CEFAGE-UE Working Paper
2011/06

Can Vertical Separation Reduce Non-Price Discrimination and Increase Welfare?

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February 11, 2011

Abstract

We investigate if vertical separation reduces non-price discrimination and increases welfare. Consider an industry consisting of a vertically integrated firm and an independent retailer, which requires access to the vertically integrated firm's wholesaler services. The wholesaler can degrade the quality of input it supplies to either of the retailers. Discrimination occurs if one of the retailers is supplied an input of lower quality than its rival. We show that separation of the vertically integrated firm reduces discrimination against the independent retailer, although it does not guarantee no-discrimination. Furthermore, with separation, the wholesaler may discriminate against the vertically integrated firm's retailer. Vertical separation impacts social welfare through two effects. First, through the double-marginalization effect, which is negative. Second, through the quality degradation effect, which can be positive or negative. Hence, the net welfare impact of vertical separation is negative or potentially ambiguous.

Keywords: Vertigal integration, vertical separation, non-price discrimination

*We thank M. Armstrong, D.-S. Jeon, M. Peitz, and T. Valletti for useful comments. The opinions expressed in this article reflect only the authors' views, and in no way bind the institutions to which they are affiliated.

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1 Introduction

The monopolist owner of a bottleneck input, which is also present in the retail market, may have the incentive and the ability to discriminate against retail entrants, to limit competition in the retail market. Regulators have resorted to various measures to prevent discrimination. Seemingly, those measures are relatively successful in preventing price-discrimination, but less so in preventing non-price-discrimination. Non-price-discrimination is hard to detect. In addition, even when abuses are detected, it takes time for regulators to solve the problem.

Many cases of non-price discrimination were brought before the European Commission and the national courts in Europe. For example, telecommunications incumbents were accused of: unjustifiable delays in processing entrants' orders, failure to provide information necessary to ensure that the entrants' services had the required quality, or providing poor quality or even non-functioning unbundled lines.[1]

Several forms of separation between the wholesale and retail units of telecommunications, energy, or railroad incumbents, ranging from accounting separation to structural separation, were proposed to prevent non-price-discrimination. Recently in the EU, functional separation has been at the center of several policy discussions regarding, e.g., the regulation of next generation networks.[2]

We investigate if vertical separation can: **(i)** reduce non-price-discrimination, and **(ii)** increase welfare. More specifically, we compare the incentives of a monopolist wholesaler to discriminate against retailers under vertical integration with one of the retailers and under vertical separation. In addition, we compare the welfare level under the two scenarios.

We model the industry as consisting of a vertically integrated firm, the incumbent, i.e., a firm that includes a wholesaler and a retailer, and an independent retailer, the entrant. The entrant requires access to the incumbent's wholesale services. The entrant and the

[1] For a thorough list of complains see the annexes of Squire (2002).

[2] European Commission (2007) states that: the "purpose of functional separation, whereby the vertically integrated operator is required to establish operationally separate business entities, is to ensure the provision of fully equivalent access products to all downstream operators, including the vertically integrated operator's own downstream divisions". In addition to the *Equality of Access* approach, of which functional separation is part, the European Commission evaluated the impact of the following regulatory approaches to the telecommunications sector: the *Continuity* approach, consisting on maintaining the current system, and the *No-Regulation* approach, consisting on the abstention from regulatory intervention in broadband networks. For a discussion of the merits of these two approaches see, respectively, Cave and Vogelsang (2003), and Gans and King (2004).

incumbent's retailer sell horizontally and vertically differentiated products. At a cost, the incumbent's wholesaler can degrade the quality of the input it supplies to either of the retailers. Hence, the incumbent's wholesaler can discriminate against either of the retailers, by degrading the quality of the input it supplies to one of the retailers more than the quality of the input it supplies to the other retailer. The sectoral regulator sets the access price to the wholesaler's services. In order to bias the results in favor of vertical separation, we assume that there are no vertical integration economies, i.e., production costs are the same under vertical integration and vertical separation. The incumbent's wholesaler and retailer maximize their profits jointly, under vertical integration, and maximize their profits separately, under vertical separation.

Under vertical integration, if the access price is low, or, if the access price takes intermediate values and the quality of the services of the entrant relative to those of the incumbent's retailer is low, the wholesaler discriminates against the entrant. Otherwise, the wholesaler does not discriminate against either of the retailers. Under vertical separation, if the access price and the relative quality of the entrant are both low, the wholesaler discriminates against the entrant. If the relative quality of the entrant is high and the access price is low, the wholesaler discriminates against the incumbent's retailer. In either case, the objective is to maximize total sales. Otherwise, the wholesaler does not discriminate against either of the retailers. To sum up, while vertical separation can mitigate the problem of discrimination against the entrant, it does not guarantee no-discrimination. Furthermore, under vertical separation, the wholesaler may discriminate against the incumbent's retailer, when, under vertical integration, it did not discriminate against any of the retailers.

Vertical separation impacts social welfare through two effects. First, through the double-marginalization effect, which is negative. Second, through the quality degradation effect, which can be positive or negative. The latter effect is negative, if, after separation, socially undesirable degradation, i.e. discrimination against a high quality retailer, increases, or, if socially desirable degradation, i.e. discrimination against a low quality retailer, decreases. If the degradation effect is negative, the net impact on welfare of vertical separation is negative. Otherwise, the net impact on welfare of vertical separation is potentially ambiguous.

Our results are were obtained without assuming coordination or vertical integration economies, with which it would have been trivial to obtain statements about the ambiguity of the impact of vertical separation on welfare, and suggest that vertical separation should be used with extreme care, because not only it might not eliminate discrimination, but it might also generate discrimination when it did not exist under vertical integration. Furthermore,

the impact of vertical separation on social welfare is, at best, potentially ambiguous.

The remainder of the article is organized as follows. Section 2 inserts our article in the literature. Section 3 presents the model. Section 4 characterizes the game's equilibrium. Section 5 presents a policy discussion and concludes. All proofs are in the Appendix.

2 Literature Review

Our research relates to two literature branches: **(i)** non-price discrimination, and **(ii)** vertical separation.[3]

The extensive literature on non-price discrimination analyzes the conditions under which a vertically integrated firm, which is a monopolist in the wholesale market, has incentives to discriminate against independent retailers. With the exception of Mandy and Sappington (2001) and Reitzes and Woroch (2007), this literature focuses on cost-increasing discrimination, instead of demand-reducing discrimination. However, the examples of non-price discrimination of section 1 that motivate our analysis, and that, e.g., functional separation proposes to solve, are about demand-reducing discrimination. Economides (1998), Mandy (2000), and Weisman and Kang (2001) consider the case where retailers compete in quantities. These articles show that an independent retailer will not be discriminated against, only if it is substantially more efficient than the integrated firm's retailer. Weisman (1995), Beard et al. (2001), and Kondaurova and Weisman (2003) consider the case where retailers compete in prices. These articles show that the incentive to discriminate by the vertically integrated firm is higher, the larger the cross-price elasticities of demand are. Mandy and Sappington (2001) examines the incentives for both cost-increasing discrimination and demand-reducing discrimination. It finds that cost-increasing sabotage is, typically, profitable both when firms compete in prices and when firms compete in quantities. In contrast, demand-reducing sabotage is often profitable when firms compete in quantities, but unprofitable when firms compete in prices. Bustos and Galetovic (2009) concludes that when cost-increasing discrimination is possible, a monopolist wholesaler prefers to vertically integrate even in the presence of vertical diseconomies. Sand (2004) shows that the incentives for non-price discrimination are lower, the higher the access price is. Hence, the socially optimal access price under non-price discrimination is higher than without non-price discrimination. Reitzes and Woroch (2007) examines the application of parity rules, and concludes that with

[3]There is a literature branch that analyses the impact of vertical integration on product variety. E.g., Kuhn and Vives (1999) shows that vertical integration increases total output and decreases variety as well as price, which implies that when variety is socially excessive, vertical integration increases welfare.

cost-based pricing parity, the wholesale monopolist has incentives to inefficiently downgrade the quality of its retail rival's services, and to excessively upgrade the quality of its retail affiliate services.

Regarding the second literature strand, De Bijl (2005) presents a framework to assess whether vertical separation is socially desirable. It argues that although vertical separation has the potential benefit of eliminating the vertically integrated firm's incentives to discriminate against its retail market rivals, it also has costs. Some of these costs are those of its implementation, the potential loss of synergies and economies of scope, and a possible decrease in investment incentives for both the vertically integrated firm and its competitors. Hence, separation should be applied as a remedy only when there is a persistent bottleneck, and no alternative regulatory regime is effective, and subject to a cost-benefit analysis. Chen and Sappington (2008) shows that, in general, vertical integration increases incentives for cost innovation, if retailers compete on quantities, but can reduce incentives for cost innovation, if retailers compete in prices. Buehler et al. (2004) compares the incentives of an incumbent to invest under vertical separation and integration. It concludes that, typically, investment is higher under vertical integration. Biglaiser and Degraba (2001) examine the consequences of allowing a monopolist wholesaler to vertically integrate with a retailer and compete with users of its wholesale services when the input has a regulated price above cost. They show that welfare increases with integration due to the elimination of the double-marginalization effect. However, they do not consider non-price discrimination.

Chikhladze and Mandy (2009) is the paper closest to ours. The article characterizes the socially optimal access price and level of vertical integration and identifies a trade-off between them. Our article differs from Chikhladze and Mandy (2009) in that we analyze the socially optimal vertical integration decision, but to isolate the effect of vertical separation, we assume that the access price is the same under vertical integration and separation. Moreover, our model assumes a different demand setup, two part retail tariffs, a different type of discrimination, and allows for both retailers to be discriminated against.

According to Chikhladze and Mandy (2009), with symmetric firms, full vertical integration is optimal when a high access price is needed to control sabotage, whereas limiting vertical integration is optimal when exogenous sabotage costs are sufficiently high to make a low access price socially desirable. Likewise, in our setting a high access price decreases the incentives to non-price discriminate the entrant, as the vertically integrated firm has high wholesale profits from the entrant's retail sales. However, under vertical separation the wholesaler may still have incentives to non-price discriminate one of the retailers when the

access price is low.

3 The Model

3.1 Environment

Consider an industry that consists of two overlapping markets: the wholesale market and the retail market. The wholesale market produces an input indispensable to supply services in the retail market. We refer to the price of the wholesale market as the *access price*. In the wholesale market there is a monopolist firm, the *wholesaler*, denoted by w. In the retail market there are two firms: the *incumbent's retailer*, denoted by r, and the *entrant*, denoted by e. The incumbent's retailer and the entrant sell horizontally and vertically differentiated products.

Initially, the wholesaler and the incumbent's retailer are vertically integrated. However, they may become vertically separated. We refer to the integrated entity as the *incumbent*, denoted by v. We index firms with subscript $j = w, r, e, v$. A *sectoral regulator* oversees the industry.

To lighten the exposition, we present the expressions for the upper and lower bounds on most of the model's parameters in appendix A. The expressions for the parameter thresholds that are needed to characterize the equilibrium are presented in the corresponding proofs in appendix B.

3.2 Sectoral Regulator

The regulator decides if the incumbent remains vertically integrated, or is separated into the wholesaler and the incumbent's retailer. To isolate the effect of vertical separation, we assume that the access price, which we denote by α on $[\underline{\alpha}, \overline{\alpha})$, is the same under vertical integration and separation.

3.3 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. In terms of their most-preferred product, consumers are uniformly distributed along a Hotelling line segment of length 1 (Hotelling, 1929). A consumer whose most-preferred product is x has a desutility cost of tx if he buys instead the product located at 0, with

t on $[\underline{t}, +\infty)$.[4] Similarly to Biglaiser and DeGraba (2001), we assume that each consumer selects only one firm and has a demand function for retail services given by $y_j(p_j; \Delta_j, \theta_j) = (z - p_j)\,\Delta_j\,(1 - \theta_j)$, where: **(i)** y_j is the number of units of retail services purchased from firm $j = r, e$, **(ii)** z is a positive parameter, **(iii)** p_j is the per unit price of retail services of firm j, **(iv)** Δ_j is the *relative quality* parameter that takes value 1 for products sold by the incumbent and takes value Δ on $\left(0, \overline{\Delta}\right)$, for products sold by the entrant, and **(v)** θ_j on $[0, 1]$ is the *quality degradation* level. The relative quality parameter measures the quality of services of the entrant with respect to the retail services of the incumbent's retailer.[5] The lower limit on t and the upper limit on Δ ensure that, in equilibrium, there is a duopoly both under vertical integration and separation. The quality degradation level measures the quality of the input supplied by the wholesaler and is observed by all players. We will say that there is degradation of the quality of the input of the entrant, or, for short, that there is *degradation of the quality* of the entrant, if $\theta_e > 0$. Similarly for the incumbent's retailer. We will say that there is *quality discrimination*, if the quality degradation levels of the two retailers are different, $\theta_r \neq \theta_e$, and that there is *quality discrimination against* retailer $j = r, e$, if the quality degradation level of retailer j is higher than the quality degradation level of retailer j': $\theta_j > \theta_{j'} \geq 0$. Let $\boldsymbol{\theta} := (\theta_r, \theta_e)$ and let $\Theta := (1 - \theta_r) - \Delta\,(1 - \theta_e)$, denote a parameter that measures the *net quality advantage* of the incumbent's retailer over the entrant.[6]

Denote by $S_j := V + (z - p_j)^2\,\Delta_j\,(1 - \theta_j)\,/2$, the consumer surplus from purchasing from firm j, gross of fixed payments and of the desutility costs of not buying its most-preferred product, where V is a fixed benefit from subscribing to either firm. We assume that V is sufficiently high to guarantee that: the market is always covered, and for any level of quality degradation both firms have a positive market share. The remaining surplus depends on the number of units bought. We assume also that quality degradation affects only the variable component of consumer surplus.

