

# **Monetary Policy and Rational Asset Price Bubbles**

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## Abstract

I examine the impact of alternative monetary policy rules on a rational asset price bubble, through the lens of an OLG model with nominal rigidities. A systematic increase in interest rates in response to a growing bubble is shown to enhance the fluctuations in the latter, through its positive effect on bubble growth. The optimal monetary policy seeks to strike a balance between stabilization of the bubble and stabilization of aggregate demand. The paper's main findings call into question the theoretical foundations of the case for "leaning against the wind" monetary policies.

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# 1 Motivation

The spectacular rise in housing prices in many advanced economies and its subsequent collapse is generally viewed as a key factor underlying the global financial crisis of 2007-2009, as well as a clear illustration of the dangers associated with asset pricing bubbles that are allowed to go unchecked.

The role that monetary policy should play in containing such bubbles has been the subject of a heated debate, well before the start of the recent crisis. The consensus view among most policy makers in the pre-crisis years was that central banks should focus on controlling inflation and stabilizing the output gap, and thus ignore asset price developments, unless the latter are seen as a threat to price or output stability. Asset price bubbles, it was argued, are difficult –if not outright impossible– to identify or measure; and even if they could be observed, the interest rate would be too blunt an instrument to deal with them, for any significant adjustment in the latter aimed at containing the bubble may cause serious "collateral damage" in the form of lower prices for assets not affected by the bubble, and a greater risk of an economic downturn.<sup>1</sup>

But that consensus view has not gone unchallenged, with many authors and policy makers arguing that the achievement of low and stable (goods price) inflation is not a guarantee of financial stability and calling for central banks to pay special attention to developments in asset markets.<sup>2</sup> Since episodes of rapid asset price inflation often lead to a financial and economic

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<sup>1</sup>See, e.g., Bernanke (2002) and Kohn (2006, 2008) for a central banker's defense of this view. Bernanke and Gertler (1999, 2001) provide a formal analysis in its support.

<sup>2</sup>See, e.g., Borio and Lowe (2002) and Cecchetti et al. (2000).for an early exposition of that view.

crisis, it is argued, central banks should act pre-emptively in the face of such developments, by raising interest rates sufficiently to dampen or bring to an end any episodes of speculative frenzy—a policy often referred to as "leaning against the wind." This may be desirable –it is argued– even if that intervention leads, as a byproduct, to a transitory deviation of inflation and output from target. Under this view, the losses associated with those deviations would be more than offset by the avoidance of the potential fallout from a possible future bursting of the bubble, which may involve a financial crisis and the risk of a consequent episode of deflation and stagnation like the one experienced by Japan after the collapse of its housing bubble in the 90s.<sup>3</sup>

Independently of one's position in the previous debate, it is generally taken for granted (a) that monetary policy can have an impact on asset pricing bubbles and (b) that a tighter monetary policy, in the form of higher short-term nominal interest rates, may help disinflate such bubbles. In the present paper I argue that such an assumption is not supported by economic theory and may thus lead to misguided policy advice. The reason for this can be summarized as follows: in contrast with the fundamental component of an asset price, which is given by a discounted stream of payoffs, the bubble component has no payoffs to discount. The only equilibrium requirement on its size is that the latter grow at the rate of interest, at least in expectation. As a result, any increase in the (real) rate engineered by the central bank will tend to *increase* the size of the bubble, even though its objective may have been exactly the opposite. Of course, any decline observed in the asset

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<sup>3</sup>See Issing (2009) or ECB (2010) for an account and illustration of the gradual evolution of central banks' thinking on this matter as a result of the crisis.

price in response to such a tightening of policy is perfectly consistent with the previous result, since the fundamental component will generally drop in that scenario, possibly more than offsetting the expected rise in the bubble component.

Below I formalize that basic idea by means of a simple asset pricing model, with an exogenous real interest rate. That framework, while useful to convey the basic mechanism at work, it fails to take into account the bubble's general equilibrium effects as well as the possible feedback from the bubble to interest rates implied by alternative monetary policies. That concern motivates the development of a dynamic general equilibrium model that allows for the existence of rational asset pricing bubbles and where nominal interest rates are set by the central bank according to some stylized feedback rule. The model assumes an overlapping generations structure, as in the classic work on bubbles by Samuelson (1958) and Tirole (1985). This is in contrast with the vast majority of recent macro models, which stick to an infinite-lived representative consumer paradigm, and in which rational bubbles can generally be ruled out under standard assumptions.<sup>4</sup> Furthermore, and in contrast with the earlier literature on rational bubbles, the introduction of nominal rigidities (in the form of prices set in advance) makes room for the central bank to influence the real interest rate and, through it, the size of the bubble. While deliberately stylized, such a framework allows me to analyze rigorously the impact of alternative monetary policy rules on the equilibrium dynamics of asset price bubbles. In particular, it makes it possible to assess the consequences of having a central bank use its interest rate policy to

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<sup>4</sup>See, e.g., Santos and Woodford (1997).

counteract asset price bubbles in a systematic way, as has been proposed by a number of authors and commentators.<sup>5</sup>

The paper's main results can be summarized as follows:

- Monetary policy cannot affect the conditions for existence (or non-existence) of a bubble, but it can influence its short-run behavior, including the size of its fluctuations.
- Contrary to the conventional wisdom a stronger interest rate response to bubble fluctuations (i.e. a "leaning against the wind policy") may raise the volatility of asset prices and of their bubble component.
- The optimal policy must strike a balance between stabilization of current aggregate demand—which calls for a positive interest rate response to the bubble—and stabilization of the bubble itself (and hence of future aggregate demand)—which would warrant a negative interest rate response to the bubble. If the average size of the bubble is sufficiently large the latter motive will be dominant, making it optimal for the central bank to lower interest rates in the face of a growing bubble.

The paper is organized as follows. In Section 2 I present a partial equilibrium model to illustrate the basic idea. Section 3 develops an overlapping generation model with nominal rigidities, and Section 4 analyzes its equilibrium, focusing on the conditions under which the latter may be consistent with the presence of rational bubbles. Section 5 describes the impact on that

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<sup>5</sup>The work of Bernanke and Gertler (1999, 2001) is in a similar spirit. In their framework, however, asset price bubbles are not fully rational, and the optimal policy analysis not fully microfounded.

equilibrium of monetary policy rules that respond systematically to the size of the bubble. Section 6 analyzes the optimal central bank response to the bubble. Section 7 discusses some of the caveats of the analysis and presents some tentative empirical evidence. Section 8 concludes.

## 2 A Partial Equilibrium Example

The basic intuition behind the analysis below can be conveyed by means of a simple, partial equilibrium asset pricing example. Consider an economy with risk neutral investors and an exogenous time-varying (gross) riskless rate  $R_t$ . Let  $Q_t$  denote the period  $t$  price of an infinite-lived asset, yielding a dividend stream  $\{D_t\}$ . In equilibrium the following difference equation must hold:

$$Q_t R_t = E_t\{D_{t+1} + Q_{t+1}\}$$

In the absence of further equilibrium constraints,<sup>6</sup> we can decompose the asset price into two components: a *fundamental* component  $Q_t^F$  and a *bubble* component  $Q_t^B$ . Formally,

$$Q_t = Q_t^F + Q_t^B$$

where the fundamental component is defined by the present value relation

$$Q_t^F = E_t \left\{ \sum_{k=1}^{\infty} \left( \prod_{j=0}^{k-1} (1/R_{t+j}) \right) D_{t+k} \right\} \quad (1)$$

The bubble component, defined as the deviation between the asset price and its fundamental value, must satisfy:

$$Q_t^B R_t = E_t\{Q_{t+1}^B\} \quad (2)$$

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<sup>6</sup>Transversality conditions generally implied by optimizing behavior of infinite-lived agents are often used to rule out such a bubble component (see, e.g., Santos and Woodford (1997)). On the other hand models with an infinite sequence of finite-lived agent types, as the one developed below, lack such transversality conditions.

It is easy to see that, *ceteris paribus*, an increase in the interest rate (current or anticipated) will lower  $Q_t^F$ , the fundamental value of the asset. On the other hand, the same increase in the interest rate will raise the expected growth of the bubble component, given by  $E_t\{Q_{t+1}^B/Q_t^B\}$ . Note that the latter corresponds to the bubble's expected return, which must equate the interest rate under the risk neutrality assumption made here. Hence, under the previous logic, any rule that implies a systematic positive response of the interest rate to the size of the bubble, will tend to amplify the movements in the latter—an outcome that calls into question the conventional wisdom about the relation between interest rates and bubbles.

Changes in interest rates, however, may affect the bubble through a second channel: the eventual comovement between the (*indeterminate*) innovation in the bubble with the surprise component of the interest rate. To see this note that the evolution over time of the bubble component of the asset price above is given by the process

$$Q_t^B = Q_{t-1}^B R_{t-1} + \xi_t$$

where  $\{\xi_t\}$  a zero mean martingale-difference process, which may or may not be related to fundamentals.<sup>7</sup> The dependence on the latter process is a reflection of the *inherent indeterminacy of the bubble size*. As a result, the contemporaneous impact of an interest rate increase on the size of the bubble depends on what one assumes regarding the correlation between the interest rate innovation,  $R_t - E_{t-1}\{R_t\}$ , and the martingale-difference variable  $\xi_t$ . Thus, and without loss of generality, one can write

$$\xi_t = \xi_t^* + \psi_r(R_t - E_{t-1}\{R_t\})$$

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<sup>7</sup>Formally,  $\{\xi_t\}$  satisfies  $E_{t-1}\{\xi_t\} = 0$  for all  $t$ .

where  $\{\xi_t^*\}$  is a zero-mean martingale-difference process orthogonal to interest rate innovations at all leads and lags, i.e.  $E\{\xi_t^* R_{t-k}\} = 0$ , for  $k = 0, \pm 1, \pm 2, \dots$ . Note that neither the sign nor the size of  $\psi_r$  are pinned down by the theory. Accordingly, the impact of an interest rate innovation (or of any other shock) on the bubble is, in principle, indeterminate.

In what follows I assume that  $\{\xi_t\}$  is a "pure" sunspot shock, i.e. one orthogonal to fundamentals (i.e.,  $\psi_r = 0$  in the formulation above). This seems a natural benchmark assumption. In that case a change in the interest rate does not affect the current size of the bubble, but only its expected growth rate. Most importantly, the previous discussion makes clear that any case for "leaning against the wind" policies based on a negative value for  $\psi_r$  would rest on extremely fragile grounds, at least from the viewpoint of economic theory.

The relation between monetary policy and asset price bubbles illustrated by the simple example in the present section is at odds with the conventional wisdom, which invariably points to an interest rate hike as the natural way to disinflate a growing bubble. One might argue that the partial equilibrium nature of the previous example may be misleading in that regard, by not taking into account the existence of aggregate constraints that may impose limits on the size of the bubble and hence on its survival. Furthermore, the type of policy intervention considered (i.e. an exogenous change in the real rate) is arguably less relevant than a policy rule determining the systematic response of the nominal interest rate to movements in the size of the bubble.

The remainder of the paper provides an example of possible failure of the conventional wisdom regarding the effects of leaning against the wind

policies. The analysis is grounded in a general equilibrium setting, with the central bank following a well defined interest rate rule and, hence, is immune to the potential criticisms mentioned above.

