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ORDERED SEARCH AND EQUILIBRIUM OBFUSCATION

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Abstract. This paper demonstrates the incentives for an oligopolist to obfuscate by deliberately increasing the cost with which consumers can locate its product and price. Consumers are allowed to choose the optimal order in which to search firms and firms are able to influence this order through their choice of search costs and prices. Competition does not ensure market transparency - for a large range of parameters, equilibrium search costs are positive and asymmetric across firms. Intuitively, an obfuscating firm can soften the competition for consumers with low time costs by inducing the remaining consumers to optimally first search its rival.

1. Introduction

This paper studies the strategic incentives for an oligopolist to manipulate the ease with which consumers can locate its product and price. Within a standard framework, we show how a firm may want to deliberately limit its information provision by increasing its cost of being searched. Indeed, for a large range of parameter values, asymmetric equilibria are documented where only one firm selects the minimum level of search costs and where the other obfuscates by choosing a level of search costs that is strictly higher. An obvious interpretation of the model is in terms of firms' advertising choices, but more generally it may provide an explanation for why some firms choose to locate in obscure positions such as backstreets or at the bottom of search results, and why some firms may intentionally pick inefficient store layouts or website designs.

Three main contributions of the paper can be identified. First, in contrast to much of the previous obfuscation literature, we demonstrate that the incentives to limit market

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information can extend to cases where all consumers are fully rational. Second, by showing that positive, asymmetric search costs may be the outcome of an endogenous market process, we add support to the importance of search models, but offer some concern over the frequency with which such models assume search costs are symmetric across firms (e.g. Varian 1980, Stahl 1989 and Janssen and Moraga-González 2004). Third, to analyse these issues, the paper makes an original step by allowing consumers to choose not only the number of firms to search, but also the order in which to search them. Several recent models have stressed the significance of search order, but assume that the order is either exogenous (Arbatskaya 2007, Armstrong et al 2007), or based on relative advertising levels (Haan and Moraga-González 2007)[1]. Here, we allow consumers to choose their *optimal* search order and let firms influence this order through their choice of search costs and prices.

The paper presents a model of a homogenous good duopoly where prior to selecting prices, firms choose the length of time required to locate its product and price. Consumers are fully rational and divided into two types that either have positive or zero per-unit costs of time. When the time required to locate products and prices is non-zero and equal across firms, as assumed within standard search models, firms employ identical mixed-strategy pricing distributions in order to resolve the tensions produced by the consumer heterogeneity. When, however, firms are allowed to pre-commit to the time required to locate their products and prices, we show that equilibria exist where one firm chooses to obfuscate and where the other, now 'prominent', firm does not.

Intuitively, the act of obfuscation by any given firm induces the consumers with positive time costs to optimally first search its rival and makes them less willing to make a second search. This increases the rival's incentives to raise its price (distribution) and benefits the obfuscating firm by softening the competition for the remaining consumers with zero time costs. The prominent firm faces no incentive to raise its search cost in addition to its rival as this can only reduce its profits by deterring consumers from entering the market. Nor does the prominent firm face any incentive to counter the obfuscating firm's strategy by providing consumers with information about its rival because the action of the obfuscating firm increases its rival's equilibrium profits and prices by an amount

[1] In a related vein, Hortaçsu and Syverson (2004) empirically demonstrate that asymmetric sampling probabilities are useful in explaining pricing patterns in the mutual fund industry.

larger than the increase in its own profits and prices, (while also reducing the expected welfare for all consumers). Such a mechanism may offer one possible explanation for why a single insurance company, Direct Line, recently made a public announcement to pull out of all price comparison sites, while following a strategy of 'bringing down the cost of car insurance'[2], or why a cut price retailer, TK Max, appears to deliberately shelve its clothing lines in a disorganised manner. In the latter part of the paper, the model is extended to the case of n firms. A full equilibrium analysis is difficult, but we show that the region of parameters where full obfuscation must form part of equilibrium remains non-empty for any finite number of firms. Simulations also show that this region of parameters appears to shrink as the number of competitors increases, offering a tentative suggestion that equilibrium search costs may be declining in the number of competitors.

In a pioneering paper, Bakos (1997) emphasised the possibility that firms have an incentive to soften competition by collectively increasing search costs[3]. More recently, however, the literature, as popularised by Ellison and Ellison (2005)[4], has focussed on understanding whether the incentives to obfuscate can extend to the level of an individual firm. In contrast to the standard results that suggest a firm will reveal all information if it costless to do so (Grossman 1981, Milgrom 1981), the literature has shown equilibrium conditions under which a firm can profitably limit its information provision. The mechanism presented within this paper however, differs from previous findings in that it does not rely on bounded consumer rationality or the existence of add-on goods. Gabaix and Laibson (2006) present an equilibrium where all firms obfuscate by 'shrouding' a high add-on price. A fraction of myopic consumers do not foresee the existence of add-ons and compare firms only on the basis of observable base prices alone, leading firms to profitably conceal high add-on prices and cross-subsidise low base good prices. In a related paper, where firms are presented with the option of setting hidden fees on top of observable prices, Garrod (2007) shows that, if some consumers are naive, no equilibrium exists where all firms commit to zero fees. Like the current paper, Ellison (2005) documents the existence of equilibrium obfuscation in a model with fully rational consumers. Specifically, consumers

[2] See http://www.directline.com/motor/welcome.htm and http://www.directline.com/motor/newimprovedcar.htm, 26 February 2008.

[3] Although Janssen and Non (2008) show this incentive may not always hold as the relationship between market search costs and prices or profits may be non-monotonic when a form of advertising is introduced.

[4] A discussion in this paper loosely inspired the framework of the current paper.

are of either high or low type, with high types being less likely to switch between firms and more willing to purchase add-ons. The concealment of add-on prices acts to soften competition due to an adverse selection effect where firms are deterred from reducing base prices by the possibility of attracting a disproportionate amount of low-types who only buy the base good at a below optimum price.

Three papers present results that are closely related to ours. Janssen and Non (2008) provide a model where consumers are similarly heterogeneous, but where firms face a binary decision of whether or not to reduce their own search costs to zero through costly advertising. In a duopoly game where advertising and pricing strategies must be chosen simultaneously, they characterise a symmetric equilibrium where firms mix over advertising such that the (expected) level of search costs is non-zero. More generally, and in order to provide a clearer link with the standard search cost literature, our model treats the choice of search costs or information provision as a continuous variable. Our results confirm that this can indeed be effectively reduced to a binary decision over whether to become fully transparent or to fully obfuscate, but we also show how the results need not rely on positive costs of influencing search costs and demonstrate the importance of asymmetric equilibria by introducing, and endogenising, search order. Similar to our set-up, both Ireland (1993) and Zettelmeyer (2000) allow firms to pre-commit to strategies that influence the cost at which consumers can obtain information. In Ireland, firms choose the proportion of (identical) consumers to inform of their existence, with consumers only being able to then (costlessly) buy from firms that they are aware of. He finds that only one firm perfectly informs the market, while the rest choose to inform only half of the consumers. Zettelmeyer's duopoly model assumes that consumers have full price information but can only realise their firm-specific product match value by incurring a search cost. Without fully endogenising consumers' search decisions, he shows that parameters can be chosen such that only one firm chooses a zero search cost in equilibrium.

Finally, in the light of behavioural industrial organisation, it is tempting to reinterpret the model as one in which consumers differ in their cognitive ability, rather than their costs of time, and where firms obfuscate by making their prices harder to understand, rather than harder to find. Such an interpretation of the search cost literature has been discussed in Ellison and Ellison (2005) and Ellison (2006), but, as explained in Section 6, we fail to find such arguments convincing in regard to the current model.

The paper continues by outlining the model in section 2 and analysing its equilibria in sections 3 and 4. Section 5 presents some extensions. It discusses some of the potential obstacles that a firm may face when trying to obfuscation and offers an extension of the model to the case of n firms. Section 6 discusses the model in a behavioural context, as outlined above, and section 7 concludes.

2. The Model

Let there be two firms, $i = 1, 2$, that each sell a single homogeneous good at zero production costs to a unit mass of consumers who each have a unit demand and a maximum willingness to pay of $V > 0$. In order to buy from firm i, it is assumed that a consumer must necessarily incur s_i units of time searching for the location of firm i's product and price. Consumers vary in their per-unit cost of time and remain indistinguishable to firms. In particular, we introduce an extreme form of consumer heterogeneity by assuming that a proportion of consumers, $\alpha \in (0, 1)$, labelled as 'costly-shoppers', have a positive cost of time that we normalise to unity, and the remaining proportion of 'shoppers', $\beta = 1 - \alpha$, have a zero cost of time (or enjoy shopping such that they always choose to search). Consequently, while costly-shoppers face a search cost for firm i equal to s_i, shoppers always face zero search costs. This latter assumption is relaxed in section 5.

