# Sticky Rents and the Stability of Housing Cycles

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# Sticky rents and the stability of housing cycles*

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**Abstract**

This paper presents a simple model of housing cycles in a two good economy. We establish that nonsubstitutability between housing services and other commodities along with high degree of heterogeneity in home ownership may lead to a stable spiral path converging to a steady-state equilibrium of rental price and housing stock. Key conditions for stability are a sufficiently sticky rent adjustment dynamics and a relatively rapid depreciation of per capita housing stock.

*JEL classification:* C62; D11; D51; R21; R31

*Keywords:* Housing cycles, Sticky rents, Stability of equilibrium, Leontief utility

## 1 Introduction

In the last decade, we have witnessed a very long boom episode followed by a sharp and a prolonged bust in the prices of houses in many parts of the world. In the World Economic Outlook 2008, IMF (2008) reports the percent increase in house prices, during the period 1997 to 2007 that is not accounted for by the fundamental drivers of house prices, ranging from around 30 percent to around 10 percent for the countries Ireland, the Netherlands, the United Kingdom, Austria, France, Norway, Denmark, Belgium, Spain, Sweden, Italy, Japan, and the United States. Over a broader time interval,

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Shiller (2005) shows that inflation-adjusted U.S. home prices increased 0.7% per year from 1940 to 2004. Before the bust that started in year 2007, the perception of a seemingly endless housing boom was widespread. The average annual increase in the Case-Shiller home price index for the United States during the 2002-2006 period in contrast was above 12%.

Many policy related reasons, like low interest for a prolonged time period, have been put forward in order to explain these significant and prolonged price swings. The tax deductibility of mortgage payments in the U.S. is another frequently cited distortion. A legal system that facilitates 'strategic defaults' or so called 'jingle mails' in case of negative equity is a third commonly cited reason. Presence and widespread promotion of adjustable rate mortgages is a fourth frequently cited culprit. But in fact in many different countries without such legal structures or economic policies, we observe similar prolonged boom-bust cycles in their housing markets as well.

The natural question then becomes whether there is a more fundamental structural economic reason that produces these cycles, a reason, which is not captured by the economic models of housing that we currently use. In this regard, Shiller (1990) brings to our attention the observation that the economic theorist's understanding of 'equilibrium in the housing market' is not shared at all by the man on the street. Prices rise because there is 'shortage of supply' is the typical answer when people with little economic training are asked about the reason behind endured periods of price increases. But we, economists by training, know that prices always adjust so that demand and supply are always equal to each other. But then given what has been and currently is happening in the housing market, could it be true that it is in fact economic theory that is missing something that the man on the street actually knows?

The answer is 'no' for economic theory in general, but 'yes' in particular for the Marshallian models of housing demand. The Marshallian partial equilibrium model is missing two significant aspects. The first important aspect that is missing is the presence of rental contracts that obviously avoid immediate price adjustment for rents. Second is the possibility a housing demand curve that may be positively sloped at some range of prices. The first aspect may give rise to a temporary disequilibrium and a mismatch between demand and supply for a relatively long period. The second aspect may give rise to the presence of more than one equilibrium house price. If you have more than one equilibrium, we know that necessarily at least one of them turns out to be unstable. But can a demand curve ever be positively sloped?

Marshallian partial equilibrium theory says no, while general equilibrium theory admits this possibility. The intuition is the income effect that comes with higher prices, in case there is a group of people specialized in providing housing services.

Thanks to seminal works in general equilibrium theory, like Arrow and Hurwicz (1958) and Hahn (1958), we have known for the last five decades that uniqueness and stability of competitive general equilibrium is an almost impossible luxury to have. Uniqueness and stability of equilibrium hinges upon a condition called 'gross substitutability'. This condition basically requires all goods to be highly substitutable with every other good in the economy. This is something very unlikely to be true, if not impossible.

Substitution between housing services and items like food and energy is not very likely. If in addition there is a high degree of specialization in provision of say food (or energy) items versus housing services, a simple algebra can show that three rather than one price that equate demand and supply is a possibility. In fact, this exercise has been carried out by Başçı and Saglam (2008). Out of the three market clearing rental rates the lowest and highest ones turn out to be stable, while the one in the middle is unstable. The range between the two stable equilibria depends on two things: the extent of specialization and degree of substitutability between housing and other commodities. This has an interesting implication. Extreme specialization that usually comes together with trade and capital account liberalization may indeed be the fundamental structural reason for higher amplitude of worldwide housing cycles.

