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Preface

This book, first published in 2009, stems from research that I began more than three decades ago when I was then working as group economist for the Babcock International Group. Prior to that, my formal university education had included degrees in engineering and management science – the latter in particular covering economics and operations research. What started out as a train of curiosity into parallels between the disciplines of economics and thermodynamics soon developed into something deeper.

Following publication of two peer-reviewed papers of mine on the subject in the journal *Energy Economics*, I was greatly encouraged in my research by other trans-disciplinary researchers with a similar interest, in particular, Dr László Kapolyi, who was then Minister for Industry of the Hungarian government, a member of the Hungarian Academy of Science and a member of the Club of Rome.

Not being based at a university and with no research grant at my disposal, my main thrust at that time had been to make a career as director of a consultancy and expert witness business and therefore, until more recently, opportunities to spend time on research had been few. Nevertheless, by the turn of the millennium I was able to find time alongside my consultancy to return to some research, and in 2007 published another peer-reviewed paper in the *International Journal of Exergy* entitled ‘A Thermodynamic Theory of Economics’, which was followed up with several working papers on monetary aspects and energy models. Interest in this work has been high, spurred on no doubt by general worldwide interest in energy and climate change.

This book and third edition is an attempt to bring together all the facets of the research into a coherent whole. Topics covered include the gas laws, the distribution of income, the 1st and 2nd Laws of Thermodynamics applied to economics, economic processes and elasticity, entropy and utility, production and consumption processes, reaction kinetics, empirical monetary analysis of the UK and USA economies, interest rates, discounted cash flow, bond yield and spread, unemployment, principles of entropy maximization and economic development, the cycle, empirical analysis of the relationship between world energy resources, climate change and economic output, and last aspects of sustainability.
Further developments have been added since the first and second editions, in particular, thoughts on production and entropy maximisation, order and disorder and relationships to the living world, which has necessitated re-organisation of some of the chapters. The chapter on money has been updated to incorporate empirical analyses of the recent upheavals in world economic activity from 2008 to 2011, though the conclusions reached have not changed, indeed, they have been reinforced.

The findings, interpretations and conclusions of this book are entirely those of my own, based on the research that I have conducted. While I have made every effort to be diligent and accurate, readers should satisfy themselves as to logic and veracity of the conclusions drawn. I hope that this third edition represents an improvement and advancement on earlier editions, but would welcome nevertheless any feedback, discussions and corrections on points that readers may have.

I am indebted to my wife Alison for all her support and for providing an atmosphere conducive to my research.

John Bryant
CHAPTER 7 INVESTMENT AND ECONOMIC ENTROPY

At chapter 4 a discussion was set out concerning the principle of maximising entropy gain, by which decision makers seek to maximise benefits to their businesses and organisations. This principle relates also to that of utility, discussed at chapter 3, by which consumers endeavour to maximise benefits to themselves from among a range of opportunities on which to spend a limited income. In this chapter we outline the way in which this principle interacts with investment decision making, in particular, discounted cash flow, bonds and yield.

It will be recalled from equation (4.25) that entropy change was related to rate of change in active output value less the rate of change in any impacting constraint toward inactiveness, all provided that rates of change in prices were equalised between output and constraint.

\[
ds = k_{\text{money}} \left[ \frac{dG_{oa}}{G_{oa}} - \frac{dG_{oc}}{G_{oc}} \right] \quad \text{[subject to } \frac{dP}{P} \text{ active } = \frac{dP}{P} \text{ inactive]}
\]

And in terms of money this equation became that stated at equation (5.22):

\[
ds_{\text{money}} = \frac{dG}{G} - \frac{dI}{I}
\]

(7.1)

Where \( ds \) was entropy change, \( dG/G \) was the rate of change in output value and \( I \) was a cumulative index of investment value that increased each period/year by the prevailing interest rate \( i \) at any time, which can go up and down. Thus \( di/I = i \). Further, turning the equation round, one could project:

\[
\frac{dG}{G} = \frac{dI}{I} + ds_{\text{money}}
\]

(7.2)