Viewing information provision as a longer-run decision variable, consider the following two-stage game. In stage 1, each firm simultaneously selects its own search cost, s_i, by choosing the length of time required to locate its product and price[5]. To ensure that the results are not influenced by any cost motivations, it is assumed that firms may influence their own search costs at zero cost. In the simplest setting of the model, the firms are free to select any positive level of search costs, $s_i \geq 0$, but more generally, we assume that search costs can not be set below some minimum level of market friction that could be thought to vary across different market scenarios, $s_i \geq m$, with $m \in [0, V]$. Consumers are fully rational, with the ability to compute all equilibrium strategies and understand all identified prices at zero cost.

In Stage 2, each firm's choice of search cost becomes common knowledge. Implicitly, we therefore assume that all consumers i) are always aware of the existence of each firm, although this can be relaxed somewhat as in section 5, and ii) are always able to assess

[5]One may wish to think of interpretations other than time. Instead, consumers could differ, say, in their cost of effort and firms would then choose the effort required to find their product and price.

the time required to search each firm. For example, in the case where a firm obfuscates by refraining from advertising, we require consumers to know how long it will take to search the firm unaided. The players then choose the following strategies simultaneously. Firms select their own (possibly degenerate) pricing distribution, $F_i(p)$, with support $[\underline{p}_i, \overline{p}_i]$, from which a price is drawn, p_i, and the two types of consumers form conjectures about the firms' pricing strategies (which are correct in equilibrium) and select their optimal search strategies.

We now proceed by characterising some general features of the game's equilibria, before then focussing on a specific range of parameters for which the analysis remains tractable.

3. Stage 2 Analysis

3.1. Optimal Search Strategies. This section begins to analyse the equilibria of Stage 2 by first considering consumers' optimal search strategies, given correct conjectures about each firm's choice of search cost and pricing strategy. While the optimal behaviour of the shoppers simply involves buying from the firm with the lowest price realisation (if lower or equal to V), and randomising in the case of a price tie, the search strategy for the costly-shoppers may not be trivial. Indeed, in contrast to standard symmetric search models where costly-shoppers need only decide how many firms to search, here costly-shoppers must also decide in which order to search the firms. To characterise an optimal *ordered* search strategy, we rely on Weitzman (1979) which presents the solution to a generalised dynamic search problem akin to the one considered here. The costly shoppers' optimal search strategy can be simplified to two rules.

Lemma 1. *Given correct conjectures about the firms' pricing distributions, $F_i(p)$, and search costs, s_i, $\forall i = 1, 2$, the costly-shoppers' optimal ordered search strategy consists of the use of the following two rules, where the reservation price of firm i, r_i^*, is the value of r_i that satisfies $\int_{\underline{p}_i}^{r_i}(r_i - p)dF_i(p) = s_i$:*

Stopping Rule: Continue to search only when i) all previously discovered prices and ii) the outside option price of V are higher than the lowest remaining reservation price.

Selection Rule: If a firm is to be searched, it should be the unsearched firm with the lowest reservation price.

Proof. See Weitzman (1979). ■

Intuitively, a reservation price forms an index of the net gains from searching a given firm. The reservation price of firm i, r_i^*, is simply the level of a (fictitious) previously discovered price at which a costly-shopper would view the marginal benefit of searching firm i, $\int_{\underline{p_i}}^{r_i}(r_i - p)dF_i(p)$, as equal to its marginal cost, s_i. For the standard symmetric case, the optimal strategy collapses into the familiar result - costly-shoppers should only begin search if the common reservation price is lower than their valuation, V, and stop searching only on the discovery of a price below the common reservation price, where the marginal net benefit from further search becomes negative. For the asymmetric case, Weitzman's logic implies that a costly-shopper should begin search only if the lowest reservation price is lower than their valuation V, select the firms in order of ascending reservation prices and continue searching until a price is found that is lower than all the reservation prices of the remaining unsearched firms and their valuation, V. While a firm's reservation price is increasing in its search cost, as one would expect, there is no necessary relationship between a firm's reservation price and its expected price. In general terms, reservation prices may be ranked only in terms of first-order stochastic dominance, that is $r_i < r_j$ if $F_i(p) \geq F_j(p)\ \forall p$ and $s_i = s_j$, and in the sense that a mean preserving spread will lower a firms' reservation price. Consumers will prefer to first search firms that exhibit low search costs and price distributions with 'low' prices and high variance.

3.2. Optimal Pricing Strategies. Having established the consumers' optimal strategies, we now complete the analysis of Stage 2 by characterising the firms' equilibrium pricing strategies for a given choice of search costs. In general, it is well known that the existence of an atom of shoppers will provide an incentive for firms to deviate from any pure strategy pricing equilibrium and so we will consider equilibrium pricing strategies in the form of price distributions. In particular, any equilibrium pricing distribution must satisfy the following conditions, i) given the behaviour of all other players, each firm must strictly prefer prices within its equilibrium support and expect to receive constant equilibrium profits over all such prices, $E(\pi_i) = \overline{\pi}_i\ \forall p_i \in [\underline{p_i}, \overline{p}_i]\ \forall i$ and $E(\pi_i) < \overline{\pi}_i\ \forall p_i \notin [\underline{p_i}, \overline{p}_i]$ $\forall i$, ii) each firm's pricing distribution must be well behaved, $F_i(\underline{p_i}) = 0$, $F_i(\overline{p_i}) = 1$ and $F_i'(p) \geq 0$, $\forall p_i \in [\underline{p_i}, \overline{p}_i]\ \forall i$, and iii) each firm's pricing distribution, $F_i(p; r_i^*)$, must be consistent with the consumers' conjectured value of the firm's reservation price, r_i^* and vice versa.

This section first finds the pricing equilibria treating the costly-shoppers' initial search decisions as exogenous, as summarised in Lemma 2, before then completing the equilibria by fully endogenising the costly-shoppers' strategies in Lemmas 3 and 4. As well as making the exposition clearer, the use of Lemma 2 as an intermediate step is useful in making comparisons with the previous literature where consumers are often treated as making a first search for free[6].

Specifically, let us firstly assume that the costly-shoppers' initial search decisions are exogenous in the sense that that $\alpha_i \in [0, \alpha]$ costly-shoppers decide to first search Firm i but behave optimally thereafter - searching firm $j \neq i$ iff $r_j^* < \min\{p_i, V\}$. Without loss of generality, let $\alpha_1 \geq \alpha_2$ and $\alpha_1 + \alpha_2 \leq \alpha$. We now proceed to derive the unique pricing equilibrium under two specific conditions that are both later demonstrated to hold within the set of full game equilibria.

As a first step, we know that by setting $\overline{p}_1 = \min\{r_2^*, V\}$, Firm 1 can guarantee an equilibrium profit of $\overline{\pi}_1 = \alpha_1 \overline{p}_1$ by ensuring that its share of costly-shoppers, α_1, find it optimal to buy without further searching Firm 2. As a result, Firm 1 will never wish to price below a lower price bound of $\underline{p_1} = (\alpha_1/(\alpha_1 + \beta))\overline{p}_1$ as even if it were to sell at this price to both its costly-shoppers and all the shoppers, it would still find it more profitable to sell only to its costly-shoppers at $\overline{p}_1$. Now, provided our first condition holds, i) $(\alpha_2 + \beta)\underline{p_1} \geq \alpha_2 \min\{r_1^*, V\}$, Firm 2 will prefer to price at $\underline{p_1}$ to guarantee the custom of both its share of costly-shoppers and the shoppers in order to earn a maximin profit of $\overline{\pi}_2 = (\alpha_2 + \beta)\underline{p_1}$, rather than pricing at the $\min\{r_1^*, V\}$ to ensure only the custom of its costly-shoppers. Further, if our second condition also holds, ii) $r_1^* \geq \overline{p}_1$ if $\alpha_2 > 0$, then Firm 2 will be willing to compete for the shoppers within $p \in [\underline{p_1}, \overline{p}_1)$, without the possibility that its costly-shoppers would wish to search Firm 1. The equilibrium pricing distributions on $p \in [\underline{p_1}, \overline{p}_1)$ must now satisfy $\overline{\pi}_i = E(\pi_i) = p[\alpha_i + \beta(1 - F_j(p))]$ $\forall i \neq j$. While Baye et al (1992) show that games of this sort can display a continuum of (asymmetric) mixed strategy equilibria in the n firm context, they also show that the equilibrium pricing distributions, $F_1(p)$ and $F_2(p)$, are unique for the case of two firms as considered here. By finding such distributions, one can find that Firm 1 has a mass point at $\overline{p}_1$, such that $E(\pi_2(p_2 \simeq \overline{p}_1)|F_1(p)) = \overline{\pi}_2$ as required.