### 3 Asset Pricing Bubbles in a Simple OLG Model with Nominal Rigidities

As a laboratory for the analysis of the impact of monetary policy on asset pricing bubbles I develop a highly stylized overlapping generations model without capital and where labor is supplied inelastically. In equilibrium, aggregate employment and output are shown to be constant, as in an endowment economy. The assumptions of monopolistic competition and price setting in advance, however, imply that monetary policy is not neutral. In particular, by influencing the path of the real interest rate, the central bank can affect real asset prices (including those of bubbly assets) and, as a result, the distribution of consumption across cohorts and welfare.

#### 3.1 Consumers

Each individual lives for two periods. Individuals born in period  $t$  seek to maximize expected utility

$$\log C_{1,t} + \beta E_t \{\log C_{2,t+1}\}$$

where  $C_{1,t} \equiv \left( \int_0^1 C_{1,t}(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  and  $C_{2,t+1} \equiv \left( \int_0^1 C_{2,t+1}(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  are the bundles consumed when young and old, respectively. Note that, in each period, there is a continuum of differentiated goods available, each produced by a different firm, and with a constant elasticity of substitution given by  $\epsilon$ .

Goods (and the firms producing them) are indexed by  $i \in [0, 1]$ . The size of each cohort is constant and normalized to unity.

Each individual is endowed with the "know-how" to produce a differentiated good, and with that purpose he sets up a new firm. That firm becomes productive only after one period (i.e. when its founder is old) *and* only for one period; after that it no longer produces any output.<sup>8</sup> An individual born in period  $t$  and setting up firm  $i \in [0, 1]$  can raise funds by selling stocks at a price  $Q_{t|t}(i)$ . Each stock is a claim to a share in the firm's one-period ahead dividend,  $D_{t+1}(i)$ , but it can also be traded at  $t + 1$  and subsequent periods at a price  $Q_{t+k|t}(i)$ , for  $k = 1, 2, \dots$ . Note that after one period the stock become a pure bubble, since it no longer constitutes a claim on any future dividends.<sup>9</sup> Henceforth, and to simplify the notation, I drop the firm subindex  $i$  if not strictly needed.

Each young individual sells his labor services inelastically, for a (real) wage  $W_t$ . With that income and the proceeds from the sale of his firm's equity, he consumes  $C_{1,t}$  and purchases two types of assets: (i) one-period nominally riskless discount bonds yielding a nominal return  $i_t$  and (ii) shares in new and old firms, in quantities  $S_{t|t-k}$  and prices  $Q_{t|t-k}$ , for  $k = 0, 1, 2, \dots$ , where the  $t - k$  subindex refers to the period of creation of the corresponding firm.

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<sup>8</sup>This is just a convenient device to avoid having infinite-lived firms, whose market value would not be bounded under the conditions that make it possible for a bubble to exist.

<sup>9</sup>The previous assumption allows me to have an asset (stocks) whose price potentially has both a fundamental and a bubble component (albeit they coexist only transitorily).

Accordingly, the budget constraint for the young at time  $t$  is given by:

$$\int_0^1 \frac{P_t(i)C_{1,t}(i)}{P_t} di + \frac{Z_t}{P_t} \exp\{-i_t\} + \sum_{k=0}^{\infty} Q_{t|t-k} S_{t|t-k} = W_t + Q_{t|t}$$

where  $P_t \equiv \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$  is the aggregate price index and  $Z_t$  denotes the quantity of nominally riskless one-period discount bonds purchased at a price  $\exp\{-i_t\}$ , with  $i_t$  being the (continuously compounded) yield on those bonds (henceforth referred to as the nominal interest rate).

When old, the individual consumes all his wealth, which includes the dividends from his portfolio of stocks, the market value of that portfolio, and the payoffs from his maturing bond holdings. Formally,

$$\int_0^1 \frac{P_{t+1}(i)C_{2,t+1}(i)}{P_{t+1}} di = \frac{Z_t}{P_{t+1}} + D_{t+1}S_{t|t} + \sum_{k=0}^{\infty} Q_{t+1|t-k} S_{t|t-k}$$

The optimal allocation of expenditures across goods yields the familiar demand functions:

$$C_{1,t}(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_{1,t} \quad (3)$$

$$C_{2,t+1}(i) = \left( \frac{P_{t+1}(i)}{P_{t+1}} \right)^{-\epsilon} C_{2,t+1} \quad (4)$$

for all  $i \in [0, 1]$ , which in turn imply  $\int_0^1 \frac{P_t(i)C_{1,t}(i)}{P_t} di = C_{1,t}$  and  $\int_0^1 \frac{P_{t+1}(i)C_{2,t+1}(i)}{P_{t+1}} di = C_{2,t+1}$ .

The remaining optimality conditions associated with the consumer's problem take the following form:

$$C_{1,t} = (1/(1 + \beta)) (W_t + Q_{t|t}) \quad (5)$$

$$\exp\{-i_t\} = E_t \{ \Lambda_{t,t+1}(P_t/P_{t+1}) \} \quad (6)$$

$$Q_{t|t-k} = \begin{cases} E_t \{ \Lambda_{t,t+1} (D_{t+1} + Q_{t+1|t}) \} & \text{for } k = 0 \\ E_t \{ \Lambda_{t,t+1} Q_{t+1|t-k} \} & \text{for } k = 1, 2, \dots \end{cases} \quad (7)$$

and where  $\Lambda_{t,t+1} \equiv \beta(C_{1,t}/C_{2,t+1})$  is the relevant stochastic discount factor. Note that once a firm has paid its one-time dividend, its shares become a pure bubble, whose market price reflects investors' expectations of the (properly discounted) price at which they will be able to resell it in the future, as made clear by equation (7).

Finally, and for future reference, I define the real interest rate as

$$r_t \equiv i_t - E_t \{ \pi_{t+1} \}$$

where  $\pi_t \equiv \log(P_t/P_{t-1})$  is the rate of inflation between  $t-1$  and  $t$ .

### 3.2 Firms

Each individual, endowed with the "know-how" to produce a differentiated good, sets up a firm that becomes productive after one period (when its founder is "old"). Then the firm operates under the technology:

$$Y_t(i) = N_t(i) \quad (8)$$

where  $Y_t(i)$  and  $N_t(i)$  denote firm  $i$ 's output and labor input, respectively, for  $i \in [0, 1]$ . After its operational period (i.e., once its founder dies) the firm becomes unproductive (with its index  $i$  being "transferred" to a newly created firm).

Each firm behaves as a monopolistic competitor, setting the price of its good in order to maximize its value subject to the demand constraint  $Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$ , where  $C_t \equiv C_{1,t} + C_{2,t}$ . If one assumes that firms set the price of their good after the shocks are realized, then they choose a price  $P_t^*$  equal

to a constant gross markup  $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$  times the nominal marginal cost  $P_t W_t$ . Hence, under flexible prices:

$$P_t^* = \mathcal{M} P_t W_t$$

In a symmetric equilibrium,  $P_t^* = P_t$ , thus implying a constant real wage  $W_t = 1/\mathcal{M}$ .

I introduce nominal rigidities by assuming that the price of each good is set in advance, before the shocks are realized. Thus, the price of a good that will be produced and sold in period  $t$ , denoted by  $P_t^*$ , is set at the end of  $t-1$  in order to maximize the firm's fundamental value

$$E_{t-1} \left\{ \Lambda_{t-1,t} Y_t \left( \frac{P_t^*}{P_t} - W_t \right) \right\}$$

subject to the demand schedule  $Y_t = \left( \frac{P_t^*}{P_t} \right)^{-\epsilon} C_t$ . The implied optimal price setting rule is then given by

$$E_{t-1} \left\{ \Lambda_{t-1,t} Y_t \left( \frac{P_t^*}{P_t} - \mathcal{M} W_t \right) \right\} = 0 \quad (9)$$

### 3.3 Stock Prices: Fundamental and Bubble Components

For future reference, it is convenient to define at this stage the two components that constitute the price of a stock, as well as its aggregate counterparts. For a firm created in period  $t$ , the initial fundamental value, denoted by  $Q_{t|t}^F$ , is given by its expected discounted dividend. For firms created in earlier periods and unproductive in period  $t$ , the fundamental value is zero. Formally,

$$Q_{t|t-k}^F = \begin{cases} E_t \{ \Lambda_{t,t+1} D_{t+1} \} & \text{for } k = 0 \\ 0 & \text{for } k = 1, 2, \dots \end{cases} \quad (10)$$

Letting  $Q_t^F \equiv \sum_{k=0}^{\infty} Q_{t|t-k}^F$  denote the *aggregate* fundamental value of existing stocks, it follows trivially that

$$Q_t^F = E_t \{ \Lambda_{t,t+1} D_{t+1} \} \quad (11)$$

The bubble component of a stock, denoted by  $Q_{t|t-k}^B$ , is defined as the difference between its market price and its fundamental component. Given (7) and (10), we have:

$$Q_{t|t-k}^B = E_t \{ \Lambda_{t,t+1} Q_{t+1|t-k}^B \} \quad (12)$$

for  $k = 0, 1, 2, \dots$  and all  $t$ . Note also that free disposal requires that  $Q_{t|t-k}^B \geq 0$  for  $k = 1, 2, \dots$ . Thus, it follows from (12) that  $Q_{t|t}^B \geq 0$  as well. In other words, the market price of a stock cannot lie below its fundamental value.

For notational convenience I henceforth use  $U_t \equiv Q_{t|t}^B \geq 0$  to denote the initial bubble component in the price of a stock introduced in period  $t$ . For simplicity, I assume that such a bubble component is identical across stocks issued in period  $t$ , and refer to it as the *new bubble*. The analysis below is simplified, with little loss of generality, by assuming that  $u_t \equiv \log U_t$  follows an i.i.d. process with mean  $u$  and variance  $\sigma_u^2$ .

Let  $B_t \equiv \sum_{k=1}^{\infty} Q_{t|t-k}^B$  denote the aggregate market value of stocks introduced in earlier periods, and which currently constitute a pure bubble (since they will not yield any future dividends). Thus  $B_t$  can be thought of as the current size of the *pre-existing* bubble. One can then use (12) to derive an equation describing the dynamics of the aggregate bubble:

$$Q_t^B \equiv B_t + U_t = E_t \{ \Lambda_{t,t+1} B_{t+1} \} \quad (13)$$

### 3.4 Monetary Policy

The central bank is assumed to set the short-term nominal interest rate  $i_t$  according to the following rule:

$$i_t = r + E_t\{\pi_{t+1}\} + \phi_\pi \pi_t + \phi_b \widehat{q}_t^B \quad (14)$$

where  $\widehat{q}_t^B \equiv \log(Q_t^B/Q^B)$  is the log deviation of the bubble from its steady state value. Note that under the above rule the *real* interest rate responds systematically to fluctuations in inflation and the size of the bubble, with a strength indexed by  $\phi_\pi$  and  $\phi_b$ , respectively.<sup>10</sup>

Note that, by adopting the specification above I abstract from the difficulties in identifying the presence of a bubble and determining its size that undoubtedly arise in practice and which constitute one of the arguments made by critics of "leaning against the wind" policies. The focus of the analysis below is thus the desirability of having the central bank respond to bubble fluctuations, leaving aside practical questions of implementation.