On the other hand, several studies, including Blanchard and Kiyotaki (1987), Calvo (1983) and Mankiw (1985), reminded economists of various institutional reasons for the presence of 'sticky prices' and the macro economic significance of this stickiness. In the housing market for instance, the presence of 'rental contracts' is an obvious obstacle against immediate market clearing. Indeed, a careful look at the subitems of the consumer price index in the U.S., or elsewhere, shows prolonged periods of real rent increases as well as decreases. The price adjustments here are actually observed not to be rapid at all. In short, not the economic theory itself but the missing link between sophisticated general equilibrium models that admit multiple equilibria and the speedy advice needs of policy makers fulfilled by rather simplistic Marshallian models seems to be the problem in modelling the housing markets.

In this paper, we attempt to provide a possible theoretical explanation

for the relation between rent stickiness and stability of housing cycles by analysing the equilibrium dynamics of a simple two-good economy with incomplete markets and strong income effects.

In our model, we focus solely on the determination of rental rates by recognising the importance of the present level and future growth prospects of rents in valuation of homes.[1] We consider a dynamic setup that uses the assumption of zero substitutability between housing and other commodities, an extreme among many cases studied in the static general equilibrium model of Başçı and Saglam (2008).[2] Başçı and Saglam (2008) assume a low degree of substitutability between housing services and other commodities together with a low share of owner-occupied houses, and establish that regions or economies with a low share of owner-occupied houses are more prone to housing cycles that are driven by locally unstable rent dynamics.

Our model involves two producer-consumers who can produce either housing services or a nonhousing good.[3] Each consumer derives utility from the consumption of both goods, while substitution between the goods is negligible.[4] Besides, intertemporal trade is not possible either, as we focus on an incomplete market setup. On the supply side, we assume that the stock of (per capita) housing depreciates at a constant instantaneous rate while investments by one of the consumers in the housing sector leads to an instantaneous gross addition through an investment function of the rental price. On the other hand, for the dynamic adjustment of housing rents, we consider a tâtonnement process, in which rents tend to increase in response to

[1] See Smith and Smith, 2006.

[2] The only other general equilibrium asset pricing with housing model that we are aware of is by Piazzessi et al. (2007). But they assert uniqueness of equilibrium upfront via a high substitution elasticity parameter.

[3] A basic reason for heterogeneity in house ownership is the life cycle. JCHS (2008) classifies, in The State of the Nation's Housing 2008, homeownership rates in the United States in 2007 by age groups. For ages under 35, the homeownership rate is 41,7 while the figure becomes 67,8 for ages 35-44, 75,4 for ages 45-54, 80,6 for ages 55-64, 82,0 for ages 65-74, and 78,7 for ages 75 and over. Such heterogeneity yields a strong income effect for home owners due to changes in rental rates, which has also been identified empirically (see, for example, Kenny, 1999). Başçı and Saglam (2008) elaborate on the theoretical possibility of a positively sloped demand function for housing services when there are strong income effects stemming from endowment heterogeneity. Dusansky and Koç (2007) obtain the same theoretical possibility in an intertemporal model of consumption, which is also supported by their econometric analysis of housing data from Florida.

[4] For a discussion on the poor evidence in the literature for a low degree of substitution between housing and nonhousing services, see Başçı and Saglam (2008).

the current level of excess demand for housing.

We establish that nonsubstitutability between housing and nonhousing services along with heterogeneity of individuals with respect to their endowments (supplies) leads to a stable (spiral) path converging to a steady-state equilibrium if the rental price of housing is sufficiently sticky or the depreciation of housing is sufficiently high. We also present some results for the comparative statics of the steady-state equilibrium.

The organization of the paper is as follows. Section 2 introduces the model and Section 3 presents our results. Finally, Section 4 concludes.

# 2 Model

We consider an exchange economy involving two producer-consumers who are indexed by $i = 1, 2$ and live over the time interval $[0, \infty)$. Time is denoted by $t$. At each time, there are two goods in the economy, indexed by $g = n, h$ with $n$ and $h$ denoting nonhousing and housing respectively. We normalize the price of nonhousing to 1 at each time point, and denote by $r(t)$ the relative real rental rate of housing at time $t$.[5]

Let $S_i^g(t)$ denote the supply of good $g$ produced by individual $i$ at time $t$. We assume that at each time point housing is produced by individual 1, while nonhousing by individual 2, solely; i.e., $S_1^n(t) = 0$ and $S_2^h(t) = 0$ for all $t$. We normalize the supply of nonhousing, $S_2^n(t)$, to 1 for all $t$; hence the variable $S_1^h(t)$ denotes the relative supply of housing in the economy at time $t$.