Finally, to find the reservation prices that are consistent with equilibrium, one can

[6] Janssen et al (2005) first departed from the standard assumption by allowing consumers' first search to be costly in the case where search costs are symmetric and exogenous. We replicate some of their results below.

insert the equilibrium price distributions into $\int_{\underline{p_i}}^{r_i} F_i(p)dp = s_i$, which, on simplification, can provide an explicit expression for r_2^* iff $r_2^* \leq V$, but not for r_1^*. The pricing equilibrium can then be summarised by Lemma 2.

Lemma 2. *Given a level of search costs, $s_2 \geq s_1 \geq m$, and an exogenous set of initial search decisions, $\{\alpha_1, \alpha_2\}$, with $\alpha_1 \geq \alpha_2$ and $\alpha_1 + \alpha_2 \leq \alpha$, the following pricing equilibrium is unique, provided that i) $(\alpha_2 + \beta)\underline{p_1} \geq \alpha_2 \min\{r_1^*, V\}$ and ii) $r_1^* \geq \overline{p}_1$ if $\alpha_2 > 0$.*

$$F_1(p) = 1 - \left(\tfrac{(\alpha_2+\beta)\underline{p_1}}{\beta p} - \tfrac{\alpha_2}{\beta}\right) \quad p \in [\underline{p_1}, \overline{p_1})$$

$$f_1(\overline{p}_1) = \left(\frac{\alpha_1 - \alpha}{\alpha_1 + \beta}\right)$$

$$F_2(p) = 1 - \left(\tfrac{(\alpha_1+\beta)\underline{p_1}}{\beta p} - \tfrac{\alpha_1}{\beta}\right) \quad p \in [\underline{p_1}, \overline{p_1})$$

$$\underline{p_1} = \left(\tfrac{\alpha_1}{\alpha_1+\beta}\right)\overline{p}_1, \quad \overline{p}_1 = \min\{r_2^*, V\}$$

$$\overline{\pi}_1 = \alpha_1 \overline{p}_1, \quad \overline{\pi}_2 = (\alpha_2 + \beta)\underline{p_1}$$

where r_i^* is the value of r_i such that $\int_{\underline{p_1}}^{r_i} F_i(p)dp = s_i$ if it exists, with $r_i^* = \infty$ if not

$$\text{and where} \quad r_2^* = \left(\tfrac{\beta s_2}{\beta + \alpha_1 \ln(\alpha_1/(\alpha_1+\beta))}\right) \quad \text{if } r_2^* \leq V$$

Further intuition of Lemma 2 will be gained after discussing Lemmas 3 and 4, but at this point it is useful to note that the proposed pricing equilibrium extends the results of a collection of standard symmetric search models to a more general asymmetric case. Indeed, the Narasimhan (1988) model can be reproduced if $s_1 = s_2 = \infty$, a duopoly version of Stahl (1989) is generated when $s_1 = s_2$ and $\alpha_1 = \alpha_2 = (\alpha/2)$, and a duopoly version of Varian (1980) can be constructed if $s_1 = s_2 = \infty$ and $\alpha_1 = \alpha_2 = (\alpha/2)$.

Lemmas 3 and 4 now complete the characterisation of the Stage 2 equilibria by endogenising the costly-shoppers' initial search decisions (while ensuring that the conditions for the validity of the pricing equilibrium in Lemma 2 are met). The equilibrium initial search decisions, $\alpha_i^*(s_1, s_2)$ $\forall i = 1, 2$, are shown to depend on the relative magnitude of s_1 and s_2. Lemma 3 first takes the case when s_1 is small relative to V such that all the costly-shoppers always participate in the market. Figure 1 illustrates its findings with some example parameters ($s_1 = m = 1$, $V = 10$ and $\alpha = \beta = 0.5$).

Insert Figure 1 here.

Lemma 3. *Given a level of search costs such that* $m \leq s_1 \leq s_2$ *and* $s_1 \leq ((1-\alpha)+\alpha\ln\alpha)V$*, the unique Stage 2 equilibria can be described by Lemma 2, with initial search decisions* $\{\alpha_1, \alpha_2\}$ *such that* $\alpha_1 = \alpha_1^*$ *and* $\alpha_2 = \alpha_2^*$*, where* $\alpha_1^* = \max\{\frac{(s_2-\beta s_1)}{(s_1+s_2)}, \alpha\}$ *and* $\alpha_2^* = \min\{0, \frac{(s_1-\beta s_2)}{(s_1+s_2)}\}$.

Proof. See Appendix: Claims 1-3. ■

To explain the intuition of the Stage 2 equilibria when s_1 is small relative to V, first consider the case of symmetric search costs, where $s_2 = s_1 = m$, as first analysed by Janssen et al (2005). Here, Lemmas 2 and 3 suggest there exists an equilibrium where the two firms use an identical pricing distribution, earn equal profits, $(\alpha\beta s_i)/(2\beta+\alpha\ln(\alpha/(1+\beta)))$ and receive exactly half of the costly-shoppers' first searches, $\alpha_1^* = \alpha_2^* = (\alpha/2)$. Both firms choose not to price above the symmetric reservation price in order to prevent their costly-shoppers from searching elsewhere, but probabilistically employ lower prices to compete for the shoppers. The costly-shoppers behaviour remains consistent with equilibrium as $r_1^* = r_2^* \leq V$ ensures the consumers are willing to make a first search and are indifferent about which firm to search first. Although not shown by Janssen et al, we also demonstrate this equilibrium to be unique as any other division of the costly-shoppers must be inconsistent with equilibrium[7].

Now consider the case of asymmetric search costs and focus on the case where the difference between the search costs is large, $s_2 > (s_1/\beta)$. In equilibrium, all the costly-shoppers decide to first visit the prominent firm with the lower search cost, Firm 1, such that $\alpha_1^* = \alpha$ and $\alpha_2^* = 0$. The fact that it is more costly for the costly-shoppers to visit Firm 2 allows Firm 1 to price quite highly and earn relatively high profits. Firm 2, with no costly-shoppers, has no such incentive and can, instead, exert some form of market power over the shoppers as Firm 1 is relatively unwilling to select low prices. In equilibrium, one can show that the price distributions can be ranked in terms of stochastic dominance, $F_1(p) \leq F_2(p)\ \forall p \in [\underline{p_1}, \overline{p_1}]$ and that Firm 1's price distribution exhibits a mass point at its upper bound (as shown in Figure 2 with the same parameters as those used in Figure 1). One can also show that Firm 1 earns higher equilibrium profits, $\overline{\pi}_1 > \overline{\pi}_2 > 0$. Despite Firm 1's higher prices, the costly-shoppers' initial search behaviour remains consistent

[7] Baye et al (1992) use a similar logic to propose a unique symmetric equilibrium within Varian's (1980) search model.

with the equilibrium due to the offsetting effect of Firm 2's large search cost and the low value of s_1 relative to V, ensuring $r_1^* < r_2^*, V$.

Insert Figure 2 here.

When the difference between the search costs is smaller, with $s_2 \in (s_1, (s_1/\beta))]$, an equilibrium exists where Firm 1 receives a relatively large, but not complete, share of the costly-shoppers' first searches with $\alpha_1^* \in ((\alpha/2), \alpha]$, $\alpha_2^* = [0, (\alpha/2))$ and $\alpha_1^* + \alpha_2^* = \alpha$. Firm 1 again employs a stochastically dominant price distribution but the unique equilibrium division of costly-shoppers neutralises any incentives for the firms to deviate and ensures the consumers are indifferent between first searching the two firms with $r_1^* = r_2^* \leq V$. As Figure 1 illustrates, one can show that $\partial\alpha_1^*/\partial s_2 > 0$ and $\partial\alpha_2^*/\partial s_2 < 0$.