[4] Value tx is usually refered to as the *transportation cost*.

[5] The services supplied by the entrant are of higher quality than those supplied by the incumbent' retailer, if $\Delta > 1$, or of lower quality, if $\Delta < 1$.

[6] After quality degradation, if $\Theta > 0$, the quality of the services of the incumbent's retailer is higher than the quality of the services of the entrant, and if $\Theta < 0$, it is lower and it is the entrant who has a *net quality advantage* over the incumbent.

3.4 Firms

The wholesaler has a constant marginal cost normalized to 0. In addition to the access price, the retailers have constant marginal costs also normalized to 0. The retailers' marginal cost equals the regulated access price, α.

The wholesaler can degrade the quality of the inputs it supplies to the retailers at a cost of: $C(\boldsymbol{\theta}) = \frac{\beta}{2t}(\theta_r - \theta_e)^2$. Typically, quality discrimination is forbidden by sectoral regulation, and if detected is subject to a fine. Hence, the quality degradation cost can be thought of as the expected fine paid by the wholesaler. The quality degradation cost function has two important properties: **(i)** the cost is positive only if there is discrimination, and **(ii)** the cost is increasing in the difference between the quality degradation levels. The first property follows when the regulator only detects quality degradation if, given Δ, retailers have products of different quality.[7] The second property follows when either the probability of the regulator detecting quality discrimination is increasing in the difference of values of the quality degradation levels,[8] or the penal code involves a punishment increasing in the degree of the offense.

To ensure that the wholesaler's problem is concave in θ_r given θ_e and vice-versa, and that, in equilibrium, $\theta_j < 1$, $j = r, e$, let β be on $\left(\underline{\beta}, +\infty\right)$, with $\underline{\beta} > 0$.

The incumbent's retailer is located at point 0 and the entrant at point 1 of the line segment where consumers are distributed.

Firms charge consumers two-part *retail tariffs*, denoted by $T_j(y_j) = F_j + p_j y_j$, $j = r, e$, where F_j on $[0, +\infty)$ is the fee of firm j, and $\mathbf{F} := (F_r, F_e)$

The consumer share of firm $j = r, e$, derived in the Appendix, is given by:

$$\sigma_j(\mathbf{F}; \boldsymbol{\theta}, \Delta) = \begin{cases} \frac{1}{2} + \frac{2(F_e - F_r) + (z - p_r)^2(1 - \theta_r) - (z - p_e)^2\Delta(1 - \theta_e)}{4t} & j = r \\ \frac{1}{2} - \frac{2(F_e - F_r) + (z - p_r)^2(1 - \theta_r) - (z - p_e)^2\Delta(1 - \theta_e)}{4t} & j = e. \end{cases} \tag{1}$$

with $\sigma_r(\mathbf{F}; \boldsymbol{\theta}, \Delta) + \sigma_e(\mathbf{F}; \boldsymbol{\theta}, \Delta) = 1$.

[7] Assume that each retailer's quality depends on three elements: an industry wide shock, a retailer specific shock symmetric across retailers, and an action taken by the wholesaler. The regulator does not observe the shocks or the wholesaler's action. In addition, the uncertainty about the industry wide shock is substantially larger than the uncertainty about the retailer specific shocks. In these circumstances, it is reasonable for the regulator to infer that there was discrimination only if the retailers' quality degradation levels differ.

[8] Given the assumption of footnote 7, it may occur that $\theta_r = \theta_e$, even when there is quality discrimination, and it may also occur that $\theta_r \neq \theta_e$, even when there is no quality discrimination. However, the larger $|\theta_r - \theta_e|$, the larger the probability that there was quality discrimination.

Under vertical integration, the profits of firm $j = v, e$ for the whole game are:

$$\pi_v = [p_r y_r + F_r]\,\sigma_r + \alpha y_e \sigma_e - C, \tag{2}$$

$$\pi_e = [(p_e - \alpha)\,y_e + F_e]\,\sigma_e. \tag{3}$$

Under vertical separation, the profits of firm $j = w, r, e$ for the whole game are:

$$\pi_w = \alpha\,[y_e \sigma_e + y_r \sigma_r] - C, \tag{4}$$

$$\pi_r = [(p_r - \alpha)\,y_r + F_r]\,\sigma_r, \tag{5}$$

$$\pi_e = [(p_e - \alpha)\,y_e + F_e]\,\sigma_e. \tag{6}$$

3.5 Timing of the Game

The game unfolds as follows. In stage 1, the sectoral regulator decides whether to separate the incumbent into the wholesaler and the incumbent's retailer. In stage 2, the incumbent, under vertical integration, or the wholesaler, under vertical separation, decides the levels of quality degradation of the inputs it supplies. In stage 3, the incumbent and the entrant, under vertical integration, or the incumbent's retailer and the entrant, under vertical separation, choose retail tariffs.

3.6 Equilibrium Definition

The sub-game perfect Nash equilibrium is: **(i)** a decision of whether to separate the incumbent, **(ii)** a decision on the levels of quality degradation, and **(iii)** a pair of retail tariffs such that:

(E1) the retail tariffs maximize the firms' profits, given the market structure, and the quality degradation decision;

(E2) the decision on the level of quality degradation maximizes the incumbent's profit under vertical integration, or the wholesalers' profit under vertical separation, given the optimal retail tariffs;

(E3) the decision of whether to separate the incumbent maximizes social welfare, given the optimal decision on the levels of quality degradation and the optimal retail tariffs.

4 Equilibrium

In this section, we characterize the equilibrium of the game, which we construct by backward induction. When necessary, we use superscripts i and s to denote variables or

functions associated with vertical integration and vertical separation, respectively.

4.1 Stage 3: The Retail Game

Next, we characterize the equilibria of the retail tariffs game under: **(i)** vertical integration and **(ii)** vertical separation.

We start with the following Lemma.

Lemma 1: *In equilibrium, firms set the marginal price of the two-part retail tariff at marginal cost.* ■

As usual with two-part tariffs, firms set the variable part of the retail tariff at marginal cost, to maximize gross consumer surplus, and then try to extract this surplus using the fixed fee. Hence: $p_r^i = 0 \leq \alpha = p_e^i = p_r^s = p_e^s$.[9] Given Lemma 1, from now on we only discuss the determination of the fixed fees. The next Lemma presents the equilibrium fixed fees.

Lemma 2: *In equilibrium:*

(i) *under vertical integration, the incumbent and the entrant set the fixed fees:*

$$F_j^i(\boldsymbol{\theta}; \alpha, \Delta) = \begin{cases} t + \frac{z^2\Theta + \alpha(1-\theta_e)\Delta(6z-5\alpha)}{6} & j = r \\ t - \frac{z^2\Theta + \alpha^2(1-\theta_e)\Delta}{6} & j = e. \end{cases}$$

and the equilibrium consumer share of the incumbent's retailer is:

$$\sigma_r^i(\mathbf{F}; \boldsymbol{\theta}, \Delta) = \frac{1}{2} + \frac{z^2\Theta + \alpha^2\Delta(1-\theta_e)}{12t}$$

(ii) *under vertical separation, the incumbent's retailer and the entrant set the fixed fees:*

$$F_j^s(\boldsymbol{\theta}; \alpha, \Delta) = \begin{cases} t + \Theta\frac{(z-\alpha)^2}{6} & j = r \\ t - \Theta\frac{(z-\alpha)^2}{6} & j = e. \end{cases}$$

and the equilibrium consumer share of the incumbent's retailer is:

$$\sigma_r^s(\mathbf{F}; \boldsymbol{\theta}, \Delta) = \frac{1}{2} + \frac{(z-\alpha)^2\Theta}{12t}$$

■

[9] Under vertical integration, even if the regulator forces the incumbent to sell, at the same price, access to the entrant and the incumbent's retailer, this latter payment constitutes only an internal transfer, and therefore the incumbent does not take it into account when maximizing its profit.

Under vertical integration, the first-order conditions with respect to the fixed fee for the incumbent and the entrant are, respectively:

$$\frac{\partial \sigma_r}{\partial F_r} F_r + \sigma_r + \alpha y_e \frac{\partial \sigma_e}{\partial F_r} = 0, \tag{7}$$

$$\frac{\partial \sigma_e}{\partial F_e} F_e + \sigma_e = 0, \tag{8}$$

while under vertical separation, the first-order condition for firm $j = r, e$ is similar to equation (8).

Under vertical separation there is the usual trade-off between profit margin and volume of sales, as inspection of equation (8) reveals. Under vertical integration, the incumbent, when compared to the entrant, has an additional upward pressure on its fixed fee, given by the additional term in (7). By hiking its retailer's fixed fee, the incumbent increases the entrant's consumer share, and hence, its own wholesale revenues. We call *wholesale effect* to this upward pressure on the incumbent retailer's fixed fee.

The next remark collects some results on how fixed fees and market shares vary with the access price.

Remark 1: *Under vertical separation:* ***(i)*** $F_e^s < F_r^s$, *if and only if,* $\Theta > 0$; ***(ii)*** $\frac{\partial F_r^s}{\partial \alpha} < 0 < \frac{\partial F_e^s}{\partial \alpha}$, *if and only if,* $\Theta > 0$; *and* ***(iii)*** $\frac{\partial \sigma_r^s}{\partial \alpha} < 0 < \frac{\partial \sigma_e^s}{\partial \alpha}$, *if and only if,* $\Theta > 0$**.**

Under vertical integration: ***(i)*** $\frac{\partial F_e^i}{\partial \alpha} < 0$, *for all* α*; and* $\frac{\partial F_r^s}{\partial \alpha} < 0$, *if* α *is on* $(3z/5, z)$, *and* $\frac{\partial F_r^s}{\partial \alpha} > 0$, *if* α *is on* $[0, 3z/5)$; *and* ***(ii)*** $\frac{\partial \sigma_r^s}{\partial \alpha} < 0 < \frac{\partial \sigma_r^s}{\partial \alpha}$, *for all* Θ. ■

Under vertical separation, the two retailers are symmetric with respect to costs. The firm with a *net quality advantage* over the other will set a higher fixed fee and will have a larger equilibrium consumer share. The higher α is, the lower the number of units consumed, and the smaller the differences in consumer surplus that result from quality differences. Thus, the higher α is, the smaller the differences in the fixed fees and in the consumer shares are.

Under vertical integration there is an asymmetry in the retailers' marginal costs, and hence, on the retail marginal price. This means that, all else constant, consumers purchase a smaller number of units from the entrant and have a lower surplus. Hence, if the incumbent's fixed fee is larger than the entrant's under separation, it is also larger under integration. An increase in the access price decreases the fixed fee set by entrant, but it may increase or decrease the fixed fee of the incumbent's retailer.[10] Contrary to the case of separation, a

[10] F_r^i may decrease with α because the wholesale effect may decrease with α if α is sufficiently large.

higher α always implies an increase in the incumbent's retailer consumer share, regardless of which firm has a net quality advantage.

The next remark collects some results on how fixed fees and market shares vary with the quality degradation levels.

Remark 2: *Under vertical separation:* ***(i)*** $\frac{\partial F_j^s}{\partial \theta_j} < 0 < \frac{\partial F_j^s}{\partial \theta_{j'}}$*; and* ***(ii)*** $\frac{\partial \sigma_j^s}{\partial \theta_j} < 0 < \frac{\partial \sigma_j^s}{\partial \theta_{j'}}$.

Under vertical integration: ***(i)*** $\frac{\partial F_r^i}{\partial \theta_r} < 0 < \frac{\partial F_e^i}{\partial \theta_r}$; $\frac{\partial F_e^i}{\partial \theta_e} < 0$ *and* $\frac{\partial F_r^i}{\partial \theta_e} < 0$*, if* α *is on* $(z/5, z)$*, and* $\frac{\partial F_r^i}{\partial \theta_e} > 0$*, if* α *is on* $[0, z/5)$*; and* ***(ii)*** $\frac{\partial \sigma_j^i}{\partial \theta_j} < 0 < \frac{\partial \sigma_j^i}{\partial \theta_{j'}}$. ■

Under vertical separation, inspection of equation (1) shows that degradation of the quality of one retailer, shifts consumers to the other retailer. The per consumer demand of the retailer whose quality is degraded falls, forcing it to reduce its fixed fee, while allowing the other retailer to increase its fixed fee.

