compares four alternatives specifications. Two of these, models (k) and (m), simply iterate; they substitute the one-step inflation forecast from the appropriate *MA*(4) model back into the observable equations. In the case of the gap model (k), we use model (b) to estimate one-quarter inflation forecasts. These then replace our approximation (12), permitting a re-estimation of the same model. Our motivation is that the fit should improve with model-consistent expectations. The surprising result is that the *MA* approximation fits these data slightly better; the same inference can be drawn from a comparison of models (e) and (m). These results confirm the conventional wisdom that naive expectation models of are not necessarily inferior to more sophisticated ones.

Table 4. Alternative expectation specifications, single-lag models
171 observations, 1967Q3-2010Q1 (z statistics in parentheses)

	(k)	(l)	(m)	(n)
model	one-step forecast, gap target	new Keynesian, gap target	one-step forecast, growth target	new Keynesian, growth target
stickiness parameter		0.388		0 (imposed)
		(2.615)		
Phillips curve slope ψ	0.214	0.297	0.003	0.003
	(2.312)	(1.566)	(0.094)	(0.094)
inflation target $\hat{\pi}$	3.907	3.846	0.512	0.512
	(1.649)	(1.203)	(0.001)	(0.001)
log likelihood	-758	-757	-699	-699

To further investigate agent expectations we develop a version of Calvo's (1983) stochastic price adjustment derivation of the Phillips curve. This sticky price model is renown for its elegant microfoundations. It assumes that $(1-\eta)$ is the probability that a firm (or agent) can adjust its price in the current period. It specifies that the current aggregate price and marginal costs determine the optimum price $\ln\left(p_t^*\right)$ for the typical firm.[22] Furthermore, under certain conditions it can be argued that marginal cost is proportional to the output gap, giving the optimal price in a relation similar to the Lucas supply curve.[23]

[22] Under imperfect competition the profit-maximizing price is a markup of marginal cost. Some authors develop further microfoundations, assuming an economy of monopolistically competitive firms providing a continuum of differentiated consumer goods; see for example, Gali (2008).

[23] There is doubt in the empirical literature about the cost-gap link. Gali and Gertler (1999) report consistent results for an approximation of marginal cost, but not for the output gap, while Rudd and Whalen (2006) find that neither variable can explain observed inflation.

Since firms may be unable to change their prices for some time, they forecast future conditions, weighing each future period according to the probability that their price will remain fixed.[24]

$$\ln\left(p_t^*\right)=(1-\eta)\sum_{\tau=0}^{\infty}\eta^{\tau}E_{t-1}^{a}\left(\ln\left(p_{t+\tau}\right)+\psi x_{t+\tau}\right)+\varepsilon_t\,.$$

We use ψ again to define the optimum price-gap relation. We now interpret ε_t as an exogenous price shock, added to account for all other factors affecting the pricing decision. It is appropriate to incorporate information that was public at the time these decisions were taken. Customary derivations date expectations from the current quarter, but we specify an expectation of date *t*-1 for the pricing decision because aggregate prices are published as advance estimates about three months after the current date.

The aggregate price level combines the firms who reset their price in the current period with those who set prices previously. Eliminating the unobserved optimum price, it can be shown that aggregate inflation is given as

$$\pi_t=\frac{(1-2\eta)}{(1-\eta)}E_{t-1}^{a}\pi_t+\frac{\eta}{(1-\eta)}E_{t-1}^{a}\pi_{t+1}+(1-\eta)\psi E_{t-1}^{a}x_t+\varepsilon_t. \qquad (13)$$

This new Keynesian Phillips curve involves forecasts of two inflation rates and of the output gap. Equation (13) is unconventional; usually expectations are dated in the t^{th} quarter so that $E_{t-1}^{a}\pi_t=\pi_t$, $E_{t-1}^{a}x_t=x_t$, and $E_{t-1}^{a}\pi_{t+1}=E_t^{a}\pi_{t+1}$. Under these assumptions, (13) simplifies to the usual

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[24] It is appropriate for firms to discount future profits. But since this complicates the result, we follow Froyen and Guender (2007) by weighting all periods equally, except for the probability of price resetting. Our results suggest that this neglect of discounting is reasonable; model (l) estimates the average length of price fixity at about 2.6 quarters.

$$\pi_t = E_t^a \pi_{t+1} + \frac{(1-\eta)^2 \psi}{\eta} x_t + \frac{1-\eta}{\eta} \varepsilon_t . \qquad (14)$$

Several empirical studies of the new Keynesian curve have studied regressions specified according to (14), often adding lagged inflation variables to examine the issue of whether this *ad hoc* extension is needed to explain inflation inertia. Invoking rationality, they assume that inflation forecasts are on average accurate. But since replacing $E_t^a \pi_{t+1}$ with π_{t+1} introduces an endogenous variable on the right hand side of the regression, they specify a list of lagged instruments to mitigate endogeneity bias.[25]

Our method does not involve instrumental variables. We estimate a new Keynesian model by a two-step procedure: first we use the backward-looking model (b) to estimate the unobserved expectations, and then we use these forecasts to estimate sticky-price versions of our 2-equation model, substituting (13) for the backward-looking Phillips curve. For the gap-target objective, the observable equations become

$$\pi_t = \frac{\frac{1-2\eta}{1-\eta} E_{t-1}^a \pi_t + \frac{\eta}{1-\eta} E_{t-1}^a \pi_{t+1} + \left((1-\eta)\psi\right)^2 \hat{\pi}}{1+\left((1-\eta)\psi\right)^2} + \varepsilon_t$$

$$g_t = g_t^* - x_{t-1} - \frac{(1-\eta)\psi\left(\frac{1-2\eta}{1-\eta} E_{t-1}^a \pi_t + \frac{\eta}{1-\eta} E_{t-1}^a \pi_{t+1} - \hat{\pi}\right)}{1+\left((1-\eta)\psi\right)^2} + \xi_t$$

[25] For example Gali and Gertler (1999).