Individuals derive utility from the consumption of both goods; however substitution between housing and nonhousing is not possible. Let $c_i^g(t)$ denote the time-$t$ consumption of good $g$ by individual $i$, whose utility is represented by a Leontief function

$$U_i(c_i^n(t), c_i^h(t)) = \min\{c_i^n(t), a_i c_i^h(t)\},$$

where $a_i > 0$. The choice problem faced by consumer $i$ at time $t$ is given by:[6]

$$\max_{c_i^n(t), c_i^h(t)} U_i(c_i^n(t), c_i^h(t))$$

[5] As for housing services, we consider only pure dwelling services, i.e. what the rent is paid for, excluding costs of utilities.

[6] Since we consider an incomplete market setup, intertemporal trade is not possible.

$$\text{s.t.} \quad c_i^n(t) + r(t)c_i^h(t) = S_i^n(t) + r(t)S_i^h(t)$$

We denote the solution to the problem of consumer $i$ at time $t$ by the pair $\langle c_i^n(r(t)), c_i^h(r(t))\rangle$ for a given rental rate $r(t)$.

It is clear that an interior solution, obtained for any positive rental rate $r(t)$, must satisfy

$$c_i^n(r(t)) = a_i c_i^h(r(t)) \tag{1}$$

for individual $i$.

**Definition 1.** Consumers have *endowment bias* if for any $t$, $c_1^n(r(t))/c_1^h(r(t)) < S_2^n(t)/S_1^h(t) < c_2^n(r(t))/c_2^h(r(t))$ for any positive $r(t)$.

Consumers will have endowment (supply) bias if at each time they plan to consume relatively more from the endowed (supplied) good for each positive rental rate.[7] Here, we immediately note that condition (1) implies that consumers will have endowment bias if $1/a_2 < S_1^h(t) < 1/a_1$ for all $t$.

# 3 Results

Using the budget constraints of the consumers along with the normalization $S_2^n(t) = 1$ for all $t$, we find the optimal quantity of housing demanded by consumers 1 and 2 at a given positive rental rate $r(t)$ at time $t$ as

$$c_1^h(r(t)) = \frac{r(t)}{a_1 + r(t)} S_1^h(t) \tag{2}$$

and

$$c_2^h(r(t)) = \frac{1}{a_2 + r(t)} \tag{3}$$

respectively. Then, at time $t$, the aggregate demand for housing at a given positive rental rate $r(t)$ becomes

$$D^h(r(t)) = c_1^h(r(t)) + c_2^h(r(t)) = \frac{r(t)}{a_1 + r(t)} S_1^h(t) + \frac{1}{a_2 + r(t)}. \tag{4}$$

[7] The term 'endowment bias' was introduced into the housing cycle/bubble literature by Başçı and Saglam (2008), with an address to the presence of transaction costs in trade as a possible interpretation. For a well-known use of endowment bias in the theory of finance, in the sense of attributing a higher value to an asset due to one's ownership of it, see Kahneman et al. (1991).

Now, define the excess housing demand $X^h(r(t)) = D^h(r(t)) - S_1^h(t)$. Equating it to zero we obtain the equilibrium rental rate:

$$r^e(t) = -a_2 + \frac{a_2 - a_1}{1 - a_1 S_1^h(t)} \tag{5}$$

As already shown by Basci and Saglam (2008), the rental rate satisfying (5) is positive and clears the goods markets if consumers have endowment bias.[8]

We henceforth assume that consumers have endowment bias, and start to analyze the equilibrium dynamics of the model to show the possibility of a housing cycle. On the supply side, we assume that the stock of housing depreciates[9] at a constant instantaneous rate $\delta \geq 0$ while investments of individual 1 to the housing sector leads to an instantaneous gross addition of $I(r(t))$, through the investment function $I$ that admits as its argument the variable $r(t)$. We assume $I'(r) = \theta > 0$ and $I''(r) = 0$ for $r > 1$. Moreover, investment is irreversible,[10] i.e., $I(r) = 0$ for $r \leq 1$. Then, the supply of housing follows the below law of motion:

$$\dot{S}_1^h(t) = \begin{cases} \theta(r(t) - 1) - \delta S_1^h(t) & \text{if } r > 1 \\ -\delta S_1^h(t) & \text{if } r \leq 1 \end{cases} \tag{6}$$

For the dynamic adjustment of the housing rents, we consider the following tâtonnement process:

$$\dot{r}(t) = \gamma X^h(r(t)) = \gamma \left( \frac{1}{a_2 + r(t)} - \frac{a_1}{a_1 + r(t)} S_1^h(t) \right) \tag{7}$$

where $\gamma > 0$ is a constant. Here, we have implicitly assumed that the logical time of the iterations performed by a Walrasian auctioneer coincides with the real time $t$.