Under vertical integration, degradation against the incumbent's retailer forces it to reduce its fixed fee and allows the entrant to increase its fixed fee, by the mechanism described in the previous paragraph. Similarly, degradation against the entrant forces the entrant to reduce its fixed fee. However, degradation of the quality of the entrant has a more complex impact on the fixed fee of the incumbent's retailer. On the one hand, it increases the demand of the incumbent's retailer, which leads to a higher fixed fee. On the other hand, it decreases the magnitude of the wholesale effect, because each of the entrant's consumers purchases a smaller number of units. This leads to a lower fixed fee. If the wholesale margin is low, the reduction in the number of units purchased by each consumer that selects the entrant is less relevant. Hence, the fixed fee increases with degradation of the quality of the entrant. If the wholesale margin is high, the opposite occurs.

4.2 Stage 2: The Quality Degradation Decision

Next, we characterize the optimal quality degradation decision, first under vertical integration, and afterwards under vertical separation.

4.2.1 Integration

In stage 2, the incumbent's profit function is:

$$\pi_v = F_r^i(\boldsymbol{\theta}; \alpha, \Delta)\sigma_r(\mathbf{F}^i(\boldsymbol{\theta}; \alpha, \Delta); \boldsymbol{\theta}, \alpha, \Delta) + \alpha y_e(\boldsymbol{\theta}; \alpha, \Delta)\sigma_e(\mathbf{F}^i(\boldsymbol{\theta}; \alpha, \Delta); \boldsymbol{\theta}, \alpha, \Delta) - C(\boldsymbol{\theta}).$$

Denote by $\Delta^i(\alpha)$, the lowest level of the relative quality parameter on $\left(0, \overline{\Delta}\right)$, if it exists,

for which it is profit maximizing for the incumbent not to degrade the quality of the entrant. In addition, let α_1^i and α_2^i be defined by, respectively, $\Delta^i(\alpha_1^i) - \overline{\Delta}(\alpha_1^i) \equiv 0$ and $\Delta^i(\alpha_2^i) \equiv 0$.

The next Lemma characterizes the incumbent's optimal quality degradation decision.

Lemma 3: *Under vertical integration there are two equilibria:*
(i) *There is no quality degradation, i.e.,* $\theta_e^i = \theta_r^i = 0$*, if and only if,* (α, Δ) *is on* $\left[\max\left\{\underline{\alpha}, \alpha_1^i\right\}, \alpha_2^i\right) \times \left[\Delta^i(\alpha), \overline{\Delta}(\alpha)\right) \cup \left[\alpha_2^i, z\right) \times \left(0, \overline{\Delta}(\alpha)\right)$.
(ii) *There is quality degradation against the entrant, i.e.,* $\theta_e^i > 0 = \theta_r^i$*, if and only if,* (α, Δ) *is on* $\left[\underline{\alpha}, \max\left\{\underline{\alpha}, \alpha_1^i\right\}\right) \times \left(0, \overline{\Delta}(\alpha)\right) \cup \left[\max\left\{\underline{\alpha}, \alpha_1^i\right\}, \alpha_2^i\right) \times \left(0, \Delta^i(\alpha)\right)$. ■

The first-order condition for the degradation of the entrant's quality, θ_e, is:

$$\alpha(1-\sigma_r)\frac{\partial y_e}{\partial \theta_e} + \left(F_r^i - \alpha y_e\right)\left(\frac{\partial \sigma_r}{\partial F_e^i}\frac{\partial F_e^i}{\partial \theta_e} + \frac{\partial \sigma_r}{\partial \theta_e}\right) = \frac{\partial C}{\partial \theta_e},$$

and involves the usual trade-off between marginal revenue and marginal cost.

Inspection of the marginal revenue shows that quality degradation against the entrant impacts the incumbent's profits through two effects: **(i)** reduces the entrant's per consumer demand, $\frac{\partial y_e}{\partial \theta_e} < 0$, **(ii)** increases the consumer share of the incumbent's retailer, $\frac{\partial \sigma_r}{\partial \theta_e} + \frac{\partial \sigma_r}{\partial F_e^i}\frac{\partial F_e^i}{\partial \theta_e} > 0$, if $\alpha < z/2$, and decreases it otherwise.[11]

The first effect decreases the wholesale profits. As noted in Remark 2, the second effect transforms wholesale customers into retail customers, or vice-versa, and has an ambiguous sign that depends on the access price. For a sufficiently low access price, given that each retail customer results in more profits for the vertically integrated incumbent than each wholesale customer at $\theta_e = \theta_r = 0$, the marginal benefit involves a trade-off between lower wholesale and higher retail profits. For a sufficiently high access price, the second effect is also negative, and there is no trade-off.

To lighten the exposition, in the remainder of the text we will often refer to the values of some parameters as being "high" or "low", instead of the intervals to which they belong. The meaning of "high" or "low" will vary according to the context. We refer the reader to the Lemmas and Propositions to the exact statement of our results.

If α is low, the incumbent always degrades the quality of the entrant. The incumbent earns a low wholesale margin with each of the entrant's consumers. Hence, it does not loose

[11] All else constant, discrimination against the entrant increases the consumer share of the incumbent's retailer. However, it also reduces the entrant's equilibrium fee, which decreases the consumer share of the incumbent's retailer. The combined effect is positive if and only if $\alpha < z/2$.

substantial wholesale profits by degrading the quality of the entrant, even when the entrant has large per consumer sales.

If α takes intermediate values, the incumbent does not degrade the quality of the entrant when the entrant's relative quality is high, and degrades the quality of the entrant when the entrant's relative quality is low. In the former case the entrant has larger sales than the incumbent's retailer. Degrading the quality of the entrant would imply loosing large wholesale profits.

If α is high, the incumbent never degrades the quality of the entrant. It is not profitable to sacrifice the high access margin the incumbent receives for each unit sold by the entrant, even when the entrant has small per consumer sales.

Finally, the incumbent never degrades the quality of its own retailer. Increasing the retail price it charges has the same impact on the entrant's number of consumers as degrading the quality it offers, and it is cheaper.

4.2.2 Separation

In stage 2, the wholesaler's profit function is:

$$\pi_w = \alpha y_r(\boldsymbol{\theta};\alpha)\sigma_r(\mathbf{F}^s(\boldsymbol{\theta};\alpha,\Delta);\boldsymbol{\theta},\Delta,\alpha) + \alpha y_e(\boldsymbol{\theta};\alpha,\Delta)\sigma_e(\mathbf{F}^s(\boldsymbol{\theta};\alpha,\Delta);\boldsymbol{\theta},\Delta,\alpha) - C(\boldsymbol{\theta}).$$

Denote by $\Delta_e^s(\alpha)$ the lowest level of the relative quality parameter on $\left(0,\overline{\Delta}(\alpha)\right)$, if it exists, for which it is profit maximizing for the wholesaler not to degrade the quality of the entrant, and denote by $\Delta_r^s(\alpha)$ the highest level of the relative quality parameter on $\left(0,\overline{\Delta}(\alpha)\right)$, if it exists, for which it is profit maximizing for the wholesaler not to degrade the quality of the incumbent's retailer. In addition, let α_1^s and α_2^s be defined by, respectively, $\Delta_e^s(\alpha_1^s) \equiv 0$ and $\Delta_r^s(\alpha_2^s) - \overline{\Delta}(\alpha_2^s) \equiv 0$.

The next Lemma presents the wholesaler's optimal quality degradation decision.

Lemma 4: *Under vertical separation there are three equilibria:*
(i) *There is no quality degradation, i.e., $\theta_e^s = \theta_r^s = 0$, if and only if, (α,Δ) is on $[\max\{\alpha_1^s,\alpha_2^s,\underline{\alpha}\}, z) \times \left(0,\overline{\Delta}(\alpha)\right) \cup [\underline{\alpha},\max\{\alpha_2^s,\underline{\alpha}\}) \times (0,\Delta_r^s(\alpha)] \cup [\underline{\alpha},\max\{\alpha_1^s,\underline{\alpha}\}) \times \left[\Delta_e^s(\alpha),\overline{\Delta}(\alpha)\right)$.*
(ii) *There is quality degradation against the entrant, i.e., $\theta_e^s > 0 = \theta_r^s$, if and only if, (α,Δ) is on $[\underline{\alpha},\max\{\alpha_1^s,\underline{\alpha}\}) \times (0,\Delta_e^s(\alpha))$.*
(iii) *There is quality degradation against the incumbent's retailer, i.e., $\theta_r^s > \theta_e^s = 0$, if and only if, (α,Δ) is on $[\underline{\alpha},\max\{\alpha_2^s,\underline{\alpha}\}) \times \left(\Delta_r^s(\alpha),\overline{\Delta}(\alpha)\right)$.* ■

The first-order condition for the degradation of the quality of firm j, θ_j, is:

$$\alpha\sigma_j \frac{\partial y_j}{\partial \theta_j} + \alpha(y_j - y_{j'}) \frac{d\sigma_j}{d\theta_j} = \frac{\partial C}{\partial \theta_j},$$

and involves the usual trade-off between the marginal revenue and marginal cost.

Quality degradation against retailer j impacts the wholesaler's revenue through two effects: **(i)** reduces retailer j's per consumer demand, $\frac{\partial y_j}{\partial \theta_j} < 0$, **(ii)** decreases the consumer share of retailer j, $\frac{d\sigma_j}{d\theta_j} < 0 < \frac{d\sigma_{j'}}{d\theta_j}$.

The first effect, which represents the losses with infra-marginal consumers, reduces the profit the wholesaler obtains from retailer j. The second effect reduces the profit the wholesaler obtains if retailer j sells more units to each consumer than retailer j', and increases the profit the wholesaler obtains otherwise. It follows that the wholesaler will never degrade the services of the retailer that sells more units per consumer. Assuming that retailer j is the one that has a net quality disadvantage, the magnitude of σ_j in the first term varies directly with α, as mentioned in Remark 1. This means that the first effect of the degradation of the quality of retailer j, which is negative, is stronger when α is high. Furthermore, the magnitude of $\frac{d\sigma_j}{d\theta_j}$ in the second term, the positive effect, varies inversely with α. If α is very high, few consumers change of supplier due to quality degradation. This happens because with a high α fewer units are purchased by each consumer and hence, a quality decrease does not translate into a large surplus loss. Hence, if α is high, the wholesaler never discriminates. If α is low and the differences in quality are small, the wholesaler does not degrade quality either, and tries to maximize the number of units sold by both retailers. If α is low and the entrant's relative quality is low, the wholesaler degrades the quality of the entrant. On the other hand, if α is low and the entrant's relative quality is high, the wholesaler degrades the quality of the incumbent's retailer. Indeed, given that retail profits are not part of the wholesaler's objective function and the access price is fixed, it only maximizes wholesale sales, and thus it prefers that the retailer which sells more units also has more consumers.

4.2.3 Comparison

We start by presenting the following Corollary.

Corollary 1: ***(i)*** *The set of parameter values for which there is quality degradation against the entrant under vertical separation is a strict subset of the set of parameter values for which there is quality degradation against the entrant under vertical integration.* ***(ii)*** *The level of degradation against the entrant is no smaller under vertical integration than under vertical separation.* ■

There is no quality degradation against the entrant under vertical separation, if there was no quality degradation under vertical integration. Furthermore, when there was degradation against the entrant under vertical integration, the level of degradation against the entrant decreases after vertical separation. Indeed, the incumbent has no smaller incentives to degrade the quality of the entrant than an independent wholesaler, since the entrant is a rival on the retail market.

Define $\Delta_r^s(\alpha)^+ \equiv \lim_{\varepsilon \to 0^+} (\Delta_r^s(\alpha) + \varepsilon)$. Comparison of Lemmas 3 and 4 leads to the next Lemma.

Corollary 2: ***(i)*** *There is no quality degradation under both vertical integration and separation, if and only if,* (α, Δ) *is on* $[\max\{\underline{\alpha}, \alpha_1^i\}, z) \times \left[\max\{\Delta^i(\alpha), 0^+\}, \min\left\{\Delta_r^s(\alpha)^+, \overline{\Delta}(\alpha)\right\}\right)$.

(ii) *There is no quality degradation under vertical integration, and there is quality degradation against the incumbent's retailer under vertical separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \max\{\underline{\alpha}, \alpha_2^s\}) \times \left[\max\{\Delta^i(\alpha), \Delta_r^s(\alpha)^+\}, \overline{\Delta}(\alpha)\right)$.