Lemmas 2 and 3 suggest that the firm with the lower search costs, Firm 1, is first searched by the (majority of) costly-shoppers, earns higher profits and selects 'higher' prices than its rival. The relationship between prominence and prices remains contested within the literature. The results of Ireland (1993) and Arbatskaya (2007) are also consistent with the idea that prominence or higher informative advertising intensity is associated with higher prices. Arbatskaya demonstrates that (pure strategy) prices are declining in an (exogenous) search order within a homogenous goods market where consumers' search costs are distributed with an atomless distribution, but where each consumer faces the same search cost for each firm. However, other papers have shown that this relationship need not always hold. Armstrong et al (2007) show that prices can be increasing in search order if the firms exhibit random product differentiation because the prominent firm's residual demand can be more elastic than its rivals' due to a higher composition of 'fresh' consumers that have yet to search the entire market. Robert and Stahl (1993) find an inverse relation between a firm's individual level of informative advertising and their prices, while Non and Janssen (2008) find no consistent relationship.

Lemma 4 now analyses the remaining cases where s_1 is large relative to V.

Lemma 4. *Given a level of search costs such that $m \leq s_1 \leq s_2$ and $s_1 > ((1-\alpha) + \alpha \ln \alpha)V$, any Stage 2 equilibria must be described by Lemma 2, with initial search decisions $\{\alpha_1, \alpha_2\}$ such that $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$, where $\alpha_1^* \leq \max\{\frac{(s_2-\beta s_1)}{(s_1+s_2)}, \alpha\}$ and $\alpha_2^* \leq \min\{0, \frac{(s_1-\beta s_2)}{(s_1+s_2)}\}$.*

Proof. See Appendix: Claims 4-6. ■

Fully characterising the equilibria in such situations is analytically difficult, but in order to proceed we need only show that the initial search decisions must be bounded by the proportions found in Lemma 3. In the symmetric case, once the search cost becomes large relative to V, the full symmetric participation of the costly-shoppers can no longer be supported as the costly-shoppers will prefer to exit the market, $r^* > V$. Instead, as in Janssen et al (2005, Proposition 2), the equilibrium must involve the costly-shoppers mixing between searching and not participating in the market, in order to lower equilibrium prices such that $r^*(\alpha_1, \alpha_2) = V$ with $\alpha_1^* = \alpha_2^* < (\alpha/2)$. Similarly, in the asymmetric cases, some non-participation is necessary to ensure prices are low enough to allow the costly-shoppers to be indifferent over entering the market in equilibrium, but the exact level of participation is difficult to present analytically due to the implicit definition of Firm 1's reservation price.

4. Stage 1 Analysis

We are now ready to examine the firms' equilibrium choice of search costs. To do so, and to avoid the tractability problems demonstrated in Lemma 4 we restrict parameter values to those consistent with Condition A, see Figure 3. Akin to standard market coverage assumptions, Condition A ensures V is sufficiently large (relative to the natural component of search costs, m) such that all consumers participate in equilibrium. Note that Condition A is satisfied for all parameter values when $m = 0$, and is likely to be satisfied more generally when the fraction of costly-shoppers, α, is relatively small.

$$(m/V) < (1-\alpha) + \alpha \ln \alpha \qquad \text{(Condition A)}$$

Insert Figure 3 here.

For convenience, we will maintain the notational assumption that $s_1 \leq s_2$, but we will also define $\overline{s}_2(\alpha_1^*) = [1 + (\alpha_1^*/\beta)\ln(\alpha_1^*/(\alpha_1^* + \beta))]V$ as the value of s_2 that produces an equilibrium value of $r_2^* = V$, for a given s_1. We shall then state that Firm 2 chooses to 'fully obfuscate' if it sets $s_2 \geq \overline{s}_2(\alpha_1^*)$ such that the costly-shoppers face no strict incentive to search it[8]. Proposition 1 now provides the main result of the paper.

Proposition 1. *When Condition A holds, there exists no game equilibrium with full transparency, where $s_1 = s_2 = m$. Instead, there exists an infinite number of outcome-equivalent asymmetric equilibria where one firm, say Firm 2, fully obfuscates with $s_2 \geq \overline{s}_2(\alpha_1^* = \alpha) = [1 + (\alpha/\beta)\ln\alpha]V > m$ and where its rival does not obfuscate, with $s_1 = m$. Relative to no obfuscation, these equilibria yield larger profits for both firms and lower surplus for both types of consumers.*

Proof. See Appendix. ■

Contrary to initial intuition, Proposition 1 demonstrates that competition may not be enough to ensure market transparency. Even when firms are presented with costless opportunities to improve consumers' information, the market equilibria involve imperfect information provision. Indeed, Proposition 1 suggests that one firm will always choose to fully obfuscate in order to soften competition by remaining hard to find[9].

To gain an understanding for this result, consider the simplest case where $m = 0$ such that Condition A holds for all $\alpha \in (0, 1)$. Here, if neither firm obfuscates, the costly-shoppers can identify both prices costlessly and Bertrand competition ensures that both firms earn zero profits. First, imagine an increase in Firm 2's search costs, given $s_1 = m$. This deters the willingness of the costly-shoppers to visit Firm 2 and results in two effects. Firm 1 receives an increased share of the costly-shoppers' first searches. In this special case when $m = 0$, any increase in s_2 makes it optimal for *all* of the costly-shoppers to first search Firm 1, (but more generally, increases in s_2 will make it optimal for an increasing share of costly-shoppers to choose Firm 1 rather than Firm 2). Further, Firm 1 is increasingly able to set higher prices without its costly-shoppers further searching Firm 2. This effect continues until $s_2 = \overline{s}_2(\alpha)$, where Firm 2 is said to fully obfuscate, and where Firm 1 is able to set $\overline{p}_1 = r_2^* = V$. At this point, Firm 1 has its weakest incentive

[8] This terminology is a little crude. Despite 'fully' obfuscating, by assumption, it is still true that the shoppers are willing to search the firm.

[9] Such an equilibrium asymmetry is related to the results in Ireland (1993) and Zettelmeyer (2000). This asymmetry is not present in a symmetric equilibrium, where, in the vein of Janssen and Non (2008), the firms randomise between $s_i = m$ and $s_i \geq \overline{s}_i(.)$. However, it is clear that the profits from such an equilibrium will be weakly dominated by the profits gained within the asymmetric pure strategy equilibria.

to compete and the subsequent pricing equilibrium allows Firm 2 to set its prices above zero, albeit probabilistically lower than Firm 1's, with $F_2(p) \geq F_1(p)$. Further increases in s_2 above $\overline{s}_2(\alpha)$ have no effect on equilibrium pricing and so for any $s_2 \geq \overline{s}_2(\alpha)$, the firms' profits equal $\overline{\pi}_1 = \alpha V > \overline{\pi}_2 = \beta\alpha V > 0$, such that Firm 2 strictly prefers to fully obfuscate.

Second, consider why the behaviour of the costly-shoppers is consistent. After obfuscation, the costly-shoppers realise Firm 2 has a lower expected price, but optimally prefer to first search Firm 1. In this special case when $m = 0$, this is trivial as it is costless to visit Firm 1, but more generally it is optimal as the high costs of searching Firm 2 ensure $r_1^* < r_2^*$. Costly-shoppers also prefer to first search Firm 1 rather than choosing not to search at all. Again this is trivial in this special case, but more generally it is true due to Condition A's restriction that s_1 is small enough relative to V, with $r_1^* \leq V$.

Third, consider why firm Firm 1 has no strict incentive to obfuscate in addition to Firm 2. As further explained in the proof, any increase in Firm 1's search costs either has no effect or reduces its profits. A large increase in s_1 such that $s_1 \geq s_2$ cannot be optimal as we know the firm with the lower search costs always earns higher profits. A moderate increase in s_1 acts to reduce demand and profits as constly-shoppers are deterred from making an initial search. A small increase in Firm 1's search costs such that all the costly-shoppers continue to search has no effect on its profits[10].

Finally, all consumers are indifferent over the set of full obfuscation equilibria with $s_2 \geq \overline{s}_2(\alpha)$ and $s_1 = m$, such that the set of equilibria are outcome-equivalent. This can be seen by noting that the the surplus of the costly-shoppers and shoppers under full obfuscation, expressed respectively as $CS_{CS} = V - m - E(p_1)$ and $CS_S = V - E(\min\{p_2, p_1\})$, and the equilibrium price distributions are all independent of s_2 for $s_2 \geq \overline{s}_2(\alpha)$. However, it is clear that all consumers are made strictly worse off in such equilibria relative to the no obfuscation case due to the the increase in prices.

