Small sample power of tests of normality when the alternative is an α-stable distribution

John C. Frain. *

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Abstract

This paper is a Monte-Carlo study of the small sample power of six tests of a normality hypotheses when the alternative is an α-stable distribution with parameter values similar to those estimated for monthly total returns on equity indices. In these circumstances a sample size of 2oo is required to detect departures from normality. In most cases only small samples of consistent monthly data on such total returns are available and these are not sufficient to differentiate between normal and α-stable distributions.

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*Comments are welcome. My email address is `frainj@tcd.ie`. This document is work in progress. Please consult me before quoting. Thanks are due to Prof. Antoin Murphy and to Michael Harrison for help and suggestions and to participants at a seminar in TCD for comments received. Any remaining errors in the paper are my responsibility. I would also like to thank my wife, Helen, for her great support and encouragement.

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1 Introduction

In this paper I give an account of a series of simulations to measure the power of various tests of the null hypothesis of normality when the alternative is an α-stable distribution. Large samples of high frequency financial data generally reject this null (see, for example, Rachev and Mittnik (2000) and Frain (2006)). These tests applied to the smaller samples of monthly data, aggregated from the same daily data, do not always reject normality. For example, when the the six normality tests examined here are applied to one hundred months of daily observations of total returns on six equity indices the normality hypothesis is overwhelmingly rejected by tests for all six indices. When the six tests are applied to monthly aggregates derived from the six indices the null of normality is accepted in fifteen of the thirty six cases.

A property of the α-stable distribution is that aggregated monthly data, derived from α-stable distributed daily data, have an α-stable distribution with the same α parameter. The apparent failure of monthly data to reject the normality hypothesis has been taken as an indication that the daily data can not have an α-stable distribution. The tests examined here are shown to be of low power when applied to the short samples of monthly data typically available from aggregated daily data. Thus, failure to reject normality in these cases can not be seen as a rejection of the non-normal α-stable distribution.

Mandelbrot, in a series of papers published in the early 60's, was the first to suggest that financial returns had an α-stable distribution (Mandelbrot (1997) reprints many of his original papers and Mandelbrot and Hudson (2004) is a non-technical account of this work). The standard references on the mathematical properties of α-stable processes are Zolotarev (1986), Janicki and Weron (1994), Samorodnitsky and Taqqu (1994), Uchaikin and Zolotarev (1999) and Rachev and Mittnik (2000). An α-stable process depends on four parameters

α The stability parameter which describes the weight of the tails of the distribution. $(0 < \alpha \leq 2)$. The smaller the value of α the heavier the tails.

β A skewness parameter. $(-1 \leq \beta \leq 1)$. If $\beta = 0$ the distribution is symmetric otherwise it is skewed.

γ A spread parameter similar to the variance of a normal distribution. $(0 < \gamma)$

δ A location parameter. $(-\infty < \delta < \infty)$.

A normal distribution is an α-stable distribution with $\alpha = 2$. In this case the β parameter is redundant. γ[1] and δ correspond to the variance and mean of the normal.

The distribution of high frequency financial returns has tails that are fatter than would be expected by a normal distribution (i.e. $\alpha < 2$). The α-stable distribution appears to

[1]For the usual parametrization of α-stable and normal distributions $\sqrt{2}\gamma = \sigma$

fit the data well. In an examination of the distribution of total daily returns on 6 equity indices[2] I found values of α in the range 1.65 to 1.73 and small negative values for the skew parameter.

Section 2 gives details of the way the α-stable data were simulated and describes the six tests of normality that I have applied to sample sizes of 50, 100 and 200 and 3 values of each of the α β, γ and δ parameters. These sample sizes are typical of those that might be encountered when monthly data are derived from daily data. Detailed results are reported in Section 3 and in the Appendix. These results are summarized in Section 4.

Section 3 also details the results of applying the normality tests to aggregated monthly series of 50, 100 and 200 observations derived from the daily returns used in the earlier analysis.

The values of the β, γ and δ parameters used do not have a large effect on the analysis. In general the tests wrongly accept normality far too often and results are satisfactory only for $\alpha = 1.6$. The Pearson and Cramer-von Mises tests are unsatisfactory in all cases while the Lilliefors (Kolmogorov-Smirnov) test is satisfactory only for a sample size of 200 and an α parameter of 1.6. The Jarque-Bera and Shapiro-Wilk test can differentiate with $\alpha = 1.6$ and a sample size of greater than 100, with $\alpha = 1.7$ and a sample size of 200. The Jarque-Bera can also detect the departure from normality for $\alpha = 1.8$ and a sample size of 200. The measured relative power of these normality tests are specific to the alternative of an α-stable distribution and should not be regarded as measures of the relative merit of the tests against other alternatives.

[2]The total returns indices examined included the ISEQ, CAC40, DAX30, FTSE100, Dow Jones Composite (DJC) and S&P500. The estimation period was from October 1959 to September 2005 for the DAX30 and from the late 1970s to September 2005 for the other indices

2 The Tests

2.1 Simulations

The α-stable random numbers used in this exercise were generated using the α-stable random number generator in the Rmetrics (Wuertz (2005)) package which is part of the R (R Development Core Team (2006)) statistical package. The method used is a variation of that proposed by Chambers et al. (1976) as extended by Weron (1996a,b). Let θ have a uniform distribution on (-$\frac{\pi}{2}$, $\frac{\pi}{2}$) and w have an exponential distribution with mean 1. If

$$X = C_{\alpha,\beta} \left(\frac{\sin(\alpha(\theta + \theta_0))}{cos(\theta)^{\frac{1}{\alpha}}} \right) \left(\frac{\cos(\theta - \alpha(\theta + \theta_0)}{w} \right)^{\frac{1-\alpha}{\alpha}}$$

where

$$C_{\alpha,\beta} = \left(1 + \beta^2 \tan^2 \left(\frac{\pi\alpha}{2}\right)\right)^{\frac{1}{2\alpha}} \quad \text{and}$$
$$\theta_0 = \frac{\arctan(\beta \tan \frac{\pi\alpha}{2})}{\alpha}$$

then X has an α-stable distribution with stability parameter[3] α for $\alpha \neq 1$, skewness parameter β, spread parameter 1 and location parameter 0. The transformation of variables ($Y = \gamma X + \delta$) produces an α-stable variable with arbitrary spread (γ) and location (δ) parameters.

For each of three values[4] of the α-stable parameter (1.6, 1.7 and 1.8), three values of the skewness parameter, β, (0, -0.075 and -0.150), three values of the spread parameter, γ, (2.7, 3.6 and 4.5) and three values of the mean parameter δ (0.44, 0.88, 1.32) samples of 50, 100 and 200 observations were drawn. Each of these 243 experiments was replicated 1000 times. Six tests for normality were applied to each of the 243,000 samples. As a control on the process the simulations were repeated for a normal distribution with corresponding mean and variance.

The tests used were

1. Anderson-Darling
2. Cramer-von Mises
3. Lilliefors (Kolmogorov-Smirnov)
4. Pearson (χ^2 Goodness of Fit)
5. Shapiro-Wilk

[3] when $\alpha = 1$ use

$$X = \frac{2}{\pi} \left[\left(\frac{\pi}{2} + \beta\theta \right) \tan\theta - \beta \log \left(\frac{\frac{\pi}{2} w \cos\theta}{\frac{\pi}{2} + \beta\theta} \right) \right]$$

[4] The ranges of values for each parameter are the monthly equivalent of those found in Frain (2006)

6. Jarque-Bera

A brief summary of each test follows. For an extended account of testing for normality see Thode (2002)

2.2 Lilliefors (Kolmogorov-Smirnov) Test

The first three normality tests considered here are based on the difference between the empirical distribution function (EDF) and the normal distribution function. If the order statistics of a random sample of size n are given by $x_{(1)}$, $x_{(2)}$, $\ldots x_{(n)}$, the EDF is given by

$$F_n(x) = \begin{cases} 0 & x < x_{(1)} \\ i/n & x_{(i)} \leq x < x_{(i+1)} \quad i = 1, \ldots, n-1 \\ 1 & x_{(n)} \leq x \end{cases} \tag{1}$$

If $\Phi()$ is the standard normal distribution function and X has a normal distribution with mean μ and variance σ^2 the corresponding values of the distribution function are given by

$$q_i = \Phi([x_{(i)} - \mu]/\sigma) \tag{2}$$

The Kolmogorov-Smirnov test statistic is based on the maximum difference between the EDF and the q_i. Thus if

$$\begin{aligned} D^+ &= \max_{i=1,\ldots,n}[i/n - q_i] \\ D^- &= \max_{i=1,\ldots,n}[q_i - i/n] \\ D &= max[D^+, D^-] \end{aligned} \tag{3}$$

The Kolmogorov-Smirnov test has been extended by Lilliefors (1967) to the case where the mean and variance are unknown and the estimated test statistic is based on the usual estimates of the mean and variance. See also Stephens (1974) and Thode (2002).

2.3 Cramer-von Mises Test

A class of EDF tests proposed by Anderson and Darling (1952) is defined by

$$W_n^2 = n \int_{-\infty}^{\infty} |F_n(x) - F(x)|^2 \psi[F(x)] dF \tag{4}$$

where $F()$ is the hypothesized distribution function and $\psi()$ is a non-negative weight function. For certain weight functions, including $\psi = 1$ and $\psi(t) = 1/[t(1-t)]$, it is possible to derive explicit limit distributions of this statistic. The Cramer-von Mises statistic uses the first of these weight functions and is given by

$$W^2 = \frac{1}{12n} + \sum \left(q(i) - \frac{2i-1}{2n} \right) \tag{5}$$

with the modification

$$W^{2*} = (1.0 + 0.5/n)W^2$$

accounting for differences in sample size when using tabulated critical values.

2.4 Anderson-Darling Test

The Anderson-Darling test uses the weighting function $\psi(t) = 1/[t(1-t)]$ in equation (4). This gives the test statistic

$$A^2 = -n - n^{-1} \sum_{i=1}^{n} [2i-1][\log(p_{(i)}) + \log(1 - p_{(n-i+1)})] \tag{6}$$

where $p_{(i)} = \Phi([x_{(i)} - \hat{\mu}]/\hat{\sigma})$ and $\hat{\mu}$ and $\hat{\sigma}$ are estimated values of the mean and standard deviation.

The modification

$$A^{2*} = (1.0 + 0.75/n + 2.25/n^2)A^2 \tag{7}$$

allows the standard critical values to be applied to all sample sizes. The Anderson-Darling test gives more weight to the tails of the distribution than the Cramer-von Mises test and may therefore be better able to differentiate between normal and α-stable distributions.

2.5 Pearson (χ^2 Goodness of Fit) Test

The Pearson test is the traditional test of goodness of fit. The observations are divided into k intervals. Let O_i and E_i be the observed and expected number in the i^{th} interval. The Pearson test is

$$P = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{8}$$

The test is implemented here by dividing the samples of 50, 100 and 200 into 10, 13 and 17 equally probable intervals. P is distributed asymptotically as χ^2 with $k-3$ degrees of freedom, where k is the number of intervals used in the calculation of P. Since the advent of specific tests for a null of a normal distribution the Pearson test is not generally used.