(iii) *There is quality degradation against the entrant under vertical integration, and there is no quality degradation under vertical separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \alpha_2^i) \times \left[\max\{\Delta_e^s(\alpha), 0^+\}, \min\left\{\Delta^i(\alpha), \Delta_r^s(\alpha)^+, \overline{\Delta}(\alpha)\right\}\right)$.

(iv) *There is quality degradation against the entrant under both vertical integration and separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \max\{\underline{\alpha}, \alpha_1^s\}) \times (0, \Delta_e^s(\alpha))$. ■

Figures 1 and 2 illustrate Corollary 2 for the cases where z is on $\left(0, \sqrt{3t}\right)$ and z is on $\left(\sqrt{3t}, +\infty\right)$, respectively. In both figures, $j \to j'$ means "under vertical integration, firm j is discriminated against, while under vertical separation firm j' is discriminated against", with $j, j' = e, r, n$, and where "n" means "no firm is discriminated against".[12]

[*Figure* 1]

[*Figure* 2]

If α is high, there is no quality degradation under either vertical integration or vertical separation. Both the incumbent or the independent wholesaler do not want to degrade the quality of any of the retailers since it would involve losing the high access margin, independently of the number of units sold by each retailer.

We now turn to Figure 1. If Δ takes intermediate values, there is no quality degradation in both scenarios. If α takes low values, and if Δ is high, the incumbent does not degrade

[12] In these Figures we consider that $\underline{\alpha} < \max\{\alpha_1^s, \alpha_2^s, \alpha_1^i\}$. Note that α_1^s is on $(-\infty, 0)$, if z is on $\left(0, \sqrt{3t}\right)$, and α_2^s is on $(-\infty, 0)$, if z is on $\left(\sqrt{3t}, +\infty\right)$.

the quality of the entrant, but the independent wholesaler degrades the quality of the incumbent's retailer, since it has a smaller per consumer demand. If α is low, and if Δ is low, the incumbent degrades the quality of the entrant, but the independent wholesaler does not degrade the quality of any of the retailers.

We now turn to Figure 2. If α is low, and if Δ is low, both the incumbent and the independent wholesaler degrade the quality of the entrant. If Δ is high, the incumbent degrades the quality of the entrant, while the independent wholesaler does not degrade the quality of any of the retailers, for a low α.

To sum up, these results question the common wisdom that vertical separation is a good policy instrument to eliminate quality discrimination by a vertically integrated firm against retail entrants. While it is true that in some circumstances vertical separation does reduce quality discrimination, in other circumstances it has no impact on quality discrimination, and in yet other circumstances it can even increase quality discrimination.

4.3 Stage 1: The Separation Decision

Next, we first identify the two effects of vertical separation on welfare, the *double-marginalization effect* and the *quality degradation effect*, and afterwards we characterize the socially optimal decision of whether to separate vertically the incumbent.

Denote by W the social welfare, i.e., the sum of the firms' profits, consumer surplus and the regulator's revenues. The degradation cost is a transfer from either the incumbent or the independent wholesaler to the regulator, and is, hence, neutral in terms of welfare.

Social welfare is given by:

$$W^h\left(\boldsymbol{\theta};\alpha,\Delta\right)=\begin{cases}\frac{5\left[z^2\Theta+\alpha^2\Delta(1-\theta_e)\right]^2}{144t}+\frac{z^2[\Theta+2\Delta(1-\theta_e)]-\alpha^2\Delta(1-\theta_e)}{4}-\frac{1}{4}t & h=i\\ \frac{(5z+7\alpha)(z-\alpha)^3\Theta^2}{144t}+\frac{\left(z^2-\alpha^2\right)[\Theta+2\Delta(1-\theta_e)]}{4}-\frac{1}{4}t & h=s.\end{cases}$$

The next Lemma compares the welfare levels under vertical integration and separation, keeping the quality degradation levels constant.

Lemma 5: *Keeping quality degradation levels constant, i.e., for $\theta_j^s=\theta_j^i$, $j=r,e$, welfare decreases with the vertical separation of the incumbent.* ■

For the incumbent, the marginal cost of its retailer is 0, since α is only an internal transfer. However, with vertical separation, the wholesaler and the incumbent's retailer maximize their profits separately, rather than jointly. Therefore, the α charged by the

wholesaler is a marginal cost not only for the entrant but also to the incumbent's retailer. From Lemma 1, firms set the marginal retail price at marginal cost. Consequently, vertical separation leads the incumbent's retailer to increase its marginal retail price from $p_r = 0$ to $p_r = \alpha$. Given the quality degradation levels, this price induces consumers to buy less units, which reduces welfare.

To sum up, vertical separation has a *double-marginalization effect*, which has a negative impact in welfare.

Let Θ_j^h denote the value for the *net quality advantage* of the incumbent's retailer over the entrant such that the derivative of W^h with respect to θ_j is zero.[13] The next Lemma analyzes the impact on welfare of quality degradation under both vertical integration and separation, when only the quality of one of the retailers is degraded, i.e., when either $\theta_r = 0$ or $\theta_e = 0$.

Lemma 6: ***(i)*** *When $\theta_r = 0$, under vertical separation, quality degradation against the entrant increases welfare if $\Delta < 1 - \Theta_e^s$, and decreases welfare if $0 > 1 - \Theta_e^s$, for all θ_e on $[0, 1]$.*

(ii) *When $\theta_e = 0$, under vertical separation, quality degradation against the incumbent increases welfare if $\Delta > 1 + \Theta_e^s$, and decreases welfare if $\Delta < \Theta_e^s$, for all θ_r on $[0, 1]$.*

(iii) *When $\theta_r = 0$, under vertical integration, quality degradation against the entrant increases welfare if $\Delta < 1 - \Theta_e^i(0)$, and decreases welfare if $0 > 1 - \Theta_e^i(0)$, for all θ_e on $[0, 1]$.*

(iv) *When $\theta_e = 0$, under vertical integration, quality degradation against the incumbent increases welfare if $\Delta > 1 - \Theta_r^i(0)$, and decreases welfare if $\Delta < -\Theta_r^i(1)$, for all θ_r on $[0, 1]$.* ■

Consider first the case of vertical separation. Assume that the incumbent's retailer has a net quality advantage over the entrant, i.e., assume that $\Theta > 0$. Then, the incumbent has a higher market share and more consumers have a higher desutility cost of not buying their most-preferred product. Quality degradation against the entrant has two effects: **(i)** first, it makes some consumers switch from the entrant to the incumbent's retailer, and **(ii)** second, it makes the consumers that remain with the entrant purchase a smaller amount. The first effect has two opposite signed impacts on welfare. On the one hand, it has a positive impact

[13] In particular, Θ_j^i is a function of θ_r and hence we denote it by $\Theta_j^i(\theta_r)$, $j = r, e$. It should also be noted that $\Theta_r^s = -\Theta_e^s$.

because some consumers change from the lower quality retailer to the higher quality retailer. On the other hand, it has a negative impact as it increases the desutility costs of not buying their most-preferred product by moving the indifferent consumer further to the right. The positive impact dominates, and therefore, the first effect is positive whereas the second effect is negative. Since the consumers who change from the entrant to the incumbent's retailer benefit from an increase in quality, the first effect may dominate the second, and degradation against the entrant may increase welfare, provided that Θ is large enough. Assume now that the entrant has a net quality advantage over the incumbent's retailer, i.e., that $\Theta < 0$. Then, the first effect is as described above, but with the signs reversed, and the second effect is negative. Hence, the two effects are negative and quality degradation against the entrant decreases welfare. Exactly the same description applies for quality degradation against the incumbent's retailer.

We now turn to the case of vertical integration. The result is qualitatively similar to the case above. There are, however, the following differences on the effects of quality degradation: (**i**) there is no deadweight loss for those consumers that change from the high marginal price entrant to the incumbent, (**ii**) the number of consumers that change of retailer after quality degradation against the incumbent is larger because quality degradation affects a larger number of units, and (**iii**) the indifferent consumer is not necessarily located closer to the lower quality firm.[14]

For sufficiently high quality differences, it is welfare increasing to degrade the quality of the retailer that has the lowest relative quality. Under these circumstances, quality degradation induces consumers to switch to the highest quality retailer, where they buy a higher number of units. Interestingly, the possibility that discriminatory quality degradation may be welfare increasing is absent of most policy discussions about vertical separation, where it is usually assumed that discriminatory quality degradation reduces welfare.[15] However, the logic underlying welfare increasing quality degradation is quite transparent.

To sum up, vertical separation has a *quality degradation effect*, which may have a positive or a negative impact on welfare. If vertical separation decreases quality degradation when quality degradation is welfare decreasing, or if vertical separation increases quality degrada-

[14] This happens, for instance, i) because the entrant may have quality advantage but also charges a positive marginal price, or ii) there is the wholesale effect that leads to a higher fixed fee by the incumbent even when the quality levels are similar.

[15] A literature initiated by Bergstrom and Varian (1985) showed, in various contexts, that increasing the asymmetry among firms with respect to some parameter, e.g., costs, while keeping the average constant may increase welfare. In our context discrimination affects both the average and the variance.

tion when quality degradation is welfare increasing, then the vertical separation effect has a positive impact on welfare. Otherwise, the vertical separation effect has a negative impact on welfare.

The next Proposition brings together the *double-marginalization effect* and the *quality degradation effect*, analyzed in Lemmas 5 and 6, respectively, and presents sufficient conditions for vertical integration to be socially preferable to vertical separation.

Proposition 1: *In equilibrium, vertical separation of the wholesaler and the incumbent's retailer is socially preferable if:*

(i) *Parameter values are as defined in Corollary 2 (i).*

(ii) *Parameter values are as defined in Corollary 2 (ii).*

(iii) *Parameter values are as defined in Corollary 2 (iii) or (iv) and* Δ *is on* $(0, 1 - \Theta_e^i(0)]$. ■

When there is no quality degradation under vertical integration or separation, i.e., in the areas "$n \rightarrow n$" of Figures 1 and 2, the only effect of vertical separation is the *double marginalization effect*. Hence, the impact of vertical separation on welfare is negative.

When there is no quality degradation under vertical integration and there is quality degradation against the incumbent's retailer under vertical separation, if Δ takes low values in the area "$n \rightarrow r$" of Figure 1., i.e. $\Delta < \Theta_e^s$, the *quality degradation effect* is negative. Hence, the impact of vertical separation on welfare is negative. If Δ takes higher values, the *quality degradation effect* may be positive, since increasing θ_r may be welfare increasing for some values of θ_r. However, the *quality degradation effect* is dominated by the *double marginalization effect*, and the impact of vertical separation on welfare is negative.

When there is quality degradation against the entrant under vertical integration and there is lower or eventually no quality degradation under vertical separation, i.e., in the areas "$e \rightarrow e$" and "$e \rightarrow n$" of Figures 1 and 2, if Δ is low, the *quality degradation effect* is negative, since welfare increases with θ_e. Hence, the impact of vertical separation on welfare is negative. Otherwise, the *quality degradation effect* is positive, and the impact of vertical separation on welfare is potentially ambiguous.

To sum up, these results question the common wisdom that vertical separation is a good policy instrument to eliminate quality degradation by a wholesaler against retail entrants. Not only vertical separation is not guaranteed to eliminate quality degradation, but also the impact of quality degradation on welfare is potentially ambiguous. Recall that we have

ignored in our analysis the existence of coordination or vertical integration economies. The presence of these economies would make the case for vertical separation less compelling.

5 Discussion and Conclusions

In this article, we investigate if vertical separation reduces non-price discrimination and increases welfare.

Vertical separation of vertically integrated incumbents has been proposed to prevent various forms of discrimination against retail entrants. In particular, functional separation has been at the center of several policy discussions regarding the regulation of next generation networks. The reasoning underlying these proposals seems to be based in the three following assertions. First, the monopolist owner of a bottleneck input, which is also present in the retail market, may have the incentive and the ability to discriminate against retail entrants, to limit competition in the retail market. Second, vertical separation of the incumbent into wholesale and retail units would eliminate the wholesaler's incentives to discriminate against retail entrants. Third, eliminating discrimination would increase welfare.

Our analysis shows that while the first assertion is true, the other two assertions are false in general. There are reasons related only to profit maximization that might lead an independent wholesaler to discriminate against some types of retailers. Hence, vertical separation is not guaranteed to eliminate discrimination. In fact, under vertical separation, discrimination against entrants might persist, and there might even be discrimination against the vertically integrated firm's retailer when there was no discrimination under vertical integration. Furthermore, discrimination is not always socially undesirable. Discrimination against a high quality retailer reduces welfare. However, discrimination against a low quality retailer may increase welfare. Hence, even when vertical separation does eliminate discrimination, it is not guaranteed that it will increase welfare.