An overall intuition for the result can also be gained by considering the role of the shoppers. The existence of the shoppers is potentially damaging for industry profits because it generates incentives for firms to undercut each other and engage in tougher

[10] This leads to the possibility of other equilibria where Firm 2 fully obfuscates and where Firm 1 obfuscates by some small amount. Such equilibria would be outcome-equivalent to those described in Proposition 1 in terms of prices and profits and would only involve lower levels of costly-shopper welfare.

price competition[11]. Here however, a firm's commitment to individual obfuscation can minimise this damage by (partially) sorting the consumer types across the two firms, such that its rival has a reduced incentive to compete. This logic is very similar to the mechanism that underlies standard results in vertical product differentiation (Shaked and Sutton 1982). In parallel, if consumers vary in their taste for high quality (or low search costs), a duopolist may be able to profit from choosing a lower level of quality (or higher level of search costs) than that of its rival.

Finally, the role of Condition A is worth underlining. Condition A maintains full participation by ensuring that the share of costly-shoppers is relatively low and that the level of natural search costs is low relative to V. Proposition 1 then implies that the incentives to fully obfuscate will be likely to exist in markets with lower levels of natural search costs, as consistent with the idea that firms had an incentive to restore market frictions following the introduction of internet commerce (Ellison and Ellison 2005). It remains difficult to fully characterise the game equilibria when Condition A is not satisfied. In such cases, equilibria may not always be consistent with full participation and the analysis is limited by the intractability problems described above. However, it seems intuitive that the incentives to obfuscate may still exist for at least some region of the remaining parameter space, albeit at a level possibly lower than full obfuscation.

5. Extensions

This section first offers a discussion of some of the possible obstacles that a firm may face in trying to obfuscate. It then provides an extension of the model to the case of n firms.

5.1. Barriers to Obfuscation. A first barrier to obfuscation may be due to practical reasons. A firm may simply be unable, or find it too costly, to increase its search cost to the extent required for full obfuscation, $\overline{s}_2(\alpha)$. If we assume that there is some ceiling to search costs, $\widehat{s}_2 \in (m, \overline{s}_2(\alpha))$, then with the logic of Proposition 1 it is easy to show that Firm 2 will elect to choose $\widehat{s}_2$. Prices and profits will strictly increase but by an amount less than under full obfuscation.

[11] Ellison's (2005) analysis of obfuscation in markets with add-ons demonstrates that firms can use the existence of shoppers (or low types in his model) to actually increase profits by creating an adverse selection effect that deters price cuts.

Second, in some circumstances it is possible that an obfuscating firm may face a tension between making itself harder to find and ensuring that consumers still know of its existence. While, within the obfuscation equilibria, the costly-shoppers need not know of the existence of the obfuscating firm (because even if they did they would not consider visiting it) the shoppers' awareness is vital. Such a tension may be a rationale for the use of uninformative advertising where a firm simply states its brand name but leaves the consumer uninformed about its location or price.

Third, for obfuscation to be profitable, shoppers must not only be aware of the obfuscating firm, but they must also be willing to spend the increased amount of time required to search the firm. Within the current assumptions, this is trivial as the shoppers have a zero cost of time, but more generally, the shoppers may be unwilling to buy from a firm that obfuscates too heavily. However, a simple example illustrates that the incentives to obfuscate are likely to remain as long as some non-zero proportion of shoppers are still willing to incur the increased search cost. If one redefines the proportion of shoppers and costly-shoppers as $\beta(s)$ and $1-\beta(s)$ respectively, where $\beta(s)=\beta$ if $s_1=s_2=m$ but $\beta(s)=b\beta$ if $s_2>m$, with $b\in(0,1]$, then a version of Condition (A) can be rewritten, $(m/V)<f(b,\beta)$, such that a region of parameters where full obfuscation exists can still be identified, $f(b,\beta)>0\ \forall b>0$, but where this region becomes smaller as the number of shoppers willing to search declines, $f_b'(b,\beta)<0$. In the extreme case where this example can not apply and where all consumers have a positive cost of time, then, in line with the Diamond paradox (1971), the firms are unable to commit to anything other than the monopoly price, and for any $m>0$ the market breaks down.

Viewing the game in a broader context, let us now consider the possible obstacles that Firm 2 may face in trying to obfuscate that derive from the incentives for other players to counter its strategy by perhaps, helping to inform the costly-shoppers of its location and price. Consider Firm 1's incentives. In Gabaix and Laibson (2006) the rival firm has no incentive to inform the myopic consumers of the obfuscating firm's concealed add-on price because, even when informed, the consumers would still prefer to buy from the obfuscating firm in order to obtain a lower base price, while substituting away from the add-on. Here, Firm 1 also has no incentive to inform the costly-shoppers, but this is for a different reason. Obfuscation by Firm 2 softens competition and strictly increases Firm 1 profits.

Now consider the shoppers' incentives. In the Gabaix and Laibson model, the non-myopic consumers have no incentive to inform the myopes, as the profits from the myopic consumers are used to subsidise the obfuscating firms' base prices in a way that increases the non-myopes' welfare. Here, there is no such incentive and the shoppers would indeed like to inform the costly-shoppers of Firm 2's location and price in order to strengthen competition and lower both firms' prices. However, there are several reasons to expect that such communication may be limited. Listening or waiting to observe other consumers' actions may also be time consuming activities, consumers' social networks may be segmented by type, and indeed, the very possibility of communication may deter any search activities due to a free-rider problem (as explored by Galeoitti 2006).

Finally, there may be incentives both here and in Gabaix and Laibson's model for third party to sell information to the costly-shoppers. Such incentives relate to the existence of gatekeepers, as studied by Baye and Morgan (2001). Further studying the equilibrium effects of obfuscation in the presence of such intermediaries is left for future research, but we note that any such agent has to avoid the potential paradox of being unable to sell any information if by doing so, it makes the market perfectly competitive (as argued in Ellison and Ellison (2005)).

5.2. Obfuscation Equilibria with More Than Two Firms. We now consider a brief extension in order to show that the incentives for individual level obfuscation can still occur in markets with a larger number of firms. Rather than taking the more comprehensive approach of the previous sections, we simply aim to characterise a set of parameters where obfuscation must form part of the market equilibria. Specifically, we consider the set of parameters where a firm can profitability deviate from the no-obfuscation outcome while still ensuring the full participation of the costly-shoppers.

Proposition 2. *The region of parameters where any equilibrium must involve obfuscation is non-empty for any finite number of firms.*

Proof. See Appendix. ■

With a similar logic to the duopoly case, the appendix demonstrates that full obfuscation can be profitable because it can induce a firm's rivals to increase their prices and reduce their willingness to compete for the shoppers. Unlike the duopoly case however, where obfuscation was profitable for all values of α when the natural level of search costs was zero it is clear that profitable obfuscation now requires $m > 0$. When $m = 0$, any individual increase in search costs is unable to raise prices as the remaining firms are still engaged in Bertrand competition. A similar argument also rules out the profitability of obfuscation when $n = \infty$. More generally however, the profitability of obfuscation is constrained by the willingness of the costly-shoppers to still participate in the market. The appendix shows that full participation can always be guaranteed if the proportion of shoppers is sufficient to constrain post-obfuscation prices.

It is difficult to show how the conditions for profitable obfuscation (under full participation) vary with an increase in the number of competitors analytically, but simulations suggest that the region of parameters where the conditions are satisfied may shrink as the number of competitors increases (See Figure 4 in the Appendix). Intuitively, this pattern derives from a well known feature of the Stahl (1989) model that shows that an increase in the number of competitors actually increases equilibrium prices because a price cut becomes less likely to win the custom of the shoppers. Thus, when n is larger, obfuscation is more likely to raise prices beyond the point where the full participation of the costly-shoppers is optimal. The difficulty of analysing outcomes outside Condition A limits our ability to make a general statement about equilibrium obfuscation, but the simulations provide a tentative suggestion that increases in the number of competitors might reduce the level of equilibrium search costs.