[8] The proof follows from (5), Definition 1, and Walras' Law.

[9] The depreciation rate $\delta$ can also be interpreted as the population growth rate in a demographic model of housing.

[10] Such an investment function can be obtained as an equilibrium behavior in an intertemporal model that involves individuals of type 1 who derive utility from leisure and decide in each period how much leisure to convert (through a housing production function) into housing. Clearly, the comparison between the marginal utility of leasure that can simply be normalized to 'one' and the rental price of housing would lead to the assumed discontinuity in the marginal propensity to investment.

The equations (6) and (7) together form transition equations for the state variables $S_1^h(t)$ and $r(t)$.

Setting $\dot{S}_1^h = 0$ when $r > 1$, the equation of the first phase locus is implicitly defined by the condition that depreciated housing stock be equal to the new addition to the housing stock. Modifying this condition yields

$$S_1^h(t) = \frac{\theta}{\delta}(r(t) - 1). \tag{8}$$

Clearly, $\dot{S}_1^h = 0$ locus slopes up as drawn in Figure 1. From (6),

$$\frac{\partial \dot{S}_1^h}{\partial S_1^h} = -\delta < 0; \tag{9}$$

so $S_1^h$ increases over time above the $\dot{S}_1^h = 0$ locus, as indicated by the horizontal arrows of motion in Figure 1.

The second phase locus ($\dot{r} = 0$) is defined by the condition that the excess housing demand be zero or the rental rate $r(t)$ be equal to $r^e(t)$. So, we plot $\dot{r} = 0$ locus as equation (5) over the range $1/a_2 < S_1^h(t) < 1/a_1$, where individuals have endowment bias. From (7),

$$\begin{aligned} \frac{\partial \dot{r}}{\partial S_1^h} &= \gamma \left( \frac{\partial D^h(r)}{\partial S_1^h} - 1 \right) \\ &= -\frac{\gamma a_1}{a_1 + r} < 0; \end{aligned} \tag{10}$$

so $r$ increases over time above the $\dot{r} = 0$ locus, as indicated by the vertical arrows of motion in Figure 1.

Inserting the two phase loci leads to us to state our first result.

**Proposition 1.** *The steady-state equilibrium of the described economy is described by*

$$r^* = \frac{\delta}{2\theta a_1} + \frac{1 - a_2}{2} + \sqrt{\left( \frac{\delta}{2\theta a_1} + \frac{1 - a_2}{2} \right)^2 + \frac{\delta}{\theta} + a_2} \tag{11}$$

and

$$(S_1^h)^* = \frac{\theta}{\delta}(r^* - 1). \tag{12}$$

Using (11)-(12), we immediately obtain the following results for the comparative statics of the steady-state equilibrium.

**Corollary 1.** *The steady-state rental rate of housing $r^*$ and the steady-state housing stock $(S_1^h)^*$ are decreasing in $a_1$, $a_2$, $\theta$ and increasing in $\delta$.*

The phase diagram of the dynamical system represented by (6)-(7) is plotted in Figure 1.

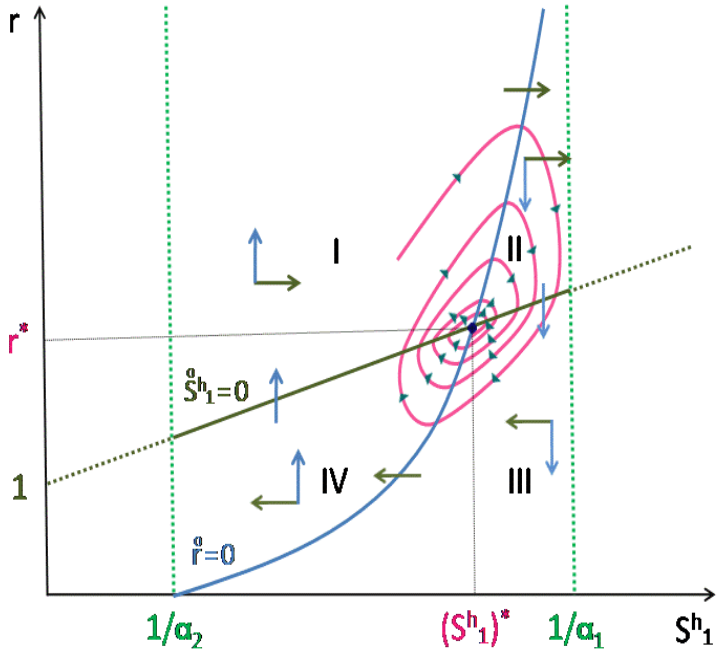



Figure 1: Phase Diagram

We notice that on a trajectory starting from any point in Region I, the increase in the housing stock is insufficient to meet the rise in the demand for housing, thus excess demand for housing pushes the rental rate further up and builds up a bubble. As the trajectory hits Region II, the rental rate of housing reaches to its highest level while the demand for housing collapses. The new additions to the housing stock in Region II can now create an