2.6 Shapiro-Wilk Test

If the data are a good fit to a normal distribution then the plot of $x_{(i)}$ against $\Phi(i/n)$ will be close to a straight line. The Shapiro-Wilk test is a measure of this fit based on a generalized least squares regression using the covariance matrix of the order statistics. Due to difficulties in calculating this covariance matrix the Shapiro-Wilk test was originally available only for sample sizes up to 50. The difficulty being partially due to the fact that a separate covariance matrix had to be calculated for each sample size. Initially the Shapiro-Wilk test allowed smaller samples to be tested for normality than the previous Pearson test. Various approximations are now available that allow the test to be used for samples up to 5000. See Royston (1982a,b, 1995)

2.7 Jarque-Bera Test

The Jarque-Bera test is probably the normality test best known to economists and is often used as a test of the normality of residuals. If m_i is the i^{th} moment about the mean of a sample then the skewness ($b_1^{1/2}$) and kurtosis (b_2) are defined by

$$b_1^{1/2} = \frac{m_3}{m_2^{3/2}} \quad \text{and} \quad b_2 = \frac{m_4}{m_2^2} \tag{9}$$

For a sample of size n from a normal distribution $b_1^{1/2}$ is asymptotically normal with mean zero and variance 6/n. For finite samples the variance of $b_1^{1/2}$ is better given by[5]

$$c_1 = \frac{6(n-2)}{(n+1)(n+3)}$$

In the same circumstances the distribution of b_2 is asymptotically normal with mean 3 and variance 24/n. For finite samples the mean c_2 and variance c_3 of b_2 are given by

$$c_2 = \frac{3(n-1)}{(n+1)}$$
$$c_3 = \frac{24n(n-2)(n-3)}{n+1)^2(n+3)(n+5)}$$

The Jarque-Bera statistic is given by

$$JB = n\left(\frac{(b_1^{1/2})^2}{6} + \frac{(b_2-3)^2}{24}\right)$$

which under the null hypothesis of normality has an asymptotic χ^2 distribution with 2 degrees of freedom. In finite samples the skewness and kurtosis are not independent and the JB statistic converges slowly to it asymptotic limit. Two solutions have been proposed. First the JB statistic may be modified by replacing the asymptotic means

[5]For details see Thode (2002)

and variances by their values in finite samples and defining an adjusted Jarque-Bera (AJB) statistic.

$$AJB = \left(\frac{(b_1^{1/2})^2}{c_1} + \frac{(b_2 - c_2)^2}{c_3} \right)$$

The AJB and JB statistics have the same asymptotic distribution. For both the JB and AJB statistics critical values have been estimated by Weurtz and Katzgraber (2005) using a large sample Monte Carlo simulation. A comparison of the simulated and asymptotic critical values for the sample sizes used here is given in the table below.

Critical Values of Jarque-Bera test of normality

	Simulated		
Sample Size	JB	AJB	Asymptotic
50	4.98	6.55	5.99
100	5.43	6.32	5.99
200	5.68	6.15	5.99

Thus inference based on the asymptotic distribution of the standard JB statistic will tend to accept normality to often. Inference based on the asymptotic distribution of the adjusted statistic tents to reject normality to often. In the simulations in this paper inferences were based on the simulated distribution of the standard Jarque-Bera statistic. To enable some comparisons to be made, both JB and AJB tests on total returns on equity indices both JB and AJB statistics are reported along with their finite sample probabilities as derived in Weurtz and Katzgraber (2005).

Tables 1, 2 and 3 contain both JB and AJB tests on monthly returns and the significance levels given there are based on those in Weurtz and Katzgraber (2005). In all cases both tests lead to the same conclusion.

3 Results

The results of the simulations of the tests on the α-stable samples are shown in Tables 4 to 12 and summarized in Figures 1, 2 and 3. The control tests on the normal distribution are given in Table 13. Each of these 729 experiments described in Section 2.1 was replicated 1000 times. Each replication consisted of the generation of a pseudo random sample of the selected size from an α-stable distribution with the appropriate parameters. The six tests detailed in Section 2 were then applied to the random sample. The number of times that the normality assumption was accepted, at the test size specified, over 1000 replications is recorded in each case.

Thus the figure of 318 at the top of column 5 of table 4 indicates that normality was accepted in 318 of the 1000 replications when an Anderson-Darling test of size 5% was used. The power of the test may be approximated as 68%. Similarly in 363, 423, 530, 280 and 225 from the 1000 replications normality was accepted at the the 5% size when, respectively, the Cramer-von Mises, Lilliefors, Pearson, Shapiro-Wilk and Jarque-Bera tests were applied. The numbers in these tables may be regarded as an estimate of the numbers of false acceptances of normality that may be found in applications of the test in the circumstances of the simulation. Smaller numbers are better.

The results of applying the tests to simulated data drawn from a normal distribution are given in Table 13. The results in this table show that there are no significant size distortions in any of the tests examined at the sample sizes considered.

3.1 Discussion of Results

The data in the tables show that the power[6] of the tests varies with α, the sample size and the test size. In the ranges examined the other three parameters are not as important. I have adopted the somewhat arbitrary definition of a satisfactory test as one of size 5% with power greater than 90%. A stricter definition would restrict the number of satisfactory tests while a more liberal approach would lead to a greater number of satisfactory outcomes.

Using this definition no test is satisfactory for a sample size of 50. The Jarque-Bera test outperforms the others with an average power of 76% for $\alpha = 1.6$ dropping to an average of under 50% for $\alpha = 1.8$

For a sample size of 100 the Jarque-Bera test is again best in all cases. For $\alpha = 1.6$ the average power of the test is 94%. This figure falls to 86% and 70% for α of 1.7 and 1.8, respectively. The Shapiro-Wilk and Anderson-Darling tests have power close to 90% when $\alpha = 1.6$ and the size of the test is 5%.

For a sample size of 200 and $\alpha = 1.6$ the power of the Jarque-Bera, Shapiro-Wilk, Anderson-Darling and Lillifors (Kologmorov-Smirnov) tests are good, with average pow-

[6]The power of the test is estimated as $1 - \frac{\text{number normality accepted}}{1000}$

ers of 1.00, 0.99, 0.99, and 0.96 respectively. In this case the average power of the Pearson and Cramer-von Mises tests are 0.89 and 0.71 respectively.

For a sample size of 200 and $\alpha = 1.7$ the the Jarque-Bera, Shapiro-Wilk and Anderson-Darling tests have powers of 0.98, 0.96, and 0.92 respectively. For a sample size of 200 and $\alpha = 1.8$ the average power the Jarque-Bera test is just under 0.90.

The Pearson and Cramer-von Mises tests are not satisfactory in any case. The Jarque-Bera test is the most satisfactory.

The measured relative power of these normality tests are specific to the alternative of an α-stable distribution and should not be regarded as measures of the relative merit of the tests against other alternatives.

3.2 Application of tests to monthly Total Returns Equity Indices

Tables 1, 2 and 3 show the results of applying the 6 tests examined to monthly total returns on equity indices for periods of 50, 100 and 200 months, respectively, up to end August 2005. The total returns equity indices included are those for the CAC40, DAX30, FTSE100, ISEQ, Dow Composite (DCI) and the S&P500. Corresponding calculations for daily data show an overwhelming rejection of normality in all cases. For the samples of 50, 100 and 200 months there are 11, 15 and 9 acceptances of the null hypothesis of normality from the 36 tests completed in each case. Given the possible common trends in the series one can not regard them as independent samples but as an illustration of the application of the earlier results in this paper.

Of the 9 acceptances of normality in the 200 month samples all but one are in the Pearson or Lilliefors tests which have been shown to have poor power. For the 100 month samples again the majority of rejections are in these two tests but, in this case, all tests show at least one acceptance of normality.

Figure 1: Power of Normality Tests when the alternative is α-Stable in sample size 50

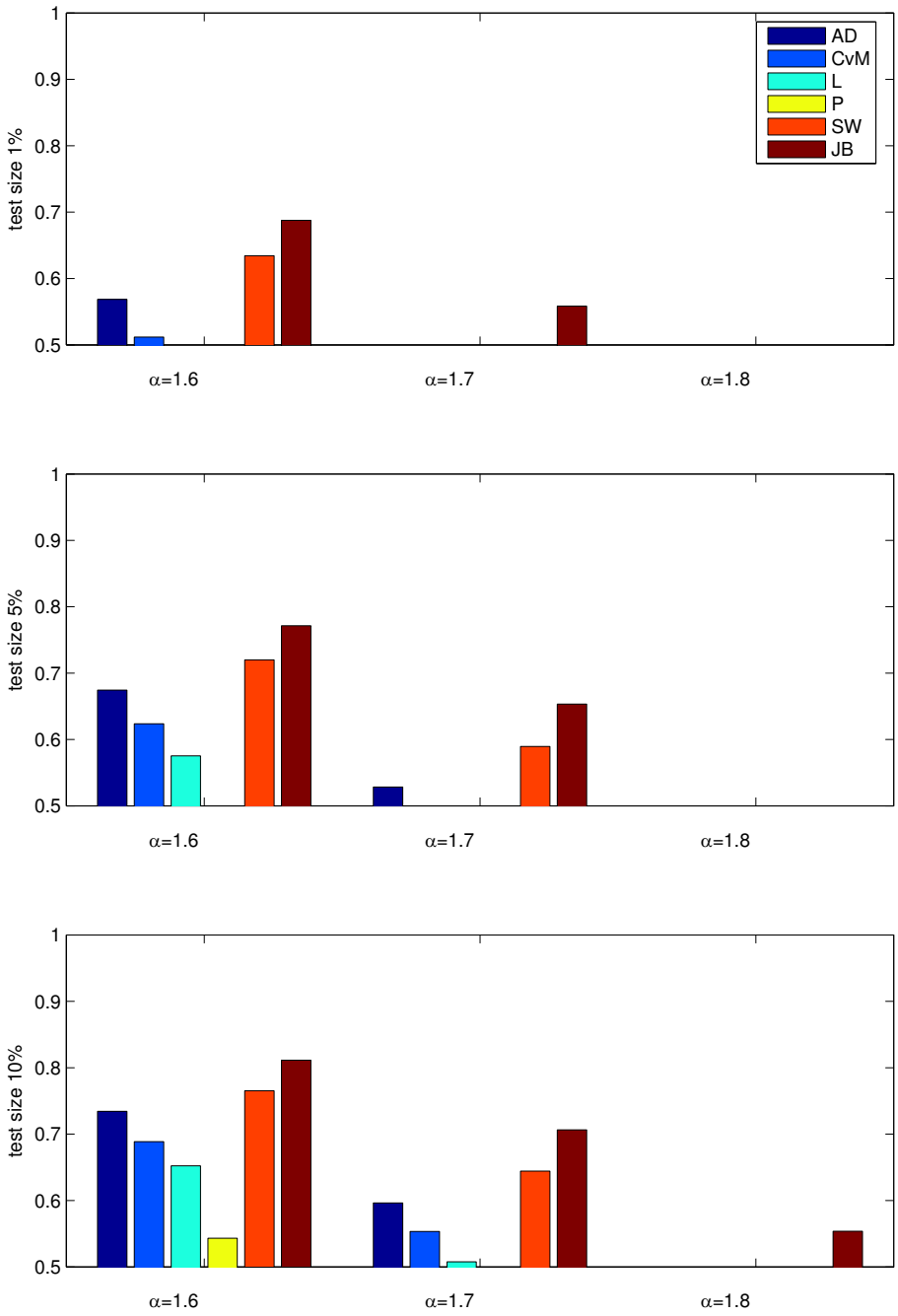


Figure 2: Power of Normality Tests when the alternative is α-Stable in sample size 100