6. A Behavioural Interpretation?

As mentioned in the introduction, there is a temptation to reinterpret the presented model as one which consumers differ in their cognitive ability rather than their costs of time, and where firms may choose to make their price information complex, rather than obscure. Such an interpretation is briefly discussed within Ellison and Ellison (2005) and Ellison (2006), and can also be traced back as a motivation for the pioneering search model of Salop and Stiglitz (1977). At least within the current model, and maybe more generally, there exists several problems that hinder such an interpretation. First, search models assume search costs are a necessary pre-purchase expenditure. While this seems reasonable for most (but not all) cases when search costs refer to the costs of information gathering, as consumers must, for example, find the restaurant or navigate the website in order to buy the product, it appears less reasonable when search costs refer to cognitive calculations. Indeed, it might be rational for a consumer to buy a product without making such calculations if the price was sufficiently complex, in which case the predicted outcomes of search models would no longer apply. Second, in reference to the current model, it seems more reasonable that firms can pre-commit to a level of 'information' search costs rather than 'cognitive' search costs. In most (but not all) cases, the cost of reversing a level of 'information' search costs is significant, as in the case of advertising or location choice, but the cost of modifying the way in which a price is presented is likely to be trivial. In a game where prices and search costs are chosen simultaneously, it is clear that Proposition 1 no longer holds and instead, an equilibrium with zero obfuscation must exist. Finally, the 'cognitive' search cost interpretation appears to generate a methodological incompatibility. One must assume that a consumer must expend some costly resources to merely understand a price, while maintaining that the consumer can infer all other players' strategies and calculate an optimal strategy based on a set of reservation prices with potentially non-analytical solutions at zero cost[12]. While the topic of spurious market complexity is of huge importance, it would appear that more radical revisions to standard search models are needed for its study. Spiegler's (2006) study of how firms can profitably obfuscate by randomising their offerings across multiple product dimensions when consumers make naïve comparisons is an exciting step in this direction.

7. Conclusion

This paper has analysed the incentives for firms to obfuscate within a duopoly framework where consumers can choose not only the number of firms to search, but also the order in which to search. For many parameter values, the model predicts that one firm will have a profitable incentive to enhance its own costs of being searched. By doing so, a firm can induce consumers with high costs of time to search its now 'prominent' rival and thereby soften the competition for the remaining consumers.

The paper's findings suggest a government intervention to provide better market information would improve competition and enhance consumer welfare. However, it remains unclear to what extent such an intervention would be necessary. Indeed, while it has been

[12] That is, the calculation of whether to understand a price is more complex than understanding the price itself. Such infinite regress problems are discussed in Conlisk (1996).

shown that both an obfuscating firm and its rival benefit from obfuscation, questions remain over the incentives for other market players to counter such strategies by improving consumers' information. To this end, further research into the effects of intermediaries and information transmission within consumer networks would appear fruitful.

Finally, while the model is quite general in the sense that it leaves the exact mechanism by which firms can obfuscate unspecified, the model can not be applied to a context where firms increase search costs by reducing product availability. Daughety and Reinganum (1991) analyse this possibility and show the existence of a symmetric duopoly equilibrium where firms either increase or reduce availability relative to the monopoly level. Further work to investigate obfuscation of this form, and/or in the context where firms sell multiple products would also be useful.

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8. Appendix:

The proofs make use of the following definition. $\overline{s}_2(\alpha_1^*) = [1 + (\alpha_1^*/\beta)\ln(\alpha_1^*/(\alpha_1^* + \beta))]V$ defines the value of s_2 that produces an equilibrium value of $r_2^* = V$, for a given s_1, such that the costly-shoppers face no strict incentive to search Firm 2.

Lemma 3. *Given a level of search costs such that* $m \leq s_1 \leq s_2$ and $s_1 \leq ((1-\alpha) + \alpha \ln \alpha)V$, *the unique Stage 2 equilibria can be described by Lemma 2, with initial search* *decisions* $\{\alpha_1, \alpha_2\}$ such that $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$, where $\alpha_1^* = \max\{\frac{(s_2-\beta s_1)}{(s_1+s_2)}, \alpha\}$ *and* $\alpha_2^* = \min\{0, \frac{(s_1-\beta s_2)}{(s_1+s_2)}\}$.

Proof: See Claims 1-3.

Claim 1. *If* $s_1 = s_2 \leq ((1-\alpha)+\alpha\ln\alpha)V$, *the unique Stage 2 equilibrium has* $\alpha_1^* = \alpha_2^* = (\alpha/2)$.

Proof. For existence, note that when $\alpha_1^* = \alpha_2^* = (\alpha/2)$, the costly-shoppers are willing to search but are indifferent between the two firms as $r_i^* = ((2\beta s_i)/(2\beta + \alpha\ln\left(\frac{\alpha}{1+\beta}\right))$ $\forall i = 1, 2$, and $r_i^* \leq V$ if $s_i \leq k = (1 + (\alpha/2\beta)\ln\alpha)V$, with $k > ((1-\alpha)+\alpha\ln\alpha)V$ $\forall\alpha \in (0, 1)$. For uniqueness, note that any other $\{\alpha_1, \alpha_2\}$ fails to produce a pricing equilibrium consistent with the behaviour of the costly-shoppers. If $\alpha_i > \alpha_j$, we know that this would imply $r_i^* > r_j^*$, as $F_j(p) \geq F_i(p)$ $\forall p \in [\underline{p_i}, \overline{p_i}]$, such that the α_i costly-shoppers would wish to search Firm j instead. If $\alpha_1 = \alpha_2 < (\alpha/2)$, then we know $r^* \leq V$, such that all the costly-shoppers would wish to search. Finally, one can check that the conditions for the validity of the pricing equilibrium in Lemma 2 are met with equality. ■

Claim 2. *When* $s_2 \in (s_1, (s_1/\beta)]$ *and* $s_1 \leq ((1-\alpha)+\alpha\ln\alpha)V$, *the unique Stage 2 equilibrium has* $\alpha_1^* = (s_2 - \beta s_1)/(s_1 + s_2)$ *and* $\alpha_2^* = \alpha - \alpha_1^* = (s_1 - \beta s_2)/(s_1 + s_2)$.

Proof. We know $\alpha_1 = \alpha_2 = (\alpha/2)$ cannot be consistent with $s_2 > s_1$ in equilibrium as this must imply $r_2^* > r_1^*$ such that the α_2 consumers would wish to search Firm 1 instead. Neither can $\alpha_1 = \alpha$ be consistent with equilibrium because (as we later show in Claim 3) for $r_1^* < r_2^*$, it is required that $s_2 > (s_1/\beta)$. Instead, the only behaviour of the costly-shoppers that can be consistent with equilibrium must involve them dividing between the two firms in unequal proportions, such that i) $r_1^* = r_2^*$ and ii) $r_2^* \leq V$. To find such proportions[13], note that $F_1(p) = F_2(p) + (\frac{(\alpha_2-\alpha_1)}{\beta}) + \left(\frac{(\alpha_1-\alpha_2)\underline{p_1}}{\beta p}\right)$ and so together with $s_2 = \int_{\underline{p_1}}^{r_2} F_2(p)dp$, one can show that $s_2 + \int_{\underline{p_1}}^{r_2}(\frac{(\alpha_2-\alpha_1)}{\beta}) + \left(\frac{(\alpha_1-\alpha_2)\underline{p_1}}{\beta p}\right) dp = \int_{\underline{p_1}}^{r_2} F_1(p)dp$. The RHS can then be decomposed into $\int_{r_1}^{r_2} F_1(p)dp + \int_{\underline{p_1}}^{r_1} F_1(p)dp = \int_{r_1}^{r_2} F_1(p)dp + s_1$ and so with some further simplification one can obtain $\int_{r_1}^{r_2} F_1(p)dp = s_2\left(\frac{\alpha_2+\beta}{\alpha 1+\beta}\right) - s_1$. Under the assumption of condition ii) to imply $\alpha_2^* + \alpha_1^* = \alpha$, condition i) will then be met if $s_2\left(\frac{\alpha_2+\beta}{\alpha 1+\beta}\right) = s_1$ which implies unique values of $\{\alpha_1, \alpha_2\}$, $\alpha_1^* = (s_2 - \beta s_1)/(s_1 + s_2)$ and $\alpha_2^* = \alpha - \alpha_1^*$. To ensure condition ii) we need $r_2^* \leq V$ for all $s_2 \in (s_1, (s_1/\beta)]$. This will

[13] I thank John Vickers for suggesting this step.

be guaranteed if $\overline{s}_2(\alpha_1^*) \geq (s_1/\beta)$, as one can show $\delta r_2^*/\delta s_2 > 0$ $\forall s_2 \in (s_1, (s_1/\beta)]$, which gives $s_1 \leq ((1-\alpha) + \alpha \ln \alpha)V$. Finally, one can check that the conditions for the validity of the pricing equilibrium are met, as $r_1^* = r_2^*$ and $\alpha_1 > \alpha_2$. ■

Claim 3. *When $s_2 > (s_1/\beta)$ and $s_1 \leq ((1-\alpha)+\alpha \ln \alpha)V$, the unique Stage 2 equilibrium has $\alpha_1^* = \alpha$ and $\alpha_2^* = 0$.*

