Scope economies, entry deterrence and welfare

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Abstract

This paper develops a model where the incumbent may expand to a second related market so as to signal the existence of scope economies and deter potential entry. We show that the incumbent only expands to another market when scope economies are large enough. Thus expansion is indeed a signal of larger economies of scope and, for certain parameter values, it leads to entry deterrence.

We show that the perfect bayesian equilibrium may involve entry accommodation, entry deterrence or a mixed strategy equilibrium. We investigate the welfare implications of prohibiting an entry deterrent expansion. In our model, such prohibition would always decrease consumer surplus. The welfare impact of preventing entry deterrence is ambiguous but negative for many parameter values.

Keywords: Scope economies; signalling; entry deterrence.

JEL classification: L14; L15; M37

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1 Introduction

In this paper we study entry deterrence when the incumbent benefits from scope economies if he expands to another product's market. The paper shows that in the presence of scope economies deterring entry may be welfare improving since it increases efficiency and generates social surplus in the new market.

We consider a two-period model where the incumbent's degree of scope economies is private information. In the first period, facing potential entry, the incumbent decides whether or not to expand to a second market. The entrant observes the incumbent's choice and decides whether to enter or not in the first market, after updating his beliefs about the magnitude of the scope economies. If entry occurs, firms compete in quantities. We characterize the equilibrium of this dynamic game and explore the welfare effects of entry deterrence under scope economies.

Our paper is related to the entry deterrence literature and, in particular, to the incomplete information models. The first entry deterrence signaling model was developed by Milgrom and Roberts (1982). In this model, a potential entrant has imperfect knowledge about the incumbent's production cost and the incumbent exploits this uncertainty by setting low prices, in order to make the entrant believe that entry is unprofitable.

In Milgrom and Roberts (1982) model the incumbent operates in a single market. Most firms, however, operate in several markets, especially when there are economies of scope to be exploited, which is the case we intend to address. An example of a limit pricing model with multimarket firms is Pires and Jorge (2011), which addresses the third-degree price discrimination policy of an incumbent that wants to deter entry. The authors show that being a multimarket incumbent facilitates entry deterrence as the incumbent can use the prices in the various markets to signal low cost. Other authors, such as Bagwell and Ramey (1988), have also explored the use of multiple signals to deter entry. They extend Milgrom and Roberts (1982) model by allowing firms to use price and advertising as potential signals.

The limit pricing strategy is usually claimed to have negative welfare effects, as it hinders competition, even though consumers benefit from temporarily low prices. Subsequent works have dealt with limit pricing in various contexts. For instance, Cabral and Riordan (1997) argue that, in the presence of learning economies, driving rivals out of the market or preventing entry may allow achieving higher efficiency levels, and thus benefit consumers. Our paper presents another circumstance where entry deterrence may be welfare improving. By expanding to another market the incumbent may benefit from economies of scope and hence become more efficient and be able to deter entry in the first market.[1] This strategy has anticompetitive effects in the first market

[1] Expanding to several markets may also be a strategy of spatial preemption, by occupying the product sprectrum so as to leave no niche for the entrant(s) (Schmalensee, 1978, Eaton and Lipsey, 1979).

but generates social surplus in the new market and increases the efficiency of the incumbent. Hence, the welfare impact of entry deterrence is ambiguous but likely to be positive.

Scope economies are usually related with the existence of inputs that may be shared among two or more production processes. These may be physical inputs, or «intangible» ones, such as, for instance, a given technology, managerial experience or a good sales team. Scope economies may arise through the fixed cost component of the multiproduct cost function (e.g. Röller and Tombak, 1990) and/or through the variable cost component (e.g. Dixon, 1994, who presents a model with deseconomies of scope). In the present paper we consider that scope economies impact on variable costs. An example of scope economies impacting through fixed costs is umbrella branding, in which brand extension allows quality signaling and thus achieving marketing economies (e.g. Choi, 1998; Cabral, 2000 and 2009).

Common examples of industries where economies of scope are relevant include telecommunications (share of inputs between long and short-distance calls, in the cellular market and even with the cable TV market, etc), transportation (share of inputs between several routes in the airline industry or by railway companies), software (share of expertise between different programs or versions), the pharmaceutical industry (share of knowledge and/or components), etc.[2] As Cantos-Sánchez *et al.* (2003) point out, in the presence of scope economies regulatory measures aimed at one of the markets may affect competition in the other(s), and thus the overall welfare effect must be considered. This is actually taken into account in the current paper.

The remainder of the article is organized as follows. The next section presents the model and simple computations regarding the last stage of the game. In Section 3 we derive the entrant's optimal strategy while Section 4 shows the incumbent's optimal strategy. Section 5 presents the Perfect Bayesian Equilibrium of the game. Section 6 studies the welfare impact of entry deterrence under scope economies. Conclusions are summarized in the final section.

2 The model and some preliminary computations

Consider a two-period model where a monopolist incumbent, firm I, faces a potential entrant, firm E. In the first period the incumbent operates only in market A and decides whether to expand to a new independent market, market B, where the firm would be a monopolist. The products sold in markets A and B can be jointly produced and there is economies of scope. The degree of economies of scope is given by $\theta \in [0,1]$. The marginal costs are equal to $c \in (0,1)$ when a single product is produced and equal to θc when the two products are produced. So the lower is θ, the stronger is the degree of scope economies. This degree is private information.

[2] See, among others, Kessides and Willig (1995) for an explanation of the existence of economies of scope in rail operations, and Banker *et al.* (1998) for evidence on scope economies in the U.S. telecommunications industry.

The entrant believes that θ is uniformly distributed on $[0,1]$ and these beliefs are assumed to be common knowledge.

In the first period, the incumbent decides whether to expand to market B (this decision is contingent on θ, the type of firm I). Firm E does not know the incumbent's type, but observes the expansion decision. In the second period, after observing the incumbent's expansion decision, the entrant updates his beliefs concerning the degree of economies of scope of the incumbent and decides whether to enter in market A with an homogenous product (this decision is contingent on whether I expands to B or not). If firm E enters, the two firms decide simultaneously their quantities. In order to simplify computations we assume that, if firm E enters, the firm learns the incumbent's degree of economies of scope, θ, before the quantity decisions are taken.

Let f^I and f^E be the expansion costs of firm I (in market B) and the entry costs of firm E (in market A), respectively and let c be the marginal costs of firm E (and of firm I when a single product is produced). The second period profits are discounted by $\delta \in (0,1]$.

We assume identical demands in the two markets. The inverse demand function is given by:

$$p = 1 - q$$

where p is the price and q is the total quantity sold in the market.

Considering the previous assumptions, let us present the second period profits and the consumer surplus under the various scenarios. Under monopoly, if I does not expand to market B, it is easy to show that his profit and consumer surplus are given by:

$$\Pi_A^m(c) = \frac{(1-c)^2}{4} \quad \text{and} \quad CS_A^m(c) = \frac{(1-c)^2}{8}$$

where the non-negativity constraint on quantity implies $c < 1$. On the other hand, if I expands to market B, his post-expansion profits and the consumer surplus in each market are given by:

$$\Pi_A^m(\theta, c) = \Pi_B^m(\theta, c) = \frac{(1-c\theta)^2}{4} \quad \text{and} \quad CS_A^m(\theta, c) = CS_B^m(\theta, c) = \frac{(1-c\theta)^2}{8}$$

The non-negativity constraint on quantity implies $c\theta < 1$, which is implied by $c < 1$ and $\theta \in [0,1]$.

The profits under duopoly depend on whether firm I expands or not to market B. If firm I does not expand, then we have a symmetric duopoly in market A. Profits and consumer surplus are given by:

$$\Pi_A^I(c) = \Pi_A^E(c) = \frac{(1-c)^2}{9} \quad \text{and} \quad CS_A(c) = \frac{2\,(1-c)^2}{9}$$

On the other hand, if firm I expands to market B, the duopoly in market A is asymmetric (firm I has marginal costs θc, while firm E has marginal costs c). The equilibrium profits and the consumer surplus are given by:

$$\begin{aligned}\Pi_A^I(\theta,c) &= \frac{(1-2\theta c+c)^2}{9},\ \Pi_A^E(\theta,c)=\frac{(1-2c+\theta c)^2}{9} \quad \text{and} \quad \Pi_B(\theta,c)=\frac{(1-c\theta)^2}{4}\\ CS_A(\theta,c) &= \frac{(2-c-\theta c)^2}{18} \quad \text{and} \quad CS_B(\theta,c)=\frac{(1-c\theta)^2}{8}\end{aligned}$$

The non-negativity constraint on the quantity of the incumbent is verified by $c, \theta < 1$. The non-negativity constraint on the quantity of the entrant further implies that $\theta > \frac{2c-1}{c}$. So the previous expression for the equilibrium profits are only relevant for $\theta > \max\left[0, \frac{2c-1}{c}\right]$. When $c > \frac{1}{2}$, for values of $\theta < \frac{2c-1}{c}$ the equilibrium is $q^E = 0$ and $\Pi_A^E(\theta,c) = 0$, whereas the incumbent's profits are $\Pi_A^I(\theta,c) = \Pi_A^m(\theta,c)$.

