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Future Changes of the Industrial Structure due to Aging and Soaring Demands for Healthcare Services in Japan
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Abstract

In order to quantify the effects of declining birthrate and changing demographic structure on the Japanese economy, we show the results of simulations by using a multi-sector dynamic general equilibrium model with overlapping generations (OLG) in an open economy. The model is constructed to incorporate substitutability between domestic products and imports and show the evolution of the industrial structure, reflecting the impacts of aging population from both supply and demand sides of the economy. Based on the scenario of increasing public demands for healthcare services, the share of healthcare sector expands to almost 2.5 times in 2050 relative to the base year 1985. The result of a simulation based on an alternative scenario where the government increases net transfer to the elderly shows smaller labor participation and GDP per capita, due to the income effects and crowding out of private capital by the increase of government debt outstanding in the long run.

JEL classification numbers: J11, H51, H68
Keywords: multi-sector OLG model, demographic change, soaring public healthcare spending

(Remark)
The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Ministry of Finance, Japan. All remaining errors are ours.

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1. Introduction

In Japan, a declining trend of population and increasing share of the elderly are apparent under the dwindling birthrate and extended longevity. While the longevity in Japan has been gradually increasing after World War II, the downward shift of the birthrate became obvious in the late 1980's. Falling birthrate may have large impacts on the future demographic structure and inter-temporal resource allocation through the expected evolution of future prices of production factor and goods and services. The impacts of such a large shock on Japan's whole economy should be carefully scrutinized under the framework of dynamic general equilibrium.

The change of demographic structure affects both supply and demand sides of the economy. On the supply side, lower birthrate will shrink labor supply and may reduce domestic production of labor intensive goods. On the demand side, there can be soaring demands for healthcare services.[1] There have been many analyses which use dynamic general equilibrium model with overlapping generations (OLG model) to analyze the impacts of Japan's demographic change.[2] However, these studies are based on one-sector, closed economy models focusing on the changes of capital-labor ratio and evolution of factor prices. In reality, such demographic change must have affected the relative prices of different goods and services subject to intensity of labor, and thus changed each agent's intra-temporal choices between labor and leisure, and consumption smoothing behaviors by households. In an open economy, declining birthrate also changes the current account dynamics, due to the change of household's inter-temporal resource allocations and the evolution of factor prices. They must have enormous impacts on the industrial structure. Fougere *et. al.* (2007) and Rausch (2009) conducted simulations to quantify these effects in Canada and Germany by utilizing a multi-sector OLG model in an open economy.[3]

In this paper, we construct an open economy, multi-sector dynamic general equilibrium model with overlapping generations (OLG) in order to simulate the impact of Japan's recent declining birthrate on Japan's whole economy. Main characteristics of our model are as follows. (1) We construct a 4-sector model (agriculture and fishery goods, other tradable goods, healthcare services and other non-tradable goods and services) and calibrate parameters based on Japan's social accounting matrix (SAM) based on 1985 Input-Output table. For inter-temporal parameters, such as a rate of time preference and a periodic time endowment, we employ the steady-state calibration procedure proposed by Rasmussen and Rutherford (2004). (2) In our model, labor participation is endogenously determined as in Rausch (2009), by

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1 In this paper, healthcare service means both medical care service and nursing care service.

2 Kawade (2003), Sato *et.al*. (2004), Ihori *et.al*. (2005), and Ihori and Bessho (2008) are the examples and others are introduced in Kawasaki and Shimasawa (2003).

3 The former uses the Canadian IO data in 1999 and the latter the German IO data in 2003.

reflecting Japan's wage profile and demographic change. (3) We incorporate an open economy setting and substitutability between domestic and imported goods. Under this setting, real exchange rate is endogenously determined to fulfill the inter-temporal resource allocation, i.e., so that the foreign net asset (or borrowing) will converge to a steady state. (4) We consider government actions as exogenous parameters, and analyze the effects of policy changes, such as increasing healthcare spending and transfers to elderly households, on economic variables. Other specification of the model follows the standard setting. Representative household in each generation lives over a deterministic lifespan, and it is assumed to be that there is no liquidity constraints and adjustment cost for investment. Markets are perfectly competitive and prices are fully elastic. Using this model, we run numerical simulations to quantify the macroeconomic effects of a birthrate shock in Japan. In particular, we focus on the transitions of key macroeconomic variables, industrial sector shares and government sector variables under two scenarios.

Our results are summarized as follows. First, under the "*baseline scenario*", where the share of healthcare sector expands to almost 2.5 times in 2050 relative to the base year, while the capital-labor ratio is projected to rise as is usual in standard OLG models, the user cost of capital is also projected to rise, due to crowding out of domestic capital by accumulation of government debt and net foreign asset. Second, under the "*increased government net transfers scenario*", where the government increases net transfer to households, labor supply (in efficiency units) and value-added per capita are smaller than the one under the baseline scenario due to income effects and crowding out of private capital by an exogenous increase of government debt outstanding.

The rest of the paper is organized as follows. In the next section, we explain the analytical framework and specification of our model. In section 3, we explain the relevant data and our calibration procedure. In section 4, we present the different policy scenarios and their simulation results. Section 5 concludes.

2. The model[4]

Since the focus of our analysis is the long-run effects of demographic impacts, the setting of the model assumes competitive markets and no restriction on mobility of labor and capital.

2.1 Households

(Figure 1 here)

[4] Our model owes a lot to a pioneering work by Rausch (2009).

A household of generation g enters labor force at year t=g, lives for a deterministic lifespan of N+1 years (after t=g), and is endowed with $\omega_{g,t}$ unit of effective time at each period of $g \le t \le g+N$. $\omega_{g,t}$ can be interpreted as the effective size of household of generation g. We assume that the time endowment within generation g does not grow over the life cycle: $\omega_{g,t}$ = $\omega_{g,t'}$ for all t and t'. Their childhood (before t=g) are fully dependent on their parents to which they neither provide utility nor impose any spending. [5]

Generation g maximizes its lifetime utility (u_g) subject to lifetime budget and a number of constraints. For simplicity, we rule out bequests and Ponzi-game situations, i.e., households do not die in debt.

$$\underset{\{c_{g,t},\,\ell_{g,t}\}}{Max}:\; u_g\left(z_{g,t}\right)=\left[\sum_{t=g}^{g+N}\theta_{g,t}\left(z_{g,t}\right)^{-\frac{1-\sigma}{\sigma}}\right]^{-\frac{\sigma}{1-\sigma}} \tag{1}$$

s.t.

$$z_{g,t}=\left[\alpha\left(c_{g,t}\right)^{-\frac{1-\nu}{\nu}}+\left(1-\alpha\right)\left(\ell_{g,t}\right)^{-\frac{1-\nu}{\nu}}\right]^{-\frac{\nu}{1-\nu}} \tag{2}$$

$$c_{g,t}=\left[\theta^{F}\left(a_{1,g,t}\right)^{-\frac{1-\kappa}{\kappa}}+\theta^{NF}\left(c_{g,t}^{NF}\right)^{-\frac{1-\kappa}{\kappa}}\right]^{-\frac{\kappa}{1-\kappa}} \tag{3}$$

$$c_{g,t}^{NF}=\left[\sum_{i\in NF}\varphi_{i,t}\left(a_{i,g,t}\right)^{-\frac{1-\sigma_{NF}}{\sigma_{NF}}}\right]^{-\frac{\sigma_{NF}}{1-\sigma_{NF}}} \tag{4}$$

$$\sum_{t=g}^{g+N}p_{c,g,t}\,c_{g,t}\le\sum_{t=g}^{g+N}\left[p_t^{w}\,\pi_{g,t}\left(\omega_{g,t}-\ell_{g,t}\right)\left(1-\tau_t^{w}\right)+p_t^{n}\zeta_{g,t}\right] \quad (5),\qquad \ell_{g,t}\le\omega_{g,t} \quad (6)$$

u_g is the lifetime utility of generation g,[6] and $z_{g,t}$ felicity aggregate of generation g at year t enters according to constant inter-temporal elasticity of substitution (CIES) function with $\theta_{g,t}$: share parameter which accounts for discounting and σ : inter-temporal elasticity of substitution. $z_{g,t}$ consists of aggregate consumption of goods and services ($c_{g,t}$) and leisure consumption ($\ell_{g,t}$), with α :value share of aggregate consumption of goods and services and ν : elasticity of substitution between aggregate consumption and leisure. $c_{g,t}$ consists of food consumption (agricultural and fishery goods) ($a_{1,g,t}$) and non-food consumption aggregate ($c^{NF}{}_{g,t}$), with θ^F: value share of food consumption, θ^{NF}: value share of non-food consumption and κ : elasticity of substitution between food and non-food consumption. $c^{NF}{}_{g,t}$ is composed of Armington

[5] An initial old generation prior to year t=0 is indexed by g=-N,$\cdots$,-1.
[6] Lifetime utility of each generation is normalized to 1 in the baseline.

products other than agricultural and fishery goods ($a_{i,g,t}$ ($i \in NF$)), with $\varphi_{i,t}$: value share of consumption of good i, and σ^{NF}: elasticity of substitution between non-food consumption. $p_{c,g,t}$ is the price of aggregate consumption of each generation g.[7] $p^w{}_t$ is the wage rate, and $\pi_{g,t}$ is an age-dependent inverted U-shaped labor efficiency profile over life cycle, while $\omega_{g,t}$ is the effective time endowment in each period. $\tau^w{}_t$ is the tax rate on labor incomes, and $\zeta_{g,t}$ the lump-sum government net transfers, which is denominated in the present value of government liability ($p^n{}_t$).[8] In this model, a household accumulates financial assets in such a manner that it purchases a single mutual fund comprised of the real capital stock, the government bond, and the net foreign assets. The mutual fund yields a constant baseline interest rate $\bar{r}$ (before tax).[9][10][11]

2.1.2 The dual cost functions

We can derive the dual cost functions of household as follows:

$$p_{u,g} = \left[\sum_{t=g}^{g+N} \theta_{g,t} \left(p_{z,g,t} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad p_{z,g,t} = \left[\alpha \left(p_{c,g,t} \right)^{1-\nu} + (1-\alpha) \left(p_{\ell,g,t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}},$$

$$p_{c,g,t} = \left[\theta^F \left(p_{A,1,t} \right)^{1-\kappa_h} + \theta^{NF} \left(p^{NF}_{c,g,t} \right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \qquad p^{NF}_{c,g,t} = \left[\sum_{i \in NF} \varphi_{i,t} \left(p_{A,i,t} \right)^{1-\sigma_{NF}} \right]^{\frac{1}{1-\sigma_{NF}}}$$

where $p_{u,g}$ is the price of lifetime utility, $p_{z,g,t}$ price of felicity, $p_{l,g,t}$ price of leisure (reservation wage), $p_{A,i,t}$ price of Armington good i, and $p^{NF}{}_{c,g,t}$ price of non-food consumption basket.