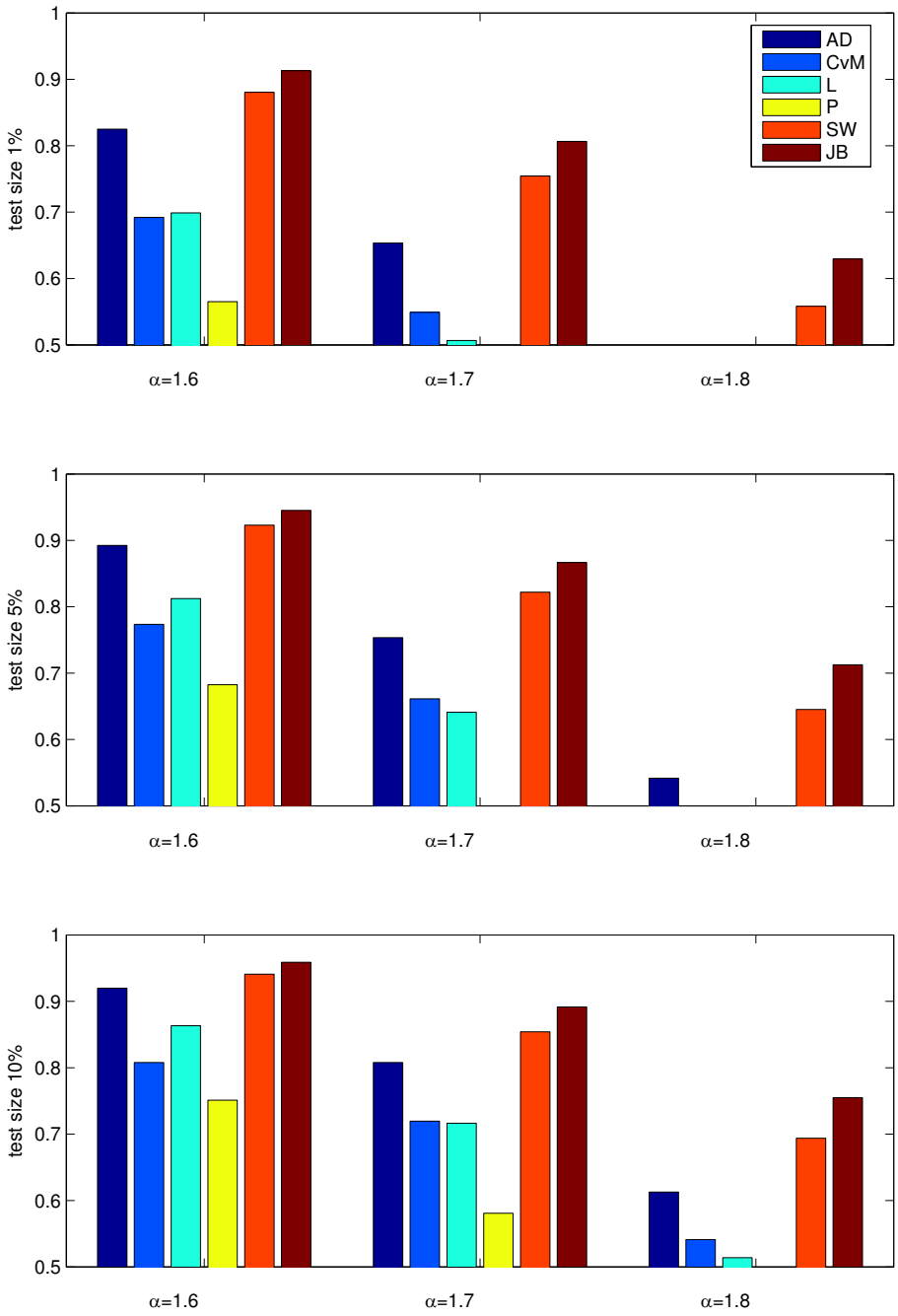
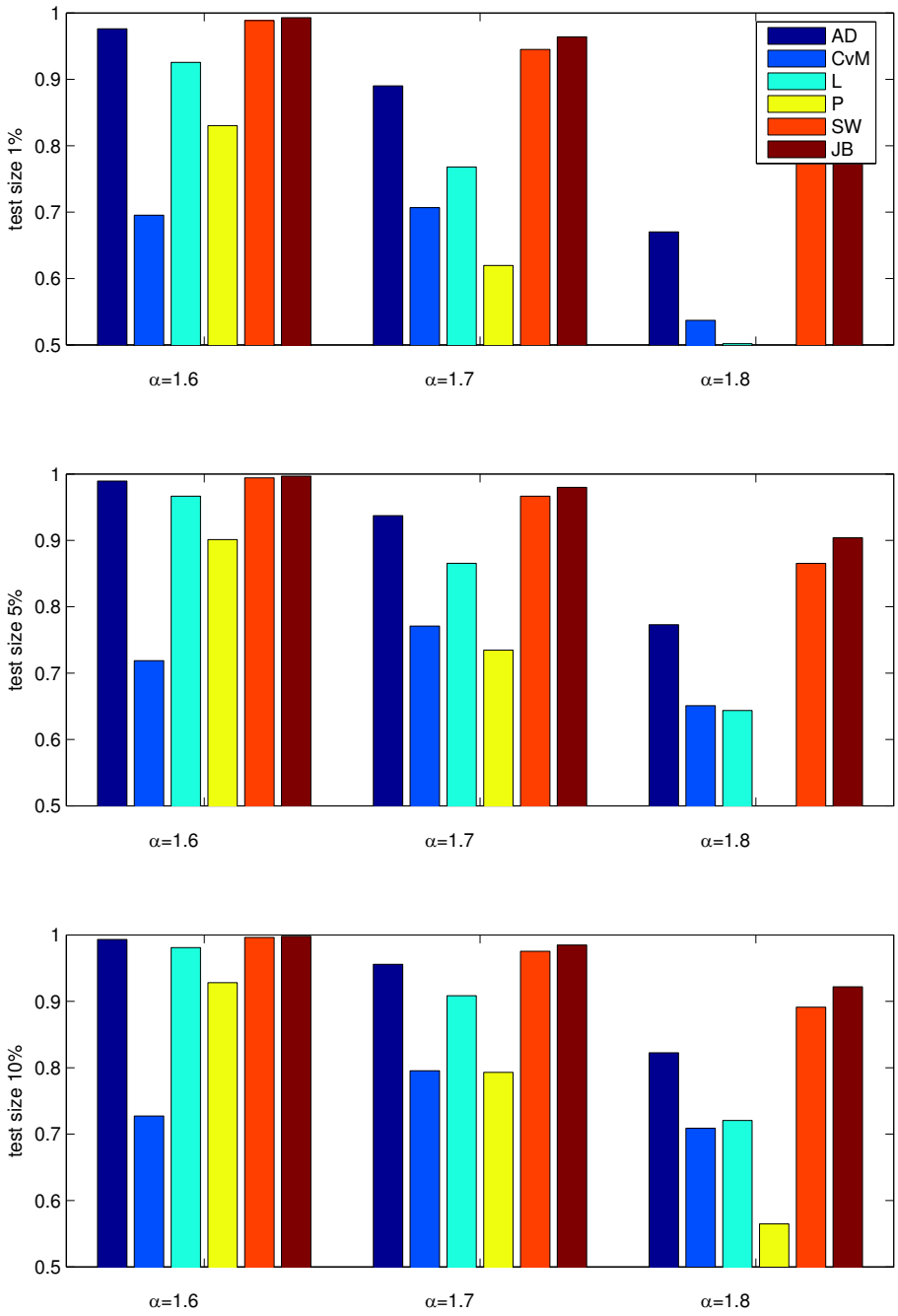


Figure 3: Power of Normality Tests when the alternative is α-Stable in sample size 200



4 Summary and Conclusions

If one regards a satisfactory test as one of size 5% with a power[7] of 90% then the only satisfactory tests are

Sample size 50 No test is satisfactory

Sample size 100 -

- For $\alpha = 1.6$ Jarque-Bera and Shapiro-Wilk tests are satisfactory.
- For $\alpha = 1.7$ No test is satisfactory
- For $\alpha = 1.8$ No test is satisfactory

Sample size 200 -

- For $\alpha = 1.6$ Jarque-Bera, Shapiro-Wilk, Anderson-Darling and Lilliefors tests are satisfactory.
- For $\alpha = 1.7$ Jarque-Bera, Shapiro-Wilk and Anderson-Darling tests are satisfactory.
- For $\alpha = 1.8$ The Jarque-Bera test was satisfactory in more than half the simulations at this level and close to satisfactory in the remailder

At the parameter values likely to fit total returns on equity indices a sample size of the order of 200 is required in order to reliably detect departures from normality using common normality tests.

The measured relative power of these normality tests do are specific to the alternative of an α-stable distribution and should not be regarded as measures of the relative merit of the tests against other alternatives.

[7]The power of a test is 1 − Prob(Type II Error). This is approximated by (1 − no of false acceptances/1000)

A Tables – Detailed Results

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Table 1: Normality Tests on Monthly Total Returns on Equity Indices for 50 months ending August, 2005

Equity Index	Summary Statistics					Normality Statistics						
	Obs.	Mean	St. dev	Skewness	Kurtosis	Anderson-Darling	Craner-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera (JB)	Jarque-Bera (AJB)
CAC40	50	-0.469	8.291	-0.876	2.485	**0.732**	**0.116**	**0.094**	**9.200**	0.949	15.435	20.877
						(.053)	(.065)	(.327)	-0.239	(.032)	(.006)	(.006)
DAX30	50	-0.124	6.071	-0.775	1.512	1.067	0.193	0.154	23.200	0.943	7.952	10.523
						(.008)	(.006)	(.004)	(.001)	(.018)	(.023)	(.023)
FTSE	50	0.137	4.236	-1.024	1.592	1.845	0.285	0.169	36.700	0.900	11.842	15.037
						(.000)	(.000)	(.001)	(.000)	(.000)	(.011)	(.012)
ISEQ	50	0.271	5.490	-0.947	0.326	1.486	0.246	0.152	16.800	0.916	7.093	8.153
						(.001)	(.001)	(.005)	(.019)	(.002)	(.028)	(.036)
DCI	50	0.361	4.392	-1.106	2.186	**0.736**	**0.095**	**0.106**	**8.000**	0.933	16.755	21.696
						(.052)	(.129)	(.167)	(.333)	(.007)	(.005)	(.006)
S&P	50	-0.091	4.380	-0.336	0.827	0.833	0.139	0.129	16.400	**0.963**	**1.711**	**2.560**
						(.030)	(.032)	(.036)	(.022)	(0.121)	(.284)	(0.189)

(Data in bold face indicate acceptance of normality hypothesis at 5% level)

Table 2: Normality Tests on Monthly Total Returns on Equity Indices for 100 months ending August, 2005

Equity Index	Summary Statistics					Normality Statistics						
	Obs.	Mean	St. dev	Skewness	Kurtosis	Anderson-Darling	Craner-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera (JB)	Jarque-Bera (AJB)
CAC40	100	0.720	6.275	-0.628	0.527	0.901	0.160	0.102	**14.140**	0.970	7.186	7.979
						(.021)	(.017)	(.012)	(.167)	(.021)	(.030)	(.033)
DAX30	100	0.332	7.747	-0.750	1.800	0.825	0.126	**0.071**	**11.800**	0.966	20.467	23.819
						(.032)	(049)	(.247)	(.299)	(.010)	(.003)	(.003)
FTSE100	100	0.409	4.325	-0.711	0.600	1.121	0.163	0.091	**12.320**	0.961	9.240	10.242
						(.006)	(.016)	(.041)	(.264)	(.005)	(.019)	(.021)
ISEQ	100	0.938	5.519	-0.822	0.993	1.165	0.195	0.105	**12.060**	0.960	14.188	15.909
						(005)	(.006)	(.009)	(.281)	(.004)	(.007)	(.008)
DCI	100	0.626	4.424	-0.815	1.305	**0.674**	**0.073**	**0.057**	**4.780**	0.959	16.559	18.850
						(.076)	(.253)	(.597)	(.905)	(.004)	(.005)	(.006)
S&P500	100	0.483	5.055	-0.499	0.293	**0.536**	**0.072**	**0.077**	**15.440**	**0.978**	**4.222**	**4.644**
						(.166)	(.260)	(.153)	(.117)	(.094)	(.074)	(.083)

(Data in bold face indicate acceptance of normality hypothesis at 5% level)

Table 3: Normality Tests on Monthly Total Returns on Equity Indices for 200 months ending August, 2005

Equity Index	Summary Statistics					Normality Statistics						
	Obs.	Mean	St. dev	Skewness	Kurtosis	Anderson-Darling	Craner-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera (JB)	Jarque-Bera (AJB)
CAC40	200	0.745	5.689	-0.550	0.519	1.085	0.195	0.081	**17.600**	0.979	11.832	12.544
						(.007)	(.006)	(.002)	(.226)	(.004)	(.010)	(.011)
DAX30	200	0.640	6.58	-0.908	2.710	2.073	0.319	0.086	24.400	0.952	83.942	90.586
						(.000)	(.000)	(.001)	(.041)	(.000)	(.000)	(.000)
FTSE100	200	0.850	4.271	-0.303	0.646	1.083	0.181	**0.062**	**16.580**	0.986	6.014	6.668
						(.008)	(.009)	(.061)	(.279)	(.040)	(.044)	(.043)
ISEQ	200	1.013	5.287	-0.455	1.411	1.236	0.188	**0.062**	**13.180**	0.974	21.889	23.996
						(.003)	(.007)	(.061)	(.512)	(.001)	(.002)	(.002)
DCI	200	0.924	4.060	-0.763	1.387	1.177	0.193	0.066	**16.580**	0.978	33.681	36.110
						(.004)	(.007)	(.032)	(.279)	(.000)	(.000)	(.000)
S&P500	200	0.871	4.311	-0.548	0.911	0.809	**0.118**	**0.053**	**19.779**	0.980	15.929	17.174
						(.036)	(0.062)	(.187)	(.137)	(.006)	(.005)	(.005)