We derived our results assuming that there are no vertical integration economies and ignoring the costs of the separation process. Either of these two factors only makes vertical separation less socially desirable.

The possibility that discrimination might be socially desirable has another important consequence. Contrary to what several authors have argued, e.g., Sand (2004), the socially optimal access price with non-price discrimination may be lower than without non-price discrimination. It is true that when discrimination is socially undesirable, a high access price decreases the wholesaler's incentives to discriminate. However, when discrimination

against a low quality retailer is socially desirable, which happens if the quality asymmetry among retailers is high, the socially optimal access price may be lower than without non-price discrimination, in order to induce discrimination by the wholesaler

To be sure, there are circumstances where vertical separation can be an appropriate remedy for a competition problem. In other words, there are circumstances where vertical separation does eliminate discrimination, and in doing so increases welfare. However, there are also circumstances where vertical separation has socially undesirable effects. Hence, before adopting these market engineering measures, policy makers should make sure they understand clearly the consequences of their actions.

Appendix A

In this appendix, we present the parameters and their admissible range:

Parameter	Range	Description
α	$[\underline{\alpha}, \overline{\alpha})$	unit access price
z	$(0, +\infty)$	individual consumer demand intercept
t	$[\underline{t}, +\infty)$	unit desutility or transportation cost
Δ	$\left[0, \overline{\Delta}\right]$	entrant's quality level
β	$\left[\underline{\beta}, +\infty\right)$	discrimination cost parameter
V	$[\underline{V}, +\infty)$	consumer valuation for access

The assumption that $\alpha < \overline{\alpha} := z$ ensures that the access prices is sufficiently low so that when retailers set unit price at marginal cost, consumers will choose a strictly positive number of units.

The assumption that $\alpha > \underline{\alpha} := \frac{\left(6t-z^2\right)z}{(30t+z^2)}$ ensures that under vertical integration the wholesaler does not have incentives to fully degrade the entrant's quality and, at the same time, degrade the incumbent's quality to minimize costs. This assumption does not qualitatively change the results and limits the number of candidates for equilibria which simplifies the exposition considerably. If one excluded this assumption, one could have a situation in which there is quality degradation against the entrant under vertical integration, and there is quality degradation against the incumbent's retailer under vertical separation. See the end of the proof of Corollary 2 in appendix B.

The assumption that $t > \underline{t} := z^2/6$ ensures that even when there is full discrimination against the entrant under vertical integration, the entrant will still serve a positive number of consumers. As the desutility costs of not buying their most-preferred product are sufficiently high, some consumers will still prefer to purchase from the firm that is discriminated against. This implies that under vertical separation the entrant will also serve a positive number of consumers when being fully discriminated against.

The assumption that $\Delta < \overline{\Delta} := \overline{\Delta}(\alpha) := \frac{6t}{z^2-\alpha^2}$ ensures that even when there is full discrimination against the incumbent under vertical integration, the incumbent will still serve a positive number of consumers. As the entrant's quality advantage is sufficiently low, some consumers will still prefer to purchase from the incumbent that is discriminated against. This implies that under vertical separation the incumbent will also serve a positive number of consumers when being fully discriminated against.

The assumption that $\beta > \underline{\beta} := \frac{\max\left\{z^4, \Delta^2\left(z^2-\alpha^2\right)^2, z^2\Delta\left(z^2-\alpha^2\right)-6t\Delta(z-\alpha)(5\alpha-z), \frac{6\alpha\Delta(z-\alpha)}{\Delta(z-\alpha)(5\alpha-z)+z^2}z^4\right\}}{36}$ ensures the following: (i) the first two inequalities ensure that the wholesaler's objective

function is concave in θ_r given θ_e and vice-versa and (ii) the last two inequalities ensure that, in equilibrium, $\theta_j < 1$, $j = r, e$, i.e., that no firm will be discriminated to the extent that consumers have no valuation for its production.

The assumption that $V > \underline{V}$ ensures that the market is fully covered.

Appendix B

Proof of Lemma 1: See Biglaiser and DeGraba (2001). ■

Proof of Lemma 2: Assuming that the market is fully covered, a consumer located at x is indifferent between both firms if and only if

$$V + S_r - tx - F_r = V + S_e - t(1-x) - F_e.$$

This implies that the indifferent consumer is located at:

$$x = \frac{1}{2} + \frac{F_e - F_r + S_r - S_e}{2t} = \frac{1}{2} + \frac{2\left(F_e - F_r\right) + \left(z - p_r\right)^2\left(1 - \theta_r\right) - \left(z - p_e\right)^2\Delta\left(1 - \theta_r\right)}{4t}.$$

For the integration scenario, and according to Lemma 1, we have $p_r = 0$ and $p_e = \alpha$. Substituting this on the profit functions (2) and (3) and solving the system of first-order conditions we obtain:

$$\begin{aligned} F_r^i(\boldsymbol{\theta}; \alpha, \Delta) &= t + \frac{z^2\left(1 - \theta_r\right)}{6} + \frac{\Delta\left(1 - \theta_e\right)\left(5\alpha - z\right)\left(z - \alpha\right)}{6} \\ F_e^i(\boldsymbol{\theta}; \alpha, \Delta) &= t - \frac{z^2\left(1 - \theta_r\right)}{6} + \frac{\Delta\left(1 - \theta_e\right)\left(z - \alpha\right)\left(z + \alpha\right)}{6}, \end{aligned}$$

and $\sigma_r^i(\boldsymbol{\theta}; \alpha, \Delta) = \frac{1}{2} + \frac{z^2(1-\theta_r) - \Delta(1-\theta_e)(z-\alpha)(z+\alpha)}{12t}$. For $0 < \sigma_r^i < 1$ for all $\boldsymbol{\theta}$, we need $t > \underline{t}$ and $\Delta < \overline{\Delta}$.

For the separation scenario, we have $p_r = p_e = \alpha$. Substituting the indifferent consumer on profit functions (5) and (6) and solving the system of first-order conditions we obtain:

$$\begin{aligned} F_e^s(\boldsymbol{\theta}; \alpha, \Delta) &= t - \frac{\left(z - \alpha\right)^2\left(\left(1 - \theta_r\right) - \Delta\left(1 - \theta_e\right)\right)}{6} \\ F_r^s(\boldsymbol{\theta}; \alpha, \Delta) &= t + \frac{\left(z - \alpha\right)^2\left(\left(1 - \theta_r\right) - \Delta\left(1 - \theta_e\right)\right)}{6}, \end{aligned}$$

and $\sigma_r^s(\boldsymbol{\theta}; \alpha, \Delta) = \frac{1}{2} + \frac{(z-\alpha)^2((1-\theta_r) - \Delta(1-\theta_e))}{12t}$. The conditions for $0 < \sigma_r^i < 1$,for all $\boldsymbol{\theta}$, imply that $0 < \sigma_r^s < 1$, for all $\boldsymbol{\theta}$. ■

Proof of Lemma 3: Let $q_j = 1 - \theta_j$, $j = r, e$. Given the retail equilibrium, the integrated incumbent's profit is given by:

$$\begin{aligned}\pi_w^i(\mathbf{q}) &= \left(t + \frac{z^2 q_r}{6} + \frac{\Delta q_e (5\alpha - z)(z - \alpha)}{6}\right)\left(\frac{1}{2} + \frac{z^2 q_r - (z^2 - \alpha^2)\Delta q_e}{12t}\right) \\ &\quad + \alpha\left(\frac{1}{2} - \frac{z^2 q_r - (z^2 - \alpha^2)\Delta q_e}{12t}\right)(z - \alpha)\Delta q_e - \frac{\beta}{2t}(q_r - q_e)^2.\end{aligned}$$

The incumbent's problem is then to $\max \pi_w^i(\mathbf{q})$ subject to $q_r \leq 1$, $q_e \leq 1$, $q_r \geq 0$, $q_e \geq 0$.

The corresponding Lagrangian function is

$$L(\mathbf{q}, \lambda_r, \lambda_e) = \pi_w^i(\mathbf{q}) + \lambda_r(1 - q_r) + \lambda_e(1 - q_e)$$

and, in addition to $q_j \leq 1, q_j \geq 0, \lambda_j \geq 0$, $j = r, e$, the Kuhn-Tucker conditions are:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_j} - \lambda_j \leq 0 \tag{9}$$

$$q_j\left(\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_j} - \lambda_j\right) = 0 \tag{10}$$

$$\lambda_j(1 - q_j) = 0. \tag{11}$$

a) Consider the candidate $q_r = q_e = 1$. Then, conditions (11) holds trivially and condition (10) implies condition (9). From (10) we obtain, respectively:

$$\begin{aligned}\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} &= \frac{6t + z^2(1 - \Delta) + \Delta\alpha^2}{36t} z^2 = \lambda_r^* \\ \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} &= -\frac{6t(z - 5\alpha) + (z + \alpha)(z^2(1 - \Delta) + \Delta\alpha^2)}{36t}(z - \alpha)\Delta = \lambda_e^*.\end{aligned}$$

Both λ_r^* and λ_e^* are non negative if and only if $\Delta \leq \frac{6t + z^2}{z^2 - \alpha^2}$ and $\Delta \geq \Delta^i(\alpha) := 1 + \frac{1}{z^2 - \alpha^2}\left(\alpha^2 - \frac{5\alpha - z}{z + \alpha} 6t\right)$, respectively.

The first condition holds trivially given our assumption that $\Delta < \overline{\Delta}(\alpha)$. With respect to the second, note that $\Delta^i(\alpha) \leq 0$ if and only if $\alpha \geq \alpha_2^i := \frac{6tz + z^3}{30t - z^2}$ and that $\Delta^i(\alpha) > \overline{\Delta}(\alpha)$ if and only if $\alpha < \alpha_1^i := \frac{z^3}{36t - z^2}$.[16] Thus, $q_r = q_e = 1$ verifies the necessary conditions for a maximum when $\alpha \geq \alpha_2^i$ or when $\alpha_1^i < \alpha < \alpha_2^i$ and $\Delta^i(\alpha) < \Delta < \overline{\Delta}(\alpha)$.

b) Consider the candidate $q_r = 1$ and $0 < q_e < 1$. Then, $\lambda_e = 0$, and conditions (9) and (10) become, respectively:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} - \lambda_r = 0 \text{ and } \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} = 0,$$

[16] By definition, $\overline{\Delta}(\alpha_1^i) = \Delta^i(\alpha_1^i)$. Inspection of $\overline{\Delta}$ reveals that it is an increasing function of α. Furthermore, $\Delta^i(\alpha)$ is decreasing in α.

which yield

$$\begin{aligned} q_e^* &= \frac{36\beta - z^2\Delta\left(z^2-\alpha^2\right) + 6t\Delta\left(z-\alpha\right)\left(5\alpha - z\right)}{36\beta - \Delta^2\left(z+\alpha\right)^2\left(z-\alpha\right)^2} \\ \lambda_r^* &= \frac{q_e^*\left(36\beta - \Delta z^2\left(z^2-\alpha^2\right)\right) + z^2\left(6t+z^2\right) - 36\beta}{36t}. \end{aligned}$$

Clearly, $q_e^* > 0$ because, given our assumption that β is on $\left(\underline{\beta}, +\infty\right)$, the numerator and denominator are both positive. Also, $q_e^* < 1 \Leftrightarrow \Delta < \Delta^i(\alpha)$.

As $\frac{\partial q_e^*}{\partial \beta} > 0 \Leftrightarrow \Delta < \Delta^i(\alpha)$ and $\frac{\partial \lambda_r^*}{\partial q_e^*} = \frac{36\beta - \Delta z^2\left(z^2-\alpha^2\right)}{36t} > 0$ a sufficient condition for $\lambda_r^* \geq 0$ is that $\frac{\frac{0 - z^2\Delta\left(z^2-\alpha^2\right) + 6t\Delta(z-\alpha)(5\alpha-z)}{0-\Delta^2(z+\alpha)^2(z-\alpha)^2}\left(36\beta - \Delta z^2(z-\alpha)(z+\alpha)\right) + z^2\left(6t+z^2\right) - 36\beta}{36t} > 0 \Leftrightarrow \alpha\Delta z^2 t\left(z-\alpha\right)\left(z+\alpha\right) + \beta\left(6t\left(z-5\alpha\right) + z^2\left(z+\alpha\right) - \Delta\left(z-\alpha\right)\left(z+\alpha\right)^2\right) > 0$.