2.1.3 The demand functions

Making the associated indirect utility function and applying the Roy's identity to it, we can derive the demand functions of household as follows.

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[7] The price of consumption aggregate $p_{c,g,t}$ can be generation dependent in a setting where we allow the consumption baskets to vary across generations.
[8] Regarding the numeraire of the model, Rausch (2009) use the price of foreign goods ($p^f{}_t$), but we introduce the price of government liability ($p^n{}_t$) in order to allow the price of foreign goods can change to adjust the future trade balances by the shock. The price of government liability is displayed in present value, as well as all other prices.
[9] Endowment of initial assets are only given to the generations prior to year t=0.
[10] Since all the prices are shown in present value, they act as a discounting operator and prices decline with the baseline after-tax interest rate ($\overline{rn}$) in steady-state equilibrium.
[11] When we conduct an actual simulation, the CES type (Leontief or Cobb-Douglas) utility, production and cost (pricing) functions and their associated optimal equations are normalized with respect to their baseline values to gain computational efficiency. This normalization, which is called "calibrated share form" in Rausch (2009), has another advantage that weighting parameters of the CES functions in calibrated share form come down to value shares which we can easily calibrate. The explanation is detailed in Appendix 1.

$$z_{g,t} = u_g \left(\frac{p_{u,g}}{p_{z,g,t}} \right)^{\sigma} \qquad (7) \qquad c_{g,t} = z_{g,t} \left(\frac{p_{z,g,t}}{p_{c,g,t}} \right)^{\nu} \qquad (8)$$

$$\ell_{g,t} = z_{g,t} \left(\frac{p_{z,g,t}}{p_{\ell,g,t}} \right)^{\nu} \qquad (9) \qquad a_{i,g,t} = c_{g,t} \left(\frac{p_{c,g,t}}{p_{A,i,t}} \right)^{\kappa} \quad for \;\; i = 1 \;\; (food) \qquad (10)$$

$$a_{i,g,t} = c_{g,t} \left(\frac{p_{c,g,t}}{p^{NF}_{c,g,t}} \right)^{\kappa} \left(\frac{p^{NF}_{c,g,t}}{p_{A,i,t}} \right)^{\sigma_{NF}} \quad for \;\; i \in NF \;\; (non-food) \qquad (11)$$

The lifetime utility of generation g (u_g) can be given by:

$$u_g = \frac{m_g}{p_{u,g}} \quad , \qquad m_g = \sum_{t=g}^{g+N} \left[p^w_t \; \pi_{g,t} \left(\omega_{g,t} - \ell_{g,t} \right) \left(1 - \tau^w_t \right) + p^n_t \zeta_{g,t} \right] \qquad (12)$$

where m_g is a level of lifetime income of generation g in present value.

2.1.4 Price of leisure (reservation wage)

The utility maximization yields the following first order condition which determines the price of leisure (reservation wage) $p_{l,g,t}$, where ϕ_t (≥ 0) is a Lagrange multiplier with respect to constraint (6):

$$p_{\ell,g,t} \;=\; p_{c,g,t} \left(\frac{\partial u_g}{\partial \ell_{g,t}} \right) \Bigg/ \left(\frac{\partial u_g}{\partial c_{g,t}} \right) \;=\; p^w_t \; \pi_{g,t} \left(1 - \tau^w_t \right) \;+\; \phi_t \; p_{c,g,t} \Bigg/ \left(\frac{\partial u_g}{\partial c_{g,t}} \right) \qquad (13)$$

If constraint (6) strictly holds with inequality (i.e. $\ell_{g,t} < \omega_{g,t}$), labor supply by household $lab_{g,t}$ ($\equiv \omega_{g,t} - \ell_{g,t}$) is strictly positive and the Lagrange multiplier ϕ_t becomes zero. In this case, the price of leisure (reservation wage) $p_{l,g,t}$ coincides with the effective market wage $p^w_t \pi_{g,t} (1 - \tau^w_t)$.

On the other hand, if constraint (6) strictly holds with equality (i.e. $\ell_{g,t} = \omega_{g,t}$), labor supply by household $lab_{g,t}$ is zero and the Lagrange multiplier ϕ_t becomes strictly positive. This implies that a household endogenously chooses the optimal retirement age when the price of leisure (reservation wage) exceeds the effective market wage.

2.2 Firms

2.2.1 Derivation of unit cost functions and factor demands

(Figure 2 here)

A representative firm in *i*-th sector (*i*=1: agriculture and fishery, i=2: tradable, i=3: non-tradable, i=4: healthcare service) uses capital (K_{it}) and labor service (L_{it}) to produce value-added (VA_{it}) upon the CES technology with elasticity of substitution $\sigma_{VA,i}$. Aside from it, intermediate composite (M_{it}) is produced by combining intermediate input *j* (A_{jit}) in fixed proportions (i.e. Leontief technology). Then, the value-added (VA_{it}) and the intermediate composite (M_{it}) are merged into gross output (Y_{it}) in fixed proportions.

We assume that capital and labor are mobile across sectors and that the markets of the primary factors are perfectly competitive. Cost minimization of the firms yields the following unit cost functions in calibrated share form:

$$c_{VA,\,i,t} = \left[\beta_i^L \left(p_t^w \right)^{1-\sigma_{VA,\,i}} + \left(1-\beta_i^L\right)\left(p_t^r \right)^{1-\sigma_{VA,\,i}} \right]^{\frac{1}{1-\sigma_{VA,\,i}}} \qquad (14)$$

$$c_{M,i,t} = \sum_j \frac{p_{A,\,j,t}}{\chi_{j,\,i}} \qquad (15)$$

$$c_{Y,\,i,t} = \beta_i^{VA}\, c_{VA,\,i,t} + \left(1-\beta_i^{VA}\right) c_{M,i,t} \qquad (16)$$

$c_{VA,i,t}$ is the dual unit cost function for producing the value-added with β_i^L: benchmark value share of labor incomes in sector i and p^r_t: user cost of capital (before capital tax). $c_{M,i,t}$ is the dual unit cost function for producing the intermediate composite, with χ_{ji}: fixed quantity of intermediate inputs *j* in sector *i* and $p_{A,j,t}$: price of Armington good *j*. $c_{Y,i,t}$ is the dual unit cost function for producing the gross output with β_i^{VA}: benchmark value share of value-added in gross output.

Applying the Shephard's Lemma, we can derive the demand functions for the primary factors:

$$K_{i,t} = VA_{i,t}\left(\frac{c_{VA,\,i,t}}{p_t^r}\right)^{\sigma_{VA,\,i}} \qquad (17) \qquad L_{i,t} = VA_{i,t}\left(\frac{c_{VA,\,i,t}}{p_t^w}\right)^{\sigma_{VA,\,i}} \qquad (18)$$

$$A_{j,i,t} = \left(\frac{1}{\chi_{j,i}}\right) M_{i,t} \qquad (19)$$

2.2.2 Accumulative motion of capital stock and investment equation

Accumulative motion of capital stock is given by equation (20).[12] Because the firms incur no

[12] To secure the well-behaved movement of capital stock at the post-terminal date, it is determined so that the growth rate of investment is equal to that of the potential GDP at the terminal date. This technique is called "the state variable targeting method", which is proposed by Lau *et*. *al*. (2002).

adjustment costs with respect to investment, the arbitrage condition for a purchase price of capital is given by equation (21).

$$K_{t+1} = (1-\delta)K_t + I_t\ , \quad K_t = \sum_{i=1}^{I} K_{i,t} \qquad (20)$$

$$p_t^k = p_t^r\_at + (1-\delta)\, p_{t+1}^k \qquad (21) \qquad\qquad p_t^r = p_t^r\_at \; + \; \tau^r\, \overline{r} \qquad (22)$$

K_t is aggregate capital stock at the beginning of period t, and I_t is aggregate private investment. δ is depreciation rate, and $p^r{}_t\_at$ represents the user cost of capital (after capital tax), while $p^k{}_t$ is purchase price of capital. τ^r is time-invariant tax rate on capital incomes.

We adopt the Leontief technology to produce aggregate investment (I_t) by combining Armington good i (I_{it}, investment good i). The associated cost function for producing aggregate investment (I_t) and the factor demand function for Armington good i (I_{it}, investment good i) are given as follows:

$$p_{t+1}^k = \sum_i \beta_i^{IA}\ p_{A,i,t} \qquad (23) \qquad\qquad I_{i,t} = \beta_i^{IA}\ I_t \qquad (24)$$

where $\beta^{IA}{}_i$ is the benchmark share of Armington good i (I_{it}, investment good i) in aggregate investment (I_t).

2.3 The government

The government collects taxes from labor incomes (Tax_L_t), capital incomes (Tax_K_t) and sector productions (Tax_Y_t : indirect taxes except consumption tax and tariff duty), with which it covers its spending. The net transfer to household in each generation ($\zeta_{g,t}$) is given according to each generation's share in the total population and total sum is $NTRF_t$.

$$Tax_t = Tax\_L_t + Tax\_K_t + Tax\_Y_t\,, \quad Tax\_L_t = \tau_t^w\, p_t^w L_t\,, \quad Tax\_K_t = \tau^r\, \overline{r}\ p_t^k\ K_t\,,$$

$$Tax\_Y_t = \sum_i t_{Y,i}\ p_{Y,i,t}\, Y_{i,t}\ , \quad NTRF_t = \sum_{g=t-N}^{t} \zeta_{g,t}$$

Tax_t is the total tax revenue and L_t is total labor supply. $t_{Y,i}$ represents time-invariant tax rate on sector productions, while $p_{Y,i,t}$ is price of gross output. Primary balance of the government budget (PB_t) can be written as follows:

$$PB_t = Tax_t - \left(p_t^{Gov}\, G_t + p_t^n\ NTRF_t \right) \qquad (25)$$

where G_t is government consumption and investment, and $p^{Gov}{}_t$ is its price. G_t, $NTRF_t$ and the target ratio of the government net debt to GDP ($tar\_debt\_gdp_t$) are the policy variables that the government can control exogenously. As is adopted in Kumhof *et al*. (2010), the government net debt outstanding is assumed to develop in accordance with its target ratio:

$$p_t^n\ GB_t = tar\_debt\_gdp_t \times \sum_i \left(p_{i,t}^{VA} \times VA_{i,t} \right) \qquad (26)$$

where GB_t is the government net debt outstanding in current value, at the beginning of period t, and $p^{VA}{}_{i,t}$ is price of value-added.

The tax rate on capital incomes (τ^r) and that on sector productions ($t_{Y,i}$) are assumed to be kept constant over time. The tax rate on labor incomes ($\tau^w{}_t$) is endogenously determined so that the government budget constraint (in present value) is always satisfied:[13]

$$p_t^n\ GB_{t+1} + PB_t = \left(1 + rn\_f_t\right) p_t^n\ GB_t \qquad (27)$$

where rn_f_t ($\equiv (1-\tau^r)\times r\_f_t$) is the benchmark risk-free rate on the government bond (net out of capital income tax).

We adopt the Leontief technology to produce the government consumption and investment (G_t) by combining Armington good i (G_{it}, government spending of good i). The associated cost function for producing the government consumption and investment (G_t) and the factor demand function for Armington good i are given as follows:

$$p_t^{Gov} = \sum_i \beta_{i,t}^{GA}\ p_{A,i,t} \qquad (28) \qquad\qquad G_{i,t} = \beta_{i,t}^{GA}\ G_t \qquad (29)$$

where $\beta^{GA}{}_{i,t}$ is the time-varying benchmark share of Armington good i in G_t.

2.4 External sector

2.4.1 Exports (destination of domestic products)

(Figure 3 here)

We adopt a small open economy setting and assume that foreign and domestic goods are imperfect substitutes (Armington assumption). A trading firm distributes domestic outputs (Y_{it}), after paying the production taxes, to either the domestic market (H_{it}, hereafter we call this home consumption of good i) or the export market (EX_{it}, in foreign currency). It maximizes the sales $S_{HX,i,t}$ with a CET (constant elasticity of transformation) technology:

$$Max:\ S_{HX,i,t} = p_t^f\ EX_{i,t} + p_{H,i,t}\ H_{i,t} \quad \text{s.t.} \quad Y_{i,t} = \left[\beta_i^X \left(EX_{i,t}\right)^{\frac{1+\sigma_{HX}}{\sigma_{HX}}} + \beta_i^H \left(H_{i,t}\right)^{\frac{1+\sigma_{HX}}{\sigma_{HX}}} \right]^{\frac{\sigma_{HX}}{1+\sigma_{HX}}}$$

where $p^f{}_t$ is the price of foreign goods or the real exchange rate (in present value), $p_{H,i,t}$ price of home consumption, σ_{HX} elasticity of transformation, β_i^X benchmark value share of good i

[13] We impose the constraint on the government bond outstanding at the post-terminal date to be consistent with a final steady state.

destined for the export market (EX_{it}) in domestic production of good i (Y_{it}), and $\beta_i^H (\equiv 1-\beta_i^X)$ benchmark value share of home consumption (H_{it}) in domestic production (Y_{it}). The real exchange rate (price of foreign goods) p^f_t is the yen-denominated value of 1 unit of foreign currency. [14] The sales maximization yields the following pricing equation:[15]

$$p_{Y,i,t}\left(1+t_{Y,i}\right)=\left[\beta_i^X\left(p_t^f\right)^{1+\sigma_{HX}}+\beta_i^H\left(p_{H,i,t}\right)^{1+\sigma_{HX}}\right]^{\frac{1}{1+\sigma_{HX}}} \qquad (30)$$

Applying the Shephard's Lemma to equation (30), we obtain the supply functions of good i destined for the export market EX_{it} and for the domestic market H_{it}:

$$EX_{i,t}=Y_{i,t}\left[\frac{p_{Y,i,t}\left(1+t_{Y,i}\right)}{p_t^f}\right]^{-\sigma_{HX}} \qquad (31)$$

$$H_{i,t}=Y_{i,t}\left[\frac{p_{Y,i,t}\left(1+t_{Y,i}\right)}{p_{H,i,t}}\right]^{-\sigma_{HX}} \qquad (32)$$

2.4.2 Imports (production of Armington goods)

An assembly firm produces Armington goods (A_{it}) by combining home consumption (H_{it}) with imported goods (IM_{it}, in foreign currency). It minimizes the associated cost TC_{Ait} with a CES technology:

$$Min:\ TC_{A,i,t}=p_{H,i,t}\ H_{i,t}+p_t^f\ IM_{i,t} \quad \text{s.t.} \quad A_{i,t}=\left[\beta_i^{AH}\left(H_{i,t}\right)^{\frac{\sigma_{AR,i}-1}{\sigma_{AR,i}}}+\beta_i^{AM}\left(IM_{i,t}\right)^{\frac{\sigma_{AR,i}-1}{\sigma_{AR,i}}}\right]^{\frac{\sigma_{AR,i}}{\sigma_{AR,i}-1}}$$

where β_i^{AH} is the benchmark value share of home consumption goods (H_{it}) in Armington production (A_{it}), $\beta_i^{AM} (\equiv 1-\beta_i^{AH})$ benchmark value share of imported goods (IM_{it}) in Armington production (A_{it}), and $\sigma_{AR,i}$ elasticity of substitution between home (H_{it}) and imported goods (IM_{it}). A cost minimization yields the following pricing equation:

$$p_{A,i,t}=\left[\beta_i^{AH}\left(p_{H,i,t}\right)^{1-\sigma_{AR,i}}+\beta_i^{AM}\left(p_t^f\right)^{1-\sigma_{AR,i}}\right]^{\frac{1}{1-\sigma_{AR,i}}} \qquad (33)$$

Applying the Shephard's Lemma to equation (33), we obtain the factor demands for home consumption goods (H_{it}) and imported goods (IM_{it}):

$$H_{i,t}=A_{i,t}\left(\frac{p_{A,i,t}}{p_{H,i,t}}\right)^{\sigma_{AR,i}} \qquad (34)$$

[14] A rise in p^f_t represents the yen's depreciation against the foreign currency.
[15] This pricing equation is derived from the first order and zero profit conditions of the sales maximization.

$$IM_{i,t} = A_{i,t}\left(\frac{p_{A,i,t}}{p_t^f}\right)^{\sigma_{AR,i}} \qquad (35)$$

2.4.3 Determination of the real exchange rate (price of foreign goods)

We assume that the after-tax interest rate on foreign assets rn_ex_t always coincides with a sum of the after-tax domestic risk-free interest rate (rn_f_t) and the risk premium on foreign assets ($prem_t$). The parity condition of no-arbitrage implies that the real exchange rate in current value $cv\_p^f{}_t$ comes down to a constant value of p^f, in which $cv\_p^f{}_t = cv\_p^f{}_{t+1} = \cdots = p^f$ holds.[16] Therefore, the real exchange rate in present value $p_t{}^f$ is given as follows.

$$p_t^f = p^f \Big/ \prod_{j=0}^{t}\left(1 + rn\_ex_j\right)$$

Without any shocks in the economy, p^f is set to be unity. If shocks hit the economy, p^f is endogenously determined so that it satisfies the following identity which relates trade balance and accumulation of the net foreign assets:[17]

$$\left(1 + rn\_ex_0\right) p_0^f \; NFA_0 + \sum_{t=0}^{T} TB_t = p_T^f \; NFA_{T+1}, \qquad TB_t = p_t^f\left(EX_t - IM_t\right) \qquad (36)$$

where NFA_t is the net foreign assets outstanding in foreign currency and in current value, at the beginning of period t, and TB_t is trade balance (in domestic currency).

2.5 Market clearing conditions

The market clearing condition for Armington good i is given as follows:

$$A_{i,t} = C_{A,i,t} + I_{i,t} + G_{i,t} + \sum_j A_{j,i,t}\,, \qquad C_{A,i,t} \equiv \sum_{g=t-N}^{t} a_{i,g,t} \qquad (37)$$

where C_{Ait} is a summation of consumption of Armington good i over households. The market clearing condition for capital goods and labor (in efficiency unit) are given as follows.

$$K_t = \sum_{i=1}^{I} K_{i,t} \qquad (38) \qquad\qquad L_t \equiv \sum_{g=t-N}^{t} lab_{g,t}\,\pi_{g,t} = \sum_{i=1}^{I} L_{i,t} \qquad (39)$$

The market clearing condition for home consumption of good i is given as follows.

$$H_{i,t} = Y_{i,t}\left(\frac{p_{Y,i,t}\left(1 + t_{Y,i}\right)}{p_{H,i,t}}\right)^{-\sigma_{HX}} = A_{i,t}\left(\frac{p_{A,i,t}}{p_{H,i,t}}\right)^{\sigma_{AR,i}} \qquad (40)$$

[16] See Rausch (2009) for details.

[17] We impose the constraint on the net foreign assets at the post-terminal date to be consistent with a final steady state.

2.6 Steady-state equilibrium and paths of transition

2.6.1 Steady-state equilibrium

Evolution of new generation's time endowment in efficiency unit (ω_g) can be decomposed into that of generation size (not in efficiency unit) $fert_g$ and that of labor-augmented productivity (cohort effects of productivity amelioration) $lamda\_c_g$:[18]

$$\omega_{g+1} = \left(1+\gamma_g\right)\omega_g\,, \qquad \gamma_g = fert_g + lamda\_c_g \qquad (41)$$

Steady-state equilibrium can be characterized as a condition in which the demographic structure is stationary:

$$fert_g = pop\_dot_t = const \qquad \text{for all } g \text{ and } t \qquad (42)$$

where pop_dot_t is a growth rate of total population (not in efficiency unit). As for the evolution of prices, all the prices declines with the after-tax interest rate $\overline{rn}$ as mentioned before.

2.6.2 Transition paths

The whole simulation term consists of (a) an initial steady state (t=-N to t=0), (b) transition periods (t=0 to t=T) and (c) a final steady state (t=T to t=T+N). In a dynamic simulation, we give the model exogenous shocks and analyze how the economy develops over the transition periods until it reaches the final steady state.

In order to solve the dynamic paths, we utilize the initial steady state (t=-N to t=0) as an initial condition and the final steady state (t=T to t=T+N) as a terminal condition. The terminal generations, which enters the economic life from year t=T-N+1 to t=T, will be alive during the final steady state (t=T to t=T+N). To simplify our analysis, we make the economy reach its final steady state at least in year t=T-N+1 by securing sufficiently long transitional periods.

3. Data and Calibration

3.1 Steady state growth rate and base year

In the following simulation, a risk-free interest rate (r_f_t) is set to 6.0% in the initial steady state and 3.0% in the final steady state. A rate of return on foreign assets (r_ex_t) is always set to 10.0%, and the benchmark rate of return on domestic real asset ($\overline{r}$) is always set to 6.0%. A potential GDP growth rate (gdp_dot_t) is set to 1.5% in the initial steady state (population growth 0.75% + labor-augmented productivity growth 0.75%) and 0.75 % in the final steady state

[18] As we assume that time endowment of each generation $\omega_{g,t}$ does not change over life cycle, we can simply denote it as ω_g.

(population growth 0.0% + labor-augmented productivity growth 0.75%).[19]

Since we focus on Japan's demographic shock of birthrate decline in the middle of 1980's, we choose 1985 as base year in the simulation, i.e., initial steady state. This choice of base year has a direct benefit for the calibration procedure: the realized government primary balance in 1985 was in surplus and public debt position was consistent with steady-state equilibrium. Hence, adjusted social accounting matrix (SAM), which is needed to implement calibration for various parameters, can be easily constructed without unrealistic adjustment on government expenditures and revenues by assuming a steady state.

3.2 Calibration of parameters for households

(Figure 4 here)

We assume that households enter economic life at the age of 20 and have a deterministic lifespan of 55 years. We first estimate an age-dependent inverted U-shaped wage profile in Japan based on the data set "Basic Survey on Wage Structure in 1985 (Japan Ministry of Health, Labor and Welfare)". Following Auerbach and Kotlikoff (1987), we estimate the wage profile as a quadratic curve:

$$\log\left(\pi_a\right) = c_0 + c_1 \times \left(a - 20\right) + c_2 \times \left(a - 20\right)^2 \qquad (43)$$

where a denotes age (a=20,$\cdots$,74) and π_a the wage rate of a worker at the age of a.

In this simulation, an economic agent begins to participate in the labor market at the age of a=20. Therefore, the term "$a-20$" reads as years of working experience. The expected signs of coefficients are c_1>0 and c_2<0. The estimation results are satisfactory: c_0=0.692, c_1=0.0444 and $c_2 = -8.60 \times 10^{-4}$, all of which are statistically significant and consistent with the sign conditions. In running simulations, we normalize the wage profile at the age of 20 ($\pi_{a=20}$) to be one. Figure 4 shows the calibrated wage profile in Japan.

(Table 1 here)

(Figure 5 here)

The benchmark share parameters (*alpha*, *theta_F*, *theta_NF*), and the benchmark tax rate on labor incomes (*tau_w*), productions (*t_Y*(*i*)) and capital incomes (*tau_r*) are calibrated so that

[19] It means that we exclude equilibrium under dynamic inefficiency in advance.

they are compatible with the Japanese Input-Output table in the base year 1985.[20] Table 1 summarizes these values. As for the government net transfers ($NTRF_t$), 62.2% of them is distributed, as pension payments, to the households at the age of 60 or over, and the rest to all of the households according to each generation's share in the total population.[21]

Subsequently, we employ the steady-state calibration procedure proposed by Rasmussen and Rutherford (2004). We assume (contra-factually) that the economy is initially on a balanced growth path in 1985, and a rate of time preference and periodic time endowment for the initial generation are calibrated so that they can be consistent with the consumption-saving choices in 1985. We then calibrate a consumption profile of the reference generation in the initial steady state and see the endogenous retirement decision at the age of around 64. Figure 5 shows the calibrated consumption-saving profile of the reference generation.

3.3 Calibration of parameters for firms

(Table 2 here)

In order to calibrate the parameters for production, we construct Japan's SAM with 4 aggregate sectors (agriculture and fishery goods, tradable goods, non-tradable goods and services, healthcare service) from Input-Output table data. The benchmark share parameters (*beta_L*(*i*), *beta_VA*(*i*), *beta_IA*(*i*), *beta_GA*(*i*), *beta_X*(*i*), *beta_AM*(*i*)) and a rate of depreciation (*delta*) are calibrated so that they are compatible with constructed SAM data.[22] The parameters of elasticity of substitutions in the production function (*sigma_VA*(*i*), *sigma_AR*(*i*), *sigma_HX*) are set in reference to Rausch (2009) and GTAP model (3rd. edition). Table 2 summarizes these values.

3.4 Demographic structure in Japan

(Figure 6 - 8 here)

The population dynamics of our simulation is based on the data set "the Population Projections for Japan in 1985 (the National Institute of Population and Social Security

[20] Calibration of the value share of aggregate consumption (*alpha*) is based on the "Survey on Time Use and Leisure Activities in 1986 (Ministry of Internal Affairs and Communications)". The parameters of elasticity of substitutions in the utility function (*sigma*, *nu*, *kappa*, *sigma_NF*) are set in reference to Rausch (2009).

[21] 62.2% is the ratio of pension payments to the total government net transfers in 1985.

[22] The rate of depreciation (*delta*) is calibrated so that it also satisfies the capital accumulative equation in the initial steady state.

Research)". The growth rate of cohort size of generation g (i.e. $fert_g$ in equation (41)) is utilized to construct the growth rate of total population (not in efficiency units). From g=2006 to 2050, the projected values from the above data set are used,[23] and from g=2051 to 2065, $fert_g$ are linearly extrapolated so that $fert_{g=2065}$ becomes zero. From g=2066 to 2200, $fert_g$ are all set to zero.[24]

The demographic transition in this model and the official projection are slightly different because of our assumption of the deterministic lifespan from age 20 to 74. Nevertheless, the both trends seem to coincide very well. The growth rate and the level of total population (not in efficiency unit) are shown in Figure 6 and Figure 7 respectively. The shift of demographic structure are shown in Figure 8, in which we call the ages of 20-44 cohorts the younger, 45-64 the middle and 65-74 the elder generations.

4. Simulations under different policy scenarios

4.1 Policy Scenarios on government expenditures

As mentioned in 2.3, government consumption and investment (G_t) and net transfers ($NTRF_t$) are assumed to be policy variables controlled exogenously by the government. We explain how to set these variables under different scenarios.

[*baseline scenario*]

Since we are going to focus on the impacts of demographic change on industrial structure, we have to incorporate the soaring demands for healthcare sector along with aging in baseline scenario. In Japan, per-capita medical cost for the elderly is much higher than for the young, and major portion of the demand for healthcare service is financed by government expenditure. Therefore, the ratio of the government consumption and investment of healthcare service to GDP has to be constructed so that it does reflect the increasing proportion of the elderly.

(Figure 9 here)

The amount of public healthcare spending Gov_EX("*med*", t), is calculated as follows:

$$Gov\_EX\left("med",\ t\right) = \sum_{age\_group} Med\_PC\left(age\_group,\ t\right) \times pop\left(age\_group,\ t\right) \qquad (44)$$

[23] An adjustment of the *Hinoeuma* outlier (a steep drop of the birth rate in 1966) is done to the growth rate of cohort size of generation g=1986 ($fert_{g=1986}$).

[24] From g=1985 to 2005, the adopted $fert_g$ are the realized values.

where $Med\_PC(age\_group, t)$ denotes the per-capita medical cost for each age group in year t. age_group is divided into 3 groups, the younger, the middle and the elder generations. $pop(age\_group, t)$ is the total population of each age group in year t.

We assume that the per-capita medical cost grows as the labor-augmented productivity, which is consistent with the balanced growth path. Actual figures of the per-capita medical cost in the base year 1985 are calculated by the National Cost of Medical Care (Japan Ministry of Health, Labor and Welfare) and the IO data, which is shown in Figure 9.

For other goods and services, the ratios of the government consumption and investment to GDP are fixed to those in the benchmark year 1985, as well as government net transfers.

[*"increased government net transfers" scenario*]

As an alternative policy scenario, we incorporate the recent increasing trend of net transfers to households in Japan until 2010. Such trend is attributed to the increase of social benefits payment in cash, mainly public pension and child benefits. However, such increasing trend of government transfer spending has not been covered by the corresponding increase of government revenues, and they have resulted in the increasing trend of government debt outstanding. In order to reflect this trend, paths of government net transfer (including pension payments) and government debt outstanding are set exogenously. We adopt the smoothed series of the realized public net transfer for the term when the actual data are available, and after 2011, it is set to accommodate the possibility of convergence of government debt.[25] Other than transfers, the ratios of the government consumption and investment to GDP are the same as those in the baseline scenario.

4.2 Simulation results

In this subsection, we show the simulation results on the macroeconomic effects of declining birthrate and rapid population aging in Japan. The simulation is based on two scenarios: *baseline scenario* and "*increased government net transfers*" *scenario*. Exogenous forces given to the model are demographic change (a growing proportion of the elderly with declining birthrate) and the government expenditures. The whole simulation term is from 1985 to 2200 (i.e. T=2200), as we mentioned above, and we regard base year (1985) as the initial steady state and capture the quantitative changes by looking at the transitional path due to the demographic shock on (a) key macroeconomic variables (per-capita variables, factor prices), (b) sector shares and (c) net government debt and average tax rate.

[25] Since such an increasing trend of net transfer is not sustainable policy, we assume to cut the transfer by 40% between 2011 and 2015, and expand the amount by the growth rate of GDP after 2016.

4.2.1 Simulation results of the *baseline scenario*

(a) Key macroeconomic variables

(Table 3 here)

Table 3 shows the percentage changes of the key macroeconomic variables with respect to the base year 1985. Labor supply (in efficiency units) is projected to be an inverted U-shape because of the gradual decline of population growth from 0.75% to -0.2% under tenacious productivity growth of 0.75%. Value-added per capita is projected to decrease because production of value-added is linked to labor supply which declines more rapidly than total population. Capital-labor ratio is projected to rise because the scarcity of labor (i.e. denominator) outweighs the decline of domestic capital stock (i.e. numerator). The latter is due to lower saving rate with the advent of aging society. Investment to value-added ratio is projected to decrease because the higher capital-labor ratio reduces the need for firms to invest in new physical capital. Real wage is projected to rise due to relative scarcity of labor, and consumption to value-added ratio is projected to rise because of increasing share of the elder people who will dissave and consume more than the younger people.

The noteworthy result of the simulation can be seen in the path of user cost of capital in Table 3. In the aftermath of the birthrate shock, it declines due to relative abundance of capital to labor, as is usually predicted in standard OLG models in a closed economy. But, in the long run, it is projected to rise. The reason is that the accumulation of domestic capital is crowded out by accumulation of government debt and net foreign asset.

(Table 4 here)

Table 4 shows the saving rate of households. It is calculated by dividing the amount of household saving by the after tax incomes from labor, asset and transfers. A growing proportion of the elder generation lowers the saving rate of households.

(b) Sector shares

(Table 5 here)

Table 5 shows the sector share of output in total domestic output until 2050. In the aftermath of the birthrate decline shock, the share of “agriculture and fishery” and “tradable goods” sectors slightly increase, and then turn to contract in the long run, while other sectors show the opposite motion. In 2050, the sector share of “healthcare service” is projected to expand to 8.0% from 3.3% in 1985.

(Figure 10-1 here)
(Figure 10-2 here)

The increasing share of healthcare service is mainly due to the soaring government demands for it. Figure 10-1 shows the path of total government spending relative to GDP and its component (government net transfers, and government consumption and investment). Figure 10-2 illustrates the breakdown of the government consumption and investment. We find that the total government spending increases due to the soaring public healthcare spending: it expands by around 10% of GDP from 1985 to 2050.

However, the increasing share of non-tradable goods and services and healthcare service does not only arise from increasing demand due to aging population. Along with the decline of labor participation, scarcity of labor and increasing wage will change the relative price of domestically produced tradable goods to imported goods. Therefore, tradable goods will be substituted by imports, which results in the contraction of domestic production in these sectors in the long run. On the other hand, “non-tradable goods and services” and “healthcare service” are difficult to be imported and there has to be a labor influx from tradable sectors into non-tradable sectors to fulfill domestic demand for them. In order to enable economic agents to implement such reallocation of resources in the future, i.e., increase of imports in the phase of declining labor participation, net foreign asset must be piled up beforehand, by increasing export in the aftermath of the shock.

(c) net government debt and average tax rate

(Figure 11 here)

Figure 11 shows the transitional paths of net government debt and average tax rate to GDP. In this simulation, the path of net government debt is set exogenously to achieve the convergence to a final steady state. By looking at the figure, we can see that net government debt reaches 60% of GDP.

4.2.2 Simulation results of the “*increased government net transfers*” *scenario*

In reality, Japan’s government has adopted to increase the net transfer to the elderly people along with the decline of birthrate. This policy has not only resulted in accumulation of government debt outstanding, but also worked as a wedge to labor supply. In order to check the effects of this policy, we show the differences of simulation results in the "*increased government net transfers*" compared to the baseline one.

(Table 6 here)

(Table 7 here)

Table 6 shows the deviations of key macroeconomic variables from the baseline scenario. The numbers of labor supply (in efficiency units) and value-added per capita are smaller than the one under the baseline scenario. These results are due to the increase of net transfer to households (income effects) and "crowding out" of private capital by exogenous increase of government debt outstanding. In Table 7, we can see that the saving rate of households is smaller than the one under the baseline scenario in the early half of the 21th century.

(Table 8 here)

Table 8 shows the level of the sector share of output in total domestic output. We see the similar patterns of sector transitions as in the baseline scenario, but the magnitude of the shift of sector share from tradable sectors to non-tradable sectors is slightly larger in the long run.

(Figure 12 here)

Figure 12 shows the transitional paths of net government debt and average tax rate to GDP. By looking at the figure, we can see that net government debt reaches 140% of GDP.

5. Conclusions

In this paper, we have shown the quantitative impacts of Japan's recent declining birthrate on Japan's whole economy by using the framework of dynamic general equilibrium incorporating multi-sector and overlapping generations. We construct a 4-sector model (agriculture and fishery goods, other tradable goods, healthcare services and other non-tradable goods and services) following Rausch (2009), which incorporates substitutability between domestic and imported goods, and calibrate parameters based on Japan's Input-Output table.

We consider government actions as exogenous parameters, and analyze the effects of different policy scenarios, such as increasing healthcare spending and transfers to elderly households.

Under the "*baseline scenario*", where the share of healthcare sector expands to almost 2.5 times in 2050 relative to the base year, while capital-labor ratio is projected to rise as is usual in standard OLG models, user cost of capital is also projected to rise, due to crowding out of domestic capital by accumulation of government debt and net foreign asset.

Under the "*increased government net transfers scenario*", where the government increases net transfer to households, labor supply (in efficiency units) and value-added per capita are smaller than the one under the baseline scenario due to income effects and crowding out of private capital by exogenous increase of government debt outstanding. The comparison between the baseline scenario and the alternative one implies that the choice of government policy regarding the transfers to households along with aging society will have large impacts on labor participation, production and paths of trade balances in the long run.

Our model is constructed to draw the transitional paths of stock and flow variables to reach the final steady state where stock variables, such as government debt outstanding and net foreign assets, are consistent with flow variables, such as primary deficits and trade balances. Therefore, the effects of demographic shocks and government policies on the macroeconomic variables can be simulated in a comprehensive manner.

This model can be expanded to allow heterogeneous income classes within each generation, as in Rausch (2009), to analyze the intra-generational impacts of demographic change, or incorporate productivity shocks in different sectors to analyze their impacts on different generations. In order to expand the scenarios on government actions which reflect actually adopted policies and feasible fiscal consolidation in the future, introduction of time-varying value-added tax rate is an important step. Since the adjustment of tax rate on labor income in the simulations in this paper is much more distortionary on the labor participation than the adjustment of value-added tax rate, it is difficult to set various scenarios only upon the adjustment of labor income tax. Those expansions remain future tasks.

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Appendix 1: A CES production function in calibrated share form[26]

A cost minimization problem subject to the CES production technology can be written as:[27]

$$Min: \ C = \sum_{i=1}^{n} p_i \ X_i \qquad \text{s.t.} \qquad Y = \left[\sum_{i=1}^{n} \alpha_i \ X_i^{-\frac{1-\sigma}{\sigma}} \right]^{-\frac{\sigma}{1-\sigma}} \tag{45}$$

where C is a total cost, p_i a price of factor i, X_i input of factor i, Y production, α_i a weight parameter of factor i, σ (>0) an elasticity of substitution between production factors. The first order condition can be derived as follows.

$$\frac{\frac{\partial Y}{\partial X_i}}{p_i} = \frac{\frac{\partial Y}{\partial X_j}}{p_j} \qquad \text{or} \qquad \frac{\alpha_i \ X_i^{-\frac{1-\sigma}{\sigma}-1}}{p_i} = \frac{\alpha_j \ X_j^{-\frac{1-\sigma}{\sigma}-1}}{p_j} \tag{46}$$

Substituting the baseline values (denoted in upper bar) into F.O.C. (46), we obtain α_i as follows.

$$\frac{\alpha_i \ \overline{X}_i^{-\frac{1-\sigma}{\sigma}-1}}{\overline{p}_i} = \frac{\alpha_j \ \overline{X}_j^{-\frac{1-\sigma}{\sigma}-1}}{\overline{p}_j} = \mu \qquad \therefore \ \alpha_i = \mu \ \overline{p}_i \ \overline{X}_i^{1/\sigma}$$

Substituting α_i into the CES production function yields:

$$Y = \kappa \left[\sum_{i=1}^{n} \theta_i \left(\frac{X_i}{\overline{X}_i} \right)^{-\frac{1-\sigma}{\sigma}} \right]^{-\frac{\sigma}{1-\sigma}} \quad \left(\ \kappa \equiv \left(\mu \ \overline{C} \right)^{-\frac{\sigma}{1-\sigma}} \ \right), \qquad \theta_i \equiv \frac{\overline{p}_i \overline{X}_i}{\sum_{i=1}^{n} \overline{p}_i \overline{X}_i} = \frac{\overline{p}_i \overline{X}_i}{\overline{C}} \tag{47}$$

where θ_i is defined as a cost share of factor i in the baseline. Substituting the baseline values into Eq(47), we can easily verify $\kappa = \overline{Y}$. Therefore, using the cost share parameter θ_i, the CES production function in (45) can be written as follows:

$$\frac{Y}{\overline{Y}} = \left[\sum_{i=1}^{n} \theta_i \left(\frac{X_i}{\overline{X}_i} \right)^{-\frac{1-\sigma}{\sigma}} \right]^{-\frac{\sigma}{1-\sigma}} \tag{48}$$

Formula (48) is called "a production function in calibrated share form". Advantage of this representation is that we can easily calibrate the deep parameters in the production function by using a cost share of factor i in the baseline θ_i.

[26] This argument can be easily applied to utility maximization problem of household.
[27] As is well known, the CES production technology encompasses the Leontief (σ =0) and the Cobb-Douglas (σ =1) functions.

Appendix 2: Simulation results of the "*without age-dependent public spending*" *scenario*

In Appendix 2, we show the simulation results based on the "*without age-dependent public spending*" *scenario*. In this scenario, the ratios of the government consumption and investment of all goods and services (agriculture and fishery goods, tradable goods, non-tradable goods and services, healthcare service) and the government net transfers (including pension payments) to GDP are fixed to those in the benchmark year 1985, i.e. the public spending is not dependent on the population aging as shown in Figure A1.

(Figure A1 here)

(Table A1 here)
(Table A2 here)
(Table A3 here)

Table A1 shows the percentage changes of the key macroeconomic variables with respect to the base year 1985. Results are almost the same as the baseline scenario. Table A2 shows the saving rate of households. Table A3 shows the level of the sector share of output in total domestic output. The large difference from the baseline scenario is that the expansion of the "healthcare service" sector is rather mild: it expands to around 1.2 times in 2050 relative to the base year 1985.

[Table 1]

Deep parameters (households)

sigma	Inter-temporal elasticity of substitution	[from Rausch(2009)]	0.8
nu	Elasticity of substitution between commodity and leisure	[from Rausch(2009)]	0.6
kappa	Elasticity of substitution between food and non-food consumption	[set by authors]	0.5
sigma_NF	Elasticity of substitution between non-food consumption	[set by authors]	0.5
alpha	Value share of commodity consumption aggregate	[set by authors]	0.5
theta_F	Value share of food consumption	[from IO data (1985)]	0.042
theta_NF	Value share of non-food consumption	[from IO data (1985)]	0.958
tau_w	Tax rate on labor incomes (in the baseline)	[from IO data (1985)]	0.263
t_Y(i)	Tax rate on productions (i=1)	[from IO data (1985)]	0.014
	(i=2)		0.046
	(i=3)		0.024
	(i=4)		0
tau_r	Tax rate on capital incomes	[set by authors]	0.239

(note) i=1 refers to agriculture and fishery goods, i=2 tradable goods, i=3 non-tradable goods and services, and i=4 healthcare service.

[Table 2]

Deep parameters (producers)

sigma_VA(i)	Elasticity of substitution between capital and labor (i=1)	[from GTAP model]	0.560
	(i=2, 3, 4)		1.260
sigma_AR(i)	Elasticity of substitution between home and imported goods (i=1)	[from GTAP model]	2.200
	(i=2)		2.800
	(i=3, 4)		1.900
sigma_HX	Elasticity of transformation between home and export	[from Rausch(2009)]	2.000
beta_L(i)	Share of labor income in the value-added (i=1)	[from IO data (1985)]	0.170
	(i=2)		0.519
	(i=3)		0.625
	(i=4)		0.710
beta_VA(i)	Share of the value-added aggregate in gross output (i=1)	[from IO data (1985)]	0.555
	(i=2)		0.269
	(i=3)		0.596
	(i=4)		0.585
beta_IA(i)	Share of Armington goods in aggregate investment (i=1)	[from IO data (1985)]	0
	(i=2)		0.516
	(i=3)		0.484
	(i=4)		0
beta_GA(i)	Share of Armington goods in aggregate government spending (i=1)	[from IO data (1985)]	0
	(i=2)		0.037
	(i=3)		0.696
	(i=4)		0.267
beta_X(i)	Share of export in domestic production (i=1)	[from IO data (1985)]	0
	(i=2)		0.098
	(i=3)		0.016
	(i=4)		0
beta_AM(i)	Share of imported goods in Armington production (i=1)	[from IO data (1985)]	0.171
	(i=2)		0.097
	(i=3)		0.015
	(i=4)		0
delta	Rate of depreciation	[set by authors]	0.0469

(note) Refer to Table 1.

[Table 3]

Percentage changes of the key macroeconomic variables (*baseline scenario*)

variables / year	1985	1990	2000	2010	2020	2030	2040	2050
Labor supply (in efficency units) (%)	0.00	7.09	22.03	30.13	26.46	21.56	9.36	−2.52
Value-added per capita (%)	0.00	0.11	0.47	1.90	−0.60	−3.56	−8.57	−12.13
Capital-labor ratio (%)	0.00	2.23	4.83	9.11	18.71	21.86	23.39	16.47
Investment to value-added ratio (%)	0.00	−1.21	−7.22	−13.02	−20.02	−28.65	−45.74	−44.65
User cost of capital (after tax) (%)	0.00	−3.14	−5.43	−5.70	−4.72	7.79	24.17	46.82
Wage rate (%)	0.00	0.45	1.95	5.84	14.67	24.95	36.28	41.15
Consumption to value-added ratio (%)	0.00	0.38	1.09	2.30	6.79	10.08	15.97	18.71

(note) Figures above are percentage changes with respect to the base year 1985 (%). "Per capita" means that a variable is divided by the sum of time endowment (in efficiency units) across all generations. Labor supply is in efficiency units. Real factor prices are deflated by the reference price $p\_n(t)$.

[Table 4]

Saving rate of households (*baseline scenario*)

year	1985	1990	2000	2010	2020	2030	2040	2050
Saving rate of households (%)	10.92	10.26	8.88	8.79	3.44	0.02	−5.51	−6.84

(note) The saving rate of households is calculated in such a manner that the amount of household saving is divided by the sum of labor, asset and transfer incomes (after tax).

[Table 5]

Level of the sector share of output in total domestic output (*baseline scenario*)

sectors / year	1985	1990	2000	2010	2020	2030	2040	2050
Agriculture and fishery (%)	2.75	2.77	2.77	2.77	2.73	2.60	2.45	2.33
Tradable goods (%)	42.52	42.62	42.27	41.74	40.48	38.39	35.79	34.57
Non-tradable goods (%)	51.43	51.37	51.09	50.71	51.52	52.33	53.61	55.09
Medical and nursing care services (%)	3.30	3.24	3.86	4.78	5.26	6.68	8.15	8.01

[Table 6]

Key macroeconomic variables of "*increased government net transfers*" *scenario* relative to *baseline scenario*

variables / year	1985	1990	2000	2010	2020	2030	2040	2050
Labor supply (in efficency units) (%)	0.41	−0.88	−6.53	−5.05	−0.87	−0.90	−2.75	−0.27
Value−added per capita (%)	0.23	−1.12	−5.78	−6.15	−4.39	−3.09	−3.70	−2.68
Capital−labor ratio	−0.03	−0.04	0.13	−0.21	−0.68	−0.44	−0.20	−0.46
Investment to value−added ratio	−1.41	−1.15	−2.30	−1.13	0.64	0.46	−0.76	−0.28
User cost of capital (after tax)	−1.02	−0.95	−0.92	−0.62	−0.42	−0.82	−1.09	−1.03
Real wage rate	−0.09	−0.08	−0.06	−0.07	−0.11	−0.13	−0.13	−0.16
Consumption to value−added ratio	0.88	0.62	1.50	0.44	−1.37	−1.87	−1.19	−1.44

(note) The numbers of labor supply (in efficiency units) and value-added per capita are expressed in deviation ratio by a percentage point, and others in difference from the baseline scenario.

[Table 7]

Saving rate of households ("*increased government net transfers*" *scenario*)

year	1985	1990	2000	2010	2020	2030	2040	2050
Saving rate of households (%)	12.47	9.97	0.35	0.75	−1.28	−4.52	−6.34	3.40

(note) Refer to Table 4.

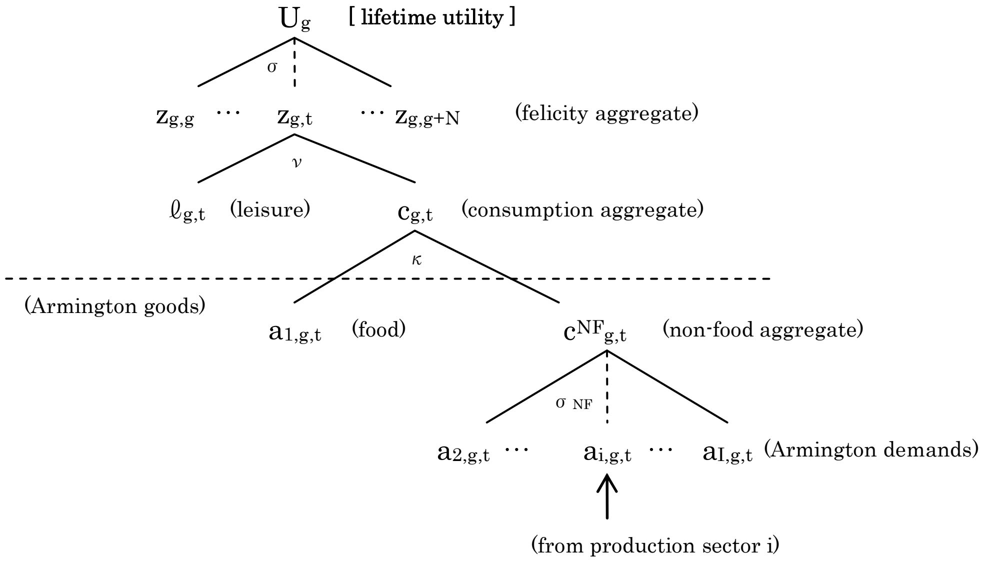
[Table 8]

Level of the sector share of output in total domestic output ("*increased government net transfers*" *scenario*)

sectors / year	1985	1990	2000	2010	2020	2030	2040	2050
Agriculture and fishery (%)	2.78	2.77	2.69	2.64	2.59	2.51	2.37	2.26
Tradable goods (%)	42.65	42.44	40.89	40.21	39.46	37.83	35.01	34.06
Non−tradable goods (%)	51.27	51.51	52.33	52.06	52.47	52.80	54.17	55.48
Medical and nursing care services (%)	3.30	3.28	4.09	5.09	5.47	6.85	8.44	8.20

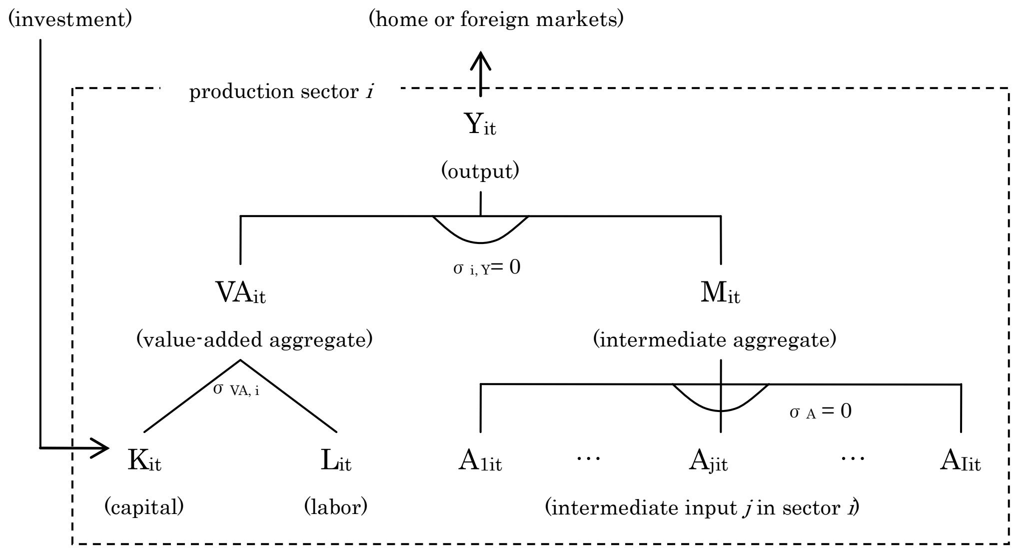
[Figure 1]

Nesting structure of utility



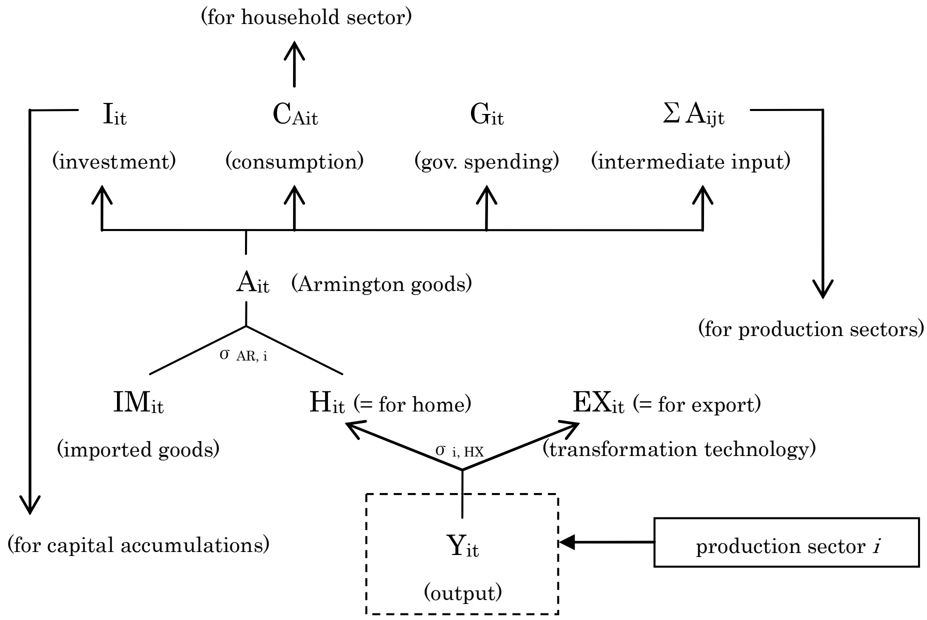
[Figure 2]

Nesting structure of production



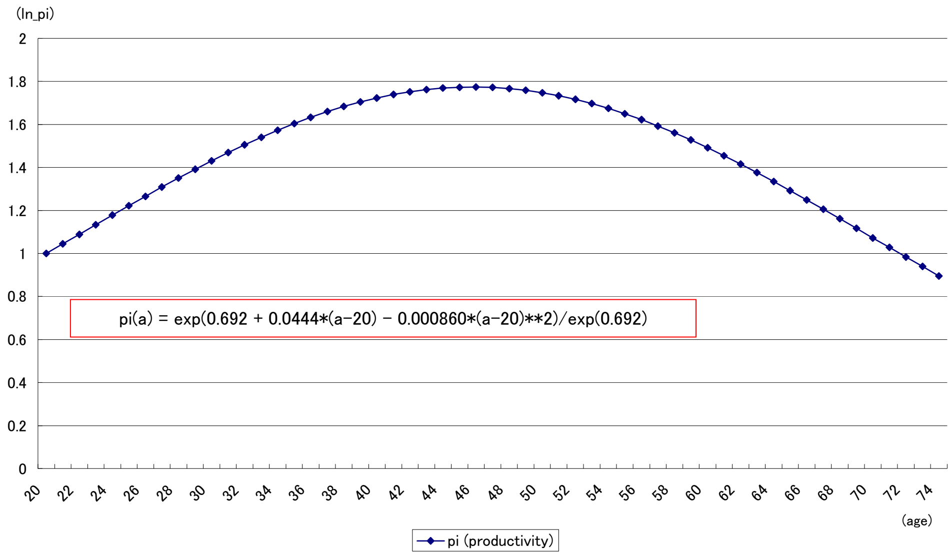
[Figure 3]

Production of Armington goods and its flow



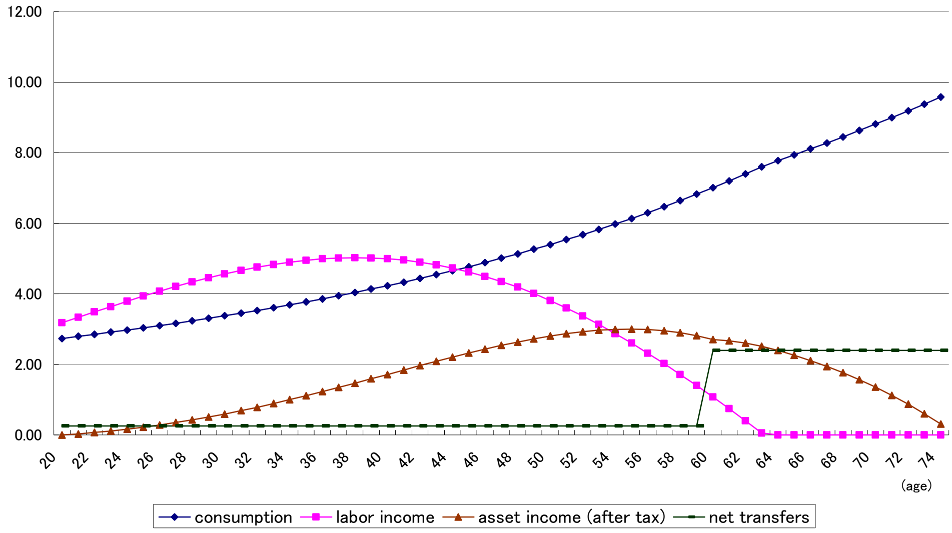
[Figure 4]

Calibrated age-dependent wage profile in Japan (the reference generation)



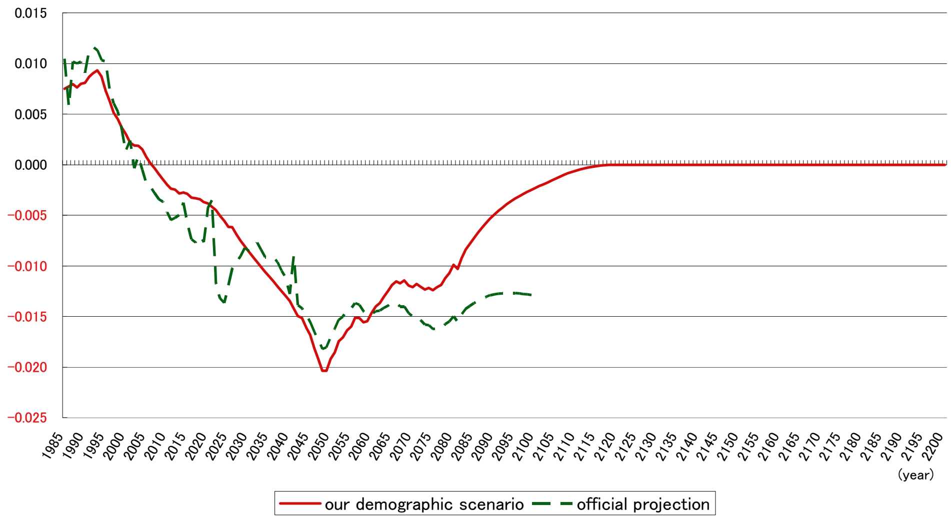
[Figure 5]

Calibrated consumption-saving profile in Japan (the reference generation)



[Figure 6]

Growth rate of total population (age of 20-74, not in efficiency unit)

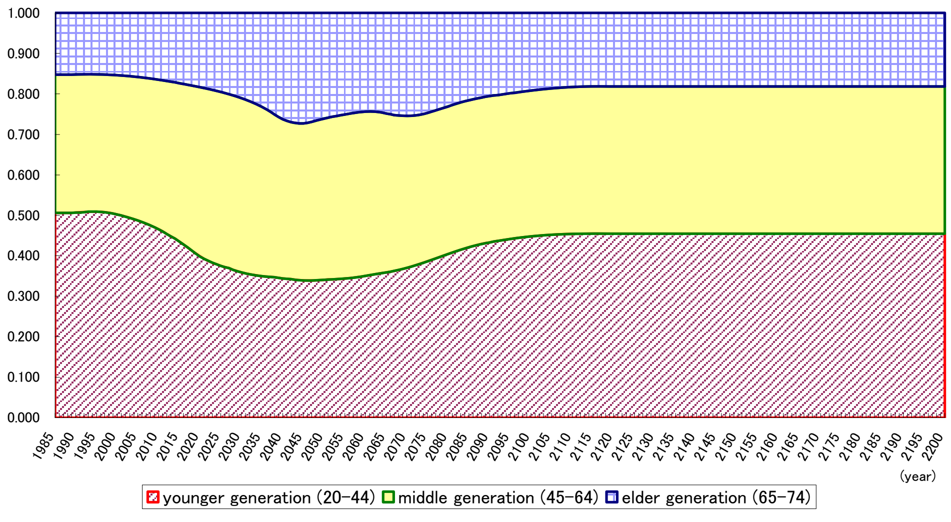


[Figure 7]

Level of total population (age of 20-74, not in efficiency unit)

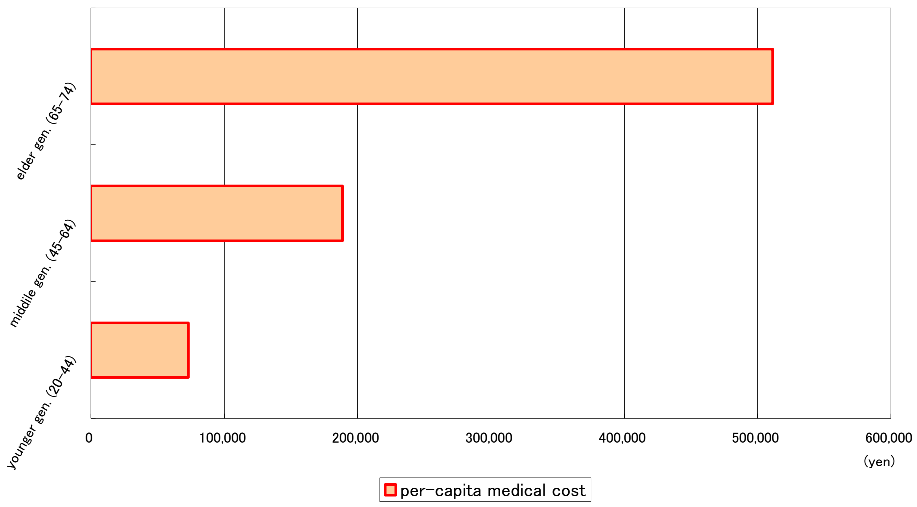
[Figure 8]

Shift of demographic structure



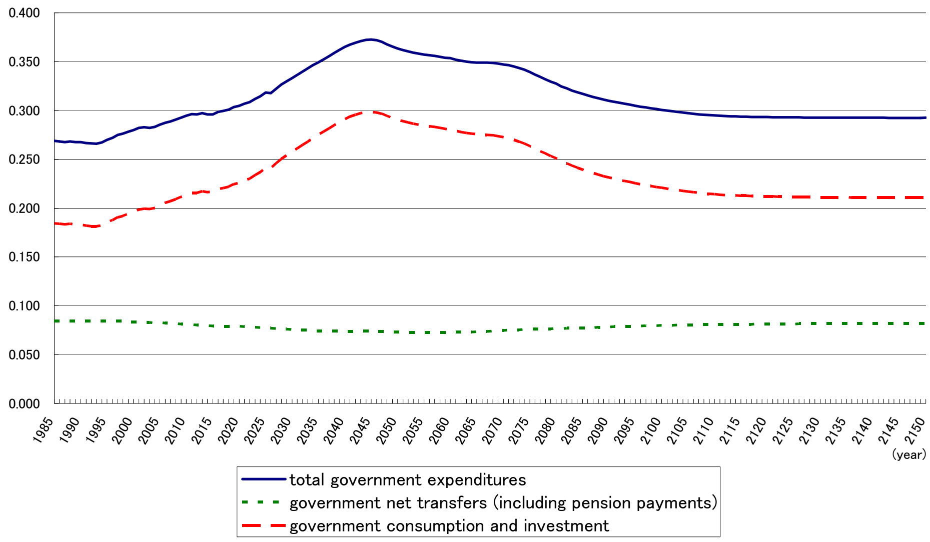
[Figure 9]

Per-capita medical cost by generations (in 1985)



[Figure 10-1]

Transitions of the government expenditures (to GDP, *baseline scenario*)

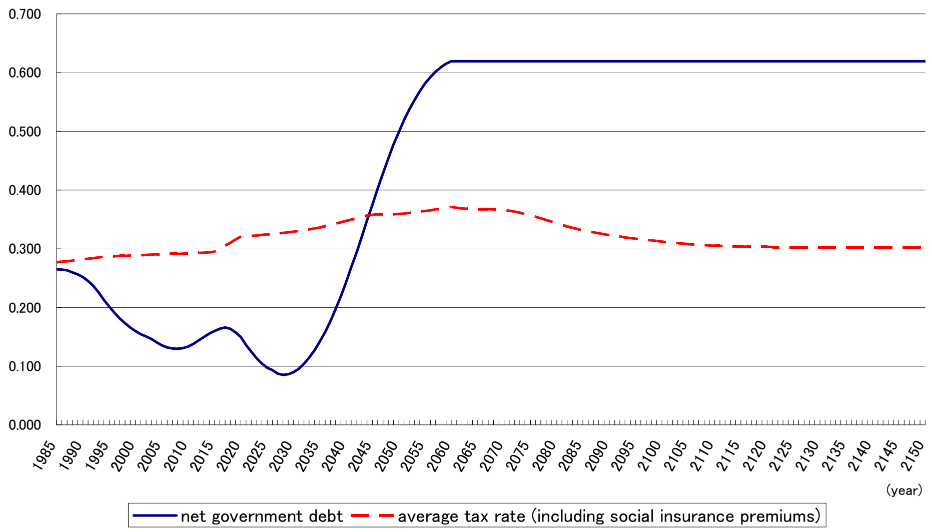


[Figure 10-2]

Breakdown of the government consumption and investment (to GDP, *baseline scenario*)

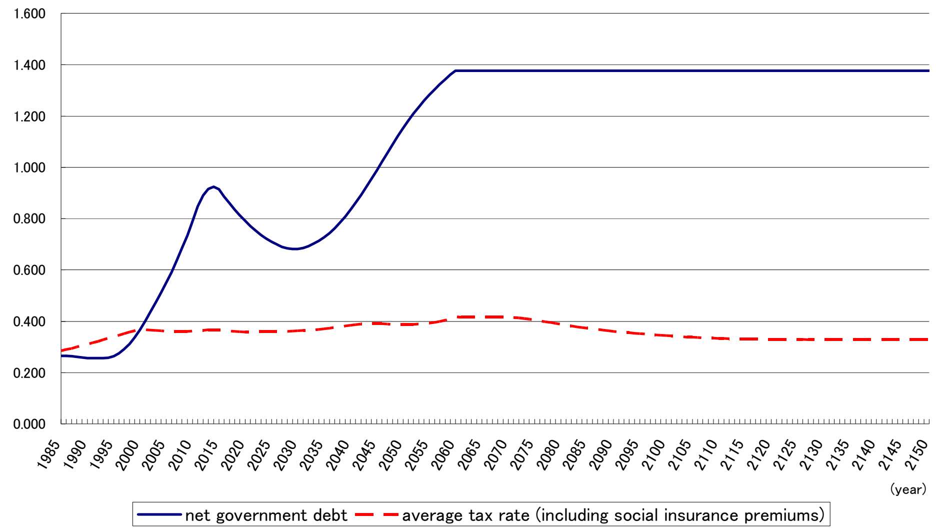
[Figure 11]

Transition of net government debt and average tax rate (to GDP, *baseline scenario*)



[Figure 12]

Transition of net government debt and average tax rate (to GDP, “*increased government net transfers*” *scenario*)



[Table A1]

Percentage changes of the key macroeconomic variables ("*without age-dependent public spending*" *scenario*)

variables ／ year	1985	1990	2000	2010	2020	2030	2040	2050
Labor supply (in efficency units) (%)	0.00	7.42	22.01	29.43	29.13	21.84	9.23	−1.57
Value−added per capita (%)	0.00	0.20	−0.17	0.73	−0.27	−3.96	−8.63	−10.30
Capital−labor ratio (%)	0.00	1.70	3.26	7.47	13.75	19.71	23.09	19.27
Investment to value−added ratio (%)	0.00	−2.10	−8.63	−13.31	−18.96	−28.40	−40.84	−38.70
User cost of capital (after tax) (%)	0.00	−2.61	−4.34	−5.81	−4.48	2.22	14.96	33.22
Real wage rate (%)	0.00	0.26	0.80	3.51	9.36	17.99	27.91	33.87
Consumption to value−added ratio (%)	0.00	0.59	3.13	6.35	11.09	17.96	26.13	28.48

(note) Refer to Table 3.

[Table A2]

Saving rate of households ("*without age-dependent public spending*" *scenario*)

year	1985	1990	2000	2010	2020	2030	2040	2050
Saving rate of households (%)	10.17	9.88	8.10	7.33	5.38	0.98	−4.41	−3.89

(note) Refer to Table 4.

[Table A3]

Level changes of the sector share of output in total domestic output ("*without age-dependent public spending*" *scenario*)

sectors ／ year	1985	1990	2000	2010	2020	2030	2040	2050
Agriculture and fishery (%)	2.74	2.76	2.79	2.83	2.83	2.78	2.69	2.59
Tradable goods (%)	42.39	42.46	42.33	42.30	41.64	40.17	38.31	37.34
Non−tradable goods (%)	51.55	51.46	51.53	51.51	52.08	53.41	55.11	56.07
Medical and nursing care services (%)	3.31	3.31	3.35	3.36	3.45	3.64	3.90	4.00

[Figure A1]

Transitions of the government expenditures (to GDP, “*without age-dependent public spending*” *scenario*)