(Data in bold face indicate acceptance of normality hypothesis at 5% level)

Table 4: Simulation of 5% Normality tests on α-stable samples of size 50 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	318	363	423	530	280	225
1.6	0	2.7	0.88	325	386	448	555	286	238
1.6	0	2.7	1.32	353	399	430	565	303	251
1.6	0	3.6	0.44	301	348	400	532	262	207
1.6	0	3.6	0.88	362	416	461	576	303	234
1.6	0	3.6	1.32	329	377	418	526	288	244
1.6	0	4.5	0.44	354	397	458	554	283	226
1.6	0	4.5	0.88	323	377	431	546	274	225
1.6	0	4.5	1.32	313	372	433	529	265	208
1.6	-0.075	2.7	0.44	309	371	420	522	282	232
1.6	-0.075	2.7	0.88	305	359	390	512	265	215
1.6	-0.075	2.7	1.32	344	395	432	535	298	231
1.6	-0.075	3.6	0.44	316	370	418	539	264	216
1.6	-0.075	3.6	0.88	343	388	436	562	305	250
1.6	-0.075	3.6	1.32	339	394	433	541	289	228
1.6	-0.075	4.5	0.44	334	378	432	544	305	249
1.6	-0.075	4.5	0.88	323	372	407	538	275	223
1.6	-0.075	4.5	1.32	337	378	430	551	283	242
1.6	-0.15	2.7	0.44	322	372	430	517	278	232
1.6	-0.15	2.7	0.88	345	394	434	558	306	237
1.6	-0.15	2.7	1.32	305	348	416	535	268	225
1.6	-0.15	3.6	0.44	340	390	434	543	280	241
1.6	-0.15	3.6	0.88	311	372	420	522	274	226
1.6	-0.15	3.6	1.32	308	372	409	517	270	221
1.6	-0.15	4.5	0.44	300	351	392	529	243	200
1.6	-0.15	4.5	0.88	327	379	425	528	263	225
1.6	-0.15	4.5	1.32	305	345	407	522	267	221
1.7	0	2.7	0.44	500	538	583	656	415	345
1.7	0	2.7	0.88	477	522	576	693	423	351
1.7	0	2.7	1.32	464	510	560	668	394	343
1.7	0	3.6	0.44	440	498	536	650	392	328
1.7	0	3.6	0.88	473	508	559	664	414	338
1.7	0	3.6	1.32	470	521	591	671	419	351
1.7	0	4.5	0.44	464	529	576	667	403	336
1.7	0	4.5	0.88	481	523	588	674	417	346

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 50 (1000 replications) *continued*

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.7	0	4.5	1.32	477	526	577	696	435	360
1.7	-0.075	2.7	0.44	470	505	569	669	410	332
1.7	-0.075	2.7	0.88	479	532	574	664	410	345
1.7	-0.075	2.7	1.32	454	496	549	662	407	348
1.7	-0.075	3.6	0.44	468	514	581	664	406	332
1.7	-0.075	3.6	0.88	442	483	553	655	386	330
1.7	-0.075	3.6	1.32	479	522	582	680	399	342
1.7	-0.075	4.5	0.44	496	535	581	677	429	360
1.7	-0.075	4.5	0.88	498	551	604	693	419	354
1.7	-0.075	4.5	1.32	465	511	558	680	399	334
1.7	-0.15	2.7	0.44	468	511	567	675	423	366
1.7	-0.15	2.7	0.88	463	514	574	676	405	328
1.7	-0.15	2.7	1.32	462	517	579	675	400	353
1.7	-0.15	3.6	0.44	478	529	574	690	417	354
1.7	-0.15	3.6	0.88	499	541	605	695	427	382
1.7	-0.15	3.6	1.32	458	511	571	673	415	346
1.7	-0.15	4.5	0.44	451	493	535	665	392	336
1.7	-0.15	4.5	0.88	476	527	590	693	407	354
1.7	-0.15	4.5	1.32	482	530	575	694	422	364
1.8	0	2.7	0.44	601	650	707	773	534	469
1.8	0	2.7	0.88	625	661	704	767	566	506
1.8	0	2.7	1.32	649	691	727	785	573	512
1.8	0	3.6	0.44	631	685	721	782	559	497
1.8	0	3.6	0.88	658	693	717	797	589	534
1.8	0	3.6	1.32	634	676	718	782	568	525
1.8	0	4.5	0.44	626	668	707	774	561	490
1.8	0	4.5	0.88	624	671	699	783	549	470
1.8	0	4.5	1.32	640	681	720	789	582	524
1.8	-0.075	2.7	0.44	639	679	722	805	563	485
1.8	-0.075	2.7	0.88	619	657	704	772	561	492
1.8	-0.075	2.7	1.32	638	674	713	783	556	502
1.8	-0.075	3.6	0.44	647	680	709	763	563	509
1.8	-0.075	3.6	0.88	626	661	709	781	570	492
1.8	-0.075	3.6	1.32	643	686	718	806	574	524
1.8	-0.075	4.5	0.44	623	665	719	795	562	517
1.8	-0.075	4.5	0.88	654	688	728	782	578	520

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 50 (1000 replications) *continued*

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	650	690	714	789	582	521
1.8	-0.15	2.7	0.44	654	679	724	798	584	530
1.8	-0.15	2.7	0.88	598	646	692	769	542	483
1.8	-0.15	2.7	1.32	657	690	727	785	583	507
1.8	-0.15	3.6	0.44	623	656	691	762	555	492
1.8	-0.15	3.6	0.88	641	685	746	786	578	510
1.8	-0.15	3.6	1.32	648	688	714	770	580	497
1.8	-0.15	4.5	0.44	638	681	728	789	571	510
1.8	-0.15	4.5	0.88	619	662	704	786	556	494
1.8	-0.15	4.5	1.32	628	653	701	773	573	512

Table 5: Simulation of 5% Normality tests on α-stable samples of size 100 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	134	229	217	343	98	67
1.6	0	2.7	0.88	110	215	177	307	74	52
1.6	0	2.7	1.32	103	216	181	314	78	55
1.6	0	3.6	0.44	93	201	177	319	68	51
1.6	0	3.6	0.88	113	229	189	309	81	51
1.6	0	3.6	1.32	117	227	198	324	77	49
1.6	0	4.5	0.44	104	242	195	332	76	52
1.6	0	4.5	0.88	94	209	159	281	68	49
1.6	0	4.5	1.32	95	225	176	313	75	55
1.6	-0.075	2.7	0.44	119	249	186	317	90	68
1.6	-0.075	2.7	0.88	110	216	199	323	78	58
1.6	-0.075	2.7	1.32	100	225	180	318	69	47
1.6	-0.075	3.6	0.44	121	242	187	304	97	71
1.6	-0.075	3.6	0.88	116	243	195	350	82	57
1.6	-0.075	3.6	1.32	98	220	182	305	63	45
1.6	-0.075	4.5	0.44	115	242	193	303	81	61
1.6	-0.075	4.5	0.88	115	240	196	348	74	52
1.6	-0.075	4.5	1.32	89	196	200	333	62	46
1.6	-0.15	2.7	0.44	119	248	198	330	83	64
1.6	-0.15	2.7	0.88	98	221	185	297	73	52
1.6	-0.15	2.7	1.32	101	225	181	298	79	51
1.6	-0.15	3.6	0.44	101	218	176	307	73	56
1.6	-0.15	3.6	0.88	97	202	182	320	68	55
1.6	-0.15	3.6	1.32	106	229	190	324	73	50
1.6	-0.15	4.5	0.44	110	239	199	329	74	61
1.6	-0.15	4.5	0.88	114	227	199	328	85	52
1.6	-0.15	4.5	1.32	113	241	169	297	82	52
1.7	0	2.7	0.44	254	351	366	506	202	146
1.7	0	2.7	0.88	273	354	371	505	202	141
1.7	0	2.7	1.32	215	317	333	482	155	116
1.7	0	3.6	0.44	246	351	367	497	175	122
1.7	0	3.6	0.88	240	320	352	483	158	109
1.7	0	3.6	1.32	231	332	358	513	169	130
1.7	0	4.5	0.44	240	338	353	493	170	127
1.7	0	4.5	0.88	264	342	373	528	180	135

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 100 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.7	0	4.5	1.32	262	348	375	498	183	128
1.7	-0.075	2.7	0.44	234	318	347	486	164	120
1.7	-0.075	2.7	0.88	257	317	338	498	191	138
1.7	-0.075	2.7	1.32	250	342	377	519	188	153
1.7	-0.075	3.6	0.44	246	334	350	494	187	148
1.7	-0.075	3.6	0.88	240	344	344	504	181	137
1.7	-0.075	3.6	1.32	263	353	359	497	170	126
1.7	-0.075	4.5	0.44	235	330	343	502	163	127
1.7	-0.075	4.5	0.88	246	340	365	509	186	140
1.7	-0.075	4.5	1.32	235	322	345	496	171	132
1.7	-0.15	2.7	0.44	247	337	361	519	185	144
1.7	-0.15	2.7	0.88	219	320	353	491	174	134
1.7	-0.15	2.7	1.32	230	334	337	489	150	109
1.7	-0.15	3.6	0.44	272	365	387	518	191	145
1.7	-0.15	3.6	0.88	232	322	345	482	179	134
1.7	-0.15	3.6	1.32	268	360	385	523	194	145
1.7	-0.15	4.5	0.44	248	346	366	503	181	144
1.7	-0.15	4.5	0.88	257	352	381	503	191	144
1.7	-0.15	4.5	1.32	254	358	360	494	169	124
1.8	0	2.7	0.44	453	531	572	670	345	274
1.8	0	2.7	0.88	472	538	577	672	364	269
1.8	0	2.7	1.32	419	505	548	656	331	270
1.8	0	3.6	0.44	465	525	563	686	365	304
1.8	0	3.6	0.88	445	511	543	665	341	287
1.8	0	3.6	1.32	472	546	579	670	361	284
1.8	0	4.5	0.44	463	533	575	682	370	298
1.8	0	4.5	0.88	447	509	556	669	346	291
1.8	0	4.5	1.32	487	551	590	693	373	305
1.8	-0.075	2.7	0.44	465	537	582	672	359	309
1.8	-0.075	2.7	0.88	441	523	561	668	339	267
1.8	-0.075	2.7	1.32	463	534	579	674	352	271
1.8	-0.075	3.6	0.44	479	536	595	695	375	310
1.8	-0.075	3.6	0.88	461	525	565	680	344	267
1.8	-0.075	3.6	1.32	434	509	581	678	350	288
1.8	-0.075	4.5	0.44	461	553	579	690	359	305
1.8	-0.075	4.5	0.88	447	521	559	668	352	288

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 100 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	445	535	578	671	349	270
1.8	-0.15	2.7	0.44	472	534	581	699	365	285
1.8	-0.15	2.7	0.88	452	525	578	677	348	278
1.8	-0.15	2.7	1.32	457	534	585	693	368	297
1.8	-0.15	3.6	0.44	449	525	549	661	334	267
1.8	-0.15	3.6	0.88	472	543	581	708	359	295
1.8	-0.15	3.6	1.32	461	530	559	663	353	294
1.8	-0.15	4.5	0.44	475	550	594	691	342	274
1.8	-0.15	4.5	0.88	448	525	562	682	370	308
1.8	-0.15	4.5	1.32	470	548	574	695	366	304