But this is implied by $6t\left(z-5\alpha\right) + z^2\left(z+\alpha\right) - \Delta\left(z-\alpha\right)\left(z+\alpha\right)^2 > 0 \Leftrightarrow \Delta < \Delta^i(\alpha)$. Thus, $q_e < 1 = q_r$ is a candidate when $\alpha < \alpha_1^i$ or when $\alpha_1^i < \alpha < \alpha_2^i$ and $\Delta < \Delta^i(\alpha)$.

c) Consider the candidate $q_e = 1$ and $0 < q_r < 1$. Then, $\lambda_r = 0$, and conditions (9) and (10) become, respectively:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} = 0 \text{ and } \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} - \lambda_e = 0,$$

from where we obtain,

$$\begin{aligned} q_r^* &= \frac{36\beta + 6tz^2 - \Delta z^2\left(z^2-\alpha^2\right)}{36\beta - z^4} \\ \lambda_e^* &= \frac{\beta\left(6t\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right) + \left(z^2 - z^2\Delta + \Delta\alpha^2\right)^2\right) - \alpha\Delta z^4 t\left(z-\alpha\right)}{t\left(36\beta - z^4\right)}. \end{aligned}$$

Clearly $q_r^* > 0$ given our assumption on β. In order to have $q_r^* < 1$ we need that $36\beta + 6tz^2 - \Delta z^2\left(z^2-\alpha^2\right) < 36\beta - z^4 \Leftrightarrow \Delta > \frac{6t+z^2}{\left(z^2-\alpha^2\right)}$ which violates our assumption on Δ.

d) Consider all the candidates with $q_r = 0$. Then $\lambda_r = 0$ and it must be that:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} \leq 0 \Leftrightarrow q_e\left(36\beta - \Delta z^2\left(z^2-\alpha^2\right)\right) + 6tz^2 \leq 0,$$

which is impossible. If $\left(36\beta - \Delta z^2\left(z^2-\alpha^2\right)\right) > 0$ it is trivial. If $36\beta - \Delta z^2\left(z^2-\alpha^2\right) < 0$ we have that $q_e\left(36\beta - \Delta z^2\left(z^2-\alpha^2\right)\right) + 6tz^2 > 1\left(36\beta - \Delta z^2\left(z^2-\alpha^2\right)\right) + 6tz^2 = 36\beta + z^2\left(6t - \Delta\left(z^2-\alpha^2\right)\right) > 0$.

e) Consider the candidate $q_e = 0$ and $0 < q_r < 1$. Then $\lambda_r = \lambda_e = 0$ and it must be that:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} = 0 \text{ and } \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} \leq 0,$$

from where we obtain:

$$\begin{aligned} q_r^* &= \frac{6tz^2}{36\beta - z^4} < 1 \\ 6\beta\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right) - \alpha\Delta z^4\left(z-\alpha\right) &\leq 0. \end{aligned}$$

A sufficient condition for $6\beta\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right)-\alpha\Delta z^4\left(z-\alpha\right)>0$ is that $z^2+\Delta\left(z-\alpha\right)\left(5\alpha-z\right)>0$ and $\beta>\frac{\alpha\Delta z^4(z-\alpha)}{6(\Delta(z-\alpha)(5\alpha-z)+z^2)}$, which is true, given our assumption that β is on $\left(\underline{\beta},+\infty\right)$. In turn, sufficient conditions for $\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right)>0$ are that $\left(5\alpha-z\right)>0$ or $\left(5\alpha-z\right)<0$ and $\Delta<\frac{-z^2}{(z-\alpha)(5\alpha-z)}$. But this is always true because $\frac{6t}{z^2-\alpha^2}\leq\frac{-z^2}{(5\alpha-z)(z-\alpha)}\Leftrightarrow\alpha\geq\underline{\alpha}$.[17]

f) Consider the candidate $q_e=0$ and $q_r=1$. Then $\lambda_e=0$ and it must be that:

$$\left(\frac{\partial\pi_w^i(\mathbf{q})}{\partial q_r}-\lambda_r\right)=0\text{ and }\frac{\partial\pi_w^i(\mathbf{q})}{\partial q_e}\leq0,$$

from where we obtain

$$\begin{aligned}\lambda_r^* &= \frac{-36\beta+6tz^2+z^4}{36t}\\ \left(36\beta-\Delta\left(z-\alpha\right)6t\left(z-5\alpha\right)-\Delta\left(z-\alpha\right)z^2\left(z+\alpha\right)\right) &\leq 0.\end{aligned}$$

The last inequality violates our assumption on β.

g) Consider the candidate $0<q_e<1$ and $0<q_r<1$. Then $\lambda_r=\lambda_e=0$ and

$$\frac{\partial\pi_w^i(\mathbf{q})}{\partial q_j}=0,$$

from where we obtain

$$\begin{aligned}q_e^* &= \frac{t\left(\alpha\Delta z^4\left(z-\alpha\right)-6\beta\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right)\right)}{\beta\left(z^2-z^2\Delta+\Delta\alpha^2\right)^2}\\ q_r^* &= \frac{t\left(\alpha\Delta^2z^2\left(z+\alpha\right)\left(z-\alpha\right)^2-6\beta\left(\Delta\left(z-\alpha\right)\left(5\alpha-z\right)+z^2\right)\right)}{\beta\left(z^2-z^2\Delta+\Delta\alpha^2\right)^2}.\end{aligned}$$

Our assumption on β implies $q_e^*<0$, which is impossible.

Finally, we compare the threshold $\underline{\alpha}$, with α_1^i and α_2^i.

Let $\underline{\alpha}:=\frac{\left(6-x^2\right)x}{(30+x^2)}\sqrt{t}$, where $x=\frac{z}{\sqrt{t}}$.

i) $\alpha_1^i:=\frac{z^3}{36t-z^2}=\sqrt{t}\frac{x^3}{36-x^2}$. Therefore $\alpha_1^i-\underline{\alpha}=72\left(x^2+30\right)^{-1}\left(x-6\right)^{-1}\left(x+6\right)^{-1}\left(3-x^2\right)x$. Hence, $\alpha_1^i<\underline{\alpha}$ if and only if $x<\sqrt{3}$.

ii) $\alpha_2^i:=\frac{6tz+z^3}{30t-z^2}=\sqrt{t}\frac{6x+x^3}{30-x^2}$. Therefore $\alpha_2^i-\underline{\alpha}=72\left(x^2+30\right)^{-1}\left(30-x^2\right)^{-1}x^3>0$. Hence, $\alpha_2^i>\underline{\alpha}$ for all parameter values. ■

Proof of Lemma 4. Let $q_j=1-q_j$, $j=r,e$. Given the retail equilibrium, the incumbent's wholesale unity profit is given by:

$$\pi_w^s=\alpha\left(z-\alpha\right)\left(q_r\left(\frac{1}{2}+\frac{\left(z-\alpha\right)^2\left(q_r-\Delta q_e\right)}{12t}\right)+\Delta q_e\left(\frac{1}{2}-\frac{\left(z-\alpha\right)^2\left(q_r-\Delta q_e\right)}{12t}\right)\right)-\frac{\beta}{2t}(q_r-q_e)^2.$$

[17] We have: $\frac{\partial\underline{\alpha}}{\partial z}=-\left(30t+z^2\right)^{-2}\left(z^4-180t^2+96tz^2\right)$. Moreover, $\frac{\partial^2\underline{\alpha}}{\partial z^2}=72\left(30t+z^2\right)^{-3}\left(z^2-90t\right)tz<0$. Hence, $\underline{\alpha}$ is maximized at $z^2=\left(6\sqrt{69}-48\right)t$ and the maximum is $\underline{\alpha}^{\max}=0.177\,23\sqrt{t}$. A sufficient condition for $\alpha>\underline{\alpha}$ is then $\alpha>0.177\,23\sqrt{t}$.

The Kuhn-Tucker conditions are the same as in Lemma 3.

a) $q_r = q_e = 1$. Then the conditions (9) and (10) become, respectively:

$$\begin{aligned}\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} &= \frac{\alpha(z-\alpha)\left((z-\alpha)^2(1-\Delta)+3t\right)}{6t} = \lambda_r^* \\ \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} &= \frac{\alpha\Delta(z-\alpha)\left((z-\alpha)^2(\Delta-1)+3t\right)}{6t} = \lambda_e^*.\end{aligned}$$

Both λ_r^* and λ_e^* are non negative if and only if $\Delta < \Delta_r^s(\alpha) := 1 + \frac{3t}{(z-\alpha)^2}$ and $\Delta > \Delta_e^s(\alpha) := 1 - \frac{3t}{(z-\alpha)^2}$, respectively.

Inspection of the functions shows that $\Delta_r^s(\alpha)$ is increasing in α and that $\Delta_e^s(\alpha)$ is decreasing in α. Therefore, we have $\Delta_e^s(\alpha) > 0$ if and only if $\alpha < \alpha_1^s := z - \sqrt{3t}$. Moreover, since $\Delta_e^s(0) < \overline{\Delta}(0)$, we conclude that $\Delta_e^s(\alpha) < \overline{\Delta}(\alpha)$ for all α.

Note that $\overline{\Delta}(\alpha) - \Delta_r^s(\alpha) = -1 - \frac{3(3\alpha - z)t}{(z-\alpha)^2(z+\alpha)}$ Clearly, $\frac{\partial(\overline{\Delta}(\alpha) - \Delta_r^s(\alpha))}{\partial \alpha} = -\frac{\left(z^2+3\alpha^2\right)6t}{(z-\alpha)^3(z+\alpha)^2} < 0$ and $\overline{\Delta}(0) - \Delta_r^s(0) = \frac{3t}{z^2} - 1$ and $\lim_{\alpha \to z}\left[\overline{\Delta}(\alpha) - \Delta_r^s(\alpha)\right] = -\infty$. Thus, if $\frac{3t}{z^2} - 1 < 0 \Leftrightarrow z > \sqrt{3t}$ we have that $\overline{\Delta}(\alpha) < \Delta_r^s(\alpha)$ for all α. Otherwise, there exists an α_2^s on $[0, z]$ such that for α on $[0, \alpha_2^s]$ we have $\overline{\Delta}(\alpha) \geq \Delta_r^s(\alpha)$.

Thus, $q_r = q_e = 1$ is a candidate when $\alpha \geq \max\{\alpha_1^s, \alpha_2^s\}$, or $\alpha \in (\alpha_1^s, \alpha_2^s)$ and $\Delta \in (0, \Delta_r^s(\alpha)]$, or $\alpha \in (\alpha_2^s, \alpha_1^s)$ and $\Delta \in \left[\Delta_e^s(\alpha), \overline{\Delta}(\alpha)\right)$.

b) $q_r = 1$ and $0 < q_e < 1$. Then, $\lambda_e^* = 0$, and conditions (9) and (10) become, respectively:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} - \lambda_r = 0 \text{ and } \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} = 0,$$

which yield

$$\begin{aligned}\lambda_r^* &= \frac{\left(\alpha(\alpha - z)\left((z-\alpha)^2(\Delta q_e^* - 1) - 3t\right) - 6\beta(1 - q_e^*)\right)}{6t} \\ q_e^* &= \frac{6\beta - \alpha\Delta(z-\alpha)\left((z-\alpha)^2 - 3t\right)}{6\beta - \alpha\Delta^2(z-\alpha)^3}.\end{aligned}$$

Clearly $q_e^* > 0$ and $\lambda_r^* \geq 0 \Leftrightarrow q_e \geq \frac{3\alpha t(z-\alpha) + \alpha(z-\alpha)^3 - 6\beta}{\alpha\Delta(z-\alpha)^3 - 6\beta}$. Hence, we must have:

(i) $q_e^* < 1 \Leftrightarrow \Delta < \Delta_e^s(\alpha)$.

(ii) $q_e^* \geq \frac{\alpha 3t(z-\alpha) + \alpha(z-\alpha)^3 - 6\beta}{\alpha\Delta(z-\alpha)^3 - 6\beta} \Leftrightarrow 6\beta \geq \frac{6\alpha\Delta^2 t(z-\alpha)^3}{3t(\Delta+1) + (z-\alpha)^2(\Delta-1)^2}$. From the second order conditions we have $6\beta > \alpha\Delta^2(z-\alpha)^3$, which is implied by our assumptions that β is on $\left(\underline{\beta}, +\infty\right)$. Note that $\alpha\Delta^2(z-\alpha)^3 > \frac{6\alpha\Delta^2 t(z-\alpha)^3}{t(3\Delta+3) + (z-\alpha)^2(\Delta-1)^2} \Leftrightarrow \left(3t + (z-\alpha)^2(\Delta-1)\right)(\Delta-1) \geq 0$. If $\Delta \geq 1$ this is always true. If $\Delta < 1$ the condition becomes $\Delta < \Delta_e^s(\alpha)$.

Thus, $q_e < 1 = q_r$ is a candidate when $0 < \Delta < \Delta_e^s(\alpha)$ which is possible if $\alpha < \alpha_1^s$.

c) $q_e = 1$ and $0 < q_r < 1$. Then, $\lambda_r^* = 0$, and conditions (9) and (10) become, respectively:

$$\frac{\partial \pi_w^i(\mathbf{q})}{\partial q_r} = 0 \text{ and } \frac{\partial \pi_w^i(\mathbf{q})}{\partial q_e} - \lambda_e = 0,$$

which yield

$$\begin{aligned}\lambda_e^* &= \frac{\left(\alpha\Delta\left(z-\alpha\right)\left(3t+\left(\alpha-z\right)^2\left(\Delta-q_i^*\right)\right)+6\beta\left(q_i^*-1\right)\right)}{6t}\\ q_r^* &= \frac{6\beta-\alpha\left(z-\alpha\right)\left(\Delta\left(z-\alpha\right)^2-3t\right)}{6\beta-\alpha\left(z-\alpha\right)^3}.\end{aligned}$$

Clearly $q_r^*>0$ and $\lambda_e^*\geq 0 \Leftrightarrow q_r \geq \frac{\alpha\Delta(z-\alpha)\left(3t+\Delta(z-\alpha)^2\right)-6\beta}{\alpha\Delta(z-\alpha)^3-6\beta}$. Hence, we must have:

(i) $q_r^*<1 \Leftrightarrow \Delta>\Delta_r^s(\alpha)$.

(ii) $q_r^*\geq \frac{\alpha\Delta(z-\alpha)\left(3t+\Delta(z-\alpha)^2\right)6\beta}{\alpha\Delta(z-\alpha)^3-6\beta} \Leftrightarrow 6\beta \geq \frac{6\alpha\Delta t(z-\alpha)^3}{3t(\Delta+1)+(\alpha-z)^2(\Delta-1)^2}$. From the second order conditions we have $6\beta>\alpha\Delta\left(z-\alpha\right)^3$, which is implied by our assumptions that β is on $\left(\underline{\beta},+\infty\right)$. Note that $\alpha\Delta\left(z-\alpha\right)^3 > \frac{6\alpha\Delta t(z-\alpha)^3}{3t(\Delta+1)+(\alpha-z)^2(\Delta-1)^2} \Leftrightarrow \left(3t+\left(\alpha-z\right)^2\left(\Delta-1\right)\right)\left(\Delta-1\right)>0$. If $\Delta>1$ this is always true. If $\Delta<1$ the condition becomes $\Delta>\Delta_r^s(\alpha)$.

Thus, $q_r<1=q_e$ is a candidate when $\alpha<\alpha_2^s$ and $\Delta_r^s(\alpha)<\Delta<\overline{\Delta}(\alpha)$.

d) Consider all the candidates with $q_r=0$. Then $\lambda_r=0$ and it must be that

$$\frac{\partial\pi_w^s(\mathbf{q})}{\partial q_r}\leq 0,$$

or $3\alpha t\left(z-\alpha\right)+q_e\left(6\beta-\alpha\Delta\left(z-\alpha\right)^3\right)\leq 0$, which is impossible.

e) Consider all the candidates with $q_e=0$. Then $\lambda_e=0$ and it must be that

$$\frac{\partial\pi_w^s(\mathbf{q})}{\partial q_e}\leq 0,$$

or $3\alpha\Delta t\left(z-\alpha\right)+q_r 6\beta-\alpha\Delta\left(z-\alpha\right)^3\leq 0$, which is impossible.

f) Consider the candidate $0<q_e<1$ and $0<q_r<1$. Then $\lambda_r=\lambda_e=0$ and

$$\frac{\partial\pi_w^s(\mathbf{q})}{\partial q_j}=0,$$

from where we obtain

$$\begin{aligned}q_r &= \frac{t\left(\Delta^2\alpha\left(z-\alpha\right)^3-3\left(\Delta+1\right)\beta\right)}{\beta\left(\Delta-1\right)^2\left(z-\alpha\right)^2}\\ q_e &= \frac{t\left(\Delta\alpha\left(z-\alpha\right)^3-3\left(\Delta+1\right)\beta\right)}{\beta\left(\Delta-1\right)^2\left(z-\alpha\right)^2}.\end{aligned}$$

Given our assumption that β is on $\left(\underline{\beta},+\infty\right)$, we always have $q_j<0$.

Finally, we compare the threshold $\underline{\alpha}$, with α_1^s and α_2^s.

i) $\alpha_1^s := z-\sqrt{3t}=\sqrt{t}(x-\sqrt{3})$. Therefore $\alpha_1^s-\underline{\alpha}=\left(x^2+30\right)^{-1}\left(24x+2x^3-30\sqrt{3}-x^2\sqrt{3}\right)$. Hence, $\alpha_1^s>\underline{\alpha}$ if and only if $x>1.871\,6$.

ii) $\alpha_2^s := \frac{1}{3}x+\sqrt[3]{\sqrt{\frac{16}{9}x^4-12x^2+27}-\frac{8}{27}x^3}-\frac{3-\frac{4}{9}x^2}{\sqrt[3]{\sqrt{\frac{16}{9}x^4-12x^2+27}-\frac{8}{27}x^3}}$. We can show numerically that, $\alpha_2^s>\underline{\alpha}$ if and only if $x<1.495\,9$. ■

Proof of Corollary 1: We need to compare $\Delta^i(\alpha)$ and $\Delta_e^s(\alpha)$. Recall that both are decreasing function of α with $\Delta_e^s(0) < \Delta^i(0)$ and that $\Delta_e^s(z-\sqrt{3t}) = 0$ and $\Delta^i(\frac{6tz+z^3}{30t-z^2}) = 0$.

Let

$$\Delta^i(\alpha) - \Delta_e^s(\alpha) = \frac{3\left(3(\frac{z}{\sqrt{t}})^2 - 10\frac{z}{\sqrt{t}}\frac{\alpha}{\sqrt{t}} + 11\frac{\alpha^2}{t}\right) + \frac{\alpha^2}{t}\left((\frac{z}{\sqrt{t}})^2 - \frac{\alpha^2}{t}\right)}{(z+\alpha)^2(z-\alpha)^2}t^2.$$

The sign of $\Delta^i(\alpha) - \Delta_e^s(\alpha)$ depends only on the sign of the numerator which is a U-shaped parabola in $x = \frac{z}{\sqrt{t}}$ with no real roots, given the assumption that $t > \underline{t}$. Hence, $\Delta^i(\alpha) > \Delta_e^s(\alpha)$ for all α on $(0, z)$.

Moreover, when there is quality degradation against the entrant under both scenarios, i.e. when $\Delta < \Delta_e^s(\alpha)$, we have:

$$\begin{aligned}\theta_e^i - \theta_e^s \;=\; & \frac{(z-\alpha)\Delta}{\left(36\beta - \Delta^2(\alpha-z)^2(z+\alpha)^2\right)\left(6\beta - \alpha\Delta^2(z-\alpha)^3\right)} \times \\ & \left(6\beta\left(-6t(2\alpha - z) + \left(z^3 - 6\alpha^3 + 12z\alpha^2 - 5z^2\alpha\right) - \Delta(z-\alpha)\left(z^2 - 4z\alpha + 7\alpha^2\right)\right)\right. \\ & \left.-\alpha\Delta^2(z-\alpha)^2\left(3t\left(3z^2 - 10z\alpha + 11\alpha^2\right) + \alpha^2\left(z^2 - \alpha^2\right)\right)\right).\end{aligned}$$

This is positive if the numerator is positive. It can be shown that the sign of the coefficient of β is positive. Hence, $\theta_e^i - \theta_e^s > 0 \Longleftrightarrow \beta > \widetilde{\beta}(\Delta) := \frac{\alpha\Delta^2(\alpha-z)^2\left(3t\left(3z^2-10z\alpha+11\alpha^2\right)+\alpha^2\left(z^2-\alpha^2\right)\right)}{6(-6t(2\alpha-z)+(z^3-6\alpha^3+12z\alpha^2-5z^2\alpha)-\Delta(z-\alpha)(z^2-4z\alpha+7\alpha^2))}$. This is increasing on Δ and it can be shown that $\widetilde{\beta}(\Delta_e^s(\alpha)) < z^4/36$, thus $\widetilde{\beta}(\Delta) < \underline{\beta}$ and $\theta_e^i > \theta_e^s$ is always true. ■

Proof of Corollary 2:

(i) *There is no quality degradation under both vertical integration and separation, if and only if,* (α, Δ) *is on* $\left[\max\{\underline{\alpha}, \alpha_1^i\}, z\right) \times \left[\max\{\Delta^i(\alpha), 0^+\}, \min\left\{\Delta_r^s(\alpha)^+, \overline{\Delta}(\alpha)\right\}\right)$.

From Lemma 3 and 4, this happens when the following conditions hold simultaneously:

$$\begin{aligned}\Delta^i(\alpha) &\leq \Delta < \overline{\Delta}(\alpha) \\ \Delta_e^s(\alpha) &\leq \Delta \leq \Delta_r^s(\alpha).\end{aligned}$$

The first inequality is only possible if $\alpha > \max\{\alpha_1^i, \underline{\alpha}\}$ and the second is always possible since $\Delta_r^s(\alpha) > \max\{0, \Delta_e^s(\alpha)\}$.

(ii) *There is no quality degradation under vertical integration, and there is quality degradation against the incumbent's retailer under vertical separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \alpha_2^s) \times \left[\max\{\Delta^i(\alpha), \Delta_r^s(\alpha)^+\}, \overline{\Delta}(\alpha)\right)$.

From Lemma 3 and 4, this happens when the following conditions hold simultaneously:

$$\begin{aligned}\Delta^i(\alpha) &\leq \Delta < \overline{\Delta}(\alpha) \\ \Delta_r^s(\alpha) &< \Delta < \overline{\Delta}(\alpha).\end{aligned}$$

The first inequality is only possible if $\alpha > \max\{\alpha_1^i, \underline{\alpha}\}$ and the second one if $\alpha < \alpha_2^s$. Hence, we need that $\max\{\alpha_1^i, \underline{\alpha}\} < \alpha < \alpha_2^s$.

Let $z < \sqrt{3t}$. Then $\max\{\alpha_1^i, \underline{\alpha}\} = \underline{\alpha}$ and we need that $\alpha_2^s > \underline{\alpha}$ which is true if and only if $z < 1.4959\sqrt{t}$ as shown in the Proof of Lemma 4. Let $z > \sqrt{3t}$. Then $\max\{\alpha_1^i, \underline{\alpha}\} = \alpha_1^i$ and we need that $\alpha_2^s > \alpha_1^i$ which is true if and only if $z < 1.589\,0\sqrt{t}$, which is impossible.[18] Hence, this case occurs for $z < 1.4959\sqrt{t}$, $\underline{\alpha} < \alpha < \alpha_2^s$ and $\max\{\Delta^i(\alpha), \Delta_r^s(\alpha)\} < \Delta < \overline{\Delta}(\alpha)$. Note that $\Delta_r^s(\alpha) > 0$.

(iii) *There is quality degradation against the entrant under vertical integration, and there is no quality degradation under vertical separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \alpha_2^i) \times \left[\max\{\Delta_e^s(\alpha), 0^+\}, \min\left\{\Delta^i(\alpha), \Delta_r^s(\alpha)^+, \overline{\Delta}(\alpha)\right\}\right)$.

From Lemma 3 and 4, this happens when the following conditions hold simultaneously:

$$\begin{aligned} 0 &< \Delta < \Delta^i(\alpha) \\ \Delta_e^s(\alpha) &\leq \Delta \leq \Delta_r^s(\alpha). \end{aligned}$$

The first inequality is only possible if $\underline{\alpha} < \alpha < \alpha_2^i$. It is always true that $\underline{\alpha} < \alpha_2^i$. The second inequality is always possible as $\Delta_r^s(\alpha) > \max\{0, \Delta_e^s(\alpha)\}$ for all parameter values. As $\Delta_r^s(\alpha) > 0$ these intervals overlap provided that $\Delta^i(\alpha) > \max\{0, \Delta_e^s(\alpha)\}$ which is always true.

(iv) *There is quality degradation against the entrant under both vertical integration and separation, if and only if,* (α, Δ) *is on* $[\underline{\alpha}, \alpha_1^s) \times (0, \Delta_e^s(\alpha))$.

From Lemma 3 and 4, this happens when the following conditions hold simultaneously:

$$\begin{aligned} 0 &< \Delta < \Delta^i(\alpha) \\ 0 &< \Delta < \Delta_e^s(\alpha). \end{aligned}$$

The first inequality is only possible if $\underline{\alpha} < \alpha < \alpha_2^i$. It is always true that $\underline{\alpha} < \alpha_2^i$. The second inequality is only possible if $\underline{\alpha} < \alpha < \alpha_1^s$ which is possible if and only if $z > 1.8716\sqrt{t}$ as shown in the Proof of Lemma 4. Note that $\Delta^i(\alpha) > \max\{0, \Delta_e^s(\alpha)\}$ and $\alpha_1^s < \alpha_2^i$ are always true.

(v) *(Proof of claim in appendix A:) There is quality degradation against the entrant under vertical integration, and there is quality degradation against the incumbent's retailer under vertical separation, if* (α, Δ) *is on* $(0, \min\{\alpha_3^i, \alpha_2^s\}) \times \left(\Delta_r^s(\alpha), \min\left\{\Delta^i(\alpha), \overline{\Delta}(\alpha)\right\}\right)$.