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<sup>10</sup>As an alternative I have also analyzed the specification

$$i_t = r + \phi_\pi \pi_t + \phi_b \widehat{q}_t^B$$

The main qualitative results obtained under (14) carry over to this alternative specification, though the analysis is (algebraically) more complicated in the latter case.

Similarly, the more general rule

$$i_t = r + \phi_\pi \pi_t + \phi_b \widehat{b}_t + \phi_u \widehat{u}_t$$

does not yield any further insights and can be shown to collapse to rule of the form (14) under the optimal policy. In order to keep the algebra as simple as possible I stick to (14) in what follows.

## 4 Equilibrium

In the present section I derive the model's remaining equilibrium conditions. The clearing of the market for each good requires that  $Y_t(i) = C_{1,t}(i) + C_{2,t}(i)$  for all  $i \in [0, 1]$  and all  $t$ . Letting  $Y_t \equiv \left( \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$  denote aggregate output, we can use the consumer's optimality conditions (3) and (4) to derive the aggregate goods market clearing condition:

$$\begin{aligned} Y_t &= C_{1,t} + C_{2,t} \\ &= \left( \frac{1}{1+\beta} \right) (W_t + Q_t^F + U_t) + (D_t + B_t) \end{aligned} \tag{15}$$

where the second equality follows from (5) and the fact that  $C_{2,t} = D_t + B_t$ . The latter result is a consequence of the market clearing requirement that  $Z_t = 0$  for all  $t$ , since all households in a given cohort are identical (and hence do not trade any assets among themselves), and there is no room for credit transactions between households from different cohorts.

Labor market clearing implies

$$\begin{aligned} 1 &= \int_0^1 Y_t(i) di \\ &= (C_{1,t} + C_{2,t}) \int_0^1 (P_t(i)/P_t)^{-\epsilon} di \\ &= Y_t \end{aligned} \tag{16}$$

where the third equality follows from (15) and the fact that all firms set identical prices in equilibrium. Thus, aggregate output supply is constant and equal to unity.

Adding the budget constraint of the two cohorts coexisting in period  $t$  we obtain  $C_{1,t} + C_{2,t} = W_t + D_t$ , which combined with (15) and (16) implies

$$D_t + W_t = 1 \tag{17}$$

Finally, evaluating the optimal price-setting condition at the symmetric equilibrium and using  $C_{2,t} = D_t + B_t$  we have:

$$E_{t-1} \{ (1/(D_t + B_t)) (1 - \mathcal{M}W_t) \} = 0 \quad (18)$$

Equations (15) through (18), combined with (11), (13) and (14) introduced earlier, describe the equilibrium dynamics of the model economy. In order to make some progress in describing those dynamics, however, the analysis below focuses on the log-linearized system around a deterministic steady state. I start by characterizing the latter.

#### 4.0.1 Steady State

Next I consider a deterministic steady state in which all real variables are constant with  $U_t = U \equiv E\{U_t\}$ , and study under what conditions a positive bubble may arise in that steady state. The following equations characterize the steady state values of the model's main aggregate variables:

$$Y = 1$$

$$W = 1/\mathcal{M}$$

$$D = 1 - 1/\mathcal{M}$$

$$Q^F = (1/R)(1 - 1/\mathcal{M})$$

$$Q^B = U/(1 - R) \quad (19)$$

$$B = (R/(1 - R)) U \quad (20)$$

$$R = (1/\beta)(1 - 1/\mathcal{M} + B)/(1/\mathcal{M} - B) \equiv \mathcal{R}(B) \quad (21)$$

$$\pi = 0$$

where  $R \equiv \exp\{r\}$  can be interpreted the steady state (gross) real interest rate.

As made clear by (19) and (20), in order for a well defined steady state with a positive bubble (henceforth, a "bubbly steady state") to exist we require that  $R \in (0, 1)$ , thus implying a negative (net) real interest rate. The need for this condition is well understood: in its absence the bubble would grow unboundedly, so no steady state would exist. Furthermore, that unbounded growth in the size of the bubble would eventually lead to a violation of the resource constraint, and it would thus be inconsistent with equilibrium.<sup>11</sup> Note also that non-negativity of consumption of the young cohort requires that  $B \in (0, 1/\mathcal{M})$ , i.e. the size of the existing bubble cannot be larger than the resources of the young. Combining both requirements with steady state condition (21) allows us to state the following Lemma

*Lemma 1:* A necessary and sufficient condition for the existence of a bubbly steady state is given by

$$\mathcal{M} < 1 + \beta \tag{22}$$

*Proof:* (Necessity) Note that  $\mathcal{R}(B)$ , as defined in (21), is a continuous, strictly increasing function of  $B$ . In order for a bubbly steady state to exist we must have  $\mathcal{R}(0) = (\mathcal{M}-1)/\beta < 1$ , for otherwise  $R > 1$  for any  $B \in (0, 1/\mathcal{M})$ . (Sufficiency) If  $\mathcal{M} < 1 + \beta$ , then  $\mathcal{R}(0) = (\mathcal{M} - 1)/\beta < 1 \in (0, 1)$ , implying that  $\mathcal{R}(B) < 1$  for some  $B \in (0, 1/\mathcal{M})$ .

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<sup>11</sup>As is well known, the introduction of secular productivity growth makes it possible to reconcile the existence of a bubbly steady state with a positive real interest rate (see, e.g. Tirole (1985)). See below for further discussion.

*Remark #1.* Note that (22) is equivalent to  $\mathcal{R}(0) < 1$ , which corresponds to a negative (net) interest rate in the bubbleless steady state. The latter is in turn associated with a Pareto suboptimal allocation since it implies  $1/C_1 < \beta/C_2$ . and, hence, the possibility of making all cohorts better off by transferring resources from the young to the old (which is what a bubble does). A similar condition holds in the models of Samuelson (1958) and Tirole (1985).

*Remark #2.* Condition (22) implies an upper bound  $B_U \equiv 1/\mathcal{M} - 1/(1 + \beta)$  on the steady state size of the bubble, determined by  $\mathcal{R}(B_U) = 1$ . Note that  $B_U < 1/\mathcal{M}$ , i.e. this new upper bound is more stringent than the one associated with a non-negative consumption for the young.

*Remark #3.* The existence of a bubbly steady state implies the existence of a continuum of them, represented by the set  $\{(B, R) | R = \mathcal{R}(B), B \in (0, B_U)\}$ . That set is represented by the solid line in Figure 1, under the assumption that  $\beta = 1$  and  $\mathcal{M} = 1.2$ . I henceforth refer to the latter as the baseline calibration.<sup>12</sup>

#### 4.0.2 Extension: The Case of Positive Deterministic Growth

The analysis above has been conducted under the assumption of a stationary technology. Consider instead a technology  $Y_t(i) = A_t N_t(i)$  with constant productivity growth, i.e.  $A_t = \Gamma^t$  and  $\Gamma > 1$ . It is easy to check that under this modified technology the model above implies the existence of an equilibrium with balanced growth. In particular, it can be easily shown that all the equilibrium conditions derived above still hold, with the original real variables

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<sup>12</sup>None of the qualitative results emphasized below hinge on the particular calibration used, as long as it satisfies the condition for existence of a bubbly steady state.

(output, consumption, dividend, wage, stock prices, and, eventually, bubble size) now normalized by parameter  $A_t$ , and with  $R_t$  being replaced with  $\tilde{R}_t \equiv R_t/\Gamma$ . Accordingly, a bubble can exist along the balanced growth path (i.e. a steady state of the normalized system) only if  $\tilde{R} < 1$  or, equivalently,  $R < \Gamma$ , i.e. as long as the real interest rate is below the economy's growth rate. Such a bubble would be growing at the same rate as the economy. An analogous result was shown in Samuelson (1958) and Tirole (1985), among others. That extension allows one to reconcile the existence of a bubbly equilibrium with the steady state (net) real interest rate being positive.

#### 4.0.3 Linearized Dynamics

Next I linearize the model's equilibrium conditions around the zero inflation steady state and analyze the resulting system of difference equations. Unless otherwise noted I use lower case letters to denote the log of the original variable, and the symbol  $\hat{\cdot}$  to indicate the deviation from the corresponding steady state value. The resulting equilibrium conditions are:

$$0 = \hat{q}_t^F + \beta R \hat{d}_t + \epsilon B(1 + \beta) R \hat{b}_t + \epsilon B(1 - R) \hat{u}_t \quad (23)$$

$$\hat{q}_t^B = E_t\{\hat{b}_{t+1}\} - \hat{r}_t \quad (24)$$

$$\hat{q}_t^F = E_t\{\hat{d}_{t+1}\} - \hat{r}_t \quad (25)$$

$$\hat{q}_t^B = R \hat{b}_t + (1 - R) \hat{u}_t \quad (26)$$

$$\hat{r}_t = \hat{i}_t - E_t\{\pi_{t+1}\} \quad (27)$$

$$\hat{i}_t = E_t\{\pi_{t+1}\} + \phi_\pi \pi_t + \phi_b \hat{q}_t^B \quad (28)$$

Under flexible prices the real wage and the aggregate dividend are constant, implying

$$\widehat{w}_t = \widehat{d}_t = 0 \quad (29)$$

On the other hand, under sticky prices, log-linearization of (18) yields

$$E_{t-1}\{\widehat{w}_t\} = E_{t-1}\{\widehat{d}_t\} = 0 \quad (30)$$

i.e. both wages and dividends remain, in expectation, at their steady state value. Finally, note that one can combine (24) and (26) to obtain

$$\widehat{b}_t = R\widehat{b}_{t-1} + (1 - R)\widehat{u}_{t-1} + \widehat{r}_{t-1} + \xi_t \quad (31)$$

where  $\{\xi_t\}$  is an arbitrary martingale-difference process (i.e.  $E_{t-1}\{\xi_t\} = 0$  for all  $t$ ). As discussed above, and in order to avoid embedding in the model an arbitrary link between monetary policy and the size of the bubble, I assume that  $\xi_t$  is an exogenous sunspot shock. By making this assumption I force monetary policy to influence the size of the bubble only through the interest rate channel and not through an (arbitrary) indeterminacy channel.

## 4.1 Natural Equilibrium

I refer to the equilibrium under flexible prices as the *natural* equilibrium, and denote the corresponding equilibrium values with a superscript "n". As discussed above, when firms can adjust freely their prices once the shocks are realized, they optimally choose to maintain a constant gross markup  $\mathcal{M}$ . This, in turn, implies that the wage and dividend remain constant at their steady state values. As a result, the goods market clearing condition (23), combined with (25), implies:

$$\widehat{r}_t^n = \epsilon B(1 + \beta)R\widehat{b}_t^n + \epsilon B(1 - R)\widehat{u}_t \quad (32)$$

The previous condition makes clear that the real interest rate is, as expected, independent of monetary policy under flexible prices. Plugging the previous result in (31):

$$\widehat{b}_t^n = \chi \widehat{b}_{t-1}^n + (1 - R)(1 + \epsilon B) \widehat{u}_{t-1} + \xi_t$$

where  $\chi \equiv R(1 + \epsilon B(1 + \beta))$ . Stationarity of the bubble requires  $\chi \in [0, 1)$ , which I henceforth assume.<sup>13</sup> Note that the latter condition will always be satisfied for a sufficiently small steady state bubble  $B$ , given the continuity and monotonicity of  $\mathcal{R}(B)$  and the fact that  $\lim_{B \rightarrow 0} \chi = \mathcal{R}(0) < 1$  holds whenever a bubbly steady state exists (as assumed here). Furthermore, note that  $\mathcal{R}(\overline{B})(1 + \epsilon \overline{B}(1 + \beta)) = 1$  implicitly defines an upper bound  $\overline{B} > 0$  on the size of the steady state bubble consistent with stationarity of bubble fluctuations. It can be easily checked that the upper bound implied by the previous stationarity requirement is tighter than the one associated with the existence of a deterministic bubbly steady state, i.e.  $\overline{B} < B_U \equiv 1/\mathcal{M} - 1/(1 + \beta)$ .<sup>14</sup> The circled locus in Figure 1 displays the subset of bubbly steady states that are consistent with stationary fluctuations in the size of the bubble.