Table 6: Simulation of 5% Normality tests on α-stable samples of size 200 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	11	271	47	102	5	1
1.6	0	2.7	0.88	16	283	34	112	11	6
1.6	0	2.7	1.32	15	287	37	99	9	8
1.6	0	3.6	0.44	9	280	33	84	5	3
1.6	0	3.6	0.88	10	288	30	106	7	4
1.6	0	3.6	1.32	15	294	34	96	6	3
1.6	0	4.5	0.44	13	270	42	104	9	6
1.6	0	4.5	0.88	14	288	26	83	9	5
1.6	0	4.5	1.32	12	270	34	98	3	3
1.6	-0.075	2.7	0.44	10	279	38	102	4	3
1.6	-0.075	2.7	0.88	9	276	30	106	7	3
1.6	-0.075	2.7	1.32	7	286	22	88	4	1
1.6	-0.075	3.6	0.44	9	288	31	91	6	4
1.6	-0.075	3.6	0.88	12	276	27	87	7	4
1.6	-0.075	3.6	1.32	12	289	32	104	7	4
1.6	-0.075	4.5	0.44	14	294	45	124	9	6
1.6	-0.075	4.5	0.88	14	295	46	113	7	2
1.6	-0.075	4.5	1.32	7	262	28	88	3	2
1.6	-0.15	2.7	0.44	8	277	31	105	3	2
1.6	-0.15	2.7	0.88	8	272	27	87	5	3
1.6	-0.15	2.7	1.32	9	291	32	93	4	4
1.6	-0.15	3.6	0.44	8	262	43	116	5	3
1.6	-0.15	3.6	0.88	10	260	34	102	7	2
1.6	-0.15	3.6	1.32	5	295	23	91	2	0
1.6	-0.15	4.5	0.44	13	285	35	90	5	1
1.6	-0.15	4.5	0.88	10	283	34	103	3	1
1.6	-0.15	4.5	1.32	6	294	28	93	4	2
1.7	0	2.7	0.44	56	224	128	259	30	19
1.7	0	2.7	0.88	58	244	141	264	35	20
1.7	0	2.7	1.32	64	256	132	272	33	20
1.7	0	3.6	0.44	61	233	133	275	29	21
1.7	0	3.6	0.88	59	238	151	267	26	20
1.7	0	3.6	1.32	59	217	125	234	28	25
1.7	0	4.5	0.44	68	236	138	253	34	21
1.7	0	4.5	0.88	57	241	131	259	26	18

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 200 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.7	0	4.5	1.32	61	244	128	269	30	18
1.7	-0.075	2.7	0.44	75	211	156	308	44	29
1.7	-0.075	2.7	0.88	66	242	147	288	41	26
1.7	-0.075	2.7	1.32	72	226	140	272	38	18
1.7	-0.075	3.6	0.44	66	224	135	258	30	13
1.7	-0.075	3.6	0.88	66	228	130	245	37	21
1.7	-0.075	3.6	1.32	52	211	111	246	28	15
1.7	-0.075	4.5	0.44	68	228	133	265	41	23
1.7	-0.075	4.5	0.88	71	214	143	272	41	24
1.7	-0.075	4.5	1.32	67	238	133	258	32	16
1.7	-0.15	2.7	0.44	52	218	129	266	30	20
1.7	-0.15	2.7	0.88	66	214	136	275	36	20
1.7	-0.15	2.7	1.32	55	203	134	264	35	19
1.7	-0.15	3.6	0.44	57	240	124	253	35	21
1.7	-0.15	3.6	0.88	55	225	112	232	27	13
1.7	-0.15	3.6	1.32	79	255	157	296	40	23
1.7	-0.15	4.5	0.44	68	222	141	276	36	27
1.7	-0.15	4.5	0.88	60	234	133	270	32	20
1.7	-0.15	4.5	1.32	57	219	131	267	28	14
1.8	0	2.7	0.44	225	348	364	505	128	84
1.8	0	2.7	0.88	247	375	360	508	151	107
1.8	0	2.7	1.32	230	365	349	515	133	94
1.8	0	3.6	0.44	241	351	370	551	149	101
1.8	0	3.6	0.88	230	345	366	541	133	98
1.8	0	3.6	1.32	232	347	360	513	156	114
1.8	0	4.5	0.44	240	367	363	503	131	101
1.8	0	4.5	0.88	243	354	349	519	142	101
1.8	0	4.5	1.32	203	323	344	507	114	79
1.8	-0.075	2.7	0.44	233	359	365	523	139	97
1.8	-0.075	2.7	0.88	233	363	357	493	115	84
1.8	-0.075	2.7	1.32	242	380	366	536	146	110
1.8	-0.075	3.6	0.44	227	340	345	524	134	89
1.8	-0.075	3.6	0.88	239	353	369	498	137	101
1.8	-0.075	3.6	1.32	238	377	379	511	161	108
1.8	-0.075	4.5	0.44	234	346	375	523	135	107
1.8	-0.075	4.5	0.88	208	335	306	477	118	84

Continued on next page

Simulation of 5% Normality tests on α-stable samples of size 200 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	200	315	342	504	119	90
1.8	-0.15	2.7	0.44	217	341	366	523	133	101
1.8	-0.15	2.7	0.88	226	362	354	515	138	96
1.8	-0.15	2.7	1.32	195	309	313	474	123	90
1.8	-0.15	3.6	0.44	191	312	343	509	106	72
1.8	-0.15	3.6	0.88	219	347	347	496	126	93
1.8	-0.15	3.6	1.32	239	354	354	526	151	107
1.8	-0.15	4.5	0.44	224	341	367	519	135	92
1.8	-0.15	4.5	0.88	219	333	360	518	137	91
1.8	-0.15	4.5	1.32	257	384	389	537	146	100

Table 7: Simulation of 1% Normality tests on α-stable samples of size 50 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	417	489	541	642	368	312
1.6	0	2.7	0.88	440	505	570	695	381	327
1.6	0	2.7	1.32	447	493	555	674	389	335
1.6	0	3.6	0.44	403	456	540	647	345	300
1.6	0	3.6	0.88	470	525	593	697	401	340
1.6	0	3.6	1.32	417	481	543	652	365	319
1.6	0	4.5	0.44	448	494	549	652	371	318
1.6	0	4.5	0.88	438	499	563	667	360	302
1.6	0	4.5	1.32	432	481	552	668	351	298
1.6	-0.075	2.7	0.44	426	489	560	662	366	316
1.6	-0.075	2.7	0.88	411	465	538	668	335	291
1.6	-0.075	2.7	1.32	443	503	546	660	383	323
1.6	-0.075	3.6	0.44	432	485	551	662	358	293
1.6	-0.075	3.6	0.88	439	491	570	685	388	337
1.6	-0.075	3.6	1.32	445	510	571	670	383	319
1.6	-0.075	4.5	0.44	438	488	565	677	372	330
1.6	-0.075	4.5	0.88	420	477	540	665	350	305
1.6	-0.075	4.5	1.32	439	488	559	664	379	315
1.6	-0.15	2.7	0.44	425	485	548	636	364	322
1.6	-0.15	2.7	0.88	447	507	563	676	391	342
1.6	-0.15	2.7	1.32	419	464	535	649	341	299
1.6	-0.15	3.6	0.44	446	507	555	672	372	311
1.6	-0.15	3.6	0.88	429	482	554	646	351	308
1.6	-0.15	3.6	1.32	416	481	549	649	353	295
1.6	-0.15	4.5	0.44	404	461	515	634	342	269
1.6	-0.15	4.5	0.88	443	497	558	643	366	309
1.6	-0.15	4.5	1.32	415	476	535	637	349	300
1.7	0	2.7	0.44	590	640	694	764	511	436
1.7	0	2.7	0.88	589	646	704	806	531	456
1.7	0	2.7	1.32	571	614	676	766	478	420
1.7	0	3.6	0.44	567	613	660	758	490	434
1.7	0	3.6	0.88	566	611	684	787	503	441
1.7	0	3.6	1.32	585	642	715	780	504	435
1.7	0	4.5	0.44	584	635	698	764	499	434
1.7	0	4.5	0.88	596	645	698	777	512	440

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Simulation of 1% Normality tests on α-stable samples of size 50 (1000 replications) *continued*

α-Stable Parameters				Number of replications where normality hypothesis accepted					
α	β	γ	δ	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
1.7	0	4.5	1.32	597	649	708	805	520	460
1.7	-0.075	2.7	0.44	576	624	676	773	512	444
1.7	-0.075	2.7	0.88	580	638	687	772	490	439
1.7	-0.075	2.7	1.32	562	600	671	767	484	432
1.7	-0.075	3.6	0.44	585	636	690	779	515	438
1.7	-0.075	3.6	0.88	561	598	672	762	486	411
1.7	-0.075	3.6	1.32	584	638	689	783	500	435
1.7	-0.075	4.5	0.44	600	647	700	777	527	458
1.7	-0.075	4.5	0.88	612	658	704	784	517	451
1.7	-0.075	4.5	1.32	585	634	696	787	500	421
1.7	-0.15	2.7	0.44	581	632	711	772	503	454
1.7	-0.15	2.7	0.88	581	624	699	789	507	441
1.7	-0.15	2.7	1.32	591	640	706	790	498	439
1.7	-0.15	3.6	0.44	605	640	719	807	522	460
1.7	-0.15	3.6	0.88	608	660	708	793	527	473
1.7	-0.15	3.6	1.32	592	637	699	774	500	446
1.7	-0.15	4.5	0.44	552	603	666	773	480	431
1.7	-0.15	4.5	0.88	597	647	706	790	507	440
1.7	-0.15	4.5	1.32	588	641	693	788	507	456
1.8	0	2.7	0.44	725	763	816	874	629	570
1.8	0	2.7	0.88	731	765	812	858	664	594
1.8	0	2.7	1.32	751	792	833	876	663	607
1.8	0	3.6	0.44	750	792	820	871	664	591
1.8	0	3.6	0.88	754	781	813	864	675	618
1.8	0	3.6	1.32	738	773	819	861	666	601
1.8	0	4.5	0.44	741	773	817	854	648	594
1.8	0	4.5	0.88	720	757	811	864	640	578
1.8	0	4.5	1.32	734	774	827	882	665	609
1.8	-0.075	2.7	0.44	749	786	836	899	671	592
1.8	-0.075	2.7	0.88	717	751	806	853	646	590
1.8	-0.075	2.7	1.32	734	776	817	887	659	592
1.8	-0.075	3.6	0.44	748	781	829	872	668	594
1.8	-0.075	3.6	0.88	721	767	816	863	648	601
1.8	-0.075	3.6	1.32	751	774	833	881	677	616
1.8	-0.075	4.5	0.44	740	779	828	882	652	591
1.8	-0.075	4.5	0.88	741	777	821	870	673	607

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Simulation of 1% Normality tests on α-stable samples of size 50 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	748	768	808	864	664	622
1.8	-0.15	2.7	0.44	758	789	835	879	675	613
1.8	-0.15	2.7	0.88	715	745	793	857	625	568
1.8	-0.15	2.7	1.32	755	784	829	873	678	622
1.8	-0.15	3.6	0.44	716	749	803	863	642	583
1.8	-0.15	3.6	0.88	746	783	831	876	670	604
1.8	-0.15	3.6	1.32	733	776	807	858	665	596
1.8	-0.15	4.5	0.44	747	778	824	866	660	599
1.8	-0.15	4.5	0.88	739	775	814	880	658	587
1.8	-0.15	4.5	1.32	723	767	817	872	652	594

Table 8: Simulation of 1% Normality tests on α-stable samples of size 100 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	6	191	20	268	3	52
1.6	0	2.7	0.88	14	181	27	247	7	42
1.6	0	2.7	1.32	10	184	27	245	7	37
1.6	0	3.6	0.44	3	167	14	256	2	45
1.6	0	3.6	0.88	7	189	20	240	3	39
1.6	0	3.6	1.32	10	195	24	260	4	38
1.6	0	4.5	0.44	6	204	18	257	4	38
1.6	0	4.5	0.88	7	179	13	218	7	38
1.6	0	4.5	1.32	9	193	21	251	3	35
1.6	-0.075	2.7	0.44	6	206	21	250	3	49
1.6	-0.075	2.7	0.88	7	181	22	258	4	43
1.6	-0.075	2.7	1.32	4	193	16	250	2	36
1.6	-0.075	3.6	0.44	5	209	16	233	4	50
1.6	-0.075	3.6	0.88	9	212	18	271	6	46
1.6	-0.075	3.6	1.32	7	191	20	242	5	32
1.6	-0.075	4.5	0.44	9	213	28	235	5	45
1.6	-0.075	4.5	0.88	10	198	23	273	5	39
1.6	-0.075	4.5	1.32	5	160	16	272	1	35
1.6	0.15	2.7	0.44	4	207	13	264	1	51
1.6	0.15	2.7	0.88	6	180	13	226	3	39
1.6	0.15	2.7	1.32	4	191	17	235	3	38
1.6	0.15	3.6	0.44	5	194	20	229	4	42
1.6	0.15	3.6	0.88	7	177	15	249	4	41
1.6	0.15	3.6	1.32	4	190	13	254	2	41
1.6	0.15	4.5	0.44	8	206	25	252	4	49
1.6	0.15	4.5	0.88	10	192	21	262	1	33
1.6	0.15	4.5	1.32	4	206	16	225	3	39
1.7	0	2.7	0.44	45	288	92	430	23	123
1.7	0	2.7	0.88	42	286	100	430	24	108
1.7	0	2.7	1.32	42	263	82	396	24	95
1.7	0	3.6	0.44	38	285	90	418	27	102
1.7	0	3.6	0.88	40	275	96	408	22	86
1.7	0	3.6	1.32	38	255	78	437	21	108
1.7	0	4.5	0.44	50	283	94	421	24	99
1.7	0	4.5	0.88	35	293	89	444	19	117

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Simulation of 1% Normality tests on α-stable samples of size 100 (1000 replications)
continued

α-Stable Parameters				Number of replications where normality hypothesis accepted					
α	β	γ	δ	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
1.7	0	4.5	1.32	42	289	88	413	23	105
1.7	-0.075	2.7	0.44	55	273	109	398	36	97
1.7	-0.075	2.7	0.88	46	269	103	413	31	106
1.7	-0.075	2.7	1.32	53	285	97	437	26	113
1.7	-0.075	3.6	0.44	41	285	84	396	20	123
1.7	-0.075	3.6	0.88	51	279	89	417	28	115
1.7	-0.075	3.6	1.32	31	293	79	411	21	99
1.7	-0.075	4.5	0.44	53	272	98	402	31	100
1.7	-0.075	4.5	0.88	49	279	101	423	30	115
1.7	-0.075	4.5	1.32	52	260	91	417	19	102
1.7	0.15	2.7	0.44	31	290	85	438	19	116
1.7	0.15	2.7	0.88	47	259	95	413	25	111
1.7	0.15	2.7	1.32	39	276	87	392	22	92
1.7	0.15	3.6	0.44	43	307	87	448	26	116
1.7	0.15	3.6	0.88	41	265	78	404	16	112
1.7	0.15	3.6	1.32	54	300	111	440	30	122
1.7	0.15	4.5	0.44	50	279	89	422	29	119
1.7	0.15	4.5	0.88	45	286	92	423	25	121
1.7	0.15	4.5	1.32	43	298	87	426	21	104
1.8	0	2.7	0.44	175	456	285	595	101	222
1.8	0	2.7	0.88	200	471	283	606	125	228
1.8	0	2.7	1.32	182	423	288	573	107	234
1.8	0	3.6	0.44	189	455	308	605	122	271
1.8	0	3.6	0.88	185	448	293	587	107	240
1.8	0	3.6	1.32	182	472	278	594	125	235
1.8	0	4.5	0.44	187	464	297	611	110	254
1.8	0	4.5	0.88	197	442	271	586	116	237
1.8	0	4.5	1.32	148	483	257	610	95	262
1.8	-0.075	2.7	0.44	190	454	290	605	105	260
1.8	-0.075	2.7	0.88	175	456	274	595	91	231
1.8	-0.075	2.7	1.32	183	457	289	588	119	226
1.8	-0.075	3.6	0.44	173	462	263	612	106	262
1.8	-0.075	3.6	0.88	183	459	290	595	105	223
1.8	-0.075	3.6	1.32	192	432	303	577	133	245
1.8	-0.075	4.5	0.44	170	482	287	607	111	263
1.8	-0.075	4.5	0.88	168	451	246	597	97	251

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Simulation of 1% Normality tests on α-stable samples of size 100 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	167	457	255	602	98	230
1.8	0.15	2.7	0.44	158	458	289	614	110	243
1.8	0.15	2.7	0.88	184	454	272	619	111	237
1.8	0.15	2.7	1.32	156	465	231	616	103	261
1.8	0.15	3.6	0.44	146	448	257	579	84	227
1.8	0.15	3.6	0.88	171	462	263	634	103	249
1.8	0.15	3.6	1.32	188	457	291	598	126	248
1.8	0.15	4.5	0.44	164	476	289	611	104	239
1.8	0.15	4.5	0.88	169	462	278	601	107	269
1.8	0.15	4.5	1.32	207	481	319	618	114	268

Table 9: Simulation of 1% Normality tests on α-stable samples of size 200 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	24	298	90	177	11	6
1.6	0	2.7	0.88	28	313	80	183	16	10
1.6	0	2.7	1.32	31	302	79	160	17	10
1.6	0	3.6	0.44	24	301	69	148	8	5
1.6	0	3.6	0.88	26	314	74	184	15	11
1.6	0	3.6	1.32	29	313	74	159	7	6
1.6	0	4.5	0.44	28	304	85	164	20	15
1.6	0	4.5	0.88	24	302	75	155	13	9
1.6	0	4.5	1.32	22	290	74	170	13	3
1.6	-0.075	2.7	0.44	25	307	82	165	13	9
1.6	-0.075	2.7	0.88	19	300	75	177	10	9
1.6	-0.075	2.7	1.32	19	309	63	161	6	3
1.6	-0.075	3.6	0.44	18	306	58	161	10	7
1.6	-0.075	3.6	0.88	23	291	62	154	12	7
1.6	-0.075	3.6	1.32	22	311	73	186	13	9
1.6	-0.075	4.5	0.44	31	322	91	191	16	7
1.6	-0.075	4.5	0.88	30	326	89	188	14	9
1.6	-0.075	4.5	1.32	23	286	62	165	6	4
1.6	-0.15	2.7	0.44	25	302	70	179	11	8
1.6	-0.15	2.7	0.88	17	293	75	165	11	9
1.6	-0.15	2.7	1.32	23	317	70	155	8	5
1.6	-0.15	3.6	0.44	34	295	86	192	16	7
1.6	-0.15	3.6	0.88	20	277	82	176	11	9
1.6	-0.15	3.6	1.32	14	317	70	151	3	2
1.6	-0.15	4.5	0.44	24	309	69	181	13	6
1.6	-0.15	4.5	0.88	26	304	72	170	9	3
1.6	-0.15	4.5	1.32	18	316	62	164	10	5
1.7	0	2.7	0.44	106	294	228	371	51	33
1.7	0	2.7	0.88	116	304	227	392	59	37
1.7	0	2.7	1.32	109	317	242	381	56	34
1.7	0	3.6	0.44	103	307	246	387	51	35
1.7	0	3.6	0.88	111	304	247	377	47	29
1.7	0	3.6	1.32	107	279	212	355	55	33
1.7	0	4.5	0.44	115	296	219	377	56	42
1.7	0	4.5	0.88	103	306	219	364	47	27

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Simulation of 1% Normality tests on α-stable samples of size 200 (1000 replications)
continued

α-Stable Parameters				Number of replications where normality hypothesis accepted					
α	β	γ	δ	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
1.7	0	4.5	1.32	109	306	219	372	60	35
1.7	-0.075	2.7	0.44	128	273	259	428	62	47
1.7	-0.075	2.7	0.88	131	316	256	390	63	46
1.7	-0.075	2.7	1.32	121	297	235	377	60	40
1.7	-0.075	3.6	0.44	114	284	228	380	50	32
1.7	-0.075	3.6	0.88	93	279	224	357	52	39
1.7	-0.075	3.6	1.32	97	274	212	367	47	29
1.7	-0.075	4.5	0.44	108	294	226	379	71	39
1.7	-0.075	4.5	0.88	127	286	247	389	72	42
1.7	-0.075	4.5	1.32	108	297	223	378	62	31
1.7	-0.15	2.7	0.44	99	288	237	377	50	32
1.7	-0.15	2.7	0.88	111	279	220	381	53	36
1.7	-0.15	2.7	1.32	94	269	235	389	47	34
1.7	-0.15	3.6	0.44	104	305	231	370	52	40
1.7	-0.15	3.6	0.88	95	280	197	359	49	33
1.7	-0.15	3.6	1.32	127	330	262	404	62	46
1.7	-0.15	4.5	0.44	111	282	252	394	53	41
1.7	-0.15	4.5	0.88	104	290	240	381	50	30
1.7	-0.15	4.5	1.32	112	279	220	395	45	30
1.8	0	2.7	0.44	336	468	497	622	187	129
1.8	0	2.7	0.88	351	474	520	633	213	156
1.8	0	2.7	1.32	343	477	518	626	193	139
1.8	0	3.6	0.44	344	470	506	663	193	151
1.8	0	3.6	0.88	350	476	530	674	201	145
1.8	0	3.6	1.32	337	456	498	638	211	159
1.8	0	4.5	0.44	343	466	501	616	197	140
1.8	0	4.5	0.88	331	462	499	646	195	146
1.8	0	4.5	1.32	300	437	486	619	170	116
1.8	-0.075	2.7	0.44	336	454	494	647	191	135
1.8	-0.075	2.7	0.88	330	476	492	612	195	125
1.8	-0.075	2.7	1.32	346	497	530	662	199	154
1.8	-0.075	3.6	0.44	322	464	500	639	194	135
1.8	-0.075	3.6	0.88	335	463	495	623	185	137
1.8	-0.075	3.6	1.32	364	491	504	636	214	162
1.8	-0.075	4.5	0.44	341	472	505	647	197	141
1.8	-0.075	4.5	0.88	301	438	452	608	176	123

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Simulation of 1% Normality tests on α-stable samples of size 200 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	305	449	492	636	171	129
1.8	-0.15	2.7	0.44	330	472	511	631	190	145
1.8	-0.15	2.7	0.88	330	463	498	625	192	138
1.8	-0.15	2.7	1.32	277	415	440	595	169	124
1.8	-0.15	3.6	0.44	299	429	484	643	164	120
1.8	-0.15	3.6	0.88	316	457	481	630	174	137
1.8	-0.15	3.6	1.32	337	467	501	657	206	153
1.8	-0.15	4.5	0.44	321	459	493	630	179	134
1.8	-0.15	4.5	0.88	318	459	503	642	182	130
1.8	-0.15	4.5	1.32	361	488	520	637	218	150

Table 10: Simulation of 10% Normality tests on α-stable samples of size 50 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	259	311	332	442	233	195
1.6	0	2.7	0.88	271	317	374	467	253	197
1.6	0	2.7	1.32	285	338	367	476	253	210
1.6	0	3.6	0.44	234	287	327	445	221	185
1.6	0	3.6	0.88	286	336	382	487	246	197
1.6	0	3.6	1.32	270	321	343	438	249	211
1.6	0	4.5	0.44	283	335	372	486	232	182
1.6	0	4.5	0.88	266	302	353	471	234	189
1.6	0	4.5	1.32	257	304	341	449	220	168
1.6	-0.075	2.7	0.44	255	295	350	447	231	189
1.6	-0.075	2.7	0.88	252	299	324	431	224	172
1.6	-0.075	2.7	1.32	279	322	359	463	237	188
1.6	-0.075	3.6	0.44	253	293	356	460	221	176
1.6	-0.075	3.6	0.88	289	325	357	466	262	215
1.6	-0.075	3.6	1.32	284	332	354	458	238	192
1.6	-0.075	4.5	0.44	275	314	349	461	257	207
1.6	-0.075	4.5	0.88	259	303	334	459	229	180
1.6	-0.075	4.5	1.32	271	315	359	464	243	198
1.6	-0.15	2.7	0.44	263	307	346	446	243	200
1.6	-0.15	2.7	0.88	287	339	365	487	246	191
1.6	-0.15	2.7	1.32	261	303	337	454	232	179
1.6	-0.15	3.6	0.44	277	320	358	456	243	195
1.6	-0.15	3.6	0.88	255	307	335	443	223	187
1.6	-0.15	3.6	1.32	262	307	329	443	224	178
1.6	-0.15	4.5	0.44	235	275	318	435	195	152
1.6	-0.15	4.5	0.88	252	306	338	462	226	179
1.6	-0.15	4.5	1.32	250	291	330	435	223	181
1.7	0	2.7	0.44	428	483	513	572	361	298
1.7	0	2.7	0.88	405	446	490	611	359	302
1.7	0	2.7	1.32	392	437	473	582	354	289
1.7	0	3.6	0.44	386	431	470	572	343	283
1.7	0	3.6	0.88	407	443	495	574	339	275
1.7	0	3.6	1.32	401	445	501	589	366	308
1.7	0	4.5	0.44	385	425	490	583	346	278
1.7	0	4.5	0.88	412	459	496	600	364	294

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Simulation of 10% Normality tests on α-stable samples of size 50 (1000 replications) *continued*

α-Stable Parameters				Number of replications where normality hypothesis accepted					
α	β	γ	δ	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
1.7	0	4.5	1.32	419	460	501	609	372	309
1.7	-0.075	2.7	0.44	410	451	489	591	353	279
1.7	-0.075	2.7	0.88	403	441	502	605	365	298
1.7	-0.075	2.7	1.32	387	431	474	580	360	309
1.7	-0.075	3.6	0.44	408	448	493	584	346	279
1.7	-0.075	3.6	0.88	375	406	469	571	337	275
1.7	-0.075	3.6	1.32	401	455	505	613	349	290
1.7	-0.075	4.5	0.44	426	463	508	612	361	300
1.7	-0.075	4.5	0.88	414	463	525	629	370	299
1.7	-0.075	4.5	1.32	401	444	476	613	342	287
1.7	-0.15	2.7	0.44	409	454	481	580	373	315
1.7	-0.15	2.7	0.88	395	441	489	592	346	273
1.7	-0.15	2.7	1.32	387	425	487	577	349	299
1.7	-0.15	3.6	0.44	412	449	494	600	360	289
1.7	-0.15	3.6	0.88	423	472	521	606	376	317
1.7	-0.15	3.6	1.32	397	437	478	592	346	289
1.7	-0.15	4.5	0.44	386	427	462	585	344	275
1.7	-0.15	4.5	0.88	407	456	510	602	353	317
1.7	-0.15	4.5	1.32	421	465	502	607	371	300
1.8	0	2.7	0.44	532	562	625	702	471	418
1.8	0	2.7	0.88	561	593	627	705	507	453
1.8	0	2.7	1.32	574	612	647	715	515	448
1.8	0	3.6	0.44	551	601	643	703	502	428
1.8	0	3.6	0.88	583	621	641	717	530	471
1.8	0	3.6	1.32	568	605	633	692	522	465
1.8	0	4.5	0.44	561	603	632	701	504	435
1.8	0	4.5	0.88	550	595	627	718	493	425
1.8	0	4.5	1.32	583	614	651	714	535	463
1.8	-0.075	2.7	0.44	538	586	625	729	502	435
1.8	-0.075	2.7	0.88	553	587	630	698	502	443
1.8	-0.075	2.7	1.32	555	608	633	694	505	445
1.8	-0.075	3.6	0.44	569	610	642	698	511	444
1.8	-0.075	3.6	0.88	558	597	639	713	515	447
1.8	-0.075	3.6	1.32	577	614	633	719	528	461
1.8	-0.075	4.5	0.44	563	585	629	713	515	468
1.8	-0.075	4.5	0.88	577	617	660	725	517	450

Continued on next page

Simulation of 10% Normality tests on α-stable samples of size 50 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	573	610	640	721	523	457
1.8	-0.15	2.7	0.44	587	619	650	731	525	472
1.8	-0.15	2.7	0.88	535	574	611	711	480	424
1.8	-0.15	2.7	1.32	577	607	649	703	521	451
1.8	-0.15	3.6	0.44	554	596	628	691	502	419
1.8	-0.15	3.6	0.88	575	606	650	727	514	448
1.8	-0.15	3.6	1.32	581	619	642	702	522	446
1.8	-0.15	4.5	0.44	569	611	655	717	525	447
1.8	-0.15	4.5	0.88	548	585	634	713	501	437
1.8	-0.15	4.5	1.32	554	589	629	702	508	447

Table 11: Simulation of 10% Normality tests on α-stable samples of size 100 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	99	191	163	268	69	52
1.6	0	2.7	0.88	86	181	129	247	58	42
1.6	0	2.7	1.32	81	184	130	245	58	37
1.6	0	3.6	0.44	70	167	128	256	50	45
1.6	0	3.6	0.88	79	189	137	240	60	39
1.6	0	3.6	1.32	82	195	137	260	61	38
1.6	0	4.5	0.44	75	204	145	257	52	38
1.6	0	4.5	0.88	72	179	124	218	54	38
1.6	0	4.5	1.32	73	193	125	251	59	35
1.6	-0.075	2.7	0.44	88	206	145	250	70	49
1.6	-0.075	2.7	0.88	76	181	137	258	60	43
1.6	-0.075	2.7	1.32	75	193	133	250	52	36
1.6	-0.075	3.6	0.44	95	209	141	233	79	50
1.6	-0.075	3.6	0.88	97	212	144	271	67	46
1.6	-0.075	3.6	1.32	70	191	135	242	50	32
1.6	-0.075	4.5	0.44	88	213	129	235	64	45
1.6	-0.075	4.5	0.88	78	198	144	273	53	39
1.6	-0.075	4.5	1.32	60	160	142	272	48	35
1.6	-0.15	2.7	0.44	90	207	150	264	68	51
1.6	-0.15	2.7	0.88	71	180	117	226	58	39
1.6	-0.15	2.7	1.32	76	191	129	235	53	38
1.6	-0.15	3.6	0.44	81	194	132	229	55	42
1.6	-0.15	3.6	0.88	69	177	130	249	52	41
1.6	-0.15	3.6	1.32	76	190	142	254	54	41
1.6	-0.15	4.5	0.44	82	206	145	252	61	49
1.6	-0.15	4.5	0.88	89	192	149	262	68	33
1.6	-0.15	4.5	1.32	85	206	131	225	63	39
1.7	0	2.7	0.44	203	288	287	430	165	123
1.7	0	2.7	0.88	193	286	288	430	162	108
1.7	0	2.7	1.32	167	263	266	396	126	95
1.7	0	3.6	0.44	190	285	291	418	142	102
1.7	0	3.6	0.88	184	275	268	408	130	86
1.7	0	3.6	1.32	187	255	279	437	136	108
1.7	0	4.5	0.44	192	283	283	421	139	99
1.7	0	4.5	0.88	211	293	290	444	155	117

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Simulation of 10% Normality tests on α-stable samples of size 100 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.7	0	4.5	1.32	196	289	295	413	148	105
1.7	-0.075	2.7	0.44	179	273	254	398	128	97
1.7	-0.075	2.7	0.88	201	269	281	413	147	106
1.7	-0.075	2.7	1.32	191	285	297	437	155	113
1.7	-0.075	3.6	0.44	212	285	290	396	152	123
1.7	-0.075	3.6	0.88	189	279	276	417	143	115
1.7	-0.075	3.6	1.32	196	293	297	411	142	99
1.7	-0.075	4.5	0.44	183	272	278	402	136	100
1.7	-0.075	4.5	0.88	197	279	282	423	147	115
1.7	-0.075	4.5	1.32	185	260	278	417	139	102
1.7	-0.15	2.7	0.44	199	290	278	438	152	116
1.7	-0.15	2.7	0.88	181	259	269	413	143	111
1.7	-0.15	2.7	1.32	178	276	270	392	130	92
1.7	-0.15	3.6	0.44	207	307	301	448	160	116
1.7	-0.15	3.6	0.88	188	265	269	404	154	112
1.7	-0.15	3.6	1.32	196	300	309	440	159	122
1.7	-0.15	4.5	0.44	186	279	284	422	147	119
1.7	-0.15	4.5	0.88	203	286	299	423	163	121
1.7	-0.15	4.5	1.32	191	298	298	426	138	104
1.8	0	2.7	0.44	376	456	483	595	288	222
1.8	0	2.7	0.88	381	471	500	606	308	228
1.8	0	2.7	1.32	360	423	455	573	287	234
1.8	0	3.6	0.44	380	455	470	605	316	271
1.8	0	3.6	0.88	386	448	468	587	302	240
1.8	0	3.6	1.32	393	472	500	594	305	235
1.8	0	4.5	0.44	393	464	502	611	314	254
1.8	0	4.5	0.88	373	442	460	586	297	237
1.8	0	4.5	1.32	411	483	495	610	326	262
1.8	-0.075	2.7	0.44	404	454	498	605	314	260
1.8	-0.075	2.7	0.88	380	456	482	595	281	231
1.8	-0.075	2.7	1.32	391	457	502	588	302	226
1.8	-0.075	3.6	0.44	403	462	510	612	320	262
1.8	-0.075	3.6	0.88	389	459	481	595	304	223
1.8	-0.075	3.6	1.32	374	432	474	577	299	245
1.8	-0.075	4.5	0.44	384	482	490	607	322	263
1.8	-0.075	4.5	0.88	371	451	479	597	305	251

Continued on next page

Simulation of 10% Normality tests on α-stable samples of size 100 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	392	457	495	602	296	230
1.8	-0.15	2.7	0.44	400	458	498	614	307	243
1.8	-0.15	2.7	0.88	392	454	477	619	302	237
1.8	-0.15	2.7	1.32	392	465	495	616	315	261
1.8	-0.15	3.6	0.44	364	448	470	579	286	227
1.8	-0.15	3.6	0.88	392	462	495	634	320	249
1.8	-0.15	3.6	1.32	374	457	477	598	309	248
1.8	-0.15	4.5	0.44	397	476	505	611	296	239
1.8	-0.15	4.5	0.88	391	462	463	601	320	269
1.8	-0.15	4.5	1.32	414	481	499	618	324	268

Table 12: Simulation of 10% Normality tests on α-stable samples of size 200 (1000 replications)

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.6	0	2.7	0.44	6	262	20	84	3	1
1.6	0	2.7	0.88	14	276	27	89	7	3
1.6	0	2.7	1.32	10	278	27	73	7	6
1.6	0	3.6	0.44	3	269	14	65	2	2
1.6	0	3.6	0.88	7	278	20	77	3	1
1.6	0	3.6	1.32	10	287	24	68	4	1
1.6	0	4.5	0.44	6	263	18	74	4	3
1.6	0	4.5	0.88	7	282	13	58	7	4
1.6	0	4.5	1.32	9	267	21	77	3	2
1.6	-0.075	2.7	0.44	6	269	21	75	3	2
1.6	-0.075	2.7	0.88	7	271	22	73	4	1
1.6	-0.075	2.7	1.32	4	284	16	67	2	1
1.6	-0.075	3.6	0.44	5	282	16	63	4	2
1.6	-0.075	3.6	0.88	9	272	18	69	6	4
1.6	-0.075	3.6	1.32	7	282	20	77	5	3
1.6	-0.075	4.5	0.44	9	281	28	80	5	5
1.6	-0.075	4.5	0.88	10	281	23	81	5	1
1.6	-0.075	4.5	1.32	5	252	16	60	1	1
1.6	-0.15	2.7	0.44	4	266	13	74	1	2
1.6	-0.15	2.7	0.88	6	265	13	62	3	2
1.6	-0.15	2.7	1.32	4	283	17	72	3	3
1.6	-0.15	3.6	0.44	5	251	20	86	4	2
1.6	-0.15	3.6	0.88	7	250	15	69	4	2
1.6	-0.15	3.6	1.32	4	284	13	61	2	0
1.6	-0.15	4.5	0.44	8	273	25	65	4	0
1.6	-0.15	4.5	0.88	10	265	21	79	1	0
1.6	-0.15	4.5	1.32	4	288	16	63	3	1
1.7	0	2.7	0.44	45	204	92	199	23	18
1.7	0	2.7	0.88	42	217	100	195	24	15
1.7	0	2.7	1.32	42	225	82	211	24	14
1.7	0	3.6	0.44	38	208	90	213	27	16
1.7	0	3.6	0.88	40	211	96	215	22	17
1.7	0	3.6	1.32	38	200	78	186	21	14
1.7	0	4.5	0.44	50	215	94	206	24	16
1.7	0	4.5	0.88	35	220	89	203	19	12