Our use of lagging expectations is appropriate to this method; if we use (14) instead of (13) to derive the model, then we may introduce simultaneity bias because in the one-step forecast $E_t\pi_{t+1}$ assumes knowledge of the knowledge of the current dependent variables.[26]

Using this same method on the growth target model, the observable equations become

$$\pi_t = \frac{\frac{1-2\eta}{1-\eta}E^a_{t-1}\pi_t + \frac{\eta}{1-\eta}E^a_{t-1}\pi_{t+1} + \left((1-\eta)\psi\right)^2\hat{\pi}}{1+\left((1-\eta)\psi\right)^2} + \frac{(1-\eta)\psi x_{t-1}}{1+\left((1-\eta)\psi\right)^2} + \varepsilon_t$$

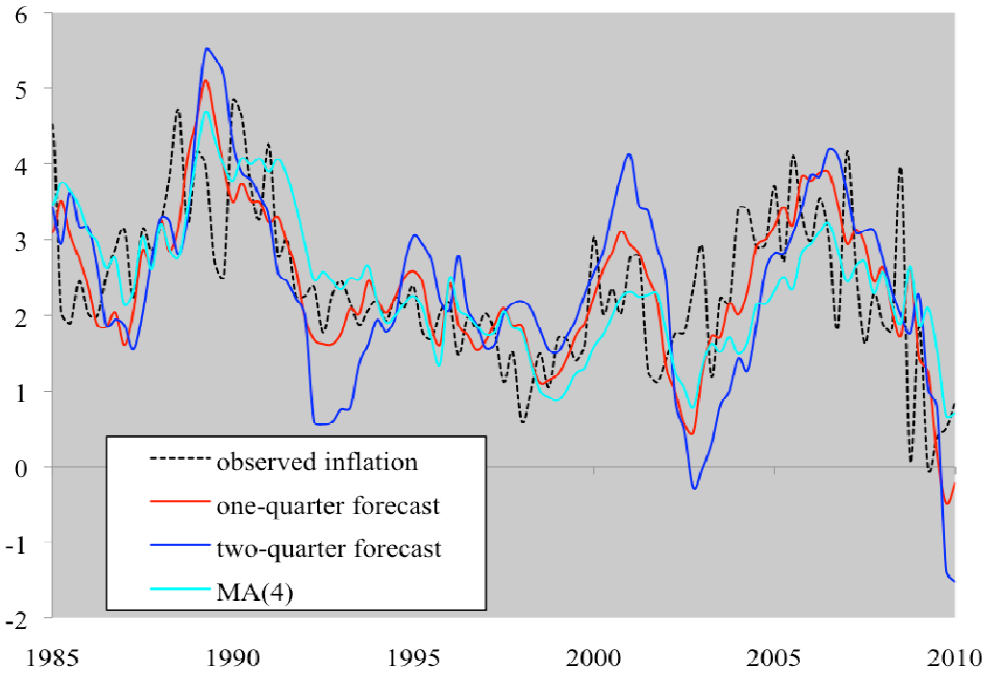
$$g_t = g^*_t - x_{t-1} - \frac{(1-\eta)\psi\left(\frac{1-2\eta}{1-\eta}E^a_{t-1}\pi_t + \frac{\eta}{1-\eta}E^a_{t-1}\pi_{t+1} - \hat{\pi}\right)}{1+\left((1-\eta)\psi\right)^2} + \frac{x_{t-1}}{1+\left((1-\eta)\psi\right)^2} + \xi_t$$

Figure 5 compares observed inflation with these one and two-quarter forecasts and with the *MA*(4) approximation. Obviously two-quarter forecasts have been less accurate, often overshooting peaks and troughs. Our results suggest that the backward-looking *MA* approximation performs better than forward-looking sticky-price versions. The sticky-price growth model (n) is a corner solution at $\eta = 0$, implying that all firms reset prices in the current period, equivalent to model (m). Table 4 shows that the growth objective assumption again performs better than the gap objective, although models (m) and (n) are suspect. With these specifications the estimated Phillips slope, $(1-\eta)\psi$, is almost zero, and so is the estimated inflation target. These results are consistent with the literature; other studies of the new Keynesian Phillips curve report unexpected negative or insignificant slope estimates.

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[26] Our methodology also differs from the econometric literature in respect of the unobserved variables; customarily the output gap is measured in a deterministic fashion, not as part of the short-run equilibrium.

Figure 5. Observed inflation and the forecasts used in model (n)



7. Conclusion

We begin with a new Keynesian characterization of stabilization policy, an admittedly *ad hoc* model involving a backward-looking Phillips curve and a simplified characterization of government behavior. We estimate this model with a real-time data set to measure the information available to agents when forming forecasts of inflation and the real economy. Overall the new Keynesian notion of activist governments who lean against the macroeconomics wind is consistent with the US evidence; we estimate an inflation target of around 4 percent with the expected Phillips curve slope. Using goodness of fit statistics to make inferences about alternative modeling assumptions, we conclude that governments have been more likely to target growth rates, than output gaps. A second finding is that the application of stabilization policy effects inflation and

output together after a single lag, although it is hard to decide whether the lag is a quarter or a year. And consistent with the literature inflation expectations are more likely to be backward rather than forward looking; the hypothesis that inflation follows the popular sticky-price scenario of stochastic price setting is poorly supported by these data.

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