

Competition and Welfare Effects of VAT Exemptions

**Helmut Dietl
Christian Jaag
Markus Lang
Urs Trinkner**

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Swiss Economics SE AG
Abeggweg 15
CH-8057 Zürich

T: +41 (0)44 500 56 20
F: +41 (0)44 500 56 21

office@swiss-economics.ch
www.swiss-economics.ch

1 Introduction

The value-added tax (VAT) has been adopted in about 130 countries over the last half-century. In particular, all OECD countries with the exception of the United States have introduced the VAT. Proponents of the VAT claim that it is a particularly effective tax that reduces the welfare costs of raising revenue. However, the tax should be raised such that it does not lead to significant distortions of competition (Keen and Lockwood 2006). Unfortunately, this is not always the case.

According to Aujean et al. (1999), distortions under the VAT arise mainly from the exemption for specified services and sectors. As Cnossen (2003) argues, the rationale for most exemptions lays in the history of their adoption and not in “their underlying economic or administrative logic”. For instance, countries that have adopted the VAT only recently (e.g., Australia, Canada or New Zealand) follow “best practice” by taxing most of the services that are exempt in countries with a long VAT history (e.g., member states of the European Union).

The postal sector is a good example of such a distortion because most member states of the European Union (EU) exempt universal postal services provided by the incumbent operator from VAT on the grounds that they are the “public postal service.” At the same time, competing postal service providers have to charge VAT at the standard rate. Recently, the European Court of Justice (ECJ) essentially confirmed this interpretation of the current VAT Directive.¹ According to the interpretation of the ECJ, universal service tariffs are VAT exempt unless prices are individually negotiated with customers. The decision of the ECJ is binding on all member states. The postal sector is therefore a good candidate to illustrate the competitive effects of VAT regimes in a comprehensive way.

The focus of our paper is on the competitive and welfare effects of different VAT regimes in sectors with asymmetric VAT exemptions. Where such exemptions are present, the competitive effects are ambiguous a priori. While an exempt firm cannot reclaim VAT paid on inputs (which is relevant for non-labor inputs only) and therefore faces higher costs ceteris paribus, an important fraction of customers of non-exempt firms will not be able to deduct VAT themselves. Hence, the exempt firm has a cost disadvantage on the one hand, and a price advantage on the other. The net effect will depend on the fraction of non-labor inputs relative to the fraction of non-rated customers.²

¹ Council Directive 77/388/EEC of 17 May 1977 on the harmonization of the laws of the EU member states relating to turnover taxes.

² Figure 1 in Section 3 illustrates this trade-off.

To analyze the effects of different VAT scenarios, we develop an analytical model that is applicable to any sector characterized by asymmetric VAT exemptions of services and activities or differentiated VAT rates. In a second step, we calibrate the model for a selected sector – namely the postal market. This approach enables us to quantify competitive and welfare effects of selected VAT regimes. The various scenarios differ regarding the firms’ fraction of labor input and their VAT status. We also take into account the fraction of non-rated customers that cannot deduct VAT themselves. The contribution of our paper is to provide a well-founded basis to assess the main competitive impact of VAT policies, while showing the consequences on overall welfare. Finally, we assess the policy trade-off between a level playing field, consumer surplus and government tax revenue.

The structure of the paper is as follows. Section 2 briefly reviews the related literature. Section 3 introduces the model framework and presents the analytical results. Section 4 describes the calibration of the model for the postal sector. Section 5 reports the simulation results. Finally, Section 6 summarizes the main findings and concludes.

2 Literature Review

Before proceeding with the model, we provide a short literature review. Cnossen (1998) examines various VAT structures and argues that the application of the VAT on public sector bodies has the advantage of confronting policymakers more directly with the full cost of public intervention. Competitive conditions are distorted if the government’s services bear a lower tax or no tax, while competing private-sector services are taxed in full. He concedes, however, that prices of government services are often regulated or subsidized which makes the levy of the VAT little more than a bookkeeping exercise, because the effect of the VAT can be exactly replicated by the exemption. Keen and Lockwood (2006) use panel data for the OECD and empirically test whether the VAT is a “money machine”. They find that countries with a VAT raise more revenue than those without. However, the effect may not be large. Keen (2007) examines in detail the critics the VAT faces today. Giesecke and Tran (2010) analyze the VAT characteristics of multi-production, legislated differences in exemption status, and industry-specific differences in the refundability of VAT paid on inputs to production and investment in a general equilibrium framework. Finally, PWC (2007) discuss various forms of market distortions resulting from VAT exemptions for financial and insurance services.

In the postal sector, VAT exemptions have been addressed in recent years. Dieke and Elixmann (2005) quantify the effect of VAT exemptions for postal operators on government

tax revenue. De Donder et al. (2009) focus on the pricing and welfare implications of changing a postal operator's VAT status. Crew et al. (2009) discuss the importance of VAT exemptions in the framework of the prospective study by PwC (2006). Relative to the work of De Donder et al. (2009), who assume that entrants act as a competitive fringe, we model profits of both the incumbent and new market entrants. This allows us to provide a more comprehensive treatment of competitive effects of VAT policies. We also provide a relevant sensitivity analysis with regards to the fraction of labor inputs and the fraction of VAT exempt customers. We show that the results are very sensitive to the operators' labor policies. Consequently, VAT exemptions have a different impact in countries with different labor regulations. Secondly, the sensitivity analysis highlights that the competitive effects will vary strongly between different customer segments. Hence, there is a second important regulatory link between VAT exemptions and uniform pricing constraints. The comprehensive treatment of competition and welfare enables us to provide guidance on how to resolve the policy trade-off between consumer surplus, government tax revenue, and a level playing field markets with VAT exemptions.

3 The Analytical Model

3.1 Notation and Assumptions

Two firms (operators) denoted by I and E offer differentiated services in the same market.³ The before-tax price of firm $i \in \{E, I\}$ is given by p_i , whereas $(1+t_i)p_i$ denotes the after-tax price of firm i , with $t_i \in [0,1]$ being the VAT rate of firm i . Moreover, each firm pays VAT given by $t \in [0,1]$ on non-labor inputs. Depending on their VAT status, firms are able to deduct the input VAT from their output VAT billed to their customers.

Following Dietl et al. (2005) and Jaag and Trinkner (2009), we define total utility of a representative customer as:⁴

$$u(x_I, x_E) = m + \alpha_I x_I - \frac{\beta}{2} x_I^2 + \alpha_E x_E - \frac{\beta}{2} x_E^2 - \varepsilon \beta x_I x_E, \quad (1)$$

where $\alpha_I, \alpha_E, \beta > 0$ and $\varepsilon \in (0,1)$. The parameter $m > 0$ characterizes the amount of money spent on other goods. x_I and x_E are the demands faced by firm I and firm E ,

³ The firms (operators) are denoted with I and E because we calibrate the model with market data from a selected sector (postal market) in Section 4. As mentioned, in postal markets the (historical) incumbent operator I is usually VAT exempt, while the entrant operator E is VAT rated.