Proof. For the behaviour of the costly-shoppers to be consistent with the proposed equilibrium, it is required that i) $r_1^* < r_2^*$ and ii) $r_1^* \leq V$. From Claim 2, we know $\int_{r_1}^{r_2} F_1(p)dp = s_2\left(\frac{\alpha_2+\beta}{\alpha 1+\beta}\right) - s_1$ such that $s_2 > (s_1/\beta)$ is required for $\alpha_1^* = \alpha$ and $\alpha_2^* = 0$, and $r_1^* < r_2^*$. Condition ii) is automatically satisfied for $s_2 \in ((s_1/\beta), \overline{s}_2(\alpha)]$ as $r_1^* < r_2^* \leq V$. It is also satisfied for $s_2 > \overline{s}_2(\alpha)$ if $s_1 \leq \beta\overline{s}_2(\alpha_1^* = \alpha) = ((1-\alpha) + \alpha \ln \alpha)V$. To see why note that $r_1^*(s_2) = r_1^*(\overline{s}_2(\alpha))$ $\forall s_2 \geq \overline{s}_2(\alpha)$ as $\overline{p}_1 = V$, and $r_1^*(\overline{s}_2(\alpha)) < r_2^*(\overline{s}_2(\alpha)) = V$ iff $\overline{s}_2(\alpha) > (s_1/\beta)$. Finally, one can check that the conditions for the validity of the pricing equilibrium are met. ■

Lemma 4. *Given a level of search costs such that $m \leq s_1 \leq s_2$* and *$s_1 > ((1-\alpha) + \alpha \ln \alpha)V$, any Stage 2 equilibria must be described by Lemma 2, with initial search decisions* $\{\alpha_1, \alpha_2\}$ such that $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$, where $\alpha_1^* \leq \max\{\frac{(s_2-\beta s_1)}{(s_1+s_2)}, \alpha\}$ *and* $\alpha_2^* \leq \min\{0, \frac{(s_1-\beta s_2)}{(s_1+s_2)}\}$.

Proof: See Claims 4-6.

Claim 4. *If $s_1 = s_2 > k = (1 + (\alpha/2\beta)\ln(\alpha/1+\beta))V$, the unique Stage 2 equilibrium has $\alpha_1^* = \alpha_2^* < (\alpha/2)$.*

Proof. From Claim 1 we know that $\alpha_i^* = (\alpha/2)$ is inconsistent with equilibrium when $s_i > k$ as $r_i^*(\alpha_i = \alpha/2) > V$. Instead the costly-shoppers must mix between making a first search to each firm with some symmetric probability, $\alpha_i < (\alpha/2)$, and not participating in the market with probability, $(1 - 2\alpha_i)$, such that $r_i^*(\alpha_i) = V$. ■

Claim 5. *When $s_2 > (s_1/\beta)$ and $s_1 > ((1-\alpha) + \alpha \ln \alpha)V$, any Stage 2 equilibrium must involve $\alpha_1^* < \alpha$ and $\alpha_2^* = 0$.*

Proof. By Claim 3, we know that if $s_2 > (s_1/\beta)$ and $s_1 > ((1-\alpha)+\alpha \ln \alpha)V$, then $V < r_1^* < r_2^*$, such that the behaviour of the costly-shoppers is inconsistent with equilibrium.

Instead, any equilibrium must involve the costly-shoppers mixing between searching Firm 1 with some probability, $\alpha_1^* < \alpha$, and not participating, such that $r_1^*(\alpha_1) = V$. ■

Claim 6. *When* $s_2 \in (s_1, (s_1/\beta)]$ *and* $s_1 > ((1-\alpha) + \alpha \ln \alpha)V$, *any Stage 2 equilibrium must involve* $\alpha_1^* < \alpha$.

Proof. If $s_1 > ((1-\alpha) + \alpha \ln \alpha)V$ then $\overline{s}_2(\alpha_1^*) < (s_1/\beta)$ at some $\alpha_1^* < \alpha$. Therefore, when $s_2 \leq \overline{s}_2(\alpha_1^*)$ the equilibrium in Claim 2 with $\alpha_1^* < \alpha$ and $\alpha_2^* = \alpha - \alpha_1^*$ still exists, but when $s_2 > \overline{s}_2(\alpha_1^*)$ it cannot, as $r_i^*(\alpha_1^*, \alpha_2^*) > V$. This cannot be sustained and some reduced participation must be required in order to ensure $r_i^*(\alpha_1^*, \alpha_2^*) = V$. ■

Proposition 1. When Condition A holds, there exists no game equilibrium with full transparency, where $s_1 = s_2 = m$. Instead, there exists an infinite number of outcome-equivalent asymmetric equilibria where one firm, say Firm 2, fully obfuscates with $s_2 \geq \overline{s}_2(\alpha_1^* = \alpha) = [1 + (\alpha/\beta)\ln \alpha]V > m$ and where its rival does not obfuscate, with $s_1 = m$. Relative to no obfuscation, these equilibria yield larger profits for both firms and lower surplus for both types of consumers.

Proof. Condition A implies $m = s_1 < ((1-\alpha) + \alpha \ln \alpha)V$ such that the first search behaviour of the costly-shoppers can be characterised by Lemma 3 rather than Lemma 4, with full participation guaranted, $\alpha_1^* + \alpha_2^* = \alpha \ \forall s_2$. Condition A also ensures that $(m/\beta) < \overline{s}_2(\alpha_1^*)$, such that the full obfuscation level of search costs can be defined as $\overline{s}_2(\alpha_1^* = \alpha) = [1 + (\alpha/\beta)\ln \alpha]V$. Consequently, Firm 2's profits, $\overline{\pi}_2 = (\alpha_2 + \beta)\underline{p_1}$ can be expanded to $\overline{\pi}_2 = (\alpha_2^* + \beta)(\frac{\alpha_1^*}{\alpha_1^*+\beta}) \min\left(\frac{\beta s_2}{\beta + \alpha_1^* \ln(\alpha_1^*/(\alpha_1^*+\beta))}, V\right)$ and displayed as the following by substituting for α_1^* and α_2^* from Lemma 3.

$$\overline{\pi}_2(s_1 = m) = \begin{cases} \frac{\alpha\beta m}{2\beta + \alpha \ln\left(\frac{\alpha}{1+\beta}\right)} & \text{if } s_2 = m \\ \left(\frac{\beta s_1(s_2 - \beta s_1)}{\beta(s_1+s_2) + (s_2 - \beta s_1)\ln\left(\frac{s_2 - \beta s_1}{s_2(1+\beta)}\right)}\right) & \text{if } s_2 \in (m, (m/\beta)] \\ \left(\frac{\beta^2 \alpha s_2}{\beta + \alpha \ln \alpha}\right) & \text{if } s_2 \in ((m/\beta), \overline{s}_2(\alpha)) \\ \beta\alpha V & \text{if } s_2 \geq \overline{s}_2(\alpha) \end{cases}$$

To show that there exists a continuum of equilibria with $s_1 = m$ and $s_2 \geq \overline{s}_2(\alpha)$ note first that by holding $s_1 = m$, $\overline{\pi}_2$ is strictly increasing in $s_2 \in (m, \overline{s}_2(\alpha)]$. Further, Firm 2 will prefer to fully obfuscate by setting any $s_2 \geq \overline{s}_2(\alpha)$ rather than setting $s_2 = m$ as it can be shown that $\overline{\pi}_2(s_2 \geq \overline{s}_2(\alpha)) > \overline{\pi}_2(s_2 = m)$. Given $s_2 \geq \overline{s}_2(\alpha)$, we know that $s_1 = m$ is a best response for Firm 1 as it is weakly dominant to set $s_1 \in [m, ((1-\alpha)+\alpha\ln\alpha)V)$ in order to earn $\overline{\pi}_1 = \alpha_1^* \overline{p}_1 = \alpha V$. This follows from Lemma 4 which suggests that full participation cannot be sustained for $s_1 > (((1-\alpha)+\alpha\ln\alpha)V$, which then implies i) $\overline{\pi}_1(s_1 = \overline{s}_2(\alpha)) \leq \alpha V/2$, ii) $\overline{\pi}_1(s_1 > \overline{s}_2(\alpha)) \leq \overline{\pi}_1 = \beta\alpha V$ and iii) $\overline{\pi}_1(s_1 \in (((1-\alpha)+\alpha\ln\alpha)V, s_2)) \leq \alpha V$. Across the continuum, the equilibria are outcome-equivalent because we first know from Lemmas 2 and 3 that the equilibrium price distributions are independent of s_2, and second, the payoffs for the firms, the costly-shoppers and the shoppers, denoted respectively as $\overline{\pi}_1 = \alpha V$, $\overline{\pi}_2 = \beta\alpha V$, $CS_{CS} = V - m - E(p_1)$ and $CS_S = V - E(\min\{p_2, p_1\})$ are also independent of s_2. Finally, it follows that both firms earn profits that are strictly larger than those obtainable under a no obfuscation outcome, as $\overline{\pi}_2 = \beta\alpha V > \overline{\pi}_i(s_2 = s_1 = m)$, and both consumer types earn lower expected surplus. This last point follows as one can still use the same expressions for consumer surplus for the no obfuscation outcome and then show that the symmetric no obfuscation price distribution is stochastically dominated by the price distributions of both firms in the obfuscation equilibria, $F(p) \geq F_2(p) \geq F_1(p)$. ■