Continued on next page

Simulation of 10% Normality tests on α-stable samples of size 200 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.7	0	4.5	1.32	42	215	88	200	23	14
1.7	-0.075	2.7	0.44	55	175	109	243	36	19
1.7	-0.075	2.7	0.88	46	208	103	218	31	19
1.7	-0.075	2.7	1.32	53	206	97	217	26	15
1.7	-0.075	3.6	0.44	41	200	84	207	20	9
1.7	-0.075	3.6	0.88	51	204	89	195	28	13
1.7	-0.075	3.6	1.32	31	193	79	192	21	14
1.7	-0.075	4.5	0.44	53	211	98	205	31	16
1.7	-0.075	4.5	0.88	49	192	101	215	30	15
1.7	-0.075	4.5	1.32	52	216	91	203	19	10
1.7	-0.15	2.7	0.44	31	189	85	208	19	18
1.7	-0.15	2.7	0.88	47	185	95	213	25	17
1.7	-0.15	2.7	1.32	39	179	87	219	22	14
1.7	-0.15	3.6	0.44	43	217	87	196	26	18
1.7	-0.15	3.6	0.88	41	206	78	164	16	13
1.7	-0.15	3.6	1.32	54	234	111	230	30	16
1.7	-0.15	4.5	0.44	50	197	89	212	29	22
1.7	-0.15	4.5	0.88	45	203	92	224	25	11
1.7	-0.15	4.5	1.32	43	193	87	199	21	11
1.8	0	2.7	0.44	175	295	285	433	101	66
1.8	0	2.7	0.88	200	311	283	426	125	89
1.8	0	2.7	1.32	182	297	288	443	107	77
1.8	0	3.6	0.44	189	297	308	469	122	84
1.8	0	3.6	0.88	185	282	293	465	107	77
1.8	0	3.6	1.32	182	298	278	425	125	86
1.8	0	4.5	0.44	187	306	297	419	110	86
1.8	0	4.5	0.88	197	306	271	434	116	86
1.8	0	4.5	1.32	148	266	257	419	95	68
1.8	-0.075	2.7	0.44	190	306	290	439	105	72
1.8	-0.075	2.7	0.88	175	312	274	432	91	69
1.8	-0.075	2.7	1.32	183	322	289	460	119	85
1.8	-0.075	3.6	0.44	173	278	263	427	106	75
1.8	-0.075	3.6	0.88	183	289	290	427	105	83
1.8	-0.075	3.6	1.32	192	320	303	435	133	94
1.8	-0.075	4.5	0.44	170	280	287	448	111	87
1.8	-0.075	4.5	0.88	168	285	246	405	97	72

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Simulation of 10% Normality tests on α-stable samples of size 200 (1000 replications)
continued

				Number of replications where normality hypothesis accepted					
α-Stable Parameters				Anderson-	Cramer-			Shapiro-	Jarque-
α	β	γ	δ	Darling	von Mises	Lilliefors	Pearson	Wilk	Bera
1.8	-0.075	4.5	1.32	167	267	255	422	98	73
1.8	-0.15	2.7	0.44	158	281	289	445	110	81
1.8	-0.15	2.7	0.88	184	304	272	437	111	84
1.8	-0.15	2.7	1.32	156	262	231	396	103	71
1.8	-0.15	3.6	0.44	146	248	257	424	84	52
1.8	-0.15	3.6	0.88	171	285	263	436	103	70
1.8	-0.15	3.6	1.32	188	296	291	448	126	87
1.8	-0.15	4.5	0.44	164	271	289	437	104	78
1.8	-0.15	4.5	0.88	169	274	278	429	107	73
1.8	-0.15	4.5	1.32	207	322	319	470	114	85

Table 13: Simulation of Normality tests on a normal distribution (1000 replications)

Simulation details				Number of replications where normality hypothesis accepted					
				Test					
sample size	test size	st.dev	mean	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
50	5	3.8	0.44	946	950	942	946	934	947
50	5	3.8	0.88	946	939	955	955	951	940
50	5	3.8	1.32	938	938	947	944	933	928
50	5	5.1	0.44	935	934	932	933	934	933
50	5	5.1	0.88	946	942	945	958	944	946
50	5	5.1	1.32	957	951	953	944	964	949
50	5	6.4	0.44	937	935	937	944	940	941
50	5	6.4	0.88	938	926	942	938	948	951
50	5	6.4	1.32	951	951	953	954	958	951
100	5	3.8	0.44	953	955	958	952	951	944
100	5	3.8	0.88	948	948	944	950	952	943
100	5	3.8	1.32	949	948	952	947	945	938
100	5	5.1	0.44	954	954	954	949	945	936
100	5	5.1	0.88	953	954	965	953	958	941
100	5	5.1	1.32	953	953	944	961	956	949
100	5	6.4	0.44	944	942	941	942	945	933
100	5	6.4	0.88	942	931	936	960	941	935
100	5	6.4	1.32	952	947	947	948	957	950
200	5	3.8	0.44	949	953	954	956	953	948
200	5	3.8	0.88	940	947	943	946	948	943
200	5	3.8	1.32	949	951	953	939	949	937
200	5	5.1	0.44	952	952	956	941	945	954
200	5	5.1	0.88	952	953	953	944	957	948
200	5	5.1	1.32	970	967	961	951	952	953
200	5	6.4	0.44	956	954	949	949	963	955
200	5	6.4	0.88	947	945	950	938	952	949
200	5	6.4	1.32	947	946	948	943	953	950
50	1	3.8	0.44	983	982	983	985	987	983
50	1	3.8	0.88	990	992	994	993	986	989
50	1	3.8	1.32	988	990	991	992	987	982
50	1	5.1	0.44	981	982	986	986	981	984
50	1	5.1	0.88	985	986	989	993	992	982
50	1	5.1	1.32	992	991	992	989	994	989
50	1	6.4	0.44	985	983	989	988	984	984
50	1	6.4	0.88	986	981	987	987	990	989

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Simulation of Normality tests on a normal distribution (1000 replications) *continued*

Simulation details				Number of replications where normality hypothesis accepted: Test					
sample size	test size	st.dev	mean	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
50	1	6.4	1.32	991	990	991	994	992	990
100	1	3.8	0.44	993	993	994	992	992	987
100	1	3.8	0.88	993	991	991	993	993	985
100	1	3.8	1.32	990	990	989	993	986	986
100	1	5.1	0.44	989	989	988	988	989	977
100	1	5.1	0.88	992	992	990	994	991	985
100	1	5.1	1.32	985	986	987	993	988	987
100	1	6.4	0.44	989	986	992	988	983	980
100	1	6.4	0.88	988	987	985	990	982	988
100	1	6.4	1.32	988	992	989	989	990	986
200	1	3.8	0.44	992	993	991	992	992	987
200	1	3.8	0.88	988	991	993	993	987	985
200	1	3.8	1.32	993	990	993	993	991	986
200	1	5.1	0.44	989	989	986	988	992	977
200	1	5.1	0.88	992	992	992	994	992	985
200	1	5.1	1.32	994	986	998	993	990	987
200	1	6.4	0.44	994	986	996	988	994	980
200	1	6.4	0.88	991	987	991	990	988	988
200	1	6.4	1.32	989	992	991	989	991	986
50	10	3.8	0.44	891	893	906	895	885	893
50	10	3.8	0.88	896	894	898	896	902	889
50	10	3.8	1.32	870	868	881	892	880	877
50	10	5.1	0.44	880	885	873	879	884	869
50	10	5.1	0.88	887	881	905	912	878	889
50	10	5.1	1.32	898	906	895	900	907	907
50	10	6.4	0.44	890	877	886	893	886	886
50	10	6.4	0.88	886	881	870	885	896	906
50	10	6.4	1.32	906	907	910	906	906	915
100	10	3.8	0.44	909	909	904	901	906	888
100	10	3.8	0.88	894	891	897	896	906	888
100	10	3.8	1.32	894	895	894	892	894	885
100	10	5.1	0.44	899	907	901	892	894	879
100	10	5.1	0.88	916	917	921	909	917	887
100	10	5.1	1.32	903	893	891	910	912	910
100	10	6.4	0.44	898	891	877	895	899	889

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Simulation of Normality tests on a normal distribution (1000 replications) *continued*

Simulation details				Number of replications where normality hypothesis accepted					
				Test					
sample size	test size	st.dev	mean	Anderson-Darling	Cramer-von Mises	Lilliefors	Pearson	Shapiro-Wilk	Jarque-Bera
100	10	6.4	0.88	885	884	881	908	894	889
100	10	6.4	1.32	895	901	891	886	905	903
200	10	3.8	0.44	913	909	905	901	902	888
200	10	3.8	0.88	893	891	879	896	896	888
200	10	3.8	1.32	900	895	892	892	889	885
200	10	5.1	0.44	909	907	903	892	904	879
200	10	5.1	0.88	908	917	913	909	913	887
200	10	5.1	1.32	922	893	911	910	913	910
200	10	6.4	0.44	895	891	899	895	896	889
200	10	6.4	0.88	895	884	893	908	905	889
200	10	6.4	1.32	905	901	894	886	908	903

References

Anderson, T. W. and D. A. Darling (1952). Asymptotic theory of certain "goodness of fit" criteria based on stochastic processes. *The Annals of Mathematical Statistics 23*(2), 193–212.

Chambers, J. M., C. L. Mallows, and B. W. Stuck (1976). A method for simulating stable random variables. *Journal of the American Statistical Association 71*, 340–344.

Frain, J. C. (2006). Total returns on equity indices, fat tails and the α-stable distribution. http://www.tcd.ie/Economics/staff/frainj/main/Stable_distribution/index.htm.

Janicki, A. and A. Weron (1994). *Simulation and ChaoticBehavior of α-Stable Stochastic Processes*. Dekker.

Lilliefors, H. W. (1967). On the Kolmogorov-Smirnov test for normality with mean and variance unknown. *Journal of the American Statistical Association 62*(318), 399–402.

Mandelbrot, B. B. (1997). *Fractals and Scaling in Finance*. Springer.

Mandelbrot, B. B. and R. L. Hudson (2004). *The (mis)Behaviour of Markets*. Profile Books.

R Development Core Team (2006). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. ISBN 3-900051-07-0.

Rachev, S. and S. Mittnik (2000). *Stable Paretian Models in Finance*. Wiley.

Royston, J. P. (1982a). Algorithm AS 181: The W test for normality. *Applied Statistics 31*(2), 176–180.

Royston, J. P. (1982b). An extension of Shapiro and Wilk's w test for normality to large samples. *Applied Statistics 31*(2), 115–124.

Royston, P. (1995). Remark AS R94: A remark on algorithm AS 181: The W-test for normality. *Applied Statistics 44*(4), 547–551.

Samorodnitsky, G. and M. S. Taqqu (1994). *Stable Non-Gaussian Random Processes Stochastic Models with Infinite Variance*. Chapman and Hall/CRC.

Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association 69*(347), 730–737.

Thode, H. C. (2002). *Testing for Normality*. Marcel Decker.

Uchaikin, V. V. and V. M. Zolotarev (1999). *Chance and Stability*. VSP, Uthrecht.

Weron, R. (1996a). Correction to: On the Chambers-Mallows-Stuck metod for simulating skewed stable random variables. Research Report HSC/96/1, Wroclaw University of Technology.

Weron, R. (1996b). On the Chambers-Mallows-Stuck method for simulating skewed random variables. *Statistics and Probability Letters 28*, 16–171.

Weurtz, D. and H. G. Katzgraber (2005). Precise finite-sample quantiles of the Jarque-Bera adjusted Lagrange multiplier test. Swiss Federal Institute of Technology, Institute for Theoretical Physics, ETH Hönggerberg, C-8093 Zurich.

Wuertz, D. (2005). Rmetrics - an environment for teaching financial engineering and computational finance with R. Institute for Theoretical Physics, Swiss Federal Institute of Technology, Zurich.

Zolotarev, V. M. (1986). *One-dimensional Stable Distributions*. Translations of Mathematical Monographs, Volume 65, American Mathematical Society.