⁴ Note the quasi-linearity of the utility specification that will later allow us to compute monetary welfare measures.

respectively. The last term in Equation (1) reflects the fact that the services offered by the two firms are differentiated products. The smaller is the parameter ε , the higher is the degree of differentiation. The parameters α_I and α_E determine the market shares of firm I and firm E , respectively, while β determines the slope of the demand functions. In our model, there are two types of customers which are VAT rated and VAT exempt respectively. For simplicity, we aggregate these to one representative customer. Her/his budget constraint is given by:

$$y = m + x_I [\gamma p_I(1+t_I) + (1-\gamma)p_I] + x_E [\gamma p_E(1+t_E) + (1-\gamma)p_E], \quad (2)$$

where $\gamma \in [0,1]$ denotes the fraction of VAT exempt, “non-rated” customers,⁵ while $(1-\gamma)$ is the proportion of customers that are VAT rated. The latter type of customers can reclaim the VAT they paid on their products because these products are an input into their own production processes. Reclaiming VAT is not possible for VAT exempt customers. Thus, for VAT rated customers, the before-tax price p_i is relevant, while for VAT exempt customers, the after-tax price $p_i(1+t_i)$ from firm i is relevant. The model specification presumes that the fraction of non-exempt letters is the same for both firms.

From Equations (1) and (2), we determine the demand faced by firm i in the following lemma.

Lemma 1

The linear demand function for the product of firm (operator) i is given by:

$$x_i = \frac{1}{\beta(1-\varepsilon^2)} \left[\alpha_i - \varepsilon \alpha_j - p_i(1+\gamma t_i) + \varepsilon p_j(1+\gamma t_j) \right], \quad (3)$$

with $i, j \in \{E, I\}$ and $i \neq j$.

Proof: See Appendix A.1.

Note that the slopes of the linear demand functions are equal for both firms. We further see that the demand of firm i decreases in own prices p_i , while it increases in the price p_j of the other firm j . Demand is also positively related to a higher degree of product differentiation (a smaller ε): i.e., $\partial x_i / \partial \varepsilon < 0$.

If firm i is VAT exempt (i.e., $t_i = 0$), it does not charge VAT to its customers while it charges VAT to its customers if it is VAT rated (i.e., $t_i > 0$). The effect of different VAT

⁵ More precisely, γ represents the fraction of demand from its VAT exempt business (e.g., banking), while $(1-\gamma)$ represents ordinary commercial business activities that are rated.

regimes t_i, t_j on x_i , i.e., $\frac{dx_i}{dt_i} = \frac{\partial x_i}{\partial t_i} + \frac{\partial x_i}{\partial p_i} \frac{\partial p_i}{\partial t_i} + \frac{\partial x_i}{\partial p_j} \frac{\partial p_j}{\partial t_i}$ will depend on the second-order effects of a change in the tax rate t_i . However, these second-order effects are ambiguous, as we will see below.

On the cost side, firm i is assumed to face three types of costs: (i) fixed costs F_i , (ii) constant marginal upstream costs u_i (e.g., collection and sorting in the postal sector), and (iii) constant marginal downstream costs d_i (e.g., mail delivery in the postal sector). The fraction of the fixed costs that is non-labor is denoted by $\mu_i^F \in (0,1)$, while $\mu_i \in (0,1)$ stands for the fraction of marginal upstream and downstream costs that is non-labor costs. Note that firm i has to pay VAT, given by t , on the fraction of non-labor costs derived from fixed costs, upstream and downstream costs independent of its VAT status.

We thus specify total costs faced by firm (operator) $i \in \{E, I\}$ as:

$$C_i = \left[1 + \mu_i^F (t - t_i) \right] F_i + 1 + \mu_i (t - t_i) (u_i + d_i) \cdot x_i, \quad (4)$$

with $t \geq t_i$ and demand x_i given by Equation (3). For notational simplicity, we denote the sum $u_i + d_i$ of marginal upstream and downstream costs with c_i in the subsequent analysis: i.e., $c_i = u_i + d_i$.

The cost function shows that the VAT status crucially determines the costs faced by firm i . For example, if firm i is VAT rated with $t_i = t$ it can reclaim the VAT it has paid on inputs such that $C_i^r = F_i + c_i x_i$. Conversely, if firm i is non-rated, i.e., VAT exempt with $t_i = 0$, it cannot reclaim the VAT it has paid on inputs such that $C_i^{nr} = (1 + \mu_i^F t) F_i + (1 + \mu_i t) c_i x_i$.

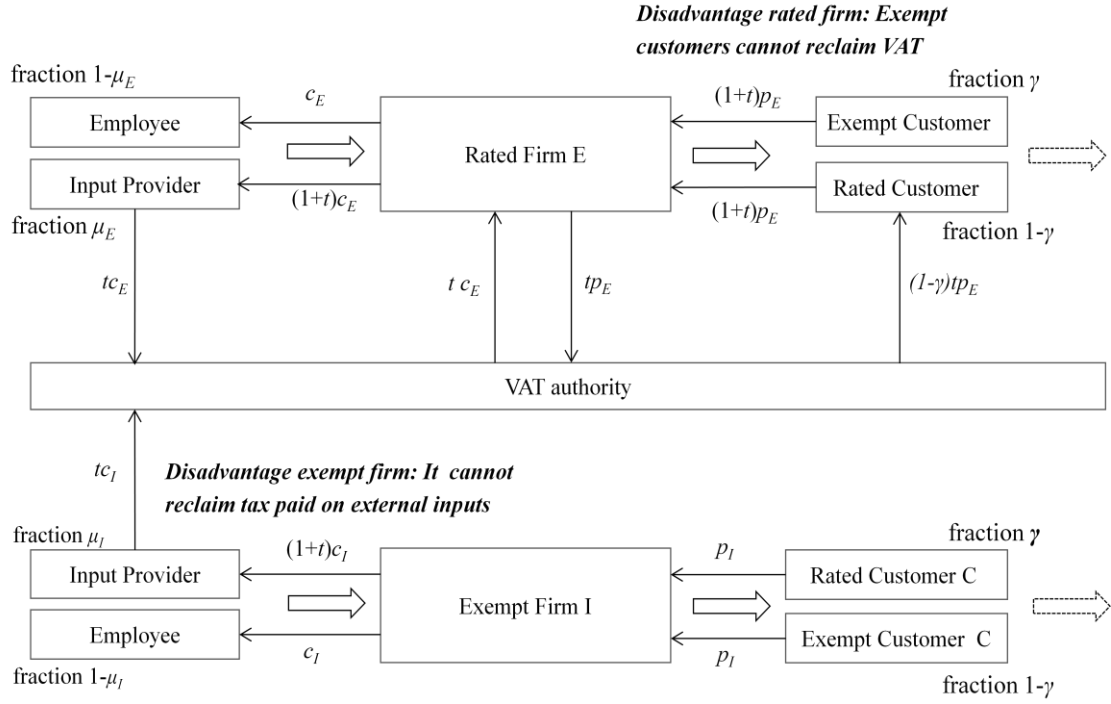
The (net of tax) profit of firm (operator) i thus amounts to:

$$\begin{aligned} \pi_i &= p_i x_i - 1 + \mu_i (t - t_i) c_i \cdot x_i - \left[1 + \mu_i^F (t - t_i) \right] F_i \\ &= \frac{p_i - 1 + \mu_i (t - t_i) c_i}{\beta(1 - \varepsilon^2)} \left[\alpha_i - \varepsilon \alpha_j - p_i (1 + \gamma t_i) + \varepsilon p_j (1 + \gamma t_j) \right] - \left[1 + \mu_i^F (t - t_i) \right] F_i, \end{aligned}$$

with $i, j \in \{E, I\}$ and $i \neq j$.

The model framework basically reflects the situation as shown in Figure 1 below. It is generally applicable to any industry with asymmetric VAT exemptions. Firm I is exempt from VAT ($t_I = 0$), whereas firm E is fully rated ($t_E = t$). This will be Scenario A in the subsequent analysis.

Figure 1: Overview of VAT Flows



3.2 Optimality Conditions

To derive its optimal pricing, firm i solves the maximization problem $\max_{p_i \geq 0} \pi_i$. The corresponding first-order conditions are computed as:⁶

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{\beta(1-\varepsilon^2)} \left[\alpha_i - (1+\gamma t_i) 2p_i - (1+\mu_i(t-t_i))c_i - \varepsilon(\alpha_j - p_j(1+\gamma t_j)) \right] = 0.$$

From the first-order conditions, we obtain the reaction function $p_i(p_j)$ for firm i as:

$$p_i(p_j) = \frac{1}{2} \left[1 + \mu_i(t-t_i) c_i + \frac{\alpha_i - \varepsilon \alpha_j}{1 + \gamma t_i} + \varepsilon \frac{1 + \gamma t_j}{1 + \gamma t_i} p_j \right],$$

with $i, j \in \{E, I\}$ and $i \neq j$. Solving the system of reaction functions yields the equilibrium before-tax price p_i^* of firm (operator) i as:

$$p_i^* = \frac{1}{4-\varepsilon^2} \left[2 1 + \mu_i(t-t_i) c_i + \varepsilon \frac{1 + \gamma t_j}{1 + \gamma t_i} 1 + \mu_j(t-t_j) c_j + \frac{\alpha_i(2-\varepsilon^2) - \alpha_j \varepsilon}{1 + \gamma t_i} \right], \quad (5)$$

with $i, j \in \{E, I\}$ and $i \neq j$.

⁶ The second-order conditions for a maximum are satisfied since $\frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{2(1+t_i\gamma)}{\beta(1-\varepsilon^2)} < 0$.

It is intuitive that the before-tax price p_i^* of firm i increases with higher own marginal upstream and downstream costs c_i . Similarly, it is straightforward that p_i^* also increases with higher marginal costs c_j of the competitor j .

We further derive that a higher tax rate t_j of competitor j induces a higher before-tax price p_i^* of firm i if the proportion γ of VAT exempt customers is sufficiently large with $\gamma > \frac{\mu_j}{1+\mu_j(t-2t_j)}$. Note that firm j 's amount of input tax (measured by μ_j) plays a crucial role. As γ is usually significantly larger than zero (see calibration section), the condition will be satisfied if μ_j is sufficiently close to zero (e.g., an operator mainly employing labor directly instead of via subcontractors). In such a case, an increase of firm j 's tax t_j will degrade its competitive position, as non-rated customers face higher prices that cannot be compensated for by input tax deductions. Hence, firm i will be able to increase its price profitably. On the other hand, the effect of a higher own tax rate t_i on the own before-tax price p_i^* is ambiguous. It depends on μ_i, γ and the relative tax status of the two firms.

To derive further insights, we analyze two scenarios: in Scenario A, firm I is VAT exempt, while firm E is VAT rated. In Scenario B, both firms are VAT rated.

3.3 Scenario A: Firm I is VAT exempt and Firm E is VAT rated

In Scenario A we assume that firm I is VAT exempt, i.e., $t_I = 0$, while firm E is VAT rated, i.e., $t_E = t > 0$. Since firm E is VAT rated, it can reclaim the VAT it has paid on inputs, while this is not possible for the VAT exempt firm I .

From Equation (5), we compute the before-tax price p_I^A of firm (operator) I in Scenario A by setting $t_I = 0$ and $t_E = t$:

$$p_I^A = \frac{1}{4-\varepsilon^2} \left[2 \frac{1+\mu_I t}{1+\gamma t} c_I + \varepsilon \frac{1+\mu_I t}{1+\gamma t} c_E + \alpha_I (2-\varepsilon^2) - \alpha_E \varepsilon \right].$$

Similarly, the before-tax price p_E^A of firm (operator) E is computed as:

$$p_E^A = \frac{1}{4-\varepsilon^2} \left[2 c_E + \varepsilon \frac{1}{1+\gamma t} \frac{1+\mu_I t}{1+\gamma t} c_I + \frac{\alpha_E (2-\varepsilon^2) - \alpha_I \varepsilon}{1+\gamma t} \right].$$

Substituting equilibrium prices (p_I^A, p_E^A) in the demand functions given by Equation (3), produces equilibrium demands (x_I^A, x_E^A) in Scenario A as:

$$x_I^A = \frac{(2 - \varepsilon^2)(\alpha_I - c_I(1 + \mu_I t)) + \varepsilon(c_E(1 + \gamma t) - \alpha_E)}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)},$$

$$x_E^A = \frac{(2 - \varepsilon^2)(\alpha_E - c_E(1 + \gamma t)) + \varepsilon(c_I(1 + \mu_I t) - \alpha_I)}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)}.$$

Note the close similarity of the two demand functions; only μ_I and γ are switched. The functions illustrate the trade-off that we have discussed: input tax disadvantage vs. output tax advantage. With symmetric costs and demands, firm I will have a larger market share whenever $\gamma > \mu_I$. Note that the parameter μ_I depends on the labor policy of the firm. *Ceteris paribus*, being VAT exempt will make it more profitable for the firm to employ workers directly than via subcontracting as compared to being VAT rated.

In the next proposition, we analyze the effect of a higher tax rate t on equilibrium prices and demands in Scenario A:

Proposition 1

(i) A higher tax rate t always yields an increase in the before-tax price p_I^A of the VAT exempt firm I , while the before-tax price p_E^A of the VAT rated firm E decreases for a reasonable range of parameters, i.e., $\frac{\partial p_E^A}{\partial t} < 0 \Leftrightarrow \alpha_I < \frac{\gamma - \mu_I}{\gamma} c_I + \frac{2 - \varepsilon^2}{\varepsilon} \alpha_E$.

(ii) A higher tax rate t induces a decrease in the equilibrium demands (x_I^A, x_E^A) of the VAT exempt firm and the VAT rated firm if $\frac{c_E}{c_I} \frac{\gamma}{\mu_I}$ is in a reasonable range of parameters, i.e.,

$$\left(\frac{\partial x_I^A(t)}{\partial t} < 0 \text{ and } \frac{\partial x_E^A(t)}{\partial t} < 0 \right) \Leftrightarrow \frac{c_E}{c_I} \frac{\gamma}{\mu_I} \in \frac{\varepsilon}{2 - \varepsilon^2}, \frac{2 - \varepsilon^2}{\varepsilon}.$$

Proof: See Appendix A.2.

Part (i) of the proposition shows that the before-tax price of the VAT exempt firm I always increases in the tax rate. If the model parameters are within a reasonable range, then the before-tax price of the VAT rated firm E decreases in the tax rate. This result can be explained by two effects: (a) as firm I cannot deduct VAT, higher taxes will directly lead to higher production costs. (b) a higher tax rate will increase firm I 's output tax advantage as the increased VAT rate is directly price relevant for firm E 's non-rated customers. Under reasonable calibration assumptions (i.e., minimal amount of non-rated customers relative to the size of μ_E), firm E will be forced to reduce prices to offset the increase in taxes without gaining market shares in return. Marginally, firm I is able to increase prices. Hence, the two

effects always have the same direction for the VAT exempt firm, while they are ambiguous for the VAT rated firm.

Both effects will – under reasonable calibration assumptions – negatively affect demand. Analytically, this is shown in part (ii) of Proposition 2: for a reasonable parameter constellation of $\frac{c_E}{c_I} \frac{\gamma}{\mu_I}$ both types of firms will respond to a higher tax with lower quantities.

3.4 Scenario B: Both Firms are VAT rated

In Scenario B, we assume that firm I and firm E are VAT rated: i.e., $t_I = t_E \equiv t > 0$. It follows that both firms can reclaim the VAT they have paid on inputs.

From Equation (5), we compute the before-tax price p_I^B of firm (operator) I in Scenario B by setting $t_I = t_E \equiv t$:

$$p_I^B = \frac{1}{4 - \varepsilon^2} \left[2c_I + \varepsilon c_E + \frac{\alpha_I(2 - \varepsilon^2) - \alpha_E \varepsilon}{1 + \gamma t} \right],$$

Similarly, the before-tax price p_E^B of firm E is given by:

$$p_E^B = \frac{1}{4 - \varepsilon^2} \left[2c_E + \varepsilon c_I + \frac{\alpha_E(2 - \varepsilon^2) - \alpha_I \varepsilon}{1 + \gamma t} \right].$$

Substituting equilibrium prices (p_I^B, p_E^B) in the demand functions given by Equation (3), produces equilibrium demands (x_I^B, x_E^B) in Scenario B as:

$$x_I^B = \frac{(2 - \varepsilon^2)(\alpha_I - c_I(1 + \gamma t)) + \varepsilon(c_E(1 + \gamma t) - \alpha_E)}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)},$$

$$x_E^B = \frac{(2 - \varepsilon^2)(\alpha_E - c_E(1 + \gamma t)) + \varepsilon(c_I(1 + \gamma t) - \alpha_I)}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)}.$$

Except for the cost structure and the market shares, the two equilibrium demands are now equal and independent of the fraction of VAT rated inputs. Hence, this VAT regime does not distort competition between the two firms (operators); consequently, Scenario B can be seen as the benchmark case for Scenario A's market distortions driven by firm I 's VAT exemption.

In the next proposition, we analyze the effect of a higher tax rate t on the equilibrium prices and demands in Scenario B.

Proposition 2

(i) A higher tax rate t yields a decrease in the before-tax prices (p_I^B, p_E^B) of firm I and firm E if the ratio of market sizes α_I / α_E is within a reasonable range of parameters, i.e.,

$$\left(\frac{\partial p_I^B(t)}{\partial t} < 0 \text{ and } \frac{\partial p_E^B(t)}{\partial t} < 0 \right) \Leftrightarrow \frac{\alpha_I}{\alpha_E} \in \frac{\varepsilon}{2-\varepsilon^2}, \frac{2-\varepsilon^2}{\varepsilon}.$$

(ii) A higher tax rate t yields a decrease in the equilibrium demands (x_I^B, x_E^B) of firm I and firm E if the ratio of cost parameters c_I / c_E is within a reasonable range of parameters, i.e.,

$$\left(\frac{\partial x_I^B(t)}{\partial t} < 0 \text{ and } \frac{\partial x_E^B(t)}{\partial t} < 0 \right) \Leftrightarrow \frac{c_I}{c_E} \in \frac{\varepsilon}{2-\varepsilon^2}, \frac{2-\varepsilon^2}{\varepsilon}.$$

Proof: See Appendix A.3.

A higher VAT tax rate will decrease before-tax prices for both firms in Scenario B under reasonable market conditions. While the fraction of firm I's non-labor inputs μ_I is no longer relevant as the firm can now deduct input taxes too, a tax increase will lead to higher prices for the non-rated customer segment. To offset some of the resulting volume reductions, the firm will be forced to reduce their before-tax prices *ceteris paribus*.

In equilibrium, total demand will decrease for both firms because the increase in VAT introduces a new cost for non-rated customers. Moreover, a higher parameter γ reinforces the negative effect of t on the equilibrium demands for both firms. Note that if $\gamma = 0$ then the tax rate t has no effect on the equilibrium demands.

3.5 Comparison of Scenarios A and B

Comparing Scenarios A and B, we can establish Proposition 3.

Proposition 3

(i) The before-tax price of the entrant is lower in Scenario A than in Scenario B if and only if the proportion γ of VAT exempt customers is lower than the fraction μ_I of upstream and downstream costs that is non-labor, i.e., $p_E^A < p_E^B \Leftrightarrow \gamma < \mu_I$.

(ii) The before-tax price of firm I is higher in Scenario A than in Scenario B for a reasonable range of parameters, i.e., $p_I^A > p_I^B \Leftrightarrow \mu_I > \gamma \frac{\varepsilon(\alpha_E - c_E(1 + \gamma t)) - (2 - \varepsilon^2)\alpha_I}{2c_I(1 + \gamma t)}$.

Proof: See Appendix A.4.

Part (i) of Proposition 3 shows that the relation between the fraction of non-labor upstream and downstream costs and the proportion of VAT exempt customers crucially determines

whether the before-tax price of firm E is higher in Scenario A or B. Hence, if γ is smaller than μ_I , firm I 's VAT exemption will translate into a disadvantage from firm E 's point of view and force firm E to reduce prices *ceteris paribus*. Note that with regard to postal markets, this is the likely scenario as incumbents (firm I) often have a high percentage of labor costs (i.e., $\mu_I > 0.5$), while the fraction of non-rated customers does usually not exceed 50% (i.e., $\gamma \leq 0.5$).

Part (ii) of Proposition 3 mirrors the results of Part (i). While firm E will be forced to decrease prices *ceteris paribus*, firm I will be able to increase its price under reasonable calibration assumptions. Hence, VAT exemptions are likely to strengthen the competitive position of firm I .

4 Calibration

To predict competitive and welfare effects more precisely, we simulate the analytical model presented in Section 3 by using stylized market data for a selected sector with asymmetric VAT exemptions. We have chosen the postal sector because it provides a good example of such a distortion. In particular, we calibrate the model to reflect the B2C bulk mail market. This is the mail segment in which competition is most likely to take place in a fully liberalized postal market. Firm I represents the incumbent operator (universal service provider), while firm E is the entrant operator. Scenario A represents the standard situation in the EU postal markets, in which only the historical (incumbent) operator is VAT exempt, while the entrant operator has to charge VAT at the standard rate. In Scenario B, both the incumbent and the entrant operator are VAT rated.

To calibrate parameters α_I and β , we assume that incumbent I as a monopolistic operator in the market would deliver 1 billion bulk mail items at an average price of 0.35 units of money per item with a point-price-elasticity of -0.5.⁷ Note that the demand function given in Equation (3) is linear, which results in the price-elasticity decreasing in quantity.

Parameters α_i influence the size of the market of the two operators' services. By setting $\alpha_I > \alpha_E$, we can include effects like customer inertia, reputation effects, switching costs, or even quality differences in favor of the incumbent (universal service provider). Formally, we define ϕ as the percentage of total demand the incumbent serves if the entrant offers the very same price for its services ("incumbent advantage"). For the calibration, we evaluate demand

⁷ For more calibration details, see Dietl et al. (2005, 2010).

given in Equation (3) with a price of 0.35 for both operators and solve the resulting equation system. We obtain:

$$\phi = \frac{x_I}{x_I + x_E}; \quad p_I = p_E = p = 0.35; \quad \alpha_E = \frac{1}{1 - \varepsilon + \frac{\varepsilon}{\phi}} \alpha_I (\varepsilon - 1 + \frac{1}{\phi}) + p(1 - \varepsilon)(2 - \frac{1}{\phi}) .$$

In the simulation, we set $\phi = 0.8$, that is, the entrant would get 20% of the market if it were to offer the very same services as the incumbent. A value of 50% would mean that customers have symmetric preferences for the two operators.⁸

As mentioned, parameter ε measures how different the two products of the two operators are: a value close to one would mean that the products are very close substitutes. Consistent with previous model calibrations (e.g., Dietl et al. 2005), we set $\varepsilon = 0.75$.

Parameter γ represents the fraction of VAT rated customers. The value varies across mail segments. For example, in the C2C segment, γ is close to zero as private customers cannot reclaim VAT. For the Base Case, we assume $\gamma = 0.5$ to reflect the bulk mail market. The value is in line with the current situation, e.g., in the German letters market, where DPWN reported a 50% fraction of non-rated customers in 2009.

On the supply side, cost needs to be differentiated in the three dimensions: variable/fixed, upstream/downstream and labor/non-labor costs. The latter is relevant for the deduction of input VAT because non-labor costs are VAT rated. In the monopolistic calibration benchmark, we assume costs of 250 million units of money excluding input taxes. In line with the demand calibration, the cost structure of the incumbent is calibrated for a hypothetical monopolistic situation. Thereby, we assume a reasonable rate of return such that the initial price of 0.35 represents a rate-of-return regulated monopoly.⁹

Table 1 shows the major cost assumptions. With these we are able to compute the necessary parameters to calibrate the two cost functions as introduced in Equation (4).

Table 1: Major Cost Assumptions Base Case

	Incumbent	Entrant
Fraction of fixed costs	50%	20%
μ^F (fraction of VAT rated fixed costs)	$=\mu_I$	$=\mu_E$
μ (fraction of VAT rated variable costs)	30%	70%
Efficiency premium upstream	-	10%
Efficiency premium downstream	-	30%
Wage premium	-	-

⁸ See Jaag (2007) for a discussion of consumers' switching behavior in the mail market.

⁹ By doing so, we are able to report the results for the monopoly benchmark. As we are interested in competitive effects mainly, we will not report these results.

We assume that the entrant pursues a different business model in the Base Case with less fixed costs because it makes use of subcontracting in delivery, rendering the cost structure more flexible (variable) and yielding a larger fraction of VAT rated inputs. We also assume that the entrant is more efficient upstream (by more strongly incentivizing digital sorting) and downstream (with a reduced delivery frequency). The lead example of such a business model is the Dutch company Sandd. Similar models can be found in other liberalized postal markets. To illustrate the distortive effects of VAT with regards to outsourcing, we assume that both players pay equal wage rates.

The quasi-linear model framework allows for a computation of overall welfare by adding up consumer surplus, operators' profits and governmental tax revenues. The effect of changing postal VAT regimes on governmental tax revenues can be computed as follows. In the case that the USP is VAT exempt, the total VAT tax base is the value of the USP's input goods plus the product value of the USP's customers' output that is VAT rated. If the USP is VAT rated, the tax base is the value of the USP's output to VAT exempt customers in addition to the product value of the USP's customers' output that is VAT rated. Whether the difference in the two cases is positive thus depends on the USP's value added and the fraction of VAT rated customers. It is positive if the fraction of VAT exempt customers is larger than the inverse of the USP's relative value added. In the next section, we will compute the relevant overall welfare measures.

5 Numerical Results

The calibrated model allows for some insight into the overall competitive and welfare consequences of various tax regimes using the example of the postal sector. In addition, we perform sensitivity analysis and derive recommendations for regulators, market players, and VAT authorities. Note that the quantitative results presented in this section serve as rough guidelines only.¹⁰

We report simulation results for the two Scenarios A (incumbent is VAT exempt, $t_I = 0$) and B (both operators fully rated at $t_I = t_E = 20\%$). We are interested in (i) competitive effects measured by market shares, prices, and profits; (ii) welfare effects; and (iii) changes in collected VAT. We compute the latter against a benchmark scenario where both operators are VAT exempt ($t_I = t_E = 0$).

¹⁰ More detailed simulation results are provided in Dietl et al. (2010).

5.1 Base Case

Table 2 reports the results for the Base Case as introduced in Section 4. The Base Case includes three different VAT scenarios. In Scenario 0, both operators are VAT exempt. Scenario A is the standard case in the European postal markets where only the historical (incumbent) operator is VAT exempt. In Scenario B, both operators are fully rated. The results that illustrate the competitive effects are shown in the upper part of the table, while those that show the welfare effects are reported in the lower part.

Table 2: Simulation Results Base Case

Simulation Results Case x ($\mu I=0.3$; $\mu E=0.7$)			
	Scenario 0	Scenario A	Scenario B
Individual VAT rate I (t_I)	0%	0%	20%
Individual VAT rate E (t_E)	0%	20%	20%
Competitive Effects			
Incumbent Market Share	64.8%	64.1%	63.7%
Price Ratio (I/E) excl. VAT	118%	131%	120%
Price Ratio (I/E) incl. VAT	118%	109%	120%
Profit Ratio (I/E)	1755%	455%	212%
Profit I	21'979'609	20'051'777	11'013'273
Profit E	1'252'698	4'410'232	5'196'978
Welfare Effects			
Overall Price Level excl. VAT	0.33	0.32	0.30
Overall Price Level incl. VAT	0.33	0.34	0.36
Operator Profits	23'232'307	24'462'009	16'210'251
Consumer Surplus	372'849'153	375'231'196	372'755'232
Incremental Government Tax Revenue	-	-3'582'967	6'069'580
Overall Welfare	396'081'460	396'110'238	395'035'064

The comparison of Scenario A and B illustrates the competitive distortions of asymmetric VAT exemptions.

In the Base Case, Scenario A is more favorable for the incumbent. Compared to Scenario B where both operators are fully rated, the incumbent's profit increases substantially while the entrant's profit decreases slightly. Both price and profit ratios are substantially higher for the incumbent in Scenario A, meaning that the incumbent can charge higher prices in Scenario A in relative terms and earn a higher profit at the same time. Despite its higher price level in Scenario A, the incumbent achieves a higher market share. The figures show that the tax exemption is distorting competition significantly.¹¹

¹¹ With the following exception, the results are in line with recent decisions of Deutsche Post DHL to reduce its letter prices for business customers significantly in light of the new VAT regime in Germany as of July 1, 2010. Deutsche Post announced net price decreases equal to the VAT rate itself, which is significantly more than we predict in our simulation.

Nevertheless, Scenario A exhibits a slightly higher overall welfare level than Scenario B.¹² There are two opposite welfare effects at work: As a result of the incumbent's VAT rating, the average marginal tax rate in the industry increases. This lowers welfare. However, as the two operators are equally rated, there is a level playing field and competition is more intense, which increases welfare. While incremental profits are roughly compensated for by opposite incremental tax effects (the profit decrease of the incumbent in Scenario B equals roughly the tax increase of the tax authority), customers are slightly better off in Scenario A. The positive effect comes from the 50% non-rated customers who face lower net prices than in Scenario B.¹³

The comparison of Scenario 0 and Scenario B illustrates the distortive impact of VAT on internal labor policies. If both operators are exempt from VAT (Scenario 0), there is a strong incentive to use internal workforce, as input VAT on outsourced labor cannot be deducted and hence outsourcing is less attractive *ceteris paribus*. Consequently, the entrant's competitive position is much stronger in Scenario B where input VAT can be deducted. Note that we treat the operators' cost structures as exogenous in both scenarios. In practice, operators would be likely to optimize their labor policy and increase the share of internal workforce in Scenario 0, while Scenario B will be competitively neutral in this regard. To sum up, abolishing the incumbent's VAT exemption levels the playing field (in the market itself and in the outsourcing market), while it slightly decreases overall welfare in the Base Case.

5.2 Effect of Different Cost Structures μ_I and μ_E

Our analytical results presented in Section 3 indicate that the effects crucially depend upon the relative magnitude of the parameters μ_i and γ . While γ is exogenously-given, the cost structure μ_i can be optimized by the operators. The fraction of rated inputs for the entrant, μ_E , is not relevant for the entrant's decision making if it is fully rated (i.e. in Scenarios A and B): a higher value of μ_E means larger VAT expenses, which, however, can be fully deducted from the VAT billed to the customers. For the tax authority, the net effect matters, as we

¹² Our welfare results differ from those reported by De Donder et al. (2009), which yield higher welfare in Scenario B. While the authors report higher consumer surplus in Scenario A too, they multiply government tax revenues by 1.3 to reflect the shadow cost of public funds and therefore find higher overall welfare in Scenario B. As we are interested in the relative effects of the postal sector, we weigh all three components of welfare equally and generally do not account for second-order effects in other parts of the economy.

¹³ Note that this effect stems from the fact that we do not allow for price differentiation between customer segments. Hence, the operators are forced to balance over the two customer segments yielding lower net prices for the rated customers. While we could extend the model to capture the relevant effects, regulations in many countries (e.g., Germany) will not allow differentiated prices for the incumbent.

report the difference in a scenario with both operators being VAT exempt. Hence, a higher μ_E increases the input tax deduction that the entrant can reclaim.

In contrast to μ_E , changes in μ_I are of great importance for the market equilibrium in Scenario A, in which the incumbent is VAT exempt. Here, changes in μ_I are directly cost-relevant: outsourcing to equally efficient partners will increase costs by the VAT rate times the amount of the outsourced input goods. In Scenario B, μ_I is irrelevant for the market equilibrium (in analogy to μ_E above).

Table 3: Simulation Results for Different Combinations of μ_I and μ_E

	Scenario A1	Scenario A2	Scenario B
μ_I (fraction of non-rated inputs I)	0.30	0.70	[0-1]
μ_E	[0-1]	[0-1]	[0-1]
Competitive Effects			
Incumbent Market Share	64.1%	63.3%	63.7%
Price Ratio (I/E) excl. VAT	131%	132%	120%
Price Ratio (I/E) incl. VAT	109%	110%	120%
Profit Ratio (I/E)	455%	70%	212%
Profit I	20'051'777	4'205'975	11'013'273
Profit E	4'410'232	5'990'790	5'196'978
Welfare Effects			
Overall Price Level excl. VAT	0.32	0.32	0.30
Overall Price Level incl. VAT	0.34	0.34	0.36
Operator Profits	24'462'009	10'196'765	16'210'251
Consumer Surplus	375'231'196	370'295'244	372'755'232
Incremental Government Tax Revenue	-3'582'967	-3'469'367	6'069'580
Overall Welfare	396'110'238	377'022'643	395'035'064

Table 3 reports the competitive effects of different cost structures.¹⁴ Scenario A1 and A2 differ in μ_I , but not in μ_E . (i.e., we obtain the same results for all $\mu_E \in [0,1]$) Scenario A1 represents an incumbent that uses employees mainly. Scenario A2 indicates an incumbent business model with subcontractors in delivery.

While the share of non-rated inputs is generally irrelevant in Scenario B, it is of great importance in Scenario A for the operator that is VAT exempt. In contrast to the relative prices, the operators' profits change substantially when comparing Scenarios A1 and A2. The incumbent's VAT exemption is an advantage in Scenario A1 and a disadvantage in Scenario A2, where incumbent profits are lower. The results are in line with our analytical findings. Note that in Scenario A1, $\mu_I < \gamma$, while we have $\mu_I > \gamma$ in Scenario A2. We conclude that the

¹⁴ Remember that the fraction of VAT exempt (non-rated) customers is set to $\gamma = 0.5$.

net competitive effect of an asymmetric VAT exemption crucially depends on the fraction of VAT rated inputs versus the fraction of non-rated customers. In the Base Case, the latter effect compensates for the former and the exempt incumbent has a competitive advantage.

In terms of overall welfare, a higher μ_I decreases welfare in Scenario A, as the higher perceived costs of the incumbent reduces its profits and increases average prices in the market slightly (lower consumer surplus). In Scenario B, operator and consumer surplus remain unaffected. Abolishing the incumbent's VAT exemption decreases welfare in Scenario A1 ($\mu_I < \gamma$), whereas it increases welfare in Scenario A2 ($\mu_I > \gamma$). Hence, from a public policy point of view, the incumbent's VAT exemption is desirable in a scenario in which the incumbent's fraction of non-labor costs is relatively low. Conversely, if the fraction is relatively high, the VAT exemption is welfare-reducing because it induces higher prices.¹⁵

In most European countries, incumbent operators predominantly do not make use of outsourced labor (i.e., μ_I is relatively low). Hence, VAT exemptions for bulk mail can be justified from a welfare perspective in countries with a substantial fraction of non-rated customers, even though such exemptions distort competition clearly in the incumbent's favor.

5.3 Effects of Different Combinations of γ and μ_I

Our analytical results in Section 3 have shown that the competitive effects of VAT exemptions are crucially driven by the relative size of γ and μ_I . The results provided in Table 3 have confirmed the analytics.

Figure 2 provides a series of 3D-plots to provide a more comprehensive understanding of the competitive effects of the two VAT regimes in Scenarios A and B. The 3D-plots depict the operators' profits in the two dimensional space defined by $\mu_I \in [0,1]$ and $\gamma \in [0,1]$, i.e., all feasible combinations of μ_I and γ .¹⁶ Figures 2-1 and 2-2 depict the profits of the incumbent and entrant for Scenarios A and B, respectively. Figure 2-3 illustrates the profits of the two operators in Scenario A, while Figure 2-4 displays the differences in the operators' profits between Scenario A and the undistorted Scenario B.¹⁷

The figures confirm the analytical findings from Section 3 and they also replicate the results presented in Section 5.2 regarding the impact of the operators' cost structure.

¹⁵ Note that this differentiation is not captured in the simulation results reported by De Donder et al. (2009).

¹⁶ Apart from that we use in Figure 2 the same parameter constellations as in Scenarios A and B in Section 4.2.

¹⁷ In Figure 2, the operators' profits appear as planes. Nevertheless, profits are not linear in μ_I and γ ; the curvature is only very weak for $\mu_I \in [0,1]$ and $\gamma \in [0,1]$.

Figure 2: Competitive Effects of Scenario A and B

Figure 2-1: Incumbent profits in Scenarios A vs. B

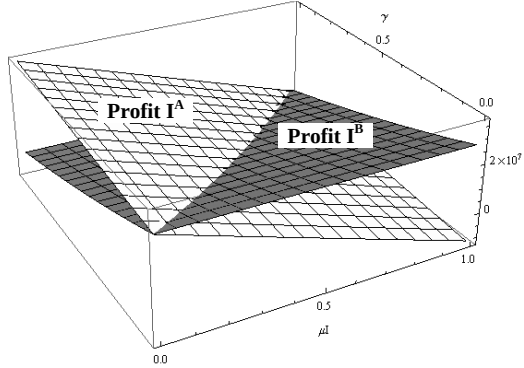


Figure 2-2: Entrant profits in Scenarios A vs. B

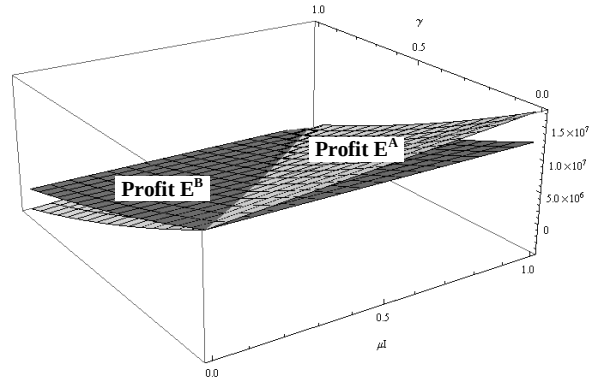


Figure 2-3: Operators' profits in Scenario A

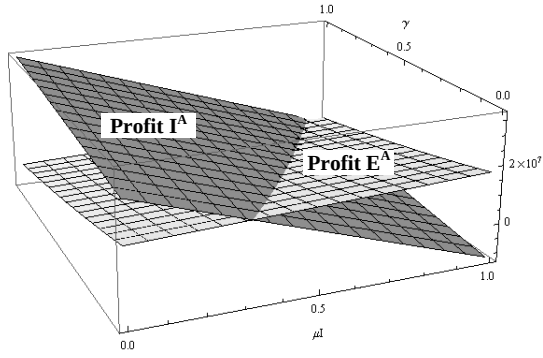


Figure 2-4: Operators' change in profits

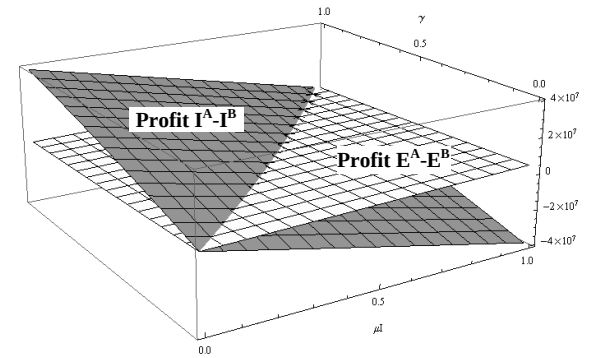


Figure 2-1 illustrates the ambiguous effect of VAT exemptions for non-rated (incumbent) operators. Such operators will be strengthened in market segments with a relatively high fraction γ of non-rated customers (e.g., industrial customers), while they will be worse off in segments with a relatively high fraction of rated customers (e.g., banks and private customers). As exempt operators will optimize their capital structure towards a low value of μ_I , they are likely to be better off in Scenario A and opt for a VAT exemption.

Figure 2-2 shows the entrant's perspective. The profits of the VAT rated are strictly decreasing in γ independently of the Scenario. It will prefer Scenario B whenever $\gamma > \mu_I$.¹⁸ As the incumbent is likely to choose a low value of μ_I in Scenario A, the entrant is likely to opt for Scenario B in policy discussions.

Figure 2-4 confirms these findings. It further shows that in the relevant parameter range (i.e., low μ_I), the competitive advantage of exempt operators in non-rated market segments is more important than their disadvantage in rated market segments.

¹⁸ Recall $p_E^A < p_E^B \Leftrightarrow \gamma < \mu_I$ from Proposition 3(i).

6 Conclusion

Distortions under the value-added tax (VAT) arise mainly from the exemption for specified services and sectors or from differentiated rates within an industry sector. The list of these exempt services and sectors include activities undertaken by public sector bodies such as medical and hospital care, welfare and social security work, provision of education and university education and the supply of certain cultural services. Furthermore, non-commercial activities carried out by public radio and television stations are also exempt from VAT. Another example is the postal sector where the universal service providers are usually VAT exempt on the grounds that they are the “public postal service”. Other postal service providers are VAT rated at the standard rate (cf. Trinkner 2009 for a recent overview).

In this paper, we develop a general model framework that can be applied to any sector where the firms competing in the market face asymmetric VAT rates. The model framework allows analyzing the effects of such asymmetric VAT regimes on market shares, optimal prices, and tax receipts analytically and by simulation. The analytical model shows how asymmetric VAT exemptions distort competition by strengthening the competitive position of non-rated firms. The net effect of VAT exemptions depends on the fraction of VAT rated inputs versus the fraction of non-rated customers. We further shed light on the main competitive impact of VAT policies, while showing the consequences on overall welfare by presenting simulation results based on a calibrated quantitative model of the postal sector.

With a reasonable model calibration, the VAT exemption positively affects profits of exempt operators and degrades profits of rated operators. Hence, it strengthens the exempt operators’ relative competitive position and results in an unlevel playing field. In the postal sector, this implies that VAT exemptions for universal service providers may reduce their burden represented by the net cost of universal service obligations.¹⁹ Our simulation results further show that the VAT exemption has a positive effect on consumer surplus. Compared to the scenario without VAT exemption, it has a small but positive welfare effect in that the marginal tax rate is lower on average.

The different VAT regimes also have an effect on the make-or-buy decisions of operators: VAT exempted operators have a higher incentive to employ their own workers instead of subcontractors and therefore raise a second set of market distortions: VAT exemptions worsen the competitive position of external suppliers.

¹⁹ See Jaag (2010) for a discussion of the unfair burden resulting from universal service obligations in the postal sector.

Our paper may help policy makers to assess the main competitive effects of VAT policies in sectors that are characterized through asymmetric VAT exemptions. Moreover, it can provide guidance on how to resolve the policy trade-off between a level playing field, consumer surplus and government tax revenue.

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A. Appendix: Proofs

A. 1 Proof of Lemma 1

The representative customer maximizes utility u under her budget constraint y and thus solves

$$\max \left\{ u(x_I, x_E) = m + \alpha_I x_I - \frac{\beta}{2} x_I^2 + \alpha_E x_E - \frac{\beta}{2} x_E^2 - \varepsilon \beta x_I x_E \right\} \text{ subject to } (x_I, x_E) \in B$$

$$\text{with } B = \{(x_I, x_E) \in \mathbb{R}_+^2 \mid y = m + p_I x_I(1 + \gamma t_I) + p_E x_E(1 + \gamma t_E)\}$$

The Lagrange function is given by

$$L(x_I, x_E, \lambda) = u(x_I, x_E) - \lambda(m + p_I x_I(1 + \gamma t_I) + p_E x_E(1 + \gamma t_E) - y)$$

By computing the first-order conditions of the Lagrange function and solving the resulting equation systems, we derive

$$x_I = \frac{1}{\beta(1 - \varepsilon^2)} (\alpha_I - \varepsilon \alpha_E - p_I(1 + \gamma t_I) + \varepsilon p_E(1 + \gamma t_E)),$$

$$x_E = \frac{1}{\beta(1 - \varepsilon^2)} (\alpha_E - \varepsilon \alpha_I - p_E(1 + \gamma t_E) + \varepsilon p_I(1 + \gamma t_I)).$$

A.2 Proof of Proposition 1

To prove part (i), we compute the partial derivative of $p_I^A(t)$ and $p_E^A(t)$ with respect to t as

$$\frac{\partial p_I^A(t)}{\partial t} = \frac{1}{4 - \varepsilon^2} (2\mu_I c_I + \varepsilon \gamma c_E) > 0,$$

$$\frac{\partial p_E^A(t)}{\partial t} = \frac{1}{4 - \varepsilon^2} \left[\frac{(\mu_I - \gamma) c_I}{(1 + \gamma t)^2} - \gamma \frac{\alpha_E(2 - \varepsilon^2) - \alpha_I \varepsilon}{(1 + \gamma t)^2} \right].$$

$$\text{We further derive } \frac{\partial p_E^A(t)}{\partial t} > 0 \Leftrightarrow \alpha_I > \alpha_I' \equiv \frac{\gamma - \mu_I}{\gamma} c_I + \frac{2 - \varepsilon^2}{\varepsilon} \alpha_E.$$

To prove part (ii), we compute the partial derivative of $x_I^A(t)$ and $x_E^A(t)$ with respect to t and further derive

$$\frac{\partial x_I^A(t)}{\partial t} = \frac{\varepsilon \gamma c_E - \mu_I(2 - \varepsilon^2) c_I}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)} < 0 \Leftrightarrow \frac{c_E}{c_I} \frac{\gamma}{\mu_I} < \frac{2 - \varepsilon^2}{\varepsilon},$$

$$\frac{\partial x_E^A(t)}{\partial t} = \frac{\varepsilon \mu_I c_I - \gamma(2 - \varepsilon^2) c_E}{\beta(4 - \varepsilon^2)(1 - \varepsilon^2)} < 0 \Leftrightarrow \frac{c_E}{c_I} \frac{\gamma}{\mu_I} > \frac{\varepsilon}{2 - \varepsilon^2}.$$

A.3 Proof of Proposition 2

To prove part (i), we compute the partial derivative of $p_I^B(t)$ and $p_E^B(t)$ with respect to t and further derive

$$\begin{aligned}\frac{\partial p_I^B(t)}{\partial t} &= -\frac{1}{4-\varepsilon^2} \left[\gamma \frac{\alpha_I(2-\varepsilon^2) - \alpha_E \varepsilon}{(1+\gamma t)^2} \right] > 0 \Leftrightarrow \frac{\alpha_I}{\alpha_E} < \frac{\varepsilon}{2-\varepsilon^2} \quad \forall \gamma > 0, \\ \frac{\partial p_E^B(t)}{\partial t} &= -\frac{1}{4-\varepsilon^2} \left[\gamma \frac{\alpha_E(2-\varepsilon^2) - \alpha_I \varepsilon}{(1+\gamma t)^2} \right] > 0 \Leftrightarrow \frac{\alpha_I}{\alpha_E} > \frac{2-\varepsilon^2}{\varepsilon} \quad \forall \gamma > 0.\end{aligned}$$

Note that if $\gamma = 0$ then the tax rate t has no effect on the equilibrium prices for both firms.

To prove part (ii), we compute the partial derivative of $x_I^B(t)$ and $x_E^B(t)$ with respect to t and further derive

$$\begin{aligned}\frac{\partial x_I^B(t)}{\partial t} &= \frac{\gamma(\varepsilon c_E - (2-\varepsilon^2)c_I)}{\beta(4-\varepsilon^2)(1-\varepsilon^2)} < 0 \Leftrightarrow \frac{c_I}{c_E} > \frac{\varepsilon}{2-\varepsilon^2} \quad \forall \gamma > 0, \\ \frac{\partial x_E^B(t)}{\partial t} &= \frac{\gamma(\varepsilon c_I - (2-\varepsilon^2)c_E)}{\beta(4-\varepsilon^2)(1-\varepsilon^2)} < 0 \Leftrightarrow \frac{c_I}{c_E} < \frac{2-\varepsilon^2}{\varepsilon} \quad \forall \gamma > 0.\end{aligned}$$

Note that if $\gamma = 0$ then the tax rate t has no effect on the equilibrium demands for both firms.

A.4 Proof of Proposition 3

To prove part (i), we compute the difference of firm E 's before-tax price in Scenarios A and B and derive

$$p_E^A - p_E^B = t \frac{\varepsilon(\mu_I - \gamma)c_I}{(4-\varepsilon^2)1+\gamma t} > 0 \Leftrightarrow \mu_I > \gamma.$$

To prove part (ii), we compute the difference of firm I 's before-tax price in Scenarios A and B as

$$p_I^A - p_I^B = t \frac{1+\gamma t (\varepsilon \gamma c_E + 2\mu_I c_I) - \gamma(\alpha_E \varepsilon - \alpha_I(2-\varepsilon^2))}{(4-\varepsilon^2)1+\gamma t} > 0$$

We further derive

$$p_I^A > p_I^B \Leftrightarrow \mu_I > \gamma \frac{\varepsilon(\alpha_E - c_E(1+\gamma t)) + (2-\varepsilon^2)\alpha_I}{2c_I(1+\gamma t)}$$