[18] Note that $\Delta_r^s(\alpha_1^i) - \overline{\Delta}(\alpha_1^i) = \frac{12t^4 z\left(13\frac{z^6}{t^3} - 11\,664 - 513\frac{z^4}{t^2} + 5832\frac{z^2}{t}\right)}{(36t - z^2)^3}$. In the relevant range for z this is positive if and only if $z > 1.589\,0\sqrt{t}$. Hence, for $z > 1.589\,0\sqrt{t}$ we have $\alpha_2^s < \alpha_1^i$.

From Lemma 3 and 4, this happens when the following conditions hold simultaneously

$$\begin{aligned} 0 &< \Delta < \Delta^i(\alpha) \\ \Delta_r^s(\alpha) &< \Delta < \overline{\Delta}(\alpha). \end{aligned}$$

The first inequality is only possible if $\underline{\alpha} < \alpha < \alpha_2^i$. It is always true that $\underline{\alpha} < \alpha_2^i$. The second inequality is only possible if $\underline{\alpha} < \alpha < \alpha_2^s$. Furthermore, the two intervals overlap if and only if $\Delta^i(\alpha) > \Delta_r^s(\alpha)$ or $\alpha < \alpha_3^i$, with $\alpha_3^i < \alpha_2^i$. As $\underline{\alpha} > \alpha_3^i$, this case is impossible.[19] Note however, that if we had not restricted α to be larger than $\underline{\alpha}$, we would have discrimination against the entrant under integration in all the cases where we have with the restriction on α. This happens because the restriction only rules out a candidate for local maximum where there is discrimination against the entrant under integration. ■

Proof of Lemma 5: We show that $W^s\left(\theta_r^s, \theta_e^s\right) - W^i\left(\theta_r^i, \theta_e^i\right) < 0$ for $\theta_r^s = \theta_r^i = \theta_r$ and $\theta_e^s = \theta_e^i = \theta_e$:

$$\begin{aligned} W^s - W^i = &\frac{5\left(z^2\left(1-\theta_r\right) - \left(z^2-\alpha^2\right)\Delta\left(1-\theta_e\right)\right)^2}{144t} + \frac{z^2\left(1-\theta_r\right) + \left(z^2-\alpha^2\right)\Delta\left(1-\theta_e\right)}{4} \\ &- \frac{\left(5z+7\alpha\right)\left(z-\alpha\right)^3}{144t}\left(\Delta\left(1-\theta_e\right) - \left(1-\theta_r\right)\right)^2 - \frac{\left(z^2-\alpha^2\right)}{4}\left(\left(1-\theta_r\right) + \Delta\left(1-\theta_e\right)\right). \end{aligned}$$

This is an inverted U-shaped parabola with no real roots. Thus $W^S - W^I < 0$ if $(432t\alpha + 288tz - 5\alpha\left(1-\theta_r\right)\left(5z+7\alpha\right)\left(z-\alpha\right)) > 0$. We know that $432t\alpha + 288tz - 5\alpha\left(1-\theta_r\right)\left(5z+7\alpha\right)\left(z-\alpha\right) > 432\frac{z^2}{6}\alpha + 288\frac{z^2}{6}z - 5\alpha\left(1-\theta_r\right)\left(5z+7\alpha\right)\left(z-\alpha\right) = 24z^2\left(2z+3\alpha\right) - 5\alpha\left(1-\theta_r\right)\left(5z+7\alpha\right)\left(z-\alpha\right) > 24z^2\left(2z+3\alpha\right) - 5\alpha\left(5z+7\alpha\right)\left(z-\alpha\right) = 35\alpha^3 - 10z\alpha^2 + 47z^2\alpha + 48z^3 > 0$ for $\alpha < z$. ■

Proof of Lemma 6: Start by noting that:

$$\begin{aligned} \frac{\partial W^s\left(\boldsymbol{\theta};\alpha,\Delta\right)}{\partial\theta_e} &> 0 \Leftrightarrow \Theta > \Theta_e^s := \frac{18t\left(z+\alpha\right)}{\left(5z+7\alpha\right)\left(z-\alpha\right)^2} \\ \frac{\partial W^s\left(\boldsymbol{\theta};\alpha,\Delta\right)}{\partial\theta_r} &> 0 \Leftrightarrow \Theta < \Theta_r^s := -\frac{18t\left(z+\alpha\right)}{\left(5z+7\alpha\right)\left(z-\alpha\right)^2} = -\Theta_e^s, \end{aligned}$$

and that

$$\begin{aligned} \frac{\partial W^i\left(\boldsymbol{\theta};\alpha,\Delta\right)}{\partial\theta_e} &> 0 \Leftrightarrow \Theta > \Theta_e^i(\theta_r) := \frac{-\alpha^2}{z^2-\alpha^2}\left(1-\theta_r\right) + \frac{18t}{5\left(z^2-\alpha^2\right)} \\ \frac{\partial W^i\left(\boldsymbol{\theta};\alpha,\Delta\right)}{\partial\theta_r} &> 0 \Leftrightarrow \Theta < \Theta_r^i(\theta_r) := \frac{-\alpha^2}{z^2-\alpha^2}(1-\theta_r) - \frac{18t}{5\left(z^2-\alpha^2\right)}. \end{aligned}$$

[19] α_3^i is implicitly defined by $\Delta^i(\alpha_3^i) - \Delta_r^s(\alpha_3^i) = 1 + \frac{1}{z^2-\alpha^2}\left(\alpha^2 - \frac{5\alpha-z}{z+\alpha}6t\right) - \left(1 + \frac{3t}{(z-\alpha)^2}\right) = 0$. We need to check the value of $\Delta^i(\underline{\alpha}) - \Delta_r^s(\underline{\alpha}) = 144\left(30t+z^2\right)^{-4}\left(z^8 - 24\,300t^4 + 36tz^6 + 3996t^3z^2 + 729t^2z^4\right)tz^2$. Note that $x^8 - 24\,300 + 36x^6 + 3996x^2 + 729x^4 < 0$ if and only if $x < 1.8630$. Hence, it is always true that $\underline{\alpha} > \alpha_3^i$ for $z < \sqrt{3t}$, i.e., for all cases in which α_3^i is relevant.

We have $\frac{\partial W^s}{\partial \theta_e} < 0$ for $\theta_r = 0$ and every θ_e on $[0,1]$ if $1 < \Theta_e^s$, and $\frac{\partial W^s}{\partial \theta_e} > 0$ for $\theta_r = 0$ and every θ_e on $[0,1]$ if $\Delta < 1 - \Theta_e^s$. Furthermore, $\frac{\partial W^s}{\partial \theta_r} < 0$ for $\theta_e = 0$ and every θ_r on $[0,1]$ if $\Delta < \Theta_e^s$, and $\frac{\partial W^s}{\partial \theta_e} > 0$ for $\theta_e = 0$ and every θ_r on $[0,1]$ if $\Delta > 1 + \Theta_e^s$.

We also have $\frac{\partial W^i}{\partial \theta_e} > 0$ for $\theta_r = 0$ and every θ_e on $[0,1]$ if $\Delta < 1 - \Theta_e^i(0) = \frac{z^2}{z^2-\alpha^2} - \frac{18t}{5(z^2-\alpha^2)}$, and $\frac{\partial W^i}{\partial \theta_e} < 0$ for $\theta_r = 0$ and every θ_e on $[0,1]$ if $1 - \Theta_e^i(0) < 0 \Leftrightarrow \frac{z^2}{z^2-\alpha^2} - \frac{18t}{5(z^2-\alpha^2)} < 0$. Finally, $\frac{\partial W^i}{\partial \theta_r} > 0$ for $\theta_e = 0$ if and every θ_r on $[0,1]$ if $\Delta > 1 - \Theta_r^i(0) = \frac{z^2}{z^2-\alpha^2} + \frac{18t}{5(z^2-\alpha^2)}$ and $\frac{\partial W^i}{\partial \theta_r} < 0$ for $\theta_e = 0$ if and every θ_r on $[0,1]$ if $\Delta < -\Theta_r^i(1) = \frac{18t}{5(z^2-\alpha^2)}$. ■

Proof of Proposition 1:

(i) Assume there is no discrimination under both vertical integration and separation. Separation results in a change in welfare given by $W^s(0,0) - W^i(0,0)$. But, from Lemma 5, $W^s(0,0) - W^i(0,0) < 0$.

(ii) Assume there is no discrimination under vertical integration, and there is discrimination against the incumbent's retailer under vertical separation. Separation results in a change in welfare given by $W^s(\theta_r^*, 0) - W^i(0,0)$. We now show that $W^s(\theta_r^*, 0) - W^i(0,0) < 0$.

Start by noting that $\frac{\partial W^s(\theta_r, 0)}{\partial \theta_r} < 0$ if $\Delta \leq \Theta_e^s$. Thus, as $W^s(0,0) - W^i(0,0) < 0$, by Lemma 5, $W^s(\theta_r^*, 0) - W^i(0,0) < 0$ for $\Delta \leq \Theta_e^s$.

Assume now that $\Delta > \Theta_e^s$. This is only possible if $\Theta_e^s < \overline{\Delta}(\alpha) \Leftrightarrow z < \left(\sqrt{6}+1\right)\alpha$. As $\Theta_e^s < \Delta < \overline{\Delta}(\alpha)$ we have that

$$W^s(1,0) - W^i(0,0) < \frac{-z^2\left(36t + 5z^2\right) + 10\overline{\Delta}(\alpha) z^2 \left(z-\alpha\right)\left(z+\alpha\right) - 4\alpha\Theta_e^{s2}\left(2z+3\alpha\right)\left(z-\alpha\right)^2}{144t}.$$

The numerator is an inverted U-shaped parabola that does not have any real roots when $z < \left(\sqrt{6}+1\right)\alpha$ and hence is always negative.

(iii) Assume that there is discrimination against the entrant under both vertical integration and separation or that there is discrimination against the entrant under vertical integration, and there is no discrimination under vertical separation. Given that separation reduces the level of degradation, we need to analyze $W^s(0, \theta_e^s) - W^i(0, \theta_e^i)$. Knowing that $W^s(0, \theta_e^s) - W^i(0, \theta_e^s) < 0$, by Lemma 5, if $\frac{\partial W^i(1, \theta_e)}{\partial \theta_e} > 0$ for every $\theta_e \in [0,1] \Leftrightarrow \Delta < 1 - \Theta_e^i(0)$, we have $W^s(0, \theta_e^s) - W^i(0, \theta_e^i) < 0$. ■

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6 Figures

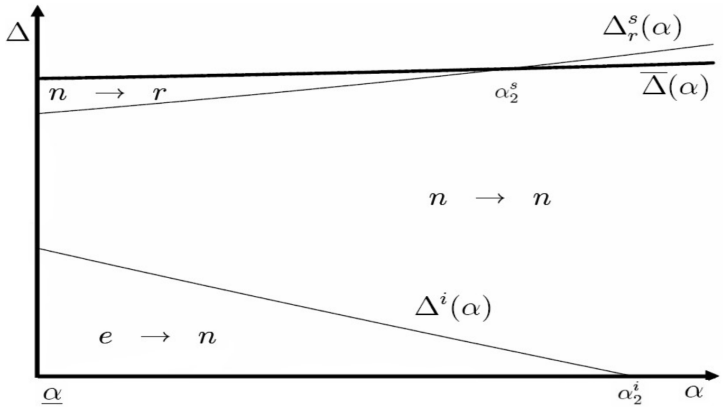


Figure 1: Change in the quality degradation decision after vertical separation for z on $\left(0, \sqrt{3t}\right)$.

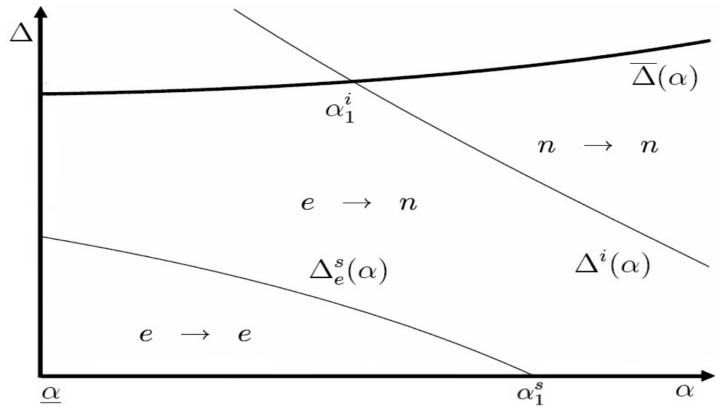


Figure 2: Change in the quality degradation decision after vertical separation for z on $\left(\sqrt{3t}, +\infty\right)$.