Note that under flexible prices, monetary policy has no influence on the evolution of the bubble, due to its inability to affect the real interest rate. Naturally, though, monetary policy can influence inflation (and other nominal variables). In particular, equilibrium inflation can be derived by combining the interest rate rule (28) with (26) and (32) to yield:

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<sup>13</sup>That stationarity assumption also justifies the use of methods based on a log-linear approximation of the equilibrium conditions.

<sup>14</sup>This can be proved by noting that (i) both  $\chi(B)$  and  $\mathcal{R}(B)$  are strictly increasing in  $B$ , (ii)  $\chi(0) = \mathcal{R}(0)$  and (iii)  $\chi(B) > \mathcal{R}(B)$  for all  $B \in (0, B_U)$ .

$$\pi_t = -(1/\phi_\pi) \left( (\phi_b - \epsilon B(1 + \beta)) R \widehat{b}_t^n + (\phi_b - \epsilon B)(1 - R) \widehat{u}_t \right)$$

Not surprisingly the impact of bubbles on inflation is not independent of the monetary policy rule. In particular, we see that some positive systematic response of the interest rate to the aggregate bubble ( $\phi_b > 0$ ) is desirable from the viewpoint of inflation stabilization. More precisely, the value of  $\phi_b$  that minimizes the variance of inflation under flexible prices is given by  $\phi_b = \epsilon B(1 + \lambda\beta) > 0$ , where  $\lambda \equiv R^2 \text{var}\{\widehat{b}_t^n\} / (R^2 \text{var}\{\widehat{b}_t^n\} + (1 - R)^2 \sigma_u^2)$ . Of course, there is no special reason why the central bank would want to stabilize inflation in the present environment, so I do not analyze this issue further here.<sup>15</sup>

## 4.2 Sticky Price Equilibrium

As discussed above, in the presence of sticky prices we have

$$E_{t-1}\{\widehat{w}_t\} = E_{t-1}\{\widehat{d}_t\} = 0 \quad (33)$$

for all  $t$ . Note also that the fact that prices are predetermined implies:

$$E_{t-1}\{\pi_t\} = \pi_t \quad (34)$$

Combining the previous equations with equilibrium conditions (24), (27) and (28) one can derive the following closed form solution for the evolution

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<sup>15</sup>It is easy to check that the central bank could fully stabilize inflation in this case if it could identify and respond separately to existing and new bubbles with a rule

$$\widehat{i}_t = \phi_\pi \pi_t + \Theta_b \widehat{b}_t + \Theta_u \widehat{u}_t$$

of the bubble (see Appendix for details):

$$\widehat{b}_t = \chi \widehat{b}_{t-1} + (\phi_b + 1)(1 - R)\widehat{u}_{t-1} + \xi_t + (\phi_b - \epsilon B(1 + \beta))R\xi_{t-1} \quad (35)$$

Thus we see that fluctuations in the size of bubble follow an ARMA(1,1) process. The persistence of those fluctuations, as measured by the autoregressive coefficient  $\chi \equiv R(1 + \epsilon B(1 + \beta))$ , is the same as in the natural equilibrium and, hence, independent of monetary policy. The latter, however, can influence the bubble's overall size and volatility through the choice of interest rate rule coefficient  $\phi_b$ , as made clear by (35). This is discussed in detail in the following section.

Through its influence on the size of the bubble  $\widehat{b}_t$  and on the fundamental component of stock prices,  $\widehat{q}_t^F = -\widehat{r}_t$ , monetary policy will in turn affect the allocation of aggregate consumption between the cohorts coexisting at any point in time, thus affecting welfare.

On the other hand, equilibrium inflation is given by the  $AR(1)$  process<sup>16</sup>

$$\pi_t = \chi \pi_{t-1} - (1/\phi_\pi) (\phi_b - \epsilon B(1 + \beta)R) (\phi_b + 1) \varepsilon_{t-1}$$

where  $\varepsilon_t \equiv R\xi_t + (1 - R)\widehat{u}_t$  is the innovation in the aggregate bubble (which in turn is a result of innovation in the pre-existing bubble as well as the new bubble). Thus, we see that inflation inherits the persistence of the aggregate bubble and is influenced by innovations in the latter as well as by the size of the new bubbles, interacting with the central bank's feedback rule.

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<sup>16</sup>See Appendix for details.

## 5 The Impact of Monetary Policy on Bubble Dynamics

As made clear by the analysis in the previous section, the *existence* of bubbles in the present model economy is *not* a monetary phenomenon. In other words, the conditions for their existence do not depend on how monetary policy is conducted.

When prices are flexible, monetary policy is neutral vis a vis the bubble: it cannot have an effect either on its size or on its persistence. Nevertheless, and given that fluctuations in the size of the bubble affect the natural rate of interest, the monetary authority may want to respond systematically to bubble developments if it wishes to stabilize inflation. In particular, it will have to raise the interest rate in response to increases in the size of the bubble.

On the other hand, in the presence of nominal rigidities, monetary policy can have an effect on the size and volatility of the *anticipated* component of the bubble,  $\widehat{b}_t^e \equiv E_{t-1}\{\widehat{b}_t\}$ . As shown in the Appendix, the latter evolves according to the simple  $AR(1)$  process:

$$\widehat{b}_t^e = \chi \widehat{b}_{t-1}^e + (\phi_b + 1)\varepsilon_{t-1} \quad (36)$$

where, again,  $\varepsilon_t \equiv R\xi_t + (1 - R)\widehat{u}_t$ .

Thus we see that the influence of monetary policy on the anticipated component of the bubble works through the choice of the interest rate coefficient  $\phi_b$ . To see how that choice influences the volatility of the aggregate bubble  $\widehat{q}_t^B$  note that (36), together with the fact that

$$\widehat{q}_t^B = R\widehat{b}_t^e + \varepsilon_t \quad (37)$$

implies

$$\text{var}\{\widehat{q}_t^B\} = \left( \frac{R^2(\phi_b + 1)^2}{1 - \chi^2} + 1 \right) \sigma_\varepsilon^2 \quad (38)$$

where  $\sigma_\varepsilon^2 \equiv R^2\sigma_\xi^2 + (1 - R)^2\sigma_u^2$  is the variance of the aggregate bubble innovation. That relation is illustrated graphically in Figure 2, which displays the standard deviation of the aggregate bubble as a function of  $\phi_b$ .<sup>17</sup>

An analysis of that relation yields several results of interest (all of which are reflected in Figure 1). Firstly, equation (38) implies that a "leaning against the wind" policy (corresponding to  $\phi_b > 0$ ) generates a larger volatility in the bubble size than a policy of "benign neglect" ( $\phi_b = 0$ ). Secondly, and conditional on  $\phi_b \geq 0$ , the stronger is the positive feedback from the bubble to the interest rate, the larger is the volatility of the former (!). Finally, the central bank can minimize the bubble volatility by setting  $\phi_b = -1 < 0$  a policy which fully stabilizes the *anticipated* component of the bubble (i.e. it implies  $\widehat{b}_t^e = 0$ , for all  $t$ ). In other words, stabilization of bubble fluctuations requires that the interest rate be *lowered* in response to positive innovations in existing or new bubbles, a finding clearly at odds with conventional wisdom.

We can also use the equilibrium expression for the fundamental stock price (as derived in Appendix 1):

$$\widehat{q}_t^F = -\epsilon B(1 + \beta)R\widehat{b}_t^e - \phi_b \varepsilon_t$$

together with the fact that

$$\widehat{q}_t = (1 - \Gamma_B)\widehat{q}_t^F + \Gamma_B\widehat{q}_t^B$$

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<sup>17</sup>The following calibration is assumed:  $\beta = 1$ ,  $\mathcal{M} = 1.2$ ,  $B = 0.1$  and  $\sigma_\xi^2 = \sigma_u^2 = 0.01$ .

where  $\Gamma_B \equiv \frac{\epsilon B}{\epsilon B + 1} \in [0, 1]$  in order to derive expressions for their respective variances. Figure 3 displays the volatility of the stock price index  $\hat{q}_t$ , together that of its fundamental and bubble components,  $\hat{q}_t^F$  and  $\hat{q}_t^B$ , as a function of coefficient  $\phi_b$ . Note that the three mappings are non-monotonic and increasing after a certain threshold (different in each case) is reached. Thus, we see that an aggressive "leaning against the wind" policy in response to bubbles may have a potentially destabilizing effect on stock prices, as well as in their fundamental and bubble components.

Equilibrium inflation in the economy with sticky prices satisfies

$$\pi_t = -(R/\phi_\pi)(\phi_b - \epsilon B(1 + \beta))\hat{b}_t^e$$

i.e., inflation is proportional to the anticipated bubble. Thus, the central bank can follow three alternative strategies if it seeks to stabilize inflation. First, it can respond very strongly to inflation itself (by setting  $\phi_\pi$  arbitrarily large, for any finite  $\phi_b$ ). Secondly, it can adjust interest rates in response to fluctuations in the bubble with a strength given by  $\phi_b = \Theta_b$  (while setting  $\phi_\pi$  at a finite value) Doing so exactly offsets the impact of the bubble on (expected) aggregate demand, thus neutralizing its impact on inflation. Note that neither of these policies eliminates fluctuations in the bubble, they just prevent the latter from affecting the aggregate price level. Finally, the central bank may choose to stabilize the anticipated component of the bubble  $\hat{b}_t^e$ , which can be achieved by setting  $\phi_b = -1$ , as discussed above. The latter result illustrates how the emergence of an aggregate bubble and the existence of fluctuations in the latter do not necessarily generate a policy trade-off

between stabilization of the bubble and stabilization of inflation.<sup>18</sup>

Note however that in the economy above, with synchronized price-setting and an inelastic labor supply, inflation is *not* a source of welfare losses. Accordingly, and within the logic of the model, there is no reason why the central bank should seek to stabilize inflation. It is also not clear that minimizing the volatility of the aggregate bubble constitutes a desirable objective in itself. In order to clarify those issues, the next section analyzes explicitly the nature of the model's implied optimal policy.

## 6 Optimal Monetary Policy in the Bubbly Economy

Next I turn to an analysis of the optimal response of monetary policy to asset price bubbles in the model economy developed above. I take as a welfare criterion the unconditional mean of an individual's lifetime utility. In a neighborhood of the steady state that mean can be approximated as

$$E\{\log C_{1,t} + \beta \log C_{2,t+1}\} \simeq \log C_1 + \beta \log C_2 - (1/2)(\text{var}\{\hat{c}_{1,t}\} + \beta \text{var}\{\hat{c}_{2,t}\})$$

where  $\hat{c}_{i,t} \equiv \log(C_{i,t}/C_i)$  for  $i = 1, 2$ .

Note that the goods market clearing condition  $C_{1,t} + C_{2,t} = 1$  implies that  $\text{var}\{\hat{c}_{1,t}\}$  is proportional to  $\text{var}\{\hat{c}_{2,t}\}$ . Thus, a central bank that seeks to maximize welfare under the criterion set above will choose the interest rate rule coefficients that minimize the variance of

$$\hat{c}_{2,t} = (1 - \Gamma_B)\hat{d}_t + \Gamma_B\hat{b}_t$$

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<sup>18</sup>The absence of a trade-off obtains when, as assumed above, bubble shocks are the only source of uncertainty in the economy. Other sources of fluctuations may require interest rate adjustments in order to stabilize inflation, which in turn may induce additional volatility in the size of the bubble.

where  $\Gamma_B \equiv \frac{\epsilon B}{\epsilon B + 1} \in [0, 1]$

That objective poses a dilemma for the central bank. To see this note that, as derived in the Appendix, dividends are given by

$$\widehat{d}_t \propto (\phi_b - \epsilon B(1 + \beta))R\xi_t + (\phi_b - \epsilon B)(1 - R)\widehat{u}_t$$

Thus, minimizing the volatility of dividends calls for setting  $\phi_b = \epsilon B(1 + \beta R^2(\sigma_\xi^2/\sigma_\varepsilon^2)) > 0$ . Note that such a policy would require adjusting the interest rate upward in response to positive bubble shocks, in order to stabilize aggregate demand and to prevent upward (downward) pressure on wages (dividends) from emerging. However, as discussed in the previous section, such a policy would amplify the impact of current bubble shocks on the *future* size of the bubble, through the effect of interest rates on bubble growth, thus contributing to destabilization of cohort-specific consumption through that channel. In fact, and as discussed above, minimizing the volatility of cohort-specific consumption resulting from bubble fluctuations calls for setting  $\phi_b = -1 < 0$ . Note finally that neither the volatility of dividends nor that of the bubble depend on the inflation coefficient  $\phi_\pi$ .

The welfare-maximizing choice of  $\phi_b$  will naturally seek a compromise between stabilization of dividends and stabilization of the bubble size. Formally, the optimal coefficient minimizes

$$var\{(1 - \Gamma_B)\widehat{d}_t + \Gamma_B\widehat{b}_t\} \propto \left( (\phi_b - \epsilon B)^2 + \frac{(\beta R \epsilon B)^2 (\phi_b + 1)^2}{1 - \chi^2} \right) \sigma_\varepsilon^2$$

Figure 4 displays the expected welfare loss as a function of  $\phi_b$ , under the model's baseline calibration. The minimum of that loss function determines

the optimal interest rate coefficient. The latter can be written as:

$$\phi_b^* = (-1)\Psi_B + \epsilon B(1 - \Psi_B) \quad (39)$$

where  $\Psi_B \equiv \frac{(\beta R \epsilon B)^2}{1 - \chi^2 + (\beta R \epsilon B)^2} \in [0, 1]$  is an increasing function of  $B$ , the steady state size of the bubble (relative to the economy's size).

Thus, the optimal strength of the central bank's response to the bubble is a nonlinear function of the average size of the latter, as well as other exogenous parameters. Figure 5 displays the optimal coefficient  $\phi_b^*$  as a function of  $B$ , under the baseline calibration for the remaining parameters. Note that the mapping is non-monotonic:  $\phi_b^*$  is shown to be first increasing, and then decreasing, in the size of the bubble. As the steady state size of the bubble approaches zero, so does the optimal coefficient, i.e.  $\lim_{B \rightarrow 0} \phi_b^* = 0$ . On the other hand, as  $B$  approaches its maximum value consistent with stationarity, the optimal coefficient converges to (minus) the corresponding interest rate, i.e.  $\lim_{B \rightarrow \bar{B}} \phi_b^* = -1 < 0$ . Hence, given a sufficiently large steady state bubble, it is optimal for the central bank to *lower* interest rates in response to a rise in the size of the bubble.

The latter finding illustrates that the optimal monetary policy strategy in response to asset price bubbles does not necessarily take the form of a "leaning against the wind" policy or one of just "benign neglect".

## 7 Discussion and Some Evidence

The main purpose of the present paper has been to call into question the theoretical underpinnings of proposals for "leaning against the wind" monetary policies with respect to asset price developments. According to those

proposals central banks should raise interest rates in the face of a developing asset price bubble, in order to tame it or eliminate it altogether. The analysis above has shown that, at least when it comes to a rational asset pricing bubble, such a policy may be counterproductive and lead instead to larger bubble fluctuations and possibly lower welfare as well. In the example economy developed above, it is generally desirable from the viewpoint of bubble stabilization (and, under some assumptions, from a welfare perspective as well) to pursue the *opposite* policy. That finding, which is a consequence of a basic arbitrage constraint that must be satisfied by a rational bubble, seems to have been ignored (or, at least, swept under the rug) by proponents of leaning against the wind policies.

To be clear, it is not my intention to suggest that policies that seek to prevent the emergence of bubbles or its excessive growth are necessarily misguided, but only to point out that certain interest rate policies advocated by a number of economists and policymakers may not always have the desired effects. There are at least three assumptions in the model above which undoubtedly play an important role in accounting for my findings. I discuss them briefly next.

Firstly, I have assumed that there is no systematic impact of interest rate surprises on the "indeterminate" component of the bubble. Some readers may find that assumption arbitrary. But it would be equally arbitrary to assume the existence of a systematic relation of a given size or sign. Whether that systematic relation exists is ultimately an empirical issue, but one that will not be settled easily given the inherent unobservability of bubbles. In any event, the analysis in the present paper points to the *fragility* of the

foundations of a leaning against the wind policy advocated on the basis of such a systematic relation.

Secondly, the asset pricing bubbles introduced in the above model economy are of the *rational* type, i.e. they are consistent with rational expectations on the part of all agents in the economy. In actual economies there may be asset price deviations from fundamentals that are different in nature from the rational bubbles considered here and for which leaning against the wind interest rate policies may have more desirable properties. Assessing that possibility would require the explicit modelling of the nature of deviations from fundamentals, and how those deviations are influenced by interest rate policy. Of course, one should not rule out the possibility that different models of non-rational bubbles may lead to entirely different implications regarding the desirability of leaning against the wind policies.

Thirdly, the analysis above has been conducted in a model economy with no explicit financial sector and no financial market imperfections (other than the existence of bubbles). In fact, the assumption of a representative consumer in each cohort implies that the only financial transactions actually carried out are the sale of stocks by the old to the young, but no credit is needed (in equilibrium) to finance such transactions. By contrast, much of the empirical and policy-oriented literature has emphasized the risks associated with the rapid credit expansion that often accompanies (and helps finance) asset price booms.<sup>19</sup> It is not clear, however, that a tighter monetary policy may be the best way to counter the credit-based speculative bubbles that may arise in this context, as opposed to a stricter regulatory and super-

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<sup>19</sup>See, e.g., Schularick and Taylor (2009).

visory framework with the necessary tools to dampen the growth of credit allocated to (potentially destabilizing) speculative activities. Further efforts at modelling explicitly the interaction of credit, bubbles and monetary policy would seem highly welcome.

What does the empirical evidence have to say about the impact of monetary policy on asset price bubbles? It is clear that any empirical analysis of that link faces many challenges. Firstly, the difficulty (or, some may say, impossibility) in identifying the bubble component of an asset certainly does not facilitate the task. Secondly, any observed comovement between asset prices and policy rates may be distorted by the presence of reverse causality, if the central bank does indeed adjust the interest rate in response to asset price movements.

Those caveats notwithstanding, I find it informative to take a look at the behavior of the interest rate and the relevant asset price during three episodes of U.S. history generally viewed (at least ex-post) as associated with the presence of a large and growing bubble, which subsequently burst: (i) the stock market boom previous to the Great Crash of October 1929, (ii) the dotcom bubble of the second half of the 1990s, and (iii) the housing bubble leading to the financial crisis of 2007-2008.

Figure 5 shows the Dow-Jones Industrial Stock Price Index along with the discount rate at the Federal Reserve Bank of New York, over the period 1927:1-1929:9. Figure 6 displays the NASDAQ Composite Index together with the Federal Funds rate target over the period 1997:1-2000:1. Finally, Figure 7 shows the Case-Shiller House Price Index (Composite 20) and, again, the Federal Funds rate target, now over the period 2002:1-2007:4. The three

episodes share two key features. First, the corresponding asset price index experiences a very fast, largely uninterrupted, rise which is hard to account for by any plausible revision of fundamentals. The subsequent collapse (not displayed in the Figures) reinforces the interpretation of much of that rise as resulting from a bubble. Secondly, in the three episodes the asset price boom is eventually accompanied with a substantial rise in the policy rate. The latter might have been partly intended to counter the growing bubble. Most importantly, however, for the purposes of the present paper, is the observation that the large increase in the policy rate (whatever its motivation) does not seem to have any significant impact on the path of asset prices.<sup>20</sup> That observation would seem to be at odds with the presumptions behind the "leaning against the wind" view, namely, that a hike in interest rates should help prick (or at least) disinflate any developing bubble (in addition to having an adverse side effect on the asset's fundamental). On the other hand it seems to be consistent with the predictions of the model above, according to which a rise in the interest rate will generally enhance the growth of the bubble and, if the latter is sufficiently large, that of the asset price as well.

## 8 Concluding Remarks

In order to do so I have developed a highly stylized overlapping generations model with monopolistic competition and price setting in advance. The over-

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<sup>20</sup>Thus, the Fed's attempt to stop the rise in stock prices through a series of interest rate increases 1928-1929 has been interpreted by a number of authors as the main factor behind the initial decline in activity during the Great Depression (see Bernanke (2002) and references therein).

lapping generations structure allows for the existence of asset price bubbles in equilibrium, as in the models of Samuelson (1958) and Tirole (1982). The introduction of nominal rigidities implies that monetary policy is not neutral. In particular, by influencing the path of the real interest rate, the central bank can affect real asset prices (including those of bubbly assets) and, as a result, the distribution of consumption across cohorts and welfare.

Two main results have emerged from the analysis of that model. First, contrary to conventional wisdom, a stronger interest rate response to bubble fluctuations (i.e. a "leaning against the wind policy") may raise the volatility of asset prices and of their bubble component. Secondly, the optimal policy must strike a balance between stabilization of current aggregate demand—which calls for a positive interest rate response to the bubble—and stabilization of the bubble itself (and hence of future aggregate demand)—which would warrant a negative interest rate response to the bubble. If the average size of the bubble is sufficiently large the latter motive will be dominant, making it optimal for the central bank to lower interest rates in the face of a growing bubble.

Needless to say the conclusions should not be taken at face value when it comes to designing actual policies. This is so because the model may not provide an accurate representation of the challenges facing actual policy makers. In particular, it may very well be the case that actual bubbles are not of the rational type and, hence, respond to monetary policy changes in ways not captured by the theory above. In addition, the model above abstracts from many aspects of actual economies that may be highly relevant when designing monetary policy in bubbly economies, including the presence of frictions and

imperfect information in financial markets. Those caveats notwithstanding, the analysis above may be useful by pointing out an potentially important missing link in the case for "leaning against the wind" policies.

## Appendix

### Appendix 1.

Combine (23), (25) and (33) to yield:

$$E_{t-1}\{\hat{r}_t\} = \epsilon B(1 + \beta) R E_{t-1}\{\hat{b}_t\} \quad (40)$$

Taking expectations on both sides of the interest rate rule:

$$E_{t-1}\{\hat{r}_t\} = \phi_\pi \pi_t + \phi_b R E_{t-1}\{\hat{b}_t\} \quad (41)$$

Combining both yields

$$\pi_t = -(R/\phi_\pi) (\phi_b - \epsilon B(1 + \beta)) E_{t-1}\{\hat{b}_t\}$$

Letting  $\varepsilon_t \equiv R\xi_t + (1 - R)\hat{u}_t$ , note that

$$\begin{aligned} \hat{r}_t &= E_{t-1}\{\hat{r}_t\} + (\hat{r}_t - E_{t-1}\{\hat{r}_t\}) \\ &= \epsilon B(1 + \beta) R E_{t-1}\{\hat{b}_t\} + \phi_b \varepsilon_t \\ &= \epsilon B(1 + \beta) R (R\hat{b}_{t-1} + (1 - R)\hat{u}_{t-1} + \hat{r}_{t-1}) + \phi_b \varepsilon_t \end{aligned}$$

It follows that

$$(1 - \epsilon B(1 + \beta) R L) \hat{r}_t = \epsilon B(1 + \beta) R (R\hat{b}_{t-1} + (1 - R)\hat{u}_{t-1}) + \phi_b \varepsilon_t$$

Combining the previous result with the bubble difference equation  $(1 - RL)\hat{b}_t = (1 - R)\hat{u}_{t-1} + \hat{r}_{t-1} + \xi_t$  yields:

$$\hat{b}_t = \chi \hat{b}_{t-1} + (\phi_b + 1)(1 - R)\hat{u}_{t-1} + \xi_t + (\phi_b - \epsilon B(1 + \beta)) R \xi_{t-1}$$

where, as above,  $\chi \equiv R(1 + \epsilon(1 + \beta)B)$  is assumed to be between zero and one.

Note that the predictable component of the bubble follows the process

$$\begin{aligned} E_{t-1}\{\widehat{b}_t\} &= \chi(E_{t-2}\{\widehat{b}_{t-1}\} + \xi_{t-1}) + (\phi_b + 1)(1 - R)\widehat{u}_{t-1} + (\phi_b - \epsilon B(1 + \beta))R\xi_{t-1} \\ &= \chi E_{t-2}\{\widehat{b}_{t-1}\} + (\phi_b + 1)\varepsilon_{t-1} \end{aligned}$$

Accordingly, and letting  $\widehat{b}_t^e \equiv E_{t-1}\{\widehat{b}_t\}$

$$\text{var}\{\widehat{b}_t^e\} = \frac{(\phi_b + 1)^2}{1 - \chi^2} \sigma_\varepsilon^2$$

where  $\sigma_\varepsilon^2 \equiv (1 - R)^2 \sigma_u^2 + R^2 \sigma_\xi^2$ .

Finally, and using the fact that  $\widehat{q}_t^B = R(\widehat{b}_t^e + \xi_t) + (1 - R)\widehat{u}_t = \widehat{b}_t^e + \varepsilon_t$ , we have

$$\text{var}\{\widehat{q}_t^B\} = \left( \frac{R^2(\phi_b + 1)^2}{1 - \chi^2} + 1 \right) \sigma_\varepsilon^2$$

Note also that we can now write the equilibrium process for inflation as:

$$\begin{aligned} \pi_t &= -(R/\phi_\pi)(\phi_b - \epsilon B(1 + \beta))\widehat{b}_t^e \\ &= \chi\pi_{t-1} - (R/\phi_\pi)(\phi_b - \epsilon B(1 + \beta))(\phi_b + 1)\varepsilon_{t-1} \end{aligned}$$

Note also that

$$\begin{aligned} \widehat{q}_t^F &= -\widehat{r}_t \\ &= -\phi_\pi\pi_t - \phi_b(R\widehat{b}_t^e + \varepsilon_t) \\ &= -\epsilon B(1 + \beta)R\widehat{b}_t^e - \phi_b\varepsilon_t \end{aligned}$$

Thus,

$$\text{var}\{\widehat{q}_t^F\} = \left( \frac{(\epsilon B(1 + \beta)R)^2(\phi_b + 1)^2}{1 - \chi^2} + \phi_b^2 \right) \sigma_\varepsilon^2$$

## Appendix 2

Goods market clearing implies

$$\begin{aligned} 0 &= \widehat{q}_t^F + \beta R \widehat{d}_t + \epsilon B(1-R)\widehat{u}_t + \epsilon B(1+\beta)R\widehat{b}_t \\ &= -\widehat{r}_t + \beta R \widehat{d}_t + \epsilon B(1-R)\widehat{u}_t + \epsilon B(1+\beta)R\widehat{b}_t \end{aligned}$$

Using the fact that  $\widehat{r}_t = (\widehat{r}_t - E_{t-1}\{\widehat{r}_t\}) + E_{t-1}\{\widehat{r}_t\} = \phi_b \varepsilon_t + \epsilon B(1+\beta)R\widehat{b}_t^e$ , one can write

$$\begin{aligned} \beta R \widehat{d}_t &= \phi_b(R\xi_t + (1-R)\widehat{u}_t) + \epsilon B(1+\beta)R\widehat{b}_t^e - (\epsilon B(1-R)\widehat{u}_t + \epsilon B(1+\beta)R\widehat{b}_t) \\ &= (\phi_b - \epsilon B(1+\beta))R\xi_t + (\phi_b - \epsilon B)(1-R)\widehat{u}_t \end{aligned}$$

Letting  $\Gamma_B \equiv \frac{\epsilon B}{\epsilon B + 1}$  we have

$$\begin{aligned} \beta R \widehat{c}_{2,t} &= \beta R((1-\Gamma_B)\widehat{d}_t + \Gamma_B \widehat{b}_t) \\ &= (1-\Gamma_B)\beta R(\widehat{d}_t + \epsilon B \widehat{b}_t) \\ &= (1-\Gamma_B)((\phi_b - \epsilon B(1+\beta))R\xi_t + (\phi_b - \epsilon B)(1-R)\widehat{u}_t + \beta R \epsilon B \widehat{b}_t) \\ &= (1-\Gamma_B)((\phi_b - \epsilon B)\varepsilon_t + \beta R \epsilon B \widehat{b}_t^e) \end{aligned}$$

implying

$$\text{var}\{\widehat{c}_{2,t}\} \propto \left[ (\phi_b - \epsilon B)^2 + \frac{(\beta R \epsilon B)^2 (\phi_b + 1)^2}{1 - \chi^2} \right] \sigma_\epsilon^2$$

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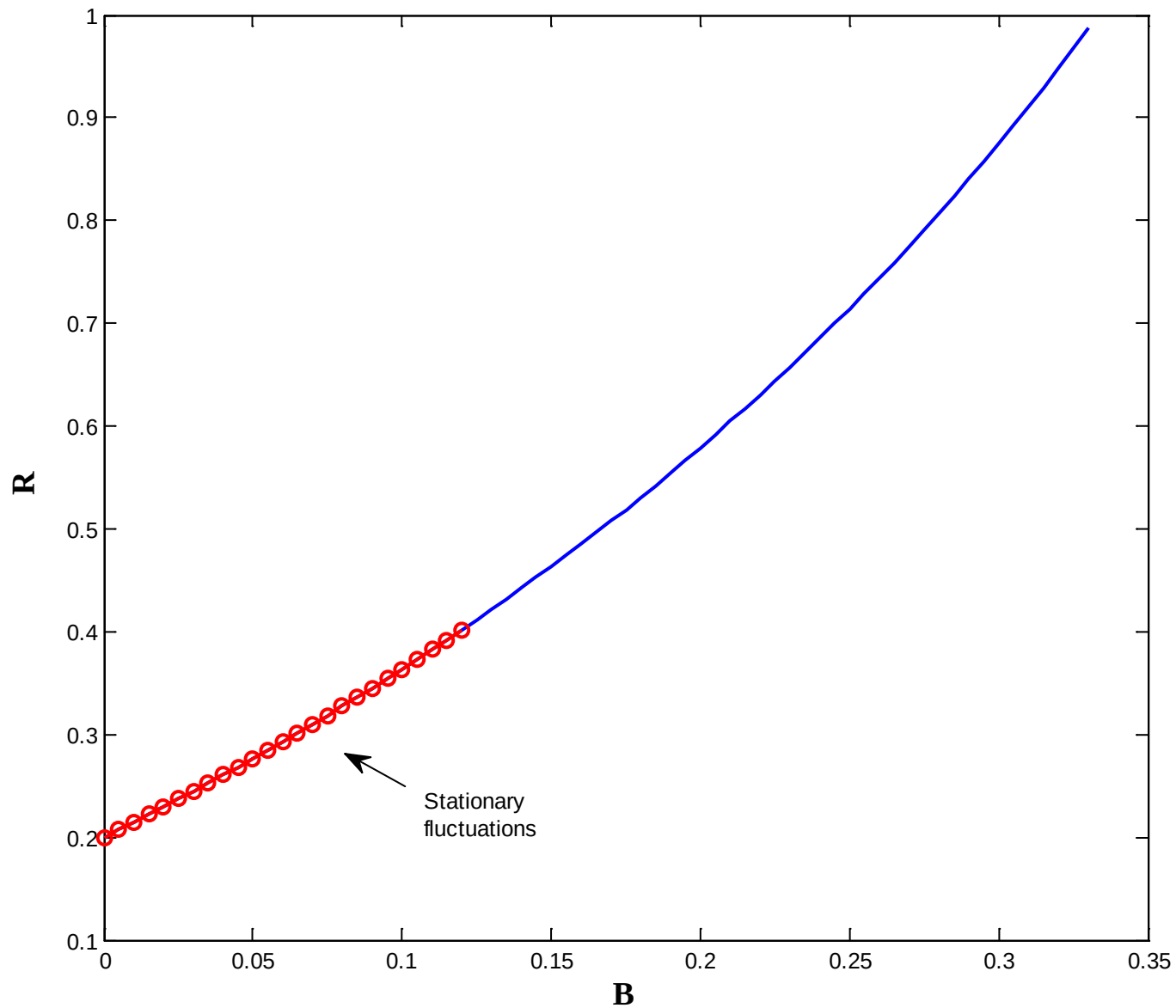
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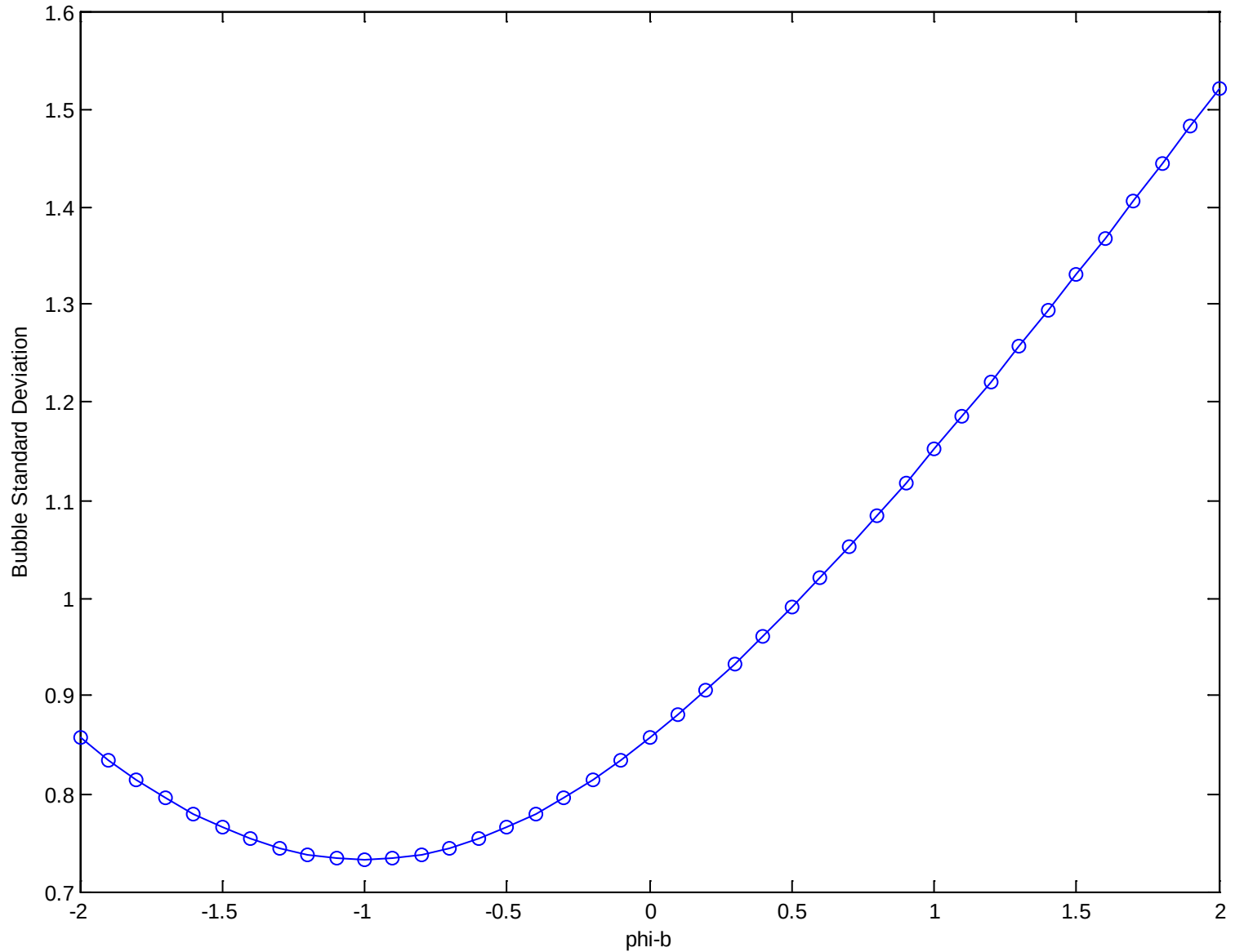
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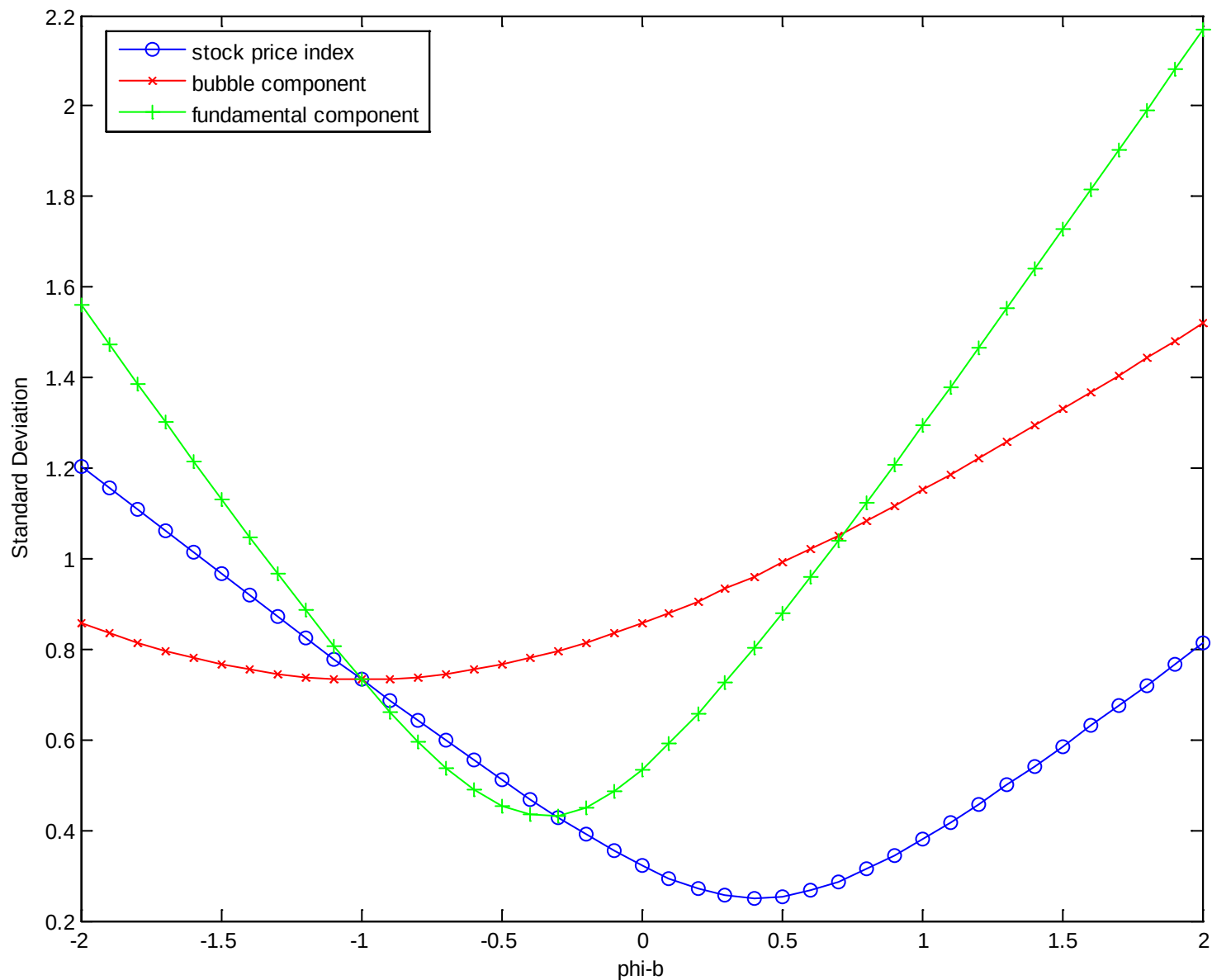
# Figure 1. Bubbly Steady States



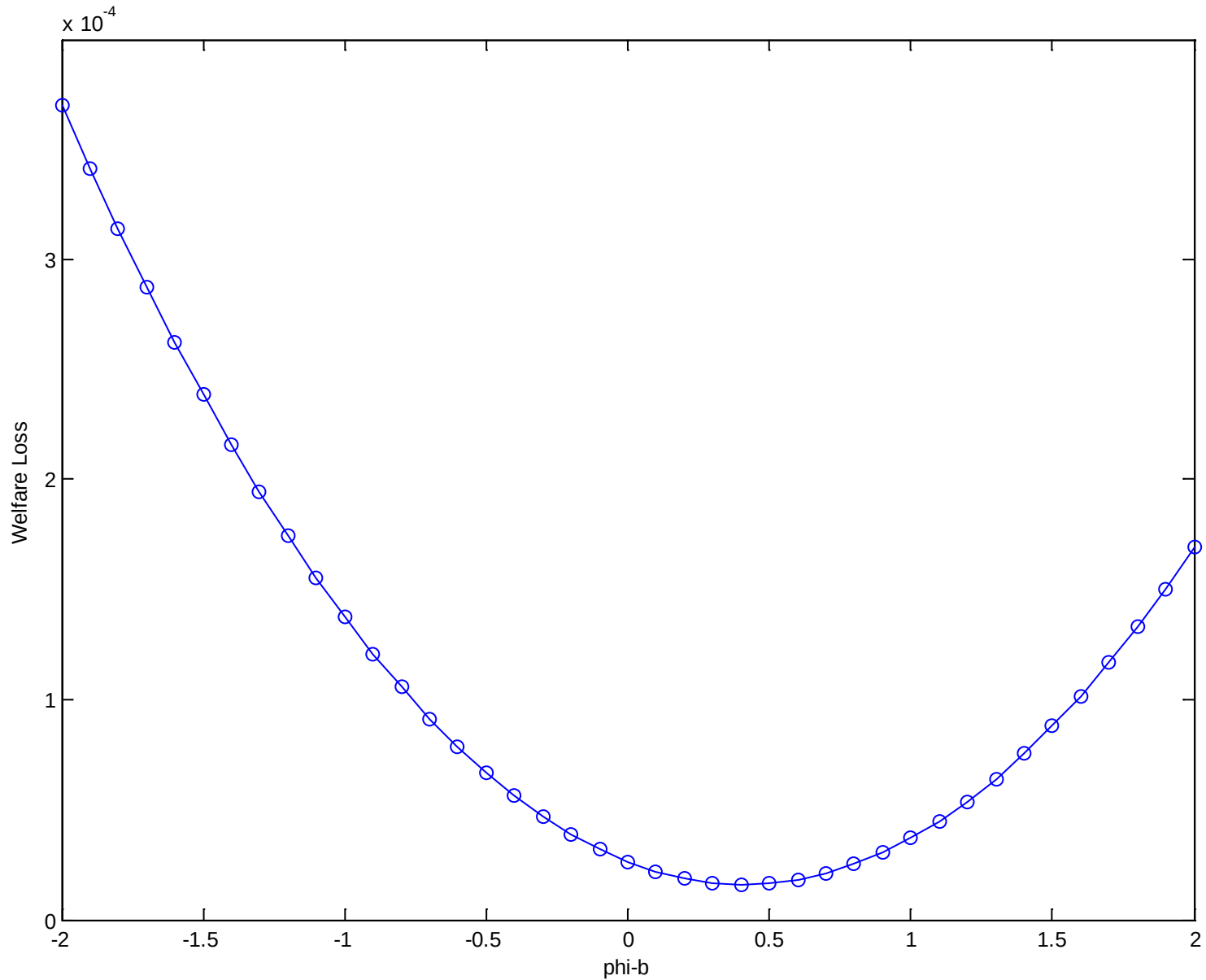
# Figure 2. Monetary Policy and Bubble Volatility



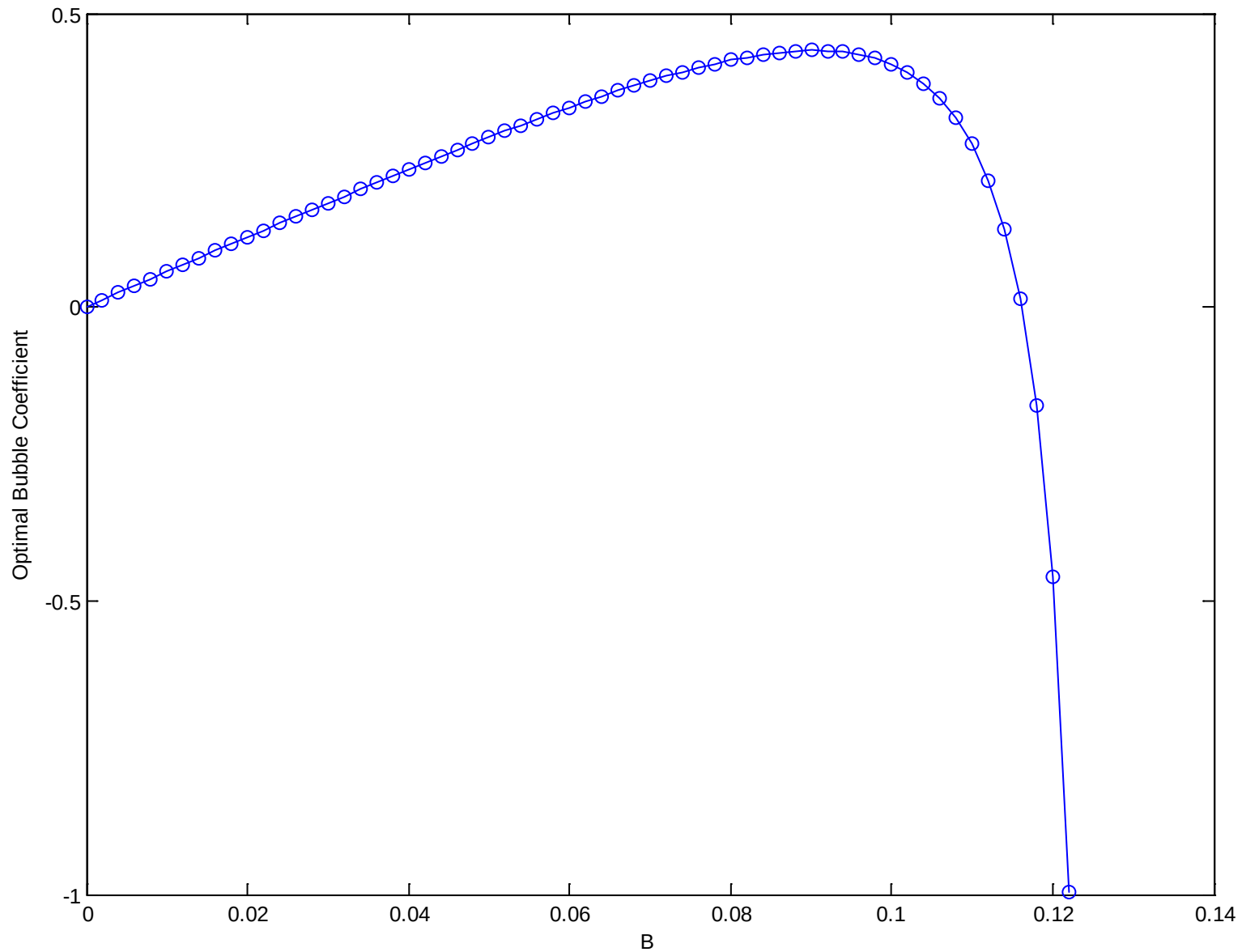
# Figure 3. Monetary Policy and the Volatility of Stock Prices and its Components



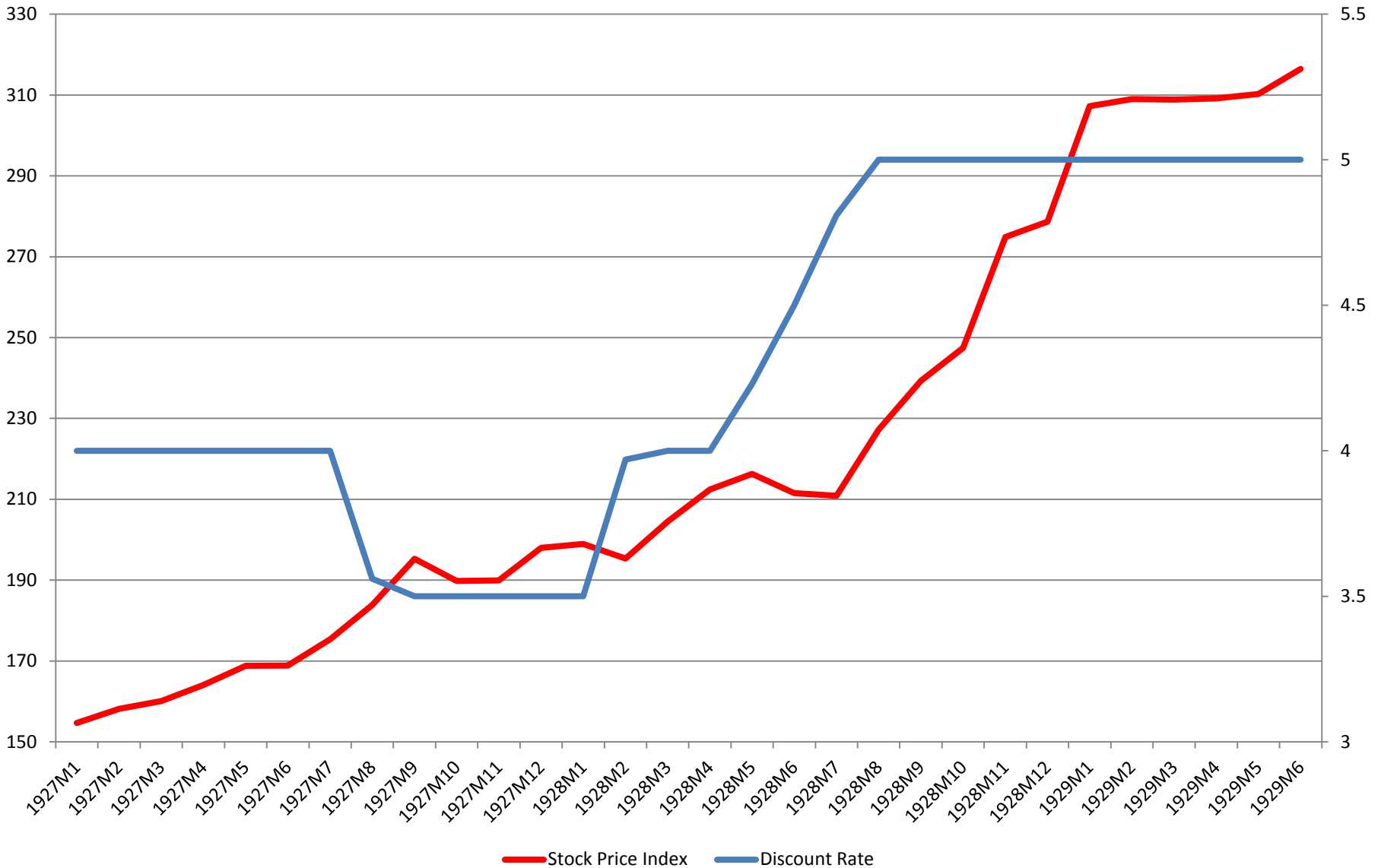
# Figure 4. Monetary Policy and Welfare Losses



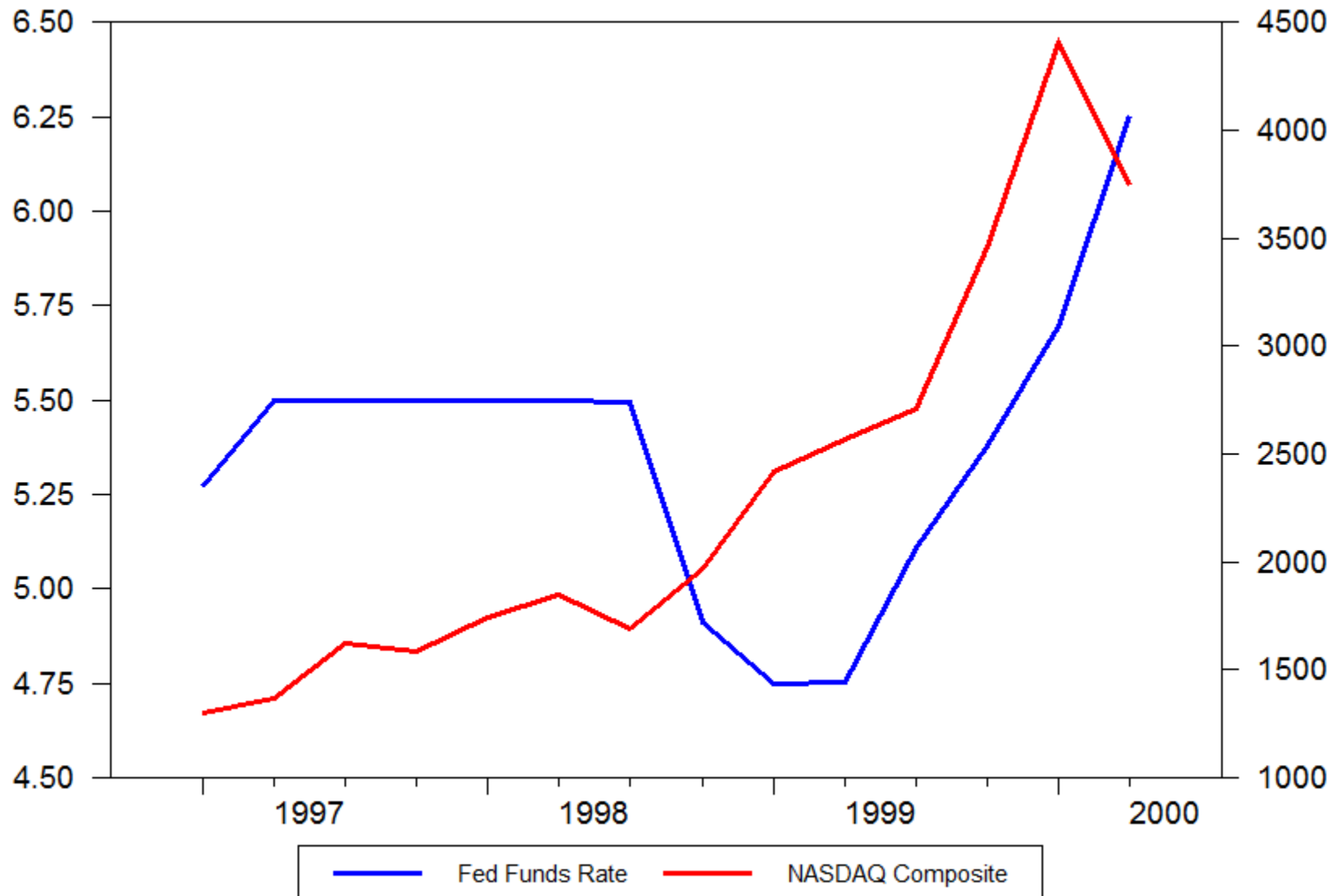
# Figure 5. Optimal Bubble Coefficient



# Figure 6. Monetary Policy and the 1928-29 Stock Market Bubble



# Figure 7. Monetary Policy and the Dotcom Bubble



## Figure 8. Monetary Policy and the Housing Bubble