This equation expresses output potential as a function of the current rate of interest, plus an entropy change indicating the utility or expectations that humankind has concerning forward economic conditions; positive when future returns might be expected to exceed current rates of interest, and negative when future returns might be expected be lower than current rates of interest. Thus the equation sets out expectations with respect to the climate for investment.
7.1 Project Investment and Discounted Cash Flow

In respect of project investments, a generalised form of investment return is of the form:

\[ \text{surplus / deficit} = -N_p + \left[ A_1 + A_2 + \ldots + A_\xi \right] \]  \hspace{1cm} (7.3)

Where \( N_p \) is the initial investment project outlay and \( A_1, A_2, \ldots \) is a stream of profits/payments received over a period of \( \xi \) years. Because such investments and projects take place into the future, it is common practice to discount each future receipt back to the time of the initial outlay using an appropriate rate of return \( r \). Thus is born the familiar equation for discounted cash flow (DCF) involving a Net Present Value (NPV):

\[ (1 + r)^\xi A_1 + (1 + r)^{\xi - 1} A_2 + \ldots + (1 + r) A_\xi + N_p = 0 \]  \hspace{1cm} (7.4)

Or in shorthand:

\[ \sum_{x=1}^{\xi} \left( \frac{A_x}{(1 + r)^x} \right) = 0 \]  \hspace{1cm} (7.5)

Hence if the net present value is positive then the project is deemed to have a chance of beating prevailing investment returns.

An alternative method of presenting the Net Present Value is by what is known as the ‘Internal Rate of Return’ or IRR, which might represent the minimum rate of return \( r \) required, such that the Net Present Value becomes zero. Thus:

\[ N_p = \sum_{x=1}^{\xi} \left( \frac{A_x}{(1 + r)^x} \right) \]  \hspace{1cm} (7.6)

Where \( r \) is the Internal Rate of Return IRR, which could be compared to prevailing investment returns to assess the profitability of the project / investment.
When comparing several projects with each other, however, it is generally recognised that the NPV form of discounted cash flow analysis at equation (7.5) is to be preferred over IRR.

With perfect knowledge, both of future rates of return and of each project return amount \( A_x \), equation (7.5) could be modified as in equation (7.7):

\[
NPV = -N_0 + \sum_{x=1}^{\xi} \left( \frac{A_x}{(1 + r_x)^x} \right)
\]  

(7.7)

Where \( r_x \) is the rate of return in each period into the future up to period \( \xi \).

Quite evidently, however, managers and decision makers are not blessed with perfect knowledge of what the future may hold, and experience suggests that it is prudent to set a project rate of return in excess of prevailing rates in order to allow for risk and uncertainty.

Figures 5.7 and 5.16 at chapter 5, for example, show several periods of economic uncertainty in the UK and USA, when the elastic index \( n \) was very high/low, and when economic entropy change \( \Delta s \) became negative. At such times there is additional risk attached to projects concerning the future, and a short term view might be preferred to a long term one.

A possible means of measuring and incorporating future uncertainty into the discount equation, therefore, is to modify the rate of return to include an entropy function. Thus at any point in time \( (t = x) \) in the future we could write:

\[
r_e = r_x + \Delta s_x
\]  

(7.8)

Where \( r_e \) is the expected rate of return.

However, given that knowledge of the future is unknown, in particular as to whether \( \Delta s_x \) is positive or negative in any year, decision makers have to base their assessment on the present. Suppose we set out two equations for equation (7.8), one for future expectations and one for the present, viz:

\[
r_e = r_x + \Delta s_x
\]

\[
r_e = r_0 + \Delta s_0
\]
Then by subtracting one from the other and assuming \( r_e \) is the same for each, we have:

\[
\begin{align*}
  r_x &= r_o + (\Delta s_0 - \Delta s_x) \\
  r_x &= r_o + \Delta s_0 \left(1 - \frac{\Delta s_x}{\Delta s_0}\right)
\end{align*}
\]  

or

\[ (7.9) \]

Thus the future rate of return at time \( t = x \) could be expressed as the present rate of return at time \( t = 0 \), plus the present entropy change modified by an entropy difference function.

It will be recalled from chapter 5 of this book that entropy change was linked to utility, and that other researchers also have highlighted similarities between the two concepts. In essence therefore, what is required for the factor in the brackets at equation (7.9) is a discounted utility or entropy function that enables decision makers to put a weight on future outcomes, given that the further ahead one attempts to look, the less will be known. Indeed that is exactly what occurs in the process of discounted cash flow and in discounting bond coupon payments to present values. For the purposes of this analysis we might posit an entropy decay function of an exponential form though, as highlighted later in the section on bonds, others might also be considered. Thus for the moment we assume:

\[
\Delta s_x = \Delta s_0 e^{-\varepsilon x}
\]

(7.10)

Where \( \varepsilon \) is a decay factor.

Figure 7.1 Exponential entropy decay function
Thus future changes in entropy are estimated as being equal to the current one modified by a decay function, expressing a view as to the extent to which the current position could be projected forward. Thus from equations (7.8) and (7.9) the expected rate of return at any point could be expressed as:

\[ r_e = r_0 + \Delta s_0 \left( 1 - e^{-\epsilon} \right) \]  

Where the decay factor $\epsilon$ is set by decision makers; having regard for current position and for the time horizon of a particular project. Clearly at times of low expectations with a negative entropy change, the expected return might be below current interest rates, and investment for the future is not promoted, and vice-versa for times of high expectations. A worked example in support of this approach will be set out later in this chapter in respect of bonds and gilt-edged stocks.

7.2 Annuities

The format of an annuity is similar in structure to that for project analysis, with the difference that all of the period payments $A$ are the same.

Thus, assuming a single rate of return, the equation for the net present value is reduced to:

\[ NPV = \frac{A}{(1 + r)^1} + \frac{A}{(1 + r)^2} + \ldots + \frac{A}{(1 + r)^\xi} \]  

(7.12)

Or in shorthand:

\[ NPV = \sum_{x=1}^{\xi} \left( \frac{A}{(1 + r)^x} \right) \]  

(7.13)

Mathematical manipulation reduces this to:

\[ NPV = \frac{A}{r} \left[ 1 - \frac{1}{(1 + r)^\xi} \right] \]  

(7.14)

Or in exponential format:
\[ \text{NPV} = \frac{A}{r} \left[ 1 - e^{-rs} \right] \]  

(7.15)

In respect of risk and uncertainty, a similar argument can be applied to an annuity as to that for a project investment, concerning the expected rate of return, in the knowledge of changes in entropy affecting matters and, similar to equations (7.8) and (7.11), we could posit an expected rate of return of the form:

\[ r_e = r_\xi + \Delta s_\xi \]
\[ r_e = r_0 + \Delta s_0 \left( 1 - e^{-c\xi} \right) \]  

(7.16)

Thus an annuity with negative entropy expectations would not likely be attractive to an investor, unless a very high yield was involved.

### 7.3 Bonds and Gilt-Edged Securities

Bonds and gilt-edged securities, issued by governments and corporations, are structured around two main types: conventional bonds (the majority) involving fixed coupon income and redemption values, and index-linked bonds involving index-linking of both coupon and redemption values in line with a consumer price index, to provide for an element of inflation-proofing.

A conventional fixed-interest bond cash-flow construction is of a similar form to that for project investments and annuities, but involving also a return of the original investment to the lender (at par) at the end of the investment period. As with an annuity, fixed coupon payments \( A \) (annual, half yearly) are paid to the lender between the beginning and end of the investment period, but the coupon payment \( A \) here is defined as a coupon rate \( c \) multiplied by the par value \( \Phi \) returnable at the end of the investment period \( \xi \), that is \( A = c\Phi \). The coupon rate \( c \) is generally set with due regard for rates of return and economic conditions prevailing at the bond issue date. Thus in cash-flow terms for annual coupons for a bond investment period of \( \xi \) years with an issue price of \( N_p \), we have:

\[ (\xi c + 1)\Phi - N_p = \text{surplus} / \text{deficit} \]  

(7.17)
The surplus/deficit from investing in a bond is generally absorbed by reformatting the equation in terms of a discount interest rate or yield to redemption \( r \), with the purchase price \( N_p \) effectively balancing out income and par value against prevailing interest rates, as follows:

\[
N_p = \frac{c\Phi}{(1+r)^1} + \frac{c\Phi}{(1+r)^2} + \ldots + \frac{c\Phi}{(1+r)^\xi} + \frac{\Phi}{(1+r)^\xi} \quad (7.18)
\]

Thus a bond price \( N_p \) above par value \( \Phi \), would imply a net income stream above prevailing interest rates, and vice-versa, a bond price below par value would imply a net income stream below prevailing interest rates. By definition for one particular bond, all components are fixed except the price \( N_p \), the period \( \xi \) remaining to maturity and the yield to redemption \( r \). Equations (7.19) set out shortened versions of equation (7.18) using standard or exponential notation:

\[
\frac{N_p}{\Phi} = \left[ \frac{1}{(1+r)^\xi} + \frac{c}{r} \left( 1 - \frac{1}{(1+r)^\xi} \right) \right]
\]

\[
\frac{N_p}{\Phi} = \left[ e^{-r\xi} + \frac{c}{r} \left( 1 - e^{-r\xi} \right) \right] \quad (7.19)
\]

By way of example, the charts at figure 7.2 illustrate the relationship between the three factors: yield to redemption \( r \), bond price \( N_p \) and maturity period \( \xi \), for a coupon rate \( c = 3\% \). In each case one variable is represented as an isopleth, linking points with the same value.
Figure 7.2(a) – (c) Conventional Bonds/Gilts. Relationships between yield to redemption, maturity period and the ratio of bond price to par value.

From equation (7.19) the current yield \( y \) of a bond is expressed as:
\[ y = \frac{c\Phi}{N_p} \] (7.20)

And the solution \( \xi = \infty \) years to maturity (an undated stock) gives:

\[ r = y = \frac{c\Phi}{N_p} \] (7.21)

The solution \( \xi = 0 \) years to maturity gives \( N_p = \Phi \), and the solution \( N_p = \Phi \) gives \( r = c \) for all \( \xi \).

Yield to redemption \( r \) is generally regarded as more meaningful than current yield, and forms a basis to compare bonds in the same class and credit quality. Index-linked bonds generally have lower yields, combined with higher purchase prices, compared to standard bonds. A yield curve can be constructed to link bonds with different maturities and coupon rates. Yield curves are mostly upward sloping as maturity lengthens, reflecting future expectations and a risk premium for holding long-term securities.

It is not always the case, however, that long-term yields exceed short term rates. On occasion, the reverse can occur, with long-term yields being below short-term rates, producing a negative spread. A negative spread is often regarded as portending an increased risk of impending recession, with investors marking down their view of the future with respect to the present.

Figures 7.3 for example, illustrates some more recent changes in terms of British conventional gilt-edged stocks. In 2007 long-term yields were at a discount to short-term yields. There followed a virulent recession, but by 2010 the yield spread had once more resumed a positive position.

Referring to equation (7.19), changes in the shape of the curves occur when different prices \( N_\xi \) of bonds are attached across the maturity periods, and coupon rates, such that short-term yields rise or decline with respect to long-term yields.

The three widely followed theories explaining the curvature of the yield curve are ‘Pure Expectations’, ‘Liquidity Preference’ and ‘Preferred Habitat’. The first theory reflects investors’ expectations of future interest rates. The second theory adds in a premium for long term rates to compensate for added risk of money being tied up for a longer period, and the third theory assumes that investors have distinct investment time
horizons and require a premium to buy outside their preferred maturity. Estrella & Trubin (2006) offer some guidelines on constructing yield curve indicators with regard to predicting future economic activity.

Figure 7.3 Yield to redemption and Coupon rates British Funds

Figure 7.4 sets out historical charts of interest yields of long and short-dated stocks in the UK and USA. Historically, yields of long-term funds in the USA have been mostly in excess of those of short term funds, reflecting a positive yield spread, but in the UK there has been a more balanced variation.
A significant body of empirical research exists, directed to estimate yield curves, based on real data of yields obtained. Chief among these are Nelson & Siegel (1987), Svensson (1994), Waggoner (1997), Anderson & Sleath (1999) and Bacon (2004). Nelson & Siegel’s model of the instantaneous forward rate curve is of the form:

\[
 f(m, \beta) = \beta_0 + \beta_1 e^{-\frac{m}{\tau}} + \beta_2 \left(\frac{m}{\tau}\right) e^{-\frac{m}{\tau}}
\]  

(7.22)

Where \( \beta_0, \beta_1 \) and \( \beta_2 \) are long-run, short-run and medium term interest rate components, \( \tau \) is a decay component and \( m \) is the maturity at which the forward rate is evaluated. Such equations do not, however, altogether
explain the forces impacting on yields, only how yields of bonds having varying coupon rates and maturity dates fit with each other. We now refer again to our thermo-economic analysis, and equation (7.2) linking output potential to current interest rate return $i (= \frac{dI}{I})$, and money entropy change $ds$.

$$\frac{dG}{G} = \frac{dI}{I} + ds_{\text{money}}$$

Turning this equation round we could write:

$$\frac{dI}{I} = \frac{dG}{G} - ds_{\text{money}}$$  \hspace{1cm} (7.23)

In respect of a bond, it was noted earlier that the coupon rate $c$ is fixed at outset, with due regard for rates of return and economic conditions prevailing at the bond issue date. For example, in 2010 conventional British funds included those with coupon rates between 2 1/4% and 12%, issued at various past dates (see figure 7.3), and figure 5.15 shows interest rates and the annualised rate of change in output value flow $\Delta G/G$ in the UK and USA. At times both variables have been high, though not necessarily equal to each other. It is unlikely that investors will be attracted to purchase newly issued bonds that do not provide a return that at least equals, if not exceeds, the current rate of return of the day – unless the bond was issued at a discount.

It is logical therefore to assume that the coupon rate of a bond will likely be set to reflect a level of return in excess of interest rates and at or about expected output value growth. We are not saying that the coupon rate $c$ always reflects the growth rate of output value flow. Clearly the latter varies a good deal over time. But we are saying that the rate set depends upon interest rates and economic conditions prevailing at the issue date, and thereafter fixed for the life of the bond. Making an assumption of this kind we could write in annualised, discrete terms for money:

$$i = \left( \frac{\Delta G}{G} \right) - \Delta s_{\text{money}}$$

and likewise for a bond with maturity of $\xi$ years:
\[ r_\xi = c - \Delta s_\xi \quad (7.24) \]

Where \( r_\xi \) is the interest rate or yield to redemption and \( \Delta s_\xi \) represents the annualised bond entropy change or disequilibrium position with respect to the coupon rate of a particular bond.

Clearly entropy change in this equation is a little different to that in a short-term monetary situation, chiefly because of changes in economic conditions \( \chi \) (in particular inflation) at the outset of a bond and during its lifetime. Figure 7.3, for example, shows how past coupon rates for British funds compare to their yield to redemption at a particular time.

It is, therefore, a matter of the extent to which the coupon rate set at the outset of a bond’s life subsequently equates to interest rates and output value growth in the future, and as an alternative we could write:

\[ r_\xi = (c - \chi) - \Delta s_{money} \quad \text{or} \quad r_\xi = c - \Delta s_\xi, \quad \text{where} \quad \Delta s_\xi = (\Delta s_{money} + \chi) \quad (7.25) \]

Even so, it is a fact that widely varying coupon rates are effectively ‘normalised’ to a yield curve as shown in figure 7.3, the changes being effected by individual entropy changes \( \Delta s_\xi \) reflected in the price \( N_P \) of each bond, with the price differing in a complex manner across the period \( \xi \).

Thus we can substitute equation (7.24) into the bond discount equation (7.19) and write for standard or exponential notation:

\[
\frac{N_P}{\Phi} = \left[ e^{-(c-\chi)\xi} + \frac{c}{(c-\Delta s_\xi)} \left( 1 - e^{-(c-\chi)\xi} \right) \right] \quad \text{or} \quad \frac{N_P}{\Phi} = \left[ \frac{c - \Delta s_\xi e^{-(c-\chi)\xi}}{c - \Delta s_\xi} \right] \quad (7.26)
\]

From equations (7.24) and (7.26), we can see that when entropy change \( \Delta s_\xi \) is zero, bond price \( N_P \) is equal to the par value \( \Phi \), and yield to redemption \( r_\xi \) is equal to the coupon rate \( c \).

When entropy change \( \Delta s_\xi \) is positive, then bond price \( N_P \) is greater than the par value \( \Phi \), and likewise yield to redemption \( r_\xi \) is less than the coupon rate.
\( \Delta s \) is negative then \( N_P < \Phi \), and yield to redemption \( r_\xi > c \).

Figure 7.5 sets out the postulated effects of entropy change on yield.

![Figure 7.5 Postulated effect of entropy change \( \Delta s \) on yield to redemption \( r \), assuming issued at par value](attachment:image)

All of the above leads to the premise that entropy change likely plays a part in determining yield to redemption, at either above or below the coupon rate, and that the relationship depends upon the time period to redemption, and upon the level of the coupon rate compared to current interest rates.

The particular problem that we now have to deal with, which cannot be entirely explained in thermodynamic terms, is that of the time dimension of entropy change; how to visualise the shape and spread of an entropy function over the period to maturity of a bond, given that little or no knowledge of the future, say fifty years ahead, will be available. All that buyers and sellers have available to them is knowledge of cumulative events to date, and long term trends (including the shape of the yield curve at any point in time) combined with a picture of the short term.

This problem was touched upon when considering discounted cash analysis earlier in this chapter, whereby an exponential discount function was used to put a weight on future outcomes. Noting that our bond equation (7.26) contains negative exponentials, it might be expected that an entropy discount function would be similarly related, as was assumed for project
and annuity investments. Sometimes, however, yield curves can have complex shapes, including humps and other non-linearities, so a decreasing exponential might not offer a ‘fit all’ solution. Frederick, Loewenstein and O’Donoghue (2002) for example believe that the Discounted Utility model introduced by Samuelson (1937) has little empirical support and that consideration should be given to other models, including hyperbolic discounting. The empirical research of Nelson & Siegel, Anderson & Sleath and others already referred to utilises complex exponential functions combined with knowledge of short, medium and long-run interest rates to derive the instantaneous forward rate curve, as illustrated earlier at equation (7.22).

For the purposes of this analysis we will continue with the simple exponential model that was assumed earlier, to see where this leads us, though a more complex approach might be considered for future work. Figure 7.6 shows the yield curve from connecting the yields of conventional British Funds at October 2010 that were illustrated at figure 7.3 and along with it a declining entropy function, such that the addition of the two equals a constant level of an average current coupon rate.

![Figure 7.6 Yield curve and entropy function British conventional funds October 2010.](image)

We might posit that the entropy function in this case could have the same form that we assumed earlier at equation (7.10):

$$\Delta s_\xi = \Delta s_0 e^{-\xi \xi}$$

Where $\Delta s_0$ is the current entropy change level and $\xi$ is some form of decay factor, dependent upon investors view on the future. However, readers
should not assume that such a function is axiomatic. A yield curve with a hump for instance will require a more complex function.

Nevertheless, pursuing the matter to a conclusion, the advantage of an entropy function of this kind, or similar, is that it can be positive or negative about the mean of the current coupon rate, resulting in a ‘flip’ of the curve, enabling the shape of the yield curve to go from ‘upward-sloping’ to ‘inverted’ and vice versa. It can meet situations where short-term economic trends change.

Similar to the approach to discounted cash flow analysis at equations (7.8) – (7.11), we could set out two equations for the yield, one for the short term, denoted by $0$, and the other for the long term, denoted by $\xi$, and then combine them as follows:

$$r_0 = c - \Delta s_0$$

$$r_\xi = c - \Delta s_\xi$$

giving:

$$r_\xi = r_0 + (\Delta s_0 - \Delta s_\xi)$$

(7.26)

The long-term yield to redemption of a bond then becomes a function of the short term rate of interest and the difference between the entropy changes associated with each. Thence by substituting in our entropy decay equation we can write:

$$r_\xi = r_0 + \Delta s_0\left(1 - e^{-\epsilon_\xi}\right)$$

(7.27)

which expresses the yield to redemption in terms of the short term interest rate, the short term entropy change and a decay function. And in the alternative, by turning this around we could also write:

$$\left(r_\xi - r_0\right) = \Delta s_0\left(1 - e^{-\epsilon_\xi}\right)$$

(7.28)

Which expresses the yield spread between long and short-term perspectives, in terms of the short term entropy change and the decay function. It is emphasised again that further research on the shape of the decay function is required in respect of more complex yield curves.

It is of interest to compare the yield spreads between long and short dated bonds, set against money entropy change for the UK and USA economies.
(based on M4 and M2 money data respectively). The yield spread data obtained for a long time series was for 20-year BGS for the UK and 10-year bonds for the USA, both set against 3-month treasury rates. If one assumed a decay rate of 5% p.a., then the discount decay factors \((1 – e^{-\epsilon x})\) applicable would be 1 for the UK and 0.5 for the USA. Figure 7.7 shows the results of this set of assumptions.

Figure 7.7 Modified Yield spread (quarterly) versus current money entropy change (4-quarter moving averages) for the UK and USA economies.

Statistical regression of the result does not produce a good correlation, though it can be seen that yield spreads appear to follow to some degree the ebb and flow of money entropy change associated with money. In particular, in the USA values of spread appear on occasion to act as a lead
factor to economic entropy change. Further work is therefore required to examine this effect in more depth, in particular lags and leads of possible impacting economic factors, improved long-run modelling of the value capacity coefficient (see figure 5.18 and following comments) and more complex decay functions, though that is beyond the scope of the research covered in this book.
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**DATA SOURCES**

BP Statistical Reviews
Energy Information Administration (EIA), US Department of Energy
IEA
Intergovernmental Panel on Climate Change:
Third & Fourth Assessment Reports.

OECD
Penn World
United Nations
USA Census Bureau
www.bea.gov
www.statistics.gov.uk
www.federalreserve.gov
## LIST OF SYMBOLS

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<th>Thermo-Economic</th>
<th>Thermodynamic</th>
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<td>t</td>
<td>Time</td>
<td>Time</td>
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<tr>
<td>P</td>
<td>Price</td>
<td>Pressure</td>
</tr>
<tr>
<td>V</td>
<td>Volume flow rate</td>
<td>Volume (3-D)</td>
</tr>
<tr>
<td>N</td>
<td>Number of stock units</td>
<td>Number of molecules</td>
</tr>
<tr>
<td>v (=V/N)</td>
<td>Specific Volume Rate</td>
<td>Specific Volume (1/density)</td>
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<tr>
<td>G (=PV)</td>
<td>Value flow rate</td>
<td>Energy</td>
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<td>k</td>
<td>Productive Content/unit</td>
<td>Boltzmann constant</td>
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<td>Nk</td>
<td>Stock Productive Content</td>
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<td>Entropy</td>
<td>Entropy</td>
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<tr>
<td>s (=S/N)</td>
<td>Entropy per unit</td>
<td>Entropy per unit</td>
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<tr>
<td>F</td>
<td>Free Value (flow)</td>
<td>Free Energy (Helmholtz)</td>
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<td>X</td>
<td>Free Value (flow)</td>
<td>Free Energy (Gibb)</td>
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<tr>
<td>f (=F/N)</td>
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<td>Free Energy per unit</td>
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<tr>
<td>C_v</td>
<td>Specific Value (Const volume)</td>
<td>Specific Heat (Const volume)</td>
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<tr>
<td>C_p</td>
<td>Specific Value (Const price)</td>
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<tr>
<td>n</td>
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<td>Q</td>
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<td>Heat Supplied/lost</td>
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<td>U</td>
<td>Internal Value (stock value)</td>
<td>Internal Energy</td>
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<td>u (=U/N)</td>
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<td>ψ</td>
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<td>ξ</td>
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