Proposition 2. *The region of parameters where any equilibrium must involve obfuscation is non-empty for any finite number of firms.*

Proof. First we show that there can exist no game equilibrium with $s_i = m\ \forall i = \{1, ...n\}$ when Conditions B and C hold. Define Condition B as $1 - \int_0^1 \frac{dy}{1+((1-\alpha)/\alpha)ny^{n-1}} - \left(\frac{(n-1)-\alpha(n-2)}{n(1-\alpha)}\right)[1 + (\frac{1}{n-2})(\psi - (n-1)\psi^{1/(n-1)}] > 0$ and Condition C as $0 < (m/V) \leq [1 + (\frac{1}{n-2})(\psi - (n-1)\psi^{1/(n-1)})]$ where $\psi = \alpha/((n-1)-\alpha(n-2))$. In the symmetric case, when $s_i = m\ \forall i = \{1, ...n\}$, Janssen et al (2005) establish that individual firm profits are no larger than $\overline{\pi}_i = (\alpha/n)(s/(1 - \int_0^1 \frac{dy}{1+((1-\alpha)/\alpha)ny^{n-1}}))$. Now consider a deviation by firm n such that $s_n > V$ and $s_a = m$ for $a = \{1, 2, ...n-1\}$. If, in equilibrium, i) $r_a^* \leq V$ and ii) $r_a^* < r_n^*$, then $\alpha_a^* = \alpha/(n-1)$ and $\alpha_n^* = 0$. It then follows from the logic of Lemma 2 that $\overline{\pi}_a = (\alpha/(n-1))\overline{p}_a$, with $\overline{p}_a = \min\{r_n^*, r_a^*, V\} = r_a^*$, and $\overline{\pi}_n = \beta\underline{p_a}$, where $\underline{p_a} = \psi r_a^*$. One can then note that $F_a(p) = 1 - (\underline{p_a}/p)^{1/(n-1)}$ and by using $\int_{\underline{p_a}}^{r_a} F_a(p)dp = m$, $r_a^* = (m/[1 + (\frac{1}{n-2})(\psi - (n-1)\psi^{1/(n-1)})]$. The Conditions then ensure that the deviation is profitable (Condition B together with $0 < (m/V)$) and that the assumption of $r_a^* \leq V$ still holds (the latter part of Condition C). It follows trivially that $r_a^* < r_n^*$. The Proposition then follows as Conditions B and C must be satisfied for any finite number of firms if α is sufficiently small - in the limit as $\alpha \to 0$, the conditions require $((n-1)/n) < 1$ and $0 < (m/V) \leq 1$, respectively. $\blacksquare$

Figure 1: Example equilibrium initial searches to Firm 1 (top), α_1^*, and Firm 2, α_2^*.

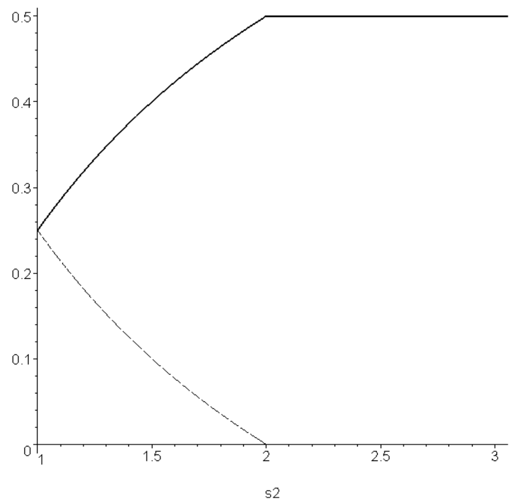

Figure 2: Equilibrium Price Distributions for Firm 1 and Firm 2 (dashed) with $s_2 > (s_1/\beta)$

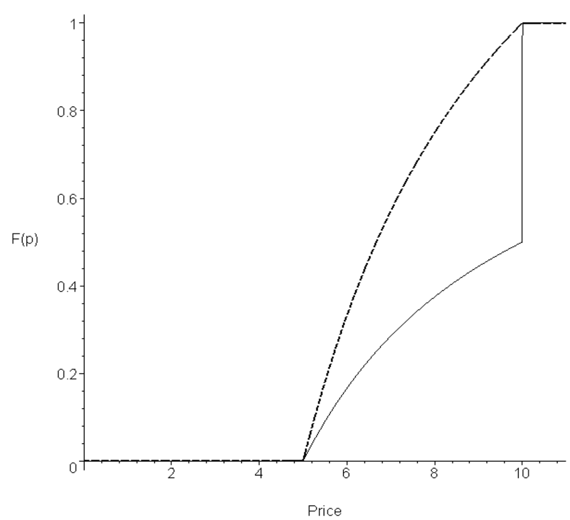

Figure 3: Condition A

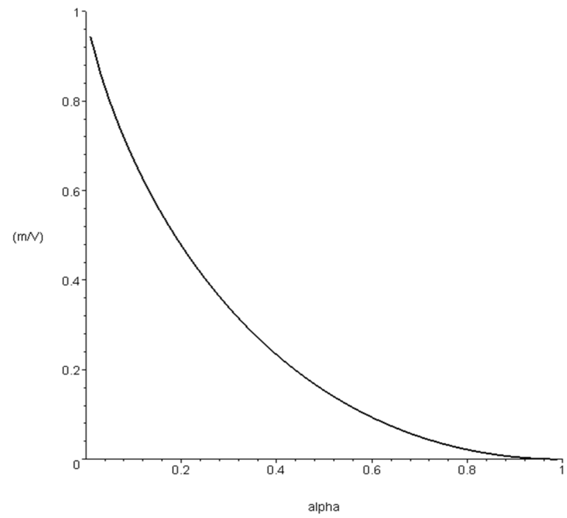

Figure 4: Simulations of Conditions B and C for $n = 3$ (top), $10, 20, 50$ and 200

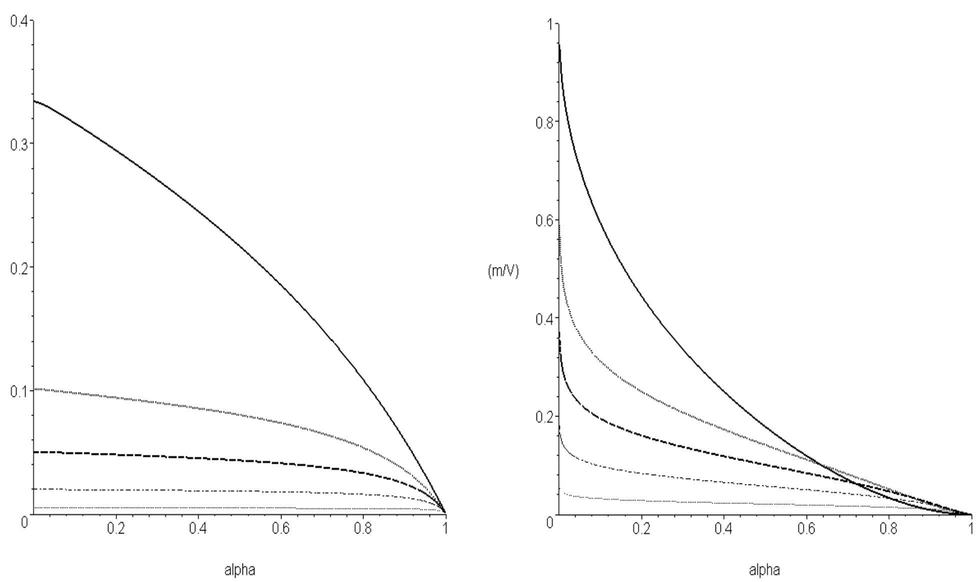

Figure 5: