Bargaining with Uncertain Value Distributions*

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Abstract

This paper studies a bargaining model in which the seller is uncertain about which distribution the buyer's values are drawn from. The distribution of the buyer's values is fixed across periods, while the buyer's values are drawn independently from the distribution each period. In the classical model of repeated bargaining where the buyer's value is drawn from a commonly known distribution and fixed across periods, the high-value buyer has a strong incentive to conceal his value, and the seller loses most of her bargaining power. An important question is whether adding a layer of uncertainty makes the high-value buyer more willing to accept high-price offers and improves the seller's revenue. We find this to be the case as long as the seller's ex ante beliefs are sufficiently optimistic.

Keywords: Repeated Bargaining, Uncertain Value Distributions, Revenue Comparison, Learning
JEL Classification: C73, D81, D82

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1 Introduction

One important question in the literature of repeated bargaining is how players' private information is revealed over time, and related to that, how economic surplus is distributed between the buyer and the seller. In this paper we examine these questions in a two-period bargaining model in which a durable-good monopolist tries to rent the durable good to a buyer in each period.[1] We adopt a framework commonly used in the previous literature where only the buyer has private information, the seller proposes a take-it-or-leave-it offer each period, and the buyer decides whether to accept or reject the offer. The classical models usually assume that the distribution of the buyer's value is common knowledge, only the buyer's value is private information, and the buyer's value is fixed across periods. Different from those models, we assume that the buyer's values are drawn from one of the two distributions independently in each period. The buyer has private information about the distribution from which his values are drawn as well as his realized valuations when each period comes about, while the seller only knows the ex ante probability of the two possible distributions. A stark result established in the classical models is that the seller has a large disadvantage and loses most of her monopoly power. We ask in this paper whether the seller's standing may be improved by introducing a layer of uncertainty about the distribution of the buyer's values and allowing the buyer's values to randomly change each period.

The literature on the Coase conjecture finds that if the durable-good monopolist sells over time and can quickly lower prices, the seller can hardly achieve profits greater than the lowest buyer valuation and the buyer obtains the entire surplus from trade in excess of his lowest valuation (Coase 1972, Fudenberg et al. 1985).[2] The intuition is that the monopolist is induced to reduce the price when facing the residual demand after having sold some quantity to high-value buyers, and rationally anticipating falling prices causes most potential buyers

[1] We will use she to denote the seller and he to denote the buyer.

[2] This result only holds under the assumption that the seller's marginal cost is lower than the buyer's lowest value, which is called the "gap" case in the literature.

to wait for a lower price in the future.

When the monopolist bargains over renting the durable good to a *non*-anonymous buyer with private value, Hart and Tirole (1988) show that the seller always offers a low price until the end of the game given any prior belief, if the horizon is long enough.[3] Intuitively, when the time horizon is long, the high-value buyer will not accept any price rejected by the low-value buyer, in order to avoid being charged with a high price in all the later periods. So the seller is not able to price discriminate and she charges a low price to both low-value and high-value buyer, until close to the end of the horizon. Therefore, if the durable-good monopolist rents the durable good to a *non*-anonymous buyer, the seller is again caught in an unfavorable position. Furthermore, Hart and Tirole (1988) show that renting is even no better than selling, and it is strictly worse if the horizon is long enough.

The basic message we get from the sale model (the Coasian dynamics) and the rental model (Hart and Tirole) is that the seller loses most of the monopoly power when facing a *non*-anonymous buyer who has private information about his fixed valuation. One earlier paper by Bulow (1982) argues that the durable-good monopolist may be better off if she chooses to rent the durable good rather than sell it, based on the assumption that the buyer is anonymous, that is, the seller can not identify the buyer's previous actions. This result is also shown in Hart and Tirole (1988). The rental model with an anonymous buyer gives the seller the highest payoff, which is equivalent to the payoff yield when the seller can commit herself to a mechanism or contract once and all.

The intuition of the seller receiving a higher payoff when facing an anonymous buyer is that the seller cannot use the information revealed by the buyer's behavior in early periods to update her belief over the buyer's value and practice price discrimination in later periods,

[3]Hart and Tirole (1988) examine the sale model and the rental model in three cases: (1) where the parties can commit themselves to a contract once and for all; (2) where the parties can only write short-term contracts which rule within a period, but cannot commit themselves between periods; (3) where parties can write a long-term contract which rules across periods, but cannot commit themselves not to renegotiate this contract by mutual agreement. The rental model without commitment as in this paper is just part of the analysis in Hart and Tirole.

and consequently the buyer is less afraid to reveal his private information and more likely to accept a high price in early periods. On the contrary, when the seller is dealing with a non-anonymous buyer, the buyer is concerned with how his action today will affect his payoff tomorrow and is less likely to accept a high price in early periods. In the extreme case as in Hart and Tirole where the buyer's value is fixed, revealing the current value for the high-value buyer means revealing all the private information on future values, so the incentive for the high-value buyer to hide his value is strong.

The main purpose of this paper is to examine whether the seller can improve her profit in a rental model with a *non*-anonymous buyer when the buyer has private information persistent across periods but his value is allowed to change each period. Our model is different from Hart and Tirole's rental model in two ways. First, we introduce an additional layer of uncertainty on the buyer's value distribution. The distribution may be either favorable or unfavorable. Both distributions can draw high value or low value, with the favorable distribution generating a high value with a higher probability. The buyer privately observes the distribution at the beginning of the game. But the seller only knows the ex ante probability of the two distributions. Second, the buyer's value is drawn from one of the two distributions independently across periods at the beginning of each period. Since the seller does not know which distribution the buyer's values are drawn from, the buyer's value is correlated across time periods from the seller's perspective. On one hand, our model maintains the feature that the buyer has strategic considerations across periods, which makes the problem still interesting and close to many real life examples where the bargaining parties are involving in a long-term relationship. On the other hand, we like to examine whether the relaxed assumption which allows the buyer's value to be redrawn is enough to provide a leeway to solve the problem of the durable-good monopolist, without necessarily assuming that the buyer's private information is completely independent across periods as in the case of anonymous buyer.

The assumption that the buyer's value distribution is uncertain can be illustrated in the following example. Imagine that an intermediate producer repeatedly rents a durable good from a monopolist, for example, a construction company rents big machines every time when a new project begins. The producer's value of consuming the durable good in each period depends on the quality of his final products in that period. The quality of the producer's final products depends on both the producer's technology and some random effects. The producer may have a superior technology or an inferior technology, and the probability of generating a high quality final product is higher with a superior technology. Both the technology and the quality of the final products are the producer's private information.

The main result we find is that the seller is indeed better off when she has sufficiently optimistic ex ante beliefs of the favorable distribution, compared to a two-period version of Hart and Tirole' (1988) model with the same ex ante probability of being a high-value buyer. The unique equilibrium outcome is for the seller to offer a high price and for the buyer with a high value to accept the offer in each period.[4] Intuitively, the buyer has no incentive to play strategically in the first period and simply adopts a strategy of truth-telling his value if the seller always offers a high price in the second period no matter whether the first-period offer is accepted or rejected. Given the buyer's strategy and that the two distributions have the same support, the seller is still optimistic enough to offer a high price after the first-period offer is rejected since her ex ante belief of the favorable distribution is sufficiently large.

When the seller has a moderate ex ante belief, however, the buyer does not always truthfully reveal his value. Multiple equilibria may exist, and some buyer types may also use a mixed strategy. The seller's revenue, however, can still be higher than that in Hart and Tirole (1988). Sufficient conditions for the seller to be better off are provided.

This paper is not the first one to introduce a buyer with randomly changing values. In the sale model, the Coase conjecture may fail to hold if we relax the assumption that the

[4]The equilibria in this paper refer to those that survive a refinement which is a variant of the D_1 criterion in signaling games (Cho and Kreps 1987, Banks and Sobel 1987). We discuss the equilibrium concept in more detail in section 2 and Appendix A.

buyer's valuation is fixed across periods.[5] Sobel (1991) shows that when there is a constant flow of new buyers, a Folk theorem holds, that is, any positive average profit less than the maximum feasible level can be attained. Blume (1990) examines a bargaining model where the low buyer type's value varies over time and the high buyer type's value stays fixed, and demonstrates that both uniqueness and Coase conjecture may fail to hold when valuations are allowed to vary randomly.

Two other papers also examine a rental model in which a non-anonymous buyer's value randomly changes over time.[6] Kennan (2001) analyzes infinitely repeated contract negotiations where the buyer has persistent (but not permanent) private information. The buyer's value is assumed to change according to a two-state Markov chain. Kennan (2001) focuses on the cyclic screening equilibria in which several pooling offers in sequence make the seller more and more optimistic and the seller makes an aggressive screening offer eventually.

Loginova and Taylor (2008) investigate a two-period model where the monopolist employs price experimentations to learn the permanent demand parameter of the buyer. Although we have benefited a lot from reading their paper, the two papers were developed independently and differ significantly in the modeling and results.

First, we assume that the value distribution may either be favorable or unfavorable, with the favorable distribution generating a high value with a higher probability. Loginova and Taylor (2008) assume that the value distribution is represented by λ, which is a continuous random variable distributed on $[0, 1]$, and a distribution represented by λ generates a high value with probability λ and a low value with probability $1 - \lambda$. We keep our model simpler so that we can completely characterize the equilibria and compare the seller's revenue with

[5] Failures of the Coase conjecture are also found when the lowest buyer valuation does not exceed the seller's cost, which is referred as the "no-gap" case in the literature (Gul et al. 1986, Ausubel and Deneckere 1989). Ausubel and Deneckere (1989) prove a Folk theorem similar to Sobel (1991).

[6] Several other papers also allow the buyer's valuations to vary over time. Blume (1998) and Battaglini (2005) study long-term contracting. Biehl (2001) analyzes a durable-goods model with anonymous buyers. Lemke (2004) presents a dynamic bargaining model in which actions in the last period affect the buyer's expected future value.

that in Hart and Tirole (1988).

Second, Loginova and Taylor (2008) assume that there is no discounting, while the discount rate is between 0 and 1 in our model. This difference has several effects on the results. First, Loginova and Taylor (2008) show that the buyer has a unique equilibrium strategy when the first-period offer is greater than the low value, by assuming the continuity of λ and no discounting. We achieve the same result after refinement. Multiple equilibrium strategies, however, reappear in Loginova and Taylor (2008) if discounting is introduced into the model. Second, Loginova and Taylor (2008) find that when all the low-value types accept an offer less than the low value, the seller never offers a first-period price that yields her valuable information about the buyer's permanent demand parament λ. In our model, the seller offers a price that yields valuable information if the discount rate is low enough or the seller's prior is high enough.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents players' equilibrium strategies in the second period. We discuss some preliminary results on screening of buyer types in Section 4. Section 5 presents the set of equilibria. Section 6 compares the seller's revenue in this model with that in Hart and Tirole (1988). Section 7 concludes. Appendix A provides a detailed discussion about equilibrium concept. All proofs are in Appendix B.

2 The Model

One buyer and one seller bargain over renting a durable good in two periods $t = 1, 2$. The seller's cost is assumed to be 0. The buyer has a positive value, v_t, in consuming the good in each period t. v_t is drawn from one of two distributions in each period: the F distribution or the G distribution. For a given distribution d, v_t equals h with probability q^d and equals l with probability $1 - q^d$. Assume that $0 < q^F < q^G < 1$ and $0 < l < h$. So the G distribution is more favorable since it has a higher probability of generating a value h. The buyer knows which one of the two distributions his values are actually drawn from as well as his current and former values. However, the seller only knows that the buyer's value is drawn each period from one of these two distributions. The ex ante probability is α for the G distribution and $1-\alpha$ for the F distribution, which is public information.

At the beginning of the game, the buyer privately observes the realization of distribution d, which is fixed throughout the game. At the beginning of each period t, the buyer's valuation v_t is drawn from the realized distribution d independently across time periods. After the buyer privately observes v_t, the seller offers a price $p_t \in \mathbb{R}$, and then the buyer chooses an action $a_t \in \{0, 1\}$, where $a_t = 1$ means acceptance and $a_t = 0$ means rejection.

Both the seller and the buyer are assumed to be risk-neutral. If the buyer accepts the seller's offer in period t, the buyer's payoff is $v_t - p_t$ and the seller's payoff is p_t in period t. They both gain nothing in period t if p_t is rejected. The two players share a common discount factor δ, and both of them maximize the discounted present value of expected payoffs.

$\theta_1 = (d, v_1)$ is referred as the buyer's type in period 1 and $\theta_2 = (d, v_1, v_2)$ as the buyer's type in period 2. Since we will focus on the buyer's first period behavior later, it is helpful to notice that there are four buyer types in period 1: (G, l), (F, l), (G, h), and (F, h). Denote h_t^S as the history observed by the seller before she announces p_t and h_t^B as the history observed by the buyer before he chooses a_t. Specifically, $h_1^S = \varnothing$, $h_1^B = (\theta_1, p_1)$, $h_2^S = (p_1, a_1)$ and $h_2^B = (\theta_2, p_1, a_1, p_2)$. A behavioral strategy for the seller, σ^S, assigns probability (or density) $\sigma^S(p_t \mid h_t^S)$ to p_t given any history h_t^S for $t = 1, 2$. A behavioral strategy for the buyer, σ^B, assigns probability $\sigma^B(a_t \mid h_t^B)$ to a_t given any history h_t^B for $t = 1, 2$. For convenience, let $\sigma^B(h_t^B) \equiv \sigma^B(a_t = 1 \mid h_t^B)$ denote the probability that the buyer accepts p_t given history h_t^B, since the buyer can only choose to accept or reject an offer.

Let $\gamma(h_t^S)$ denote the probability that the seller's belief assigns to the G distribution at the beginning of period t given history h_t^S. The seller's ex ante belief of the G distribution is α. Based on α, the seller forms her ex ante beliefs over the buyer's type θ_1. After offering p_1 and observing a_1, the seller updates her belief of θ_1, using Bayes' rule whenever possible. Then the seller's posterior belief of the G distribution is formed based on her posterior belief of θ_1. Notice that $\gamma(p_1, 0)$ and $\gamma(p_1, 1)$ denote the seller's belief of $d = G$ given that p_1 is rejected and accepted respectively.

The equilibrium concept used is strong Perfect Bayesian equilibrium. Specifically, Bayes' rule is used to update the seller's belief conditional on reaching any price p_1, even if p_1 is off the equilibrium path. We also employ a refinement which is a variant of criterion D_1 in the signalling game (Cho and Kreps 1987, Banks and Sobel 1987). In Appendix A, we formally define criterion D_1 and provide an example on how criterion D_1 can help select an equilibrium.

3 The Second-Period Equilibrium Strategies

We start the analysis from the second (last) period. Similar as in the previous literature, the equilibrium strategies in the last period are simple: the buyer accepts p_2 if and only if p_2 does not exceed v_2; the seller either offers $p_2 = l$ or $p_2 = h$ depending on whether her belief of $v_2 = h$ is less or greater than the cutoff belief l/h.[7] Since the seller's belief of $v_2 = h$ is $q^G\gamma + q^F(1-\gamma)$ given that her belief of $d = G$ is γ, the seller offers $p_2 = l$ or $p_2 = h$ depending on whether her belief of $d = G$ is less or greater than γ^*, where γ^* satisfies the equation $q^G\gamma^* + q^F(1-\gamma^*) = l/h$. Lemma 1 formally states the discussion above, in which $x(h_2^S)$ denotes the probability that the seller offers $p_2 = l$ following history h_2^S. Finally, to make the problem more interesting, we assume $q^F < l/h < q^G$.[8]

[7] As noted by Fudenberg and Tirole (1983), the buyer is indifferent between acceptance and rejection if $p_2 = v_2$, but on the equilibrium path the buyer accepts $p_2 = v_2$ with probability one. Given the buyer's strategy, it is only optimal for the seller to offer $p_2 = l$ or $p_2 = h$.

[8] The seller always offers $p_2 = h$ if the probability of drawing a value h from the F distribution q^F is greater than l/h, and always offers $p_2 = l$ if q^G is smaller than l/h, regardless of her belief of the buyer's value distribution. Only when $q^F < l/h < q^G$, the seller's strategy in the second period potentially responds to her belief of the distribution, and correspondingly the buyer has the strategic consideration across periods.

Lemma 1 *In any PBE, the buyer's strategy in the second period is*

$$\sigma^B(h_2^B) = \begin{cases} 1, & \textit{if } p_2 \leq v_2; \\ 0, & \textit{if } p_2 > v_2, \end{cases}$$

and the seller's strategy in the second period is

$$x(h_2^S) = \begin{cases} 1, & \textit{if } \gamma(h_2^S) < \gamma^*; \\ 0, & \textit{if } \gamma(h_2^S) > \gamma^*; \\ \in [0,1], & \textit{if } \gamma(h_2^S) = \gamma^*, \end{cases}$$

where $\gamma^* = (l/h - q^F)/(q^G - q^F)$.

4 Screening

In this section we discuss some preliminary observations on what kind of screening of buyer types happens in equilibrium. Recall that there are four buyer types in the first period: (G, l), (G, h), (F, l), and (F, h). The first observation is that the seller can never offer a first-period price which separates one buyer type, say (G, h), from the other three types (i.e., (G, l), (F, h), and (F, l)) in equilibrium. In some cases, however, the seller's first-period offer can separate the two buyer types with low value from the two types with high value (i.e. (G, h) and (F, h) vs. (G, l) and (F, l)). Observation 2 shows that if this kind of screening happens, the seller must be more optimistic when p_1 is accepted than rejected. Following that we further discuss in which cases the seller offers $p_2 = l$ with a higher probability when p_1 is accepted than rejected. To do that, we first divide the seller's prior belief into different ranges. We find that the seller offers $p_2 = l$ with a higher probability after acceptance than rejection of p_1 only when she has a moderate prior belief, however, when she has an extreme prior belief, she offers $p_2 = l$ with a same probability following acceptance and rejection of p_1.

We start with defining the cutoff value for each buyer type, which the buyer type is indifferent between accepting and rejecting. From the players' equilibrium strategies in the second period, the buyer's expected payoff from accepting p_1 is $v_1 - p_1 + \delta q^d x(p_1, 1)(h - l)$, where $v_1 - p_1$ is the buyer's payoff in the first period from accepting p_1 and $\delta q^d x(p_1, 1)(h-l)$ is the expected payoff in the second period. Recall that q^d is the probability for the buyer type to draw an h value and $x(p_1, 1)$ is the probability for the seller to offer $p_2 = l$ after acceptance of p_1. Correspondingly, the buyer's expected payoff from rejecting p_1 is $\delta q^d x(p_1, 0)(h - l)$, where $x(p_1, 0)$ is the probability for the seller to offer $p_2 = l$ after rejection of p_1. By comparing the payoffs from accepting and rejecting p_1, the buyer type (d, v_1) accepts p_1 with probability one if p_1 is smaller than $v_1 + \delta q^d[x(p_1, 1) - x(p_1, 0)](h - l)$ and rejects p_1 with probability one if it is greater than $v_1 + \delta q^d[x(p_1, 1) - x(p_1, 0)](h - l)$.

Definition 1 *Define $C(d, v_1) \equiv v_1 + \delta q^d[x(p_1, 1) - x(p_1, 0)](h - l)$ as the* Cutoff Value *for buyer type $\theta_1 = (d, v_1)$ given $x(p_1, 0)$ and $x(p_1, 1)$.*

Lemma 2 *In any PBE, the probability for buyer type $\theta_1 = (d, v_1)$ to accept p_1 is*

$$\sigma^B(\theta_1, p_1) = \begin{cases} 1, & \text{if } p_1 < C(d, v_1); \\ 0, & \text{if } p_1 > C(d, v_1); \\ \in [0, 1], & \text{if } p_1 = C(d, v_1). \end{cases}$$

By definition the buyer's cutoff value depends on his type (d, v_1) as well as the seller's second-period strategy. Figure 1 below describes the order of all buyer types' cutoff values regarding different strategies the seller may use in the second period. We can see that the buyer types with $v_1 = l$ always have a smaller cutoff value than types with $v_1 = h$. This is robust regardless of the value distribution d and the seller's strategy in the second period. While, if we consider the cutoff values of two buyer types who have the same v_1 but draw from different distributions, say (F, l) and (G, l), the order depends on whether the seller offers $p_2 = l$ with a higher probability after acceptance of p_1 or rejection of p_1. If the seller offers $p_2 = l$ with a higher probability when p_1 is accepted (i.e., $x(p_1, 0) < x(p_1, 1)$), type (G, l) has a larger cutoff value than type (F, l). On the opposite, if the seller offers $p_2 = l$

with a larger probability when p_1 is rejected than accepted (i.e., $x(p_1, 0) > x(p_1, 1)$), type (G, l) has a smaller cutoff value. The basic intuition is that, since type (G, l) potentially has a larger payoff in the second period, he is more willing to take an action that makes the seller pessimistic than type (F, l).

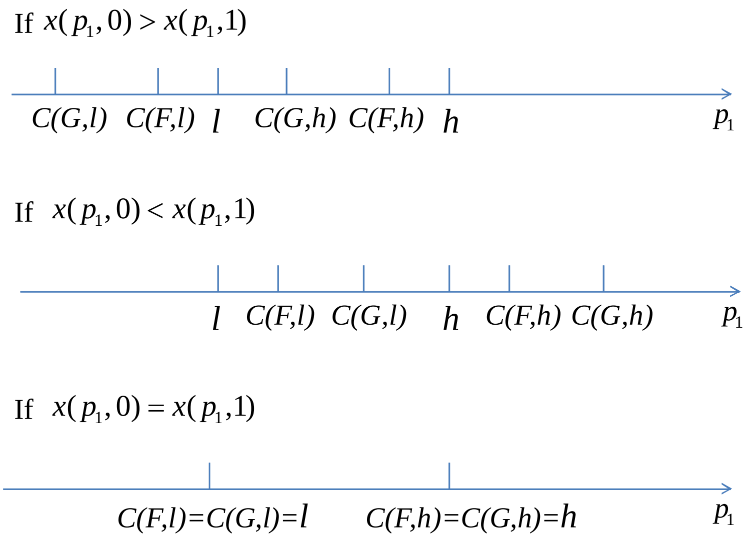

Figure 1: The Order of Cutoff Values

Next we discuss what kind of screening is possible (impossible) to happen in equilibrium. Observation 1 says that the seller can never offer a price which screens one buyer type, say (F, h), from the other three types, i.e., (G, l), (G, h), and (F, l), in equilibrium.

Observation 1: No first-period offer can screen one buyer type from the other three types in equilibrium.

To see why no separation between one buyer type and the other three types is possible, consider the following hypothetic strategy. Suppose that the seller chooses to offer a low price if the first-period price is rejected and offer a high price if it is accepted. Then for a first-period price which is below but still close to h, type (G, h) (as well as types with $v_1 = l$)

chooses to reject the offer in order to get a low price in the second period, while type (F, h) chooses to accept the offer since his chances of still having an h value in the second period are very low. However, the seller then posteriorly knows type (F, h) is the only type that accepts the offer, so she will choose to offer a low price after acceptance of p_1, which is contradictory to the proposed strategy. Using the same reasoning, we can easily show that the seller cannot screen any single buyer type out.

The key point here is that the buyer types from the G distribution have a larger incentive to take the action that induces the seller to offer a low price in the second period, the seller however becomes extremely optimistic if that action is taken by a single type drawing from the G distribution. Thus it reaches a contradiction and cannot happen in equilibrium. Importantly, Observation 1 also implies that the seller is not able to identify the buyer's distribution in equilibrium, given that the cutoff values of both l-value types are smaller than the cutoff values of both h-value types.

It turns out, however, that partial screening may still happen in equilibrium. Specifically, one feature of the equilibria is the screening between the buyer types with $v_1 = l$ and the buyer types with $v_1 = h$. The next three observations discuss this case in more details. Observation 2 shows that if the seller offers a price which screens low value buyer types and high value types, then the probability for the seller to offer $p_2 = l$ after rejection of p_1 must be at least as high as after acceptance of p_1.[9] In other words, the seller is more optimistic when p_1 is accepted than rejected. This observation is easy to verify. Since the cutoff values of both l-value types are smaller than those of both h-value types, a price can screen the l-value types from the h-value types only if the l-value types reject the offer and the h-value types accept the offer. Given that the favorable distribution G has a higher probability of generating an h value, the seller must be more optimistic after acceptance of p_1 than rejection of p_1.

[9] Actually a stronger result holds: as long as it is on the equilibrium path of the continuation game following p_1, then $x(p_1, 0) \geq x(p_1, 1)$. See Lemma 5 in Appendix B.

Observation 2: If a first-period offer screens the two buyer types with $v_1 = l$ from the two buyer types with $v_1 = h$, then the seller offers $x(p_1, 0) \geq x(p_1, 1)$ in the second period.

Next we explore how the seller's prior beliefs are related to her strategy in the second period. Given that the screening between the l-value types and the h-value types is an important feature of the equilibria, we classify the seller's ex ante beliefs according to whether the seller's posterior beliefs conditional on $v_1 = l$ and $v_1 = h$ are greater or less than γ^*, which is the cutoff belief for the seller to decide whether to offer $p_2 = l$ or $p_2 = h$.

Define functions

$$\overline{\gamma}(\alpha) \equiv \frac{\alpha q^G}{\alpha q^G + (1-\alpha) q^F},$$

and

$$\underline{\gamma}(\alpha) \equiv \frac{\alpha(1-q^G)}{\alpha(1-q^G) + (1-\alpha)(1-q^F)}.$$

Based on Bayes' rule, $\overline{\gamma}(\alpha)$ and $\underline{\gamma}(\alpha)$ are the seller's posterior beliefs of the G distribution conditional on $v_1 = h$ and $v_1 = l$ respectively.

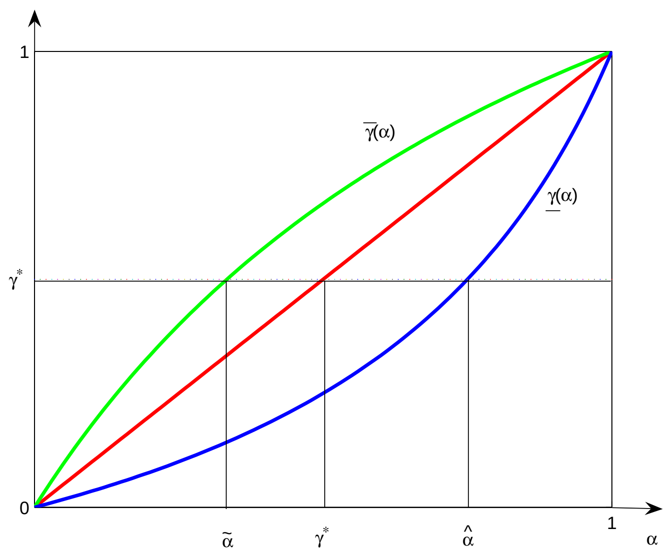

Figure 2: $\overline{\gamma}(\alpha)$ and $\underline{\gamma}(\alpha)$ with $q^F = 0.4$, $q^G = 0.8$, and $l/h = 0.6$

Figure 2 plots the functions $\overline{\gamma}(\alpha)$ and $\underline{\gamma}(\alpha)$ with $q^F = 0.4$, $q^G = 0.8$, and $l/h = 0.6$. In Figure 2, the x axis is the seller's ex ante belief α, $\widetilde{\alpha} \equiv \overline{\gamma}^{-1}(\gamma^*)$ and $\widehat{\alpha} \equiv \underline{\gamma}^{-1}(\gamma^*)$.[10] Since the favorable distribution G has a higher probability of generating a high value, the seller becomes more pessimistic when the buyer's behavior suggests $v_1 = l$, so the curve $\underline{\gamma}(\alpha)$ is below the 45^0 line. On the contrary, the seller becomes more optimistic conditional on $v_1 = h$, and therefore the curve $\overline{\gamma}(\alpha)$ is above the 45^0 line. Furthermore, when the seller has an *extreme* ex ante belief, which is smaller than $\widetilde{\alpha}$ or greater than $\widehat{\alpha}$, her posterior beliefs conditional on $v_1 = h$ and $v_1 = l$ are both below or above γ^*. In that case, the seller offers the same price even if the first-period offer screens l-value from h-value types. However, when the seller has a *moderate* ex ante belief which is between $\widetilde{\alpha}$ and $\widehat{\alpha}$, her posterior belief conditional on $v_1 = h$ is above γ^* and her belief conditional on $v_1 = l$ is below γ^*. So the seller offers different prices when the buyer's behavior suggests $v_1 = l$ or $v_1 = h$.

Observation 3: If a first-period offer screens the two buyer types with $v_1 = l$ from the two buyer types with $v_1 = h$, then the seller offers $x(p_1, 0) = 1$ and $x(p_1, 1) = 0$ in the second period when she has a moderate ex ante belief.

Observation 4: If a first-period offer screens the two buyer types with $v_1 = l$ from the two buyer types with $v_1 = h$, then the seller offers $x(p_1, 0) = x(p_1, 1)$ in the second period when she has an extreme ex ante belief.

According to the seller's ex ante belief of the G distribution, we define a seller *Pessimistic* if $0 < \alpha < \widetilde{\alpha}$, *Moderately Pessimistic* if $\widetilde{\alpha} < \alpha < \gamma^*$, *Moderately Optimistic* if $\gamma^* < \alpha < \widehat{\alpha}$, and *Optimistic* if $\widehat{\alpha} < \alpha < 1$. As in the previous literature, the knife-edge cases are omitted.

[10] Both $\overline{\gamma}(\alpha)$ and $\underline{\gamma}(\alpha)$ are continuous and increasing in α; $\underline{\gamma}(\alpha) < \alpha < \overline{\gamma}(\alpha)$ for $\alpha \in (0, 1)$; $\underline{\gamma}(\alpha) = \overline{\gamma}(\alpha) = \alpha$ for $\alpha \in \{0, 1\}$. So $\widetilde{\alpha}$ and $\widehat{\alpha}$ are well-defined and $\widetilde{\alpha} < \gamma^* < \widehat{\alpha}$.

5 The Equilibria

In this section we present the equilibria of the game with respect to different ranges of the seller's ex ante belief. When the equilibrium price in the first period is accepted by all buyer types, we call the equilibrium pooling. When the first-period offer is accepted and rejected both by more than one buyer type, we call the equilibrium semi-separating.

5.1 Seller with Extreme Ex Ante Beliefs

When the seller has a pessimistic or an optimistic belief, the buyer's strategy is to truthfully reveal his value in both periods: accept an offer no greater than his value and reject an offer otherwise.[11] Given that the two buyer types who have the same v_1 but draw from different distributions behave the same, the seller can only distinguish the buyer's value v_1 but cannot tell the buyer's distribution d. As demonstrated in Figure 2, the seller's posterior beliefs conditional on $v_1 = l$ and $v_1 = h$ are both below (above) the cutoff belief γ^* when she has a pessimistic (an optimistic) ex ante belief. Therefore, a pessimistic seller always offers $p_2 = l$ and an optimistic seller always offers $p_2 = h$ given the buyer's equilibrium strategy. Since the seller's second-period offer does not depend on the buyer's acceptance/rejection action in the first period, it can be verified that truth-telling is the buyer's equilibrium strategy in the first period.

When the seller has a pessimistic ex ante belief, she always offers $p_t = l$ in both periods on the equilibrium path and all buyer types accept the offers.

Proposition 1 (Pessimistic Seller) *When the seller is pessimistic* $(0 < \alpha < \widetilde{\alpha})$, *there is a unique* D_1 *equilibrium outcome: the seller offers* $p_t = l$ *and all buyer types accept* p_t, *for* $t = 1, 2$.

[11] Although the buyer has a unique equilibrium strategy when the seller has extreme ex ante beliefs, multiple equilibria arise since different beliefs can be assigned after all buyer types reject $p_1 \leq l$ or after all buyer types accept $p_1 > h$ to support the unique D_1 equilibrium outcome presented in Proposition 1 and 2.

When the seller has an optimistic ex ante belief, she always offers $p_t = h$ on the equilibrium path and the h-value types accept the offer in each period.

Proposition 2 (Optimistic Seller) *When the seller is optimistic* ($\widehat{\alpha} < \alpha < 1$), *there is a unique* D_1 *equilibrium outcome: the seller offers* $p_t = h$, *the buyer types with* $v_t = h$ *accept* p_t *and the buyer types with* $v_t = l$ *reject* p_t, *for* $t = 1, 2$.

The outcome in Proposition 2 is of particular interest to us. In the model of Hart and Tirole (1988), in which the buyer's value is private information but the value distribution is common knowledge, it does not happen in any equilibrium that the h-value buyer accepts $p_1 = h$ with probability one, even if the seller has a very optimistic ex ante belief of the buyer's value. The intuition is that, the seller will offer $p_2 = l$ after rejection if the h-value buyer accepts $p_1 = h$ with probability one, and then the h-value buyer has an incentive to deviate to reject $p_1 = h$. Therefore, when the seller has a sufficiently optimistic ex ante belief, introducing the uncertainty about the buyer's value distribution improves the seller's revenue. We will discuss the comparison of expected revenue between our model and Hart and Tirole's (1988) in more detail in Section 6.

5.2 Seller with Moderate Ex Ante Beliefs

Different from the case when the seller has an extreme ex ante belief, the buyer's strategy has the following features when the seller's ex ante belief is moderate. First, the buyer does not always truthfully reveal his value anymore. From Figure 2 and Observation 3, the seller offers high price in the second period conditional on buyer's behavior suggesting $v_1 = h$ and low price conditional on $v_1 = l$. This gives the l-value buyer types an incentive to signal their current values, and at the same time, gives the h-value buyer types an incentive to conceal their first-period value. Therefore, the highest price for a buyer type to accept can be lower than his value. Second, the buyer has multiple equilibrium strategies instead of unique equilibrium strategy. In particular, the two l-value buyer types may reject or accept p_1 less than l but close to l in equilibrium. Intuitively, the l-value types can signal their value by rejecting such a p_1 and have a low offer in the second period. On the other hand, it is also optimal for the l-value types to accept such a p_1 if the buyer believes that the seller offers a same price after acceptance and rejection of p_1. Finally, for some particular ranges of p_1 some buyer types may use a mixed strategy instead of a pure strategy. Figure 3 captures these features of the buyer's strategy.

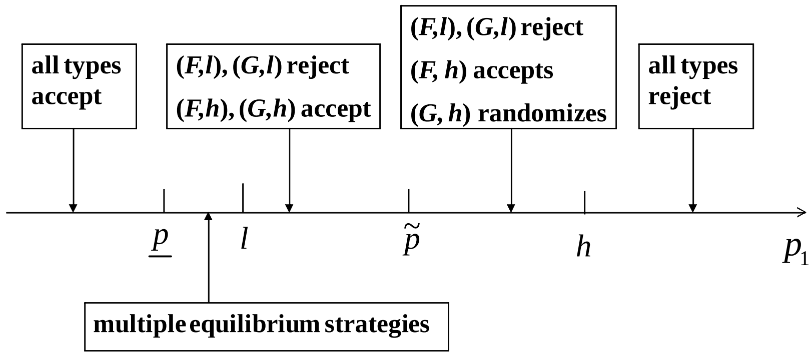

Figure 3: Buyer's Equilibrium Strategy when $\widetilde{\alpha} < \alpha < \widehat{\alpha}$

In Figure 3, all buyer types accept $p_1 \leq \underline{p} \equiv l - \delta q^F(h-l)$ and reject $p_1 > h$. The l-value types always reject $p_1 > l$. The h-value types always accept $p_1 \leq \widetilde{p} \equiv h - \delta q^G(h-l)$.

There are multiple equilibrium strategies for $p_1 \in (\underline{p}, l]$. In particular, the l-value types may reject p_1, which is less than l, in order to signal their current value and have a low price in the second period. The cutoff price $\underline{p} = l - \delta q^F(h-l)$ is the highest price that the buyer type (F, l) is willing to accept given that the seller offers $p_2 = l$ after rejection and $p_2 = h$ after acceptance. On the other hand the l-value types may also accept $p_1 \in (\underline{p}, l]$. The seller can always assign the same posterior belief after rejection as after acceptance of $p_1 \leq l$ given that all buyer types accept p_1, then all buyer types should accept p_1 less than v_1. Finally, it is also part of an equilibrium strategy that buyer type (G, l) rejects $p_1 \in (\underline{p}, l]$, the buyer types with $v_1 = h$ accept $p_1 \in (\underline{p}, l]$, and buyer type (F, l) plays mixed strategy.[12]

For $p_1 \in (\widetilde{p}, h]$ where $\widetilde{p} = h - \delta q^G(h-l)$, the buyer types with $v_1 = l$ reject p_1, buyer type (F, h) accepts p_1, and buyer type (G, h) plays mixed strategy.[13] To see the intuition, suppose all the buyer types reject the offer, then buyer type (F, h) has an incentive to deviate to accept p_1 since he gains a positive payoff in the first period and gets the low offer in the second period by revealing his distribution. Given that buyer type (F, h) accepts p_1, buyer type (G, h) has an incentive to accept p_1 as well. However, the seller's posterior belief after acceptance of p_1 is then above the cutoff belief γ^* since her ex ante belief is moderate. The seller then offers $p_2 = l$ after rejection and $p_2 = h$ after acceptance of p_1. Since p_1 is greater than $\widetilde{p}$, buyer type (G, h) then has an incentive to reject p_1. Therefore, buyer type (G, h) randomizes to accept and reject $p_1 \in (\widetilde{p}, h]$ in equilibrium.

Lemma 3 summarizes the buyer's strategy.

Lemma 3 *When the seller has a moderate prior belief ($\widetilde{\alpha} < \alpha < \widehat{\alpha}$), the buyer's strategy in a D_1 equilibrium is as follows*[14]*:*

(i) if $p_1 \leq \underline{p}$, all buyer types accept p_1;

(ii) if $\underline{p} < p_1 \leq l$, (1) all buyer types accept p_1; or (2) types (F, l) and (G, l) reject p_1, and types (F, h) and (G, h) accept p_1; or (3) type (G, l) rejects p_1, type (F, l) randomizes, and types (G, h) and (F, h) accept p_1;

(iii) if $l < p_1 \leq \widetilde{p}$, types (F, l) and (G, l) reject p_1, and types (F, h) and (G, h) accept p_1;

(iv) if $\widetilde{p} < p_1 \leq h$, types (F, l) and (G, l) reject p_1, type (G, h) randomizes, and type (F, h) accepts p_1;

[12] Notice that when $x(p_1, 0) > x(p_1, 1)$, the cutoff value of type (G, l) is smaller than that of type (F, l), so type (G, l) rejects any $p_1 \in (p_1, l]$ if type (F, l) rejects it. Therefore the cutoff price $\underline{p}$ is derived from the incentive constraint of type (F, l). Furthermore, type (F, l) is the one who may play mixed strategy.

[13] For the similar reason as for type (F, l), the cutoff value of type (G, h) is smaller than that of type (F, h) when $x(p_1, 0) > x(p_1, 1)$, so type (F, h) accepts any p_1 less than $\widetilde{p}$ if type (G, h) accepts it. Therefore the cutoff price $\widetilde{p}$ is derived from the incentive constraint of type (G, h), and type (G, h) is the one who plays mixed strategy.

[14] We require that the buyer's strategy is left continuous for the cutoff prices $p_1 \in \{\underline{p}, l, \widetilde{p}, h\}$, that is, the behavioral strategy following the cutoff prices are the same as the strategy following $p_1 - \epsilon$.

(v) if $p_1 > h$, all buyer types reject p_1.

There are several points we find important about the equilibrium strategies presented above. First, although some buyer types strategically reject an offer less than their first-period value, all buyer types truthfully reject $p_1 > v_1$, and the buyer never incurs a loss in any period in equilibrium. This is because the seller always gets more optimistic when p_1 is accepted than rejected, so no buyer type has an incentive to accept an offer larger than his value.

Second, the highest first-period offer accepted by all buyer types may be less than the buyer's lowest value l. This feature is also found by Blume (1990), Kennan (2001), and Loginova and Taylor (2008). In all these models including ours, a buyer type with a low value in the current period has a positive probability of drawing a high value in the next period, so the low-value type will reject an offer less than but close to the low value if rejection can help the seller price discriminate in the next period.

Finally, the equilibrium strategies in this paper are different from those in Kennan (2001). In Kennan (2001), the buyer's value changes according to a Markov process, so the seller's posterior belief becomes more optimistic when all buyer types accept a pooling offer, and the seller offers an aggressive screening offer following acceptance of several pooling offers when her posterior belief grows beyond some cutoff. This pattern is described as a cyclic equilibrium. In our model, the seller's posterior belief is the same as her ex ante belief after acceptance of a pooling offer. So we do not expect that the same pattern as in the cyclic equilibrium emerges in this model, even in a longer horizon.

Next we discuss the seller's optimal p_1 and conclude by describing the equilibria of the game for moderately pessimistic and moderately optimistic seller respectively. All cases presented below in Proposition 3-6 arise for a non-negligible set of parameters.[15]

[15] This is proved using Mathematica. The program is available upon request.

5.2.1 Moderately Pessimistic Seller ($\widetilde{\alpha} < \alpha < \gamma^*$)

We first provide the seller's payoffs from offering the cutoff prices $p_1 \in \{\underline{p}, l, \widetilde{p}, h\}$ according to Lemma 3, and then discuss the conditions for there to exist pooling and semi-separating equilibria for the seller with moderately pessimistic ex ante beliefs.

(1) $p_1 = \underline{p}$: The seller can always guarantee payoff U_1 by offering $p_1 = \underline{p}$ and $p_2 = l$, with p_1 and p_2 accepted by all buyer types. U_1 is the seller's lowest payoff from a pooling offer.

$$U_1 = \underline{p} + \delta l;$$

(2) $p_1 = l$: Since there are multiple equilibrium strategies for the buyer following $p_1 \in (\underline{p}, l]$, the seller's payoff from offering $p_1 = l$ depends on which strategy the buyer uses. Suppose that all buyer types choose to accept $p_1 = l$, then the seller can achieve the highest payoff from a pooling offer, U_2, by offering $p_1 = p_2 = l$ with p_1 and p_2 accepted by all buyer types.

$$U_2 = l + \delta l;$$

If buyer type (G, l) rejects p_1, buyer type (F, l) randomizes, and buyer types (F, h) and (G, h) accept p_1, then the seller's payoff from offering $p_1 = l$ is U_3.

$$U_3 = [\alpha q^G + (1-\alpha)q^F + (1-\alpha)(1-q^F)(1-X^*)]l + \delta l,$$

where X^* is the probability that buyer type (F, l) randomizes to reject p_1 to make the seller indifferent in offering $p_2 = l$ or $p_2 = h$ after acceptance of p_1,[16]

$$\gamma(p_1, 1) = \frac{\alpha q^G}{\alpha q^G + (1-\alpha)q^F + (1-\alpha)(1-X^*)(1-q^F)} = \gamma^*;$$

Notice that if both l-value buyer types choose to reject and both h-value buyer types choose to accept $p_1 = l$, then offering $p_1 = l$ is dominated by offering $p_1 = \widetilde{p}$.

[16] Since the seller is more optimistic after acceptance of p_1 than rejection of p_1, her posterior belief $\gamma(p_1, 1) > \gamma(p_1, 0)$. For a moderately pessimistic seller whose ex ante belief α is less than γ^*, it must be the case that the seller is indifferent after acceptance of p_1, i.e., $\gamma(p_1, 1) = \gamma^*$, when the buyer randomizes.

(3) $p_1 = \widetilde{p}$: Buyer types (F, l) and (G, l) reject p_1, buyer types (F, h) and (G, h) accept p_1, and the seller offers $p_2 = l$ if p_1 is rejected and $p_2 = h$ if p_1 is accepted.

$$\begin{aligned} U_4 &= [\alpha q^G + (1-\alpha)q^F]\widetilde{p} + \delta[\alpha(q^G)^2 + (1-\alpha)(q^F)^2]h \\ &\quad + \delta[\alpha(1-q^G) + (1-\alpha)(1-q^F)]l; \end{aligned}$$

(4) $p_1 = h$: The unique equilibrium strategy is for buyer type (G, h) to randomize, buyer type (F, h) to accept p_1, and buyer types (F, l) and (G, l) to reject p_1, then the seller's payoff is U_5.

$$U_5 = [\alpha q^G(1 - Y^*) + (1-\alpha)q^F]h + \delta l,$$

where Y^* is the probability that buyer type (G, h) randomizes to reject p_1 to make the seller indifferent in offering $p_2 = l$ or $p_2 = h$ after acceptance of p_1,[17]

$$\gamma(p_1, 1) = \frac{\alpha(1-Y^*)q^G}{\alpha(1-Y^*)q^G + (1-\alpha)q^F} = \gamma^*.$$

When the seller has moderately pessimistic ex ante beliefs, we find that there always exists a pooling equilibrium with $p_1 = l$ on the equilibrium path, since the highest payoff from a pooling offer, i.e. U_2, is always greater than the highest payoff from a semi-separating offer, i.e. $\max\{U_3, U_4, U_5\}$. Surprisingly, this result implies that the payoffs for a moderately pessimistic seller is no better than those for a pessimistic seller.

Recall that there are multiple equilibrium strategies for the l-value buyer types given $p_1 \in [\underline{p}, l]$: both l-value types accept p_1 or both l-value types reject p_1. When the lowest payoff from a pooling offer, i.e. U_1, is greater than $\max\{U_4, U_5\}$, any $p_1^* \in [\underline{p}, l]$ can arise in a pooling equilibrium, with both l-value buyer types accepting $p_1 \leq p_1^*$ and rejecting $p_1 > p_1^*$. When U_1 is less than $\max\{U_4, U_5\}$, we can find a pooling offer $p' \in [\underline{p}, l]$ which gives the seller the same payoff as $\max\{U_4, U_5\}$. Then any $p_1^* \in [p', l]$ can arise in a pooling equilibrium, with the l-value buyer types accepting $p_1 \leq p_1^*$ and rejecting $p_1 > p_1^*$.

[17] Similar to the case where buyer type (F, l) randomizes, the seller is indifferent after acceptance of p_1.

Proposition 3 (MP Seller: Pooling Equilibrium) *When the seller is moderately pessimistic, there always exists a pooling D_1 equilibrium with $p_1 = l$.*

(i) If $U_1 > \max\{U_4, U_5\}$, any $p_1 \in [\underline{p}, l]$ can arise in a pooling equilibrium;

(ii) If $U_1 < \max\{U_4, U_5\}$, any $p_1 \in [p', l]$, with $\underline{p} < p' < l$, can arise in a pooling equilibrium.

Proposition 4 presents the conditions for semi-separating D_1 equilibria. If the lowest payoff from a pooling offer U_1 is greater than the highest payoff from a semi-separating offer, then there is no semi-separating equilibrium. On the contrary, if U_1 is less than the highest payoff from a semi-separating offer, then semi-separating equilibria exist. Furthermore, if $p_1 = \widetilde{p}$ or $p_1 = h$ gives the highest payoff among all semi-separating offers, the equilibrium path of the semi-separating equilibria is unique. If $p_1 = l$ gives the highest semi-separating payoff, then a continuum equilibrium price $p_1 \in [p'', l]$, with $\underline{p} < p'' < l$, arises.

Proposition 4 (MP Seller: Semi-separating Equilibrium) *When the seller is moderately pessimistic, the semi-separating D_1 equilibria are characterized as follows.*

(i) If $U_1 > \max\{U_3, U_4, U_5\}$, no semi-separating equilibrium exists;

(ii) If $U_1 < \max\{U_3, U_4, U_5\} = \max\{U_4, U_5\}$, semi-separating equilibria exist and the path is unique, with $p_1 = \widetilde{p}$ or $p_1 = h$;

(iii) If $U_1 < \max\{U_3, U_4, U_5\} = U_3$, any $p_1 \in [p'', l]$, with $\underline{p} < p'' < l$, can arise in a semi-separating equilibrium, so does $p_1 = \widetilde{p}$ or $p_1 = h$ if $\max\{U_4, U_5\} > U_1$.

5.2.2 Moderately Optimistic Seller ($\gamma^* < \alpha < \widehat{\alpha}$)

In this subsection we discuss the pooling equilibria and semi-separating equilibria for a seller with moderately optimistic ex ante beliefs. Similar to last subsection, we start with the seller's payoffs from offering the cutoff prices $p_1 \in \{\underline{p}, l, \widetilde{p}, h\}$.

(1) $p_1 = \underline{p}$: V_1 is the seller's payoff from offering $p_1 = \underline{p}$ and $p_2 = h$, with p_1 accepted by all buyer types and p_2 accepted by types with $v_2 = h$. Notice that a moderately optimistic seller

offers $p_2 = h$ when all buyer types accept p_1, which is different from a moderately pessimistic seller. V_1 is the seller's lowest payoff from a pooling offer.

$$V_1 = \underline{p} + \delta[\alpha q^G + (1-\alpha)q^F]h;$$

(2) $p_1 = l$: V_2 is the seller's payoff from offering $p_1 = l$ and $p_2 = h$, with p_1 accepted by all buyer types and p_2 accepted by types with $v_2 = h$. V_2 is the seller's highest payoff from a pooling offer.

$$V_2 = l + \delta[\alpha q^G + (1-\alpha)q^F]h;$$

V_3 is the seller's payoff from offering $p_1 = l$, buyer type (G, l) rejects p_1, buyer type (F, l) randomizes, and buyer types (F, h) and (G, h) accept p_1.

$$V_3 = [\alpha q^G + (1-\alpha)q^F + (1-\alpha)(1-q^F)(1-X^{**})]l + \delta[\alpha q^G + (1-\alpha)q^F]h,$$

where X^{**} is the probability that buyer type (F, l) randomizes to reject p_1 to make the seller indifferent in offering $p_2 = l$ or $p_2 = h$ after rejection of p_1,[18]

$$\gamma(p_1, 0) = \frac{\alpha(1-q^G)}{\alpha(1-q^G) + (1-\alpha)X^{**}(1-q^F)} = \gamma^*;$$

(3) $p_1 = \widetilde{p}$: $V_4 = U_4$ is the seller's payoff when buyer types (F, l) and (G, l) reject p_1, buyer types (F, h) and (G, h) accept p_1, and the seller offers $p_2 = l$ if p_1 is rejected and $p_2 = h$ if p_1 is accepted.

$$\begin{aligned} V_4 &= [\alpha q^G + (1-\alpha)q^F]\widetilde{p} + \delta[\alpha(q^G)^2 + (1-\alpha)(q^F)^2]h \\ &\quad +\delta[\alpha(1-q^G) + (1-\alpha)(1-q^F)]l; \end{aligned}$$

(4) $p_1 = h$: V_5 is the seller's payoff from offering $p_1 = h$, buyer type (G, h) randomizes, buyer type (F, h) accepts p_1, and buyer types (F, l) and (G, l) reject p_1.

$$V_5 = [\alpha q^G(1-Y^{**}) + (1-\alpha)q^F]h + \delta[\alpha q^G + (1-\alpha)q^F]h,$$

[18] When the seller's ex ante belief is moderately optimistic, the seller is indifferent after rejection of p_1, i.e., $\gamma(p_1, 0) = \gamma^*$, in order to satisfy the posterior belief $\gamma(p_1, 1) > \gamma(p_1, 0)$ and $\alpha > \gamma^*$.

where Y^{**} is the probability that buyer type (G, h) randomizes to reject p_1 to make the seller indifferent in offering $p_2 = l$ or $p_2 = h$ after rejection of p_1,

$$\gamma(p_1, 0) = \frac{\alpha Y^{**} q^G + \alpha(1 - q^G)}{\alpha Y^{**} q^G + \alpha(1 - q^G) + (1 - \alpha)(1 - q^F)} = \gamma^*.$$

The proof of next two propositions are omitted since it is similar to the proof of Proposition 3 and 4.

Proposition 5 (MO Seller: Pooling Equilibrium) *When the seller is moderately optimistic, the pooling D_1 equilibria are characterized as follows.*

(i) If $V_1 > \max\{V_4, V_5\}$, any $p_1 \in [\underline{p}, l]$ can arise in a pooling equilibrium;

(ii) If $V_1 < \max\{V_4, V_5\} < V_2$, any $p_1 \in [p''', l]$, with $\underline{p} < p''' < l$, can arise in a pooling equilibrium;

(iii) If $V_2 < \max\{V_4, V_5\}$, no pooling equilibrium exists.

Different from the results for a moderately pessimistic seller, case (iii) in Proposition 5 implies that it is possible for a semi-separating equilibrium to emerge even if all buyer types accept $p_1 \in (\underline{p}, l]$. That is, when the seller's ex ante belief is sufficiently optimistic, the best pooling offer does not necessarily arise as an equilibrium price. This finding is different from that of Loginova and Taylor (2008). They argue that the seller never offers a first-period price that yields valuable information about the buyer's distribution in a Good equilibrium where all buyer types accept p_1 less than l. But that conclusion depends on the assumption of no discounting. We find that case (iii) of Proposition 5 arises for a non-negligible set of parameters when the discount factor is sufficiently low.

When the seller is moderately optimistic, the conditions for the semi-separating D_1 equilibria are similar to those when the seller is moderately pessimistic.

Proposition 6 (MO Seller: Semi-separating Equilibrium) *When the seller is moderately optimistic, the semi-separating D_1 equilibria are characterized as follows.*

(i) If $V_1 > \max\{V_3, V_4, V_5\}$*, no semi-separating equilibrium exists;*

(ii) If $V_1 < \max\{V_3, V_4, V_5\} = \max\{V_4, V_5\}$*, a semi-separating equilibrium exists and the path is unique, with* $p_1 = \widetilde{p}$ *or* $p_1 = h$*;*

(iii) If $V_1 < \max\{V_3, V_4, V_5\} = V_3$*, any* $p_1 \in [p'''', l]$*, with* $\underline{p} < p'''' < l$*, can arise in a semi-separating equilibrium, so does* $p_1 = \widetilde{p}$ *or* $p_1 = h$ *if* $\max\{V_4, V_5\} > V_1$*.*

6 Comparison of Expected Revenue

The most important question that this paper is concerned with is whether the seller improves her revenue and gains more monopoly power when the value distribution is the buyer's private information and the buyer's values are drawn from the distribution each period. In this section, we address this issue by comparing the seller's expected revenue in our model with that in the two-period version of Hart and Tirole's (1988) rental model in which the buyer's value distribution is common knowledge.

The two-period version of Hart and Tirole's (1988) rental model is as follows. The buyer has private information about his value, which can be either high or low. The buyer's value is drawn at the beginning of the game and is fixed once realized. In each period $t = 1$ or 2, the seller offers a rental price and the buyer decides to accept or reject the offer. Let μ denote the seller's ex ante belief that she is facing a high-value buyer.

In order to make a fair comparison, we require that the ex ante probabilities of the high-value buyer in both models be equal, that is, $\mu = \alpha q^G + (1 - \alpha) q^F$, where α is the seller's ex ante belief of the G distribution in our model.

The following proposition compares the expected revenues in the equilibria of the two models for any ex ante belief the seller can have. When the seller has an optimistic ex ante belief, her revenue in our model is higher than that in Hart and Tirole's (1988). As shown in Proposition 2, the buyer types with $v_1 = h$ accept $p_1 = h$ with probability one since the seller offers $p_2 = h$ independent of whether p_1 is accepted or rejected. In contrast, in the two-period version of Hart and Tirole's (1988) rental model, the high-value buyer rejects $p_1 = h$ with a positive probability even if the seller is optimistic enough to offer equilibrium price $p_1 = h$, since otherwise the seller offers $p_2 = l$ after rejection of p_1 and the high-value buyer has an incentive to deviate to reject p_1.

When the seller has a pessimistic or moderately pessimistic ex ante belief, there always exists a pooling equilibrium in our model where the seller offers $p_1 = p_2 = l$ and all buyer types accept the offers. This equilibrium yields the seller the same expected revenue as in Hart and Tirole (1988).

When the seller has a moderately optimistic ex ante belief, she can still be better off than in Hart and Tirole (1988) if the two distributions are sufficiently different, that is, q^F is small enough and q^G is big enough, and the seller's ex ante belief is sufficiently optimistic. However, if the seller's ex ante belief is close to the lower bound of moderately optimistic beliefs, γ^*, then the seller is worse off than in Hart and Tirole (1988).

Proposition 7 (Revenue Comparison) *If the ex ante probability of high-value buyer in the two-period version of Hart and Tirole's (1988) rental model is the same as in this model, then*

(i) For an optimistic seller, the seller's revenue is higher than in Hart and Tirole's (1988);

(ii) For a moderately optimistic seller, if q^F is small enough and q^G is big enough, there exists $\overline{\alpha} \in (\gamma^*, \widehat{\alpha})$ such that, for all $\alpha \in (\overline{\alpha}, \widehat{\alpha})$, the seller's revenue is higher than in Hart and Tirole's (1988);

(iii) For a pessimistic and moderately pessimistic seller, there always exists an equilibrium in this model which yields the same revenue as in Hart and Tirole's (1988).

From Proposition 7 we conclude that, when the seller has sufficiently optimistic ex ante beliefs, the seller is better off compared to the case that the distribution of the buyer's value is common knowledge.

7 Conclusion

In this paper we have considered a two-period repeated bargaining model where the seller offers a price to rent a durable good in each period. The buyer's value of consuming the durable good is drawn from a fixed distribution in each period. The buyer has private information not only about his value in each period, but also about the distribution which his values are drawn from.

We compare the seller's expected revenue in our model with that in the two-period version of Hart and Tirole's (1988) rental model where the distribution of the buyer's value is common knowledge, under the assumption that the ex ante probabilities of high-value buyer are the same in the two models. We find that the seller is better off with the additional layer of uncertainty about the buyer's value distribution, when she has sufficiently optimistic ex ante beliefs.

The results we found may cast some light on the longer horizon. In the current two-period model, the seller cannot perfectly learn the buyer's value distribution. It is interesting to examine whether the seller is able to learn the buyer's distribution eventually if she is allowed to employ price experimentation in a finite or an infinite horizon.

On the other hand, this model only allows the seller to rent the durable good. For future research, we are interested in investigating the case where the seller is able to adopt a more general strategy, such as selling the durable good or providing both options of selling and renting the durable good to the buyer.

8 Appendix A: Criterion D_1

In this appendix we first describe two equilibria when the seller has a pessimistic ex ante belief ($\alpha < \widetilde{\alpha}$) and show that only Equilibrium A can survive if we impose more restrictions on the seller's belief off the equilibrium path. Then we give the formal definition of criterion D_1 and present the effect of applying this criterion.

Equilibrium A: The seller always offers $p_1 = p_2 = l$; All buyer types always accept p_t if $p_t \leq v_t$ and reject p_t if $p_t > v_t$; The seller always forms a belief less than the cutoff belief γ^*.

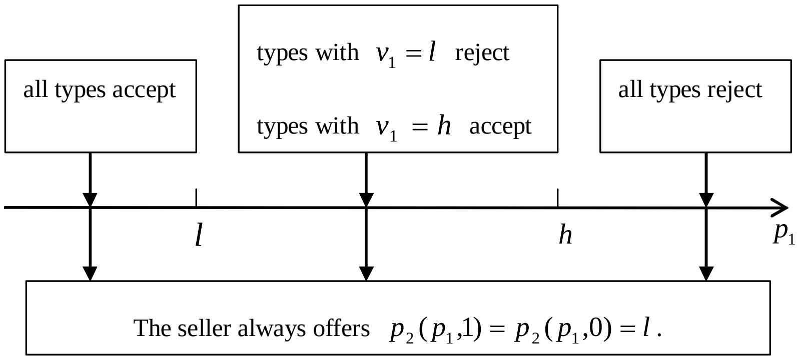

Figure A1: Equilibrium A

Equilibrium B: The seller offers $p_1 = l + \delta q^F(h-l)$, $p_2 = h$ if $p_1 \leq l + \delta q^F(h-l)$ is rejected, $p_2 = l$ otherwise; Buyer types with $v_1 = l$ accept $p_1 \leq l + \delta q^F(h-l)$ and reject $p_1 > l + \delta q^F(h-l)$; Buyer types with $v_1 = h$ accept $p_1 \leq h$ and reject $p_1 > h$; The seller forms a belief greater than the cutoff belief γ^* after $p_1 \leq l + \delta q^F(h-l)$ is rejected and a belief less than the cutoff belief γ^* otherwise.

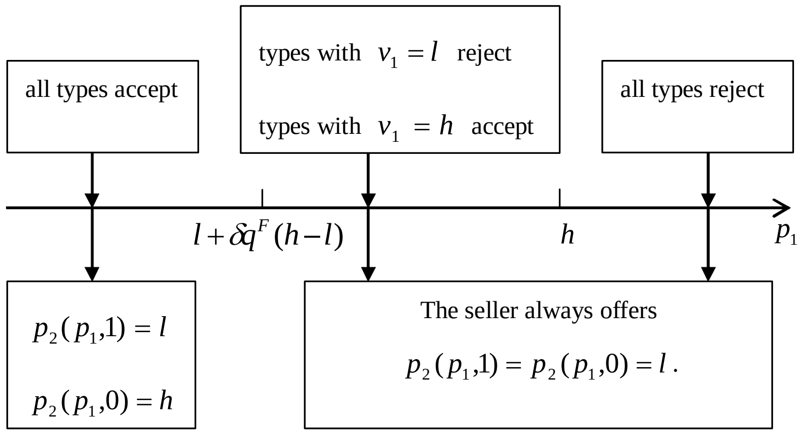

Figure A2: Equilibrium B

We have discussed Equilibrium A in Proposition 1.

In Equilibrium B, the buyer types with $v_1 = l$ are willing to accept $p_1 \in (l, l + \delta q^F(h-l)]$ and incur a loss in the first period since the seller threatens to offer $p_2 = h$ if p_1 is rejected. In equilibrium, no type rejects p_1 in this range, so the seller can assign any belief after p_1 is rejected, so $p_2 = h$ after rejection of p_1 can be sustained as part of the equilibrium strategy.

Equilibrium B, however, is not very reasonable if we impose more restrictions on the seller's belief off the equilibrium path. Suppose the seller unexpectedly observes rejection of $p_1 \in (l, l+\delta q^F(h-l)]$ and tries to figure out which type is most likely to reject the offer. First note that the buyer types with $v_1 = h$ gain a positive payoff in the first period by accepting $p_1 \in (l, l + \delta q^F(h-l)]$ and the lowest offer $p_2 = l$ after acceptance of p_1, so the types with $v_1 = h$ are strictly worse off by rejecting p_1. Second, the seller finds that if her response to rejection makes type (G, l) better off or indifferent from the equilibrium by deviation, then type (F, l) is definitely better off by deviation. Thus, type (F, l) is most likely to reject p_1 among all the buyer types. If this is true, the seller should offer $p_2 = l$ instead of $p_2 = h$ when p_1 is rejected, and Equilibrium B is no longer sequentially rational.

Equilibrium A does not have this problem. In Equilibrium A, since the seller always offers

$p_2 = l$ and the buyer always accepts $p_1 \leq v_1$, no buyer type can be better off by deviation. Thus, the criterion applied on Equilibrium B does not have any effect on Equilibrium A.

Criterion D_1 formalizes the idea discussed above. The intuition conveyed by criterion D_1 is as follows. Consider a fixed equilibrium and the out-of-equilibrium action a_1 in the continuation game following any p_1. If whenever buyer type θ_1 has an incentive to defect from the equilibrium and send the out-of-equilibrium message a_1, or is indifferent between the equilibrium and defection, some other buyer type $\widetilde{\theta}_1$ is strictly better off by defection, then the seller's beliefs should assign zero probability to type θ_1 when the seller unexpectedly observes the out-of-equilibrium action a_1. Definition 2 formally defines criterion D_1 in the continuation game.

Definition 2 (Criterion D_1 in the Continuation Game Following p_1) *Consider a fixed equilibrium on the continuation of p_1, with action $a_1 \in \{0, 1\}$ reached with zero probability. Suppose $x(p_1, 1)$ and $x(p_1, 0)$ is the seller's equilibrium strategy.*
Step 1: *Find the sets of all (mixed) responses ϕ by the seller that would cause type $\theta_1 = (d, v_1)$ to defect from the equilibrium and to be indifferent. If $a_1 = 0$ is the out-of-equilibrium action, form the sets*

$$D_{\theta_1} \equiv \{\phi : (v_1 - p_1) + \delta q^d x(p_1, 1)(h - l) < \delta q^d \phi(h - l), \phi \in [0, 1]\},$$

$$D^0_{\theta_1} \equiv \{\phi : (v_1 - p_1) + \delta q^d x(p_1, 1)(h - l) = \delta q^d \phi(h - l), \phi \in [0, 1]\}.$$

If $a_1 = 1$ is the out-of-equilibrium action, form the sets

$$D_{\theta_1} \equiv \{\phi : (v_1 - p_1) + \delta q^d \phi(h - l) > \delta q^d x(p_1, 0)(h - l), \phi \in [0, 1]\},$$

$$D^0_{\theta_1} \equiv \{\phi : (v_1 - p_1) + \delta q^d \phi(h - l) = \delta q^d x(p_1, 0)(h - l), \phi \in [0, 1]\}.$$

Step 2: *For a given out-of-equilibrium action a_1, if for some type θ_1 there exists a second type $\widetilde{\theta}_1$ with $D_{\theta_1} \cup D^0_{\theta_1} \subsetneq D_{\widetilde{\theta}_1}$, then the combination (θ_1, a_1) may be pruned from the continuation game following p_1.*

Step 3: *Check whether the fixed equilibrium is still sequentially rational given that the seller's belief is restricted to the buyer types who survive from Step 2. If not, then the equilibrium does not survive from D_1.*

Given a PBE, if the corresponding equilibrium in all the continuation games survives from D_1, then we say that the PBE survives from D_1.

Definition 3 (D_1 Equilibrium) *A PBE of the game survives from D_1 if and only if the equilibrium on the continuation of p_1 survives from D_1 for all $p_1 \in \mathbb{R}$.*

The effect of applying criterion D_1 in our model is summarized in the following lemma.

Lemma 4 *The following results hold for a PBE in the continuation game following p_1:*

(i) If $p_1 > l$ and all buyer types accept p_1, the equilibrium in the continuation game can not pass criterion D_1;

(ii) If $p_1 \leq l$ and all buyer types accept p_1, the equilibrium in the continuation game passes criterion D_1;

(iii) If $p_1 < h$ and all buyer types reject p_1, the equilibrium in the continuation game can not pass criterion D_1;

(iv) If $p_1 \geq h$ and all buyer types reject p_1, the equilibrium in the continuation game passes criterion D_1.

9 Appendix B: Proofs

In this appendix, we first prove Lemma 4, which is shown in Appendix A. Then we present two supplementary lemmas Lemma 5 and Lemma 6, which are used in the proof of other results later. Finally we proceed with the proof of results in the main text.

Proof of Lemma 4. **Part 1:** Suppose $\Psi(p_1, 1) = 1$ in the continuation game following $p_1 > l$. Then $x(p_{1,}1) > x(p_1, 0)$ and $x(p_{1,}1) = 1$ without considering the knife-edge case that $\alpha = \gamma^*$. Since $\max\{x(p_{1,}1) - x(p_1, 0)\} = 1$ and all types accept p_1, $p_1 \leq \min\limits_{(d,v_1)}\{v_1 + \delta q^d(h-l)\} = l + \delta q^F(h-l)$ by the definition of cutoff value.

Apply Definition 1 in the case that $a_1 = 0$ is the out-of-equilibrium message and form the sets D_{θ_1} and $D^0_{\theta_1}$ for each buyer type θ_1. So $D_{\theta_1} = \{\phi : \phi > x(p_1, 1) + \frac{v_1 - p_1}{\delta q^d(h-l)}, \phi \in [0, 1]\}$ and $D^0_{\theta_1} = \{\phi : \phi = x(p_1, 1) + \frac{v_1 - p_1}{\delta q^d(h-l)}, \phi \in [0, 1]\}$. Therefore, for $x(p_{1,}1) = 1$ and $p_1 \in (l, l + \delta q^F(h-l)]$, $D_{\theta_1} \cup D^0_{\theta_1} \subsetneq D_{(F,l)}$ for all $\theta_1 \neq (F, l)$. All the combinations $(\theta_1, a_1 = 0)$ with $\theta_1 \neq (F, l)$ are pruned from the game. Given the seller's belief is restricted on type (F, l) after rejection, $x(p_1, 0) = 1$ and it is contradictory to $x(p_{1,}1) > x(p_1, 0)$. So the equilibrium fails to pass criterion D_1.

Part 2: Suppose $\Psi(p_1, 1) = 1$ in the continuation game following $p_1 \leq l$. From Part 1, $D_{\theta_1} = \{\phi : \phi > x(p_1, 1) + \frac{v_1 - p_1}{\delta q^d(h-l)}, \phi \in [0, 1]\}$ and $D^0_{\theta_1} = \{\phi : \phi = x(p_1, 1) + \frac{v_1 - p_1}{\delta q^d(h-l)}, \phi \in [0, 1]\}$.

If $p_1 = l$ and $\alpha < \gamma^*$, $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for $\theta_1 \in \{(F, h), (G, h)\}$ and $D_{\theta_1} \cup D^0_{\theta_1} = \{1\}$ for $\theta_1 \in \{(F, l), (G, l)\}$.

If $p_1 = l$ and $\alpha > \gamma^*$, $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for $\theta_1 \in \{(F, h), (G, h)\}$ and $D_{\theta_1} \cup D^0_{\theta_1} = [0, 1]$ for $\theta_1 \in \{(F, l), (G, l)\}$.

If $p_1 < l$ and $\alpha < \gamma^*$, then $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for all buyer types θ_1.

If $p_1 < l$ and $\alpha > \gamma^*$, then either $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for all buyer types θ_1 or $D_{\theta_1} \cup D^0_{\theta_1} \subsetneq D_{(G,l)}$. If the latter happens, the seller's belief is restricted on type (G, l) after rejection and she offers $x(p_1, 0) = 0$. It is still sequential rational for all buyer types θ_1 to accept $p_1 < l$ given $x(p_1, 1) = x(p_1, 0) = 0$.

In all the cases above, the equilibrium passes criterion D_1.

Part 3: Suppose $\Psi(p_1, 0) = 1$ in the continuation game following $p_1 < h$. Then $x(p_{1,}0) > x(p_1, 1)$ and $x(p_{1,}0) = 1$ without considering the knife-edge case that $\alpha = \gamma^*$. Since $\max\{x(p_{1,}0) - x(p_1, 1)\} = 1$ and all types reject p_1, $p_1 \geq \max_{(d,v_1)}\{v_1 - \delta q^d(h-l)\} = h - \delta q^F(h-l)$ by the definition of cutoff value.

Apply Definition 1 in the case that $a_1 = 1$ is the out-of-equilibrium message. So $D_{\theta_1} = \{\phi : \phi > x(p_1, 0) + \frac{p_1 - v_1}{\delta q^d(h-l)}, \phi \in [0,1]\}$ and $D^0_{\theta_1} = \{\phi : \phi = x(p_1, 0) + \frac{p_1 - v_1}{\delta q^d(h-l)}, \phi \in [0,1]\}$. Then for $x(p_{1,}0) = 1$ and $p_1 \in [h - \delta q^F(h-l), h)$, $D_{\theta_1} \cup D^0_{\theta_1} \subsetneq D_{(F,h)}$ for all $\theta_1 \neq (F, h)$. All the combinations $(\theta_1, a_1 = 1)$ with $\theta_1 \neq (F, h)$ are pruned from the game. Given the seller's belief is restricted on type (F, h) after acceptance, $x(p_{1,}1) = 1$ and it is contradictory to $x(p_{1,}0) > x(p_1, 1)$. So the equilibrium fails to pass Criterion D_1.

Part 4: Suppose $\Psi(p_1, 0) = 1$ in the continuation game following $p_1 \geq h$. From Part 3, $D_{\theta_1} = \{\phi : \phi > x(p_1, 0) + \frac{p_1 - v_1}{\delta q^d(h-l)}, \phi \in [0,1]\}$ and $D^0_{\theta_1} = \{\phi : \phi = x(p_1, 0) + \frac{p_1 - v_1}{\delta q^d(h-l)}, \phi \in [0,1]\}$.

If $p_1 = h$ and $\alpha < \gamma^*$, $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for $\theta_1 \in \{(F, l), (G, l)\}$ and $D_{\theta_1} \cup D^0_{\theta_1} = \{1\}$ for $\theta_1 \in \{(F, h), (G, h)\}$.

If $p_1 = h$ and $\alpha > \gamma^*$, $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for $\theta_1 \in \{(F, l), (G, l)\}$ and $D_{\theta_1} \cup D^0_{\theta_1} = [0, 1]$ for $\theta_1 \in \{(F, h), (G, h)\}$.

If $p_1 > h$ and $\alpha < \gamma^*$, then $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for all buyer types θ_1.

If $p_1 > h$ and $\alpha > \gamma^*$, then either $D_{\theta_1} \cup D^0_{\theta_1} = \emptyset$ for all buyer types θ_1 or $D_{\theta_1} \cup D^0_{\theta_1} \subsetneq D_{(G,h)}$. If the latter case happens, the seller's belief is restricted on type (G, h) after acceptance and $x(p_1, 1) = 0$. Then it is still sequential rational for all buyer types θ_1 to reject $p_1 > h$ given $x(p_1, 1) = x(p_1, 0) = 0$.

In all the cases above, the equilibrium passes criterion D_1. ■

Lemma 5 *Let $\Psi(p_1, a_1)$ denote the probability that action a_1 is taken in the continuation game following p_1. If $\Psi(p_1, 1) \in (0, 1)$ for a given p_1 in a PBE, then $x(p_1, 0) \geq x(p_1, 1)$.*

Proof of Lemma 5. Suppose $\Psi(p_1, 1) \in (0,1)$ and $x(p_1, 0) < x(p_1, 1)$ in a PBE. Then $l < C(F,l) < C(G,l) < h < C(F,h) < C(G,h)$, and $\Psi(p_1, 1) \in (0,1)$ only if $p_1 \in [C(F,l), C(G,h)]$. We will show that it reaches a contradiction for any $p_1 \in [C(F,l), C(G,h)]$.

If $p_1 \in [C(F,l), C(G,l))$, then only type (F,l) rejects p_1 and $x(p_1, 0) = 1 \geq x(p_1, 1)$.

If $p_1 \in (C(F,h), C(G,h)]$, then only type (G,h) accepts p_1 and $x(p_1, 1) = 0 \leq x(p_1, 0)$.

If $p_1 \in (C(G,l), C(F,h))$, then $\gamma(p_1, 0) = \underline{\gamma}(\alpha) < \alpha < \overline{\gamma}(\alpha) = \gamma(p_1, 1)$ and $x(p_1, 0) \geq x(p_1, 1)$.

Denote X' as the probability for type (G,l) to reject $p_1 = C(G,l)$ and Y' as the probability for type (F,h) to reject $p_1 = C(F,h)$.

If $p_1 = C(G,l)$,

$$\gamma(p_1, 0) = \frac{\alpha X'(1-q^G)}{\alpha X'(1-q^G) + (1-\alpha)(1-q^F)} < \underline{\gamma}(\alpha)$$

and

$$\gamma(p_1, 1) = \frac{\alpha q^G + \alpha(1-X')(1-q^G)}{\alpha q^G + (1-\alpha)q^F + \alpha(1-X')(1-q^G)} > \overline{\gamma}(\alpha),$$

so $x(p_1, 0) \geq x(p_1, 1)$.

If $p_1 = C(F,h)$,

$$\gamma(p_1, 0) = \frac{\alpha(1-q^G)}{\alpha(1-q^G) + (1-\alpha)(1-q^F) + (1-\alpha)Y'q^F} < \underline{\gamma}(\alpha)$$

and

$$\gamma(p_1, 1) = \frac{\alpha q^G}{\alpha q^G + (1-\alpha)(1-Y')q^F} > \overline{\gamma}(\alpha),$$

so $x(p_1, 0) \geq x(p_1, 1)$.

Therefore, every case is contradictory to $x(p_1, 0) < x(p_1, 1)$, and the seller offers $x(p_1, 0) \geq x(p_1, 1)$ in a PBE if $\Psi(p_1, 1) \in (0,1)$. ∎

Lemma 6 *If $\Psi(p_1, 1) \in (0,1)$ for a given p_1 in a PBE, then*

(i) $x(p_1, 0) > x(p_1, 1) \Rightarrow \alpha \in [\widetilde{\alpha}, \widehat{\alpha}]$*;*

(ii) $\alpha \in (0, \widetilde{\alpha}) \cup (\widehat{\alpha}, 1) \Rightarrow x(p_1, 0) = x(p_1, 1)$*.*

Proof of Lemma 6. (i) $\Psi(p_1, 1) \in (0, 1)$ and $x(p_1, 0) > x(p_1, 1) \Rightarrow \alpha \in [\widetilde{\alpha}, \widehat{\alpha}]$.

Suppose $\Psi(p_1, 1) \in (0, 1)$, $x(p_1, 0) > x(p_1, 1)$, and $\alpha \in (0, \widetilde{\alpha}) \cup (\widehat{\alpha}, 1)$. Then $C(G, l) < C(F, l) < l < C(G, h) < C(F, h) < h$, and $\Psi(p_1, 1) \in (0, 1)$ only if $p_1 \in [C(G, l), C(F, h)]$. We will show that it reaches a contradiction for any $p_1 \in [C(G, l), C(F, h)]$.

If $p_1 \in [C(G, l), C(F, l))$, then only type (G, l) rejects p_1 and $x(p_1, 0) = 0 \leq x(p_1, 1)$.

If $p_1 \in (C(G, h), C(F, h)]$, then only type (F, h) accepts p_1 and $x(p_1, 1) = 1 \geq x(p_1, 0)$.

If $p_1 \in (C(F, l), C(G, h))$ and $\alpha < \widetilde{\alpha}$, then $\gamma(p_1, 1) = \overline{\gamma}(\alpha) < \gamma^*$ and $x(p_1, 1) = 1 \geq x(p_1, 0)$.

If $p_1 \in (C(F, l), C(G, h))$ and $\alpha > \widehat{\alpha}$, then $\gamma(p_1, 0) = \underline{\gamma}(\alpha) > \gamma^*$ and $x(p_1, 0) = 0 \leq x(p_1, 1)$.

Denote X as the probability for type (F, l) to reject $p_1 = C(F, l)$ and Y as the probability for type (G, h) to reject $p_1 = C(G, h)$.

If $p_1 = C(F, l)$,

$$\gamma(p_1, 0) = \frac{\alpha(1 - q^G)}{\alpha(1 - q^G) + (1 - \alpha)X(1 - q^F)} > \underline{\gamma}(\alpha)$$

and

$$\gamma(p_1, 1) = \frac{\alpha q^G}{\alpha q^G + (1 - \alpha)q^F + (1 - \alpha)(1 - X)(1 - q^F)} < \overline{\gamma}(\alpha).$$

So $\gamma(p_1, 1) < \overline{\gamma}(\alpha) < \gamma^*$ when $\alpha < \widetilde{\alpha}$ and $x(p_1, 1) = 1 \geq x(p_1, 0)$. $\gamma(p_1, 0) > \underline{\gamma}(\alpha) > \gamma^*$ when $\alpha > \widehat{\alpha}$ and $x(p_1, 0) = 0 \leq x(p_1, 1)$.

If $p_1 = C(G, h)$,

$$\gamma(p_1, 0) = \frac{\alpha Y q^G + \alpha(1 - q^G)}{\alpha Y q^G + \alpha(1 - q^G) + (1 - \alpha)(1 - q^F)} > \underline{\gamma}(\alpha)$$

and

$$\gamma(p_1, 1) = \frac{\alpha(1 - Y)q^G}{\alpha(1 - Y)q^G + (1 - \alpha)q^F} < \overline{\gamma}(\alpha).$$

So $\gamma(p_1, 1) < \overline{\gamma}(\alpha) < \gamma^*$ when $\alpha < \widetilde{\alpha}$ and $x(p_1, 1) = 1 \geq x(p_1, 0)$. $\gamma(p_1, 0) > \underline{\gamma}(\alpha) > \gamma^*$ when $\alpha > \widehat{\alpha}$ and $x(p_1, 0) = 0 \leq x(p_1, 1)$.

Therefore, every case is contradictory to $x(p_1, 0) > x(p_1, 1)$.

(ii) It is directly derived from Lemma 5 and (i) of Lemma 6. ■

Proof of Observation 1. **Step 1:** Suppose $x(p_1, 0) > x(p_1, 1)$. Then $C(G, l) < C(F, l) < l < C(G, h) < C(F, h) < h$. A price p_1 can screen one type from the other types only if $p_1 \in [C(G, l), C(F, l)]$ or $p_1 \in [C(G, h), C(F, h)]$.

If $p_1 \in [C(G, l), C(F, l)]$ and only type (G, l) rejects p_1, then $x(p_1, 0) = 0$, so it contradicts with $x(p_1, 0) > x(p_1, 1)$.

If $p_1 \in [C(G, h), C(F, h)]$ and only type (F, h) accepts p_1, then $x(p_1, 1) = 1$, so it contradicts with $x(p_1, 0) > x(p_1, 1)$.

Step 2: Suppose $x(p_1, 0) < x(p_1, 1)$. Then $l < C(F, l) < C(G, l) < h < C(F, h) < C(G, h)$. A price p_1 can screen one type from the other types only if $p_1 \in [C(F, l), C(G, l)]$ or $p_1 \in [C(F, h), C(G, h)]$.

If $p_1 \in [C(F, l), C(G, l)]$ and only type (F, l) rejects p_1, then $x(p_1, 0) = 1$, so it contradicts with $x(p_1, 0) < x(p_1, 1)$.

If $p_1 \in [C(F, h), C(G, h)]$ and only type (G, h) accepts p_1, then $x(p_1, 1) = 0$, so it contradicts with $x(p_1, 0) < x(p_1, 1)$.

Step 3: Suppose $x(p_1, 0) = x(p_1, 1)$. Then $C(F, l) = C(G, l) = l < C(F, h) = C(G, h) = h$.

If $p_1 < l$, all types accept p_1.

If $p_1 > h$, all types reject p_1.

If $l < p_1 < h$, both type (F, l) and (G, l) reject p_1 and both type (F, h) and (G, h) accept p_1.

If $p_1 \in \{l, h\}$, under the assumption that the buyer's strategy is left continuous, that is, same as that following $p_1 - \epsilon$ (see footnote 14 on page 18), screening one buyer type cannot happen in equilibrium. ■

Proof of Observation 2. Observation 2 is directly derived from Lemma 5. ■

Proof of Observation 3 and 4. If p_1 screens buyer types (F, l) and (G, l) from buyer types (F, h) and (G, h), then buyer types (F, l) and (G, l) reject p_1 and buyer types (F, h) and (G, h) accept p_1. By definition of $\widetilde{\alpha}$ and $\widehat{\alpha}$, the following holds. When $\widetilde{\alpha} < \alpha < \widehat{\alpha}$, $\gamma(p_1, 0) < \gamma^*$ and $\gamma(p_1, 1) > \gamma^*$, so $x(p_1, 0) = 1$ and $x(p_1, 1) = 0$. When $\alpha < \widetilde{\alpha}$, $\gamma(p_1, 0) < \gamma(p_1, 1) < \gamma^*$, so $x(p_1, 0) = x(p_1, 1) = 1$. When $\alpha > \widehat{\alpha}$, $\gamma^* < \gamma(p_1, 0) < \gamma(p_1, 1)$, so $x(p_1, 0) = x(p_1, 1) = 0$. ■

Proof of Proposition 1. **Step 1:** It is the unique D_1 equilibrium strategy for the buyer to accept p_t if and only if $p_t \leq v_t$.

Suppose $x(p_1, 0) > x(p_1, 1)$. Then $\Psi(p_1, 0) = 1$ by Lemma 6 given $\alpha < \widetilde{\alpha}$. Since $x(p_1, 0) > x(p_1, 1)$, $C(F, h) = \max_{\theta_1}\{C(\theta_1)\} < h$. By Lemma 4, a PBE cannot pass criterion D_1 if $\Psi(p_1, 0) = 1$ and $p_1 < h$. Therefore, $p_1 \geq h$ and $\Psi(p_1, 0) = 1$, i.e., all types reject $p_1 \geq h$.

Suppose $x(p_1, 0) < x(p_1, 1)$. Then $\Psi(p_1, 1) = 1$ by Lemma 6 given $\alpha < \widetilde{\alpha}$. Since $x(p_1, 0) < x(p_1, 1)$, $C(F, l) = \min_{\theta_1}\{C(\theta_1)\} > l$. By Lemma 4, a PBE cannot pass criterion D_1 if $\Psi(p_1, 1) = 1$ and $p_1 > l$. Therefore, $p_1 \leq l$ and $\Psi(p_1, 1) = 1$, i.e., all types accept $p_1 \leq l$.

Suppose $x(p_1, 0) = x(p_1, 1)$. Then all buyer types accept p_1 if and only if $p_1 \leq v_1$, otherwise the seller offers $p_1 = v_1 - \epsilon$ and no equilibrium exists.

Therefore, combining three cases above, it is the unique equilibrium strategy for the buyer to accept p_1 if and only if $p_1 \leq v_1$.

Step 2: Given the buyer's strategy, the seller offers $p_1 = l$ or $p_1 = h$, and always offers $p_2 = l$ on the equilibrium path. The respective payoffs for the seller is:

$$\begin{cases} \pi(l) = l + \delta l; \\ \pi(h) = \alpha h + \delta l. \end{cases}$$

Since $\alpha < \widetilde{\alpha} < \gamma^*$, it is optimal to offer $p_1 = l$. ■

Proof of Proposition 2. **Step 1:** It is the unique D_1 equilibrium strategy for the buyer to accept p_t if and only if $p_t \leq v_t$.

Suppose $x(p_1, 0) > x(p_1, 1)$. Then $\Psi(p_1, 1) = 1$ by Lemma 6 given $\alpha > \widehat{\alpha}$. Since $x(p_1, 0) > x(p_1, 1)$, $C(G, l) = \min_{\theta_1}\{C(\theta_1)\} < l$. By Lemma 4, a PBE can pass criterion D_1 if $\Psi(p_1, 1) = 1$ and $p_1 \leq l$. Therefore, $p_1 < C(G, l)$ and $\Psi(p_1, 1) = 1$, i.e., all types accept $p_1 < C(G, l)$.

Suppose $x(p_1, 0) < x(p_1, 1)$. Then $\Psi(p_1, 0) = 1$ by Lemma 6 given $\alpha > \widehat{\alpha}$. Since $x(p_1, 0) < x(p_1, 1)$, $C(G, h) = \max_{\theta_1}\{C(\theta_1)\} > h$. By Lemma 4, a PBE can pass criterion D_1 if $\Psi(p_1, 0) = 1$ and $p_1 \geq h$. Therefore, $p_1 > C(G, h)$ and $\Psi(p_1, 0) = 1$, i.e., all types reject $p_1 > C(G, h)$.

Suppose $x(p_1, 0) = x(p_1, 1)$. Then all buyer types accept p_1 if and only if $p_1 \leq v_1$, otherwise the seller offers $p_1 = v_1 - \epsilon$ and no equilibrium exists.

Therefore, combining three cases above, it is the unique equilibrium strategy for the buyer to accept p_1 if and only if $p_1 \leq v_1$.

Step 2: Given the buyer's strategy, the seller offers $p_1 = l$ or $p_1 = h$, and always offers $p_2 = h$ on the equilibrium path. The respective payoffs for the seller is:

$$\begin{cases} \pi(l) = l + \delta\alpha h; \\ \pi(h) = \alpha h + \delta\alpha h. \end{cases}$$

Since $\alpha > \widehat{\alpha} > \gamma^*$, it is optimal to offer $p_1 = h$. ■

Proof of Lemma 3. **Step 1:** Suppose $x(p_1, 0) < x(p_1, 1)$. Then $l < C(F, l) < C(G, l) < h < C(F, h) < C(G, h)$. $\Psi(p_1, 1) = 0$ or 1 given $\widetilde{\alpha} < \alpha < \widehat{\alpha}$ by Lemma 6. Therefore, all buyer types accept $p_1 \leq C(F, l)$ when $\widetilde{\alpha} < \alpha < \gamma^*$, and all types reject $p_1 > C(G, h)$ when $\gamma^* < \alpha < \widehat{\alpha}$. By Lemma 4, if all types accept $p_1 \in (l, C(F, l)]$, the equilibrium cannot pass criterion D_1. So all types accept $p_1 \leq l$ when $\widetilde{\alpha} < \alpha < \gamma^*$ and reject $p_1 > C(G, h)$ when $\gamma^* < \alpha < \widehat{\alpha}$, under the assumption of $x(p_1, 0) < x(p_1, 1)$.

Step 2: Suppose $x(p_1, 0) = x(p_1, 1)$. Then $C(G, l) = C(F, l) = l < C(G, h) = C(F, h) = h$. Therefore, all buyer types accept $p_1 < l$ and all types reject $p_1 > h$. At $p_1 = l$ or h, see footnote 14 on page 18. For $p_1 \in (l, h)$, types (F, l) and (G, l) reject p_1 and types (F, h) and (G, h) accept p_1, so $x(p_1, 0) = 1$ and $x(p_1, 1) = 0$ for $\widetilde{\alpha} < \alpha < \widehat{\alpha}$, which leads to a contradiction.

Step 3: Suppose $x(p_1, 0) > x(p_1, 1)$. Then $C(G, l) < C(F, l) < l < C(G, h) < C(F, h) < h$.

If $\Psi(p_1, 1) \in \{0, 1\}$, then all buyer types reject $p_1 > C(F, h)$ when $\widetilde{\alpha} < \alpha < \gamma^*$ and accept $p_1 < C(G, l)$ when $\gamma^* < \alpha < \widehat{\alpha}$. By Lemma 4, if all types reject $p_1 \in (C(F, h), h)$, the equilibrium cannot pass criterion D_1. So all buyer types reject $p_1 > h$ when $\widetilde{\alpha} < \alpha < \gamma^*$ and accept $p_1 < C(G, l)$ when $\gamma^* < \alpha < \widehat{\alpha}$.

Now consider the case $\Psi(p_1, 1) \in (0, 1)$. First consider pure strategy, i.e., suppose $x(p_1, 0) = 1$ and $x(p_1, 1) = 0$. By the proof of Observation 1, it is not possible for any p_1 to separate a single type from other types. So pure strategy is possible to be adopted for $p_1 \in (C(F, l), C(G, h)] = (\underline{p}, \widetilde{p}]$. Types (F, l) and (G, l) reject p_1, and types (F, h) and (G, h) accept p_1.

Next consider mixed strategy, i.e., $0 < x(p_1, 0) - x(p_1, 1) < 1$. Then either $x(p_1, 0) = 1$ and $x(p_1, 1) \in (0, 1)$ or $x(p_1, 0) \in (0, 1)$ and $x(p_1, 1) = 0$, since the knife-edge condition $\alpha = \gamma^*$ is omitted. The former implies $\gamma(p_1, 0) < \gamma^*$ and $\gamma(p_1, 1) = \gamma^*$, and the latter implies $\gamma(p_1, 0) = \gamma^*$ and $\gamma(p_1, 1) > \gamma^*$. Therefore, $\gamma(p_1, 1) = \gamma^*$ when $\widetilde{\alpha} < \alpha < \gamma^*$, and $\gamma(p_1, 0) = \gamma^*$ when $\gamma^* < \alpha < \widehat{\alpha}$. From Observation 1, it is not possible for type (G, l) or (F, h) to randomize, otherwise the seller can at least sometimes separate type (G, l) or (F, h) from other types. So only type (F, l) and (G, h) will play mixed strategy.

Case 1: When $\widetilde{\alpha} < \alpha < \gamma^*$, type (F, l) randomizes to reject p_1 with probability X^*, (G, l) rejects p_1, and (G, h) and (F, h) accept p_1. Then

$$\gamma(p_1, 1) = \frac{\alpha q^G}{\alpha q^G + (1-\alpha) q^F + (1-\alpha)(1-X^*)(1-q^F)} = \gamma^*.$$

Type (F, l) is indifferent from accepting and rejecting p_1, then

$$l - p_1 + \delta q^F x(p_1, 1)(h - l) = \delta q^F (h - l).$$

So type (F, l) rejects $p_1 \in (\underline{p}, l]$ with probability $X^* = 1 + \frac{q^F}{1-q^F} - \frac{\alpha q^G (1-\gamma^*)}{(1-\alpha)(1-q^F)\gamma^*}$, and the seller offers $x(p_1, 1) = 1 - \frac{l-p_1}{\delta q^F (h-l)}$, $x(p_1, 0) = 1$.

Case 2: When $\gamma^* < \alpha < \widehat{\alpha}$, type (F,l) randomizes to reject p_1 with probability X^{**}, (G,l) rejects p_1, and (G,h) and (F,h) accept p_1. Then

$$\gamma(p_1,0) = \frac{\alpha(1-q^G)}{\alpha(1-q^G)+(1-\alpha)X^{**}(1-q^F)} = \gamma^*.$$

Type (F,l) is indifferent from accepting and rejecting p_1, then

$$l - p_1 = \delta q^F x(p_1,0)(h-l).$$

So type (F,l) rejects $p_1 \in (\underline{p}, l]$ with probability $X^{**} = \frac{\alpha(1-q^G)(1-\gamma^*)}{(1-\alpha)(1-q^F)\gamma^*}$, and the seller offers $x(p_1,0) = \frac{l-p_1}{\delta q^F(h-l)}$ and $x(p_1,1) = 0$.

Case 3: When $\widetilde{\alpha} < \alpha < \gamma^*$, type (G,h) randomizes to reject p_1 with probability Y^*, (F,l) and (G,l) reject p_1, and (F,h) accepts p_1. Then

$$\gamma(p_1,1) = \frac{\alpha(1-Y^*)q^G}{\alpha(1-Y^*)q^G+(1-\alpha)q^F} = \gamma^*.$$

Type (G,h) is indifferent from accepting and rejecting p_1, then

$$h - p_1 + \delta q^G x(p_1,1)(h-l) = \delta q^G(h-l).$$

So type (G,h) rejects $p_1 \in (\widetilde{p}, h]$ with probability $Y^* = 1 - \frac{(1-\alpha)q^F\gamma^*}{\alpha q^G(1-\gamma^*)}$, and the seller offers $x(p_1,1) = 1 - \frac{h-p_1}{\delta q^G(h-l)}$ and $x(p_1,0) = 1$.

Case 4: When $\gamma^* < \alpha < \widehat{\alpha}$, type (G,h) randomizes to reject p_1 with probability Y^{**}, (F,l) and (G,l) reject p_1, and (F,h) accepts p_1. Then

$$\gamma(p_1,0) = \frac{\alpha Y^{**}q^G+\alpha(1-q^G)}{\alpha Y^{**}q^G+\alpha(1-q^G)+(1-\alpha)(1-q^F)} = \gamma^*.$$

Type (G,h) is indifferent from accepting and rejecting p_1, then

$$h - p_1 = \delta q^G x(p_1,0)(h-l).$$

So type (G,h) rejects $p_1 \in (\widetilde{p}, h]$ with probability $Y^{**} = \frac{(1-\alpha)(1-q^F)\gamma^*}{\alpha q^G(1-\gamma^*)} - \frac{1-q^G}{q^G}$ and the seller offers $x(p_1,0) = \frac{h-p_1}{\delta q^G(h-l)}$ and $x(p_1,1) = 0$.

Lemma 3 comes from the combination of three steps. ■

Proof of Proposition 3. **Step 1:** $U_2 > \max\{U_4, U_5\}$ for $\widetilde{\alpha} < \alpha < \gamma^*$.

$$\begin{aligned} & U_4 - U_2 \\ = \; & \delta(1-\alpha)q^F l(q^G - 1) + \delta\alpha q^G l(q^G - 1) \\ & + \delta(1-\alpha)q^F h(q^F - q^G) + [\alpha q^G h + (1-\alpha)q^F h - l] \\ < \; & 0 \end{aligned}$$

Each item on the right hand side of the equation is negative for $\widetilde{\alpha} < \alpha < \gamma^*$.

By plugging Y^* into the definition of U_5, $U_5 = \frac{1-\alpha}{1-\gamma^*} q^F h + \delta l$, which is decreasing in α. So

$$\begin{aligned} & \max_{\alpha \in (\widetilde{\alpha}, \gamma^*)} (U_5 - U_2) \\ < \; & \frac{1-\widetilde{\alpha}}{1-\gamma^*} q^F h - l \\ = \; & \frac{h}{q^G + q^F - l/h}(l/h - q^G)(l/h - q^F) < 0. \end{aligned}$$

Step 2: Assume all buyer types accept $p_1 \in [\underline{p}, l]$, then $p_1 = l$ is the equilibrium price given that $U_2 > \max\{U_4, U_5\}$.

(i) For an arbitrary $p_1^* \in [\underline{p}, l]$, assume all buyer types accept $p_1 \in [\underline{p}, p_1^*]$, type (F, l) and (G, l) reject $p_1 \in (p_1^*, l]$, and type (F, h) and (G, h) accept $p_1 \in (p_1^*, l]$. Then p_1^* is the optimal p_1 given $U_1 > \max\{U_4, U_5\}$.

(ii) Since $U_1 = \underline{p} + \delta l < \max\{U_4, U_5\} < U_2 = l + \delta l$, there exists $p' \in (\underline{p}, l)$ such that $u(p') = p' + \delta l = \max\{U_4, U_5\}$. For an arbitrary $p_1^* \in [p', l]$, assume all buyer types accept $p_1 \in [\underline{p}, p_1^*]$, type (F, l) and (G, l) reject $p_1 \in (p_1^*, l]$, and type (F, h) and (G, h) accept $p_1 \in (p_1^*, l]$. Then p_1^* is the optimal p_1 given $u(p') = \max\{U_4, U_5\}$. ■

Proof of Proposition 4. (i) By the definition of U_3, U_4, and U_5, they are the possibly highest payoffs when the buyer adopts any semi-separating equilibrium strategy. Since the lowest payoff from a pooling offer, U_1, is greater than $\max\{U_3, U_4, U_5\}$, there is no semi-separating equilibrium.

(ii) For all $p_1 \in (\underline{p}, l]$, the buyer can adopt two semi-separating equilibrium strategies: 1) types with $v_1 = l$ reject p_1 and types with $v_1 = h$ accept p_1, or 2) types with $v_1 = h$ accept p_1, type (G, l) rejects p_1 and type (F, l) randomizes. If the first strategy is adopted at $p_1 \in (\underline{p}, l]$, the seller's payoff by offering p_1 is dominated by U_4. If the second strategy is adopted, the payoff is dominated by $U_3 < \max\{U_4, U_5\}$. Suppose the buyer adopts either strategy, given $U_1 < \max\{U_4, U_5\}$, a semi-separating equilibrium exists and the path is unique, with $p_1 = \widetilde{p}$ or $p_1 = h$, depending on whether U_4 or U_5 is larger.

(iii) Define $U(p_1, X^*) = [\alpha q^G + (1-\alpha)q^F + (1-\alpha)(1-q^F)(1-X^*)]p_1 + \delta l$, which is increasing in p_1. First suppose $\max\{U_4, U_5\} < U_1 < U_3$. By definition $U(\underline{p}, X^*) < U_1 < U_3$. Therefore, there exists $p'' \in (\underline{p}, l)$ such that $U(p'', X^*) = U_1$. For any arbitrary $p_1^* \in [p'', l]$, assume the buyer uses the second strategy for $p_1^* \leq p''$ and uses the first strategy described in part (ii) for $p_1^* > p''$, then $p_1^* \in [p'', l]$ is the optimal p_1.

Then suppose $U_1 < \max\{U_4, U_5\} < U_3$. Since $U(\underline{p}, X^*) < U_1 < U_3$, $U(\underline{p}, X^*) < \max\{U_4, U_5\} < U_3$. Define $p'' \in (\underline{p}, l)$ such that $U(p'', X^*) = \max\{U_4, U_5\}$. If for any arbitrary $p_1^* \in [p'', l]$, the buyer uses the second strategy described in part (ii) for $p_1^* \leq p''$ and uses the first strategy for $p_1^* > p''$, then $p_1^* \in [p'', l]$ is the optimal p_1. If for any $p_1 \in (\underline{p}, l]$, the buyer uses the first strategy, then $p_1 = \widetilde{p}$ or $p_1 = h$ is optimal, depending on whether U_4 or U_5 is larger. ∎

Proof of Proposition 7. Step 1: The equilibrium in the two-period version of Hart and Tirole's (1988) rental model is as follows. In period 2, both types accept p_2 if and only if $p_2 \leq v_2$ and reject p_2 otherwise. In the first period, the l-type buyer accepts p_1 if and only if $p_1 \leq l$ and reject p_1 otherwise. If $\mu < l/h$, the h-type buyer accepts $p_1 \leq h - \delta(h-l)$ and reject $p_1 > h - \delta(h-l)$ in the first period. If $\mu > l/h$, the h-type buyer accepts $p_1 \leq h - \delta(h-l)$, randomizes to accept $p_1 \in (h - \delta(h-l), h]$ with probability $y^* = \frac{\mu h - l}{\mu(h-l)}$, and reject $p_1 > h$ in the first period. Therefore, if $\mu < l/h$, the seller offers $p_1 = p_2 = l$; if $l/h < \mu < \frac{hl+\delta l(h-l)}{hl+\delta h(h-l)}$, the seller offers $p_1 = h - \delta(h-l)$, $p_2 = h$ if p_1 is accepted, and $p_2 = l$ if p_1 is rejected; if $\mu > \frac{hl+\delta l(h-l)}{hl+\delta h(h-l)}$, the seller offers $p_1 = p_2 = h$. The corresponding revenues are as follows.

$$\pi = \begin{cases} l + \delta l, & \text{if } \mu < l/h; \\ \mu[h - \delta(h-l)] + \delta\mu h + \delta(1-\mu)l = \mu h + \delta l, & \text{if } l/h < \mu < \frac{hl+\delta l(h-l)}{hl+\delta h(h-l)}; \\ \mu y^* h + \delta\mu h = \frac{\mu h^2 - hl + \delta\mu h^2 - \delta\mu hl}{h-l} & \text{if } \mu > \frac{hl+\delta l(h-l)}{hl+\delta h(h-l)}. \end{cases}$$

This part of proof is available upon request.

Step 2: Compare the revenue in our model with that in Step 1, assuming that $\mu = \alpha q^G + (1-\alpha)q^F$.

(i) For an optimistic seller ($\alpha > \widehat{\alpha}$), there is a unique equilibrium outcome as shown in Proposition 2, and the seller's revenue in our model is

$$\begin{aligned} & (\alpha q^G + (1-\alpha)q^F)h + \delta(\alpha q^G + (1-\alpha)q^F)h \\ = \; & \mu h + \delta\mu h > \mu y^* h + \delta\mu h. \end{aligned}$$

So the seller's revenue in our model is higher than in Hart and Tirole's (1988).

(ii) For a moderately optimistic seller ($\gamma^* < \alpha < \widehat{\alpha}$), the corresponding μ is greater than l/h. Denote $W_1 = \mu h + \delta l$ and $W_2 = \frac{\mu h^2 - hl + \delta\mu h^2 - \delta\mu hl}{h-l}$. Then it suffices to compare the potential optimal revenues W_1 and W_2 in step 1 with the optimal revenue in our model. Our proof consists of the following results.

Result 1: $W_1 > V_2$ for $\gamma^* < \alpha < \widehat{\alpha}$.

$$\begin{aligned} W_1 - V_2 &= (\mu h + \delta l) - \{l + \delta[\alpha q^G + (1-\alpha)q^F]h\} \\ &= (\mu h + \delta l) - (l + \delta\mu h) = (1-\delta)(\mu h - l) > 0 \end{aligned}$$

The inequality holds since $\alpha > \gamma^*$ and $\mu > l/h$.

Result 2: $W_1 > V_4$.

$$\begin{aligned} W_1 - V_4 &= \{\mu[h - \delta(h-l)] + \delta\mu h + \delta(1-\mu)l\} \\ &\quad -\{\mu[h - \delta q^G(h-l)] + \delta[\alpha(q^G)^2 + (1-\alpha)(q^F)^2]h + \delta(1-\mu)l\} \\ &= \delta\{\mu - [\alpha(q^G)^2 + (1-\alpha)(q^F)^2]\}h - \delta\mu(1-q^G)(h-l) \\ &= \delta[\alpha q^G(1-q^G) + (1-\alpha)q^F(1-q^F)]h - \delta\mu(1-q^G)(h-l) \\ &= \delta\mu(1-q^G)h + \delta(1-\alpha)q^F(q^G-q^F)h - \delta\mu(1-q^G)(h-l) \\ &= \delta(1-\alpha)q^F(q^G-q^F)h + \delta\mu(1-q^G)l > 0. \end{aligned}$$

Result 3: $V_5 > W_2$ if $(1-q^F)(1-l/h) < q^G - l/h$.

$$\begin{aligned} V_5 - W_2 &= [\alpha q^G(1-Y^{**}) + (1-\alpha)q^F]h - \mu y^* h \\ &= \alpha(q^G-q^F)(\frac{1-q^F}{q^G-l/h} - \frac{1}{1-l/h})h + [q^F - \frac{(1-q^F)(l/h-q^F)}{q^G-l/h} + \frac{l/h-q^F}{1-l/h}]h \end{aligned}$$

If $(1-q^F)(1-l/h) < q^G - l/h$, then $V_5 > W_2$ when

$$\alpha < \frac{l/h-q^F}{q^G-q^F} - \frac{q^F}{q^G-q^F}\frac{1}{\frac{1-q^F}{q^G-l/h} - \frac{1}{1-l/h}}.$$

The RHS of the inequality is decreasing in q^G and converges to 1 when $q^G \to 1$, therefore the RHS of the inequality is greater than 1, so the inequality always holds when $(1-q^F)(1-l/h) < q^G - l/h$.

Result 4: There exists $\overline{\alpha} \in (\gamma^*, \widehat{\alpha})$ such that for $\alpha \in (\overline{\alpha}, \widehat{\alpha})$ $W_2 > W_1$, if $q^G > (1-q^F)(1-l/h) + l/h$ and $q^F < \frac{\delta(l/h)(1-l/h)}{l/h+\delta(1-l/h)}$.

$$\begin{aligned} W_2 &> W_1 \Rightarrow \mu > \frac{l/h + \delta(l/h)(1-l/h)}{l/h + \delta(1-l/h)} \\ &\Rightarrow \alpha > \frac{1}{q^G-q^F}[\frac{l/h + \delta(l/h)(1-l/h)}{l/h + \delta(1-l/h)} - q^F] \equiv \overline{\alpha} \end{aligned}$$

It is easy to show that $\overline{\alpha} > \gamma^* = \frac{l/h - q^F}{q^G - q^F}$. Next we need to show the conditions under which $\overline{\alpha} < \widehat{\alpha} = \frac{1-q^F}{(1-q^F)-(q^G-l/h)} * \frac{l/h-q^F}{q^G-q^F}$.

$$\begin{aligned} q^G - l/h &> (1-q^F)(1-l/h) \\ &\Leftrightarrow (1-q^F) - (q^G - l/h) < (1-q^F) - (1-q^F)(1-l/h) \\ &\Leftrightarrow (1-q^F)(l/h) > (1-q^F) - (q^G - l/h) \\ &\Leftrightarrow \frac{l/h - q^F}{l/h} < \frac{(1-q^F)(l/h - q^F)}{(1-q^F) - (q^G - l/h)} \end{aligned}$$

To show $\overline{\alpha} < \widehat{\alpha}$, it is sufficient to show that

$$\frac{l/h + \delta(l/h)(1-l/h)}{l/h + \delta(1-l/h)} - q^F < \frac{l/h - q^F}{l/h},$$

which is satisfied when $q^F < \frac{\delta(l/h)(1-l/h)}{l/h+\delta(1-l/h)}$.

Combining Result 1, 2, 3, and 4, we have shown that, if $q^G > (1-q^F)(1-l/h) + l/h$ and $q^F < \frac{\delta(l/h)(1-l/h)}{l/h+\delta(1-l/h)}$, there exists $\overline{\alpha} \in (\gamma^*, \widehat{\alpha})$ such that for $\alpha \in (\overline{\alpha}, \widehat{\alpha})$ $V_5 > W_2 > W_1 > \max\{V_2, V_4\}$. Therefore, V_5 is the optimal revenue in our model and it is higher than the optimal revenue in the two-period version of Hart and Tirole (1988).

(iii) For a pessimistic seller or moderately pessimistic seller ($\alpha < \gamma^*$), there always exists a pooling equilibrium in which the seller offers $p_1 = p_2 = l$ and all buyer types accept the offer as shown in Proposition 1 and 3. This equilibrium yields revenue $l + \delta l$, which is the same as in Hart and Tirole's (1988). ∎

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