Note that $\Pi_A^E(\theta,c)$ is increasing with θ in the relevant range (where quantities are positive) and that for $\theta = 1$ (no economies of scope) we are in the symmetric case and thus $\Pi_A^E(1,c) = \Pi_A^E(c)$. On the contrary, $\Pi_A^m(\theta,c)$ and $\Pi_A^I(\theta,c)$ are decreasing with θ.

3 Optimal strategy of the entrant

In this section we analyze the optimal strategy of the entrant. The entrant's strategy is contingent on whether the incumbent expands or does not expand to market B. To simplify the exposition we will assume that when the entrant is indifferent between entering or not, he enters. However under indifference any decision is optimal (entering, not entering or following any mixed strategy between entering or not).

When I does not expand to B, the optimal entry decision is the following one:

Lemma 1 *If firm I does not expand to B, then E should enter in market A if and only if*

$$\Pi_A^E(c) = \frac{(1-c)^2}{9} \geq f^E.$$

Proof. If I does not expand to B, when E enters there is a duopoly with symmetric cost and post-entry profits are given by $\Pi_A^E(c)$. As a consequence, entry in market A is optimal as long as $\Pi_A^E(c) \geq f^E$. ■

Since the entrant's profits when the incumbent benefits from scope economies are lower than when he doesn't, another immediate result is:

Lemma 2 *If it is optimal for the entrant not to enter in market A when I does not expand to B, then, regardless of beliefs, it is also optimal not to enter when I expands to B.*

Proof. Not entering when I does not expand can only be optimal for $f^E > \Pi_A^E(c) = \frac{(1-c)^2}{9}$. Since $\Pi_A^E(c) \geq \Pi_A^E(\theta,c)$ for all $\theta \in [0,1]$ (equality holds for $\theta = 1$) it follows that $f^E > \Pi_A^E(c) \Rightarrow$

$f^E > \Pi_A^E(\theta, c)$ for all $\theta \in [0,1]$. Thus, regardless of the entrant's beliefs about θ, it is optimal not to enter when I expands to B. ■

The previous result does not depend on the entrant's beliefs. However, in general, when I expands to market B the optimal decision for the entrant depends on his beliefs about the degrees of economies of scope of the incumbent. Let us assume that the entrant believes that the incumbent's types who expand to market B are the ones with larger economies of scope (latter on we will see that these beliefs are consistent with the incumbent's optimal strategy). If the entrant believes that I expands if and only if $\theta \leq \widetilde{\theta}$ where $\widetilde{\theta} \in (0,1]$, then the posterior beliefs following I's expansion to B should be that θ is uniformly distributed on $\left[0, \widetilde{\theta}\right]$. Under these circumstances the optimal decision of the entrant is:

Lemma 3 *When I expands to B, if the entrant believes that θ is uniformly distributed on $\left[0, \widetilde{\theta}\right]$ where $\widetilde{\theta} \in (0,1]$, and $\widetilde{\theta} \leq \frac{2c-1}{c}$ then the entrant should not enter in market A. Moreover, if $\widetilde{\theta} > \frac{2c-1}{c}$ the entrant should enter in market A when I expands to B if and only if:*

$$E_\theta \left[\Pi_A^E(\theta, c) | \theta \sim U\left[0, \widetilde{\theta}\right] \right] \geq f^E \quad \Leftrightarrow \quad \int_{\max\left[0, \frac{2c-1}{c}\right]}^{\widetilde{\theta}} \frac{(1-2c+\theta c)^2}{9} \frac{1}{\widetilde{\theta}} d\theta \geq f^E.$$

When I expands to B, if the entrant believes that $\theta = 0$, then he should enter in market A if and only if $c \leq \frac{1}{2}$ and:

$$\Pi_A^E(0, c) = \frac{(1-2c)^2}{9} \geq f^E$$

Proof. When $\widetilde{\theta} \leq \frac{2c-1}{c}$ the entrant's profit in case of entry is nil, thus entry cannot be profitable. When $\widetilde{\theta} > \frac{2c-1}{c}$ entry is profitable if the expected profit, given that θ is uniformly distributed on $\left[0, \widetilde{\theta}\right]$, is higher than the entry costs. Finally, when $\Pr(\theta = 0 |\ I \text{ expands to } B) = 1$, if $c > \frac{1}{2}$ the entrant's profits are $\Pi_A^E(0, c) = 0$, hence E should not enter; if $c \leq \frac{1}{2}$ the entrant's profits are $\Pi_A^E(0, c) = \frac{(1-2c)^2}{9}$ and entry is profitable if and only if $\Pi_A^E(0, c) \geq f^E$. ■

The previous lemmas show that the optimal strategy of the entrant depends on c and f^E. It is interesting to characterize the entrant's optimal strategy as a function of c and f^E.

Note that the most favorable scenario for the entrant occurs when the incumbent does not expand to market B, and thus I does not benefit from economies of scope. In this case, the two firms have symmetric costs and the post-entry profits when E enters are given by $\Pi_A^E(c)$. It is immediate that if $f^E > \Pi_A^E(c) = \frac{(1-c)^2}{9}$ the entrant does not enter even if I does not expand to B. Since E never wants to enter, entry is blockaded and consequently the incumbent can behave as a monopolist (there is no credible threat of entry).

On the other hand, the least favorable scenario for the entrant is when the incumbent expands to market B only if economies of scope are maximal ($\Pr(\theta = 0 |\ I \text{ expands to } B) = 1$). In this case, if $c > \frac{1}{2}$ the entrant's post entry profit is nil and hence he nevers enters while if $c \leq \frac{1}{2}$ his profits are (asymmetric duopoly):

$$\Pi_A^E(0,c) = \frac{(1-2c)^2}{9}$$

If E wants to enter in this case, E will always enter. This happens if $f^E < \Pi_A^E(0,c)$.

Figure 1 shows the set of points in the space (c, f^E) where the optimal decision of firm E is independent of I's expansion strategy. For $f^E > \Pi_A^E(c)$ firm E never enters no matter if I expands or not to market B, thus entry is blockaded. For $f^E < \Pi_A^E(0,c)$ firm E always enters in market A.

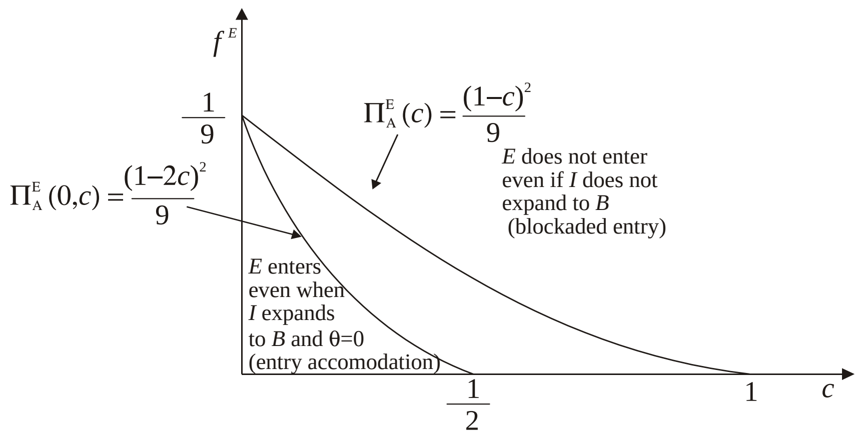

Figure 1: Set of (c, f^E) where E never enters and where E always enters in market A.

Between the two curves, E's optimal decision depends on the expansion strategy of firm I and on the entrant's beliefs after observing expansion. If firm I does not expand, firm E enters in market A. If firm I expands to market B, then firm E only enters if his expected post-entry profits, conditional on posterior beliefs, are above the entry costs.

4 Optimal strategy of the incumbent

In this section we derive the optimal strategy of the incumbent taking into account the expected strategy of the entrant. Obviously the optimal strategy depends on whether the entrant never enters in market A, always enters in market A or enters if and only if I does not expand to B (later on we will also consider the case where E follows a mixed strategy whenever I expands to B). However we will show that the optimal strategy is always of the cut-off type: incumbent's types with θ below or equal to certain cut-off value expand to market B, whereas incumbent's types with θ higher than the cut-off value do not expand to B.

4.1 No threat of entry in Market A

A monopolist incumbent with no threat of entry would enter market B if and only if

$$f^I \leq \delta(\Pi_A^m(\theta, c) - \Pi_A^m(c)) + \delta\Pi_B^m(\theta, c). \tag{1}$$

That is, the incumbent expands to market B if and only if the expansion costs are lower than the expansion benefits. The expansion benefits are equal to the discounted post-expansion profit in market B plus the discounted efficiency gain in market A. By expanding, the incumbent becomes more efficient in market A (his marginal cost decrease from c to θc) which increases the incumbent's monopoly profit in this market.

Notice that the right hand side of the previous expression is decreasing with θ. This implies that if the previous condition is satisfied for $\theta = \widehat{\theta}$ then it will also be satisfied for all $\theta < \widehat{\theta}$ and, conversely, if the condition is not satisfied for $\theta = \widetilde{\theta}$, then it will also not be satisfied for $\theta > \widetilde{\theta}$. This suggests that the optimal strategy of the incumbent is of the cut-off type:

Lemma 4 *Suppose that the incumbent expects that E never enters in market A. For given c, δ and $f^I \leq \frac{\delta\left(1+2c-c^2\right)}{4}$ there exists a cut-off value $\theta^m \in [0, 1]$ such that if $\theta \leq \theta^m$ the incumbent expands to market B while if $\theta > \theta^m$ the incumbent does not expand to market B. The value of θ^m depends on c, δ and f^I as follows:*

$$\theta^m = g(\delta, c, f^I) = \begin{cases} \frac{\sqrt{2\delta} - \sqrt{\delta(1-c)^2 + 4f^I}}{c\sqrt{2\delta}} & \text{if } f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta\left(1+2c-c^2\right)}{4} \right] \\ 1 & \text{if } \ f^I \leq \frac{\delta(1-c)^2}{4} \end{cases}$$

On the other hand, if $f^I > \frac{\delta\left(1+2c-c^2\right)}{4}$ then the incumbent does not expand to B for all $\theta \in [0, 1]$.

Proof. A monopolist incumbent with no threat of entry would enter market B if and only if condition (1) holds. Substituting the values of the profits, the condition is equivalent to:

$$f^I \leq \frac{\delta\left(1 + 2c - 4c\theta - c^2 + 2c^2\theta^2\right)}{4} \tag{2}$$

For $f^I \leq \frac{\delta(1-c)^2}{4}$ it is easy to verify that $\theta = 1$ satisfies the previous condition, thus $\theta^m = 1$. On the other hand, for $f^I > \frac{\delta\left(1+2c-c^2\right)}{4}$ the previous condition is not satisfied even for $\theta = 0$, implying that no type of incumbent wants to expand to market B. Finally, for $f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta\left(1+2c-c^2\right)}{4} \right]$ condition (2) holds in equality for

$$\theta^m = \frac{\sqrt{2\delta} - \sqrt{\delta\,(1-c)^2 + 4f^I}}{c\sqrt{2\delta}}$$

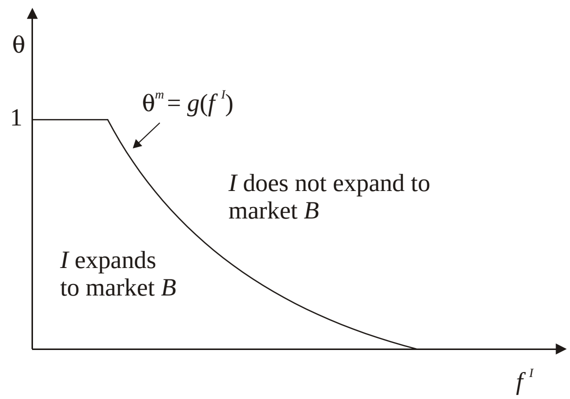

Figure 2: Optimal expansion decision with no threat of entry.

Thus the incumbent enters if and only if $\theta \leq \theta^m$. ■

Figure 2 shows the optimal expansion decision as a function of the expansion costs, f^I, and the degree of economies of scope, θ, under no threat of entry (in the figure δ and c are fixed).

To summarize, if $f^I > \frac{\delta\left(1+2c-c^2\right)}{4}$ then no type of incumbent expands to market B. On the other hand, if $f^I \leq \frac{\delta\left(1+2c-c^2\right)}{4}$ then the optimal expansion decision is of the cut-off type: below θ^m it is optimal to expand, above θ^m it is optimal not to expand. In other words, the types who expand are the ones with higher economies of scope (lower θ). Finally, for $f^I \leq \frac{\delta(1-c)^2}{4}$ all the incumbent types want to expand, which means that $\theta^m = 1$.

4.2 Threat of entry and entry deterrence

Let us now study the optimal strategy of the incumbent when he expects that E does not enter if he expands to market B but enters otherwise. Given the expected entrant's strategy, the condition for expansion to be optimal for I is:

$$f^I \leq \delta(\Pi_A^m(\theta, c) - \Pi_A^I(c)) + \delta\Pi_B^m(\theta, c) \tag{3}$$

Like before, the expansion decision is based on the comparison between expansion costs and discounted expansion benefits. However the discounted gain in market A is now larger as by expanding the incumbent remains a monopolist while if he does not expand he becomes a duopolist. Since $\delta(\Pi_A^m(\theta, c) - \Pi_A^I(c)) = \delta\left(\Pi_A^m(\theta, c) - \Pi_A^m(c)\right) + \delta\left(\Pi_A^m(c) - \Pi_A^I(c)\right)$, the discounted gain in market A can be decomposed into two components: an efficiency gain, $\delta\left(\Pi_A^m(\theta, c) - \Pi_A^m(c)\right)$, and an entry deterrence gain, $\delta\left(\Pi_A^m(c) - \Pi_A^I(c)\right)$. Thus the benefits from expansion are larger under entry deterrence than under no threat of entry.

Since the right hand side of condition (3) is decreasing with θ, it is easy to show that the incumbent follows a cut-off strategy:

Lemma 5 *Suppose that the incumbent expects that E does not enter in market A if and only he expands to B. For given* c, δ *and* $f^I \leq \frac{\delta\left(4c-2c^2+7\right)}{18}$ *there exists a cut-off value* $\theta^d \in [0,1]$ *such that if* $\theta \leq \theta^d$ *the incumbent expands to market B while if* $\theta > \theta^d$ *the incumbent does not expand to market B. The value of* θ^d *depends on* c, δ *and* f^I *as follows:*

$$\theta^d = h(\delta, c, f^I) = \begin{cases} \frac{3\sqrt{\delta}-\sqrt{2}\sqrt{\delta(1-c)^2+9f^I}}{3c\sqrt{\delta}} & \text{if } f^I \in \left(\frac{7\delta(1-c)^2}{18}, \frac{\delta\left(4c-2c^2+7\right)}{18}\right] \\ 1 & \text{if } \ f^I \leq \frac{7\delta(1-c)^2}{18} \end{cases}$$

On the other hand, if $f^I > \frac{\delta\left(4c-2c^2+7\right)}{18}$ *then the incumbent does not expand to B for all* $\theta \in [0,1]$.

Proof. Substituting the equilibrium profits in condition (3) we conclude that expansion to B is optimal as long as:

$$f^I \leq \frac{\delta\left(4c+9c^2\theta^2-18c\theta-2c^2+7\right)}{18} \tag{4}$$

For $f^I \leq \frac{7\delta(1-c)^2}{18}$ it is easy to verify that $\theta = 1$ satisfies the previous condition, therefore $\theta^d = 1$. On the other hand, for $f^I > \frac{\delta\left(4c-2c^2+7\right)}{18}$ the previous condition is not satisfied even for $\theta = 0$, implying that no type of incumbent wants to expand to market B. Finally, for $f^I \in \left(\frac{7\delta(1-c)^2}{18}, \frac{\delta\left(4c-2c^2+7\right)}{18}\right]$ condition (4) holds in equality for

$$\theta^d = \frac{3\sqrt{\delta}-\sqrt{2}\sqrt{\delta\left(1-c\right)^2+9f^I}}{3c\sqrt{\delta}}$$

Thus the incumbent enters if and only if $\theta \leq \theta^d$. ■

It is interesting to compare the cut-off values of θ in the entry deterrence case with the cut-off values in the blockaded entry case. Note that, for given δ, θ and c, the RHS of condition (3) is higher than the RHS of condition (1) since $\Pi_A^I(c) < \Pi_A^m(c)$. This implies that the cut-off level, θ^d, below which expansion to market B occurs when expansion deters entry is higher than the cut-off level when there is no threat of entry, i.e., $\theta^d > \theta^m$.

The intuition for the result is the following one. When expansion leads to entry deterrence the benefits of expanding to market B are equal to the profit in market B plus the benefit of being a monopolist with marginal costs $\theta c < c$ instead of a duopolist with costs c. On the other hand, the benefits of expanding under no threat of entry are equal to the profit in market B plus the increase in the monopoly profit when costs drop from c to θc. Since expansion is more profitable under the threat of entry, expansion will be optimal for lower economies of scope (higher θ).

4.3 Entry accommodation

If the incumbent cannot avoid entrance in market A (E enters even if I expands to B) his decision of expanding to market B or not is based on:

$$f^I \leq \delta(\Pi_A^I(\theta,c) - \Pi_A^I(c)) + \delta\Pi_B^m(\theta,c) \tag{5}$$

In this case, the expansion benefit is equal to the discounted post-expansion profit in market B plus the discounted efficiency gain in market A, considering that the incumbent is a duopolist in market A regardless of its expansion decision.

Since the right hand side of the previous condition is decreasing with θ, it is easy to show that the incumbent follows a cut-off strategy:

Lemma 6 *Suppose that the incumbent expects that E enters in market A regardless of his expansion decision. For given* c, δ *and* $f^I \leq \frac{\delta(9+16c)}{36}$ *there exists a cut-off value* $\theta^a \in [0,1]$ *such that the incumbent expands to market B if and only if* $\theta \leq \theta^a$*. The value of* θ^a *depends on* c, δ *and* f^I *as follows:*

$$\theta' = k(\delta, c, f^I) = \begin{cases} \frac{17\sqrt{\delta}+8c\sqrt{\delta}-2\sqrt{225f^I+16\delta(1-c)^2}}{25c\sqrt{\delta}} & \text{if } f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta(9+16c)}{36}\right] \\ 1 & \text{if } f^I \leq \frac{\delta(1-c)^2}{4} \end{cases}$$

On the other hand, if $f^I > \frac{\delta(9+16c)}{36}$ *then the incumbent does not expand to B for all* $\theta \in [0,1]$.

Proof. Substituting the equilibrium profits in condition (5) we conclude that expansion to B is optimal as long as:

$$f^I \leq \frac{\delta\left(9 - 25c\theta + 16c\right)\left(1 - c\theta\right)}{36} \tag{6}$$

For $f^I \leq \frac{\delta(1-c)^2}{4}$ it is easy to verify that $\theta = 1$ satisfies the previous condition, thus $\theta^a = 1$. On the other hand, for $f^I > \frac{\delta(9+16c)}{36}$ the previous condition is not satisfied for $\theta = 0$, implying that no type of incumbent wants to expand to market B. Finally, for $f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta(9+16c)}{36}\right]$ condition (6) holds in equality for

$$\theta^a = \frac{17\sqrt{\delta} + 8c\sqrt{\delta} - 2\delta\sqrt{225f^I + 16\delta\left(1-c\right)^2}}{25c\sqrt{\delta}}$$

Thus the incumbent enters if and only if $\theta \leq \theta^a$. ■

Note that, comparing with the case where expansion to B deters entry (condition (3)), the RHS of condition (5) is clearly lower (as $\Pi_A^I(\theta,c) < \Pi_A^m(\theta,c)$). Thus the cut-off value θ^a below which expansion occurs under entry accommodation is smaller than the cut-off level under entry deterrence, $\theta^a < \theta^d$.

Figure 3 shows how the expansion decision depends on the expansion costs, f^I, and the degree of scope economies, θ, for given values of the remaining parameters (c, δ). For values of $\theta > \theta^d$ the incumbent does not want to expand to market B even if by doing so it deters entry in market A. For $\theta^a < \theta \leq \theta^d$ the incumbent wants to expand to market B if that deters entry in market A but would not expand to market B if E always enters in market A. Finally, for $\theta \leq \theta^a$ the incumbent wants to expand to market B both in the case where expansion deters entry as well as in the case where E always enters in market A. The shaded area corresponds to the case where expansion to market B is just to deter entry (it would not occur under entry accommodation). Thus the shaded area is a region of «strategic expansion».

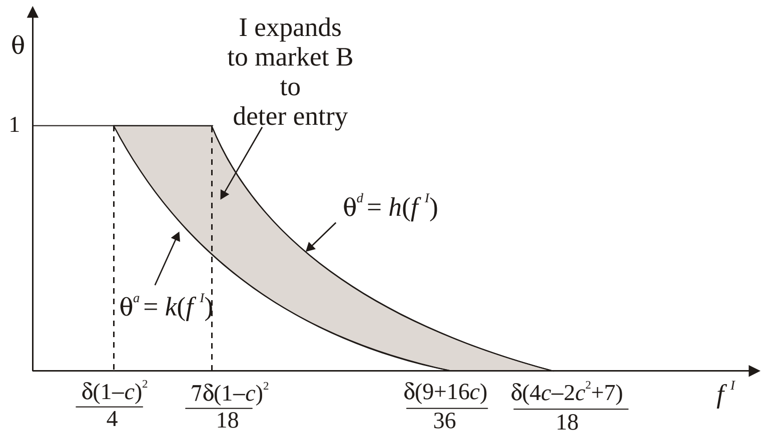

Figure 3: Comparison of cut-off values under entry deterrence and under entry accommodation.

5 Perfect Bayesian equilibrium

Having described the optimal strategies of firms I and E, we are now ready to characterize the perfect bayesian equilibrium (PBE) of the game. We restrict our analysis to the cases where $f^I \leq \frac{\delta\left(4c-2c^2+7\right)}{18}$ and $\frac{(1-2c)^2}{9} < f^E \leq \frac{(1-c)^2}{9}$ for $c \leq \frac{1}{2}$ and $f^E \leq \frac{(1-c)^2}{9}$ for $c > \frac{1}{2}$. When $f^I > \frac{\delta\left(4c-2c^2+7\right)}{18}$ the incumbent would never expand to market B and thus economies of scope would be irrelevant. Moreover, to describe the PBE when entry is blockaded or when entry always occurs regardless of the beliefs is trivial considering the analysis in the two previous sections.

Let $\underline{f}_d^E$ be the entrant's entry cost such that the entrant is indifferent between entering and not entering in market A, when the incumbent expands to B and E believes that θ is uniformly distributed on $\left[0, \theta^d\right]$. If $\theta^d > \max\left[0, \frac{2c-1}{c}\right]$, $\underline{f}_d^E$ is given by:

$$\int_{\max\left[0, \frac{2c-1}{c}\right]}^{\theta^d} \Pi_A^E(\theta, c) \frac{1}{\theta^d} d\theta = \underline{f}_d^E.$$

On the other hand, if $\theta^d \le \max\left[0, \frac{2c-1}{c}\right]$ the entrant would have $\Pi_A^E(\theta, c) = 0$ if he entered, thus $\underline{f}_d^E = 0$. Obviously, for $f^E > \underline{f}_d^E$ and the aforementioned beliefs the entrant does not enter in A when I expands to B. $\underline{f}_d^E$ can be interpreted as the minimal entry costs which are consistent with entry deterrence.

Similarly, let $\overline{f}_a^E$ be E's entry costs such that the entrant is indifferent between entering and not entering in market A, when the incumbent expands to B and E believes that θ is uniformly distributed in $[0, \theta^a]$. If $\theta^a > \max\left[0, \frac{2c-1}{c}\right]$, $\overline{f}_a^E$ is given by:

$$\int_{\max\left[0, \frac{2c-1}{c}\right]}^{\theta^a} \Pi_A^E(\theta, c) \frac{1}{\theta^a} d\theta = \overline{f}_a^E.$$

On the other hand, if $\theta^a \le \max\left[0, \frac{2c-1}{c}\right]$ the entrant would have $\Pi_A^E(\theta, c) = 0$ if he entered, thus $\overline{f}_a^E = 0$. Note that, for $f^E < \overline{f}_a^E$ the entrant enters in market A when I expands to B if he believes that θ is uniformly distributed on $[0, \theta^a]$. Thus $\overline{f}_a^E$ can be interpreted as the maximal entry costs which are consistent with entry accommodation.

It should be noted that $\underline{f}_d^E < \Pi_A^E(c) = \frac{(1-c)^2}{9}$. Since $\Pi_A^E(\theta, c) < \Pi_A^E(c)$ for all $\theta < 1$, the expected profit conditional on $\theta \le \theta^d$ is necessarily below $\Pi_A^E(c)$. Moreover since $\Pi_A^E(\theta, c)$ is increasing with θ and $\theta^a \le \theta^d$ then the expected profit conditional on $\theta \le \theta^d$ cannot be lower than the expected profit conditional on $\theta \le \theta^a$, thus $\overline{f}_a^E \le \underline{f}_d^E$ (and $\overline{f}_a^E < \underline{f}_d^E$ when $\theta^a < \theta^d$ and $\theta^d > \max\left[0, \frac{2c-1}{c}\right]$). Finally, $\overline{f}_a^E \ge \Pi_A^E(0, c)$ since $\Pi_A^E(\theta, c)$ is increasing with θ. This implies that $\overline{f}_a^E \ge \frac{(1-2c)^2}{9}$ when $c \le \frac{1}{2}$ and that $\overline{f}_a^E \ge 0$ when $c > \frac{1}{2}$.

Let us now characterize the PBE for the various levels of expansion costs: low ($f^I \le \frac{\delta(1-c)^2}{4}$), intermediate ($\frac{\delta(1-c)^2}{4} < f^I \le \frac{\delta(9+16c)}{36}$) and high ($\frac{\delta(9+16c)}{36} < f^I < \frac{\delta\left(4c-2c^2+7\right)}{18}$). Figure 3 helps understanding our definition of low, intermediate and high expansion costs. For low expansion costs all types of incumbent expand to market B both when expansion deters entry and when E always enters; i.e., $\theta^d = \theta^a = 1$. On the other hand, for intermediate expansion costs, $\theta^d > \theta^a \ge 0$. Finally, for high expansion costs, $\theta^d \ge 0$ but no type wants to expand if expansion does not deter entry.

The next proposition describes the PBE when the incumbent's expansion costs are low.

Proposition 1 *For given δ, c, f^E and $f^I \le \frac{\delta(1-c)^2}{4}$ there exists a unique PBE. In this PBE the incumbent expands to B for all $\theta \in [0,1]$ and, when I expands to B, E believes that θ is uniformly distributed on $[0,1]$. On the other hand, the entrant's equilibrium strategy depends on f^E as follows:*

1. *For $f^E \leq \overline{f}_a^E = \underline{f}_d^E$ the entrant enters in A regardless of I' s expansion decision.*

2. *For $\overline{f}_a^E = \underline{f}_d^E < f^E \leq \frac{(1-c)^2}{9}$ the entrant enters in market A if and only if the incumbent does not expand to B.*

Proof. We need to check that the incumbent's strategy is optimal given the entrant's strategy, that the entrant's strategy is optimal given beliefs and that beliefs are consistent with Bayes rule and the incumbent's equilibrium strategy. When $f^I < \frac{\delta(1-c)^2}{4}$ lemmas 5 and 6 imply that $\theta^d = \theta^a = 1$ and thus the optimal strategy of the incumbent is to expand for all $\theta \in [0,1]$, regardless of the entrant's strategy. Since all incumbent's types expand to B, expansion to B is not informative about θ, thus posterior beliefs should be equal to the prior beliefs that θ is uniformly distributed on $[0,1]$. Given these beliefs, the optimality of the entrant's strategy follows from lemmas 1 - 3. Note that $\overline{f}_a^E = \underline{f}_d^E = \int_{\max\left[0,\frac{2c-1}{c}\right]}^1 \frac{(1-2c+\theta c)^2}{9} d\theta$ since $\theta^d = \theta^a = 1$. ■

When the incumbent's expansion costs are low, the incumbent expands to market B independently of his degree of economies of scope. On the other hand, the entrant's optimal strategy depends on his entry costs. For low f^E the entrant always enters and thus the PBE involves entry accommodation. For higher values of f^E, the entrant enters if and only if I does not expand. In this case, expansion to B leads to entry deterrence.

Let us now describe the PBE for intermediate values of f^I:

Proposition 2 *For given δ, c, f^E and $\frac{\delta(1-c)^2}{4} < f^I \leq \frac{\delta(9+16c)}{36}$ there exists a unique PBE. Moreover, for each δ and c the PBE depends on f^I and f^E as follows:*

1. *For $f^E \leq \overline{f}_a^E$ the incumbent expands to market B if and only if $\theta \leq \theta^a$; if I expands to B, E believes that θ is uniformly distributed on $[0, \theta^a]$; and the entrant enters in market A regardless of I's expansion decision.*

2. *For $\underline{f}_d^E \leq f^E \leq \frac{(1-c)^2}{9}$ the incumbent expands to market B if and only if $\theta \leq \theta^d$; if I expands to B, E believes that θ is uniformly distributed on $\left[0, \theta^d\right]$; and the entrant enters in market A if and only if the incumbent does not expand to B.*

3. *Finally, for $\overline{f}_a^E < f^E < \underline{f}_d^E$ the incumbent expands to market B if and only if $\theta \leq \theta'$ where θ' is such that*

$$f^E = \int_{\max\left[0,\frac{2c-1}{c}\right]}^{\theta'} \frac{(1-2c+\theta c)^2}{9} \frac{1}{\theta'} d\theta; \tag{7}$$

if I expands to B, E believes that θ is uniformly distributed on $\left[0, \theta'\right]$; and the entrant enters in market A if I does not expand to B and enters in market A with probability β if I expands to B, where β is the solution to:

$$f^I = \delta(\beta \Pi_A^I(\theta', c) + (1-\beta)\Pi_A^m(\theta', c) - \Pi_A^I(c)) + \delta \Pi_B^m(\theta', c). \tag{8}$$

Proof. When $\frac{\delta(1-c)^2}{4} < f^I \leq \frac{\delta(9+16c)}{36}$ lemmas 5 and 6 imply that the incumbent follows a cut-off strategy for all the possible strategies of the entrant and that $0 \leq \theta^a < 1$ (θ^d is equal to 1 for $\frac{\delta(1-c)^2}{4} < f^I \leq \frac{7\delta(1-c)^2}{18}$). Considering this, the proofs of cases 1 and 2 are immediate consequences of lemmas 1-3 and lemmas 5 and 6.

In case 3 one can show that there cannot exist a PBE where the entrant follows a pure strategy when I expands. If E never enters when he observes I expanding to market B, then types $\theta \in \left[0, \theta^d\right]$ would expand to market B. However, considering the posterior beliefs, the entrant would be better off by entering as $f^E < \underline{f}_d^E$, a contradiction. Similarly, if E always enters when I expands to B, only types $\theta \in [0, \theta^a]$ want to expand to market B, but then it would be optimal for E not to enter as $f^E > \overline{f}_a^E$, a contradiction. Thus, if $\overline{f}_a^E < f^E < \underline{f}_d^E$ there does not exist a PBE where E follows a pure strategy when I expands to market B.

Let us now check the mixed strategy PBE. In order for it to be optimal for E to follow a mixed strategy when I expands to market B, firm E has to be indifferent between entering and not entering. That is, θ' has to be such that condition (7) holds.

Considering the optimal strategy of firm E (entering when I does not expand to B, entering with probability β when I expands to B), firm I should expand to market B if and only if:

$$f^I \leq \delta(\beta \Pi_A^I(\theta, c) + (1-\beta)\Pi_A^m(\theta, c) - \Pi_A^I(c)) + \delta \Pi_B^m(\theta, c) \tag{9}$$

Thus if β is the solution to equation (8), then type θ' will be indifferent between expanding or not to market B while types $\theta < \theta'$ strictly prefer to expand to B. Thus it is optimal for I to expand to B for $\theta \leq \theta'$. Finally, the belief that θ is uniformly distributed on $\left[0, \theta'\right]$ is consistent with the cut-off strategy of the incumbent. ■

When the incumbent's expansion costs are intermediate the PBE may involve entry deterrence, entry accommodation or the entrant playing a mixed strategy when the incumbent expands to B. It is worthwhile to explore how the mixed strategy PBE changes with f^E. When f^E decreases, the value of θ' that satisfies condition (7) has to decrease in order to maintain the equality (in the PBE less incumbent's types expand to B). Moreover, since the RHS of condition (8) is decreasing with β and with θ', when θ' decreases, β has to increase in order to maintain the equality. As a consequence, the lower is f^E, the higher has to be the probability of the entrant entering in market A when I expands to B. When f^E decreases to values close to $\overline{f}_a^E$, $\theta' \to \theta^a$ and $\beta \to 1$. On the other hand, when f^E tends to $\underline{f}_d^E$ the cut-off level θ' converges to θ^d and $\beta \to 0$.

Finally, the next proposition describes the PBE when the incumbent's expansion costs are high (but not so high that I never wants to expand to B):

Proposition 3 *For given δ, c, f^E and $\frac{\delta(9+16c)}{36} < f^I < \frac{\delta\left(4c-2c^2+7\right)}{18}$ there may exist multiple PBE.*

1. *For* $f^E \leq \underline{f}_d^E$ *there are the following PBE:*

   (a) *The incumbent does not expand to* B *for all* $\theta \in [0,1]$ *and the entrant enters in market* A *regardless of* I*'s expansion decision. The entrant´s equilibrium strategy can be sustained by the belief, when* I *expands to* B*, that* θ *is uniformly distributed on* $\left[0, \theta^d\right]$.[3]

   (b) *The incumbent expands to market* B *if and only if* $\theta \leq \theta'$ *where* θ' *is such that*

$$f^E = \int_{\max\left[0, \frac{2c-1}{c}\right]}^{\theta'} \frac{(1-2c+\theta c)^2}{9} \frac{1}{\theta'} d\theta;$$

   if I *expands to* B*,* E *believes that* θ *is uniformly distributed on* $\left[0, \theta'\right]$*; and the entrant enters in market* A *if* I *does not expand and enters in market* A *with probability* β *if* I *expands to* B*, where* β *is the solution to:*

$$f^I = \delta(\beta \Pi_A^I(\theta', c) + (1-\beta)\Pi_A^m(\theta', c) - \Pi_A^I(c)) + \delta \Pi_B^m(\theta', c).$$

2. *For* $\underline{f}_d^E < f^E \leq \frac{(1-c)^2}{9}$ *the incumbent expands to market* B *if and only if* $\theta \leq \theta^d$*; if* I *expands to* B*,* E *believes that* θ *is uniformly distributed on* $\left[0, \theta^d\right]$*; and the entrant enters in market* A *if and only if the incumbent does not expand to* B*.*

Proof. When $\frac{\delta(9+16c)}{36} < f^I < \frac{\delta\left(4c-2c^2+7\right)}{18}$ lemma 5 implies that $\theta^d > 0$ and lemma 6 implies that if I expects E to always enter then I does not expand for all $\theta \in [0,1]$.

To prove 1.a we just need to note that, given E's strategy, not expanding to B is indeed optimal for all $\theta \in [0,1]$. Moreover, when the incumbent does not expand to B, it is optimal for E to enter by lemma 1 and, given beliefs, it is also optimal to enter as $f^E \leq \underline{f}_d^E$.

The proof of 1.b and 2 are similar to the proofs of cases 3 and 2 in the previous proposition, respectively. ■

Although there are two possible PBE in case 1 of this proposition, the equilibrium described in 1.a can be ruled out if we impose further refinements upon off-the-equilibrium path beliefs. For instance, this equilibrium does not survive the Fudenberg and Tirole (1991) version of the divinity criterion, $D1$.

Lemma 7 *The PBE 1.a. of Proposition 3 does not satisfy criterion* $D1$ *of Fudenberg and Tirole (1991).*

[3]Note that these are off-the-equilibrium path beliefs, since in equilibrium no incumbent type is expected to expand. Off-the-equilibrium path beliefs are unrestricted and it is possible to find other beliefs that support this PBE outcome, but these beliefs satisfy the intuitive criterion.

Proof. Consider the equilibrium 1.a. of Proposition 3 and let $\delta\Pi_A^{I*}(\theta)$ be type θ's equilibrium payoff. Let e denote the off-the-equilibrium path action of expanding and $D(\theta, T, e)$ be the set of mixed strategy best responses (MBR) of the entrant when the incumbent expands if the entrant's beliefs are concentrated on the set of types T that make type θ strictly prefer expanding to his equilibrium strategy. That is:

$$D(\theta, T, e) = \bigcup_{\mu:\mu(T|e)=1} \left\{\beta \in MBR(\mu, e) : \delta\Pi_A^{I*}(\theta) < \beta\delta\Pi_A^{I}(\theta, c) + (1-\beta)\delta\Pi_A^{m}(\theta, c) + \delta\Pi_B^{m}(\theta, c) - f^I\right\}$$

and let $D^0(\theta, T, e)$ be the set of mixed strategy best responses of the entrant that make type θ exactly indifferent between expanding or not. A type θ is deleted according to criterion $D1$ when expansion is observed (action e is observed) if there exists a type θ' such that $\left\{D(\theta, T, e) \cup D^0(\theta, T, e)\right\} \subset D(\theta', T, e)$. In other words, type θ is eliminated if the set of the entrant's responses that make type θ willing to deviate is strictly smaller than the set of responses that make type θ' willing to deviate. Since the set of mixed strategies for which $\theta > 0$ wants to deviate (i.e., expand) is strictly smaller than the set of mixed strategies for which type $\theta = 0$ wants to deviate, all types except $\theta = 0$ are eliminated according to criterion $D1$. But if the entrant believes $\theta = 0$ when he observes expansion, we does not want to enter. Thus 1.a does not survive criterion $D1$. ∎

Considering this result, from hereafter, in case 1 of Proposition 3, we assume PBE 1.b.

For given δ and f^I, one can find the set of values in the space (c, f^E) which are compatible with entry deterrence, entry accommodation or with a mixed strategy PBE. Figure 4 illustrates the case where $f^I = 0$. In this case, proposition 1 applies for all $c \in (0, 1)$ and thus, for $\Pi_A^E(0, c) < f^E \leq \Pi_A^E(c)$, either we have entry accommodation or entry deterrence. The figure shows in light grey the set of values of c and f^E which are compatible with an entry deterrence PBE. Below $\underline{f}_d^E$ the entrant always enters.

Figure 5 illustrates the case of a relatively low f^I, such that for small values of c proposition 1 still applies, but for higher values of c, the relevant result is proposition 2. The set of values in the space (c, f^E) where the entry deterrence PBE equilibrium exists is represented in light grey. The region in dark grey is a region where there is a mixed strategy equilibrium. Below that, firm E always enters, hence we have entry accommodation. The curves indicating $\overline{f}_a^E$ and $\underline{f}_d^E$ depend on f^I. The higher is f^I the lower are the curves.

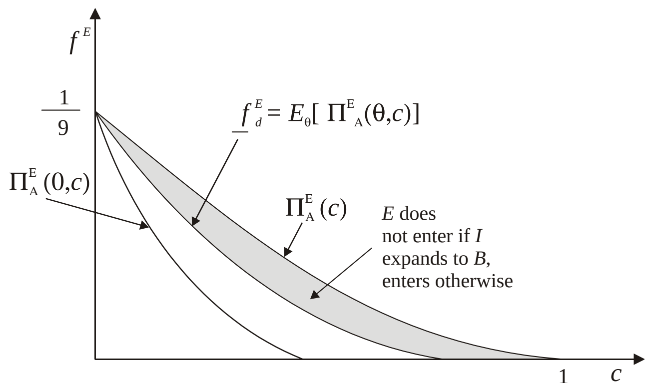

Figure 4: Set of values of c and f^E that are compatible with an entry deterrence PBE when $f^I = 0$ (in grey).

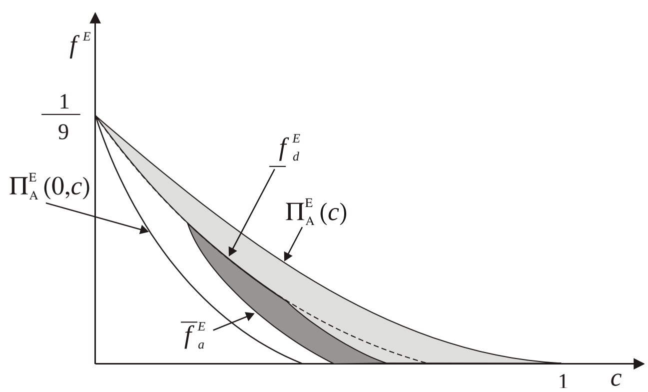

Figure 5: Entry deterrence PBE (light grey) and mixed strategy PBE (dark grey) when f^I is relatively low.

6 Should entry deterrence be prohibited?

The welfare depends on whether firm I expands or does not expand to B and on whether firm E enters or does not enter in market A. Table 1 shows the social welfare in the four possible outcomes, considering the equilibrium profits and the consumers surplus computed in section 2.

Table 1: Social welfare in each of the four possible outcomes.

	I does not expand	I expands
E does not enter	$\frac{3}{8}\delta\left(1-c\right)^{2}$	$\frac{3}{4}\delta\left(1-c\theta\right)^{2}-f^{I}$
E enters	$\frac{4}{9}\delta\left(1-c\right)^{2}-\delta f^{E}$	$\frac{\delta(1-2\theta c+c)^{2}}{9}+\frac{\delta(1-2c+\theta c)^{2}}{9}+\frac{\delta(2-c-\theta c)^{2}}{18}+\frac{3}{8}\delta\left(1-\theta c\right)^{2}-f^{I}-\delta f^{E}$

Cabral and Riordon (1997) call an action predatory if: (1) a different action would increase the likelihood that rivals remain viable, and (2) the different action would be more profitable under the counterfactual hypothesis that the rival's viability was unaffected. Our definition of entry deterrent expansion is similar:

Definition 1 *Expansion to B is entry deterrent if the following two conditions hold:*

1. *Not expanding to B would increase the probability of E entering in market A;*

2. *Not expanding to B would be optimal for I under the counterfactual hypothesis that the rival's entry decision was unaffected.*

If we use this definition of entry deterrence, only incumbent types with $\theta\in\left[\theta^{a},\theta^{d}\right]$ in the pure strategies PBE or with $\theta\in\left[\theta^{a},\theta'\right]$ in the mixed strategies equilibrium behave so as to deter entry. If there was a social planner that can control the expansion decision and who has complete information about θ, would it be beneficial to prevent these types from expanding?

Expansion to market B by types $\theta\in\left[\theta^{a},\theta'\right]$ or $\theta\in\left[\theta^{a},\theta^{d}\right]$ has anticompetitive effects in the future as it increases the likelihood of a monopoly in market A. However, expansion to B generates social surplus in B and an efficiency gain in market A, due to economies of scope. Thus, in general, the welfare impact of prohibiting entry deterrence is ambiguous.

We need to concentrate our analysis in a set of parameter values such that $\theta^{a}<\theta^{d}$ (if $\theta^{a}=\theta^{d}=1$, which happens for low f^{I}, all types expand to B even when E enters, thus I's expansion is not due to entry deterrence reasons). Consequently, we need to consider intermediate or high values of f^{I},$\frac{\delta(1-c)^{2}}{4}<f^{I}<\frac{\delta\left(4c-2c^{2}+7\right)}{18}$. Moreover we need to consider the case where $\overline{f}_{a}^{E}\leq f^{E}\leq\frac{(1-c)^{2}}{9}$ so that, in equilibrium, expansion leads E not to enter with some probability while E enters when I does not expand (in other words, we need to consider cases 2 and 3 in proposition 2 and proposition 3).

6.1 The impact of a prohibition on consumer surplus

One interesting result in our model is that consumers are always worse off if entry deterrence is prohibited:

Proposition 4 *If expansion to B is entry deterrent, then consumers would be worse off with a prohibition of entry deterrence.*

Proof. Let us consider first the case of a pure strategy PBE. If expansion by type $\theta \in \left[\theta^a, \theta^d\right]$ was prohibited, the entrant would enter but the incumbent of type $\theta \in \left[\theta^a, \theta^d\right]$ would not expand, thus the consumer surplus would be $\frac{2(1-c)^2}{9}$. On the other hand, if expansion by type $\theta \in \left[\theta^a, \theta^d\right]$ was allowed, E would not enter and consumer surplus would be $\frac{(1-c\theta)^2}{4}$. However $\frac{2(1-c)^2}{9} < \frac{(1-\theta c)^2}{4}$ since the RHS is decreasing with θ and the inequality holds for $\theta = 1$. Thus consumers would be worse off if entry deterrence was prohibited.

In the case of a mixed strategy equilibrium, the proof is similar. If entry deterrence expansion was prohibited E would enter and consumer surplus would be $\frac{2(1-c)^2}{9}$. On the other hand, if expansion by type $\theta \in \left[\theta^a, \theta'\right]$ was allowed the entrant would enter with probability β and not enter with probability $(1-\beta)$. The expected consumer surplus would be

$$\beta \frac{(2-c-\theta c)^2}{18} + (1-\beta)\frac{(1-c\theta)^2}{8} + \frac{(1-c\theta)^2}{8} \tag{10}$$

Note that (10) is decreasing with θ and for $\theta = 1$ is equal to $\left(\frac{7}{72}\beta + 1\right)\frac{(1-c)^2}{4} > \frac{(1-c)^2}{4} > \frac{2(1-c)^2}{9}$. Thus the expected consumer surplus when entry deterrent expansion is allowed is higher than when entry deterrence is prohibited. ■

The previous result is quite strong as it indicates that benefitting consumers cannot be used as an argument to prevent entry deterrent expansion. It should be noted that the result refers to the aggregate consumer surplus. Consumers in market B are clearly worse off when entry deterrence is prohibited as preventing expansion implies that market B is not served. On the other hand, consumers in market A may either be better off or worse off if entry deterrence is prevented (it depends on the probability of E entering when expansion occurs and the degree of economies of scope). However, the aggregate consumer surplus is always higher if entry deterrence is allowed.

It is interesting to notice that prohibition of entry deterrence is particularly harmful for consumers when the equilibrium is a mixed strategies equilibrium. In this case, the anticompetitive effect of expansion only occurs with probability $(1-\beta)$, hence with probability β consumers will benefit from the scope economies efficiency gain while still enjoying the advantages of competition in market A.

6.2 The impact of a prohibition on welfare

Consumers are always worse off if entry deterrence is prohibited, but in order to evaluate the welfare impact of a prohibition against entry deterrence we also need to take into account the expansion and entry costs and the post-entry profits. In the next two subsections, we evaluate the welfare impact of a prohibition against entry deterrence, and show that such prohibition may either decrease welfare or increase welfare, depending on the parameter values.

6.2.1 Prohibition of entry deterrence decreases welfare

It is relatively easy to show that, for certain parameter values, there is a welfare loss if entry deterrence is prohibited. One scenario that is favorable to show that welfare may be lower if entry deterrence is prohibited is when f^I is high, $\frac{\delta(9+16c)}{36} < f^I < \frac{\delta\left(4c-2c^2+7\right)}{18}$. In this case all types who expand, expand for entry deterrence reasons (they would not expand if E enters). Moreover the types who expand have very low θ, which implies that it is highly likely that expansion is optimal from a social point of view as economies of scope are large. The two next propositions show that when the expansion costs are high and δ is close to 1, entry deterrent expansion is welfare improving for many parameter values.

Proposition 5 *If δ is close to 1 and θ is close to 0, $\frac{(9+16c)}{36} < f^I < \frac{\left(4c-2c^2+7\right)}{18}$, $\underline{f}_d^E < f^E \leq \frac{(1-c)^2}{9}$, then a prohibition against entry deterrent expansion decreases welfare for all c.*

Proof. Under the assumptions we know that the PBE is a pure strategy entry deterrence equilibrium (see case 2 of proposition 3). We first prove that preventing entry deterrence decreases welfare for $\delta = 1$ and $\theta = 0$. The rest of the result follows from continuity. For $\delta = 1$, preventing entry deterrence decreases welfare if:

$$\frac{3}{4}(1-c\theta)^2 - f^I > \frac{4}{9}(1-c)^2 - f^E \quad \Leftrightarrow \quad \frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - f^I + f^E > 0$$

Since $f^I < \frac{\left(4c-2c^2+7\right)}{18}$ and $f^E > \underline{f}_d^E$ we know that

$$\frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - f^I + f^E > \frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - \frac{\left(4c-2c^2+7\right)}{18} + \underline{f}_d^E$$

Since $\underline{f}_d^E \geq \frac{(1-2c)^2}{9}$ for $c \leq \frac{1}{2}$,

$$\frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - \frac{\left(4c-2c^2+7\right)}{18} + \underline{f}_d^E > \frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - \frac{\left(4c-2c^2+7\right)}{18} + \frac{(1-2c)^2}{9}$$

For $\theta = 0$, the RHS of the previous is

$$\frac{1}{9}c^2 + \frac{2}{9}c + \frac{1}{36} > 0$$

which holds for all c.

When $c > \frac{1}{2}$, $\underline{f}_d^E \geq 0$, thus

$$\frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - \frac{\left(4c-2c^2+7\right)}{18} + \underline{f}_d^E > \frac{3}{4}(1-c\theta)^2 - \frac{4}{9}(1-c)^2 - \frac{\left(4c-2c^2+7\right)}{18}$$

For $\theta = 0$, the RHS of the previous is positive if and only if

$$-\frac{1}{3}c^2 + \frac{2}{3}c - \frac{1}{12} > 0$$

which holds for all $c > \frac{1}{2}$.

Thus, for all $c \in (0,1)$ we have shown that for $\delta = 1$ and $\theta = 0$:

$$\frac{3}{4}\delta(1-c\theta)^2 - \frac{4}{9}\delta(1-c)^2 - f^I + f^E > 0$$

Since the LHS is continuous with δ and θ, by the sign preserving property of continuous functions there exists an $\varepsilon > 0$ such that if $\|(\delta,\theta)-(1,0)\| < \varepsilon$ then the function in the LHS is still positive.
■

In other words, if f^I is high, for δ close enough to 1 and θ small enough preventing entry deterrence decreases welfare, for all c. When θ is small, economies of scope are very large, thus the social gains from expansion are high. Thus preventing types with very large economies of scope from expanding would decrease welfare.

The next proposition complements the previous result as it shows that, when the expansion costs are high and δ is close to 1, for c above a certain level, a prohibition against entry deterrent expansion always decreases welfare.

Proposition 6 *If δ is close to 1, $\frac{(9+16c)}{36} < f^I < \frac{\delta\left(4c-2c^2+7\right)}{18}$, $\underline{f}_d^E < f^E \leq \frac{(1-c)^2}{9}$ and $c > \frac{1}{2}\sqrt{5}-1$ a prohibition against entry deterrent expansion decreases welfare for all $\theta \in \left[0,\theta^d\right]$.*

Proof. Under the assumptions we know that the PBE is a pure strategy entry deterrence equilibrium (see case 2 of proposition 3). We first prove that preventing entry deterrence decreases welfare for $\delta = 1$ and $\theta \in \left[0,\theta^d\right]$. The rest of the result follows from continuity. For $\delta = 1$, we know that preventing entry deterrence decreases welfare if:

$$\frac{3}{4}(1-c\theta)^2 - f^I > \frac{4}{9}(1-c)^2 - f^E$$

The LHS of this condition is decreasing with θ. Thus if the condition holds for θ^d, then it holds for all $\theta \in \left[0,\theta^d\right]$. Substituting θ^d in the expression and simplifying we obtain:

$$-\frac{5}{18}(1-c)^2 + f^E + \frac{1}{2}f^I > 0$$

We know that $f^I \geq \frac{(9+16c)}{36}$ and that, for $c < \frac{1}{2}$, $\underline{f}_d^E \geq \frac{(1-2c)^2}{9}$, which implies:

$$-\frac{5}{18}(1-c)^2 + f^E + \frac{1}{2}f^I > -\frac{5}{18}(1-c)^2 + \frac{(1-2c)^2}{9} + \frac{1}{2}\frac{(9+16c)}{36}$$

Thus a sufficient condition for prohibition against entry deterrence to decrease welfare is:

$$-\frac{5}{18}(1-c)^2 + \frac{(1-2c)^2}{9} + \frac{1}{2}\frac{(9+16c)}{36} > 0$$

which holds for $c > \frac{1}{2}\sqrt{5} - 1$.

For $c > \frac{1}{2}$, we know that $\underline{f}_d^E \geq 0$. Thus a sufficient condition for prohibition against entry deterrence to decrease welfare is:

$$-\frac{5}{18}(1-c)^2 + \frac{1}{2}\frac{(9+16c)}{36} > 0$$

which holds for all $c > \frac{1}{2}$.

We showed already that for $\delta = 1$, $c > \frac{1}{2}\sqrt{5} - 1 = 0.11803$ and $\theta \leq \theta^d$ we have:

$$\frac{3}{4}\delta(1-c\theta)^2 - \delta\frac{4}{9}(1-c)^2 - f^I + \delta f^E > 0$$

Since the LHS is continuous with δ, by the sign preserving property of continuous functions there exists an $\varepsilon > 0$ such that if $\|\delta - 1\| < \varepsilon$ then the function in the LHS is still positive. Consequently, for δ close enough to 1 and $c > \frac{1}{2}\sqrt{5} - 1$, a prohibition against entry deterrent expansion decreases welfare for all $\theta \in [0, \theta^d]$. ■

It should be highlighted that the previous condition is a sufficient but not necessary condition. In other words, prohibition against entry deterrence may decrease welfare under less strict conditions. The previous result suggests that if future is very important (δ is close to 1) and f^I is high, prohibition of entry deterrence decreases welfare as long as c is not very low.

It is quite interesting that a prohibition of entry deterrence decreases welfare when expansion costs are high. At the first sight one may think that higher expansion costs make expansion less desirable in terms of social welfare. However, higher expansion costs also mean that expansion is a signal of large economies of scope since the only types who expand are the ones with low θ. Considering the large economies of scope of the types who expand, preventing these types from expanding would decrease welfare. The result is easier to be satisfied when c is high because the efficiency gain due to economies of scope is higher when the original costs are high. In addition, when c is high the potential entrant is relatively inefficient, thus the fact that there is no competition in market A is not so harmful.

6.2.2 Prohibition of entry deterrence increases welfare

To show that there are cases where prohibition of entry deterrent expansion may be welfare improving we consider a scenario where all types want to expand when expansion deters entry but some of them would not expand under entry accommodation. In this case, there are types who expand to deter entry who have very small economies of scope. But these are the types for whom the social gain from expansion are lower, and thus it is possible that welfare improves by preventing these types from expanding.

Proposition 7 *If δ is close to 1 and $c > -\frac{2}{19}\sqrt{6}+\frac{9}{19}$, $f^I = \frac{7(1-c)^2}{18}$, $f^E = \underline{f}_d^E$, a prohibition against entry deterrent expansion increases welfare for θ close to $\theta^d = 1$.*

Proof. Under the assumptions we know that the PBE is a pure strategy entry deterrence equilibrium (see case 2 of proposition 2). We first prove that preventing entry deterrence increases welfare for $\delta = 1$ and $\theta = \theta^d = 1$. The rest of the result follows from continuity. For $\delta = 1$, preventing entry deterrence increases welfare if:

$$\frac{4}{9}(1-c)^2 - f^E > \frac{3}{4}(1-c\theta)^2 - f^I$$

The RHS is decreasing with θ, thus this condition is easier to be satisfied for $\theta = \theta^d$. Let us consider $f^I = \frac{7(1-c)}{18}$. By lemma 5, this implies that $\theta^d = 1$. For $\theta = \theta^d = 1$ the previous condition is equivalent to:

$$\frac{4}{9}(1-c)^2 - \frac{3}{4}(1-c)^2 - f^E + f^I > 0 \quad \Leftrightarrow \quad -\frac{11}{36}(1-c)^2 - f^E + f^I > 0$$

For $\theta^d = 1$ and $c \leq \frac{1}{2}$, $\underline{f}_d^E$ is equal to

$$\underline{f}_d^E = \int_0^1 \frac{(1-2c+\theta c)^2}{9} d\theta = \frac{7}{27}c^2 - \frac{1}{3}c + \frac{1}{9}.$$

Thus when $f^E = \underline{f}_d^E$ and $f^I = \frac{7(1-c)^2}{18}$ the condition for entry deterrence prohibition to be optimal for $\theta = \theta^d = 1$ is:

$$-\frac{11}{36}(1-c)^2 - \frac{7}{27}c^2 + \frac{1}{3}c - \frac{1}{9} + \frac{7(1-c)^2}{18} > 0 \quad \Leftrightarrow -\frac{19}{108}c^2 + \frac{1}{6}c - \frac{1}{36} > 0$$

which holds for $-\frac{2}{19}\sqrt{6} + \frac{9}{19} < c \leq \frac{1}{2}$.

When $c > \frac{1}{2}$, $\underline{f}_d^E$ is equal to

$$\underline{f}_d^E = \int_{\frac{2c-1}{c}}^1 \frac{(1-2c+\theta c)^2}{9} d\theta = \frac{1}{27c}(1-c)^3.$$

Thus when $f^E = \underline{f}_d^E$ and $f^I = \frac{7(1-c)^2}{18}$ the condition for entry deterrence prohibition to be optimal for $\theta = \theta^d = 1$ is:

$$-\frac{11}{36}(1-c)^2 - \frac{1}{27c}(1-c)^3 + \frac{7(1-c)^2}{18} > 0 \quad \Leftrightarrow \frac{1}{108c}(13c-4)(1-c)^2 > 0$$

which holds for all $c > \frac{1}{2}$. Thus when $\delta = 1$, $f^I = \frac{7(1-c)^2}{18}$ and $f^E = \underline{f}_d^E$ it is optimal to deter entry for type $\theta = \theta^d = 1$ as long as $c > -\frac{2}{19}\sqrt{6} + \frac{9}{19}$. By continuity, preventing entry increases welfare for δ and θ close enough to 1. ■

This result tells us that entry deterrence decreases welfare if economies of scope are very small and the entrant's entry costs are the minimum consistent with entry deterrence (with slightly lower entry costs, we would not have an entry deterrence equilibrium). Thus the combination of very small economies of scope and relatively small entry costs is favorable to the prohibition of entry deterrence (the fact that f^I is relatively low is necessary for entry deterrent expansion to be optimal for types with small economies of scope).

7 Conclusion

Incumbent firms may decide to expand to a related market just to benefit from economies of scope and thus decrease unitary costs. However expansion to a related market may also be used, in certain cases, to prevent possible rivals from entering in the first market as the incumbent becomes more efficient which decreases the attractiveness of entry. In this article we explore this rationale for expanding to a related market and investigate whether such entry deterrent expansion can be welfare improving.

We develop a model where the potential entrant has incomplete information regarding the economies of scope of the incumbent. In the model, the decision of expanding to a related market can be used as a signal of the degree of economies of scope of the incumbent. We show that the incumbent's optimal strategy is always a cut-off strategy: the types who expand are the ones with higher economies of scope. Consequently, expansion is indeed a signal of larger economies of scope and, for certain parameter values, it leads to entry deterrence. For other parameter values, there is entry accommodation or a mixed strategy perfect bayesian equilibrium where, when the incumbent expands to the related market, the entrant enters in the first market with some probability. The entry deterrence equilibrium occurs for intermediate or high values of the incumbent's expansion costs and relatively high entrant's entry cost. The intuition is that for higher expansion costs, expansion is a stronger signal that economies of scope are large, which is more likely to lead the potential rival not to enter.

Since expansion to a related market may occur exclusively for efficiency reasons, expansion cannot always be classified as an anticompetitive action. We define expansion to be entry deterrent if: (i) no expansion by the incumbent would increase the probability of potential rival's entry and (ii) not expanding would be optimal for the incumbent under the counterfactual hypothesis that the rival's entry decision was unaffected. If expansion to a related market is anticompetitive, should it be prohibited?

Our results suggest that the most likely answer is no! We show that a prohibition against entry deterrent expansion decreases consumer surplus and it is likely to decrease welfare. This strong result is driven to a large extent by the fact that the incumbent's expansion to another market generates a large social surplus in the new market. But, considering the existence of economies of scope, it is possible that consumers in the first market are also better off under entry deterrence. It is true that there is less competition, which hurts consumers in this market, but the efficiency gain due to economies of scope may overwhelm the lower competition effect. We believe that the relative size of the two markets may influence whether the result holds or not. However, the fact that by expanding the incumbent is serving another market which otherwise would not be served, is one major reason why preventing entry deterrence may decrease welfare.

References

[1] Bagwell, K. and Ramey, G. (1988), Advertising and limit pricing, *Rand Journal of Economics* 19(1), 59-71.

[2] Banker, R., Chang, H.-H. and Majumdar, S. (1998), Economies of scope in the U.S. telecommunications industry, *Information Economics and Policy* 10, 253–272.

[3] Cabral, L. and Riordan, M. (1997), The learning curve, predation, antitrust and welfare, *The Journal of Industrial Economics* XLV(2), 155-169.

[4] Cabral, L. (2000), Stretching firm and brand reputation, *Rand Journal of Economics* 31(4), 658–673.

[5] Cabral, L. (2009), Umbrella branding with imperfect observability and moral hazard, *International Journal of Industrial Organization* 27, 206-213.

[6] Cantos-Sánchez, P., Moner-Colonques, R. and Sempere-Monerris, J. (2003), Competition enhancing measures and scope economies: a welfare appraisal, *Investigaciones Económicas* XXVII (1), 97-123.

[7] Choi, J. P. (1998), Brand extension as informational leverage, *Review of Economic Studies* 65, 665-669.

[8] Dixon, H. D. (1994), Inefficient diversification in multi-market oligopoly with diseconomies of scope, *Economica* 61, 213-219.

[9] Eaton, B. C. and Lipsey, R. G. (1979), The Theory of Market Pre-emption: The Persistence of Excess Capacity and Monopoly in Growing Spatial Markets, *Economica* 46, 149-158.

[10] Fudenberg, D. and Tirole, J., (1991), *Game Theory*, Cambridge, MIT Press.

[11] Kessides, I. and Willig, R. (1995), Restructuring regulation of the rail industry for the public interest, *Policy Research Working Paper* 1506, The World Bank.

[12] Milgrom, P. and Roberts, J. (1982), Limit pricing and entry under incomplete information: an equilibrium analysis, Econometrica 50(2), 443-459.

[13] Pires, C. and Jorge, S. (2011), Limit pricing under third-degree price discrimination, *International Journal of Game Theory.* DOI: 10.1007/s00182-011-0310-x

[14] Röller, L.-H. and Tombak, M. (1990), Strategic choice of flexible production technologies and welfare implications, *The Journal of Industrial Economics* XXXVIII(4), 417-431.

[15] Schmalensee, R. (1978), Entry deterrence in the ready-to-eat breakfast cereal industry, The Bell Journal of Economics 9(2), 305-327.