excess supply and the rental rate of housing starts to fall; the beginning of the collapse of the bubble. The net investment in housing eventually stops as the trajectory hits Region III and the stock of housing attains its maximum. In Region III, the demand for housing is so poor that there still remains an excess supply despite the contraction in the stock of housing. While the trajectory hits Region IV, the rental rate of housing attains its minimum level ever. In this fourth region of the spiral trajectory, the demand starts to recover while the stock of housing still remains to fall; hence the resulting excess demand for housing starts to push the rental rate up again; the rise of a new bubble albeit with a reduced amplitude. While the trajectory leaves Region IV, the stock of housing reaches to its minimum level.

To characterize conditions that will ensure the assumed convergence of the trajectory drawn in Figure 1 towards the long-run equilibrium, we linearize the differential system (6)-(7) around the steady state point $(r^*, (S_1^h)^*)$ and get the following matrix equation:

$$\begin{bmatrix} \dot{S}_1^h \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\delta & \theta \\ -\frac{\gamma a_1}{a_1+r^*} & \frac{\gamma a_1 \theta r^*}{\delta(a_1+r^*)^2} - \frac{\gamma}{(a_2+r^*)^2} \end{bmatrix} \begin{bmatrix} S_1^h - (S_1^h)^* \\ r - r^* \end{bmatrix}$$

The linearized system has a stable spiral point if the trace of the above coefficient matrix is negative; i.e.,

$$-\delta + \frac{\gamma a_1 \theta r^*}{\delta(a_1 + r^*)^2} - \frac{\gamma}{(a_2 + r^*)^2} < 0.$$

This condition can be rewritten as $\gamma < \delta / D_r^h(r^*, (S_1^h)^*)$, where the right-hand side of the inequality is independent of the parameter $\gamma$. This leads us to state the following result.

**Proposition 2.** *The steady state $(r^*, (S_1^h)^*)$ is a stable spiral if the speed of the tâtonnement adjustment is so low as to satisfy $\gamma < \delta / D_r^h(r^*, (S_1^h)^*)$.*

Apparently, when $\gamma$ is too high or when the depreciation rate $\delta$ is too low, the dynamic system has a spiral which is unstable. An explanation for why $\gamma$ can be low, i.e. rental rates are sticky, may be the presence of explicit and/or implicit contracts between owners and renters of houses.

# 4 Conclusions

In this paper, we attempted to provide a basic theoretical framework to explore the key derivers of fluctuations in rental prices and housing stock around their long-run equilibrium level in a two good economy involving two representative producer-consumers.

We have established that nonsubstitutability between housing services and other nonhousing commodities and full specialization in the production of goods altogether lead to the presence of a positively sloped aggregate demand for housing services and a locally unstable rental dynamics with a fixed supply housing. When the supply is also made endogenous, the possibility of a stable spiral converging to the long-run levels of housing stock and rental price hinges upon the rental price of housing being sufficiently sticky.

We should here emphasize that to drive our results the heterogeneity of home ownership is essential while the assumed zero substitutability between housing services and other nonhousing commodities can be relaxed by assuming sufficiently low elasticities of substitution like in the static fixed house supply model of Başçı and Saglam (2008).[11] In that case, one could expect multiple steady-state equilibria involving unstable ones.

We finally note that future research need to study the intertemporal aspects of consumer choice of housing services by explicitly introducing intertemporal trade possibilities in our model.[12] An arbitrage condition in the economy would enable one to find the linkage between the selling price/rental rate of a unit of housing and the short-run interest rate in the asset market. Hence, one may also deal in the extended model with several policy questions including how the government of the described economy can act against price and rental cycles in the housing sector.

[11] Başçı and Saglam (2008) obtain three equilibria for sufficiently low elasticities of substitution corresponding to utility functions of the CES family. As the utility functions of consumers converge to a Leontief function, the lowest of the equilibrium rental rates converges to zero while the highest one goes to infinity. Moreover, the lowest and highest of the equilibrium rental rates are found to be stable while the one in between is unstable.

[12] A pioneering work in the housing literature by Poterba (1984) already studies the dynamic equilibrium of the housing market using an asset market approach.

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